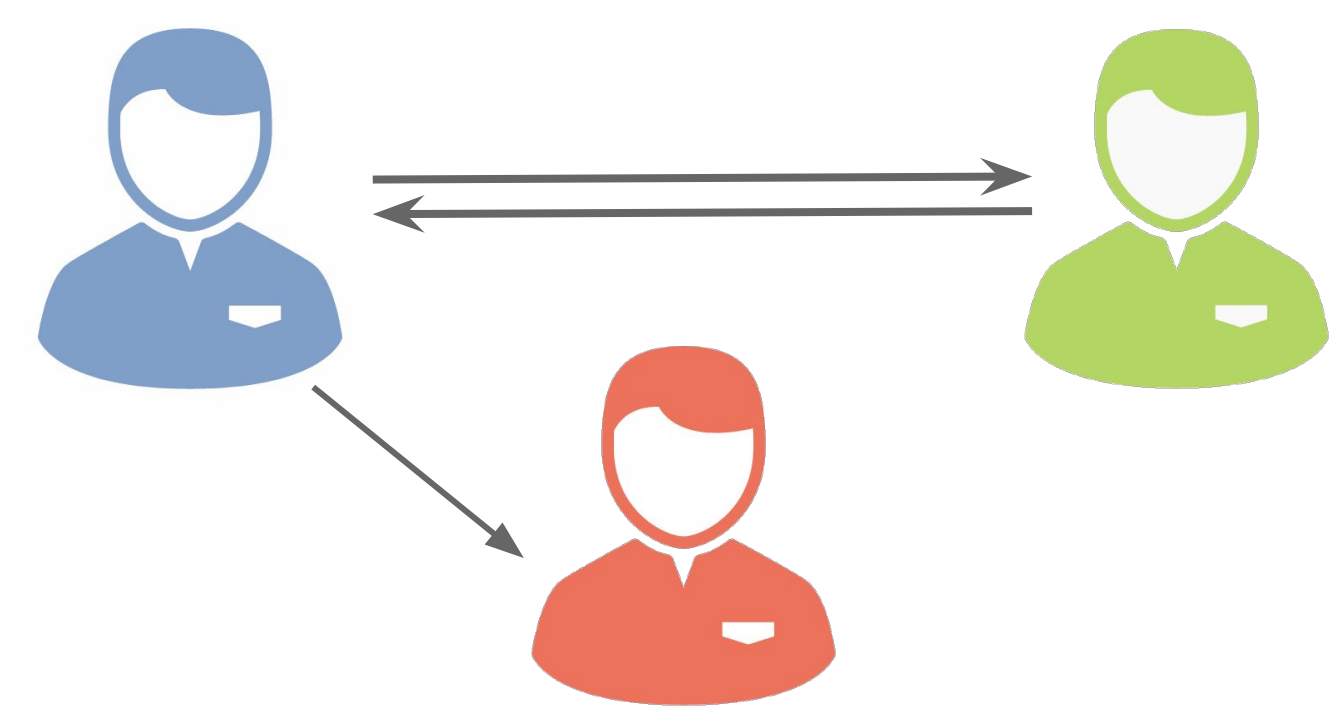
An Evaluation Framework for Granger Matrix Extraction via Point Processes

Guilherme Resende Borges
Flavio Figueiredo
Pedro O. S. Vaz de Melo
Renato Assunção

