CSC236 Week 5 Tutorial:

Substitution Method

Slides Credit: Charlene Chu

Review of Repeated Substitution

Step 1: substitute a few times to find a pattern

Step 2: guess the recurrence after k substitutions

For each base case:

Step 3: solve for k

Step 4: substitute k back into the formula to find a **potential** closed-form
("potential", because it might be wrong)

Step 5: prove that the recursive and potential closed form are equivalent using induction

Preliminary

Here is a recursively-defined function.

$$T(n) = \begin{cases} 5, & \text{if } n = 1 \\ T(n-1) + 3, & \text{if } n > 1 \end{cases}$$

Carry out the five steps for repeated substitution to prove a closed-form for this recursive definition. Split up your five steps in your submission so that we know where each step begins and ends.

Exercise 1

Here is a recursively-defined function where $c, d \in \mathbb{N}$.

$$T(n) = \begin{cases} c, & \text{if } n = 0 \\ d, & \text{if } n = 1 \\ 2T(n-1) - T(n-2) + 1, & \text{if } n > 1 \end{cases}$$

Carry out the five steps for repeated substitution to prove a closed-form for this recursive definition. Split up your five steps in your submission so that we know where each step begins and ends.

Exercise 2

Consider this function:

$$f(n) = \begin{cases} 3, & \text{if } n = 1 \\ 4f\left(\frac{n}{2}\right) + n, & \text{if } n > 1 \end{cases}$$

Assume n is a power of 2. Find and prove a closed-form for this function, following the five steps of the substitution method described in class.