PH 521: Midterm, Wednesday March 21st, 6pm to 8pm

Instructions: Please show your work. Use of any electronic device including cell phones and calculators, is prohibited. You will have 2 hours to complete this exam. This exam will count towards 20% of your final grade.

1. (30 Points) Memorized equations
   (a) Write down an expression for the complex q parameter in terms of the wavefront curvature R, the beam radius w.
   (b) Write down an expression for the Rayleigh range, z_R, in terms of the beam waist at the focus, w_0.
   (c) Write down an expression for the beam waist, $w(z)$, as a function of w_0, and z_R.
   (d) Write down an expression for the full divergence angle, Θ, as a function of w_0.
   (e) What is the ABCD matrix for propagation through a lens?
   (f) What is the ABCD matrix for propagation through free space?
   (g) If intracavity losses are small, write an approximate expression for the Finesse in terms of the total round-trip cavity loss.
   (h) If intracavity losses are small, write an approximate expression for the on-resonance buildup in terms of the total round-trip cavity loss, and input coupler transmission.
   (i) Please write down an expression which relates the cavity resonance width and the cavity free spectral range.
   (j) How is the Q parameter related to the cavity resonance width?
   (k) Provided that the linewidth of a transition is given approximately by the natural linewidth, what is the cross section for laser excitation?
   (l) How is the gain parameter related to the small signal gain parameter and the saturation intensity, I_s?

2. (20 Points) Consider the lens shown in Fig. 1. Assume that a Gaussian beam is focused at point A with waist w_0. Please use Gaussian beam propagation to find the waist size at point B. You may assume that the waist size is very small ($w_0^2 n/\lambda << 1$ meter) and make appropriate approximations.

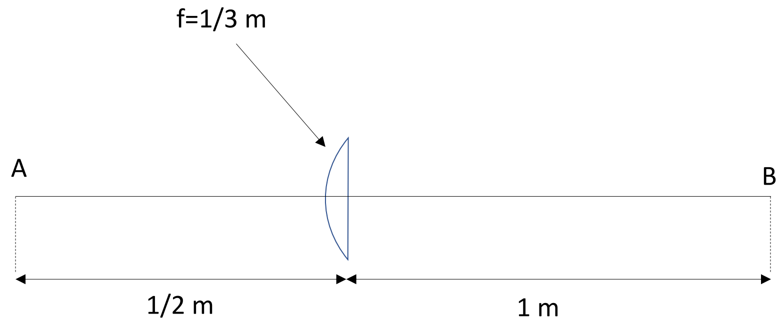

Figure 1: optics arrangement for Problem 2

3. (10 Points) Schrödinger's equation for a two-level atom yields the following two coupled differential equations for the amplitudes in state 1 and state 2:

$$i\dot{c}_1 = \Omega \cos(\vec{k}\cdot\vec{r} - \omega t) e^{-i\omega_0 t} c_2 \tag{1}$$

$$i\dot{c}_2 = \Omega \cos(\vec{k}\cdot\vec{r} - \omega t) e^{i\omega_0 t} c_1. \tag{2}$$

   Please apply the dipole approximation and the rotating wave approximation to simplify these equations, but do not attempt to solve.

4. (10 Points) To obtain inversion of a gain medium, the rate at which population is put into an excited state has to compete with the rate at which population spontaneously decays out of that state. A general rule of thumb is that the power required to build a laser at wavelength λ scales as roughly λ^{-4}. Please explain why.

5. (10 Points) Please explain qualitatively why 4-level lasers are desirable as compared with 3-level lasers.

6. (20 Points) Consider an optical cavity with mirrors spaced by 10 cm. Each mirror of the cavity has transmission losses of 3 ppm at a wavelength of 600 nm. 100 μW of optical power is coupled to the cavity (assume perfect transverse modematching). Please find

   (a) The cavity finesse.

   (b) The resonance width of the cavity

   (c) The cavity Q parameter

   (d) The intracavity power on resonance

   (e) The transmitted power through the cavity on resonance