Lasers: Assignment 4

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Problem 1 (Nagourney 3.1)

Problem Statement

A cavity is excited by a 500 nm *mode-matched laser and has resonances whose separation is* 125 MHz *and whose width is* 2.5 MHz*. Determine the following:*

(a) The cavity length ℓ;

(b) The cavity finesse $\mathscr{F}$;

(c) The Q-factor;

(d) The photon lifetime t_s.

$\boxed{\textbf{(a)}}$ Let $\not{\ell}$ denote the free spectral range. If ℓ is the round-trip cavity length (as opposed to the physical separation between the mirrors), then $\not{\ell} = c/\ell$, so

$$\begin{aligned}\ell &= \frac{c}{\not{\ell}} \\ &= \frac{3 \times 10^8\,\mathrm{m/s}}{125\,\mathrm{MHz}} \\ &\approx 2.4\,\mathrm{m}.\end{aligned}$$

$\boxed{\textbf{(b)}}$ The finesse is given by

$$\mathscr{F} = \frac{\not{\ell}}{\Delta\nu_{1/2}} = \frac{125\,\mathrm{MHz}}{2.5\,\mathrm{MHz}} \approx 50.$$

$\boxed{\textbf{(c)}}$ By definition,

$$Q = \frac{\nu}{\Delta\nu_{1/2}} = \frac{(3 \times 10^8\,\mathrm{m/s})/(500\,\mathrm{nm})}{2.5\,\mathrm{MHz}} \approx 2.4 \times 10^8.$$

$\boxed{\textbf{(d)}}$ The lifetime is

$$t_c = \frac{1}{2\pi\Delta\nu_{1/2}} = \frac{1}{2\pi(2.5\,\mathrm{MHz})} \approx 64\,\mathrm{ns}.$$

Problem 2 (Nagourney 3.2)

Problem Statement

Calculate the transmission of the electric field and intensity for a cavity having mirrors with field reflectivity (resp. transmissivity) r_1, r_2 (t_1, t_2), separation d, and containing a medium with transmission t.

As in the text, let $r_m \coloneqq r_2 t^2$, and let δ denote the round-trip phase shift through the cavity. The "one-pass" contribution to the electric field just outside the output coupler (i.e., the field ignoring contributions from multiple passes through the cavity) is

$$\mathbf{E}_1 = (t_1 t t_2) e^{-i\delta/2} \mathbf{E}_0.$$

Contributions from subsequent passes acquire factors of $r_1 r_2 t^2 e^{-i\delta} = r_1 r_m e^{-i\delta}$. Let

$$\begin{aligned} \alpha &\coloneqq t_1 t t_2 e^{-i\delta/2} \\ \beta &\coloneqq r_1 r_m e^{-i\delta}. \end{aligned}$$

By the above logic, the total field at the output coupler is then

$$\begin{aligned} \mathbf{E}_t &= \alpha(1 + \beta + \beta^2 + \cdots)\mathbf{E}_0 \\ &= \alpha\left(\frac{1}{1-\beta}\right)\mathbf{E}_0. \end{aligned}$$

The output intensity is thus

$$\begin{aligned} I_t &= I_0 \left| \frac{\alpha}{1-\beta} \right|^2 \\ &= I_0 \left[\frac{t^2 t_1^2 t_2^2}{(1 - r_1 r_m)^2 + 4 r_1 r_m \sin^2(\delta/2)} \right] \end{aligned}$$

(the denominator is the same as for the reflected and circulating intensities).

Problem 3 (Nagourney 3.3)

Problem Statement

Calculate the FSR, finesse, Q, and photon lifetime for a three-mirror ring cavity with 20 cm *arms and field reflectivities of* $r_1 = r_2 = 0.99$, $r_3 = 0.98$. *Assume* $\lambda = 600\,\mathrm{nm}$.

The round-trip optical path length is 60 cm; hence the FSR is

$$\not{\ell} = \frac{c}{\ell} = \frac{3\times 10^8\,\mathrm{m/s}}{60\,\mathrm{cm}} \approx 500\,\mathrm{MHz}.$$

The finesse is approximately given by equation 3.14 from Nagourney:

$$\begin{aligned}\mathscr{F} &\approx \frac{\pi\sqrt{r_1 r_m}}{1 - r_1 r_m} \\ &= \frac{\pi\sqrt{r_1 r_2 r_3}}{1 - r_1 r_2 r_3} \\ &\approx 78.\end{aligned}$$

The Q-factor at $\lambda = 600\,\mathrm{nm}$ is

$$Q = \mathscr{F}\left(\frac{\ell}{\lambda}\right) \approx 78\left(\frac{60\,\mathrm{cm}}{600\,\mathrm{nm}}\right) \approx 7.8\times 10^7.$$

Finally, the photon lifetime is

$$t_c = \frac{1}{2\pi\Delta\nu_{1/2}} = \frac{\mathscr{F}}{2\pi\not{\ell}} \approx 2.5\,\mathrm{ns}.$$