Murphy, Orla . "Multiple Linear Regression." Lecture Hamilton Ontario, October 3, 2019.

OLS Estimates

- The OLS estimates for $\boldsymbol{\beta}$ minimize

$$S(\boldsymbol{\beta}) = \sum_{i=1}^{n} \epsilon_i^2 = \boldsymbol{\epsilon}'\boldsymbol{\epsilon}.$$

- From Lecture 6:

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{x}'\mathbf{x}\right)^{-1}\mathbf{x}'\mathbf{y}$$

provided $(\mathbf{x}'\mathbf{x})^{-1}$ exists.

- $(\mathbf{x}'\mathbf{x})^{-1}$ will exist if $\mathbf{x}$ has full column rank (i.e., each of the columns of the matrix are linearly independent).
- The fitted values can be written as

$$\hat{\mathbf{y}} = \mathbf{x}\hat{\boldsymbol{\beta}}.$$

- The residuals are

$$\boldsymbol{e} = \mathbf{y} - \hat{\mathbf{y}}.$$

viewed as the predicted values of unknown random errors $\boldsymbol{\epsilon}$.

- The estimated Variance-Covariance matrix for $\boldsymbol{\beta}$ given by

$$\widehat{Var}(\hat{\boldsymbol{\beta}}) = \hat{\sigma}^2(\mathbf{x}'\mathbf{x})^{-1} = \hat{\sigma}^2\boldsymbol{C}.$$

- The standard errors can be found by looking at the diagonal elements of this matrix.

$$\text{se}(\hat{\beta}_j) = \hat{\sigma}\sqrt{c_{jj}}$$

for $j = 0, 1, \ldots, p$.

- The $100(1-\alpha)\%$ confidence intervals for individual regression coefficients are given by

$$\hat{\beta}_j \pm t_{(n-(p+1);\alpha/2)}\text{se}(\hat{\beta}_j) \quad j = 0, 1, \ldots, p.$$