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Homework 6 (Due: 4/25)

Show that the Fourier transforms of

(a) $f(ax)$ is $\frac{1}{a}F(\frac{u}{a})$, where a is any nonzero real number.

(b) $f(x-x_0)$ is $F(u)\exp(-j2\pi u x_0)$.

(a) $f(x) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi u x}\, dx = F(u)$

$$
\begin{aligned}
f(ax) &= \int_{-\infty}^{\infty} f(ax)\, e^{-j2\pi u x}\, dx \\
&= \int_{-\infty}^{\infty} f(ax)\, e^{-j2\pi \frac{u}{a}(ax)}\, \frac{d(ax)}{a} \qquad k = ax \\
&= \int_{-\infty}^{\infty} f(k)\, e^{-j2\pi \frac{u}{a}\cdot k}\, \frac{dk}{a} \\
&= \frac{1}{a}\int_{-\infty}^{\infty} f(k)\, e^{-j2\pi \frac{u}{a}\cdot k}\, dk \\
&= \frac{1}{a}F\left(\frac{u}{a}\right) ※
\end{aligned}
$$

(b) $f(x) = \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi u x}\, dx = F(u)$

Fourier transform of $f(x-x_0)$.

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} f(x-x_0)\, e^{-j2\pi u x}\, dx \\
&= \int_{-\infty}^{\infty} f(x-x_0)\, e^{-j2\pi u(x-x_0+x_0)}\, dx \qquad \frac{d(x-x_0)}{dx} = 1
\end{aligned}
$$

let $k = x - x_0$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} f(k)\, e^{-j2\pi u(k+x_0)}\, dk \\
&= \int_{-\infty}^{\infty} f(k)\left(e^{-j2\pi u\cdot k}\cdot e^{-j2\pi u x_0}\right) dk \\
&= e^{-j2\pi u x_0}\cdot \underbrace{\boxed{\int_{-\infty}^{\infty} f(k)\, e^{-j2\pi u\cdot k}\, dk}}_{F(u)} \\
&= e^{-j2\pi u x_0}\cdot F(u) = F(u)\exp(-j2\pi u x_0) ※
\end{aligned}
$$