Can Mathisson-Papapetrou equations give clue to some problems in astrophysics?

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Abstract.

First, we stress that for correct description of highly relativistic fermions in a gravitational field it is necessary to have an equation which in the limiting transition to the classical (non-quantum) case corresponds to the exact Mathisson-Papapetrou equations. According to these equations the spin in general relativity is Fermitransported, and the parallel transport of spin is realized only in some approximation. The traditional general-relativistic Dirac equation (1929) is based on the parallel transported spinors and does not ensure the correspondent transition. Second, because in the range of very high velocity (close to the speed of light) of a spinning particle relative to the Schwarzschild or Kerr sources the Mathisson-Papapetrou equations have the solutions which reveal that the spin-gravity interaction acts as a strong antigravity force, we suppose that this fact can be useful for explanation some astrophysical phenomena. Some association with the OPERA results is possible.

The Mathisson-Papapetrou equations are known from 1937 as the equations which describe motions of a spinning test particle (rotating test body) in a gravitational field in the framework of general relativity [1]. These equations can be written as

$$\frac{D}{ds}\left(mu^{\lambda} + u_{\mu}\frac{DS^{\lambda\mu}}{ds}\right) = -\frac{1}{2}u^{\pi}S^{\rho\sigma}R^{\lambda}_{\ \pi\rho\sigma},\tag{1}$$

$$\frac{DS^{\mu\nu}}{ds} + u^{\mu}u_{\sigma}\frac{DS^{\nu\sigma}}{ds} - u^{\nu}u_{\sigma}\frac{DS^{\mu\sigma}}{ds} = 0,$$
(2)

where $u^{\lambda} \equiv dx^{\lambda}/ds$ is the particle's 4-velocity, $S^{\mu\nu}$ is the tensor of spin, m and D/ds are, respectively, the mass and the covariant derivative with respect to the particle's proper time s; $R^{\lambda}_{\pi\rho\sigma}$ is the Riemann curvature tensor (units c = G = 1 are used); here and in the following, latin indices run 1, 2, 3 and greek indices 1, 2, 3, 4; the signature of the metric (-,-,+) is chosen.

While investigating the solutions of equations (1), (2), it is necessary to add a supplementary condition in order to choose an appropriate trajectory of the particle's center of mass. Most often conditions

$$S^{\lambda\nu}u_{\nu} = 0 \tag{3}$$

or

$$S^{\lambda\nu}P_{\nu} = 0 \tag{4}$$

are used, where

$$P^{\nu} = mu^{\nu} + u_{\lambda} \frac{DS^{\nu\lambda}}{ds} \tag{5}$$

is the 4-momentum. The condition for a spinning test particle

$$\frac{|S_0|}{mr} \equiv \varepsilon \ll 1 \tag{6}$$

must be taken into account as well, where $|S_0| = const$ is the absolute value of spin, r is the characteristic length scale of the background space-time (in particular, for the Kerr metric r is the radial coordinate), and S_0 is determined by the relationship

$$S_0^2 = \frac{1}{2} S_{\mu\nu} S^{\mu\nu}. \tag{7}$$

Instead of exact MPD equations (1) their linear spin approximation

$$m\frac{D}{ds}u^{\lambda} = -\frac{1}{2}u^{\pi}S^{\rho\sigma}R^{\lambda}_{\ \pi\rho\sigma} \tag{8}$$

is often considered. The long list of publications devoted to the Mathisson-Papapetrou equations is presented, for example, in [2–4].

Our purpose is to draw attention to the two points concerning the strict Mathisson-The first is connected with the general-relativistic Dirac Papapetrou equations. equation, which was obtained in 1929 [5], i.e., eight years before the Mathisson-Papapetrou equations. Later it was shown in many papers that the Mathisson-Papapetrou equations are, in certain sense, the classical approximation of the Dirac equation [6]. The main step in obtaining the general-relativistic Dirac equation in the curved spacetime consists in introduction the notion of the parallel transport for spinors as a generalization of this notion for tensors. However, if one want to satisfy the principle of correspondence between the general-relativistic Dirac equation and the Mathisson-Papapetrou equations, it is necessary to take into account the known fact that according to the Mathisson-Papapetrou equations the spin of a test particle is transported by Fermi, nor parallel transported (we underline that this fact was unknown in 19290. The Fermi transport coincides with the parallel transport only in some approximation, when a world line of a spinning particle practically coincides with the corresponding geodesic line, for example, in the post-Newtonian approximation. In general, by the principle of correspondence, it is necessary to know how to write the Dirac equation in the curved spacetime with the Fermi transport in the limiting transition to the classical (nonquantum) description. One can suppose that for this aim it is sufficiently to introduce the Fermi transport for spinors, instead of their parallel transport (in this sense an attempt was discussed in [7]). However, in common sense, the notation "Fermitransported spinor" cannot be introduced without violation of the Lorentz invariance. By the way, in this context it is interesting that for last years the possibility of the Lorentz invariance violation is discussed in the literature from different points of view.

In any case (with violation of the Lorentz invariance or not), it is necessary to propose a more exact equation for fermions in gravitational field than the usual general-relativistic Dirac equation (probably, this equation must be nonlinear in the ψ - function [7]).

The second point of importance connected with the Mathisson-Papapetrou equations is the properties of their solutions which describe highly relativistic motions of a spinning particle in the Schwarzschild or Kerr backgrounds. Namely, if the velocity of a particle relative to the Schwarzschild or Kerr mass corresponds to the relativistic Lorentz γ – factor of order $1/\sqrt{\varepsilon}$, where ε is determined by (6), the trajectory of this particle can significantly differ from the corresponding trajectory of a spinless particle, i.e, from the geodesic line [3, 4] (in these papers same cases are calculated when r is not much grater than the horizon radius). That is, the Mathisson-Papapetrou equations predict an interesting phenomenon: for highly relativistic spinning particles gravity becomes antigravity, at some correlation of signs of the spin and the particle's orbital velocity (for another correlation of these signs the spin-gravity interaction acts as an strong attractive force).

The necessary value of the Lorentz factor is less for the particles with higher ratio spin/mass, if r in (6) is fixed. For example, this value is less for neutrino than for electron or proton (some numerical estimates are presented in [3, 8, 9]). By the way, for neutrino near the Earth surface the value $1/\sqrt{\varepsilon}$ is of order 3×10^6 . That is, the Lorentz factor for neutrinos in the OPERA experiments is much higher.

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