Thinfilm

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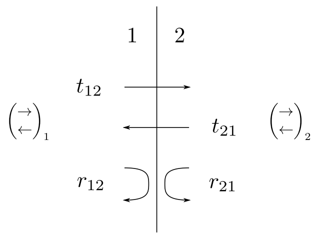
1 Matrice de transfert

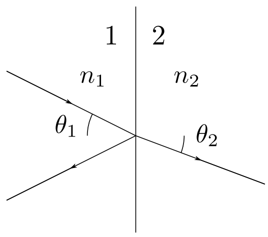


$\rightarrow_1$ est la phase et amplitude de l'onde incidente depuis la gauche. $\leftarrow_1$ est la phase et amplitude de l'onde réfléchie vers la gauche. $\rightarrow_2$ est celle de l'onde transmise et finalement $\leftarrow_2$ est nul.

$$\begin{cases} \rightarrow_2 = t_{12} \rightarrow_1 + r_{21} \leftarrow_2 & \Rightarrow \quad \rightarrow_1 = \frac{\rightarrow_2 - r_{21} \leftarrow_2}{t_{12}} \\ \leftarrow_1 = t_{21} \leftarrow_2 + r_{12} \rightarrow_1 & \Rightarrow \quad \leftarrow_1 = t_{21} \leftarrow_2 + r_{12} \frac{\rightarrow_2 - r_{21} \leftarrow_2}{t_{12}} \end{cases} \tag{1}$$

$$\begin{pmatrix} \rightarrow \\ \leftarrow \end{pmatrix}_1 = \frac{1}{t_{12}} \begin{pmatrix} 1 & -r_{21} \\ r_{12} & t_{12} t_{21} - r_{12} r_{21} \end{pmatrix} \begin{pmatrix} \rightarrow \\ \leftarrow \end{pmatrix}_2 \tag{2}$$

2 Fresnel



Polarisation perpendiculaire

$$\begin{cases} r_s = r_{12} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \\ t_s = t_{12} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{2\eta_1}{\eta_1 + \eta_2} \end{cases} \tag{3}$$

Avec $\eta = n \cos\theta$.

Polarisation parallèle

$$\begin{cases} r_p = r_{12} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} \\ t_p = t_{12} = \frac{2 n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{2 n_1 / \cos\theta_2}{\eta_1 + \eta_2} = \frac{2 \frac{n_1}{n_2} \eta_2}{\eta_1 + \eta_2} \end{cases} \tag{4}$$

Avec $\eta = n / \cos\theta$.

3 Matrice de transfert de Fresnel

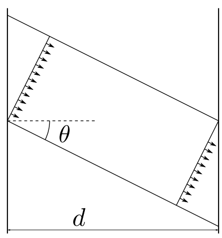
Polarisation perpendiculaire

$$\begin{aligned}
\frac{1}{t_{12}}\begin{pmatrix}1 & -r_{21}\\ r_{12} & t_{12}t_{21}-r_{12}r_{21}\end{pmatrix} &= \frac{1}{2\eta_1}\begin{pmatrix}\eta_1+\eta_2 & \eta_1-\eta_2\\ \eta_1-\eta_2 & \frac{2\eta_1 2\eta_2}{\eta_1+\eta_2}-\frac{(\eta_1-\eta_2)(\eta_2-\eta_1)}{\eta_1+\eta_2}\end{pmatrix}\\
&= \frac{1}{2\eta_1}\begin{pmatrix}\eta_1+\eta_2 & \eta_1-\eta_2\\ \eta_1-\eta_2 & \eta_1+\eta_2\end{pmatrix}\\
&= \frac{1}{2\eta_1}\begin{pmatrix}\eta_1 & 1\\ \eta_1 & -1\end{pmatrix}\begin{pmatrix}1 & 1\\ \eta_2 & -\eta_2\end{pmatrix}
\end{aligned} \tag{5}$$

