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Accessibility in complex networks

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ABSTRACT

This Letter describes a method for the quantification of the diversity of non-linear dynamics in complex networks as a consequence of self-avoiding random walks. The methodology is analyzed in the context of theoretical models and illustrated with respect to the characterization of the accessibility in urban streets.

1. Introduction

The intrinsic relationship between structure and dynamics seems to scaffold many dynamical processes in nature, from the flight of birds to the binding of proteins. Because of its inherent ability to represent and model the most diverse types of discrete structures, complex networks have received growing attention. After initially focusing attention on the characterization and modeling of the topology of interconnectivities (e.g. [1–3]), complex network research progressed steadily to encompass the relationship between structure and dynamics in the most diverse systems (e.g. [2,4,5]). Though connectivity does not completely define dynamics, it strongly affects it. This has become clear through investigations of relationships between structure and several types of dynamics, including diffusion (e.g. [6]), synchronization (e.g. [4]) and neuronal networks [7–10]. Particularly, when the dynamics is modeled in terms of random walks (e.g. [6,11]), the displacements of the respective moving agents are strongly influenced by several topological factors such as the number of connections at each node and the shortest path lengths between nodes.

With a tradition extending back over several decades, the study of the dynamics of random walks represents one of the main paradigms in statistical physics and dynamical systems. Traditional random walks are usually performed by one or more agents choosing with uniform probability between the outgoing edges at each node. Therefore, random walks represent one of the least intelligent ways to move in a network, involving no additional criterion rather than uniform chance. Still, such a dynamics is directly related to the important linear dynamics of diffusion (e.g. [12,13]), which plays an important role in a large number of dynamical processes (e.g. reaction–diffusion and Schrödinger equation). The dynamics of traditional, linear, random walks on complex networks has been investigated by several articles (e.g. [6,14–18]). Several other types of random walks have also been considered in the literature (e.g. [19–21]). For instance, the category of *self-avoiding* random walks—*path-walks* for short—represents a particularly interesting situation in which the moving agent is not allowed to return to nodes and/or edges (e.g. [19,20,22,23]).

A question of special relevance which has received relatively little attention from the literature regards the *diversity* of the obtained walks and path-walks. By diversity, it is meant how much the destination of the walks differ one another, which is immediately affected by intrinsic structure of the respective walks. In a previous work, Herrero [23] investigated the average number of self-avoiding walks defined in uniformly random and scale free networks. In the present Letter, the diversity of self-avoiding walks starting at a specific node i is quantified in terms of the entropies of the probability of visits to the destination nodes after the first h steps along self-avoiding walks, giving rise to the *diversity entropy* (see Fig. 1).

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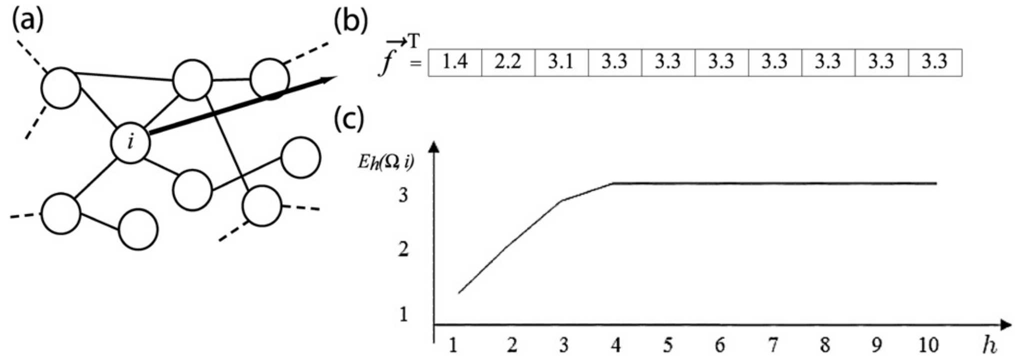

Fig. 1. Given a node i in a network (a), the entropies $E_h(\Omega, i)$ of the probabilities of visited nodes after the initial h steps (c) can be calculated by simulating several self-avoiding random walks starting from i. Therefore, a signature $\vec{f}$ (b) can assigned to each node i which expresses the diversity of paths obtained at each step h.

