Problem 1 : PROBLEM SET 2

$$p(x) = \max_{x_{ij}} \frac{1}{Z} \prod_{(i,j)\in E} \phi_{ij}(x_{ij}) \cdot \prod_{\substack{(i,k)\in E \\ (i,j)\in E}} \psi_{ij,ik}(x_{ij}, x_{ik})$$

~~Z = No. of nodes~~

For a graph with even no. of vertices, k should be greater than or equal to 2. $(k \geq 2)$ for a possible assignment.

$$\phi_{ij}(x_{ij}) = \exp(w_{x_{ij}})$$

$$\psi_{ij,ik}(x_{ij}, x_{ik}) = \mathbb{1}_{x_{ij} \neq x_{ik}}.$$

Maximal values would be <u>k and $k-1$</u> for alternate edges.

$$p(x) = \frac{1}{Z} \prod_{(i,j)\in E} \exp(w_{x_{ij}}) \cdot \prod_{\substack{(i,k)\in E \\ (i,j)\in E}} \mathbb{1}_{x_{ij} \neq x_{ik}}.$$

Alternate ~~edges are k and $k-1$~~

$$\Rightarrow \cancel{p(x) = \frac{1}{Z} e^{\frac{nk}{2}} \cdot e^{\frac{n(k-1)}{2}}}$$

$$p(x) = \frac{1}{Z} e^{w_{x_{12}}} \cdot e^{w_{x_{23}}} \cdots\cdots e^{w_{x_{n1}}} \qquad \{n = |V|\}.$$

$p(x)$ is maximized when $e^{w_{x_{12}}} \cdot e^{w_{x_{23}}} \cdots e^{w_{x_{n1}}}$ is maximized. This is maximized at alternate k and $k-1$ assignments giving,

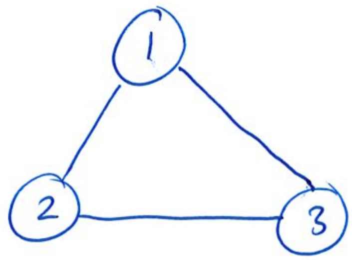
$$p(x) = \frac{1}{Z} e^{w_k} \cdot e^{w_{k-1}} \cdots\cdots e^{w_{k-1}}$$

$$= \frac{1}{Z} e^{k} \cdot e^{k-1} \cdot e^{k} \cdots\cdots e^{k-1} \qquad (n \text{ times}).$$

$$= \frac{1}{Z} \cdot e^{\frac{nk}{2}} \cdot e^{\frac{n(k-1)}{2}}.$$

$\therefore$ k and $k-1$ to alternate edges are the ~~ma~~ MAP assignment when the graph has even number of vertices.

2.



$k = 3$

$w_1 = 1, \; w_2 = 2, \; w_3 = 3.$

MAP IP :

$$p(x) = \sum_{(i,j) \in E} \sum_{x_{ij}} \tau_{ij}(x_{ij}) \log \phi_{ij}(x_{ij})$$

$$+ \sum_{\substack{(i,j) \in E \\ (i,k) \in E}} \sum_{x_{ij}, x_{ik}} \tau_{ij,ik}(x_{ij}, x_{ik}) \cdot \log\left(\psi_{ij,ik}(x_{ij}, x_{ik})\right)$$

Assignments :

$x_{12} = 1$	$\tau_{12}(1) = 1$	$\tau_{12,31}(1,3) = 1$
$x_{23} = 2$	$\tau_{23}(2) = 1$	$\tau_{12,23}(1,2) = 1$
$x_{31} = 3$	$\tau_{31}(3) = 1$	$\tau_{23,31}(2,3) = 1$

$$p(x) = 1 * \log(e^1) + 1 * \log(e^2) + 1 * \log(e^3) + 0 + 0 \ldots$$

$$+ \left[0 + \ldots + 1 * 0 + 1 * 0 + 1 * 0\right]$$

$$= \cancel{15}\; 6.$$

MAP LP :

$$p(x) = \sum_{(i,j)\in E} \sum_{x_{ij}} \tau_{ij}(x_{ij}) \log \phi_{ij}(x_{ij})$$

$$+ \sum_{\substack{(i,j)\in E \\ (i,k)\in E}} \sum_{x_{ij}, x_{ik}} \tau_{ij,ik}(x_{ij}, x_{ik}) \log\left(\psi_{ij,ik}(x_{ij}, x_{ik}) \right)$$

With the same assignments as with IP problem, we get the maximum value of $p(x)$.

$p(x) = \underline{6}.$

~~This~~ The optimal value does not change with any other assignment.