Computational Electromagnetics with Discrete Exterior Calculus

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A novel computational electromagnetic method developed with discrete exterior calculus (DEC) (M. Desbrun *et al.*, arXiv:math/0508341) on simplicial mesh is presented. As is well known, differential forms can been used to recast Maxwell's equations in a more succinct fashion, which completely separate metric-free and metric-dependent components. Instead of dealing with vectorial field as finite difference method and finite element method (FEM), DEC studies electric and magnetic fields in discrete differential forms, i.e. their integrals over simplicial structures. In DEC, electric and magnetic fields are represented as cochains, which are vectors with finite length. Differential operators, such as curl and divergence, are replaced by discrete exterior derivative, which are highly sparse matrices only containing geometrical connection information with elements 0, 1 or -1. From Yee grid, the necessity and importance of a dual mesh or dual grid has been realized. In DEC, the dual mesh is constructed by connecting circumcenters of simplicial structures (triangles in 2D and tetrahedrons in 3D). With this circumcenter dual or Voronoi dual, the discrete Hodge star operators, which map from primal cochains to dual cochains, can be constructed as diagonal matrices. *Effective dual volume* is also introduced to incorporate material information into Hodge star operators. Then constitutive relations in an inhomogeneous medium can be described with these Hodge star operators. The case when circumcenters fall outside triangles or tetrahedrons is also considered and treated appropriately. The implementation of various boundary conditions is also illustrated.

Compared to finite difference method, DEC is also based on unstructured simplicial mesh as in FEM. Therefore, our method is adaptive to arbitrary complex inhomogeneous structures. Moreover, this method also exactly preserves important structural features of Maxwell's equations as in Yee grid, e.g. Gauss' law $\nabla \cdot \mathbf{D} = \rho$. Besides, since $\nabla \times \nabla = 0$ and $\nabla \cdot \nabla \times = 0$ are naturally and exactly preserved, our method will not give rise to spurious solutions due to spurious charge. Finally, we performed several numerical calculations with this method, including hybrid modes in inhomogeneous waveguides, band structures of photonic crystals, scattering problem with absorbing boundary conditions, and modes of resonant cavities.