The diversity of walks is immediately related to a large number of important theoretical and practical aspects of complex networks structure and dynamics. To begin with, the cases in which the path-walks are found to lead to similar destinations along the steps (i.e. little diversity) imply that the agent had little choice during its motion, and therefore little path redundancy is present in the network. At the same time, such situations will also be characterized as being highly efficient as far as node coverage during the walk is concerned (relatively few edges are required while visiting several nodes). Indeed, by recalling that all nodes in a path-walk must be distinct, a path-walk involving $S + 1$ nodes will necessarily have S edges, which is the minimum number of connections required to connect those nodes. On the contrary, in case the path-walks are found to be strongly diverse, we can conclude that the progression of the agent is characterized by great freedom of choice and variety of transient dynamics. Consequently, because the path-walks cannot repeat nodes, we also have that the self-avoiding walks in this case will involve several diverse nodes. Therefore, nodes with high diversity constitute natural candidates as distributing sources (e.g. for information or mass), as several distinct destinations nodes can be effectively reached from those nodes. Because we quantify the diversity in terms of the entropy of the probabilities of reaching the possible destination nodes, this measurement reflects more properly the coverage of those nodes than would be obtained by simply taking into account the number of reachable destination nodes at each number of steps h after the beginning of the walks. Therefore, situations where some destination nodes are visited more frequently than others will necessarily imply in decrease of the diversity entropy.

It is also important to observe that the dynamics diversity of a node provides information which is complementary to other measurements of network nodes. For instance, though diversity tends to be correlated with the node degree at the initial steps of the self-avoiding walks, such a correlation can be quickly lost as a consequence of the connectivity diversity at the progressive surroundings of the initial node. The walk diversity is also distinct from the betweenness centrality (e.g. [2,3]) in the sense that chained nodes with high betweenness centrality will lead to low walk diversity. Therefore, diversity can be best thought as a novel measurement which can complement previous approaches in the characterization of the structural properties of complex networks.

Many are the interesting applications of such diversity studies to real-world problems. For instance, in case the random walks are used to model the acquisition of knowledge or cultural values by the agent (in this case each node represent a knowledge or cultural fact, e.g. [21]), the diversity measurements can provide sound basis for discussing how diverse the development of agents starting from similar backgrounds but subsequently exposed to different information will be. Several other real-world systems—including transportation, communications, energy distribution, to name just a few—can be investigated in terms of the diversity entropy. Another particularly interesting application concerns the objective quantification of the diversity of life and species along phylogenetics, as well as geographical exploration. In addition to its potential for the objective characterization of dynamics performed in complex networks, the quantification of the diversity of random walks and path-walks can also provide valuable indications about the structure of the respective networks. For instance, in case all path-walks are identical, we have a chain of nodes extending from the starting node to its very end. Contrariwise, a high diversity implies the presence of path redundancies in the network.

The diversity entropy can also be used to derive an estimation of the relative frequency of access effectively received and made by a particular node. This can be accomplished by a simple normalization, yielding the concept of *node accessibility*. More specifically, the *outward accessibility* quantifies the potential that a node i has in accessing the other nodes of the network. In an analogous fashion, the *inward accessibility* quantifies the frequency of the visits to each node i of self-avoiding walks departing from all other nodes h steps before.

The Letter starts by presenting the basics concepts, as well the definition of diversity entropy and some of its properties. It proceeds by describing the inward and outward accessibilities of the nodes. Next, the behaviour of the diversity entropy is studied considering four theoretical models of complex networks: Erdős–Rényi (ER), Barabási–Albert (BA), Watts–Strogatz (WS) and a geographical model (GG). In addition, a real example is presented, in which the outward accessibility of the urban streets of the Brazilian town of São Carlos is characterized and analyzed. It is shown how the addition of a few new connections on this network can strongly improve the accessibility of the surrounding regions of the city.

2. Basic concepts

An unweighted and undirected complex network can be represented by a matrix K, called the *adjacency matrix*. For a network with N nodes and E edges, the dimension of this matrix is equal to $N \times N$. If the nodes i and j are connected through an edge, the elements $K(i, j)$ and $K(j, i)$ of the adjacency matrix are set to 1; otherwise $K(i, j) = K(j, i) = 0$. Two nodes of the network are said to be adjacent if they share one edge. Two edges of the network are said to be adjacent if one extremity of each edge share the same node. The *degree* k_i of a node i is the number of its immediate neighbors. A *walk* over the network is composed by a sequence of adjacent nodes, starting from an initial node and proceeding through successive steps h. A *self-avoiding walk* is a walk where the nodes and edges are never repeated.