UFMG
UNIVERSIDADE FEDERAL
DE MINAS GERAIS

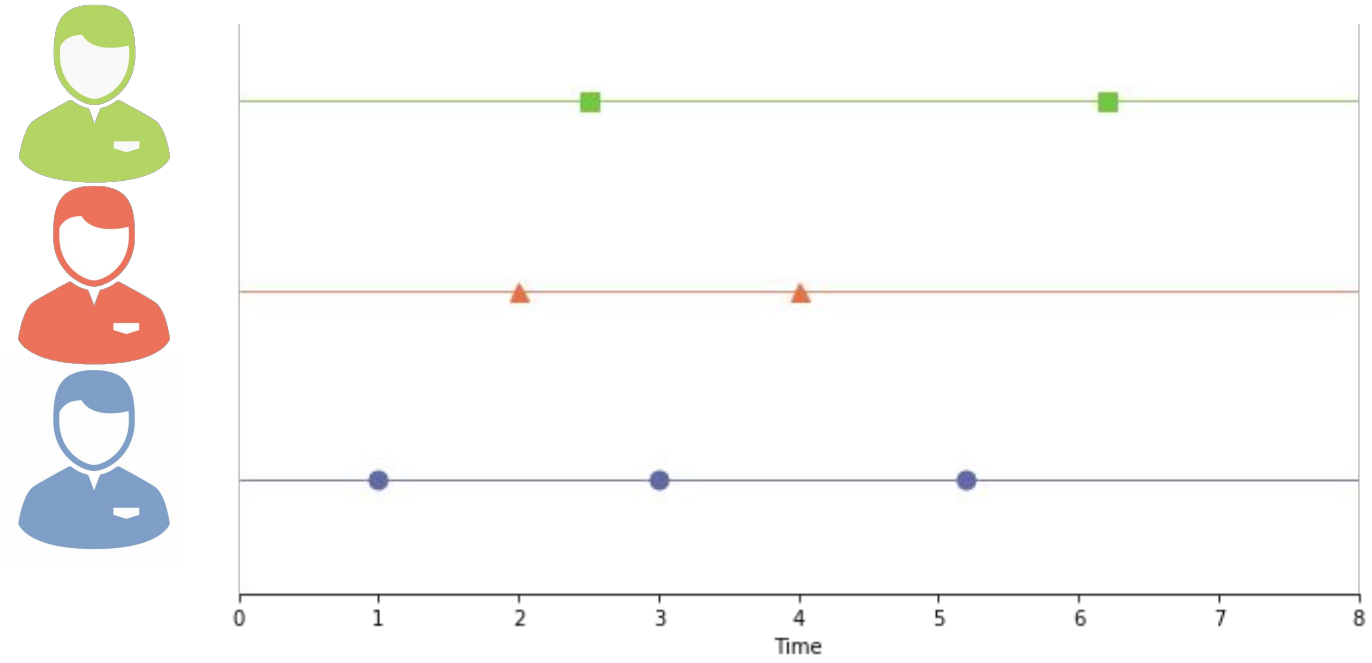
DCC
DEPARTAMENTO DE
CIÊNCIA DA COMPUTAÇÃO

Estimating a Granger Causal Graph



What is observed

What is observed

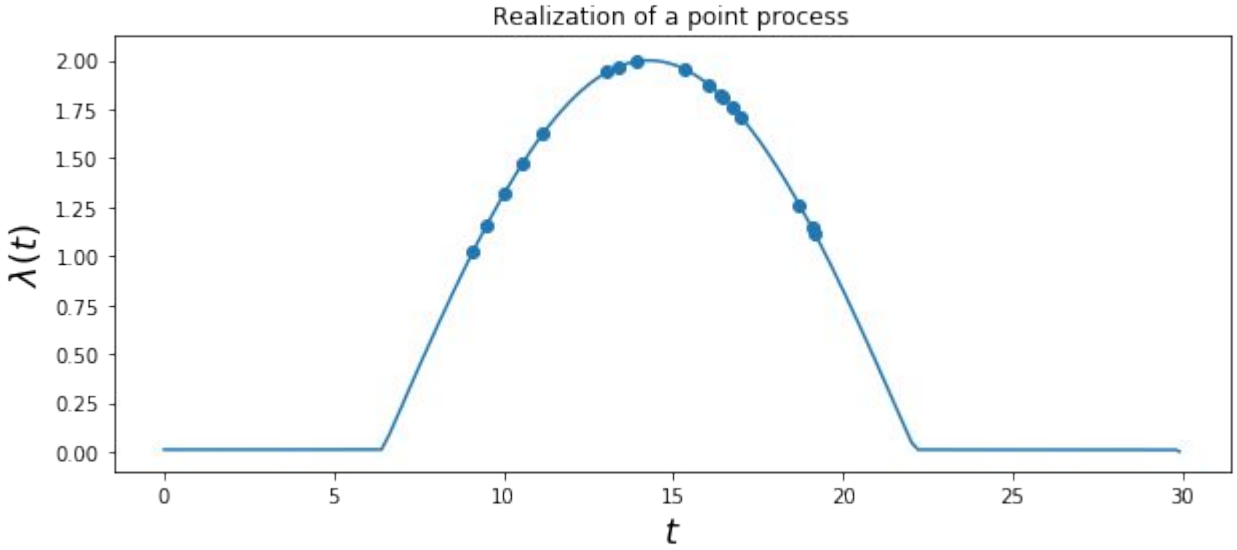


Point Processes

Data containing events occurring randomly in time is found across several fields, e. g.

