MATH22981 - Regression Supplement

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December 10th, 2016

The objective is to convert a non-linear problem to a linear problem. Some basic non-linear equations you may already know include

$$y = ax^k, \tag{1}$$
$$y = a(b^{kx}),\ b > 0 \tag{2}$$

The transformation we are applying is the natural logarithm. That is, if we take the natural logarithm of both sides of equation (1), we have

$$\begin{aligned} lny &= ln(ax^n) \\ lny &= lna + lnx^k \\ lny &= lna + klnx \\ Y &= lna + kX \end{aligned} \tag{3}$$

where $X = lnx$ and $Y = lny$. Similarly for equation (2), we have

$$\begin{aligned} lny &= ln(a(b)^{kx}) \\ lny &= lna + lnb^{kx} \\ lny &= lna + kxlnb \\ Y &= lna + (klnb)X \end{aligned} \tag{4}$$

where $X = x$ and $Y = lny$. **Keep in mind, if you are using log to some base, b, the same rules apply. However, if you are trying to solve for certain variables, you have to keep in mind the base you are working with. I chose to use the natural logarithm because the calculations are simpler.**

Using the method of least squares for equation (3), we can find our k and a value by the following formulas

$$k = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} \tag{5}$$
$$lna = \frac{\sum X^2 \sum Y - \sum X \sum XY}{n\sum X^2 - (\sum X)^2} \tag{6}$$
$$\tag{7}$$

Keep in mind, to find a, you must raise both sides apply e to both sides.

Using the method of least squares for equation (4), we can find our k and a value by the following formulas

$$
\begin{aligned}
klnb &= \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} \\
k &= \frac{1}{lnb}\frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2} \qquad (8) \\
lna &= \frac{\sum X^2 \sum Y - \sum X \sum XY}{n\sum X^2 - (\sum X)^2}
\end{aligned}
$$

$$(9)$$

Keep in mind, to find a, you must raise both sides apply e to both sides.