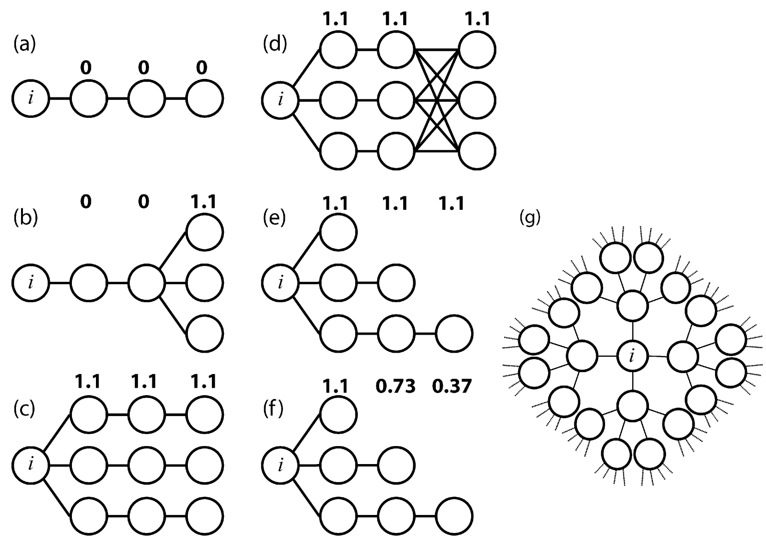

Fig. 2. Illustration of diversity entropy signatures. (a)–(f) The diversity entropy for the node i at steps $h = 1, 2$ and 3 are shown in bold. Note that for the networks shown in (a)–(e) the moving agent remains at the termination node and contributes to the probabilities and diversity for all the considered steps. A different condition is shown in (f), where the agent is eliminated when it can proceed no further, resulting in a decrease of the diversity for $h > 1$. (g) An example of a situation where the diversity entropy increases linearly with the steps h.

The *transition probability* that a moving agent node reaches a node j after departing from a node i, after h steps along a self-avoiding walk, is henceforth expressed as $P_h(j,i)$, with $h = 1, 2, \dots, H$. In order to estimate this probability, a total of M self-avoiding *random* walks, starting from the node i and proceeding H steps, are performed. Note that these walks stop when any of the following three conditions is met: (i) the walk reaches the maximum pre-defined value H of steps; (ii) the walk reaches an extremity node, i.e., a node with degree one; or (iii) the walk cannot proceed further because all of immediate neighbors of the node at step h were already visited. The probability $P_h(j,i)$ can then be estimated in terms of the number of times that the walks departing from a node i reach the node j after h steps, divided by the number of performed walks, M, where M is as large as possible. Note that $P_h(j,i)$ is typically different of $P_h(i,j)$. After the probabilities are estimated for each node, it is possible to calculate the *diversity entropy signature* $E_h(\Omega, i)$ of a node i after h steps as:

$$E_h(\Omega, i) = -\sum_{j=1}^{N} \begin{cases} 0 & \text{if } P_h(j,i) = 0, \\ P_h(j,i)\log(P_h(j,i)) & \text{if } P_h(j,i) \neq 0, \end{cases} \tag{1}$$

where Ω is the set of all nodes except i. Note that the maximum diversity at a given step h can only be achieved when all other $N-1$ nodes in the network (i.e. except the initial node) are reached with equal probabilities (i.e. $1/(N-1)$). Also, it should be observed that, once a self-avoiding walk is terminated (i.e. the moving agent can proceed no further), it is possible to assume two different situation for the moving agent: (i) it remains at the final node and contribute to the probabilities and diversities for all remaining steps; and (ii) it is eliminated from the analysis for all subsequent steps. The choice of which situation should be used depends on the problem under study and, as exemplified below, has a direct impact on the results of the analysis.

Fig. 2 illustrates several particularly relevant situations regarding diversity entropy signatures. Because of the total absence of branches, the chain network in (a) yields a completely null diversity signature. This means complete determinism in the sense that the destinations of the self-avoiding random walks starting from i will be identical for all steps h. The presence of a branch at step 3 in the structure in (b) implies the increase of the diversity entropy at this specific step. In the network in (c), the branch occurs at the first step, implying diversity entropy $E_h(\Omega, i) = -\log(1/3) \approx 1.1$, which remains constant for the two following steps (i.e. $h = 2$ and 3). Observe, in (d), that the additional all-to-all connections between the nodes in the second and third steps have no effect in changing the respective diversity entropy, as they do not affect the respective probabilities $P_3(j,i)$. The situation depicted in (e)–(f) involves self-avoiding random walks with different lengths, namely 1, 2 and 3. These examples illustrate the two possible alternatives referring to the moving agent reaching a termination node. In case (e), the moving agent is assumed to remain at its termination node for all subsequent steps, resulting in no changes in the diversity entropy values along the three initial steps. Though this assumption implies eventual degeneracy such as obtaining the same diversity entropy signatures for the structures in (c)–(d) and (e), the distinction between such cases can be easily accomplished by considering additional measurements such as the length of the walks. The other alternative, shown in (f), the agent is eliminated from the analysis after getting stuck. As consequence, the diversity entropy for $h > 1$ decreases with h when compared to the first alternative, and no degeneracy between the cases (c)–(d) and (f) is found. However, the latter choice (i.e. eliminate the stuck agents) implies that the total probability of visits will no longer be conserved as being equal to 1. The choice between these two possibilities should reflect their suitability with respect to each application.

