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Condensation of vortices and disorder parameter in 3d Heisenberg model

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The 3d Heisenberg model is studied from a dual point of view in terms of 2d solitons (vortices). It is shown that the disordered phase corresponds to condensation of vortices in the vacuum, and the critical indices are computed from the corresponding disorder parameter.

1. Introduction

The Heisenberg ferromagnet is defined by the partition function

$$Z[\beta] = \int \prod [d\Omega(x) \exp(-S) \qquad (1)$$

where

$$S = \frac{1}{2}\beta \sum_{\mu,x} [\Delta_\mu \vec{n}(x)]^2 \qquad \vec{n}^2(x) = 1 \qquad (2)$$

and $d\Omega(x)$ is the element of solid angle for the orientation of $\vec{n}$ in colour space.

The model presents a 2nd order phase transition at $\beta_c \simeq 0.7$[1]. For $\beta > \beta_c$ there is an ordered phase, with order parameter the magnetization $\langle\vec{n}\rangle \neq 0$; for $\beta < \beta_c$, $\langle\vec{n}\rangle = 0$ (disordered phase). We shall describe the system from a dual point of view, and show that in the disordered phase vortices condense. The model will be viewed as a $2+1$ dimensional euclidean field theory. Vortices will be labelled by the integer valued topological charge of the 2 dimensional spacial configurations, which is a conserved quantity, and defines a $U(1)$ symmetry of the system. A disorder parameter $\langle\mu\rangle$ will be constructed, which detects spontaneous breaking of this $U(1)$ symmetry, or condensation of vortices.

The situation is analogous to the condensation of monopoles in the confining phase of gauge theories[2,3] or to the condensation of abelian vortices in He_4[4]. To check our construction we shall extract from the numerical determination of $\langle\mu\rangle$ at the phase transition the known critical indices.

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2. The Heisenberg model as a fiber bundle.

Usually the colour frame to which the direction of $\vec{n}$ is referred is a fixed frame, independent of x

$$\vec{\xi}^0_i \; (i = 1,2,3) \quad \vec{\xi}^0_i \wedge \vec{\xi}^0_j = \vec{\xi}_k i^0 \quad \vec{\xi}^0_i \cdot \vec{\xi}^0_j = \delta_{ij}$$

A body fixed frame can be defined[5] by three unit vectors $\vec{\xi}_i(x)$ $(i = 1,2,3)$

$$\vec{\xi}_i(x) \wedge \vec{\xi}_j(x) = \vec{\xi}_k i(x) \quad \vec{\xi}_i(x) \cdot \vec{\xi}_j(x) = \delta_{ij}$$

and $\vec{\xi}_3(x) \equiv \vec{n}(x)$. The frame is defined up to an arbitrary rotation around $\vec{n}(x) = \vec{\xi}_3(x)$.

Since $\vec{\xi}^2_i = 1$

$$\partial_\mu \vec{\xi}_i(x) = \vec{\omega}_\mu \wedge \vec{\xi}_i(x)$$

or

$$\begin{aligned} D_\mu \vec{\xi}_i(x) &\equiv (\partial_\mu - \vec{\omega}_\mu \wedge)\vec{\xi}_i = 0 \\ D_\mu &= \partial_\mu - i\,\vec{\omega}_\mu \cdot \vec{T} \end{aligned} \qquad (3)$$

$(T^a)_{ij} = i\varepsilon_{iaj}$ are the generators of the $O(3)$ symmetry group. Eq.(3) is nothing but the definition of parallel transport. From eq.(3) it follows $[D_\mu, D_\nu]\,\vec{\xi}_i = 0$ or, by completeness of $\vec{\xi}_i$

$$[D_\mu, D_\nu] = \vec{T} \cdot \vec{F}_{\mu\nu}(\omega) = 0 \qquad (4)$$

$$\vec{F}_{\mu\nu}(\omega) = \partial_\mu \vec{\omega}_\nu - \partial_\nu \vec{\omega}_\mu + \vec{\omega}_\mu \wedge \vec{\omega}_\nu$$

$\vec{\omega}_\mu$ is a pure gauge, apart from singularities. The general solution of eq.(3) is then, for $\vec{n}(x) = \vec{\xi}_3(x)$

$$\vec{n}(x) = P \exp\left(i \int_{\infty,C}^{x} \vec{T}\vec{\omega}_\mu(x')dx'_\mu \right) \vec{n}_0$$

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where $\vec{n}_0$ is the value of $\vec{n}(x)$ at infinity, and the dependence on the path C is trivial, because $F_{\mu\nu}$ is a pure gauge, eq.(4). This is true apart from singularities. We will show that such singularities exist.

