# Solving Cubic Equations



**If you thought the Quadratic Formula was complicated, the method for solving Cubic Equations is even more complex. We will use the example from the Cubic Equation Calculator:**

$$2x^3 - 4x^2 - 22x + 24 = 0$$

**Cubic equations have to be solved in several steps. First we define a variable 'f':**

$$f = \frac{(3c/a) - (b^2/a^2)}{3}$$

**"Plugging in" the numbers from the above equation, we get:**

**f = ((3 * -22/2) - (16/4)) / 3 = - 12.333333...**

**Next we define 'g':**

$$g = \frac{(2b^3/a^3) - (9bc/a^2) + (27d/a)}{27}$$

**From this point on, you are expected to "plug in" the numbers:**

**g = 4.07407407407407....**

**Then we define 'h':**

$$h = (g^2/4) + (f^3/27)$$

**h = -65.333333...**

If h > 0, there is only 1 real root and is solved by another method.
(SCROLL down for this method)

For the special case where f=0, g=0 and h = 0, all 3 roots are real and equal.
(SCROLL to the bottom for this method)

When h <= 0, as is the case here, all 3 roots are real and we proceed as follows:

# ALL 3 Roots Are Real

$$i = ((g^2/4) - h)^{1/2}$$

**i = 8.33563754151978...**

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$$j = (i)^{1/3}$$

**j = 2.0275875100994063...**

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**NOTE: The following trigonometric calculations are in radians**

$$k = \text{arc cosine } (- (g / 2i))$$

**k = 1.817673356517739...**

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$$L = j * -1$$

**L = -2.0275875100994...**

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$$M = \text{cosine } (K/3)$$

**M = 0.8219949365268...**

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$$N = (\text{Square Root of } 3) * \text{sine } (K/3)$$
$$N = 0.9863939238321...$$

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$$P = (b/3a) * -1$$
$$P = 0.6666666666666...$$

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$$x_1 = 2j * \text{cosine}(k/3) \quad -(b/3a)$$
$$x_1 = 4$$

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$$x_2 = L * (M + N) + P$$
$$x_2 = -3$$

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$$x_3 = L * (M - N) + P$$
$$x_3 = 1$$

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## When Only 1 Root Is Real

$$3x^3 - 10x^2 + 14x + 27 = 0$$

$$f = \frac{(3c/a) - (b^2/a^2)}{3}$$
$$f = .962962962962962...$$

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$$g = \frac{(2b^3/a^3) - (9bc/a^2) + (27d/a)}{27}$$
$$g = 11.441700960219478...$$

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$$h = (g^2/4) + (f^3/27)$$
$$h = 32.761202560585275...$$

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$$R = -(g/2) + (h)^{1/2}$$
$$R = .002889779596782...$$

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$$S = (R)^{1/3}$$
$$S = .142436591824886...$$

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$$T = -(g/2) - (h)^{1/2}$$
$$T = -11.4445907398163...$$

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$$U = (T)^{1/3}$$
$$U = -2.25354770293599...$$

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$$X_1 = (S + U) - (b/3a)$$
$$X_1 = -1$$

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$$X_2 = -(S + U)/2 - (b/3a) + i*(S-U)*(3)^{1/2}/2$$
$$X_2 = 2.16666666666... + i*2.07498326633146$$

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$$X_3 = -(S + U)/2 - (b/3a) - i*(S-U)*(3)^{1/2}/2$$
$$X_3 = 2.16666666666... - i*2.07498326633146$$

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# When All 3 Roots Are Real and Equal

$$x^3 + 6x^2 + 12x + 8 = 0$$

$$f = \frac{(3c/a) - (b^2/a^2)}{3}$$
$$f = ((3*12/1)-(36/1)) / 3$$
$$f = 0$$

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$$g = \frac{(2b^3/a^3) - (9bc/a^2) + (27d/a)}{27}$$
$$g = ((2*216/1) - (9*6*12/1) + (27*8/1)) / 27$$
$$g = (432 - 648 + 216) / 27$$
$$g = 0$$

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$$h = (g^2/4) + (f^3/27)$$
$$h=0$$

$$x_1 = x_2 = x_3 = (d/a)^{1/3} * -1$$

$$x_1 = x_2 = x_3 = (8/1)^{1/3} * -1$$

$$x_1 = x_2 = x_3 = -2$$

RETURN TO CUBIC EQUATION CALCULATOR

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