Chapter 1

Formulae for analysing Y- and Z-circles

Here is the formula collection I have put together from [1–3, 5] while writing the code.

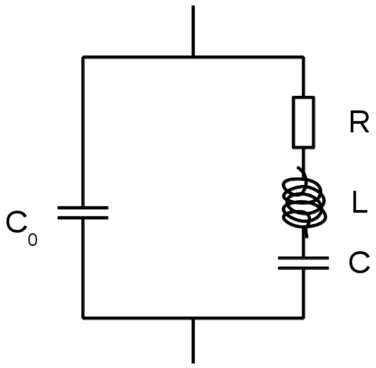

Fig. 1.1 Often viable equivalent circuit of a piezoelectric transducer.

Taking measured admittance and impedance circles as a starting point and the equivalent circuit of figure 1.1 as the model basis, a set of quantities can be calculated to characterise a given piezoelectric transducer. Table 1.2 lists several useful formulae for this sort of analysis. They have been taken from [2, 3, 5] and [1][1]. Adopting the nomenclature of [3], f_1 is the collective term for the resonance splitting up into (f_m, f_s, f_r) and f_2 denotes the antiresonance (f_a, f_p, f_n). DeAngelis & Schulze use the terms f_1 and f_2 for the top and the bottom of the admittance circle. These frequencies are labeled here as f_{mB}, the frequency of maximum susceptance B and f_{nB} the frequency of minimum B.

[1] who compiled formulae from [4, 5]

label	description
$f_1 \rightarrow (f_m, f_s, f_r)$	resonance (with $f_m \leq f_s \leq f_r$)
$f_2 \rightarrow (f_n, f_p, f_a)$	antiresonance (with $f_a \leq f_p \leq f_n$)
f_m	maximum $\lvert Y \rvert$
f_s	maximum $\mathrm{Re}(Y)$, *motional resonance, series resonance*
f_r	$\mathrm{Im}(Y) = 0$, *electrical resonance*
f_n	maximum $\lvert Z \rvert$
f_p	maximum $\mathrm{Re}(Z)$, *parallel resonance*
f_a	$\mathrm{Im}(X) = 0$, *electrical antiresonance*
order:	$f_m \leq f_s \leq f_r < f_a \leq f_p \leq f_n$
f_{mB},f_{nB},f_{mX},f_{nX}	corresponds to max B, min B, max X, min X
order:	$f_{mB} < f_s < f_{nB}$ and $f_{mX} < f_p < f_{nX}$

Table 1.1 The characteristic frequenceies of electromechanical transducers

formula	description
$k = \sqrt{1 - \frac{f_s^2}{f_p^2}} = \sqrt{\frac{C}{C_0 + C}}$	electromechanical coupling coefficient (Paraphrasing Wilson, k^2 is, in a sense, like a transduction efficiency, but more precisely, it is a characteristic of the mechanism with all dissipative processes ignored.)
$Q = Q_m = \frac{f_s}{f_{nB} - f_{mB}} = \frac{sqrt L/C}{R}$	quality factor Q or mechanical Q (higher means less losses)
$Q_e = \frac{B_s}{G_{\max}}$	electrical Q (lower means less dielectric losses, as it is a measure of the admittance circle vertical offset)
$Q_e Q_m = \frac{C_0}{C} = \frac{1-k^2}{k^2} = r$	Q-product is the same as the capacitance ratio
$R = \frac{1}{G_{\max}}$	motional resistance, represents the mechanical dissipation
$L = \frac{Q_m R}{\omega_s}$	motional inductance
$C = \frac{1}{Q_m R \omega_s}$	motional capacitance
$C_0 = \frac{f_r^2}{f_a^2 - f_r^2} C \approx \frac{B_s}{\omega_s}$	parallel capacitance, also called parasitic capacitance, it is a result of the electrode surface geometry (and sometimes cables as well), and it affects the ability of the electrical driver unit to deliver energy to the transducer
$\Gamma = C\,d/A$	motional capacitance constant (d: linear dimension $\|\|\vec{E}$, A: electrode area)
$B_s = \omega_s C_0$	susceptance of admittance circle centre (susceptance at the series resonance f_s)
$r = \frac{C_0}{C}$	capacitance ratio
$M = \frac{Q_m}{r} = \frac{1}{\omega_s C_0 R} = \frac{k^2 Q_m}{1-k^2}$	figure of merit
$\delta = \omega C_0 R$	normalised damping factor
$\Omega = \frac{f^2 - f_s^2}{f_p^2 - f_s^2}$	normalised frequency factor
$Z = \frac{1}{Y} = \frac{i}{\omega C_0} \frac{\Omega - i\delta}{1 - \Omega + i\delta}$	equation for impedance $Z(\omega)$ or admittance $Y(\omega)$
$Q = \frac{f_{\text{peak}}}{f_{\text{hdp2}} - f_{\text{hdp1}}}$	generic Q-factor of a resonant system where $f_{\text{hdp2}} - f_{\text{hdp1}}$ gives the peak width at half dissipation power which corresponds to a reduction by a factor $1/\sqrt{2}$ for many signal amplitudes

Table 1.2 Useful formulae for transducer analysis

Bibliography

[1] D.A. DeAngelis and G.W. Schulze. “Optimizing piezoelectric ceramic thickness in ultrasonic transducers”. In: *Ultrasonic Industry Association Symposium (UIA), 2010 39th Annual*. 2010, pp. 1–9.

[2] “IEEE Standard Definitions and Methods of Measurement for Piezoelectric Vibrators”. In: *IEEE Std No.177* (1966), pp. 1–19.

[3] “IEEE Standard on Piezoelectricity”. In: *ANSI/IEEE Std 176-1987* (1988), 0_1–.

[4] D. Stansfield. *Underwater electroacoustic transducers a handbook for users and designers*. English. Bath; St. Albans: Bath University Press ; Institute of Acoustics, 1991.

[5] Oscar Bryan Wilson. *Introduction to theory and design of sonar transducers*. en. Los Altos, CA: Peninsula, 1988.