Problem Set 6

CS 6347

Due: 5/5/2018 by 11:59pm

Note: all answers should be accompanied by explanations for full credit. Late homeworks cannot be accepted. All submitted code **MUST** compile/run.

Problem 1: Putting it all together (at least in theory) (100 pts)

Given a graph $G = (V, E)$, a vertex cover is a set $A \subseteq V$ such that each edge $(i, j) \in E$ has at least one vertex in the set A. To each vertex $i \in V$, associate a weight $w_i = \exp(\theta_i)$. The weight of a vertex cover is defined to be $w(A) = \prod_{i \in A} w_i$.

1. (10 pts) Construct a MRF over the graph G such that $p(A) \propto w(A)$.
2. (10 pts) Suppose that you wanted to find the minimum vertex cover on a tree, i.e., $\arg\min_{A \subset V} p(A)$. Can you do this using a message passing algorithm? Explain.
3. (20 pts) Suppose that $w_i = 1$ for all $i \in V$, and let G be a k-cycle for some $k \geq 3$. What are the fixed points of the sum-product algorithm for the graph G?
4. (10 pts) Explain how to sample from the distribution $p(\cdot)$ using Gibbs sampling. Be sure to express the updates performed by the Gibbs sampler in terms of the specific potentials in the MRF.
5. (15 pts) What is the log-likelihood for this MRF? Compute the derivative of the log-likelihood with respect to θ_i.
6. (15 pts) What is the log-pseudolikelihood for this MRF? Compute the derivative of the log-pseudolikelihood with respect to θ_i.
7. (20 pts) Let $\lambda = 100$. Suppose that we add an ℓ_2 regularizer $-\frac{\lambda}{2}||\theta||^2$ to the log-likelihood. Given the following samples of vertex covers from some unknown probability distribution over an unknown graph over 4 nodes $\{a, b, c, d\}$, find the parameters of the joint distribution that maximize the regularized log-likelihood: $\{a, d\}$, $\{a, d\}$, $\{b, c\}$. What would happen if you attempted to maximize the unregularized log-likelihood with only one sample, $\{a\}$?