ASSIGNMENT 5 - PART 1

1) M models $- \; h_1, h_2, \ldots h_M$

$\epsilon_i(x) = f(x) - h_i(x)$: Error for each model

Squared Error Expected : $E(\epsilon_i(x)^2) = E[(f(x) - h_i(x))^2]$

$$E_{avg} = \frac{1}{M} \sum_{i=1}^{M} E[\epsilon_i(x)^2]$$

$$h_{agg} = \frac{1}{M} \sum_{i=1}^{M} h_i(x)$$

$$E_{agg} = E\left[\left\{\frac{1}{M}\sum_{i=1}^{M} h_i(x) - f(x)\right\}^2\right]$$

$$= E\left[\left\{\frac{1}{M}\sum_{i=1}^{M} \epsilon_i(x)\right\}^2\right] \quad \text{—— (1)}$$

Given : $E[\epsilon_i(x)] = 0 \quad \forall\, i$ —— (2)

& $E(\epsilon_i(x) \cdot \epsilon_j(x)) = 0 \quad \forall\, i \neq j$ —— (3)

(1) $\Rightarrow$ $E_{agg} = \frac{1}{M^2} E\left[\left\{\sum_{i=1}^{M} \epsilon_i(x)\right\}^2\right]$ $\left\{\text{constant } \frac{1}{M}\right\}$

~~If M = 2 :~~

$E_{agg} =$

$$= \frac{1}{M^2} E\left[\left\{\epsilon_1(x) + \epsilon_2(x) + \ldots \epsilon_M(x)\right\}^2\right]$$

$$= \frac{1}{M^2} E\left[\left\{\epsilon_1(x)^2 + \epsilon_2(x)^2 + \ldots\right\} + \left\{\epsilon_1(x) \cdot \epsilon_2(x) + \ldots + \epsilon_{M-1}(x) \cdot \epsilon_M(x)\right\}\right]$$

{All errors have 0 mean}

Eqn (4) $\rightarrow$ $E_{agg} = \frac{1}{M^2}\left\{ E\left[\left\{\epsilon_1(x)^2 + \epsilon_2(x)^2 + \ldots \epsilon_M(x)^2\right\}\right] + E\left[(\epsilon_1(x) \cdot \epsilon_2(x))\right] + \ldots E\left[\epsilon_{M-1}(x)\, \epsilon_M(x)\right]\right.$

$$= \frac{1}{M^2}\left\{ E\left[\left\{ \varepsilon_1(x)^2 + \varepsilon_2(x)^2 + \cdots + \varepsilon_M(x)^2 \right\}\right] + 0 + \cdots + 0 \right\}$$

$\left\{ E\left[\varepsilon_i(x) \cdot \varepsilon_j(x)\right] = 0 \right\}$ from 3

$$= \frac{1}{M^2} E\left[\sum_{i=1}^{M} \varepsilon_i(x)^2\right]$$

OR $$E_{agg} = \frac{1}{M^2} \sum_{i=1}^{M} E\left[\varepsilon_i(x)^2\right]$$ {Summation property of E( )}

OR $$\boxed{E_{agg} = \frac{1}{M} E_{avg}}$$ $\left\{ \text{Given: } E_{avg} = \frac{1}{M}\sum_{i=1}^{M} E\left[\varepsilon_i(x)^2\right] \right\}$

Proved.

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Using all equations and derivations from previous question.

Eqn (4) and still ignoring all single $\epsilon_i(x)$.

$$\Rightarrow E_{agg} = \frac{1}{M^2}\left\{ E\left[\{\epsilon_1(x)^2 + \dots + \epsilon_M(x)^2\}\right] + E\left[\epsilon_1(x)\cdot\epsilon_2(x)\right] + \dots + E\left[\epsilon_{M-1}(x)\cdot\epsilon_M(x)\right] \right\}$$

$$\Rightarrow E_{agg} = \frac{1}{M^2}\left\{ M E_{avg} + \sum_{i,j} E\left[\epsilon_i(x)\cdot\epsilon_j(x)\right] \right\}$$

$$\cancel{= \frac{1}{M^2}\{\le M E_{avg}\}}$$

$$\cancel{\Rightarrow \boxed{E_{agg} \le E_{avg}}}$$

~~Proved~~.

$$E_{agg} = \frac{1}{M} E_{avg} + \frac{1}{M^2}\sum_{i,j} E\left[\epsilon_i(x)\cdot\epsilon_j(x)\right]$$

$$\le \frac{1}{M} E_{avg}$$

$$\boxed{E_{agg} \le E_{avg}}$$

Proved.