FUZZY LOGIC

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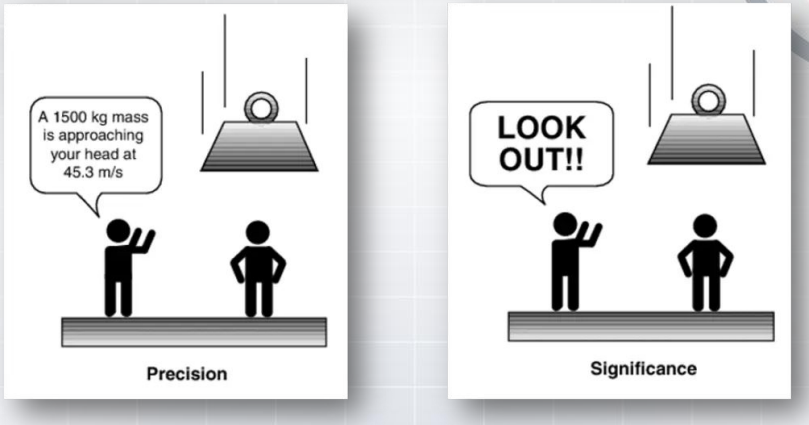
“The human brain works as a binary computer and can only analyze the exact information-based zeros and ones (or black and white). Our heart is more like a chemical computer that uses fuzzy logic to analyze information that can't be easily defined in zeros and ones.”
– Naveen Jain (founder and CEO of Bluedot)

FUZZY LOGIC

“It is not true that women think in illogical way -- the logical thinking apparatus is as appropriate for a female brain as it is for a male brain. However, there is a fundamental gender-based distinction in the functional system of this thinking apparatus: unlike male, female logic is based on fuzzy logic -- wherein each statement has got several values in such a way that if women say "No", this response doesn't mean absolute 'no-thing-ness', but implies some insensible and imperceptible features of the quite opposite response -- "Yes". The same is also true for a shift in the opposite direction of evaluation in female logic -- from "Yes" to "No". That is why it sometimes turns out to be a very difficult task to translate women's fuzzy logic to men's two-valued logic, that includes only "Yes" or "No", without a third value.”

— Elmar Hussein (University Lecturer at Baku State University)

FUZZY LOGIC



https://codecrucks.com/what-and-why-fuzzy-set/

As complexity rises, precise statements lose meaning and meaningful statements lose precision.

— Lotfi Zadeh

Fuzzy Logic

- Compared to Boolean logic, which includes the concept of truth, fuzzy logic is an improvement.
- A logical system that follows the way humans reason tends to use the word "approximate" rather than "exact."
- **A method of dealing with uncertainty that combines the real values [0, 1] and logic operations.**

Fuzzy Logic

- Fuzzy logic was first proposed by Lotfi Zadeh in a 1965 paper for the journal Information and Control. In his paper, titled "Fuzzy Sets," Zadeh attempted to reflect the kind of data used in information processing and derived the elemental logical rules for this kind of set.
- A fuzzy set of the universe U that has been grouped by the membership function μA(x), which is between [0, 1]. (Wang, 1997).

Fuzzy Logic

- The membership function of the classical set has only two values, 0 and 1, whereas the membership function of the fuzzy set is a continuous function with a range of [0, 1].

Advantages of Fuzzy Logic

1. Easy and understandable
2. Basic mathematical modeling
3. Improper data tolerance
4. Can model complex nonlinear functions
5. Apply expert knowledge without the need for training
6. Based on natural language
7. Fuzzy logic is very adaptable

