Geometric Graphity and the Gravitization of Quantum Mechanics

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GEOMETRIC GRAPHITY AND THE GRAVITIZATION OF QUANTUM MECHANICS

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Abstract. We propose and prove three theorems in an axiomatic extension of Quantum Mechanics to a noncommutative spacetime geometry. The geometry is built from the Clifford Algebras $Cl(\mathbb{R}^3)$ and $Cl(\mathbb{R}^2)$, where every point on a symplectic manifold (phase space) is replaced by a pair of algebraic operator-valued graphs, corresponding to the superimposed and collapsed basis states. The spacetime geometry is then built from entangling N independent quantum harmonic oscillators in each basis where Lorentz invariance is recovered via the transitions between these states. We derive a conformally invariant metric at the Planck scale, such that for a pair of critical rotational exponents becomes a system of two spacetime equations of state containing the Bekenstein-Hawking area entropy law. This equation describes the fundamental interactions of two masses, expressed in terms of the fundamental constants, irremovable from the vacuum and separated in magnitude by the cosmological constant, causing spacetime to carry a massive negative pressure. While these results are in direct accordance with the ΛCDM model of cosmology they are also in direct conflict with the $U(1) \times SU(2) \times SU(3)$ model of particle physics, suggesting all particle interactions can be described using the n-dimensional irreducible representations of $SU(2)$ for $n = 1, 2, 3$. We discuss these results and propose a specific experiment to falsify our claims.

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1. Introduction

1.1. **Motivation.** When we look at the cosmos at the largest length scales we see a differentiable manifold whose dynamics are governed by Einstein's Field Equations. If we zoom into the macroscopic length scales, the scales of classical mechanics, we see a differentiable manifold described by Hamilton's equations of motion. When we further zoom into the microscopic length scales, the scales of quantum mechanics, the cosmos can no longer be described by a differentiable manifold. Rather at these scales, the cosmos is described by a Hilbert space whose dynamics are governed by the Schrodinger equation. On the one hand, manifolds are metric spaces containing commutative coordinates whereas on the other hand Hilbert spaces are complex metric spaces containing noncommutative coordinates. A natural question then arises. If we zoom into the smallest possible length scales, the Planck scale, would we expect spacetime itself to be commutative or noncommutative? If it's the former there exists a deep asymmetry in the natural order where there exists a special scale in which the cosmos is described by a Hilbert space and not a differentiable manifold. We would be faced with the following question: why does there exist a special length scale that can only be described using noncommutative coordinates?

During the early days of Quantum Field Theory it was Heisenberg that first suggested the use of a noncommutative spacetime geometry at sufficiently small scales to introduce an effective UV cutoff [16]. In 1947 Synder formalized this idea by introducing a formulation of quantum mechanics on a noncommutative spacetime [15]. His reasoning was inspired by Heisenberg's commutation relations $[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$ where neither the momentum nor position variables can be considered points belonging to a smooth manifold. By extending this concept to spacetime, Synder thought it may be possible to tame the divergences that plagued QFT at the time. His work was largely ignored because of the theoretical and experimental successes of renormalization techniques. It was however unknown at the time that these renormalization techniques would ultimately fail when attempting to unify gravity within the framework of QFT. Von Neumann further formalized the idea of noncommutative phase space by introducing topological spaces whose commutative C^*-algebras of functions are replaced by noncommutative algebras [5]. Rather recently, a low energy limit of string theory proposed that the commutation relations $[\hat{x}_i, \hat{x}_j] = 0$ be replaced by $[\hat{x}_i, \hat{x}_j] = i\theta_{ij}$, where θ_{ij} is a real-valued antisymmetric matrix with units of length squared [23]. Since then noncommutative spacetime geometries have gained significant attention.

For nearly three centuries modern physics has been built on Euclidean geometry, only recently being extended to the more general ideas of manifolds and Hilbert spaces. Even more recently have commutative spacetimes been extended to their noncommutative counterparts. Prior to Quantum Mechanics the quest for physical understanding was dominated by physical intuition. Albert Einstein was truly the last great intuitive physicist with his pioneering work on Special and General Relativity. In today's day in age physical intuition is becoming harder and harder to come by – there is nothing physically intuitive about Quantum Mechanics or noncommutative structures. One can reasonably argue the zeitgeist of physical intuition has come and gone. To address this and to broaden the horizons of inquiry we will adopt the following hypotheses proposed by Tegmark [18].

1.2. **The External Reality Hypothesis (ERH).** *There exists an external physical reality completely independent of us humans.*

1.3. **The Mathematical Universe Hypothesis (MUH).** *Our external physical reality is a mathematical structure.*

By the *virtue* of their assumption we can approach quantum gravity from a purely mathematical perspective, effectively freeing ourselves from otherwise restrictive physical intuition. Our approach is in the spirit of Dirac who in 1931 expressed: "The most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalize the mathematical formalism that form the existing basis of theoretical physics, and after each success in this direction, to try to interpret the new mathematical features in terms of physical entities" [12]. From this perspective physics is the application of mathematical languages that *describe* mathematical structures that *prescribe* physical reality. To illustrate this consider the following example of Hamiltonian dynamical systems. Given the pair (Ω, T) where Ω is the state space containing $\omega = (\mathbf{p}, \mathbf{q})$ and where T are Hamilton's equations of motion

$$\dot{p}_i = -\frac{\partial H}{\partial q_i}$$
$$\dot{q}_i = \frac{\partial}{\partial p_i}$$

These systems can be studied from three different perspectives that differ in the mathematical description of Ω and T.

1. *The Language of Differential Dynamics* employs the formalisms of actions by differential maps on smooth manifolds;

2. *The Language of Topological Dynamics* employs the formalisms of actions of continuous maps on topological spaces, usually compact metric spaces and Hilbert spaces;

3. *The Language of Ergodic Theory* employs the formalisms of measure preserving actions of measurable spaces on a measure space, usually assumed to be finite.

On the one hand, General Relativity is cast within the language of differential dynamics while on the other hand Quantum Mechanics is cast within the language of topological dynamics. If we are too take the MUH seriously the cosmos cannot simultaneously be prescribed by both a manifold and a Hilbert space. We are forced to either reject General Relativity or Quantum Mechanics.

1.3.1. *Problems with the continuum. "It always bothered me that in spite of all this local business, what goes on in in no matter how, tiny a region of space and no matter how tiny a region of time, according the laws as we understand them today, takes a computing machine an infinite number of logical operations to figure out. Now how could all that be going on in that tiny [spacetime]? Why should it take an infinite amount of logic to figure out what one stinky tiny bit of spacetime is going to do? And so I've often hypothesized that the laws of physics are going to turn out to be in the end simple, like the checkerboard." - Richard P. Feynman*

Quantum Field Theory is the most precise theory ever devised, successfully predicting the anomalous magnetic dipole moment $g-2$ up to ten parts in a billion. The foundation of QFT rests upon an uncountably infinite number of degrees of freedom for which at every point in space and time a value can be assigned. Because of this QFT is computationally intractable. The computational demands of the theory increase exponentially with the number of vertices of a Feynman diagram. For a sufficiently large number of vertices no computer on earth can produce a number that can be compared to experiment. How we do know if the theory is as precise at these scales as it is when the number of vertices are small? The short answer is we don't know. What we do know is that any theory requiring an infinite number of degrees of freedom to perform a calculation will always have a scale in which computational complexity forbids any new predictions from being made. With the current state of affairs, either we will develop new novel computers that are capable of overcoming the computational hurdles or there will come a time in which QFT can offer no new predictions – at time where everything that can be computed has been computed. To address this we will adopt yet another one of Tegmark's propositions [18].

1.4. **Computable Finite Universe Hypothesis (CFUH).** *The mathematical structure that prescribes our external physical reality is discrete and defined by computable functions.*

The reason for adopting the CFUH is two-fold. It dramatically reduces the possible mathematical structures that can prescribe our universe while at the same time avoids possible inconsistencies with Gödel's Incompleteness Theorems. The CFUH furthermore opens the door to a digital rather than physical intuition. Interestingly, mathematicians have been debating these issues without reference to either physics or computation. In the Constructivist/Intuitionist school of thought, which included the likes of Kronecker, Weyl, and Goodstein, a mathematical object only exists if it can be constructed from the natural numbers in a finite number of steps [14]. In 1936 two formal systems of computation were developed independently of one another, Church's λ-calculus and the Turing machine. Shortly after their development Church and Turing proved that both of these systems were equivalent in what is now known as the Church-Turing thesis. In 1958 Curry found a remarkable connection between the typed combinators of the λ-calculus and representations of proofs in first-order logic. In 1969 Howard discovered a correspondence between deductive proofs and certain typed λ-terms [4]. The Curry-Howard correspondence draws a deep connection between mathematical proofs and computer programs. Wolfram has even gone as far as suggesting that our universe could be a finite cellular automata [24].

1.5. **Mathematical Object Space.** The MUH proposes that our universe *is* a mathematical structure, belonging to the space of all possible mathematical objects (MOS). The CFUH drastically reduces the possible number of mathematical structures, proposing that our universe is computable, belonging to the space of all possible computable mathematical objects (CMOS). To further reduce the number of candidate structures we propose that there exists a unique computable mathematical structure (COSMOS) whose free parameters are the fundamental constants, and where our *particular* universe is the one whose fundamental constants are precisely $(G, \hbar, c)$.

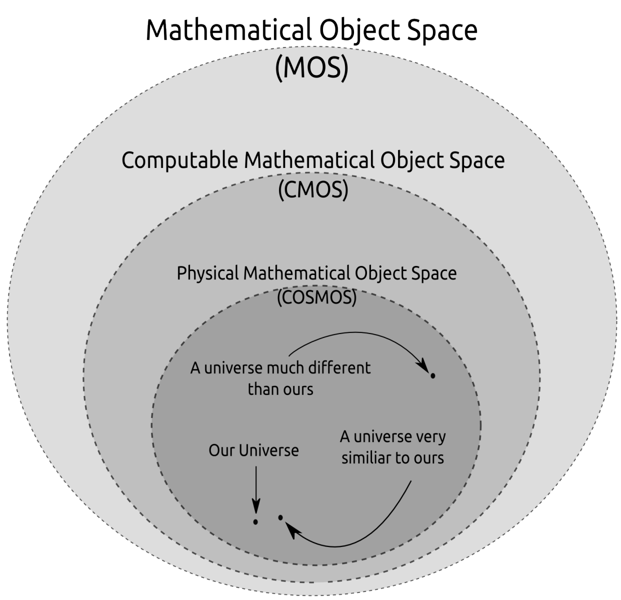

FIGURE 1.

2. Restructuring the Continuum

Networks are perhaps one of the most ubiquitous structures in nature. They appear at all length scales and throughout virtually all scientific disciplines [8]. They are aesthetic structures, neither too simple nor too complex. Likewise, the power-law is perhaps one of the most ubiquitous functions in nature. They too appear at all length scales and across virtually all scientific disciplines. Is the ubiquity of networks and power-laws unreasonable, insofar as to suggest there could exist a deep connection between the two? The answer is a resounding yes. There is a profound connection between the two and it rests at the very foundation of Geometric Graphity.

In this section we use the above intuition to restructure the continuum. We do this by analytically continuing the graph-theoretic walk, effectively 'moving' a continuum of spatial degrees of freedom to a complex phase $e^{i\theta}$ that encodes a continuum of rotational degrees of freedom with a finite number of spatial degrees of freedom specified by the graph. This procedure guarantees that no divergences of any kind can appear within the theory.

2.1. **The Graph-theoretic walk.** Within the context of Graph Theory, a walk of length k is simply an ordered list of vertices, $\{n, u_1, u_2, ..., u_{k-1}, m\}$, where the size of the sequence is the length of the walk. Graphs are visible mathematical structures, and therefore come equipped with a naturally *enumerable* metric. The adjacency relation can simply be stated as: two vertices n and m, are adjacent if and only if there exists a walk of length one from n to m. We then write the adjacency matrix as,

$$a_{nm} = \begin{cases} 1, & d(n,m) = 1 \\ 0, & d(n,m) \neq 1 \end{cases} \tag{2.1}$$

and the distance matrix of $\mathbf{A}$ as,

$$d_{nm} = \begin{cases} k, & d(n,m) = k \\ \infty, & otherwise \end{cases} \tag{2.2}$$

We can enumerate all walks of length k from n to m by multiplying the adjacency matrix with itself k times,

$$[\mathbf{A}^k]_{nm} = \sum_{u_1=1}^{N} \sum_{u_2=1}^{N} \cdots \sum_{u_{n-1}=1}^{N} \sum_{u_{n-2}=1}^{N} a_{n,u_1} a_{u_1,u_2} \ldots a_{u_{n-2},u_{n-1}} a_{u_{n-1},m} \tag{2.3}$$

A discretized version of the path integral can be obtained by summing over all walks. This however does not bear an equivalence with the Feynman path integral since there isn't a complex phase being assigned to every walk. Suppose now that we take the continuum and discretize it into an arbitrary graph G. Now suppose we take the same continuum, before discretization, and move it to k. In other words, we lift the restriction that the length of walk must be an integer and allow it to be any real number $k \in \mathbb{R}$. But wait, how could that possibly make sense? A graph has a discrete number of vertices, how can there exist continuous length walks? Well, it doesn't have to make sense to be mathematically true.