The current $J_\mu = \frac{1}{8\pi}\varepsilon_{\mu\alpha\beta}\vec{n}\cdot(\partial_\alpha\vec{n}\wedge\partial_\beta\vec{n})$ is identically conserved

$$\partial^\mu J_\mu = 0 \tag{5}$$

If we look at the theory as the euclidean version of a field theory, with euclidean time on the 3 axis, the corresponding conserved quantity is

$$Q = \frac{1}{4\pi}\int d^2x\vec{n}\cdot(\partial_1\vec{n}\wedge\partial_2\vec{n})$$

which is nothing but the topological charge of the 2 dimensional configurations of the theory. Q can assume positive and negative integer values. By use of eq.(3) and eq.(4) it is easy to show that

$$Q = \frac{1}{4\pi}\int d^2x\vec{n}\cdot(\vec{\omega}_1\wedge\vec{\omega}_2) = -\oint_C(\vec{\omega}_i\cdot\vec{n})dx^i \tag{6}$$

where the path C is the contour of the region in the 2 dimensional space (x_3 = const.) where eq.(4) holds. Since $Q = \pm n$ Eq.(6) shows that $\vec{\omega}_\mu$ is not always a pure gauge.

This can be explicitely checked on a configuration corresponding to a static 2 dimensional instanton propagating in time (vortex).

The conserved current J_μ identifies a $U(1)$ symmetry. We will show that this symmetry is Wigner in the ordered phase $\beta > \beta_c$, and is spontaneously broken in the disordered phase.

3. Disorder parameter

Let $R_q(\vec{x},\vec{y})$ be a $\vec{x}$ dependent singular rotation creating a vortex of charge q at the site $\vec{y}$ in a 2 dimensional configuration.

$$R_q(\vec{x},\vec{y})\vec{n}(\vec{x},t) \Rightarrow \vec{n}_q(\vec{x},t)$$

The creation operator of a vortex μ_q at site $\vec{y}$, time t, will be defined as

$$\mu_q(\vec{y},t) = \exp\Big\{-\beta\sum_x[(R_q^{-1}(\vec{x},\vec{y})\vec{n}(\vec{x},t+1)-\vec{n}(\vec{x},t))^2 - (\vec{n}(\vec{x},t+1)-\vec{n}(\vec{x},t))^2]\Big\} \tag{7}$$

We measure the correlator

$$\mathcal{D}(x_0) = \langle\mu_{-q}(\vec{0},x^0)\mu_q(\vec{0},0)\rangle$$

By cluster property

$$\mathcal{D}(x_0) \underset{|x_0|\to\infty}{\simeq} Ae^{-M|x_0|} + \langle\mu_q\rangle^2 \tag{8}$$

$\langle\mu_q\rangle \neq 0$ signals spontaneous breaking of the $U(1)$ symmetry (5), or condensation of vortices.

By use of the definition (7) it is easy to see that

$$\mathcal{D}(x_0) = \frac{Z[S+\Delta S]}{Z[S]}$$

where $S + \Delta S$ is obtained from S, eq.(2) by the replacement

$$[\Delta_0\vec{n}(\vec{x},0)]^2 \to [R_q^{-1}(\vec{x},0)\vec{n}(\vec{x},1) - \vec{n}(\vec{x},0)]^2$$
$$[\Delta_0\vec{n}(\vec{x},x_0)]^2 \to [R_q^{-1}(\vec{x},0)\vec{n}(\vec{x},x_0+1) - \vec{n}(\vec{x},x_0)]^2$$

and that this really amounts to have a vortex propagating from $(\vec{0},0)$ to $(\vec{0},x_0)$[2].