Disadvantages of Fuzzy Logic

1. Broad validation and verification are required
2. Human expertise and knowledge are required

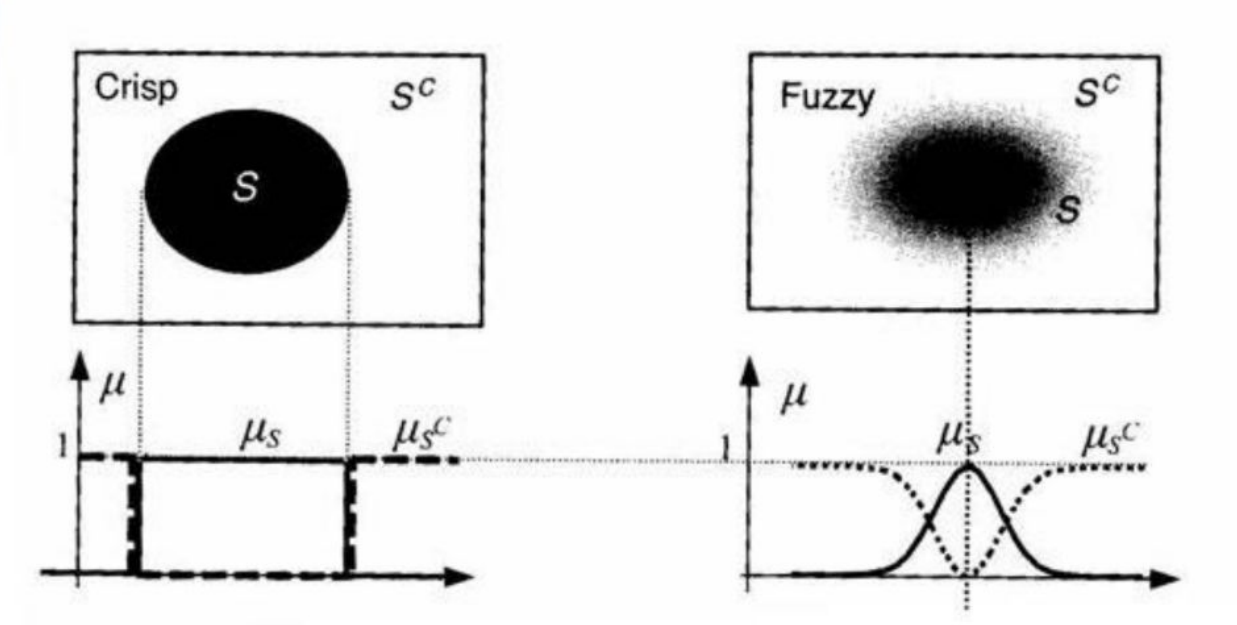
Applications of Fuzzy Logic

1. In Germany in the 1990s, a washing machine with fuzzy logic was introduced (Matsushita Electric Industry Company)
2. Automobiles with automatic transmissions. Nissan used it and saved 12-17% on gasoline.
3. The Sendai subway is automated.
4. Medical science, decision-making, economics, pattern recognition, psychology, engineering, and other fields.

Fuzzy Logic vs Probability

Fuzzy Logic	Probability
In fuzzy logic, we basically try to capture the essential concept of vagueness.	Probability is associated with events and not facts, and those events will either occur or not occur
Fuzzy Logic captures the meaning of partial truth	Probability theory captures partial knowledge
Fuzzy logic takes truth degrees as a mathematical basis	Probability is a mathematical model of ignorance

Crisp set vs Fuzzy set



Crisp set vs Fuzzy set

BASIS FOR COMPARISON	FUZZY SET	CRISP SET
Basic	Prescribed by vague or ambiguous properties.	Defined by precise and certain characteristics.
Property	Elements are allowed to be partially included in the set.	Element is either the member of a set or not.
Applications	Used in fuzzy controllers	Digital design
Logic	Infinite-valued	bi-valued

Crisp set vs Fuzzy set

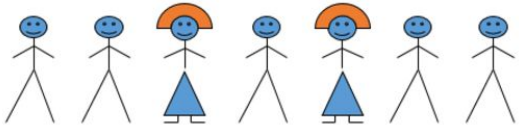
Are Alex and John good friends?

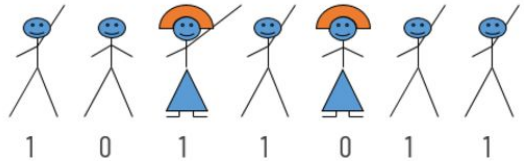
- Yes (1)
- No (0)

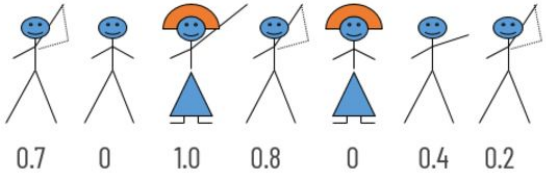
Are Alex and John good friends?

- Extremal good (1.0)
- Very good (0.6)
- Not so good (0.2)
- Not at all (0.0)

Crisp set vs Fuzzy set







Universal set (top), crisp set (middle), fuzzy set (bottom)
https://codecrucks.com/what-and-why-fuzzy-set/

We can consider the class of students as a universal set (U).

