Izvod

Sustav raspolaze s m pravila:

Ako x je A_1 i y je B_1 tada $z_1 = p_1x + q_1y + r_1$

Ako x je A_2 i y je B_2 tada $z_2 = p_2x + q_2y + r_2$

...

Ako x je A_m i y je B_m tada $z_m = p_mx + q_my + r_m$

Prijenosna funkcija koju sustav koristi definirane su izrazima:

$$\mu_{A_i}(x) = \frac{1}{1 + e^{b_i(x-a_i)}}$$

$$\mu_{B_i}(x) = \frac{1}{1 + e^{c_i(y-d_i)}}$$

Opcenito greska je definirana izrazom:

$$E_k = \frac{1}{2}(y_k - o_k)^2$$

Azuriranje parametra gradijentnim spustom:

$$\psi(t+1) = \psi(t) - \eta\frac{\delta E_k}{\delta \psi}$$

Cilj je utvrditi parcijalne derivacije po svim parametrima $a_i, b_i, c_i, d_i, p_i, q_i, r_i$
Izvod pravila za azuriranje parametara z_i:
Izlaz sustava neizrazitog upravljanja definiran je kao tezinska suma:

$$o = \frac{\sum_{j=1}^{m} \gamma_j z_j}{\sum_{j=1}^{m} \gamma_j}$$

Pri cemu je jakost paljenja i-tog pravila i u promatranom slucaju ona je jednaka

$$\gamma_i := T(A_i(x), B_i(x)) = A_i(x)B_i(x)$$

te su definirani

$$\alpha_i := A_i(x), \beta_i := B_i(x) \rightarrow \gamma_i = \alpha_i\beta_i$$

Za potrebe izracuna parcijalne derivacije funkcije pogreske po parametrima p_i, q_i i r_i posluziti cemo se pravilom ulancavanja:

$$\frac{\partial E_k}{\partial p_i} = \frac{\partial E_k}{\partial o_k}\frac{\partial o_k}{\partial z_i}\frac{\partial z_i}{\partial p_i}$$

$$\frac{\partial E_k}{\partial q_i} = \frac{\partial E_k}{\partial o_k}\frac{\partial o_k}{\partial z_i}\frac{\partial z_i}{\partial q_i}$$

$$\frac{\partial E_k}{\partial r_i} = \frac{\partial E_k}{\partial o_k}\frac{\partial o_k}{\partial z_i}\frac{\partial z_i}{\partial r_i}$$

Pri cemu su:

$$\frac{\delta E_k}{\partial o_k} = \frac{\partial}{\partial o_k}\left(\frac{1}{2}(y_k - o_k)^2\right) = -(y_k - o_k)$$

$$\frac{\partial o_k}{\partial z_i} = \frac{\partial}{\partial z_j}\left(\frac{\sum_{j=1}^{m}\gamma_j z_i}{\sum_{j=1}^{m}\gamma_j}\right) = \frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}$$

I:

$$\frac{\partial z_i}{\partial p_i} = \frac{\partial}{\partial p_i}(p_i x + q_i y + r_i) = x$$

$$\frac{\partial z_i}{\partial q_i} = \frac{\partial}{\partial q_i}(p_i x + q_i y + r_i) = y$$

$$\frac{\partial z_i}{\partial r_i} = \frac{\partial}{\partial r_i}(p_i x + q_i y + r_i) = 1$$

Pa su:

$$\frac{\partial E_k}{\partial p_i} = -(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}x$$

$$\frac{\partial E_k}{\partial q_i} = -(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}y$$

$$\frac{\partial E_k}{\partial r_i} = -(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}$$

Za izvod ostalih parametara trebati ce nam slijedece derivacije:

$$\frac{\partial o_k}{\partial \gamma_i} = \frac{\partial}{\partial \gamma_i}\left(\frac{\sum_{j=1}^{m}\gamma_j z_j}{\sum_{j=1}^{m}\gamma_j}\right) =$$

$$= \frac{\frac{\partial}{\partial \gamma_i}\left(\sum_{j=1}^{m}\gamma_j z_j\right)\left(\sum_{j=1}^{m}\gamma_j\right) - \frac{\partial}{\partial \gamma_i}\left(\sum_{j=1}^{m}\gamma_j\right)\left(\sum_{j=1}^{m}\gamma_j z_j\right)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}$$

$$= \frac{\sum_{j=1, j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}$$

$$\frac{\partial \gamma_i}{\partial \alpha_i} = \frac{\partial}{\partial \alpha_i}\left(\alpha_i\beta_i\right) = \beta_i$$

$$\frac{\partial \gamma_i}{\partial \beta_i} = \frac{\partial}{\partial \beta_i}\left(\alpha_i\beta_i\right) = \alpha_i$$

Izvod derivacija za parametre a_i i b_i:

$$\frac{\partial E_k}{\partial a_i} = \frac{\partial E_k}{\partial o_k}\frac{\partial o_k}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial a_i}$$

$$\frac{\partial E_k}{\partial b_i} = \frac{\partial E_k}{\partial o_k}\frac{\partial o_k}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial b_i}$$

Potrebno je jos izracunati slijedece:

$$\frac{\partial \alpha_i}{\partial a_i} = \alpha_i(1-\alpha_i)b_i$$

$$\frac{\partial \alpha_i}{\partial b_i} = \alpha_i(1-\alpha_i)(x-a_i)$$

Pomocu gore navedenih izraza dobivamo:

$$\frac{\partial E_k}{\partial a_i} = -(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\beta_i\alpha_i(1-\alpha_i)b_i$$

$$\frac{\partial E_k}{\partial b_i} = -(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\beta_i\alpha_i(1-\alpha_i)(x-a_i)$$

Izvod derivacija za parametre c_i i d_i analogan je izvodu za parametre a_i i b_i

$$\frac{\partial E_k}{\partial c_i} = -(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\alpha_i\beta_i(1-\beta_i)d_i$$

$$\frac{\partial E_k}{\partial d_i} = -(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\alpha_i\beta_i(1-\beta_i)(y-c_i)$$

Iz cega dobivamo izraze za azuriranje parametara:

$$a_i(t+1) = a_i(t) + \eta(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\beta_i\alpha_i(1-\alpha_i)b_i$$

$$b_i(t+1) = b_i(t) + \eta(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\beta_i\alpha_i(1-\alpha_i)(x-a_i)$$

$$c_i(t+1) = c_i(t) + \eta(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\alpha_i\beta_i(1-\beta_i)d_i$$

$$d_i(t+1) = d_i(t) + \eta(y_k - o_k)\frac{\sum_{j=1,j\neq i}^{m}\gamma_j(z_i - z_j)}{\left(\sum_{j=1}^{m}\gamma_j\right)^2}\alpha_i\beta_i(1-\beta_i)(y-c_i)$$

$$p_i(t+1) = p_i(t) + \eta(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}x$$

$$q_i(t+1) = q_i(t) + \eta(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}y$$

$$r_i(t+1) = r_i(t) + \eta(y_k - o_k)\frac{\gamma_i}{\sum_{j=1}^{m}\gamma_j}$$