CSC236 Week 9 Tutorial:

Correctness of Iterative Functions

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Preliminary

Consider the following function.

```
def mystery(a, b):
    ''' Pre:  a > 0 and b > 0'''
    x = a
    y = 0
    while x >= b:
       x -= b
       y += 1
    return x
```

Prove the following loop invariant for the while-loop: $x \geq 0$ and $a = x + yb$. Recall that to prove an invariant requires two things: that the invariant holds when the loop is reached, and that one arbitrary loop iteration maintains the invariant.

Exercise 1

```
def term_ex(x,y):
   ''' Pre:  x and y are integers >= 0 '''
   a = x
   b = y
   while a > 0 or b > 0:
      if a > 0:
         a -= 1
      else:
         b -= 1
```

Prove that $a \geq 0 \wedge b \geq 0$ is an invariant of this loop.

Then find and prove a variant for this loop.

Exercise 2

Below is a slightly modified version of the code in Exercise 1

```
def term_ex_2(x,y):
    ''' Pre:  x and y are integers >= 0 '''
    a = x
    b = y
    while a >= 0 or b >= 0:
        if a > 0:
            a -= 1
        else:
            b -= 1
```

Does this function always terminate?

If not, show why the proof of termination given for Exercise 1 fails.