Polarisation parallèle

$$\begin{aligned}
\frac{1}{t_{12}}\begin{pmatrix}1 & -r_{21}\\ r_{12} & t_{12}t_{21}-r_{12}r_{21}\end{pmatrix} &= \frac{1}{2\frac{n_1}{n_2}\eta_2}\begin{pmatrix}\eta_1+\eta_2 & \eta_1-\eta_2\\ \eta_1-\eta_2 & \frac{2\frac{n_1}{n_2}\eta_2 2\frac{n_2}{n_1}\eta_1}{\eta_1+\eta_2}-\frac{(\eta_1-\eta_2)(\eta_2-\eta_1)}{\eta_1+\eta_2}\end{pmatrix}\\
&= \frac{1}{2\frac{n_1}{n_2}\eta_2}\begin{pmatrix}\eta_1+\eta_2 & \eta_1-\eta_2\\ \eta_1-\eta_2 & \eta_1+\eta_2\end{pmatrix}\\
&= \frac{1}{2n_1}\begin{pmatrix}\eta_1 & 1\\ \eta_1 & -1\end{pmatrix}\frac{n_2}{\eta_2}\begin{pmatrix}1 & 1\\ \eta_2 & -\eta_2\end{pmatrix}
\end{aligned} \tag{6}$$

4 Matrice de transfert de propagation

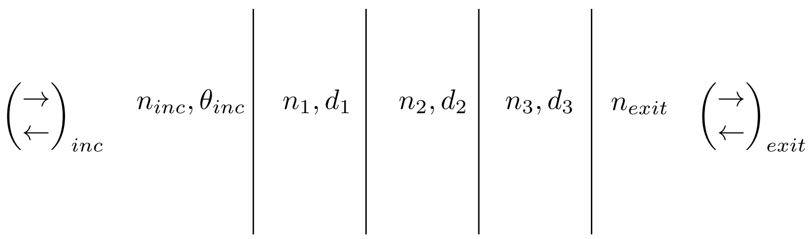


On a $t_{12} = t_{21} = \psi$ et aucune réflexion.

$$\frac{1}{t_{12}}\begin{pmatrix}1 & -r_{21}\\ r_{12} & t_{12}t_{21}-r_{12}r_{21}\end{pmatrix} = \begin{pmatrix}\psi^{-1} & 0\\ 0 & \psi\end{pmatrix} \tag{7}$$

$\psi = \exp(ikd\cos\theta) = \exp(ink_0 d\cos\theta) = \exp(in\frac{2\pi}{\lambda_0}d\cos\theta)$ où k_0 est le nombre d'onde dans le vide et λ_0 est la longueur d'onde dans le vide.

5 Multi couches



Polarisation perpendiculaire

$$\begin{aligned}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{inc} &= \frac{1}{2\eta_{inc}}\begin{pmatrix}\eta_{inc} & 1\\ \eta_{inc} & -1\end{pmatrix}\overbrace{\begin{pmatrix}1&1\\ \eta_1 & -\eta_1\end{pmatrix}\begin{pmatrix}\psi_1^{-1}&0\\0&\psi_1\end{pmatrix}\frac{1}{2\eta_1}\begin{pmatrix}\eta_1&1\\ \eta_1&-1\end{pmatrix}}^{L_1}\begin{pmatrix}1&1\\ \eta_2&-\eta_2\end{pmatrix}\begin{pmatrix}\psi_2^{-1}&0\\0&\psi_2\end{pmatrix}\\
&\frac{1}{2\eta_2}\begin{pmatrix}\eta_2&1\\ \eta_2&-1\end{pmatrix}\overbrace{\begin{pmatrix}1&1\\ \eta_3&-\eta_3\end{pmatrix}\begin{pmatrix}\psi_3^{-1}&0\\0&\psi_3\end{pmatrix}\frac{1}{2\eta_3}\begin{pmatrix}\eta_3&1\\ \eta_3&-1\end{pmatrix}}^{L_3}\begin{pmatrix}1&1\\ \eta_{exit}&-\eta_{exit}\end{pmatrix}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit}\\
&= \frac{1}{2\eta_{inc}}\begin{pmatrix}\eta_{inc}&1\\ \eta_{inc}&-1\end{pmatrix}L_1L_2L_3\begin{pmatrix}1&1\\ \eta_{exit}&-\eta_{exit}\end{pmatrix}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit}\end{aligned} \tag{8}$$

$$\begin{aligned}L &= \begin{pmatrix}1&1\\ \eta&-\eta\end{pmatrix}\begin{pmatrix}\psi^{-1}&0\\0&\psi\end{pmatrix}\frac{1}{2\eta}\begin{pmatrix}\eta&1\\ \eta&-1\end{pmatrix}\\
&= \begin{pmatrix}\frac{\psi+\psi^{-1}}{2} & -\frac{\psi-\psi^{-1}}{2\eta}\\ -\eta\frac{\psi-\psi^{-1}}{2} & \frac{\psi+\psi^{-1}}{2}\end{pmatrix}\\
&= \begin{pmatrix}\cos\delta & -\frac{i\sin\delta}{\eta}\\ -\eta i\sin\delta & \cos\delta\end{pmatrix}\end{aligned} \tag{9}$$