If we ask question "**who does have a driving licence?**",
student may or may not have driving licence.
Based on that, **the membership value assigned to student will be either 0 or 1 (crisp)**.

But if we ask the question "**who can drive well?**"
answer to this question is quite subjective.
Based on the skill of student, **the membership value of student in particular set will vary from 0 to 1**, where 0 indicates no driving skill and 1 indicate the highest level of driving skill. This is how fuzzy representation helps us to capture the uncertainty in the data.

Crisp set vs Fuzzy set

The **crisp sets have crisp boundary** (element is either inside set or outside set), and **fuzzy set is having fuzzy boundary** (element can be partially member of set)

Crisp set vs Fuzzy set (example)

"If the temperature is greater than or equal to 80 °Fahrenheit, it is considered hot; otherwise, it is considered not hot."

The temperature = 100 °Fahrenheit, so it is hot

The temperature = 80.1 °Fahrenheit, so it is hot

The temperature = 79.9 °Fahrenheit, so it is not hot

The temperature = 50 °Fahrenheit, so it is not hot

It is considered hot if the temperature ≥ 80 °Fahrenheit.

It is not hot if the temperature < 80 °F.

The crips set's membership function fails to distinguish between members of the same set.

There are some problems that are too complex to adequately define.

Fuzzy Set

Fuzzy Variables

Variables in a fuzzy system. As an example, consider the following: weight, height, temperature, and so on.

Fuzzy Set

A fuzzy set that represents a condition on a fuzzy variable.

Example:

- The **temperature variable** is classified into three fuzzy sets: **hot, warm, and cold**.

Fuzzy Set

There are two attributes to the fuzzy set.

- **Linguistics**: The identification of a representative group of circumstances or conditions based on language experience. Example: young, middle-aged, and old
- **Numerical**: a value (number) that represents the size of a variable, such as 25, 40, 50, and so on.

Fuzzy Set

Set the Universe

The overall value that a variable fuzzy can operate on

Example:

- Universe for weight variables: [1, 150]
- Universe for temperature variables: [0.100].

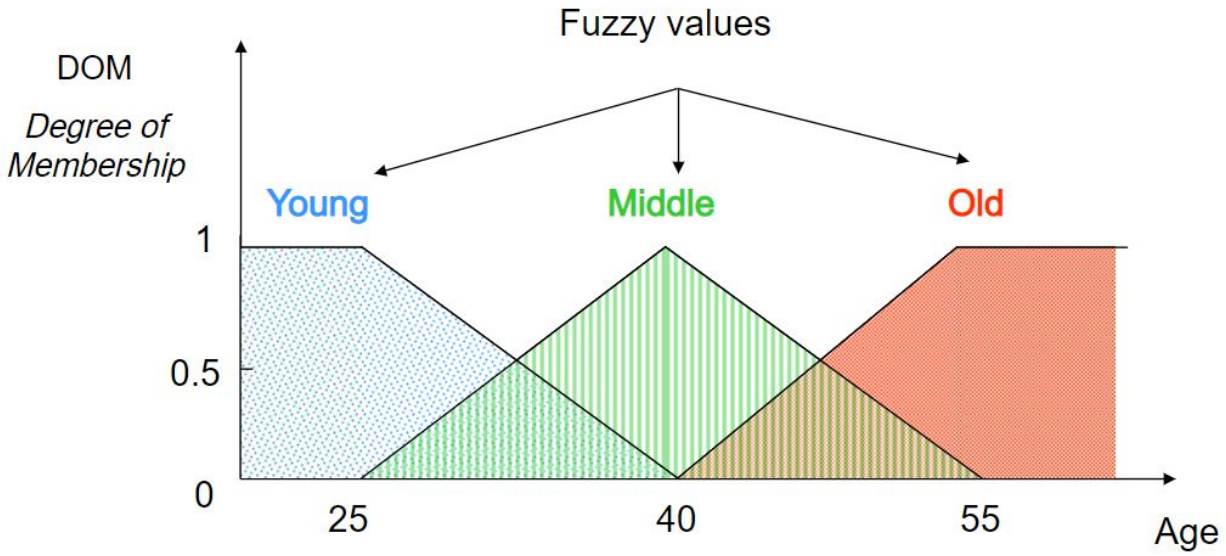
Domains

The domain of a fuzzy set is the entire universe's allowable values that can be operated in a fuzzy set.