A particularly interesting situation occurs when each node at each level h leads exclusively to a constant number $\langle k \rangle$ of new nodes in the subsequent level $h+1$, as illustrated in Fig. 2(g). In this case, $P_h(j,i) = 1/(\langle k \rangle^h)$, so that

$$E_h(\Omega, i) = -\sum_{j=1}^{\langle k \rangle^h} \log\left(\langle k \rangle^{-h}\right)/\langle k \rangle^h = h\log\left(\langle k \rangle\right). \tag{2}$$

Therefore, the diversity entropy will tend to monotonically increase (or remain null for $\langle k \rangle = 1$) with h at constant rate $\log(\langle k \rangle)$. This situation requires an infinite and completely regular network (i.e. each node has the same degree $\langle k+1 \rangle$, except the starting node, which has degree k). As complex networks are often analyzed with respect to regular or nearly regular counterparts (e.g. ER model), it is useful to consider the above configuration as a reference. For instance, the situation in which the diversity entropy tends to increase almost linearly with h along an interval can

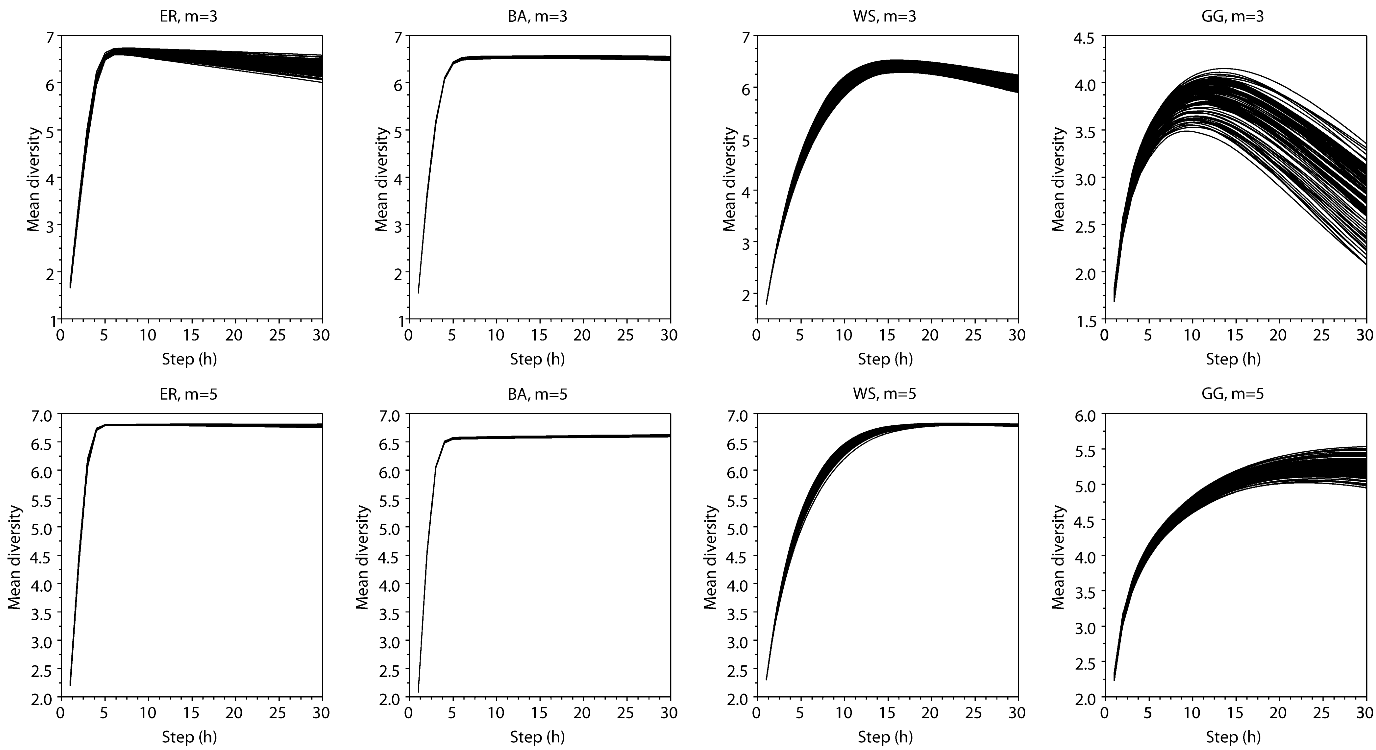

Fig. 3. The average of the diversity entropies obtained for each of the networks considered for each of the four complex networks models, assuming $N = 1000$ and $m = 3$ and $m = 5$ (i.e. $\langle k\rangle = 6$ and $\langle k\rangle = 10$, respectively).

be understood as an indication that the network is mostly regular along that interval. However, it should be born in mind that linear increase of the diversity entropy can also be caused by other structural organizations in complex networks (i.e. constant increase of entropy does not necessarily imply network degree regularity, but the latter necessarily implies linear entropy increase).

