# PhaseShifts.c explained

Lucas M. Rios, Lucas N. Abrantes, Yuri R. Tonin

June 30, 2016

## 1 Introduction

As the matter wave propagates, the particles experience a variety of interactions with their surroundings, some of which can produce phase shifts in the interference pattern at the third grating of the Mach-Zender interferometer. These interactions are modeled in the file PhaseShifts.c, and this document explains the implementation of such phase-shifting physical effects into the simulation code.

The general theoretical references are McMorran and Cronin (2008) and McMorran (2005).

## 2 Tilt

To calculate the phase shifts it's necessary to have a slit well represented, i. e., we need its dimensions known given that there might be twists and tilts in the gratings in case they are not perfectly aligned. Thus we need to know how those effects could affect our measurements.

Let's consider a tilt, that is, the planes that define the gratings are inclined in respect to the x-y plane in the x direction (vertical). Thereby as the beam propagates the "actual" slit that the wave finds in its way may be different because the some particles could encounter the wall of the tunnel inside the slit.

In figure 1 the trapezoid is the slit through which the beam propagates. In that case the grating is aligned with the x-axis (no tilt, horizontal line as a reference), thus the beams that arrive at the entrance are limited by the line $\overline{AB}$. The $\alpha$ is the wedge angle, it's more likely that in our simulation it'll equal to zero, however we want it to be general enough for different simulations.

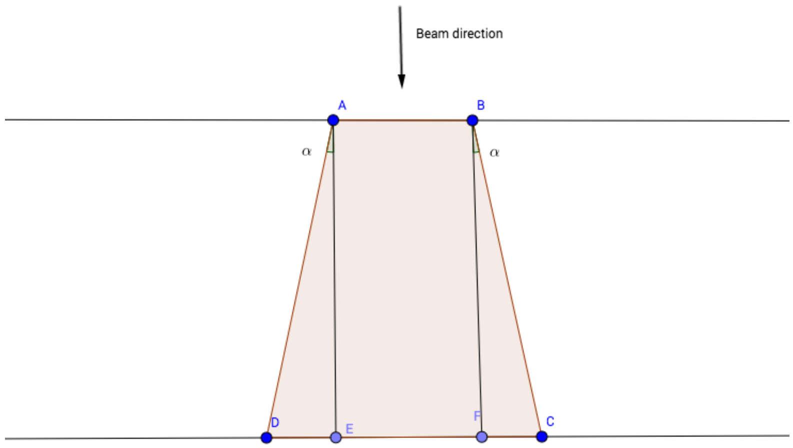


**Figure 1:** The trapezoid represents the path inside the slit. In this case there's no tilt.

Now if we introduce a tilt $\beta$ that is less or equal to $\alpha$, the new "tunnel" that the beam can travel through is still limited by the corner $A$ and $B$, see figure 2, but the width of the entrance is shorter because it's projected in the x direction it's the same length as the line $\overline{IJ}$. That is, the beam travels within the range between the two lines $\overline{AI}$ and $\overline{BJ}$. Then, if the width of the slit is $w$ ($\overline{AB}$ and $\overline{EF}$), the x positions will be in the range of the projected line equal to $w \cos\beta$ ($\overline{IJ}$).

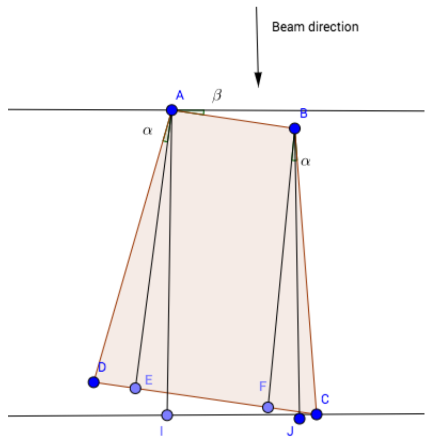


**Figure 2:** Tilt less or equal to wedge angle

If $\beta > \alpha$, then the beam travels in a space limited by the corners $A$ and $B$ (figure 3), i. e., it propagates diagonally-like in the trapezoid.