Example:

- COLD = [0.60] .60], WARM = [50,80], HEAT = [80, +∞]

Membership Function



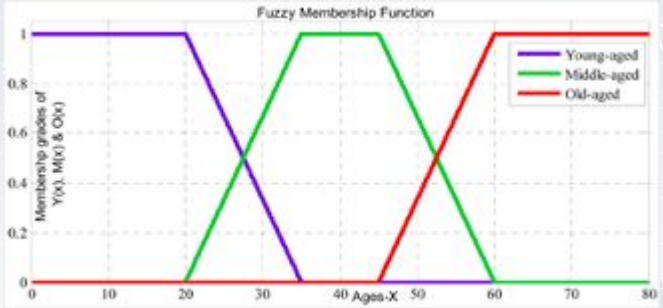
Fuzzy values have associated degrees of membership in the set.

Membership Function

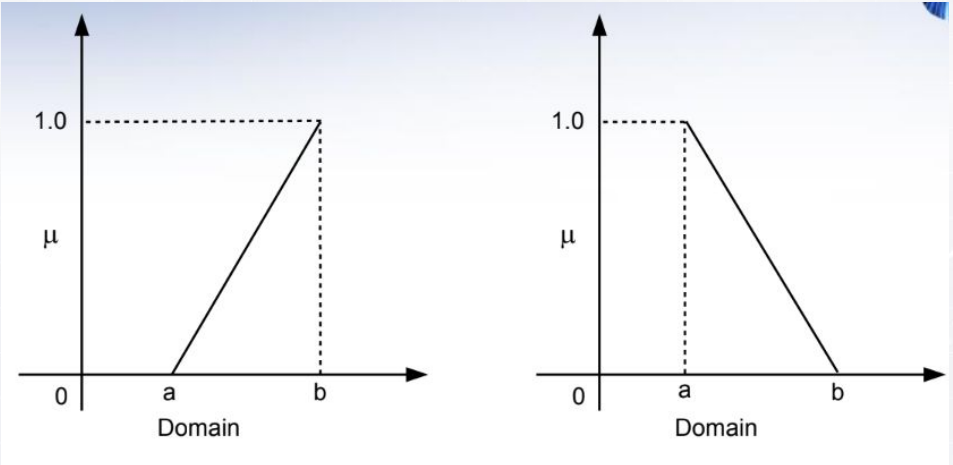
A function (curve) that depicts the mapping of data input points into membership values (degree of membership) with a range of 0 to 1.

There are several functions available:

1. Linear
2. Triangular
3. Trapezoidal
4. Sigmoid
5. Phi



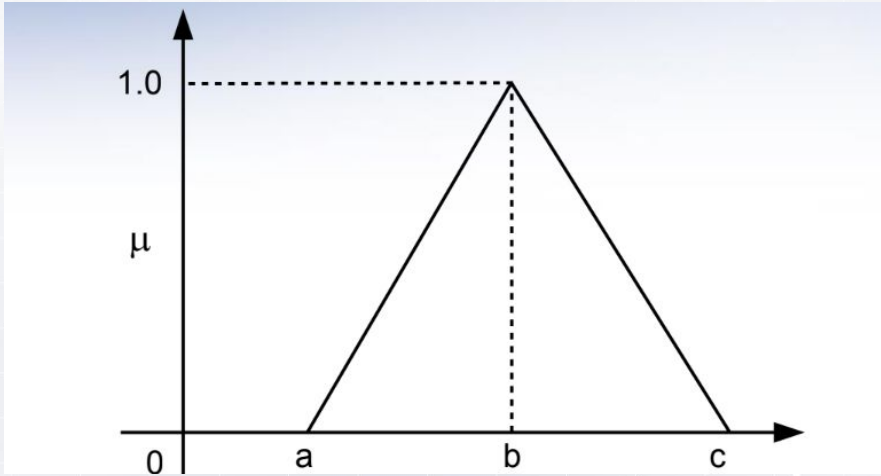
Membership Function – **Linear**



$\mu[x] = 0;\ x \le a$
$(x-a)/(b-a);\ a < x \le b$
$1;\ x > b$

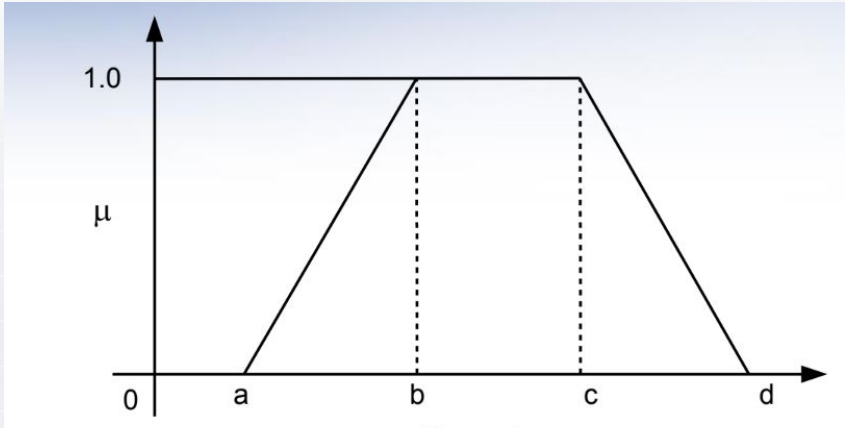
$\mu[x] = (b-x)/(b-a);\ a \le x < b$
$0;\ x \ge b$

Membership Function – **Triangular**



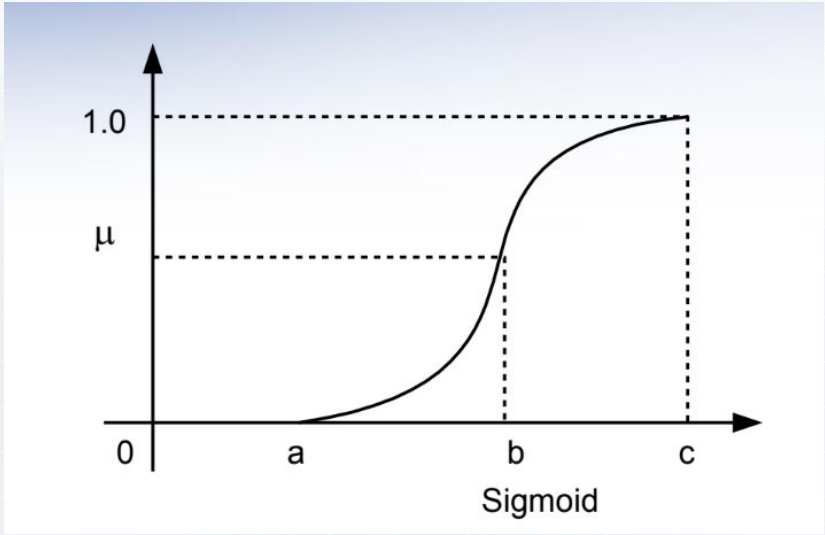
$$\mu[x] = \begin{cases} 0; & x \le a \text{ or } x \ge c \\ (x-a)/(b-a); & a < x \le b \\ (c-x)/(c-b); & b < x < c \end{cases}$$

Membership Function – **Trapezoidal**



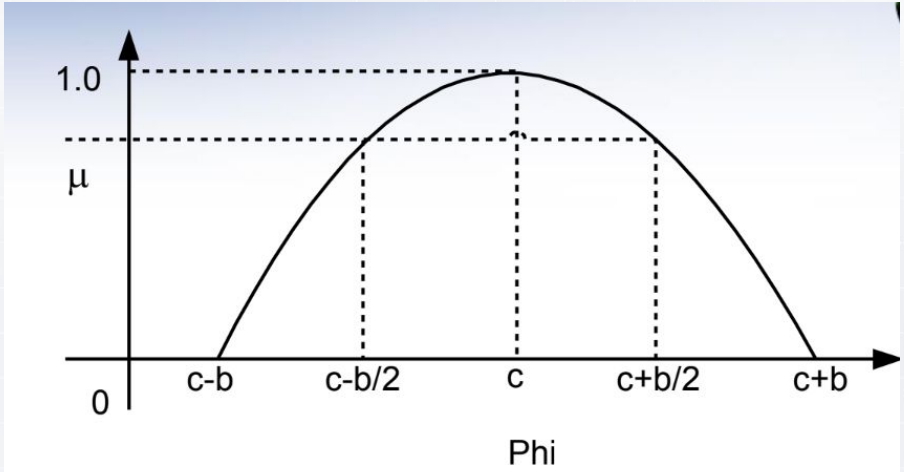
$$\mu[x] = \begin{cases} 0; & x \le a \text{ or } x \ge d \\ (x-a)/(b-a); & a < x \le b \\ 1; & b < x \le c \\ (d-x)/(d-c); & c < x < d \end{cases}$$