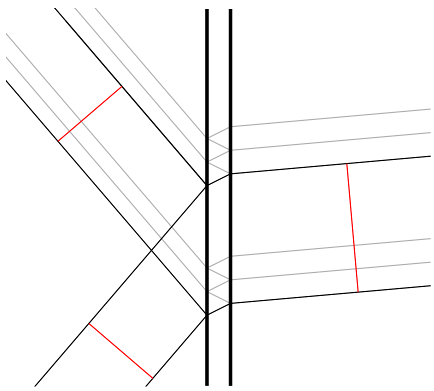
où $\delta = n\frac{2\pi}{\lambda_0}d\cos\theta$

Polarisation parallèle

$$\begin{aligned}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{inc} &= \frac{1}{2n_{inc}}\begin{pmatrix}\eta_{inc} & 1\\ \eta_{inc} & -1\end{pmatrix}\overbrace{\frac{n_1}{\eta_1}\begin{pmatrix}1&1\\ \eta_1 & -\eta_1\end{pmatrix}\begin{pmatrix}\psi_1^{-1}&0\\0&\psi_1\end{pmatrix}\frac{1}{2n_1}\begin{pmatrix}\eta_1&1\\ \eta_1&-1\end{pmatrix}}^{L_1}\frac{n_2}{\eta_2}\begin{pmatrix}1&1\\ \eta_2&-\eta_2\end{pmatrix}\begin{pmatrix}\psi_2^{-1}&0\\0&\psi_2\end{pmatrix}\\
&\frac{1}{2n_2}\begin{pmatrix}\eta_2&1\\ \eta_2&-1\end{pmatrix}\overbrace{\frac{n_3}{\eta_3}\begin{pmatrix}1&1\\ \eta_3&-\eta_3\end{pmatrix}\begin{pmatrix}\psi_3^{-1}&0\\0&\psi_3\end{pmatrix}\frac{1}{2n_3}\begin{pmatrix}\eta_3&1\\ \eta_3&-1\end{pmatrix}}^{L_3}\frac{n_{exit}}{\eta_{exit}}\begin{pmatrix}1&1\\ \eta_{exit}&-\eta_{exit}\end{pmatrix}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit}\\
&= \frac{1}{2n_{inc}}\begin{pmatrix}\eta_{inc}&1\\ \eta_{inc}&-1\end{pmatrix}L_1L_2L_3\frac{n_{exit}}{\eta_{exit}}\begin{pmatrix}1&1\\ \eta_{exit}&-\eta_{exit}\end{pmatrix}\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit}\end{aligned} \tag{10}$$

Les matrices L ont la même forme que dans le cas perpendiculaire.

6 Coefficients de transmition et reflexion



$$\vec{E} = \vec{e}\exp(i(\vec{k}\cdot\vec{x}-\omega t)) + \text{c.c.} \tag{11}$$

$$\vec{H} = \frac{1}{\omega}\vec{k}\wedge\vec{E} = \frac{n}{c}\frac{\vec{k}}{k}\wedge\vec{E} \tag{12}$$

$$\vec{S} = \vec{E} \wedge \vec{H} = \frac{n}{c}\vec{E} \wedge (\frac{\vec{k}}{k} \wedge \vec{E}) = \frac{n}{c}\vec{E}^2\frac{\vec{k}}{k} \tag{13}$$

La surface du front d'onde incident est déformé après la transmission. Sa surface peut changer :

$$\frac{S_{exit}}{S_{inc}} = \frac{\cos\theta_{exit}}{\cos\theta_{inc}} \tag{14}$$

On relie alors les coefficients r et t à la réflectance et transmittance : R et T (proportionnelles à $\vec{S}$) par :

$$R = \frac{S_{inc}\frac{n_{inc}}{c}\vec{E}_r^2}{S_{inc}\frac{n_{inc}}{c}\vec{E}_i^2} = \frac{S_{inc}\frac{n_{inc}}{c}|r|^2\vec{E}_i^2}{S_{inc}\frac{n_{inc}}{c}\vec{E}_i^2} = |r|^2 \tag{15}$$

$$T = \frac{S_{exit}\frac{n_{exit}}{c}\vec{E}_t^2}{S_{inc}\frac{n_{inc}}{c}\vec{E}_i^2} = \frac{\cos\theta_{exit}}{\cos\theta_{inc}}\frac{n_{exit}}{n_{inc}}|t|^2 \tag{16}$$