2.2. **Graphity Field Theorem.**

Theorem 2.1. *Let G be a simple non-empty graph. The power-law,*

$$\mathcal{P}[r] = Ar^{\alpha} \tag{2.4}$$

with boundary condition,

$$\mathcal{P}[0] = 0 \tag{2.5}$$

maps the (non-unique) adjacency matrix $\boldsymbol{A}(G)$ to an eigenfunction g, belonging to the complex L^2-space of functions.

Proof. Let us denote $\mathbf{A} = \mathbf{A}(G)$. It's rather trivial to show that $\mathcal{P}[\mathbf{A}]$ is complex. Since $\mathbf{A}$ is symmetric it is always diagonalizable and we can always write $\mathcal{P}[\mathbf{A}] = \mathbf{O}\mathcal{P}[\boldsymbol{\Lambda}]\mathbf{O}^{\top}$. $\boldsymbol{\Lambda}$ is the diagonal matrix of the eigenvalues of $\mathbf{A}$ and $\mathbf{O}$ is an orthogonal matrix belonging to $O(n)$, whose columns are the eigenvectors of $\mathbf{A}$. Since $\mathbf{A}$ is traceless the sum of the eigenvalues is always zero. We are therefore guaranteed at least one negative eigenvalue $(-\lambda)^{\alpha} = e^{i\pi\alpha}\lambda^{\alpha}$. In a stepwise manner we have,

1. $\mathbf{A} = \mathbf{A}^{\top} \Rightarrow \mathbf{A}$ is always diagonalizable.
2. Since $\mathbf{A}$ is always diagonalizable, $\mathcal{P}[\mathbf{A}]$ is a function of *only* the eigenvalues of $\mathbf{A}$.
3. Since $\operatorname{tr}\mathbf{A} = \sum_i \lambda_i = 0$, we are guaranteed at least one negative eigenvalue.
4. $\mathcal{P}(-\lambda) = a(-\lambda)^{\alpha} = e^{i\pi\alpha}\lambda^{\alpha}$.

Let us rewrite $\lambda^{\alpha} = e^{\alpha \ln \lambda}$ and $\theta = \alpha\pi$ and compute the spectral decomposition

$$\begin{aligned}
\mathcal{P}[\mathbf{A}] &= \mathbf{O}\mathcal{P}[\boldsymbol{\Lambda}(\theta)]\mathbf{O}^{\top} \\
&= \mathbf{O}\mathcal{P}(\boldsymbol{\Lambda}_{+})\mathbf{O}^{\top} + \mathbf{O}\mathcal{P}[\boldsymbol{\Lambda}_{-}(\theta)]\mathbf{O}^{\top} \\
&= \sum_{n=1}^{N} O_{nm}\mathcal{P}(\lambda_n^{+}) + \sum_{m=1}^{M} O_{nm}e^{i\theta}\mathcal{P}(\lambda_m^{-}) \\
&= \sum_{n=1}^{N} O_{nm}\exp\left\{\frac{\theta}{\pi}\ln(\lambda_n^{+})\right\} + \sum_{m=1}^{M} O_{nm}\exp\left\{\frac{\theta}{\pi}\big(\ln(\lambda_m^{-}) + i\pi\big)\right\}.
\end{aligned} \tag{2.6}$$

We've separated the spectrum $\boldsymbol{\Lambda}$ into its non-negative part $\boldsymbol{\Lambda}_{+}$ and its negative part $\boldsymbol{\Lambda}_{-}$,

(2.7)
$$\mathbf{\Lambda}_+ = \begin{pmatrix} \lambda_1^+ & & & & \\ & \ddots & & & \\ & & \lambda_n^+ & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix}, \quad \mathbf{\Lambda}_- = e^{i\theta} \begin{pmatrix} 0 & & & & \\ & \ddots & & & \\ & & \lambda_{n+1}^- & & \\ & & & \ddots & \\ & & & & \lambda_m^- \end{pmatrix}.$$

The eigenfunctions of the respective spectrums are

$$|g_+\rangle := \exp\left\{\frac{\theta}{\pi}\ln\left(\lambda_n^+\right)\right\}, \quad |g_-\rangle := \exp\left\{\frac{\theta}{\pi}\left(\ln\left(\lambda_m^-\right) + i\pi\right)\right\}, \tag{2.8}$$

where we define the following linear operators

$$\mathbf{L}^+ := \sum_n O_{nm}, \quad \mathbf{L}^- := \sum_m O_{nm}. \tag{2.9}$$

We can express eq (2.6) in the rather eloquent form

$$|G\rangle = \mathbf{L}^+ |g_+\rangle + \mathbf{L}^- |g_-\rangle \tag{2.10}$$

where the plus and minus sub and superscripts are representative of summation over the negative and non-negative eigenvalues. We will henceforth refer to the above as the Generalized Graphity Field Equation. Taking the real and imaginary parts we have

$$\operatorname{Re} g = \sum_{n=1}^{N} \exp\left\{\frac{\theta}{\pi}\ln\left(\lambda_n^+\right)\right\} + \sum_{m=1}^{M} \cos\theta \exp\left\{\frac{\theta}{\pi}\ln\left(\lambda_m^-\right)\right\} \tag{2.11}$$

$$\operatorname{Im} g = \sum_{m=1}^{M} i \sin\theta \exp\left\{\frac{\theta}{\pi}\ln\left(\lambda_m^-\right)\right\}. \tag{2.12}$$

We immediately see that $\operatorname{Im} g = 0$ when $\alpha = \pm 1, \pm 2, \pm 3, ...$ but is non-zero when for example $\alpha = \pm(1/2), \pm(2/3), \pm(3/4), ...$ or more generally when α is any real number. If we ask how many possible ways there are to move from one vertex to the next for a non-integral number of steps, a complex phase is assigned returning a complex number! The Graphity Field Equation is thus the analytic continuation of the graph-theoretic walk. An immediate property of the *complex walk* is that it is not possible to enumerate any of its elements. These sets act as perfect black boxes, preventing us from keeping track of any of the vertices. We are now in the position to sum over all walks in a continuous interval, thereby reconstructing Feynman's path integral as an inner-product of the above eigenfunction. As an example lets calculate all walks in the $(0, 2\pi)$ interval for some arbitrary graph G.

$$
\begin{aligned}
\langle g|g\rangle &= \langle g_+|g_+\rangle + \langle g_+|g_-\rangle + \langle g_-|g_+\rangle + \langle g_-|g_-\rangle \\
&= \frac{1}{\pi}\int_0^{2\pi} d\theta \sum_n \left[e^{2\theta\pi^{-1}\ln(\lambda_n^+)} + 2\cos\theta e^{\theta\pi^{-1}\ln(\lambda_n^+\lambda_n^-)} + e^{2\theta\pi^{-1}\ln(\lambda_n^-)} \right] \\
&= \frac{1}{\pi}\sum_n \left[\frac{\pi e^{2\theta\pi^{-1}\ln(\lambda_n^+)}}{2\ln\lambda_n^+} + \frac{e^{\theta\pi^{-1}\ln(\lambda_n^-\lambda_n^+)}(\cos\theta\ln(\lambda_n^-\lambda_n^+) + \pi\sin\theta)}{2\ln(\lambda_n^-)\ln(\lambda_n^+) + \ln^2(\lambda_n^-\lambda_n^+) + \pi^2} + \frac{\pi e^{2\theta\pi^{-1}\ln(\lambda_n^-)}}{2\ln\lambda_n^-} \right]_0^{2\pi} \\
&= \frac{1}{\pi}\sum_n \left(\frac{(e^{2\ln(\lambda_n^-\lambda_n^+)} - 1)\ln(\lambda_n^-\lambda_n^+)}{2\ln(\lambda_n^-)\ln(\lambda_n^+) + \ln^2(\lambda_n^-\lambda_n^+) + \pi^2} + \frac{\pi}{2}\frac{e^{4\ln(\lambda_n^+)} - 1}{\ln\lambda_n^+} + \frac{\pi}{2}\frac{e^{4\ln(\lambda_n^-)} - 1}{\ln\lambda_n^-} \right)
\end{aligned}
$$

We observe that the last two terms are indeterminates of the form $0/0$ when λ_n^+ and $\lambda_n^- \to 1$. Upon a change of variable the limit becomes,

$$\lim_{x\to 1}\frac{\pi}{2}\frac{x^4-1}{\ln x} = 2\pi \tag{2.13}$$

and we have our desired result

$$\langle g|g\rangle < \infty \quad \therefore g \in L^2(0,2\pi). \tag{2.14}$$

It's rather straightforward to demonstrate that the above is true for any bounded interval. We need not concern ourselves with the unbounded case since we would be integrating over an infinite number of backtracked walks. Furthermore, to extract all relevant information from the graph we need only integrate over its longest path. The proof is thus completed. □

2.3. **Geometric Graphity Identities.**

1. $\langle\delta(\theta-\pi)|G\rangle = \mathbf{A}(G)$
2. $\langle\delta(\theta+\pi)|G\rangle = \mathbf{A}^+(G)$ (Moore-Penrose Psuedo Inverse)
3. $\langle\delta(\theta)|G\rangle = \mathbf{L}$

2.4. **Geometric Graphity Metric.** We identify the eigenfunction $|g\rangle = |g_+\rangle + |g_-\rangle$ as a metric on a state space of graphs $\Omega = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, ..., \mathcal{G}_i\}$

$$d(\mathcal{G}_i, \mathcal{G}_j) = ||g_i - g_j|| = \sqrt{\langle g_j - g_i|g_j - g_i\rangle} \tag{2.15}$$

2.4.1. *Summary.* The Graphity Field Theorem (GFT) is an analytic derivation of a computationally tractable Feynman path integral where the computational complexity is at most that of diagonalizing the adjacency matrix. The remainder of this paper will be focused on the construction of the appropriate graph-theoretic coordinate system for which the GFT acts on.

3. The Axioms of Geometric Graphity

I. The Central Axiom. For every quantum mechanical operator $\hat{\mathbf{A}}$ there exists a corresponding adjacency operator $\hat{\mathbf{A}}(\mathcal{G})$ with an underlying graph $\mathcal{G}$.

II. The Geometric Algebra Axiom. The vertices of $\mathcal{G}$ are elements of the Clifford Algebra $Cl(\mathbb{R}^3) = \{1, e_1, e_2, e_3, e_1e_2, e_2e_3, e_1e_3, e_1e_2e_3\}$ where $\mathcal{V} = \{e_1, e_2, e_3\}$.

III. The State Space Axiom. The state space of graphs is $\omega = (\mathcal{G}_{\mathbf{p}}, \mathcal{G}_{\mathbf{q}})$ where the position space $(\hat{x}, \hat{y}, \hat{z})$ lies along the diagonal of $\hat{\mathbf{Q}}(\mathcal{G})$ and the momentum space $(\hat{p}_x, \hat{p}_y, \hat{p}_z)$ lies along the diagonal of $\hat{\mathbf{P}}(\mathcal{G})$.

IV. The Isotropic Axiom. There exists a morphism $\mathcal{G} \longrightarrow \mathcal{G}'$ such that $\operatorname{tr} \hat{\mathbf{Q}}(\mathcal{G}') = 0$ and $\operatorname{tr} \hat{\mathbf{P}}(\mathcal{G}') = 0$ where $(\hat{x}, \hat{y}, \hat{z}) \in \hat{\mathbf{Q}}(\mathcal{G}')$, $(\hat{p}_x, \hat{p}_y, \hat{p}_z) \in \hat{\mathbf{P}}(\mathcal{G}')$, and $\mathcal{V}' = \{e_1, e_2\}$ where $Cl(\mathbb{R}^2) = \{1, e_1, e_2, e_1e_2\}$.