- Occurrence of earthquakes and aftershocks
- Spiking neurons
- Activity in online social networks

The Conditional Intensity Function



Multivariate Point Processes

$$\lambda_a(t|\mathcal{H}(t)) = \mu_a + \sum_{b=1}^{n} \alpha_{ba}\omega_{ba}(t)$$

Baseline or endogenous intensity

Multivariate Point Processes

$$\lambda_a(t|\mathcal{H}(t)) = \mu_a + \sum_{b=1}^{n} \alpha_{ba}\omega_{ba}(t)$$

Kernel Function

Multivariate Point Processes

$$\lambda_a(t|\mathcal{H}(t)) = \mu_a + \sum_{b=1}^{n} \alpha_{ba}\omega_{ba}(t)$$

Captures the influence of b on a

Methods

Several previous efforts have been made to accomplish this task

- Problem #1: they mostly employed a different evaluation methodology
- Problem #2: it is difficult to compare newly proposed methods

Methods

We analyzed 7 models to which open source implementations are available:

- SumGaussians (SG)
- ADM4
- HawkesEM
- HawkesBasisKernels / MMEL
- Hawkes Conditional Law
- NPHC
- GrangerBusca

Methods

Method	Per Iteration Complexity
ADM4	$O(N^3K^3)$
MMEL	$O(MN^3K^2 + ML_m(NK + N^2))$
CondLaw	$O(NK^2Q + K^4Q^3)$ (non interative)
SG	$O(MN^3K^2)$
NPHC	$O(K^3)$
GBusca	$O(N(\log N + \log K))$

Metrics

1. Comparing the estimated Granger matrix with a ground truth
2. Comparing the real series of events with simulated series
    a. Evaluating the number of events generated
    b. Evaluating the distribution of inter-event times

Metrics tested for each paper

Metrics\Papers	[3]	[5]	[4]	[6]	[2]	[1]	[7]
RelErr		X	X			X	
RankCorr		X				X	X
Q-Q plot for goodness of fit					X		
Loglik		X	X	X			
RelErr Baseline			X				
L2 error of kernel	X			X			
Precision@k							X

Rank Correlation

$$MRankCorr(A,B) = \frac{1}{K}\sum_{i=0}^{K-1} RankCorr([a^i],[b^i])$$

Where $RankCorr(x,y)$ is the Kendall's tau Rank Correlation

Relative Error

$$RelErr(A,B) = \frac{1}{K^2} \sum_{i,j} \frac{|a^{ij} - b^{ij}|}{|a^{ij}|} 1_{a^{ij} \neq 0} + |b^{ij}| 1_{a^{ij} = 0}$$