Membership Function – **Sigmoid**



$$\mu[x;a,b,c]_{sigmoid} = \begin{cases} 0; & x \le a \\ 2\,((x-a)/(c-a))^2; & a < x \le b \\ 1 - 2((c-x)/(c-a))^2; & b < x < c \\ 1; & x \ge c \end{cases}$$

Membership Function – **Phi**



$$\mu[x;a,b,c]_{phi} = \mu[x;c-b,c-b/2,c]_{sigmoid};\ x \leq c$$

$$\mu[x;c,c+b/2,c+b]_{sigmoid};\ x > c$$

Fuzzy Set Operations

- Fuzzy operations are performed on fuzzy sets, whereas crisp operations are performed on crisp sets.
- Fuzzy operations are very useful in the design of a Fuzzy Logic Controller.
- The new membership value that results from operation two of the set is known as the *firing strength* or α *predicate*. There are three basic operations on a fuzzy set:
  - OR (Union)
  - AND (Intersection)
  - NOT (Complements)

Fuzzy Set Operations- **Union**

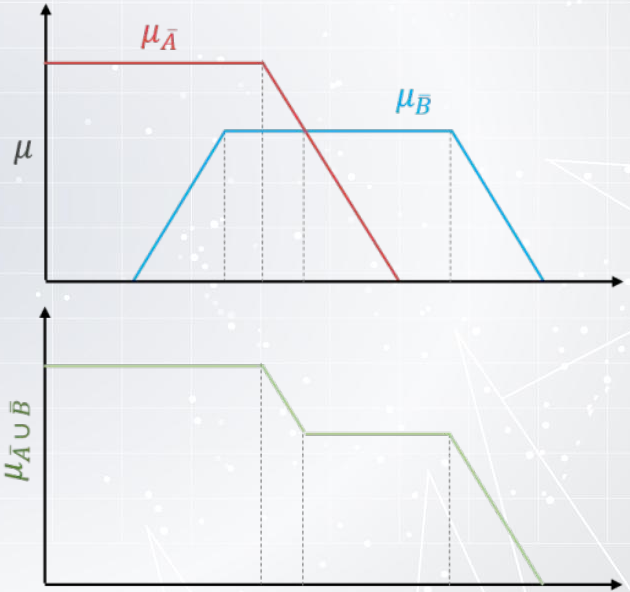
In the case of fuzzy sets, when there are common elements in both fuzzy sets, we should **select the element with the maximum membership value**.

The union of two fuzzy sets A and B is a fuzzy set C, written as **C = A ∪ B**

$C = A \cup B = \{(x, \mu A \cup B(x)) \mid \forall x \in X\}$

$\mu C(x) = \mu A \cup B(x) = \mu A(x) \vee \mu B(x)$

$= \max(\mu A(x), \mu B(x)), \forall x \in X$



Fuzzy Set Operations- **Union**

Example of Fuzzy Union:

