

CSC236 Week 10 Tutorial:

Review for Term Test 2

Exercise 1

This question concerns the following code.

```
def mystery(lst):  
    '''Pre:  lst is a list of 0s and 1s.  
    Post:  ...  
    '''  
    i = 0  
    j = 0  
    while i < len(lst):  
        if lst[i] == '0':  
            j = j + 1  
        else:  
            j = j - 1  
        i = i + 1  
    return j == 0
```

- a. Is j a variant for this loop? Explain why or why not.
- b. State and prove a loop invariant for this loop. Your invariant should be useful for proving the postcondition. Prove only the loop invariant, not loop termination or the postcondition.
- c. State and prove the postcondition for `mystery`. Your postcondition should characterize what the algorithm does.

Exercise 2

```
def count_ordered ( A ):  
    '''  
    Pre:  A is a list of numbers  
    Post: Outputs the number of pairs (i,j)  
    such that  $i < j$  and  $A[i] \leq A[j]$   
    E.g., count_ordered([3,2,5,1])=2  
    and count_ordered([10,10])=1  
    '''  
    if len(A)==0:  
        return 0  
    else:  
        count=0  
        i=1  
        while i<len(A):  
            if A[i]>=A[0]:  
                count+=1  
            i+=1  
        return count+count_ordered(A[1..len(A)-1])
```

Read the following program's specifications and code carefully. Notice both the loop and the recursive call.

- State and prove a helpful loop invariant for the while loop ("steps 1 and 2" from lecture). You may assume $i \in \mathbb{N}$ and $i \leq \text{len}(A)$ have already been proven as loop invariant (so use them freely).
- Using your loop invariant from part (a), prove the correctness of `count_ordered` according to the specifications. Once again, note that `count_ordered` is recursive, so you need to analyse a recursive call.