Instead of $\mathcal{D}(x_0)$ itself it proves convenient[2,3] to study the quantity $\rho(x_0) = \frac{1}{2}d\ln\mathcal{D}(x_0)/d\beta$. As $|x_0| \to \infty$ from eq.(8) we have for $\rho \equiv \rho(x_0 = \infty)$, $\rho \simeq d\ln\langle\mu\rangle/d\beta$. Since $\langle\mu\rangle_{\beta=0} = 1$, $\langle\mu\rangle = \exp\left[\int_0^\beta\rho(\beta')d\beta'\right]$. ρ is easier to measure and contains all the information on the transition. The behaviour of ρ is shown in fig.1.

For $\beta < \beta_c$, $\rho \to$ finite limit consistent with zero, or $\langle\mu\rangle \neq 0$, which means condensation of vortices.

For $\beta > \beta_c$ ρ can be evaluated in perturbation theory and behaves as

$$\rho = -c_1L + c_2 \qquad c_1 > 0 \tag{9}$$

L is the lattice size. Eq.(9) implies that $\langle\mu\rangle = 0$ for $\beta > \beta_c$ in the thermodynamical limit $V \to \infty$. Around β_c a finite size scaling analysis can be performed

$$\langle\mu\rangle = f\left(\frac{a}{\xi},\frac{L}{\xi}\right) \sim f\left(0,\frac{L}{\xi}\right)$$

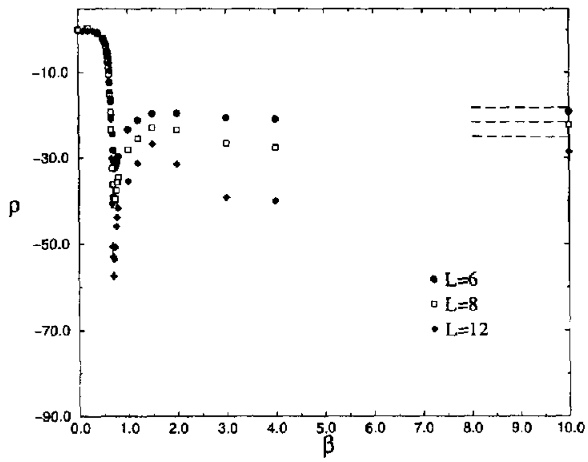

Fig.1

and since $1/\xi \simeq (\beta_c - \beta)^\nu$, we get the scaling law[2]

$$\frac{\rho}{L^{1/\nu}} = f(L^{1/\nu}(\beta_c - \beta))$$

The scaling law is verified, fig.2, and allows to extract β_c and ν

$$\begin{aligned}\beta_c &= 0.695 \pm 0.003 \; [0.6928] \\ \nu &= 0.70 \pm 0.02 \; [0.698]\end{aligned}$$

They agree with the values determined from $\langle \vec{n} \rangle$, which are indicated in parentheses[1,6].

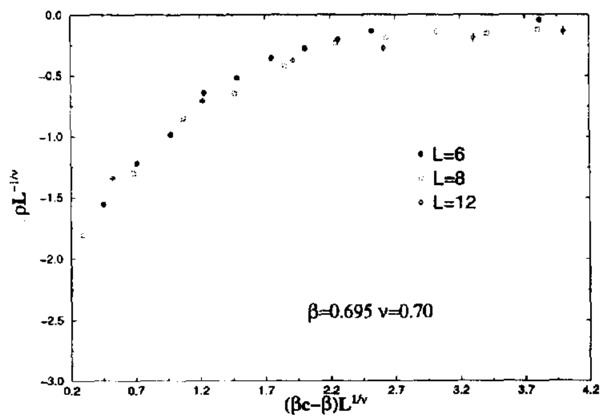

Fig.2

Conclusions

The phase transition to disorder in 3d Heisenberg model is produced by condensation of topological solitons. A disorder parameter can be defined and out of it the critical indices can be determined.

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