V. The Structure Axiom. The operator power-spectrum $\mathcal{P}[\hat{\mathbf{A}}(\mathcal{G}')]$ generates a Hilbert space $\mathcal{H}$ of dimension $|\mathcal{V}|$ with the metric eigenfunction $g' = g'_+ + g'_-$ and state space $\omega' = (\mathcal{G'}_{\mathbf{p}}, \mathcal{G'}_{\mathbf{q}})$.

4. Construction of the Graphity Geometry

In this section we will first infer the off-diagonal elements of the higher dimensional operators $\hat{\mathbf{Q}}(\mathcal{G})$ and $\hat{\mathbf{P}}(\mathcal{G})$ and then solve for $\mathcal{G}$. We then move to the derivation of the transformed operators $\hat{\mathbf{Q}}(\mathcal{G}')$ and $\hat{\mathbf{P}}(\mathcal{G}')$, inferred from the structure of $\mathcal{G}$. Finally, we solve for $\mathcal{G}'$ and discuss its physical implications.

4.1. **A Higher Dimensional Classical Mechanics.** To infer the off-diagonal elements we first begin with a generalized 2nd-rank tensor:

1. $T = T^\top$ symmetric
2. $T = OT'O^\top$ diagonalizable
3. $OO^\top = I$ orthogonal
4. $det(O) = 1$ proper,

which can be diagonalized by matrices belonging to $SO(3)$. These tensors come equipped with a set of three rotational invariants:

1. $a = det(T) = \lambda_1^2\lambda_2^2\lambda_3^2$
2. $b = \dfrac{1}{2}[(\operatorname{tr} T)^2 - \operatorname{tr} T^2] = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2$
3. $c = \operatorname{tr} T = \lambda_1^2 + \lambda_2^2 + \lambda_3^2.$

Let us suppose T' represents a *homogeneous dilation* in which all eigenvalues are degenerate $r = \lambda_1 = \lambda_2 = \lambda_3$. The above rotational invariants then become squared geometric invariants:

1. $r^3 = \sqrt{(a/3)}$
2. $r^2 = \sqrt{(b/3)}$
3. $r = \sqrt{(c/3)}$.

We now seek a particular 2nd-rank tensor such that its trace is the Euclidean inner-product. Therefore, our tensor must take the form of the outer-product of the position and momentum vectors

$$\mathbf{q} \otimes \mathbf{q} = \begin{bmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{bmatrix}, \quad \mathbf{p} \otimes \mathbf{p} = \begin{bmatrix} p_x^2 & p_x p_y & p_x p_z \\ p_y p_x & p_y^2 & p_y p_z \\ p_z p_x & p_z p_y & p_z^2 \end{bmatrix}, \tag{4.1}$$

which are otherwise known as gyration tensors. Upon diagonalizing the above we see $x = \lambda_1,\ y = \lambda_2$, and $z = \lambda_3$. Therefore $\mathbf{q}\cdot\mathbf{q} \longrightarrow \operatorname{tr} T'_q$ and $\mathbf{p}\cdot\mathbf{p} \longrightarrow \operatorname{tr} T'_p$ constitute a canonical transformation. It's rather straightforward to demonstrate that if we replace phase space with its higher dimensional analogue $(\mathbf{q}\otimes\mathbf{q}, \mathbf{p}\otimes\mathbf{p})$ we can simply trace out classical mechanics for systems that are quadratic in position and momentum. This higher dimensional classical theory contains additional degrees of freedom, describing the shape or topology of phase space. As we prove below, these additional degrees of freedom are necessary for the gravitization of quantum mechanics.

4.2. **The Holographic Theorem.**

Theorem 4.1. *There exists a graph $\mathcal{G}_{\hat{\mathbf{q}}}$ and $\mathcal{G}_{\hat{\mathbf{p}}}$ with the self-edges $\mathcal{E}_{self} = (\hat{x}, \hat{y}, \hat{z})$, $\mathcal{E}_{self} = (\hat{p}_x, \hat{p}_y, \hat{p}_z)$, and with the vertices $\mathcal{V} = (e_1, e_2, e_3)$ such that the morphisms $\mathcal{G}_{\hat{\mathbf{q}}} \longrightarrow \mathcal{G}'_{\hat{\mathbf{q}}}$ and $\mathcal{G}_{\hat{\mathbf{p}}} \longrightarrow \mathcal{G}'_{\hat{\mathbf{p}}}$ provides a double cover of $\mathcal{E}_{self}$ with the reduced vertex set $\mathcal{V} = (e_1, e_2)$.*

Proof. Let us consider our above tensor where we replace the $q = (x, y, z)$ classical real-valued coordinates with the quantum operator-valued coordinates $\hat{q} = (\hat{x}, \hat{y}, \hat{z})$

$$\hat{\mathbf{q}} \otimes \hat{\mathbf{q}} = \begin{bmatrix} \hat{x}^2 & \hat{x}\hat{y} & \hat{x}\hat{z} \\ \hat{y}\hat{x} & \hat{y}^2 & \hat{y}\hat{z} \\ \hat{z}\hat{x} & \hat{z}\hat{y} & \hat{z}^2 \end{bmatrix}. \tag{4.2}$$

We now seek a composite operator containing both the operator-valued coordinates $(\hat{x}, \hat{y}, \hat{z})$ and the orthonormal basis (e_1, e_2, e_3). We want to construct this operator such that its trace is $\hat{\mathbf{q}} = \hat{x}e_1 + \hat{y}e_2 + \hat{z}e_3$, with the noncommutative basis:

1. $e_i^2 = 1$
2. $e_i e_j = -e_j e_i$
3. $i_2 = e_1 e_2 \equiv e_{1\cdot 2}, \quad i_3 = e_1 e_2 e_3 \equiv e_{1\cdot 2\cdot 3}$,

such that we can define a product of these operators which gives us $\hat{\mathbf{q}} \otimes \hat{\mathbf{q}}$. If we define the following 'square root vector'

$$\sqrt{\hat{\mathbf{q}} \cdot \mathbf{e}} := (\sqrt{\hat{x}e_1}, \sqrt{\hat{y}e_2}, \sqrt{\hat{z}e_3}), \tag{4.3}$$

we can construct our composite operator with the following outer-product

$$\pm(\sqrt{\hat{\mathbf{q}} \cdot \mathbf{e}}) \cdot \mathbf{e} \otimes \pm\sqrt{\hat{\mathbf{q}} \cdot \mathbf{e}} = \begin{bmatrix} \hat{x}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{y}e_2}e_1 & \sqrt{\hat{x}e_1}\sqrt{\hat{z}e_3}e_1 \\ \sqrt{\hat{y}e_2}\sqrt{\hat{x}e_1}e_2 & \hat{y}e_2 & \sqrt{\hat{y}e_2}\sqrt{\hat{z}e_3}e_2 \\ \sqrt{\hat{z}e_3}\sqrt{\hat{x}e_1}e_3 & \sqrt{\hat{z}e_3}\sqrt{\hat{y}e_2}e_3 & \hat{z}e_3 \end{bmatrix}. \tag{4.4}$$

We then can solve for $\mathcal{G}$ by identifying that the algebraic constants under the radical give us an adjacency relation where the algebraic constants outside of the radical give us the orientation of this relation. Therefore $\mathcal{G} = \mathcal{K}^3$, a complete directed graph on 3 vertices, which we call the 3-block. Let us denote the adjacency relation as $e_i \sim e_j \equiv e_{ij}$, then

$$\mathcal{K}^3 = (\mathcal{V}, \mathcal{E}) = \big(\{e_1, e_2, e_3\}, \{e_{ij}, e_{ii}\}\big), \quad val(e_{ii}) \in \mathbb{R}^3 \Rightarrow \mathbb{R}^3 \cong \mathcal{E}_{self}(\mathcal{K}^3). \tag{4.5}$$

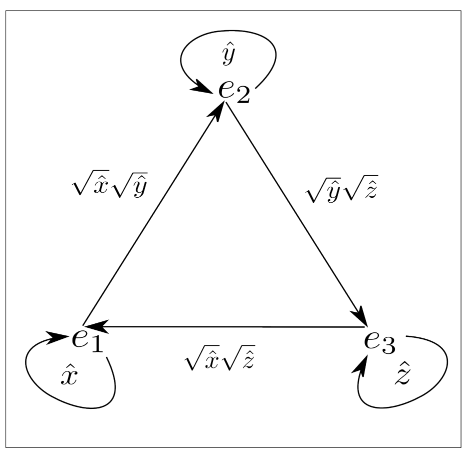

FIGURE 2. The 3-block is an algebraic operator-valued graph. $\mathbb{R}^3$ is recovered at its vertices where its edges represents an additional internal structure of the continuum encoding the spin-1/2 degree of freedom.

The ± 1 pre-factor in eq (3.4) indicates that it transforms as a spinor. We can re-write eq (3.4) as,

$$\hat{\mathbf{Q}}(\mathcal{K}^3) := e^{i_3\delta s}(\hat{\mathbf{q}} \cdot \mathbf{e})^s \cdot \mathbf{e} \otimes e^{i_3\delta s}(\hat{\mathbf{q}} \cdot \mathbf{e})^s \quad \text{for} \quad s = (1/2), \quad \delta = 2\pi \to -1, 4\pi \to +1$$

where the transpose flips the orientation of the edges, sending $s \mapsto -s$

$$\hat{\mathbf{Q}}^{\top}(\mathcal{K}^3) := e^{-i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^{-s}\otimes e^{-i_3\delta s}(\hat{\mathbf{q}}\cdot\mathbf{e})^{-s}\cdot\mathbf{e} \;\text{ for }\; s=(-1/2), \quad \delta = 2\pi \to -1, 4\pi \to +1.$$

We can recover the gyration tensor through the schur-product (element-wise product) of the adjacency matrix of 3-block with itself

$$\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3) = \hat{\mathbf{q}}\otimes\hat{\mathbf{q}}. \tag{4.6}$$

Given the 3D quantum harmonic oscillator (or any quadratic Hamiltonian),

$$\hat{H} = \frac{1}{2m}\hat{\mathbf{p}}\cdot\hat{\mathbf{p}} + \frac{1}{2}m\omega^2\hat{\mathbf{q}}\cdot\hat{\mathbf{q}} \tag{4.7}$$

we can define a new composite operator, which we call the Graphtonian

$$\begin{aligned}\hat{\mathrm{G}}(\mathcal{K}^3) &:= \frac{1}{2m}\hat{\mathbf{P}}(\mathcal{K}^3)\circ\hat{\mathbf{P}}(\mathcal{K}^3) + \frac{1}{2}m\omega^2\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3)\\ &= \frac{1}{2m}\begin{bmatrix}\hat{p}_x^2 & \hat{p}_x\hat{p}_y & \hat{p}_x\hat{p}_z\\ \hat{p}_y\hat{p}_x & \hat{p}_y^2 & \hat{p}_y\hat{p}_z\\ \hat{p}_z\hat{p}_x & \hat{p}_z\hat{p}_y & \hat{p}_z^2\end{bmatrix} + \frac{1}{2}m\omega\begin{bmatrix}\hat{x}^2 & \hat{x}\hat{y} & \hat{x}\hat{z}\\ \hat{y}\hat{x} & \hat{y}^2 & \hat{y}\hat{z}\\ \hat{z}\hat{x} & \hat{z}\hat{y} & \hat{z}^2\end{bmatrix},\end{aligned} \tag{4.8}$$

where

$$\begin{aligned}\hat{p} = -i_3\hbar\frac{\partial}{\partial x} &= -e_1e_2e_3\hbar\frac{\partial}{\partial x}\\ \hat{x} &= x\\ \hat{H} &= \operatorname{tr}\hat{\mathrm{G}}(\mathcal{K}^3)\\ &= \operatorname{tr}\hat{\mathrm{G}}\end{aligned} \tag{4.9}$$

