ACM/IDS 104 APPLIED LINEAR ALGEBRA
PRACTICE PROBLEMS FOR LECTURE 1

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Each lecture is accompanied by two practice problems: a somewhat easier, more practical *Problem A*, and a more difficult, more conceptual *Problem B*. The main goal of the practice problems is threefold: to help you better understand the material covered in the corresponding lecture, to help you prepare to solve problems in problem sets and exams, and to accommodate the diversity of students' math backgrounds by providing both easier and more challenging problems. These problems are for self-practice: they will not be graded, and the solutions—posted on Piazza—also illustrate the expected level of rigor for problem sets and exams.

Problem 1A. Gaussian Elimination
Let A be the following nonsingular matrix:

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 1 & 2 & 5 \\ 1 & 1 & 4 \end{bmatrix} \tag{1}$$

(a) Reduce A to an upper triangular matrix U with non-zero diagonal elements using elementary row operations.
(b) Find a permutation matrix P and a special lower triangular matrix L such that $PA = LU$.

Problem 1B. Uniqueness of the Permuted LU Decomposition for Fixed P
Let A be a nonsingular matrix. In general, the permuted LU decomposition $PA = LU$, where P is a permutation matrix, L is special lower triangular, and U is upper triangular with non-zero diagonal elements, is not unique[1]. However, if P is fixed, then the decomposition is unique. Namely, prove that if

$$PA = LU \quad \text{and} \quad PA = \tilde{L}\tilde{U}, \tag{2}$$

where $L, \tilde{L}$ are special lower triangular and $U, \tilde{U}$ are upper triangular with non-zero diagonal elements, then $L = \tilde{L}$ and $U = \tilde{U}$.

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[1]It can be shown that there exist other $\tilde{P}, \tilde{L}, \tilde{U}$ such that $\tilde{P}A = \tilde{L}\tilde{U}$ precisely when $\tilde{P}A$ has all leading principal minors (determinants of the top-left $k \times k$ submatrices, $k = 1, \ldots, n$) non-zero.