CSC236 Week 4 Tutorial:

Asymptotic Notations

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Review of Definitions

Big Oh Definition Function $f(n) = O(g(n))$ (“f is big oh of g“) iff

(i) There is some positive $n_0 \in \mathbb{N}$

(ii) There is some positive $c \in \mathbb{R}$

such that $\forall n \geq n_0, f(n) \leq cg(n)$
This means that $g(n)$ is an **upper bound** on $f(n)$. (e.g., $100n + 10000 = O(n^2)$.)

Big Omega Definition Function $f(n) = \Omega(g(n))$ iff

(i) There is some positive $n_0 \in \mathbb{N}$

(ii) There is some positive $c \in \mathbb{R}$

such that $\forall n \geq n_0, cg(n) \leq f(n)$
This means that $g(n)$ is a **lower bound** on $f(n)$. (e.g., $n^3 + 4n^2 = \Omega(n^2)$)

Big Theta Definition Function $f(n) = \Theta(g(n))$ iff
$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.
This means that $g(n)$ is a **tight bound** on $f(n)$. (e.g., $n^2 + 5n + 7 = \Theta(n^2)$)

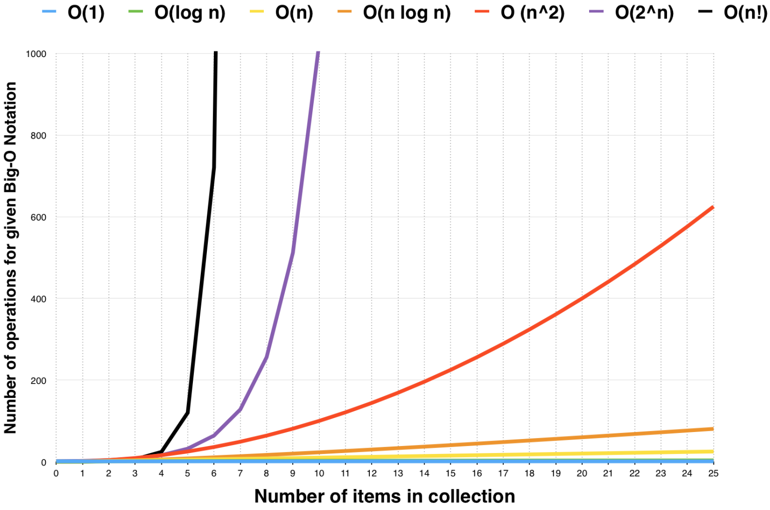
Review of Inequalities

If $x < y$, then $-x > -y$.

If $x < y$, then $\frac{1}{x} > \frac{1}{y}$.

So we can remove negative terms from big oh (upper bound) proofs and remove positive terms from big omega (lower bound) proofs.

Comparing Big Oh



Exercise 1

Prove that $\sqrt{7x^2 + 4x} = \Theta(x)$.

Exercise 2

Prove that $3n^3 - 5n^2 + 4n$ is $O(n^3)$.

Note: The constants (n_0 and c) don't have to be the best or smallest ones.

Exercise 3

Show that $(n \log n - 2n + 13) = \Omega(n \log n)$.

Note: In this course, all logs with no base are assumed base 2.