$C = A \cup B = \{(x, \mu A \cup B(x)) \mid \forall x \in X\}$

$A = \{ (x1, 0.2), (x2, 0.5), (x3, 0.6), (x4, 0.8), (x5, 1.0) \}$

$B = \{ (x1, 0.8), (x2, 0.6), (x3, 0.4), (x4, 0.2), (x5, 0.1) \}$

$\mu A \cup B(x1) = \max(\mu A(x1), \mu B(x1)) = \max \{ 0.2, 0.8 \} = 0.8$

$\mu A \cup B(x2) = \max(\mu A(x2), \mu B(x2)) = \max \{ 0.5, 0.6 \} = 0.6$

$\mu A \cup B(x3) = \max(\mu A(x3), \mu B(x3)) = \max \{ 0.6, 0.4 \} = 0.6$

$\mu A \cup B(x4) = \max(\mu A(x4), \mu B(x4)) = \max \{ 0.8, 0.2 \} = 0.8$

$\mu A \cup B(x5) = \max(\mu A(x5), \mu B(x5)) = \max \{ 1.0, 0.1 \} = 1.0$

So, $A \cup B = \{ (x1, 0.8), (x2, 0.6), (x3, 0.6), (x4, 0.8), (x5, 1.0) \}$

Fuzzy Set Operations- **Intersection**

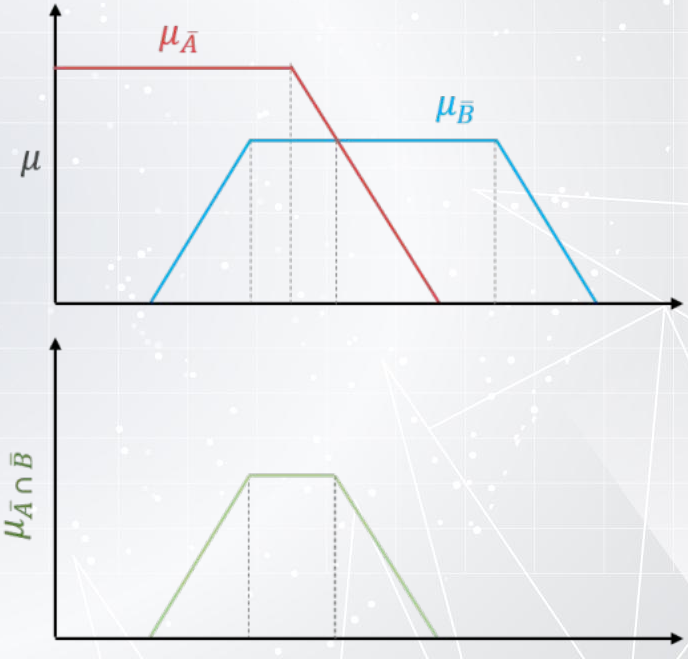
In the case of fuzzy sets, when there are common elements in both fuzzy sets, **we should select the element with minimum membership value**.

The intersection of two fuzzy sets A and B is a fuzzy set C, written as **C = A ∩ B**

$C = A \cap B = \{(x, \mu A \cap B\ (x)) \mid \forall x \in X\}$

$\mu C(x) = \mu A \cap B\ (x) = \mu A(x) \wedge \mu B(x)$

$= \min(\ \mu A(x), \mu B(x)\ ),\ \forall x \in X$



Fuzzy Set Operations- **Intersection**

Example of Fuzzy Intersection:

$C = A \cup B = \{(x, \mu A \cup B(x)) \mid \forall x \in X\}$

$A = \{ (x1, 0.2), (x2, 0.5), (x3, 0.6), (x4, 0.8), (x5, 1.0) \}$

$B = \{ (x1, 0.8), (x2, 0.6), (x3, 0.4), (x4, 0.2), (x5, 0.1) \}$

$\mu A \cap B(x1) = \min(\mu A(x1), \mu B(x1)) = \max \{ 0.2, 0.8 \} = 0.2$

$\mu A \cap B(x2) = \min(\mu A(x2), \mu B(x2)) = \max \{ 0.5, 0.6 \} = 0.5$

$\mu A \cap B(x3) = \min(\mu A(x3), \mu B(x3)) = \max \{ 0.6, 0.4 \} = 0.4$

$\mu A \cap B(x4) = \min(\mu A(x4), \mu B(x4)) = \max \{ 0.8, 0.2 \} = 0.2$

$\mu A \cap B(x5) = \min(\mu A(x5), \mu B(x5)) = \max \{ 1.0, 0.1 \} = 0.1$

So, $A \cap B = \{ (x1, 0.2), (x2, 0.5), (x3, 0.4), (x4, 0.2), (x5, 0.1) \}$

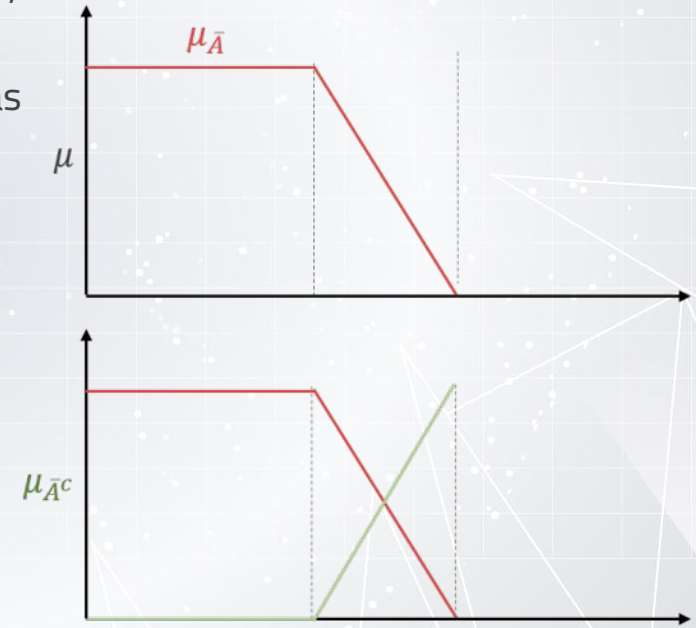
Fuzzy Set Operations- **Complement**

Fuzzy complement is **identical to crisp complement operation**. The membership value of every element in the fuzzy set is complemented with respect to 1, i.e. it is subtracted from 1.

