CSC236 Week 7 Tutorial:

Divide-and-Conquer and Master Theorem

Slides Credit: Charlene Chu

Preliminary

For each of the following recurrences, do one of two things. If the master theorem applies, indicate which case applies and state the asymptotic runtime obtained using the master theorem. If the master theorem does not apply, briefly indicate why it does not apply and go no further.

A. $T(n) = 5T(2n/5) + n$

B. $T(n) = 4T(n/2) + 15n^3 + 4n^2 + n + 4$

C. $T(n) = T(n/2)$

Exercise 1

Consider the simple problem of computing the result of 42 raised to the power of n, i.e., 42^n. Suppose that your programming language does not have an exponent operator, so you have to write an algorithm to compute it. The runtime of your algorithm is evaluated by how many multiplications need to be performed. First consider the naive algorithm and its runtime, and then develop a divide-and-conquer algorithm that is an order of magnitude faster. Give a recurrence for the worst-case runtime of your algorithm in terms of n, and find the tightest asymptotic runtime of your algorithm.

Exercise 2

For each of the following recurrences, explain why the Master Theorem cannot be used.

(a) $T(n) = nT(n/2) + 2$

(b) $T(n) = T(n-1) + 3n^2$

(c) $T(n) = T(n/2) + \log n$

(d) $T(n) = 2T(n/4) + 3T(n/3) + 1$

(e) Finding a closed form expression for $T(n) = 2T(n/2) + 4n$