Additionally, in order to complement the concept of diversity, we propose the following two additional measurements: the *inward and outward accessibility*. Both these features can be seen as a normalization of the diversity entropy. In the case of the *outward accessibility*, $OA_h(i)$, it can quantify the diversity of the access of a node i with respect to all remainder nodes in the network after h steps, being defined as:

$$OA_h(i) = \frac{\exp(E_h(\Omega, i))}{N-1} \tag{3}$$

where the exponential is required in order to obtain a measurement which, after normalization by the number $N-1$ of remainder nodes in the network, can be interpreted as a relative frequency or 'probability'. On the other hand, the *inward accessibility*, $IA_h(i)$, quantifies the frequency of accesses to the node i from all of the other nodes of the network after h steps, and can be expressed as:

$$IA_h(i) = \frac{\exp(E_h(i, \Omega))}{N-1} \tag{4}$$

where $E_h(i, \Omega)$ is given as:

$$E_h(i, \Omega) = -\sum_{j=1}^{N} \begin{cases} 0 & \text{if } P_h(i,j) = 0, \\ (\frac{P_h(i,j)}{N-1}) \log(\frac{P_h(i,j)}{N-1}) & \text{if } P_h(i,j) \neq 0. \end{cases} \tag{5}$$

In the next section it is shown an analysis of the diversity entropy considering theoretical models of complex networks, followed by an application of the concept of outward accessibility to the study of the accessibility in an urban street network.

3. Diversity entropy for complex network models

Four theoretical models of complex networks are considered in the present work—Erdős–Rényi (ER), Barabási–Albert (BA), Watts–Strogatz (WS) and a geographical model (GG). The ER, BA and WS networks are grown in the traditional way (e.g. [1–4,24]). The GG networks in this work are obtained by distributing N nodes within a square with uniform probability and connecting all nodes which are closer than a minimal distance d. All networks considered in this analysis have $N \approx 1000$ and $m \approx 3$ or $m \approx 5$ (m is the number of spokes in the added nodes in the BA model), with average degree $\langle k\rangle \approx 2m$. The approximations are a consequence of the statistical variability of the models. For the same reason, the number of nodes N can vary slightly for the GG networks. Because the average degrees considered in this work are relatively large (well above the percolation critical value for ER), most of the nodes in each network belong to the largest connected component, which has been considered for all the analyses reported in this Letter. The total of rewirings used in the WS case was equal to $0.1E$.

The averages of the diversity entropy signatures were obtained for the realizations of each of the four considered network models. More specifically, a total of 200 realizations were performed for each of the four complex network models considering two average degrees ($\langle k\rangle \approx 6$, $\langle k\rangle \approx 10$). For each of such realizations, 10 000 random path-walks were performed starting from each of the nodes, and the respective entropies $E_h(\Omega, i)$ were estimated for $h = 1, 2, \ldots, 30$. The average of these diversity entropy values, for each realization, are shown in Fig. 3 for $m = 3$ and $m = 5$.

A series of interesting results can be inferred from the curves in these figures. First, observe that two general behaviors can be distinguished in both figures: the steeper increase of the diversity entropy with h obtained for the ER and BA models as opposed to the more gradual increase verified for the WS and GG models. These two types of transient dynamics can be observed for both $m = 3$ and $m = 5$. In the case of the transient evolutions observed for the ER and BA models, the diversity entropy tended to