Recall that the structure axiom demands $\mathcal{P} : \mathcal{G}' \longrightarrow \mathcal{H}$, which generates a complex Hilbert space from a graph $\mathcal{G}'$ if and only if $\operatorname{tr}\hat{\mathbf{A}}(\mathcal{G}') = 0$. Therefore, the morphism $\mathcal{G} \longrightarrow \mathcal{G}'$ must cause the Hamiltonian to vanish. To solve for $\mathcal{G}'$ we thus require

$$\operatorname{tr}\hat{\mathbf{Q}}(\mathcal{K}^3)\circ\hat{\mathbf{Q}}(\mathcal{K}^3) = \hat{\mathbf{q}}\cdot\hat{\mathbf{q}} = 0 \;\text{ and }\; \operatorname{tr}\hat{\mathbf{P}}(\mathcal{K}^3)\circ\hat{\mathbf{P}}(\mathcal{K}^3) = \hat{\mathbf{p}}\cdot\hat{\mathbf{p}} = 0, \tag{4.10}$$

which forms a 2-dimensional operator-valued isotropic Hilbert space, a sufficient condition for the construction of spinors. We can write down a matrix $\hat{\mathbf{Q}}(\mathcal{G}')$ that represents $\hat{\mathbf{q}}$ in complex 3-space. This matrix admits a factorization as an outer-product

$$\hat{\mathbf{Q}}(\mathcal{G}') = 2\begin{bmatrix}\hat{\zeta}_0\\ \hat{\zeta}_1\end{bmatrix}\begin{bmatrix}\hat{\zeta}_0 & \hat{\zeta}_1\end{bmatrix}, \tag{4.11}$$

yielding the overdetermined system of equations

$$\begin{aligned} \hat{\zeta}_0^2 - \hat{\zeta}_1^2 &= \hat{x} \\ i_2(\hat{\zeta}_0^2 + \hat{\zeta}_1^2) &= \hat{y} \\ -2\hat{\zeta}_0\hat{\zeta}_1 &= \hat{z} \end{aligned} \tag{4.12}$$

with the solutions

$$\hat{\zeta}_0 = \pm\sqrt{\frac{\hat{x} - i_2\hat{y}}{2}}, \quad \hat{\zeta}_1 = \pm\sqrt{\frac{-\hat{x} - i_2\hat{y}}{2}}. \tag{4.13}$$

We solve eq. (4.13) by taking the Pauli vector

$$\vec{\sigma} = \sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3, \tag{4.14}$$

and dotting it with $\hat{\mathbf{q}}$, giving

$$\hat{\mathbf{Q}}(\mathcal{K}^2) = \hat{\mathbf{q}} \cdot \vec{\sigma} = \begin{bmatrix} -\hat{z} & \hat{x} - i_2\hat{y} \\ \hat{x} + i_2\hat{y} & \hat{z} \end{bmatrix}. \tag{4.15}$$

Thus, under the following map,

$$\hat{\mathbf{Q}}(\mathcal{K}^3) \longrightarrow \hat{\mathbf{Q}}(\mathcal{K}^2), \quad \hat{\mathbf{P}}(\mathcal{K}^3) \longrightarrow \hat{\mathbf{P}}(\mathcal{K}^2) \tag{4.16}$$

the Graphtonian becomes

$$\begin{aligned} \hat{\mathrm{G}} &= \frac{1}{2m}\hat{\mathbf{p}} \cdot \vec{\sigma} + \frac{1}{2}m\omega^2\hat{\mathbf{q}} \cdot \vec{\sigma}, \\ &= \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x - i_2\hat{p}_y \\ \hat{p}_x + i_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega\begin{bmatrix} -\hat{z} & \hat{x} - i_2\hat{y} \\ \hat{x} + i_2\hat{y} & \hat{z} \end{bmatrix} \\ &= \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x - e_1e_2\hat{p}_y \\ \hat{p}_x + e_1e_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega\begin{bmatrix} -\hat{z} & \hat{x} - e_1e_2\hat{y} \\ \hat{x} + e_1e_2\hat{y} & \hat{z} \end{bmatrix} \\ &= \frac{1}{2m}\begin{bmatrix} -\hat{p}_z & \hat{p}_x + e_2e_1\hat{p}_y \\ \hat{p}_x + e_1e_2\hat{p}_y & \hat{p}_z \end{bmatrix} + \frac{1}{2}m\omega\begin{bmatrix} -\hat{z} & \hat{x} + e_2e_1\hat{y} \\ \hat{x} + e_1e_2\hat{y} & \hat{z} \end{bmatrix} \end{aligned} \tag{4.17}$$

where

$$\begin{aligned}
\hat{p} &= -i_2\hbar\frac{\partial}{\partial x} = -e_1e_2\hbar\frac{\partial}{\partial x} \\
\hat{x} &= x \\
\hat{H}_{grav} &= \operatorname{tr}\hat{\mathrm{G}}(\mathcal{K}^2). \\
&= \operatorname{tr}\hat{\mathrm{G}}' \\
&= 0
\end{aligned} \tag{4.18}$$

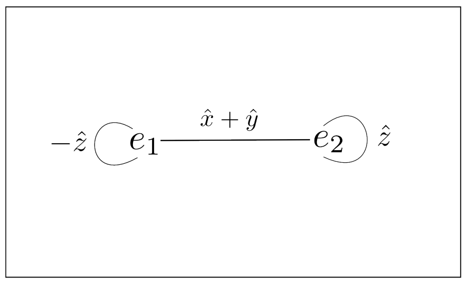

Figure 3. The 2-block is a superimposed 3-block, which can collapse into either a spin $+1/2$ or $-1/2$ 3-block.

$$\mathcal{K}^2 = \big(\{e_1, e_2\}, \{e_1, e_2\}, \{e_{11}, e_{22}\}\big),\ \ val(e_{11}) \in \mathbb{R},\ \ val(e_{22}) \in \mathbb{R} \Rightarrow \mathcal{E}_{self}(\mathcal{K}^2) \cong \mathbb{R}^2.$$

$\square$

5. A New Cosmic Arena

The blockmorphism $\mathrm{B} : \mathcal{K}^3 \longrightarrow \mathcal{K}^2$ maps a topological phase space to a relativistic phase space with an internal spin-1/2 angular momentum $\hat{p} \cdot \vec{\sigma}$ and an internal spin-1/2 angular position $\hat{q} \cdot \vec{\sigma}$. We can write $\hat{\mathbf{q}} \otimes \hat{\mathbf{q}} = X\Lambda X^{-1}$ where $X \in SO(3)$ and $\hat{\mathbf{q}} \cdot \vec{\sigma} = X'\Lambda' X'^{-1}$ where $X' \in SU(2)$. Therefore, we have the following mapping

$$(\underbrace{\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}}_{SO(3)}, \underbrace{\hat{\mathbf{q}} \otimes \hat{\mathbf{q}}}_{SO(3)}) \longrightarrow (\underbrace{\hat{\mathbf{p}} \cdot \vec{\sigma}}_{SU(2)}, \underbrace{\hat{\mathbf{q}} \cdot \vec{\sigma}}_{SU(2)}),$$

which generates the group-theoretic representation of 4-dimensional Euclidean space

$$\begin{aligned}
&SU(2) \times SU(2) \longrightarrow SO(4) \\
&SO(4) \longrightarrow SO(3) \times SO(3),
\end{aligned} \tag{5.1}$$

thereby rendering the Geometric Graphity Lorentz invariant. We now have all of the necessary tools to build a dynamic noncommutative spacetime. We can entangle N independent quantum harmonic oscillators with the following set of rules:

1. e_i's can only be glued (entangled) to other e_i's;

2. either all e_i's of a pair of 2-blocks or 3-blocks are glued together or none are;
3. if a pair of 3-blocks are glued together they must have opposite orientations (spin 1/2 or -1/2) on their internal edge set.

Since there are two possible orientations of the 3-block, we identify the 2-block as the 3-block's superposition state. Let us now define the direct sum with respect to the above as the gluing of the respective e_i's; for example:

$$\begin{aligned} \mathcal{K}^2 \oplus \mathcal{K}^2 &= \mathcal{C}^4 \quad \text{(block-4-cycle)} \\ \mathcal{K}^3 \oplus \mathcal{K}^3 &= \mathcal{Y}^3 \quad \text{(block-3-prism)} \end{aligned} \tag{5.2}$$

such that the adjacency relation is defined as $e_i^n \sim e_j^m \equiv e_{ij}^{nm}$, where the lower indices index the Clifford basis and the upper indices index the oscillators. For the block-4-cycle we write the vertex (2D Clifford basis), internal edge (spin-1/2 degrees of freedom), external edge (entanglement degrees of freedom), and the self-edges (2D spatial degrees of freedom) sets as

$$\begin{aligned} \mathcal{V}'^n &\equiv e_i^n = \{e_1^n, e_2^n\} \\ \mathcal{E}'_{int} &\equiv e_{ij}^{nn} = \{e_{12}^{22}, e_{12}^{11}\} \\ \mathcal{E}'_{ext} &\equiv e_{ii}^{nm} = \{e_{11}^{21}, e_{22}^{21}\} \\ \mathcal{E}'_{self} &\equiv e_{jj}^{mm} = \{e_{11}^{11}, e_{22}^{11}, e_{11}^{22}, e_{22}^{22}\} \end{aligned} \tag{5.3}$$

$$\mathcal{C}^4 := \left(\mathcal{V}'^1, \mathcal{V}'^2, \mathcal{E}'_{int}, \mathcal{E}'_{ext}, \mathcal{E}'_{self}\right) = \left(e_i^1, e_i^2, e_{ij}^{nn}, e_{jj}^{nm}, e_{ii}^{mm}\right) \tag{5.4}$$

For the block-3-prism we write the vertex (3D Clifford basis), internal edge (spin-1/2 degree's of freedom), external edge (entanglement degrees of freedom), and the self-edges (3D spatial degrees of freedom) sets as

$$\begin{aligned} \mathcal{V}^n &\equiv e_i^n = \{e_1^n, e_2^n, e_3^n\} \\ \mathcal{E}_{int} &\equiv e_{ij}^{nn} = \{e_{12}^{11}, e_{13}^{11}, e_{23}^{11}, e_{12}^{22}, e_{23}^{22}, e_{13}^{22}\} \\ \mathcal{E}_{ext} &\equiv e_{jj}^{nm} = \{e_{11}^{12}, e_{22}^{12}, e_{33}^{12}\} \\ \mathcal{E}_{self} &\equiv e_{ii}^{mm} = \{e_{11}^{11}, e_{22}^{11}, e_{33}^{11}, e_{11}^{22}, e_{22}^{22}, e_{33}^{22}\} \end{aligned} \tag{5.5}$$

$$\mathcal{Y}^3 := \left(\mathcal{V}^1, \mathcal{V}^2, \mathcal{E}_{int}, \mathcal{E}_{ext}, \mathcal{E}_{self}\right) = \left(e_i^1, e_i^2, e_{ij}^{nn}, e_{jj}^{nm}, e_{ii}^{mm}\right) \tag{5.6}$$

Thus far we've established the commutation relation on the internal edge set $\mathcal{E}_{int}$ which we've defined as the Clifford basis. The non-commutativity of the internal edge set implies that the positions of each oscillator cannot be measured simultaneously with respect to all axis. The external edge set $\mathcal{E}_{ext}$ entangles N independent frames of reference by coupling the internal spin-1/2 degrees of freedom for each oscillator. We define the commutation relation on $\mathcal{E}_{ext}$ for the 2-block as

$$[e^n, e^m] = i_2 \mathbf{A}^{nm}, \tag{5.7}$$

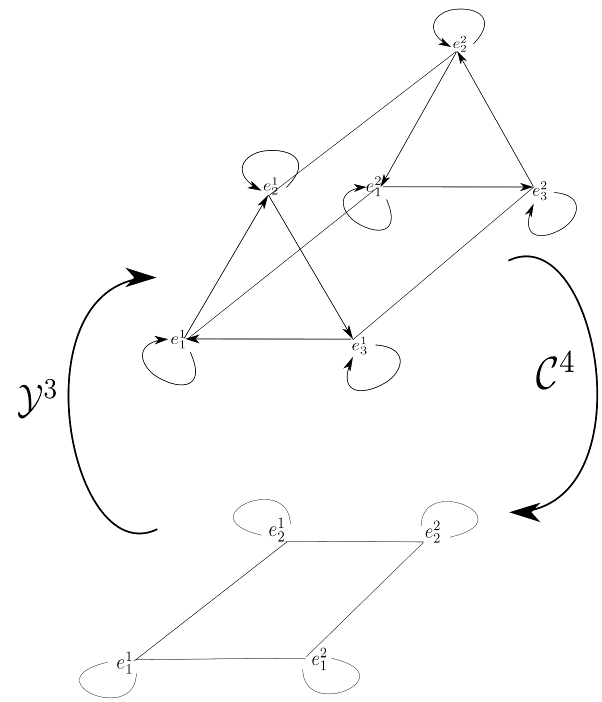

FIGURE 4. The block-4-cycle is a superposition of two block-3-prisms. We can think of the blockmorphism $\mathrm{B} : \mathcal{C}^4 \longrightarrow \mathcal{Y}^3$ as a precipitation-like event, where the block-4-cycle precipitates off of the boundary of the block-3-prism and into its bulk space. The inverse blockmorphism $\mathrm{B} : \mathcal{Y}^3 \longrightarrow \mathcal{C}^4$ can be thought of as a condensation-like event, where the block-3-prism in the bulk space condensates on the surface of its boundary. Alternatively, we can think of the condensation-like event as an optimal lossless compression algorithm on $(\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}, \hat{\mathbf{q}} \otimes \hat{\mathbf{q}})$ to $(\hat{\mathbf{p}} \cdot \vec{\sigma}, \hat{\mathbf{q}} \cdot \vec{\sigma})$, and the precipitation-like event as an optimal lossless decompression algorithm from $(\hat{\mathbf{p}} \cdot \vec{\sigma}, \hat{\mathbf{q}} \cdot \vec{\sigma})$ to $(\hat{\mathbf{p}} \otimes \hat{\mathbf{p}}, \hat{\mathbf{q}} \otimes \hat{\mathbf{q}})$.

