PROBLEM SET 1

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Question 1: Prove that $4^n+15n-1$ is divisible by 9 for all $n \geq 1$, using simple inductions.

ANSWER: P(n): $4^n + 15n - 1$ is divisible by 9 $\forall n \geq 1$.
Base case: If $n = 1$ we get $4 + 15 - 1 = 18$ which is divisible by 9. So the base case holds.
Induction hypothesis: Now we assume it holds for $n = k$.
Induction step: Prove for $n = k + 1$. So we have to prove $4^{k+1} + 15(k + 1) - 1$ is divisible by 9
$4^{k+1} + 15(k + 1) - 1 = 4 * 4^k + 15k + 15 - 1 = 3 * 4^k + 4^k + 15k - 1 + 15$. We already know that $4^k + 15k - 1$ divisible by 9, so now we only need to prove that the rest is also divisible by 9. Take $3 * 4^k + 15$. We can write it as $3(4^k + 5)$ which is divisible by 3. Now we only need to prove that $4^k + 5$ is divisible by 3.

P(n): $(4^n + 5)$ is divisible by 3
Base case: When $n = 1$, $4 + 5 = 9$, divisible by 3. Base case holds.
Induction hypothesis: Assume it holds for n=k, so assume $(4^k + 5)$ is divisible by 3.
Now we prove for $n = k + 1$. $4^{k+1} + 5 = 4 * 4^k + 5 = 3 * 4^k + 4^k + 5$. Since $3 * 4^k$ is divisible by 3 and $4^k + 5$ is divisible by 3 from the induction hypothesis, $(4^n + 5)$ is divisible by 3.

Going back to the first induction $4^{k+1} + 15(k + 1) - 1$ is divisible by 9.
$\therefore$ *Q.E.D*

Question 2a: Consider strings made up only of the characters 0 and 1; these are binary strings. A binary pa

ANSWER: f(n): Number of binary palindromes of length $2n$, for $n \geq 0$.

$$f(0) = 1 \qquad f(1) = 2 \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \qquad f(2) = 4 \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad f(3) = 8 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

f(n) $= 2^n$

Question 2b: Prove that your formula is correct for all $n \geq 0$, using simple induction.

ANSWER:

- P(n): The number of binary palindromes of length 2n, for $n \geq 0$ is 2^n
- Base case holds since $f(0) = 1 = 2^0$
- Induction hypothesis: Assume it is true for $n = k$
- Prove for $n = k + 1$

$f(k) = 2^k$ Prove $f(k + 1) = 2^{k+1}$

After each calculation of a palindrome, we know the length is 2n. This means we can take the length of the previous palindrome and multiply it by 2 to find the length of the next palindrome.

$$f(k) * 2 = 2^{k+1}$$
$$2^k * 2 = 2^{k+1}$$
$$2^{k+1} = 2^{k+1}$$

$\therefore QED.$