1 Section 11.2

- Recall, a sequence $\{a_n\}_{n=1}^{\infty}$ is a list of numbers. This form is also known as an infinite sequence. For example, $\{a_1, a_2, a_3, \dots\} = \{1, 2, 3, \dots\}$ is an example of a sequence. Now, suppose we want to add up the terms of an infinite sequence.

$$a_1 + a_2 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n$$

  We call the right-hand-side (RHS) an infinite series, or just series. Sometimes authors drop indices, i.e. $\sum a_n$, and let the context decide whether or not the sum is infinite or finite. I will sometimes do that so be careful!

- For a sequence $\{a_n\}_{n=1}^{\infty}$, let

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n.$$

  This is called the n-th partial sum of an infinite series. We can form a sequence using these partial sums, denoted by $\{s_n\}_{n=1}^{\infty}$. We call this the sequence of partial sums.

- The (infinite) series $\sum_{n=1}^{\infty} a_n$ is convergent if the sequence of partial sums $\{s_n\}_{n=1}^{\infty}$ is convergent (i.e. the limit as $n \to \infty$ of s_n exists) and so, $\sum_{n=1}^{\infty} a_n = s$.

- Theorem 1 [Theorem 6, Page 713, 8E]:

$$\text{If the series } \sum_{n=1}^{\infty} a_n \text{ is convergent, then } \lim_{n\to\infty} a_n = 0.$$

  WARNING:

  - This result assumes your given series is convergent. If your series does not meet this criteria, you cannot use this theorem!
  - This result does not give you any results about convergence of the series. The result gives you information about the sequence terms.

- Divergence Test:

$$\text{If } \lim_{n\to\infty} a_n \text{ DNE or} \neq 0, \text{ then the series } \sum_{n=1}^{\infty} a_n \text{ is divergent.}$$

  - You must check the assumptions before applying this test. In other words, you must show the limit either does not exist or is non-zero.
  - If the limit is zero, this theorem does not tell us anything about the convergence or divergence of the series.

2 Section 11.3

- On the Integral Test:
  - You do not have to use the interval $[1, \infty)$. In general, we can have $[c, \infty)$.
  - You must check that $f(x)$ is a continuous, positive, and decreasing function on $[c, \infty)$. Do not skip ahead as this is all part of the information.
  - If your $f(x)$ is decreasing on another interval, say $[3, \infty)$, but your given series begins at, say $n = 1$. You can instead use the interval $[3, \infty)$. It is fine to subtract off finitely many terms.
- On the Remainder Estimate for the Integral Test:
  - Recall, $R_n = s - s_n$. Remember, s is the value for which the series sums to (or converges to) and s_n is the n-th partial sum.
  - The interpretation is that we are essentially taking the difference between the true value of the sum and an estimated value of the sum. So, R_n is the error with respect to the approximation determined by the n-th partial sum, s_n.

3 Section 11.4

- On the Comparison Test (CT):
  - Given: $\sum a_n, \sum b_n$ are series with positive terms, i.e. $a_n, b_n \geq 0$.
  - (Convergence) Must check that $\sum b_n$ is convergent and $0 \leq a_n \leq b_n$ for all n.
  - (Divergence) Must check that $\sum b_n$ is divergent and $0 \leq b_n \leq a_n$ for all n.
- On the Limit Comparison Test (LCT):
  - In the book, this test only covers the case when the limit is a positive number!
  - See Chris Mclean's Lecture 9 for other cases and some examples.