The complement of fuzzy set A, denoted by AC, is defined as

AC = {(x, μAC (x)) | ∀ x ∈ X}

AC (x) = 1 – μA(x)



Fuzzy Set Operations- **Complement**

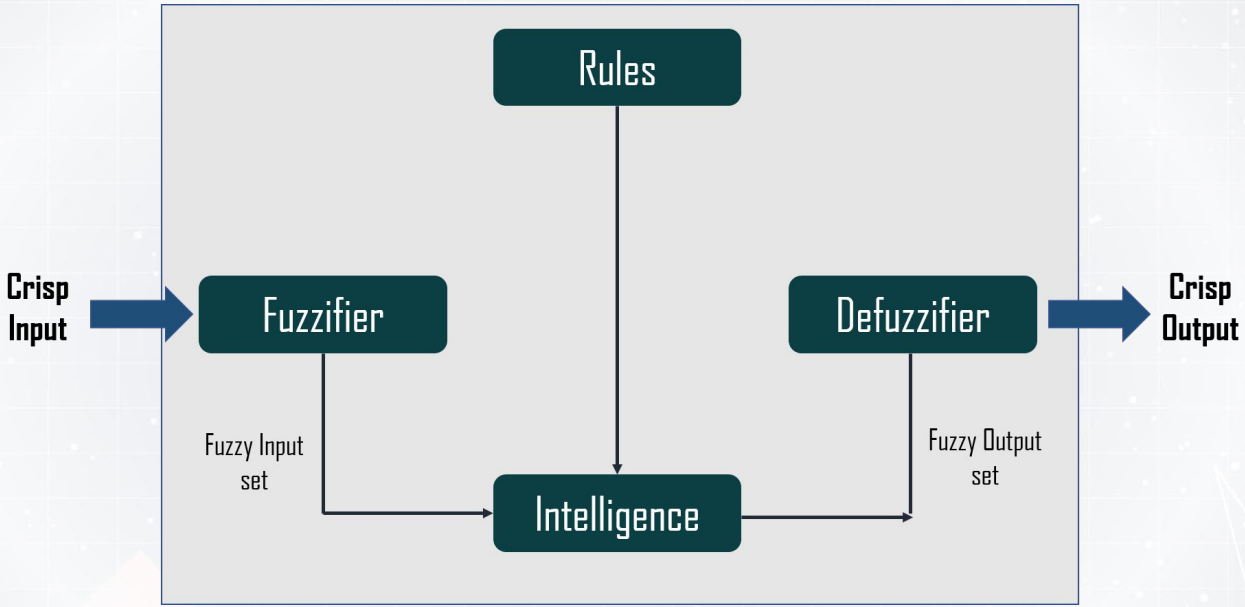
Example of Fuzzy Complement:

$AC(x) = 1 - \mu A(x)$

A = { (x1, 0.2), (x2, 0.5), (x3, 0.6), (x4, 0.8), (x5, 1.0) }

AC = { (x1, 0.8), (x2, 0.5), (x3, 0.4), (x4, 0.2), (x5, 0.0) }

Architecture of a Fuzzy Logic System



https://www.edureka.co/blog/fuzzy-logic-ai/

Architecture of a Fuzzy Logic System

The architecture of a fuzzy logic system consists of four components:

Fuzzification

This module converts input or crisp numbers into fuzzy sets. For example, categorizing “age” variables as young (Y), middle-aged (M), and old (O).

Rule Base

All of the if-then rules and conditions proposed by experts to control the decision-making system are stored in the rule base.

Architecture of a Fuzzy Logic System

Inference Enginee

The inference engine processes all information in the fuzzy logic system, so it is the most important component. This enables the user to determine the degree of correspondence between the current fuzzy input and the rules. Following degree matching, the system determines which rule to add based on the input field.

The following are the popular approaches to fuzzy inference systems: Mamdani fuzzy inference system, Takagi-Sugeno fuzzy inference system, Tsukamoto fuzzy inference system

Defuzzification

This component takes the fuzzy set input generated by the inference engine and converts it to crisp values. This is the last step in the fuzzy logic system development process. Crips value is a type of value that users can accept.

THANKS!

See You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