and for the 3-block

$$[e^n, e^m] = i_3 \mathbf{A}^{nm}, \tag{5.8}$$

where

$$\mathbf{A} = \begin{cases} 1, & n \sim m \\ 0, & \text{otherwise} \end{cases} \tag{5.9}$$

and where the uncertainty of the positions along each axis of each oscillator is given by

$$\Delta e^n \Delta e^m = l_p^2. \tag{5.10}$$

5.1. **The Problem of Time.** We can immediately identify the following conservation laws within the Geometric Graphity framework:

$$\Delta \mathcal{V} = 0 \quad \text{(Conservation of Basis)}, \tag{5.11}$$

$$\Delta \mathcal{E}_{int} = 0 \quad \text{(Conservation of Spin)}. \tag{5.12}$$

The number of vertices of the 2-block and the 3-block must always remain constant and the internal spin-1/2 degrees of freedom, too, must always remain constant. Furthermore, for any configuration of the entanglement structure $\mathcal{E}_{ext}$, the total energy of the system will always remain constant

$$\forall \Delta \mathcal{E}_{ext}, \quad \Delta \operatorname{tr} \hat{\mathrm{G}}(\mathcal{K}^3) = 0 \;\; \text{(Local Conservation of Energy)} \tag{5.13}$$

and therefore,

$$\Delta \mathcal{E}_{ext} \cong t \quad \text{(Emergence of Time)}. \tag{5.14}$$

If we introduce the hyper-edge $\mathcal{E}_{charge} \equiv e_q$, we can identify all permutations of $\mathcal{V}$ along with the empty set as the elementary charges

$$q = \begin{cases} +1, & \{e_1, e_2, e_3\} \\ +1/3, & \{e_1, e_3, e_2\} \\ +2/3 & \{e_2, e_1, e_3\} \\ -1/3, & \{e_2, e_3, e_1\} \\ -2/3, & \{e_3, e_1, e_2\} \\ -1, & \{e_3, e_2, e_1\} \\ 0, & \{\emptyset\} \end{cases} \tag{5.15}$$

The Hamiltonian then remains invariant under the following transformations:

1. $\hat{\mathcal{Q}}(\mathcal{K}^3) \longrightarrow \hat{\mathcal{Q}}^\top(\mathcal{K}^3)$ Spin Conjugation
2. $\hat{\mathcal{Q}}(\mathcal{K}^3) \longrightarrow -\hat{\mathcal{Q}}(\mathcal{K}^3)$ Charge Conjugation
3. $\mathcal{E}_{self} \longrightarrow -\mathcal{E}_{self}$ Parity Conjugation

which we will refer to as Charge-Parity-Spin (CPS) Symmetry. We've identified that changes in the entanglement structure of the system *is* time, where the actions on the system are the gluing and cutting of external edges. Running time backwards is thus tantamount to the undoing of those actions. Therefore, we arrive at a time-charge-entanglement relation

$$\Delta^q \mathcal{E}_{ext} = qt \tag{5.16}$$

where the above CPS Symmetry is enhanced to a Mirror Symmetry with

4. $\Delta \mathcal{E}_{ext} \longrightarrow \Delta^{-1} \mathcal{E}_{ext}$ Time Inversion.

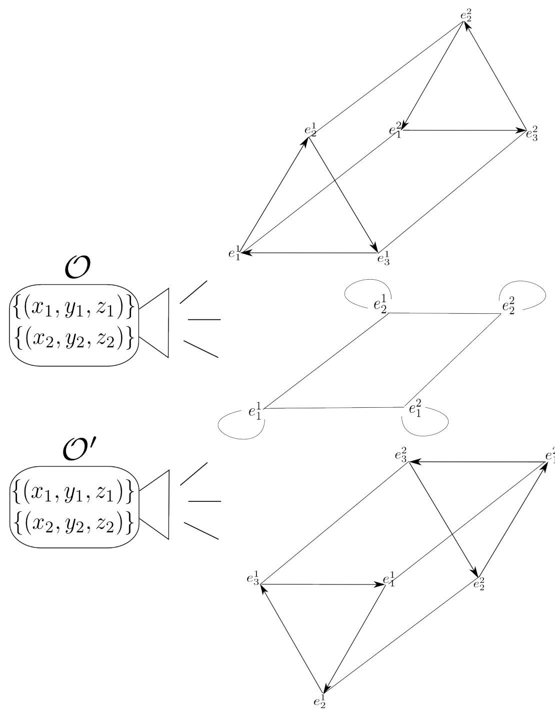

FIGURE 5. An observer and their mirror image moving the system from the ground energy state (2-block) to the next energy state (3-block) by storing position data.

5.2. **The Problem of Measurement.** We've demonstrated a local conservation of energy in the 2-block and 3-block basis states. Transitions between these states, described by the blockmorphism, has the following Hamiltonian representation $\hat{H}_{grav}\Psi \longrightarrow \hat{H}\Psi$ yielding $0 \longrightarrow E\Psi$. This is in clear violation of a global conservation of energy. One could argue that this isn't a problem by invoking the energy-time uncertainty relation. This however does not work in the Graphity framework since time only emerges on the 3-block side of the theory. On the the 2-block side, the entanglement structure can propagate but the system remains frozen in its ground energy state. The time-energy uncertainty relation only becomes meaningful after a transition from a 2-block to a 3-block. Furthermore, the Hilbert spaces of the respective basis states are incommensurable. On the one hand, we have a 2-dimensional Hilbert space on the 2-block side, whereas on the other hand, we have an infinite dimensional Hilbert space on the 3-block side. For Geometric Graphity to remain consistent, there must exist a unique Hilbert space $\mathcal{H}$ such that $\mathcal{H}(\mathcal{K}^2) = \mathcal{H}^2$ is the bounded region of $\mathcal{H}(\mathcal{K}_3) = \mathcal{H}^3$. Therefore, $\mathcal{H}$ must be commensurable (finite in index)

$$comm(\mathcal{H}) = [\mathcal{H} : \mathcal{H}^2 \cap \mathcal{H}^3] < \infty, \tag{5.17}$$

which implies that the blockmorphism be replaced with

$$\texttt{measure}[\mathcal{O}] : \begin{bmatrix} -\hat{z} & \hat{x} - i_2\hat{y} \\ \hat{x} + i_2\hat{y} & \hat{z} \end{bmatrix} \longrightarrow \left\{ \begin{bmatrix} 0 & \sqrt{x}\sqrt{y} & \sqrt{x}\sqrt{z} \\ \sqrt{y}\sqrt{x} & 0 & \sqrt{y}\sqrt{z} \\ \sqrt{z}\sqrt{x} & \sqrt{z}\sqrt{y} & 0 \end{bmatrix} \middle| \, \mathcal{O} \supset (x, y, z) \right\}, \tag{5.18}$$

where an observer $\mathcal{O}$ measures $\hat{\mathbf{q}} \cdot \vec{\sigma}$, sending it to the collapsed state $\mathcal{Q}(\mathcal{K}_3) := \mathcal{Q}(\mathcal{K}^3) - diag(x, y, z)$, such that the observer $\mathcal{O}$ is a proper superset of the measured quantities (x, y, z). Recall that the blockmorphism is 2-to-1 where $\hat{\mathcal{Q}}(\mathcal{K}^3) \circ \hat{\mathcal{Q}}(\mathcal{K}^3) \longrightarrow \hat{\mathbf{q}} \cdot \vec{\sigma}$ and so the measuremorphism is 2-to-1. This together with the Mirror Symmetry result implies the existence of an observer in a mirror universe, where all spin up measurements become spin down measurements, all matter is replaced with anti-matter, all positions are reflected, and time is reversed. We can restate the above as the *Principle of Observation*:

for every observation $\mathcal{O}$ there exists an equal and opposite observation $\mathcal{O}'$.

This ensures a global conservation of energy where our toy universe exactly cancels with its mirror image. From the frog's perspective, both observers will experience identical universes evolving forward in time, causally disconnected from one another. From the bird's perspective, this universe is an eternal and unchanging mathematical structure. This however does not mean our toy universe in any way requires the existence of the observer. This universe without observers remains stuck in the 2-block basis state, where the entanglement structure evolves independently of the observer, but forever remains frozen in its ground energy state. The role of the observer is then to move the system from its ground energy state to a higher energy state by decompressing a 2-block into a 3-block and by storing the self-edges $\mathcal{E}_{self}$. It is in this capacity that the observer can be any arbitrary information processing and storage device with a finite memory.

5.3. **Origins of an Expanding Universe.** Suppose we prepare our toy universe in a state that minimizes entropy. This can be done by gluing N 2-blocks together, such that any 2-block can be reached in a single step

$$\mathfrak{S}(N) := \bigoplus_{i=1}^{N} \mathcal{K}_i^2, \quad \mathcal{H}_{sys} = \bigotimes_{i=1}^{N} \mathcal{H}(\mathcal{K}_i^2) \tag{5.19}$$

If we make N copies

$$\{\mathfrak{S}_1(N), \mathfrak{S}_2(N), ..., \mathfrak{S}_N(N)\}, \tag{5.20}$$

then each $\mathfrak{S}(N)$ acts as the unit cell of a "false singularity" where the pairwise distance between every $\mathfrak{S}(N)$ is zero. Specifying the current state of the entire system requires only a single unit cell, and so the system is in a minimum state of entropy. Specifying the next state of the system requires the diagonalization of the adjacency matrix of the unit cell. Since this matrix has the maximum number of off-diagonal elements it is in a maximum state of computational complexity. Let's suppose $N = 3$ and we make a cut producing a $\mathcal{K}^2$ and a $\mathcal{C}^4$ on the boundary with the corresponding $\mathcal{K}_3$ and $\mathcal{Y}_3$ in the bulk. Let's assume the distances on the boundary, the bulk, and the self-edges are all proportional

$$d(\mathcal{K}^2, \mathcal{C}^4) \propto d(\mathcal{K}_3, \mathcal{Y}_3) \propto d(\mathcal{E}_{self}(\mathcal{K}_3), \mathcal{E}_{self}(\mathcal{Y}_3)). \tag{5.21}$$

Since $||g(\mathcal{C}^4)|| > ||g(\mathcal{K}^2)||$ the distance between the two disconnected subgraphs increases as does the distance between the self-edges. We can now imagine ways of cutting and gluing external edges on the boundary that results in either dilations or contractions in the bulk. Dilations can be generated for pairs of disconnected subgraphs G and H, such that $|V(G)|$ is strictly increasing with respect to $|V(H)|$, where contractions can be generated when $|V(G)|$ is approaching $|V(H)|$. If at

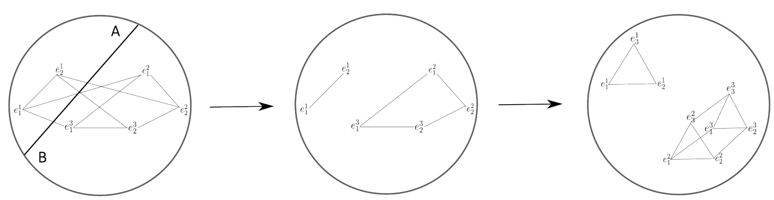

FIGURE 6. A cut of $\mathfrak{S}(3)$ into two subsystems A and B.

every iteration we double the amount of disentanglement between G, the rest of the system, and H and the rest of the system, then both observers will experience an acceleration away from each other. Alternately, if we double the amount of entanglement then they will experience an acceleration towards each other. For this toy universe to exhibit features of our expanding universe, there must not only exist anisotropies in the microscopic structure of spacetime, but also an intrinsic mass accompanying the spin angular position. Furthermore, since the observer is finite in memory this toy universe must also be finite.

