Problem Set 2

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Question 1: Prove if $7 \in F$, $w, v \in F$ then $u + v \in F$, and nothing else is in F. Use structural induction to prove that $\forall w \in F$, $w\ mod\ 7 = 0$.

ANSWER: P(F): All elements of F are multiples of 7.
Base case: If there is only 1 element in the set F. This element must be 7 and $7\ mod\ 7 = 0$
$\therefore$ the base case holds.
Induction: Let $u, w \in F$. We can assume $u\ mod\ 7 = 0$ and $w\ mod\ 7 = 0$ (IH). Since we know we have 2 multiples of 7, we can infer that:
$a7 = u$ and $b7 = w$ where $a, b \in \mathbb{R}$.
$u + w = a7 + b7$
$u + w = (a + b)7$
$\therefore$ We create another multiple of 7.
Q.E.D

Question 2a: Prove by complete induction that $\forall n \geq 1, f(n)\ mod\ 7 = 3$.

$$f(n) = \begin{cases} 10, & if \mathbf{n = 1} \\ 24, & if \mathbf{n = 2} \\ (f(n-1))^2 + f(n-2) + 5, & if \mathbf{n} \geq 3 \end{cases}$$

ANSWER: P(n): $f(n)\ mod\ 7 = 3\ \forall\ n \in \mathbb{N}$.
Base case:

$n = 1$	$n = 2$	$n = 3$
$10\ mod\ 7 = 3$	$24\ mod\ 7 = 3$	$(24^2 + 10 + 5)\ mod\ 7$
		$591\ mod\ 7 = 3$

$\therefore$ Base case holds. Induction: IH. $\forall i\ \in \mathbb{N}, i\ mod\ 7 = 3$
Find f(n + 1):
$= ((f(n + 1 - 1))^2 + f(n - 1) + 5)\ mod\ 7$
$= ((f(n))^2\ mod\ 7) + (f(n - 1)\ mod\ 7) + (5\ mod\ 7)$
$= 2 + 3 + 5$
$= 10\ mod\ 3$
$= 3$
$\therefore$ the statement holds for $f(n + 1)$ QED.