$au \in G = T D_4$

$$[\pi(au) f](x) = f(u^{-1} a^{-1} x u)$$

$$[\pi(au)\pi(bv) f](x) = [\pi(bv) f](u^{-1}a^{-1} x u)$$

$$= f(v^{-1} b^{-1} u^{-1} a^{-1} x u v)$$

$$= f((uv)^{-1} u b^{-1} u^{-1} a^{-1} \times uv)$$

$$= [\pi(aubv) f](x)$$

$$[\pi(au) g](xw) = g(u^{-1} a^{-1} x u \;\; u^{-1} w)$$

$$[L f](yw) = \int_T dx \quad K(yw, x) f(x)$$

$$[L[\pi(au) f]](yw) = \int_T dx \quad K(yw, x) f(u^{-1} a^{-1} x u)$$

$$= \int_T dx \;\; K(yw, auxu^{-1}) f(x)$$

$$[\pi(au)[L f]](yw) = \int_T dx \;\; K(u^{-1}a^{-1} yw, x) f(x)$$

$$\Rightarrow K(yw, x) = K(u^{-1}a^{-1} yw, u^{-1}a^{-1} x u)$$

$$= K(e, w^{-1} y^{-1} x w)$$

$a = y$
$u = w$

$$[Lf](yw) = \int_T dx \; K(w^{-1}y^{-1}xw)\, f(x)$$

$$[Lf](yw) = \int_T dx \sum_v K(w^{-1}y^{-1}xw\, w^{-1}v)\, f(xv)$$

$$\boxed{[Lf](y) = \int_T dx \sum_v K(v^{-1}y^{-1}x\, v)\, f(xv)}$$

$$[L\,\pi(a)f](y) = \int_T dx \sum_v K(v^{-1}y^{-1}x\, v)\, f(u^{-1}a^{-1}x\, u\, u^{-1}v)$$

$$= \int_T dx \sum_v K(v^{-1}(u^{-1}y^{-1}\, a\, u)x\, v)\, f(x\, v)$$

$$= [Lf](u^{-1}a^{-1}y\, u) = \pi(a)[Lf](y)$$