Results

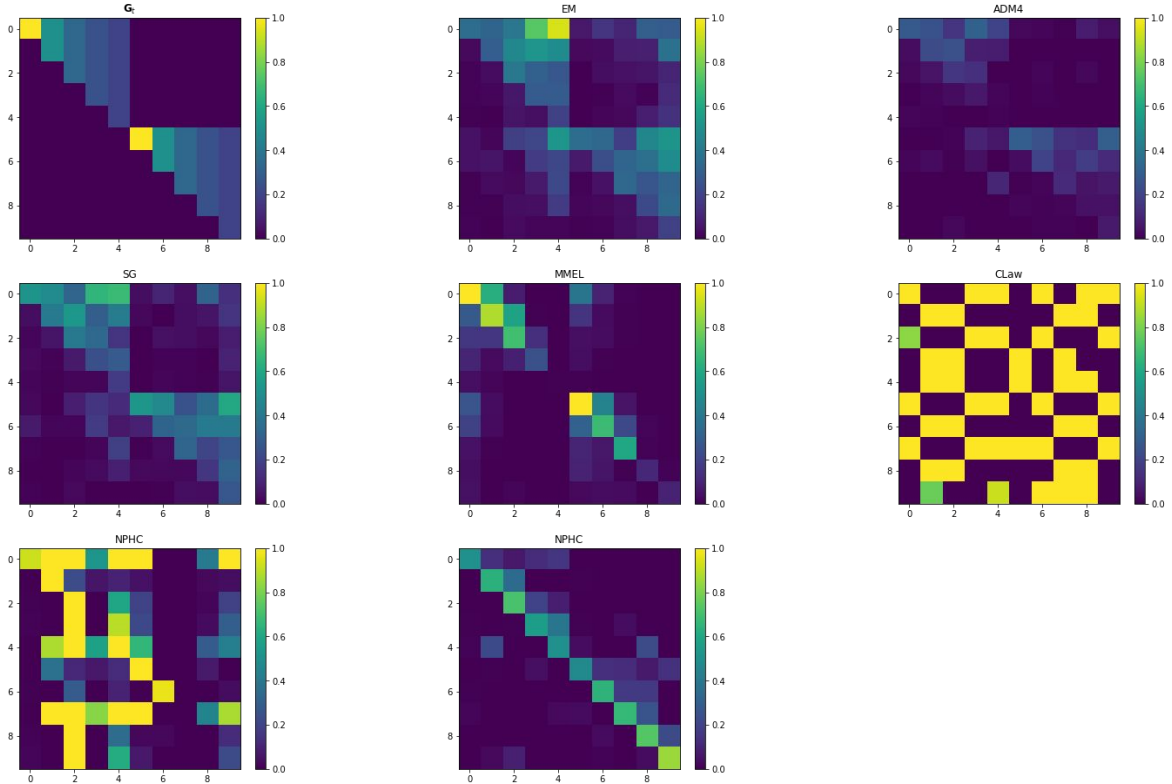
Memetracker (15-day) dataset

RankCorr	P@3	RelErr
GBusca(0.29)	GBusca(0.44)	GBusca(88.88)
NPHC(0.20)	ADM4(0.09)	SG(577.74)
MMEL(0.20)	SG(0.07)	NPHC(2027.59)
ADM4(0.19)	MMEL(0.07)	MMEL(4088.59)
SG(0.14)	NPHC(0.05)	CLaw(7900.25)
CLaw(0.09)	CLaw(0.0)	ADM4(2872960.85)

Results Granger-Simulated

RankCorr	P@3	RelErr	MSE	FMatch	MAPE	KolDiff	CorrDiff	VarDiff
GBusca(0.69)	GBusca(0.47)	ADM4(0.15)	ADM4(1430.78)	ADM4(0.82)	ADM4(0.16)	ADM4(0.14)	GBusca(0.12)	ADM4(86.00)
SG(0.59)	MMEL(0.40)	SG(0.16)	MMEL(28572.92)	GBusca(0.66)	GBusca(0.30)	GBusca(0.15)	SG(0.18)	GBusca(119.08)
HkEM(0.55)	ADM4(0.23)	HkEM(0.21)	SG(28610.89)	MMEL(0.49)	MMEL(0.44)	SG(0.23)	ADM4(0.18)	MMEL(377.20)
ADM4(0.54)	SG(0.20)	MMEL(0.22)	GBusca(44502.65)	SG(0.48)	SG(0.48)	MMEL(0.24)	MMEL(0.21)	SG(650.02)
MMEL(0.34)	CLaw(0.17)	GBusca(0.27)	HkEM(81174.62)	HkEM(0.32)	HkEM(0.75)	HkEM(0.33)	HkEM(0.23)	HkEM(4044.50)
NPHC(0.09)	HkEM(0.10)	NPHC(1.44)						
CLaw(-0.12)	NPHC(0.10)	CLaw(9.85)						