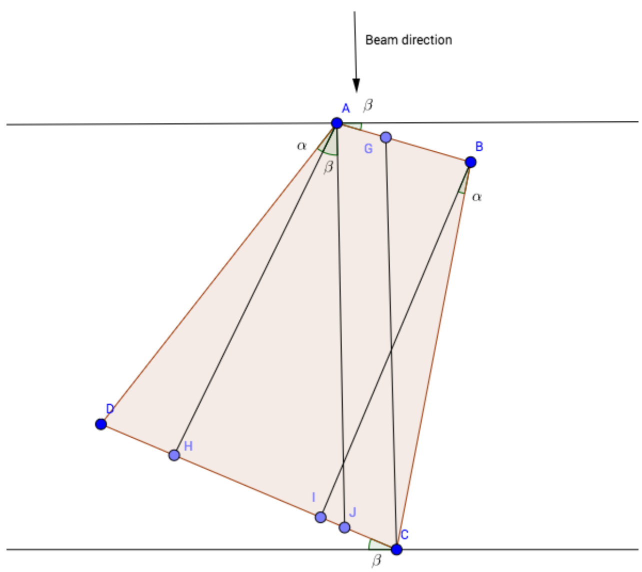


**Figure 3:** Tilt greater than wedge angle

Therefore, in this case the line that we must project is the $\overline{JC}$. Then we're set with the task to find out its length. The length of the slit is $l$ ($\overline{AH}$ and $\overline{BI}$). Thus we're able to find $\overline{HJ}$ because we know a angle and a side of the triangle

$\overline{AHJ}$.

$$\overline{HJ} = l \tan\beta \tag{1}$$

For the same reasons we have:

$$\overline{IC} = l \tan\alpha \tag{2}$$

Thereby, we can say that:

$$\overline{JC} = w + l(\tan\alpha - \tan\beta) \tag{3}$$

Then if we use the a coordinate system where $A$ position is $-\frac{w}{2}\cos\beta$, the range of x positions are:

$$x_{min} = -\frac{w}{2}\cos\beta \tag{4}$$

$$x_{max} = \begin{cases} \frac{w}{2}\cos\beta, & \beta \leqslant \alpha \\ \frac{w}{2}\cos\beta + l(\tan\alpha - \tan\beta)\cos\beta, & \beta > \alpha \end{cases} \tag{5}$$

This is where our calculation disagrees with our reference and we're unsure if we've made a mistake. In McMorran (2005) equation 8 there is no $\cos\beta$ multiplying the $l$ factor, unlike our equation 5 second case. It could be some sort of approximation.

# 3 Gravity

Kaplan et al. (2016) states that the muonium beam gravitational phase shift is given by

$$\phi_{grav} = \frac{2\pi\, g\, t^2}{d} \tag{6}$$

where $g$ is the gravitational acceleration, $t$ is the time spent in free fall and $d$ is the period of the gratings. A derivation of this formula can be found in M. K. Oberthaler (2002).

The above expression is implemented verbatim in the code.

# 4 Van der Waals

Although muonium atoms are neutral, they might experience a sudden momentary polarization that can create attractive interactions between neutral matter as explained in Perreault and Cronin, which discuss phase shift in matter wave due to atom-surface interaction.

The phase shift is given by:

$$\phi_{vdw} = -\frac{C_3 l}{\hbar v r^3} \tag{7}$$

Where $C_3$ is a coefficient that describes the strength of the interaction (we're assuming that the muonium and hydrogen are similar), $l$ is is the interaction length (in the code it's the grating thickness, but we're not sure if it should be), $\hbar$ is the reduced Planck's constant, $v$ is the particle velocity and $r$ is the distance to the surface (since the slit has two surfaces it's necessary to consider the distance from each one).