Problem Set 3

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Question 1: $n\ log(0.5n^2 + 10) - 2n + 7log(n) \in \Theta(n\ log\ n)$

ANSWER:

$$\begin{aligned}
&n\ log(0.5n^2 + 10) - 2n + 7log(n)\\
<\ & n\ log(0.5n^2 + 10) + 7log(n)\\
<\ & n\ log(0.5n^2 + 10n^2) + 7log(n)\\
=\ & 2n\ log(10.5n) + 7log(n)\\
=\ & 2n\ log(n) + 2n\ log(10.5) + 7log(n)\\
<\ & 2n\ log(n) \qquad n_0 = 11\\
\therefore\ & n_0 = 11, C = 2\ and \in \mathcal{O}(n\ log(n))
\end{aligned}$$

$$\begin{aligned}
&n\ log(0.5n^2 + 10) - 2n + 7log(n)\\
>\ & n\ log(0.5n^2 + 10) - 2n\\
>\ & n\ log(0.5n^2) - 2n\\
=\ & 2n\ log(0.5n) - 2n\\
>\ & 2n\ log(n) - 2n\\
>\ & 2n\ log(n) \qquad n_0 = 3\\
>\ & 2n\ log(n)\\
\therefore\ & n_0 = 3, C = 2\ and \in \Omega(n\ log(n))
\end{aligned}$$

Since $\mathcal{O}(n\ log(n)) = \Omega(n\ log(n))$ we can conclude $n\ log(0.5n^2 + 10) - 2n + 7log(n) \in \Theta(n\ log\ n)$. Q.E.D.

Question 2: Given $f(n) \in \mathcal{O}(n^2)$ and $g(n) \in \mathcal{O}(n^2)$, use the definition of $\mathcal{O}$ to prove that: $42 \cdot f(n) + 236 \cdot g(n) \in \mathcal{O}(n^2)$

ANSWER: Using the definition of $\mathcal{O}$:

$$\begin{aligned}
&f(n) \in \mathcal{O}(n^2)\\
\implies\ & f(n) \leq cn^2 \qquad c \in \mathbb{R}\\
\implies\ & 42f(n) \leq cn^2
\end{aligned}$$

$$\begin{aligned}
&g(n) \in \mathcal{O}(n^2)\\
\implies\ & g(n) \leq cn^2 \qquad c \in \mathbb{R}\\
\implies\ & 236g(n) \leq cn^2
\end{aligned}$$

This can then be used in the problem question:

$42 \cdot f(n) + 236 \cdot g(n) \leq 42cn^2 + 236cn^2$

$42 \cdot f(n) + 236 \cdot g(n) \leq n^2(42c + 236c)$

The definition of $\mathcal{O}$ states that $\mathcal{O}$ is a constant value multiplied by a function of n which is greater then a given function. This line states just that

$\therefore 42 \cdot f(n) + 236 \cdot g(n) \in \mathcal{O}(n^2)$

Q.E.D.