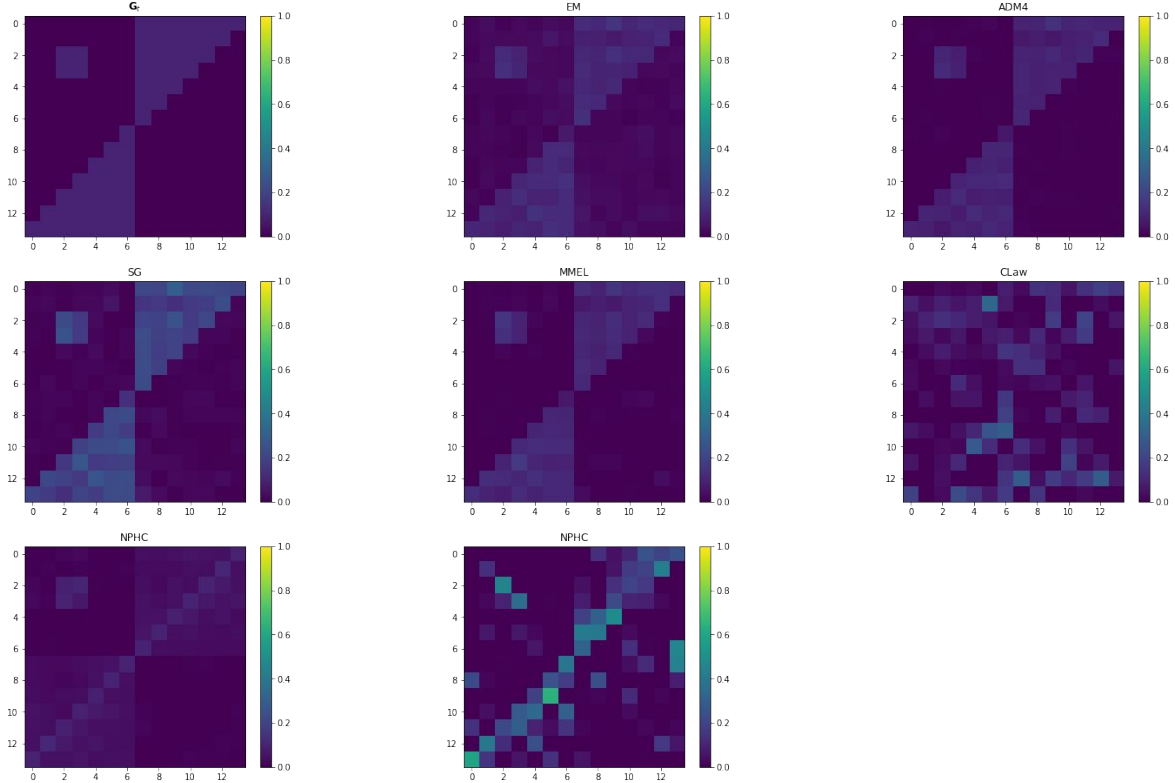
Results Granger-Simulated



Results HawkesExp-Simulated

RankCorr	P@3	RelErr	MSE	FMatch	MAPE	KolDiff	CorrDiff	VarDiff
ADM4(0.63)	GBusca(0.36)	ADM4(0.04)	ADM4(460.11)	ADM4(0.98)	HkEM(0.07)	ADM4(0.08)	HkEM(0.08)	ADM4(985.08)
HkEM(0.62)	MMEL(0.21)	HkEM(0.04)	HkEM(508.50)	HkEM(0.98)	ADM4(0.07)	MMEL(0.08)	MMEL(0.08)	HkEM(1243.60)
MMEL(0.62)	ADM4(0.19)	NPHC(0.13)	MMEL(918.46)	MMEL(0.89)	MMEL(0.09)	HkEM(0.08)	ADM4(0.09)	MMEL(1376.41)
SG(0.62)	NPHC(0.17)	MMEL(0.19)	SG(3342.82)	SG(0.52)	SG(0.21)	SG(0.12)	SG(0.09)	SG(3691.79)
GBusca(0.56)	SG(0.14)	CLaw(0.29)	GBusca(28964980.00)	GBusca(0.00)	GBusca(20.45)	GBusca(0.79)	GBusca(0.11)	GBusca(4543.58)
NPHC(0.56)	HkEM(0.12)	SG(0.33)						
CLaw(0.19)	CLaw(0.07)	GBusca(0.42)						

Results HawkesExp-Simulated



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