CSC236 Week 8 Tutorial:

Correctness of Recursive Functions

Slides Credit: Charlene Chu

Preliminary

Here is code for a recursive function that finds the minimum element of a list.

```
def rec_min(A):
    if len(A) == 1:
       return A[0]
    else:
       m = len(A) // 2
       min1 = rec_min(A[0..m-1])
       min2 = rec_min(A[m..len(A)-1])
       return min(min1, min2)
```

State preconditions and postconditions for this function. Then, prove that this algorithm is correct according to your specifications.

Exercise 1

For all strings u, v, we say that v is the *reversal* of u, denoted $v = u^R$, if

$$|u| = |v| \wedge \forall 0 \leq i \leq |u| - 1,\ u_i = v_{|u|-1-i}$$

where $|u|$ denotes the *length* of u, and u_i is the i-th character of u. We also assume that strings are indexed from 0 to the length of the string minus 1.

For example, `"abcde" = ("edcba")`^{R}. Consider the algorithm below that reverses a string u:

Exercise 1 (cont'd)

```
def rev(u):
    l = len(u)
    if l < 2:
       return u
    else:
       m = l // 2
       v = rev(u[0..m-1])
       w = rev(u[m..len(u) - 1])
       return wv
```

where $u[i..j]$ is the substring of u from position i to position j (both inclusive). The goal is to prove that algorithm `rev` correctly reverses a string.

Write pre- and postconditions for the given function `rev`, and state a precise statement for correctness of `rev`.

Then, show that `rev` is correct according to your statement.

Exercise 2

Consider a recursive selection sort algorithm:

```
def recSS(A, i):
    if i < len(A) - 1:
       # Find the minimum element in A[i:]
       small = i
       for j in range(i + 1, len(A)):
          if A[j] < A[small]:
             small = j
       # Swap A[i] and A[small]
       temp = A[i]
       A[i] = A[small]
       A[small] = temp

       # Sort the remainder of the list
       recSS(A, i + 1)
```

Write pre- and postconditions for `recSS`. Can you argue informally why this program is correct, assuming the loop does indeed find the minimum element in $A[i..\texttt{len}(A)]$?