

## Outlier detection

- Generally, outliers will have a large externally studentized residual in absolute value.
- **Recall:**  $h_{ii} = [\mathbf{x}(\mathbf{x}'\mathbf{x})^{-1}\mathbf{x}']_{ii}$  is a diagonal entry of the hat matrix and measures how much  $y_i$  contributes to the estimation of  $\hat{y}_i$ . We call  $h_{ii}$  **leverages**.
- Typically, observations with

$$|r_i^*| = \left| \frac{e_i}{\hat{\sigma}_{(i)}\sqrt{1-h_{ii}}} \right| > 2$$

are considered to be possible outliers.

- In SAS, the `RStudentByLeverage` plot can help us to visualize observations exceed this cutoff.
- **Note:** If you decide to make any modifications to the model (e.g., transformations to variables), you should check again for outliers.

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## High-Leverage Points I

- Observations that are outliers in  $X$ -space are considered **high-leverage points**.
- Outliers in  $X$ -space are high leverage values  $h_{ii}$ .
- It can be shown that  $\sum h_{ii} = p + 1$ , therefore the average value for  $h_{ii}$  is  $(p + 1)/n$ .
- Observations with  $h_{ii}$  greater than  $2(p + 1)/n$  are generally considered to be high-leverage points.
- A plot of  $h_{ii}$  against the index  $i$  can be useful for this purpose.

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