1. Correction to Problem 5 of 1stSampleTest #2

   Recall, the problem statement: Find the Taylor Series for $f(x) = \frac{1}{\sqrt{x}}$ centered at $a = 4$.

   From my solution, the $(n+1)$-th derivative should have been given by

$$f^n(x) = \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n \cdot x^{(2n+1)/2}},\ n = 1, 2, \ldots$$

   Since $a = 4$, the coefficient is given by

$$\begin{aligned} c_n &= \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n \cdot (4)^{(2n+1)/2}} \\ &= \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^n \cdot 2^{2n+1}} \\ &= \frac{(-1)^n \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}{2^{3n+1}}, n = 1, 2, \ldots \end{aligned}$$

   Recall, the Taylor Series of $f(x)$ centered at a is given by

$$f(x) = \sum_{i=0}^{\infty} c_i (x-a)^i = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

   which can be rewritten as

$$f(x) = \frac{f^{(0)}(a)}{0!} + \sum_{i=1}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i$$

   Putting everything together, we obtain

$$\begin{aligned} f(x) &= \frac{1}{\sqrt{4}} + \sum_{i=1}^{\infty} \frac{(-1)^i \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2i-1)}{2^{3i+1}}(x-4)^i \\ &= \frac{1}{2} + \sum_{i=1}^{\infty} \frac{(-1)^i \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2i-1)}{2^{3i} \cdot 2}(x-4)^i \\ &= \frac{1}{2}\left[1 + \sum_{i=1}^{\infty} \frac{(-1)^i \cdot 1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2i-1)}{2^{3i}}(x-4)^i\right] \end{aligned}$$

2. 2ndSampleTest #2 (Winter 2017), Problem 6

   Find the sum of the series

$$\sum_{n=1}^{\infty} n\left(\frac{x}{2}\right)^{n-1}$$

   at $x = 1$.

   Let $f(x) = \sum_{n=1}^{\infty} n(\frac{x}{2})^{n-1}$. Define $g(x) = \int f(x)\, dx$. Then

$$\begin{aligned} g(x) &= \int \sum_{n=1}^{\infty} n\left(\frac{x}{2}\right)^{n-1} dx \\ &= \sum_{n=1}^{\infty} \int \frac{n}{2^{n-1}} x^{n-1} dx \\ &= \sum_{n=1}^{\infty} \frac{1}{2^{n-1}} x^{n} \\ &= \sum_{n=1}^{\infty} 2 \cdot \left(\frac{x}{2}\right)^{n} \\ &= \frac{2}{1-(x/2)},\ |x| < 2 \end{aligned}$$

To simplify the calculation, multiply numerator and denominator by 2. This gives us

$$g(x) = \frac{4}{2-x},\ |x| < 2$$

To get back $f(x)$, we differentiate $g(x)$ since $g'(x) = f(x)$.

$$f(x) = \frac{d}{dx}\left[\frac{4}{2-x}\right] = \frac{4}{(2-x)^2}$$

At $x = 1/2$, we obtain the value

$$f(1/2) = \frac{4}{(2-(1/2))^2} = \frac{4}{(3/2)^2} = \frac{16}{9}$$