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### Harmonious numbers

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**Amicable numbers** are pairs of numbers such that the sum of the proper divisors of each one is equal to the other. The smallest such pair of numbers is 220 and 284.

Your great aunt Maude has long believed that it is a fundamental error to consider 1 to be a proper divisor of any number<sup>1</sup>, so she calls a pair of numbers *harmonious* if the sum of the proper divisors in her sense of either one is equal to the other. She has tasked you with providing a catalogue of all pairs of harmonious numbers where the smaller one is less than 2,000,000.

Note that the difference between *harmonious* and *amicable* pairs is just in how we compute the “sum of the proper divisors”. For harmonious pairs we exclude 1 from the sum of proper divisors.

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### Task

Write a program that computes all pairs of *harmonious* numbers and outputs them, one pair to a line, separated by a space, to `stdout`. The smaller number of each pair should be listed first, and there should be no duplicates.

The pairs must be written in increasing order of the smaller number. That is, looking down the output, the first column increases.

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### Standards

For an achieved standard the program must operate as specified to compute the list of harmonious pairs whose first number is at most 100,000.

Merit criteria include a program that computes the list of harmonious pairs whose first number is at most 2,000,000.

There are no clear excellence criteria for this étude.

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### Objectives

1.2, 2.6-2.9, 3.5, 3.7

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<sup>1</sup>She claims, quite reasonably, that if  $a$  is a proper divisor of  $n$  with  $ab = n$  then  $b$  should also be a proper divisor. So, since  $n$  is not considered a proper divisor of  $n$ , neither should 1 be.

(Individual)