Math 515-1: Derived commutative rings
Problem set 02

Definition. A symmetric monoidal structure on an ∞-category $\mathcal{C}$ is a coCartesian fibration $\mathcal{C}^{\otimes} \to \mathbf{Fin}^{\mathrm{part}}$ with an identification $\mathcal{C}^{\otimes}_{*} \simeq \mathcal{C}$ and such that for each $I \in \mathbf{Fin}^{\mathrm{part}}$ the induced functor

$$\prod_{i \in I} \rho_{i,*} \colon \mathcal{C}^{\otimes}_{I} \to \prod_{i \in I} \mathcal{C}^{\otimes}_{\{i\}}$$

is an equivalence.

1. Construct the Cartesian symmetric monoidal structure on $\mathbf{Set}$.

2. Let R be a commutative ring and let $\mathbf{Mod}_R^{\heartsuit}$ be the abelian category of static R-modules. Construct the usual symmetric monoidal structure on $\mathbf{Mod}_R^{\heartsuit}$ as a coCartesian fibration $\mathbf{Mod}_R^{\heartsuit,\otimes} \to \mathbf{Fin}^{\mathrm{part}}$.

3. Construct the Cartesian symmetric monoidal structure on $\mathbf{Cat}_{\infty}$. Show that it restricts to a symmetric monoidal structure on $\mathbf{Ani}$. Hint: attempt to use the homotopy coherent nerve construction.

4. Show that the inert and active maps in $\mathbf{Fin}^{\mathrm{part}}$ form a factorization system (see Tag 04PD of Kerodon).

Department of Mathematics, Northwestern University
antieau@northwestern.edu