Polarisation perpendiculaire Si on prend $\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit} = \begin{pmatrix}t\\0\end{pmatrix}$ et $\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{inc} = \begin{pmatrix}1\\r\end{pmatrix}$ on a alors :

$$\begin{pmatrix}1/t\\r/t\end{pmatrix} = \frac{1}{2\eta_{inc}}\begin{pmatrix}\eta_{inc} & 1\\ \eta_{inc} & -1\end{pmatrix} L_1L_2L_3\begin{pmatrix}1\\ \eta_{exit}\end{pmatrix} \tag{17}$$

Si on définit

$$\begin{pmatrix}b\\c\end{pmatrix} = L_1L_2L_3\begin{pmatrix}1\\ \eta_{exit}\end{pmatrix} \tag{18}$$

$$\begin{aligned}\begin{pmatrix}1/t\\r/t\end{pmatrix} &= \begin{pmatrix}\frac{b+c/\eta_{inc}}{2}\\ \frac{b-c/\eta_{inc}}{2}\end{pmatrix}\\ \Rightarrow &\begin{cases} t = \frac{2}{b+c/\eta_{inc}}\\ r = \frac{b-c/\eta_{inc}}{2}\frac{2}{b+c/\eta_{inc}} = \frac{b-c/\eta_{inc}}{b+c/\eta_{inc}}\end{cases} \quad \Rightarrow \quad T = \frac{\eta_{exit}}{\eta_{inc}}\left|\frac{2}{b+c/\eta_{inc}}\right|^2\end{aligned} \tag{19}$$

Polarisation parallèle Si on prend $\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{exit} = \begin{pmatrix}t\\0\end{pmatrix}$ et $\begin{pmatrix}\rightarrow\\ \leftarrow\end{pmatrix}_{inc} = \begin{pmatrix}1\\r\end{pmatrix}$ on obtient :

$$\begin{pmatrix}1/t\\r/t\end{pmatrix} = \frac{1}{2n_{inc}}\begin{pmatrix}\eta_{inc} & 1\\ \eta_{inc} & -1\end{pmatrix} L_1L_2L_3\frac{n_{exit}}{\eta_{exit}}\begin{pmatrix}1\\ \eta_{exit}\end{pmatrix} \tag{20}$$

On peut écrire différemment le terme suivant :

$$\begin{aligned}\frac{1}{n_{inc}}\frac{n_{exit}}{\eta_{exit}} &= \frac{1}{\eta_{inc}\cos\theta_{inc}}\frac{n_{exit}}{n_{exit}/\cos\theta_{exit}}\\ &= \frac{1}{\eta_{inc}}\frac{\cos\theta_{exit}}{\cos\theta_{inc}}\end{aligned} \tag{21}$$

Pour avoir au final :

$$\begin{pmatrix}1/t\\r/t\end{pmatrix} = \frac{\cos\theta_{exit}}{\cos\theta_{inc}}\frac{1}{2\eta_{inc}}\begin{pmatrix}\eta_{inc} & 1\\ \eta_{inc} & -1\end{pmatrix} L_1L_2L_3\begin{pmatrix}1\\ \eta_{exit}\end{pmatrix} \tag{22}$$

$$\begin{aligned}\begin{pmatrix}1/t\\r/t\end{pmatrix} &= \frac{\cos\theta_{exit}}{\cos\theta_{inc}}\begin{pmatrix}\frac{b+c/\eta_{inc}}{2}\\ \frac{b-c/\eta_{inc}}{2}\end{pmatrix}\\ \Rightarrow &\begin{cases} t = \frac{2}{b+c/\eta_{inc}}\frac{\cos\theta_{inc}}{\cos\theta_{exit}}\\ r = \frac{b-c/\eta_{inc}}{b+c/\eta_{inc}}\end{cases} \quad \Rightarrow \quad T = \frac{\eta_{exit}}{\eta_{inc}}\left|\frac{2}{b+c/\eta_{inc}}\right|^2\end{aligned} \tag{23}$$