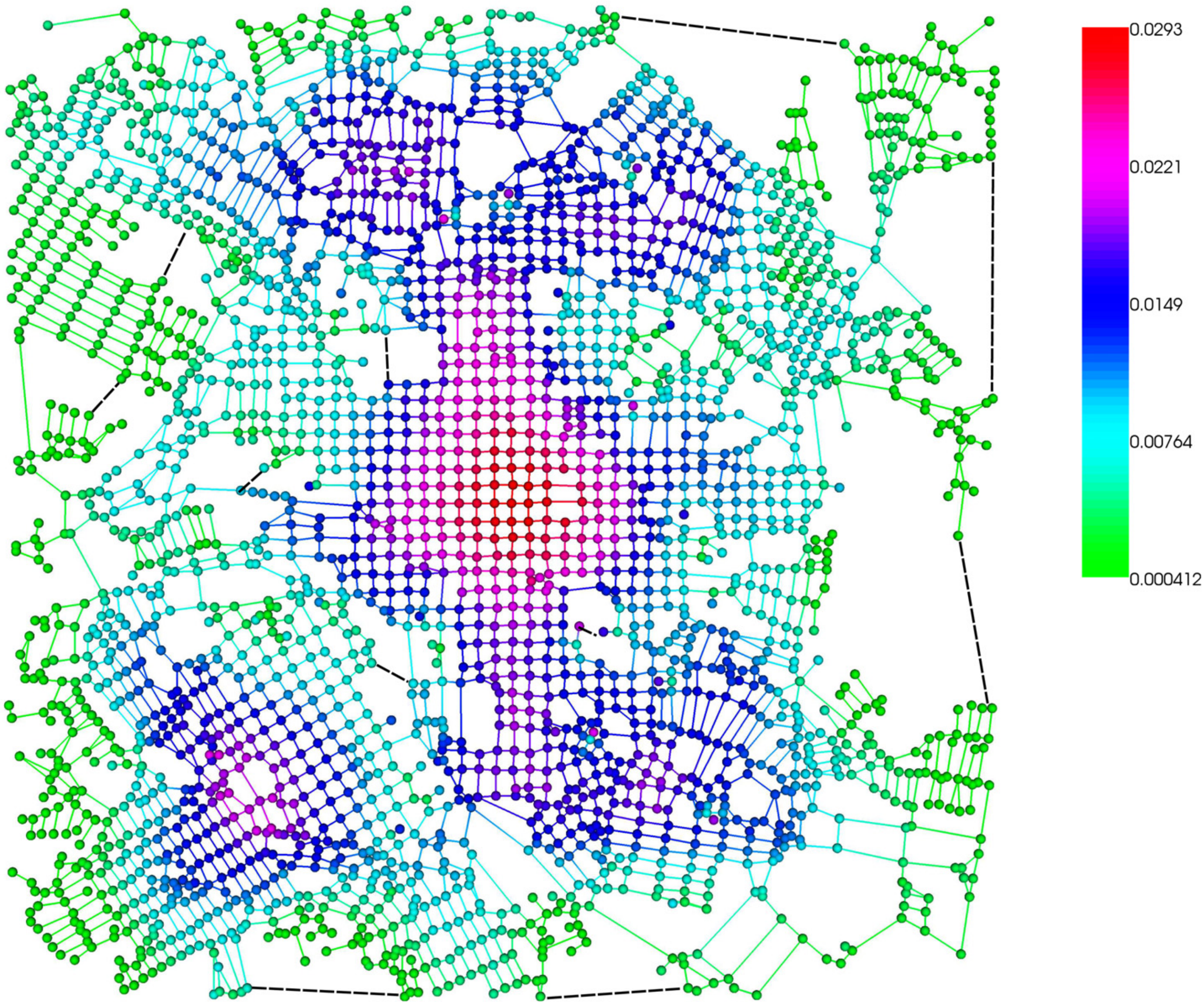

Fig. 4. Network of urban streets of São Carlos, Brazil. The color of the nodes indicates their respective averaged outward accessibility accordingly to the legend at the right-hand side. The dashed lines represent the hypothetical additional edges aimed at improving the accessibility. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

reach stabilization near a higher plateau (with diversity entropy approximately equal to 6.2, very near to the theoretical maximum of the diversity, $\log(N-1)$) after the three or four initial steps. This suggests that the self-avoiding paths in these networks tend to reach most nodes after just a few steps. The more gradual increase of entropy observed for the WS and GG models indicates that the moving agent takes substantially more time to cover a smaller portion of the nodes during the transient dynamics. This is a consequence of the fact that, though nearly regular (i.e. similar degrees for all nodes), these two types of networks are characterized by having pairs of nodes which are either connected through many short paths (adjacent nodes) or virtually unconnected. More informally, given two nodes i and j of a network, the *adjacency* between them can be quantified in terms of the number of short (i.e. up to a maximum length) paths interconnecting those nodes; the higher this number, the more adjacent the pair of nodes is.

Another interesting result regards the reduction of the diversity entropy found for large values of h for the ER, WS and GG models considering $m = 3$. The main reason for this reduction is that many self-avoiding walks are trapped before fully completing the h steps, diminishing the number of accessible nodes and consequently reducing the diversity entropy. Note that this result is particularly accentuated for the GG networks, where the geographical constraints induces the formation of bottlenecks in the network, isolating many regions and not allowing that a self-avoiding walk, once in a region, reaches others regions of the network.

4. Characterization and improvements of accessibility in an urban streets network

The second analysis presented in the current work applies the concept of *accessibility* in order to quantify in an objective and comprehensive way the outward accessibilities of each node of a town (i.e. each intersection or beginning of routes). The proposed methodology is illustrated with respect to a real-world network representing the Brazilian town of São Carlos. Complex networks have been used to characterize important topological, dynamical and spatial properties of cities (e.g., [25–29]). In this work, image processing and analysis methods were used to transform the plan of the town into a respective geographical planar network, where the nodes represent the crossings or beginning of routes, while the edges correspond to the streets. Fig. 4 shows the network derived from the central part of the town. This network has $N = 2812$ nodes and $E = 4713$ edges.