5.4. **The Cosmological Theorem.**

Theorem 5.1. *The Cosmological constant Λ sets the bound of a universe that is finite both in space $l_p \leq l \leq l_b$ and in time $t_p \leq t \leq t_b$.*

Proof. From the **Structure Axiom** the operator-valued power-spectrum generates the Hilbert space $\mathcal{H}(\mathcal{K}^2)$,

$$\begin{aligned} \mathcal{P}[\hat{\mathcal{Q}}(\mathcal{G})] &= \hat{\mathcal{Q}}^{(\delta/2\pi)}(\mathcal{G}) \\ &= (\hat{\mathbf{q}} \cdot \hat{\sigma})^{(\delta/2\pi)}. \end{aligned} \tag{5.22}$$

From the **Graphity Field Theorem** we can re-write the above as the Graphity Field Equation and solve for the metric eigenfunction by the spectral decomposition

$$\begin{aligned} |\mathcal{G}\rangle &= \mathbf{L}^+ |g_+\rangle + \mathbf{L}^- |g_-\rangle \\ &= \sum_i U_{ij} |g_i\rangle + \sum_j U_{ij} |g_j\rangle \\ &= \sum_i U_{ij} \exp\left\{ \frac{\delta}{2\pi} \ln \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2} \right\} \\ &\quad + \sum_j U_{ij} \exp\left\{ \frac{\delta}{2\pi} \Big(\ln \big(\sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2} \big) + i_2 \pi \Big) \right\} \end{aligned} \tag{5.23}$$

giving us

$$|g\rangle = |g_+\rangle + |g_-\rangle = e^{(\delta/2\pi) \ln \sqrt{\hat{x}^2+\hat{y}^2+\hat{z}^2}} (1 + e^{i_2 \delta/2}). \tag{5.24}$$

Substituting $l_p = \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2}$ in the above gives us the metric at the Planck scale

$$g(\delta) = l_p^{(\delta/2\pi)} (1 + e^{i_2 \delta/2}). \tag{5.25}$$

Expanding this equation out we have

$$g(\delta) = \exp\left\{ \frac{\delta}{4\pi} \ln \Big(\frac{G\hbar}{c^3} \Big) \right\} + \exp\left\{ \frac{\delta}{4\pi} \Big(\ln \Big(\frac{G\hbar}{c^3} \Big) + 2 i_2 \pi \Big) \right\}. \tag{5.26}$$

If we restrict the rotational exponent δ to only take on values contained in the set *const.* $= (\Lambda, G, c, \hbar, k_B, \pi)$ we have two consistent ways of choosing δ such that g becomes an equation of state. We can choose

$$\delta_1 = -\pi k_B \Lambda \quad \text{or} \quad \delta_2 = -\frac{\pi k_B c^3}{G\hbar}. \tag{5.27}$$

The Planckian metric evaluated at the above gives us a pair of Bekenstien-Hawking entropies where the area of a black hole is replaced with the area information content at the Planck scale $I_A = -\ln A_p$,

$$S_1 = -\frac{k_B \Lambda}{4} \ln \Big(\frac{G\hbar}{c^3} \Big), \quad S_2 = -\frac{k_B c^3}{4G\hbar} \ln \Big(\frac{G\hbar}{c^3} \Big). \tag{5.28}$$

We can now write the metric as a system of equations of state depending on pressure, area, and temperature.

$$g_{21} = g(P_{21}, A_p, T) = A_p^{(-k_B\Lambda/4)}\left(1 + \exp\left\{\frac{-2l_p e_2 e_1 P_{21}}{T}\right\}\right) \tag{5.29}$$

$$g_{12} = g(P_{12}, A_p, T) = A_p^{(-k_B c^3/4G\hbar)}\left(1 + \exp\left\{\frac{-2l_p e_1 e_2 P_{12}}{T}\right\}\right). \tag{5.30}$$

We solve for the following pressures

$$P_{21} = \frac{\Lambda\hbar a}{6\pi^2 l_p c} \quad \text{and} \quad P_{12} = -\frac{c^2 a}{6\pi^2 l_P G} \tag{5.31}$$

where T is the Unruh temperature

$$T = \frac{\hbar a}{2\pi c k_B}. \tag{5.32}$$

With the pressure-force relation

$$P = \frac{F}{A_p}, \tag{5.33}$$

and Newton's 2nd Law

$$F = ma, \tag{5.34}$$

.

we can calculate a mass corresponding to each equation of state in terms of the fundamental constants

$$m_1 = \frac{\Lambda\hbar^{3/2}G^{1/2}}{6\pi^2 c^{5/2}} \approx 0.96 \times 10^{-131}\text{kg} \tag{5.35}$$

$$m_2 = \frac{\hbar c}{6\pi^2\sqrt{G\hbar c}} \approx 3.6 \times 10^{-10}\text{kg}, \tag{5.36}$$

which allows us to express the cosmological constant as the ratio of these masses

$$\frac{m_1}{m_2} = \Lambda l_P^2. \tag{5.37}$$

We can now complete the proof by computing the Compton wavelength of m_0 and m_1

$$\lambda_2 = 6\pi^2 l_p \tag{5.38}$$

$$\lambda_1 = 6\pi^2 l_b = 6\pi^2\left(\frac{1}{\Lambda l_p}\right) \tag{5.39}$$

providing us with the relation

$$\Lambda = \frac{1}{l_p l_b}, \tag{5.40}$$

which bounds a finite universe by

$$\begin{aligned} 10^{-35} &\leq l \leq 10^{87}\ \textit{meters} \\ 10^{-44} &\leq t \leq 10^{79}\ \textit{seconds}. \end{aligned} \tag{5.41}$$

Extremis veritas mysterium tremendum et fascinosum □

6. The Structure Sector

The Cosmological Theorem predicts that the microscopic structure of spacetime is an internal spin-1/2 field with a conformally invariant metric that is a function of either the entanglement entropy S_1 or disentanglement entropy S_2 of the masses m_1 and m_2, intrinsic to the vacuum. m_1 is extraordinarily light, weighing an astonishing 10^{-96} ev$/c^2$, exerting an extremely large negative pressure $(-10^{61}$ kg$/m^2)\cdot a$ on m_2, weighing 10^{16} Gev$/c^2$ or $.033M_{pl}$, which exerts an astonishingly small positive pressure $(10^{-60}$ kg$/m^2)\cdot a$ on m_1. This implies the existence of a 'structure sector' that simultaneously describes spacetime and the internal structure of matter. We

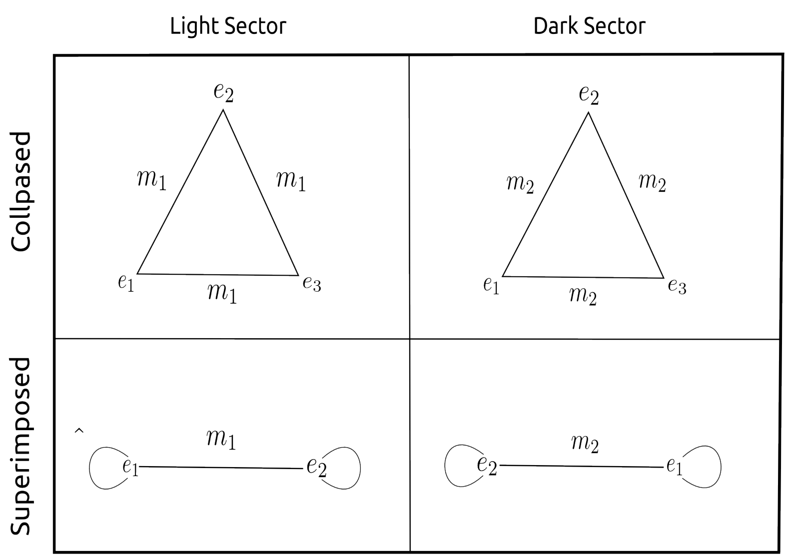

Figure 7. The Light and Dark sectors in the 2-block (superimposed) and 3-block (collapsed) basis states.

can assign m_1 and m_2 as the two possible edge weights, yielding two structure sectors that modify the masses of the fundamental fermions. These sectors describe empty space, where we can 'fill' them by embedding the masses m and charges e_q of the fundamental fermions in the above graphs, yielding a supersymmetry where every fermion has a supermassive partner with identical spin and charge.

6.1. **Construction of the Proton and its Superpartner.** In order to construct the proton we need a way of embedding the color charges r (red), $\bar{r}$ (antired), b (blue), $\bar{b}$ (antiblue), g (green), and $\bar{g}$ (antigreen). We can do this by introducing the algebraic constants e_c and $e_{\bar{c}}$ where

$$c = \begin{cases} r, & \{m_1, m_u, m_d\} \\ b, & \{m_1, m_d, m_u\} \\ g & \{m_d, m_1, m_u\} \\ \bar{r}, & \{m_d, m_u, m_1\} \\ \bar{b}, & \{m_u, m_d, m_1\} \\ \bar{g}, & \{m_u, m_1, m_d\} \end{cases}, \quad \bar{c} = \begin{cases} \bar{r}, & \{m_2, m_u, m_d\} \\ \bar{b}, & \{m_2, m_d, m_u\} \\ \bar{g} & \{m_d, m_2, m_u\} \\ r, & \{m_d, m_u, m_2\} \\ b, & \{m_u, m_d, m_2\} \\ g, & \{m_u, m_2, m_d\}, \end{cases} \tag{6.1}$$

the color charges of the Light proton correspond to anticolor charges of the Dark proton. Recall that when we were building a spacetime from entanglement relations, if two 3-blocks are glued together they must have opposite orientations on their internal edge set. In order to build a proton we must glue two up quarks and one down quark where the entanglement relations are replaced with a bosonic relation, namely the gluon. The gluon relation must carry a rule such that the spin of the proton, like all of its constituents, is spin 1/2. We can choose a rule that maps each 3-block of the proton to a vertex of a new graph whose structure is identical to the 3-block. The gluon relations then become the internal edge set of the proton which can be oriented in one of two possible ways and which can be colored with $c\bar{c}$ in one of nine possible ways.

6.2. **A New Kind of Physics.** We've opened the door to a fundamentally new kind of physics – a digital paradigm in which the CFUH banishes all UV/IR divergences and where *computability* is treated on equal footing with *falsifiability*. This story, however, is far from complete. So far, the model naturally encodes the fermions but it remains to be seen how exactly the model encodes bosons. Like the fermions the bosons too should have a unique superimposed and collapsed basis state. If we look at the operator-valued power-spectrum $\mathcal{P}[\hat{\mathbf{A}}(\mathcal{G})] = \hat{\mathbf{A}}^{(\delta s/\pi)}(\mathcal{G})$, *all* spin values are naturally encoded in the exponent. Half-integral spin values correspond to fractional length walks on $\mathcal{G}$ and are assigned a complex phase whereas integral spin values correspond to integral length walks on $\mathcal{G}$ and are assigned no phase. In the spin-1/2 setting $\hat{\mathbf{A}}(\mathcal{G}) = \hat{\mathbf{q}} \cdot \vec{\sigma}$ and so for example in the spin-1 setting we would use the spin-1 Pauli matrix to solve for $\mathcal{G}$. Indeed, this digital paradigm is in direct challenge to the $U(1) \times SU(2) \times SU(3)$ Standard Model where instead of quanta being point-like they are graph-like, suggesting we need only the n-dimensional irreducible representations of $SU(2)$ for $n = 1, 2, 3$ to describe all particle interactions including gravity. This grand simplification may one day open the door to a **Grand Unified Theorem of Everything** – a single theorem capable of predicting all observable phenomena in our universe.

6.3. **The Multiverse.** In the digital paradigm the fundamental constants $(G, \hbar, c)$ are truly free parameters whose values cannot be computed from the axioms of the theory. The fundamental irremovable vacuum masses $\dfrac{\Lambda \hbar^{3/2} G^{1/2}}{6\pi^2 c^{5/2}}$ and $\dfrac{\hbar c}{6\pi^2 \sqrt{G\hbar c}}$ provide a measure across an infinite multiverse $(G, \hbar, c) \in \mathbb{R}$ of finite universes bounded by Λ. Across this infinite multiverse there exists one and only one number that is guaranteed to be observed by all observers, the cosmological constant Λ.

