Lecture 7

Euler's formula: G connected planar graph
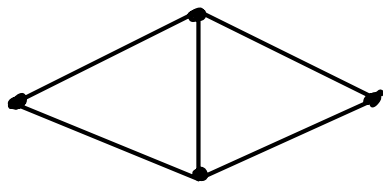
Draw it on the plane with edge crossings, called a plane graph. Let v denote the number of vertices

e

r

edges

reigons bounded by edges
(including the outer, unbounded reigon)

We have $v - e + r = 2$.

Example

$v = 4 \quad e = 5 \quad r = 3$

How many reigons in a plane graph w/ 10 vertices each of degree 3.

$$e = 10 \times \frac{3}{2} = 15$$

$$r = e - v + 2 = 15 - 10 + 2 = 7$$

Corollary: If G is a connected planar graph, $e > 1$, then $e \leq 3v - 6$. $e \geq \frac{3}{2} r$

Euler cycles

Multigraph: Allow multiple edges between a pair of vertices as well as loops.

Theorem: A multigraph is an Euler cycle iff it is connected and every vertex has even degree.

Cycle: first vertice = last vertice