The outward accessibility of each node was estimated by simulating 10 000 self-avoiding walks of length $H = 60$ starting at all nodes. Self-avoiding walks were adopted because they represent a natural simplified choice for modeling urban displacements, implying the agents to move away from their initial position in a more effective way while not repeating edges or nodes. Also, in this analysis, if a moving agent could not finish its walk, i.e., reached a stop condition before the completion of the all pre-established number of steps, it no longer contribute to the transition probabilities and the diversity entropy. This choice was made because it represents a better model than using stationary agents at the extremities of the network, as we are interesting in studying the displacement of people and transportation through the streets and roads.

In Fig. 4, the color of the nodes correspond to their respective outward accessibility, averaged over all the steps, i.e. $h = 1, 2, \ldots, 60$. An interesting result is that most part of the highly accessible nodes corresponds to the downtown São Carlos, located at the central region of the map. Another important property is the high spatial discriminative power provided by the outward accessibility measurement regarding the identification of border nodes: it can be clearly seen from Fig. 4 that the nodes situated at the border of the network have the smallest outward values, while the inner nodes have the highest outward values. Interestingly, nodes with low outward accessibility can be found even downtown. Another important property of the outward accessibility measurement is that self-avoiding walks initiating from nodes characterized by high accessibility for a given path length tend to visit all reachable nodes at that length in the shortest period of time in the average. In addition, the outward accessibility intrinsically considers the redundancy of alternative routes from the initial node to the reachable nodes. Nodes with high outward accessibility therefore have more balanced number of routes leading to the reachable nodes.

In order to gather additional insights regarding the accessibility of the investigated urban network, we also considered hypothetical new edges (i.e., new streets) connecting some nodes of the periphery and internal regions of the town (represented as dashed lines in Fig. 4). This new arrangement allowed a study of the potential impact of such new edges on the accessibility of their respective neighborhoods. The mean accessibility was estimated by considering the nodes located up to seven blocks away from the nodes that received the new connections. Fig. 5 shows these values for the original network and for the enhanced network. Note the substantial increase of 21% in the accessibility up to after approximately $h = 15$ steps for the place where the new edges were added. This result shows that major improvements of accessibility can be achieved by adding just a few streets at strategic locations.

An additional analysis was also performed in order to compare the obtained results with the *search information* S of the network, a measure proposed by Rosvall et al. [25]. In the measure, the shortest path lengths are used to quantify and compare the information needed to locate specific addresses in different cities. Briefly explained, the search information between the node s and t gives the amount of information needed to locate t departing from s through the shortest paths, and is expressed as:

$$S(s \to t) = -\log_2 \sum_{\{p(s,t)\}} P\big[p(s,t)\big] \tag{6}$$

where

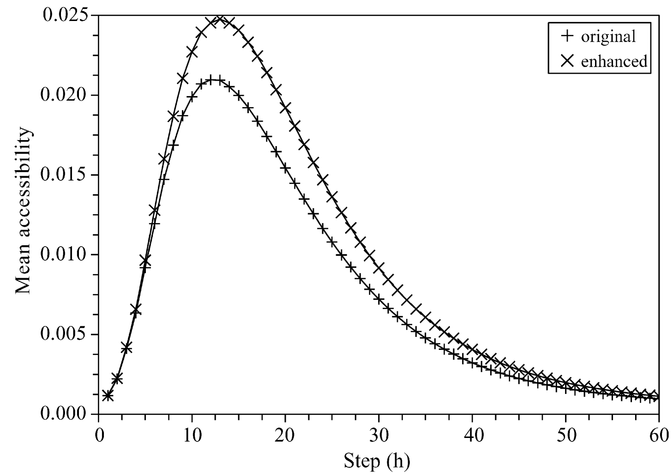

Fig. 5. Mean outward accessibility of the original network and the enhanced network. In both cases, only the nodes with up to seven edges of distance from the nodes that received the new connections were taken into account. Observe the substantial increase in the outward accessibility obtained as a consequence of the new connections.

$$P\big[p(s,t)\big] = \frac{1}{k_s} \prod_{j \in p(s,t)} \frac{1}{k_j - 1} \tag{7}$$

and $p(s,t)$ represents all the shortest paths between s and t. The average search information for a node s (also known as the access information of the node s [25]) is given by:

$$\mathcal{A}_s = \frac{1}{N} \sum_{t=1}^{N} S(s,t). \tag{8}$$

The average search information was also calculated for the network of the urban streets of São Carlos and the results are shown in Fig. 6(a). When comparing this figure with Fig. 4, it becomes clear the differences between the outward accessibility and the search information. Recall that the outward accessibility allows to identify the regions of the city with high accessibility—being the central region the most important, but with markedly presence of many others locations. Each of these regions can even be seen as the center of specific 'communities' in the network. On the other hand, the search information pattern is radial, i.e., nodes at the periphery of the network have the highest values and these values decrease almost uniformly toward the direction of the center of the network. This result is similar to the mean shortest path distance of each node, and shows that this measurement is, as proposed by Rosavall et al. [25], more appropriate to evaluate networks of urban streets where the streets are considered as nodes and the edges are established by the intersections between the streets. In this case, the distance between two regions of the city is less restrictive than the amount of information necessary to locate a specific address [25]. Fig. 6(b) reinforces the differences between the two measurements, as reflected in the low value of their Pearson correlation coefficient ($\rho = -0.57$).