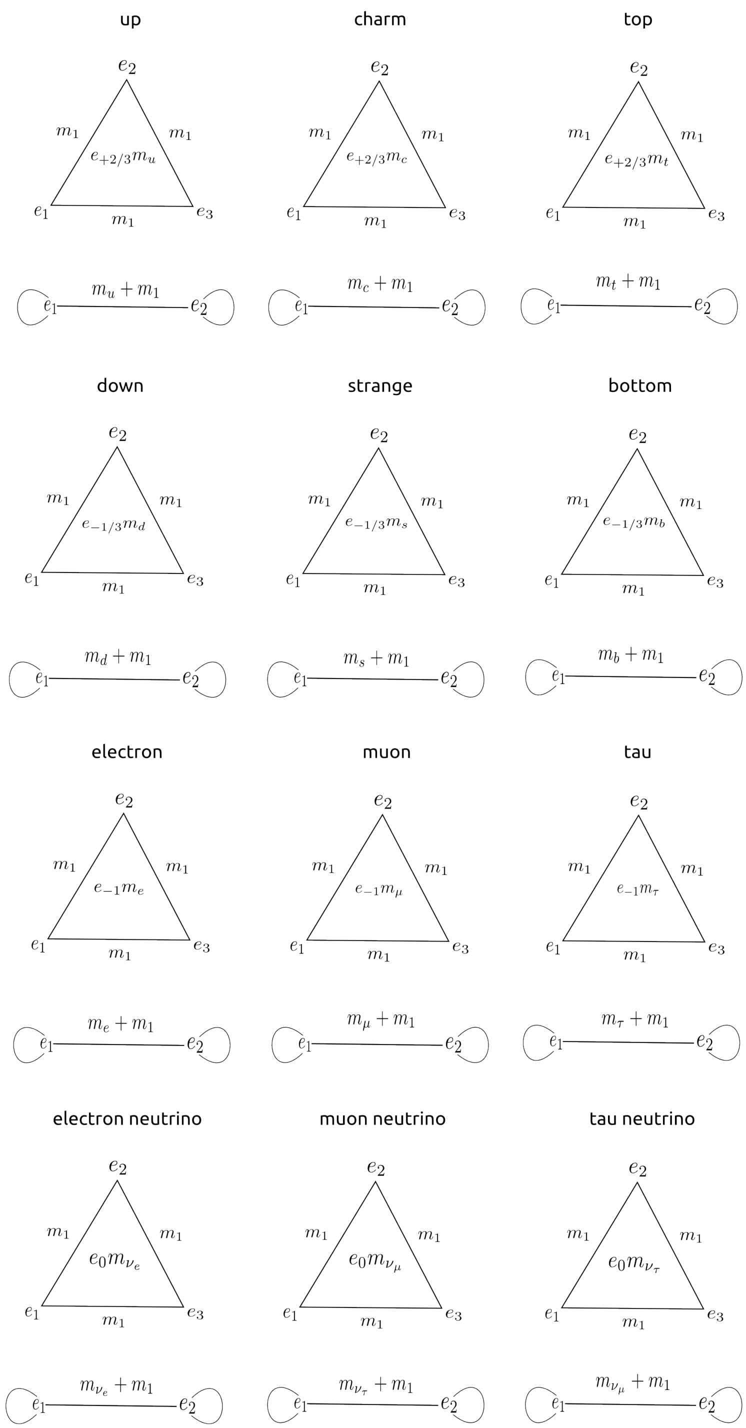

FIGURE 8. Standard Model fermions embedded in the Light sector with the modified masses $m_1 + m$.

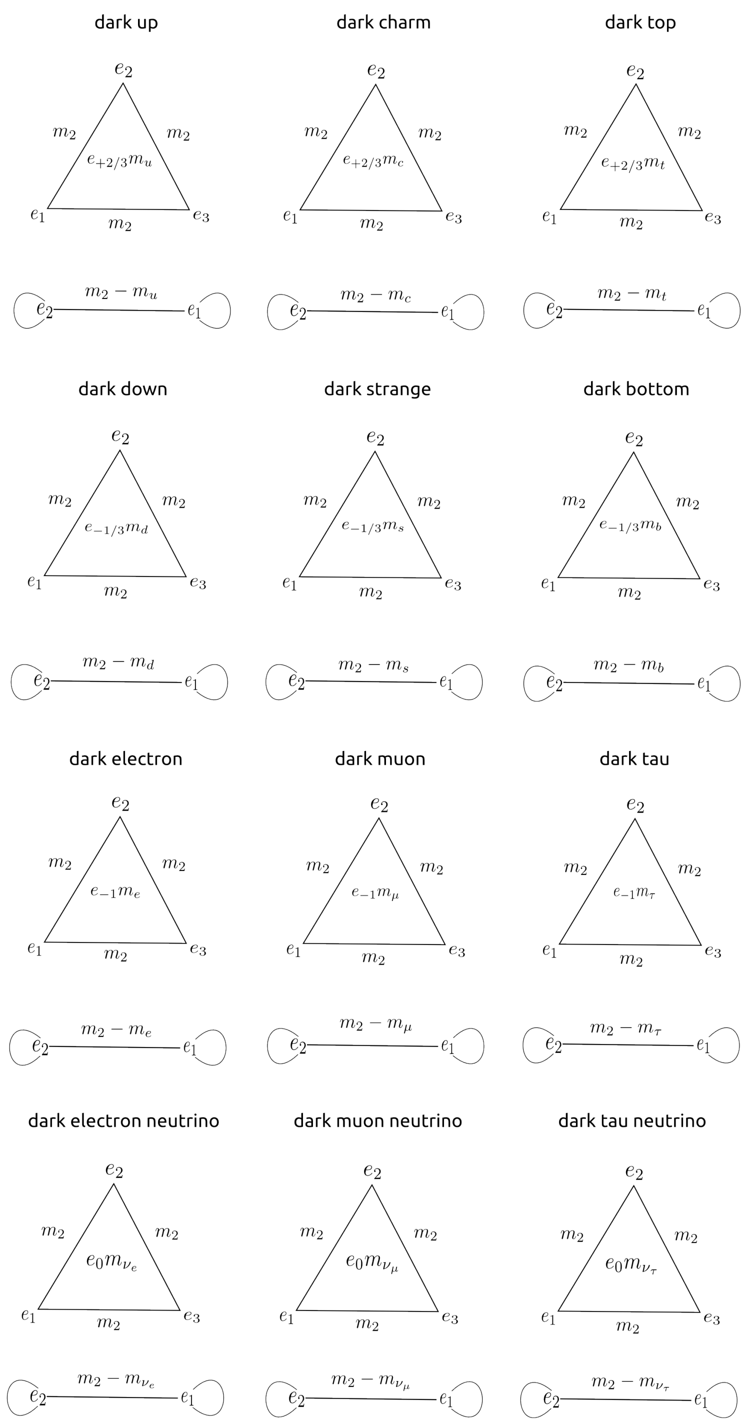

FIGURE 9. Standard Model fermions embedded in the Dark sector with the modified masses $m_2 - m$.

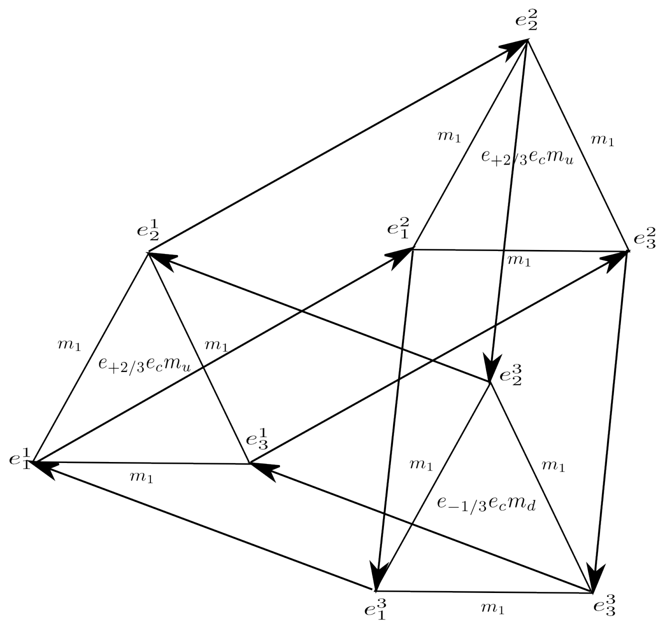

FIGURE 10. A Light sector proton in the 3-block basis state. Each of the three 3-block faces contains three degrees of freedom: charge, color charge, and mass. The internal edges carry the mass of the vacuum (either m_1 or m_2) where each of the external edges with respect to a 3-block can carry one of the 3^2 color-anticolor combinations of gluons.

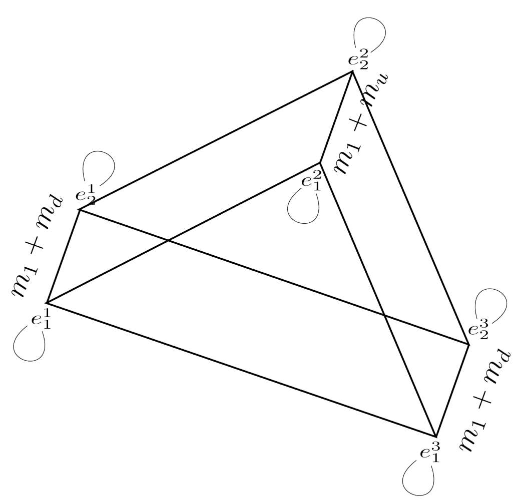

FIGURE 11. A Light sector proton in the 2-block basis state.

7. Discussion

Is our universe prescribed by a computable finite mathematical structure? this is indeed what the results suggest. The Cosmological Theorem proves that for multiverses in which the CFUH is true, the cosmological constant guarantees each universe of this multiverse to halt. In other words, the toy multiverse we are describing is indistinguishable from an eternal computer program, possessing neither a computer nor a programmer. Rather remarkably, the digital paradigm reproduces all of the same predictions of General Relativity. This can be seen from the fact that the only quantities required to calculate curvature are distances, angles, and their rates, which can be readily calculated from the metric eigenfunction. More over, we've seen both time and gravity *emerge*, where the former *is* a change in the entanglement structure and the latter *is* the entanglement rate density. In this framework, just as General Relativity predicts, space and time are apart of the same fundamental structure whose curvature is indistinguishable from gravity, so too does Geometric Graphity.

Not only does Geometric Graphity reproduce the predictions of General Relativity while maintaining Lorentz invariance, but it also predicts the existence of a new type of supersymmetry where every particle in the Standard Model has a supermassive ($\sim 0.33M_{pl}$) twin with the same charge and spin. The neutrinos of this 'Dark Sector' have all of the properties of cold dark matter. They comprise a supermassive pressureless dust that only interact gravitationally and through the weak force. The observed imbalance between Light Sector and Dark Sector particles is a reflection of the anisotropy of the microscopic structure of spacetime. This anisotropy is indistinguishable from dark energy, causing spacetime to have a massive outward pressure. In 2004 Tegmark et. al arguably reported the most precise 3D anisotropy measurements of the matter-power spectrum $\mathcal{P}(k)$ from WMAP data – a feature not predicted by the ΛCDM model of cosmology.

Furthermore, Geometric Graphity predicts that universe posses a mirror symmetry with a causally disconnected mirror image, where all spin up measurements become spin down measurements, all matter is replaced by antimatter, the positions of all particles are reflected, and time is reversed. Not only would this explain the observed lack of antimatter in our universe but it also removes the internal probabilistic nature of quantum mechanics, and does so by invoking a global conservation of energy and a precise mechanism of observation. The observer plays a critical role in the theory, effectively selecting a finite number of degrees of freedom from the continuum and storing them in memory. This process is equivalent to an optimal lossless decompression algorithm, which takes a 2-block, unzips it and stores 3 numbers in memory, leaving behind a 3-block. The observer need not be either human or conscious, merely an arbitrary information processing and storage device. Furthermore, universes without observers is entirely possible, if not inevitable. They would be frozen in the 2-block basis state where every possible configuration of the entanglement structure has a zero ground energy state. While nothing interesting can ever happen in these universes they are still internally dynamic given that the entanglement structure can evolve independent of the observer.

