CSC236 – Problem Set 2

There are two components of this problem set. The preliminary question is not marked or submitted: it is there as a suggested exercise that you should do early to make sure that you're on track. The problem set itself is what you will submit for marks.

Get in the habit of starting work early – the less time you give yourself, the more stressed you'll find yourself each week!

To avoid suspicions of plagiarism: at the beginning of your submission, **clearly state any resources (people, print, electronic) outside of your group, the course notes, and the course staff, that you consulted**.

The PDF file you submit must be typed, scanned handwritten submissions will not be marked.

Preliminary: Not Marked

This question is an opportunity for you to check your understanding of the topics and practice writing formal solutions. This is a valuable *learning opportunity* – if you see that you're at a loss, get help quickly!

Here is a recursive definition for some set T of non-empty binary trees.

- A single node is in T
- If t_1 and t_2 are in T, then the bigger tree with root r connected to the roots of t_1 and t_2 is in T
- Nothing else is in T

Use structural induction to prove the following property for each of the elements in T: there are m nodes that have two children and $m + 1$ nodes that have no children.

Problem Set: due September 30, 2016 22:00, required filename: ps2sol.pdf

Answer each question completely, always justifying your claims and reasoning. Your solution will be graded not only on correctness, but also on clarity.

Answers that are technically correct that are hard to understand will not receive full marks. Mark values for each question are contained in the [square brackets].

You may work in groups of up to THREE to complete these questions.

1. **[6]** Let set F be recursively defined as follows:
   - $7 \in F$
   - If $u, v \in F$, then $u + v \in F$
   - Nothing else is in F

   Use structural induction to prove that $\forall w \in F$, $w \bmod 7 = 0$, i.e., all elements in F are divisible by 7.

2. **[6]**
$$f(n) = \begin{cases} 10, & \text{if } n = 1 \\ 24, & \text{if } n = 2 \\ (f(n-1))^2 + f(n-2) + 5, & \text{if } n \geq 3 \end{cases}$$

   Prove by complete induction that $\forall n \geq 1, f(n) \bmod 7 = 3$.

   Hint: It is helpful to write "$x \bmod 7 = 3$" as "$\exists k \in \mathbb{N}, x = 7k + 3$".