All in all, this Letter has presented the concepts of diversity and accessibility, aimed at quantifying important aspects of the dynamics unfolding on complex systems underlain by complex networks. The potential of this approach has been substantiated with respect to theoretical models of complex networks and also for characterizing the accessibility of town of São Carlos, SP, Brazil. It should be observed that though we assumed that the dynamics was produced by self-avoiding displacements, the measurements and methodology reported in this work can be immediately applied to other types of dynamics.

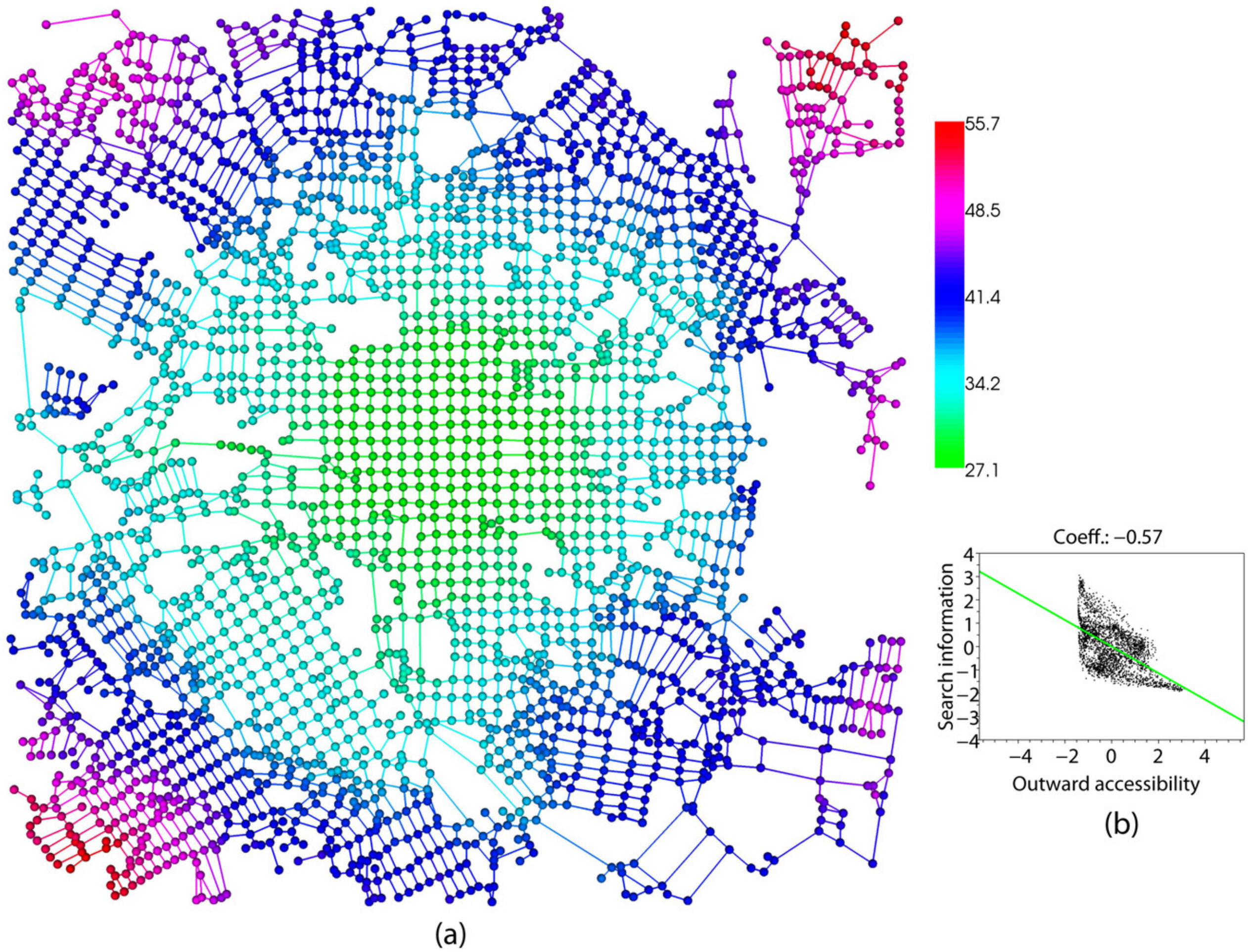

Fig. 6. (a) Average search information for the network of urban streets of São Carlos. (b) Pearson correlation coefficient between the mean outward accessibility and the average search information for each node of the network. No considerable correlation between these two measurements has been found.

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