Problem 1

1. $$\max_{x_j} b_{ij}^*(x_i, x_j)$$

$$= \max_{x_j} \left[\frac{1}{Z} \phi_i(x_i) \cdot \phi_j(x_j) \psi_{ij}(x_i, x_j) \left(\prod_{k \in N(i) \setminus j} m_{k \to i}^*(x_i) \right) * \left(\prod_{k \in N(i) \setminus i} m_{k \to j}^*(x_j) \right) \right]$$

$$= \frac{1}{Z} \phi_i(x_i) \left(\prod_{k \in N(i) \setminus j} m_{k \to i}^*(x_i) \right) \left[\max_{x_j} \phi_j(x_j) \cdot \psi_{ij}(x_i, x_j) \left(\prod_{k \in N(i) \setminus i} m_{k \to j}^*(x_j) \right) \right]$$

$$= \frac{1}{Z} b_i^*(x_i) * C(\phi, \psi)$$

{ $C(\phi, \psi)$ is a constant for any given ϕ & ψ. It is max among all values of belief of second node }

$$\max_{x_j} b_{ij}^*(x_i, x_j) \propto b_i^*(x_i)$$

2. Replacing max in above equation with sum over x_j gives.

$$\sum_{x_i} b_{ij}^*(x_i, x_j) \propto b_i^*(x_i)$$

~~2. Similar~~

3.

$$\max_x p(x) = \lim_{T\to 0} \left(\sum_x (p(x))^{1/T} \right)^T \quad \text{— (1)}$$

$$p(x) = \frac{1}{Z} \prod_{i\in V} b_i(x_i) \cdot \prod_c \frac{b_c(x_c)}{\prod_{i\in c} b_i(x_i)}$$

from sum-product we get a return value of Z and the converged beliefs $b_i^*(x_i)$ and $b_c^*(x_c)$.

$$\Rightarrow p(x) = \frac{1}{Z} \prod_{i\in V} b_i^*(x_i) \cdot \prod_c \frac{b_c^*(x_c)}{\prod_{i\in c} b_i^*(x_i)} \quad \text{— (3)}$$

replace $p(x)$ from (3) into (1) to get

$$\cancel{\max_x}\ f(T) = \cancel{\lim_{T\to 0}} \left(\sum_x (p(x))^{1/T} \right)^T$$

Use the matlab limit(f(T), T, 0) to calculate

$\max_x p(x)$.

Problem 3

$$p(x) = \frac{1}{Z} \prod_{i \in V} \exp(h_i x_i) \prod_{(i,j) \in E} \exp(J_{ij} x_i x_j)$$

$h_i = 0$ for all vertices. & $J_{ij} = J \ \forall \ i,j$

$$\Rightarrow \quad p(x) = \frac{1}{Z} \prod_{(i,j) \in E} \exp(J x_i x_j).$$

$$\log(p(x)) = -\log Z + \sum_{(i,j) \in E} J x_i x_j$$

$$= \left[\sum_m J . \left(\sum_{(i,j) \in E} x_i x_j \right) \right] - \sum_m \log(Z).$$

$$= J \left[\sum_m \sum_{(i,j) \in E} (x_i x_j) \right] - \sum_m \log(Z).$$

$Z = ?$

$J = ?$ ←

1. $(1,1,1)$

$$\overset{l(J)}{\log(p(x))} = 3J - \log(Z).$$

2. $(1,1,-1)$ $\overset{(l(J))}{\log p(x)} = -J - \log(Z)$

3. $(1,1,1)$ $\overset{l(J)}{\log p(x)} = 3J - \log(Z)$

4. $(-1,-1,-1)$ $\overset{l(J)}{\log p(x)} = 3J - \log(Z)$

5. $(1,1,-1)$ $\log p(x) = -J - \log(Z)$

Sum:

$\log l(J) = 7J - 5\log(Z).$

$$\log l(J) = 7J - 5\log(Z)$$

$$\frac{\partial \log l(J)}{\partial J} = 7 - 5 \cdot \frac{\partial \log(Z)}{\partial J} = 0$$

$$\Rightarrow 7 = 5 \cdot \frac{\partial \log Z}{\partial J}$$

$$\Rightarrow \frac{\partial \log Z}{\partial J} = \frac{7}{5}.$$

$$\frac{\partial \log Z(J)}{\partial J} = \sum_c \sum_{x_c} \cancel{f_c(x_c) \, p(x, J)}$$

$$\frac{\sum_x \left[\sum_c f_c(x_c)_i \right] \prod_c \exp(\langle J, f_c(x_c) \rangle)}{\sum_x \prod_c \exp(\langle J, f_c(x_c) \rangle)}$$

Numerators:

Sample $(1,1,1)$ = $\sum \frac{[3] \cancel{\exp(J^3)}}{}$ = $3 \exp(3J)$

Sample $(1,1,-1)$ = $[-1] \cancel{* J^3}$ = $\cancel{J^3}$ $\exp(-J)$

Sample $(1,1,-1)$ = $\cancel{3(eJ^3)}$ = $\cancel{9\exp(3J)}$ = $3\exp(3J)$

Sample $(1,1,1)$ = $\cancel{3eJ^3}$ = $\cancel{9\exp J}$ = $3\exp(3J)$

Sample $(-1,-1,-1)$ = $\cancel{eJ^3}$ = $\cancel{-3\exp(J)}$ = $-\exp(3J)$

Sample $(-1,1,-1)$ =

Denominator: $\cancel{e^{3J^3} - J^3 + 3J^3 + 3J^3 - J^3}$ = $\exp(3J) * 3 + \exp(-J) * 2$

$\cancel{= e^{7J^3}}$ $3 \cdot 2 + \exp(3J)$

$\cancel{\exp(3J - J + 3J + 3J - J)}$

$\cancel{= \exp(7J)}$

$$\frac{\partial \log Z(J)}{\partial (J)} = \frac{9\exp(3J) - 2\exp(-J) \cancel{+3}}{3\exp(3J) \cancel{\exp(7J)} + 2\exp(-J)}$$

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$$\Rightarrow \quad \frac{9\exp(3J) - 2\exp(-J)}{\cancel{\exp(7J)}\ 3\exp(3J) + 2\exp(-J)} = \frac{7}{5}$$

$$\Rightarrow \quad 45\exp(3J) - 10\exp(-J) = 21\exp(3J) + 14\exp(-J)$$

$$\Rightarrow \quad 24\exp(3J) = 24\exp(-J)$$

$$\Rightarrow \quad \exp(4J) = \frac{24}{24} = \cancel{\frac{12}{17}}\ 1$$

$$\Rightarrow \quad 4J = \cancel{\log\left(\frac{12}{17}\right)}\ \log(1)$$

$$\Rightarrow \quad J_{MLE} = \left[\frac{1}{4}\log\left(\cancel{\tfrac{12}{17}}\ 1\right)\right]$$

$$\Rightarrow \quad \boxed{J_{MLE} = 0}$$

Ans