The evolution of the entanglement relations are furthermore indistinguishable from a finite state machine, much like a cellular automata. Instead of cells of a lattice being turned on and off we have edges of a graph being turned on and off, which has the effect of either dilating or contracting spacetime. Preparing this finite state machine in a state that minimizes entropy while maximizing computational complexity allows at every iteration access to more computational power. The initial or 0th state of this toy universe is completely entangled. We know our universe had to have undergone 60 or so e-foldings immediately following the big bang. Therefore, for the toy universe to look like our universe its 1st state must have had much less entanglement than its 0th state. This finite state machine will halt in exactly 10^{79}s with a Final state in which all entanglement has been turned off.

Extraordinary claims requires extraordinary evidence. While our claims are far from speculative, rather consequences of the five fundamental axioms of the geometry, at the end of the day it is only experiment and not mathematics, no matter how rigorous or beautiful, that can validate them. That being said, Geometric Graphity is, in fact, the first falsifiable model of quantum gravity. *It's primary prediction is the existence of two fundamental masses, $m_1 \sim 10^{-131}$ kg and $m_2 \sim 10^{-10}$ kg, that cannot be removed from the vacuum, constituting the most precise prediction ever made!*

On the one hand, m_1 is so extraordinarily close to zero there will never exist an experiment that can detect it, on the other hand m_2 is as massive as a fine grain of sand and certainly can be detected, albeit probably not directly. If this mass exists its density here on earth will vary with the local gravitational field. Since we know the earth's rate of rotation (or length of day (LOD)) oscillates, so too will the gravitational constant appear to oscillate. In 2015 a study was done looking at all measurements of G spanning from 1962 to 2014 and concluded that the value of G oscillates with a period of 5.9 years and an amplitude of $(1.619 \pm 0.103) \times 10^{-14}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ resulting in a G/LOD correlation with a statistical significance of 0.99764 [2]. The G/LOD correlation, however, isn't the only phenomenon we can expect. If the vacuum has irremovable mass, momentum can be exchanged with it.

In 2016 White et. al at NASA developed an electromagnetic resonant cavity thruster that produced a consistent thrust-to-power ratio of 1.2 ± 0.1 mN/kW in vacuum [22]. They argue that the Pilot-Wave interpretation of quantum mechanics implies that the vacuum is an immutable medium in which momentum can be exchanged. Although it isn't immediately clear how they reached this conclusion, our results are more or less identical but for fundamentally different reasons. While the G/LOD correlation and the EM drive's thrust is substantial evidence in support of our claims it isn't sufficient to conclude that the vacuum has irremovable mass. We can, however, combine these two experiments to rule out false positive signals in both the G/LOD and the EM drive experiments. *If Geometric Graphity is describing our universe, precision measurements of G in the immediate vicinity of where thrust is being generated will return a G value that resonates with the EM drive.*

8. Conclusion

At the foundation of this work are three remarkably insightful propositions put forth by Tegmark. Among them was the Computable Finite Universe Hypothesis (CFUH) from which we've managed to prove the Graphity Field Theorem (GFT) which constructs a computable version of Feynman's path integral for an unknown graph-theoretic coordinate system. From the GFT we managed to sculpt out the five fundamental axioms of the geometry. From these axioms we prove the Holographic Theorem (HT) which returns an algebraic operator-valued graph-theoretic coordinate system from which physics can be computed on. Applying the HT to the GFT returns the Cosmological Theorem (CT) which proves the cosmological constant sets the bounds of a universe that is finite both in space and in time, thereby validating the CFUH. In acquiring this result we obtained a conformally invariant metric at the Planck scale, such that for a pair of critical rotational exponents becomes a system of two spacetime equations of state containing the Bekenstein-Hawking area entropy law. This equation describes the fundamental interactions of two masses, expressed in terms of the fundamental constants, irremovable from the vacuum and separated in magnitude by the cosmological constant, causing spacetime to carry a massive negative pressure. While these results are in direct accordance with the ΛCDM model of cosmology they are also in direct conflict with the $U(1) \times SU(2) \times SU(3)$ model of particle physics, suggesting all particle interactions including gravity can be described using the n-dimensional irreducible representations of $SU(2)$ for $n = 1, 2, 3$ while at the same time predicting the existence of a new supersymmetry where every fermion has a supermassive partner with the same spin and charge. Finally, we discussed two recent experiments that support our claims and propose a specific experiment to falsify them.

9. Appendix

9.1. **Restructuring the Continuum.** Suppose we have a weighted adjacency matrix whose entries are the real numbers, $a_{nm} \in \mathbb{R}$. Now suppose we 'move' $\mathbb{R}$ from the entries of the matrix to the indices of the matrix leaving behind a discrete 0-1 adjacency matrix. Raising this continuous adjacency matrix to the kth power gives us the following integral

$$[\mathcal{A}^k]_{nm} = \int du_1 \int du_2 \dots \int du_{n-1} \int du_{n-2} a_{n,u_1} a_{u_1,u_2} \dots a_{u_{n-2},u_{n-1}} a_{u_{n-1},m}$$

.

Upon perturbing the distance matrix we have

$$\begin{aligned} d_{n+\delta n, m+\delta m} &= \begin{cases} k + \delta k, & d(n+\delta n, m+\delta m) = k + \delta k \\ 0, & d(n+\delta n, m+\delta m) = 0 \end{cases} \\ &\cong \begin{cases} k : k \in \mathbb{R}, & d(n, m) = k \\ 0, & d(n, m) = 0 \end{cases} \end{aligned} \tag{9.1}$$

which yields the following relation that looks identical to the transfer matrix derivation of the Feynman path integral

$$
\begin{aligned}
[\mathbf{A}^{k+\delta k}]_{nm} &= [\mathcal{A}^k]_{n+\delta n, m+\delta m} \\
&= \int d(u_1+\delta u_1) \int d(u_2+\delta u_2) \ldots \int d(u_{n-1}+\delta u_{n-1}) \int d(u_{n-2}+\delta u_{n-2}) \\
&\quad \times a_{n,(u_1+\delta u_1)} a_{(u_1+\delta u_1),(u_2+\delta u_2)} \cdots a_{(u_{n-2}+\delta u_{n-2}),(u_{n-1}+\delta u_{n-1})} a_{(u_{n-1}+\delta u_1),m} \\
&= \int du_1 \int du_2 \ldots \int du_{n-1} \int du_{n-2} \\
&\quad \times (1+\delta) a_{n,(1+\delta)u_1} (1+\delta) a_{(1+\delta)u_1,(1+\delta)u_2} \ldots (1+\delta) a_{(1+\delta)u_{n-2},(1+\delta)u_{n-1}} (1+\delta) a_{(1+\delta)u_{n-1},m} \\
&= \int du_1 \int du_2 \ldots \int du_{n-1} \int du_{n-2} [\widehat{T} a_{n,u_1}][\widehat{T} a_{u_1,u_2}] \ldots [\widehat{T} a_{u_{n-2},u_{n-1}}][\widehat{T} a_{u_{n-1},m}].
\end{aligned}
$$

9.2. **A Higher Dimensional Classical Mechanics.**

9.2.1. *Random Matrix Theory.* Random Matrix Theory (RMT) is the generalization of probability theory to matrices. For orthogonal and unitary invariant matrices, the matrix entries with respect to the Lebesgue measures, are functions of only the eigenvalues. The joint element densities of the entries of an n x n real symmetric matrix belonging to the Gaussian Orthogonal Ensemble (GOE) is:

$$
f^{GOE}(\mathbf{A}) = \frac{1}{2^{n/2}} \frac{1}{\pi^{n/2+n(n-1)\beta/4}} exp\left(-\frac{1}{2} tr(\mathbf{A}^2)\right) \tag{9.2}
$$

where the joint eigenvalue density is,

$$
f_\beta(\lambda_1, ..., \lambda_n) = c^\beta_{GOE} \prod_{i<j}^{n} |\lambda_i - \lambda_j|^\beta exp\left(-\sum_{i=1}^{n} \frac{\lambda_i^2}{2}\right) \tag{9.3}
$$

and where,

$$
c^\beta_{GOE} = (2\pi)^{-\dfrac{n}{2}} \prod_{j=1}^{n} \frac{\Gamma(1+\dfrac{\beta}{2})}{\Gamma(1+\dfrac{\beta}{2}j)} \tag{9.4}
$$

The level-density $\rho^{\mathbf{A}}_{n,\beta}$, for an n x n Gaussian Orthogonal Ensemble is the distribution of eigenvalues chosen as an i.i.d random variable from the ensemble,

$$
\rho^{\mathbf{A}}_{n,\beta}(\lambda_k) = \int_{-\infty}^{\infty} d\lambda_{k+1} \ldots \int_{-\infty}^{\infty} d\lambda_n f_\beta(\lambda_1, ..., \lambda_n) \tag{9.5}
$$

where $\beta = 1$ corresponds to a real symmetric square matrix, $\beta = 2$ corresponds to a complex Hermitian square matrix, and $\beta = 4$ corresponds to a quaternion.

9.2.2. *Gaussian momentum and position distributions on* $(\boldsymbol{p}\otimes\boldsymbol{p}, \boldsymbol{q}\otimes\boldsymbol{q})$. For $\mathbf{q}\sim\mathbf{N}(0,1)$ and $\mathbf{p}\sim\mathbf{N}(0,1)$, the joint eigenvalue density is

$$f_1(\lambda_1,\lambda_2,\lambda_3) = c_{GOE}|\lambda_1-\lambda_2||\lambda_2-\lambda_3||\lambda_1-\lambda_3|exp\bigg(-\frac{1}{2}(\lambda_1^2+\lambda_2^2+\lambda_3^2)\bigg), \tag{9.6}$$

where the eigenvalues are written in ascending order, $\lambda_1\le\lambda_2\le\lambda_3$. To make the above integral trivial, we simply choose to write the eigenvalues in descending order $\lambda_3\le\lambda_2\le\lambda_1$. Eq. (3.1) then becomes

$$f_1(\lambda_1,\lambda_2,\lambda_3) = c_{GOE}(\lambda_3-\lambda_1)(\lambda_2-\lambda_1)(\lambda_3-\lambda_2)exp\bigg(-\frac{1}{2}(\lambda_1^2+\lambda_2^2+\lambda_3^2)\bigg). \tag{9.7}$$

Upon integrating we have

$$\begin{aligned}
\rho_{3,1}^{\mathbf{A}}(\lambda_1) &= \frac{1}{\sqrt{2\pi}}exp\bigg(-\frac{\lambda_1^2}{2}\bigg)\\
\rho_{3,1}^{\mathbf{A}}(\lambda_2) &= \frac{1}{\sqrt{2\pi}}\lambda_2^2 exp\bigg(-\frac{\lambda_2^2}{2}\bigg)\\
\rho_{3,1}^{\mathbf{A}}(\lambda_3) &= \frac{1}{\sqrt{2\pi}}exp\bigg(-\frac{\lambda_3^2}{2}\bigg).
\end{aligned} \tag{9.8}$$

Since we now know how each eigenvalue is distributed, the first invariant is now a sum of three i.i.d random variables. We will use the characteristic function, which is the Fourier transform of a probability distribution

$$\phi(k) = \int_{-\infty}^{\infty} e^{ikx}f(x)dx. \tag{9.9}$$

Upon Fourier transforming each level-density we obtain

$$\begin{aligned}
\phi_{\lambda_1}(k) &= \frac{1}{\sqrt{2\pi}}exp\bigg(-\frac{k^2}{2}\bigg)\\
\phi_{\lambda_2}(k) &= -\frac{1}{\sqrt{2\pi}}(k^2-1)exp\bigg(-\frac{k^2}{2}\bigg)\\
\phi_{\lambda_3}(k) &= \frac{1}{\sqrt{2\pi}}exp\bigg(-\frac{k^2}{2}\bigg).
\end{aligned} \tag{9.10}$$

Since the characteristic function of a sum of independent random variables is the product of characteristic functions, we obtain the first invariant's probability density by taking the following inverse Fourier transform

$$\begin{aligned}
f(\lambda_1,\lambda_2,\lambda_3) &= \int_{-\infty}^{\infty} e^{-it(\lambda_1^2+\lambda_2^2+\lambda_3^2)}\phi_{\lambda_1}(t)\phi_{\lambda_2}(t)\phi_{\lambda_3}(t)dt\\
&= \frac{1}{9\sqrt{6\pi}}\big(6+(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2\big)exp\bigg(-\frac{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2}{6}\bigg)
\end{aligned}. \tag{9.11}$$

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