Image Processing Morphological Analysis

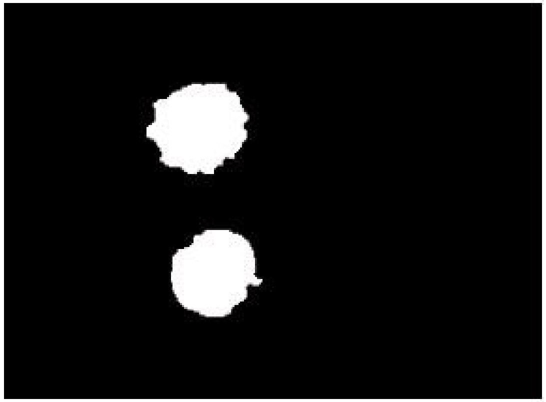
SUMMER 2025

Preview

- Morphology
  - About the **form** and **structure** of animals and plants
- Mathematical morphology
  - Using set theory
  - Extract image component
  - Representation and description of region shape

Preview (cont.)

- Sets in mathematical morphology represent objects in an image



- Example
  - Binary image: the elements of a set is the coordinate (x,y) of the pixels, in $\mathbf{Z}^2$
  - Gray-level image: the element of a set is the triple, (x, y, gray-value), in $\mathbf{Z}^3$

Outline

Binary images

- Preliminaries – set theory
- Dilation and erosion
- Opening and closing
- Hit-or-miss transformation
- Some basic morphological algorithms
- Extensions to gray-scale images

Preliminaries – set theory

- A be a set in $\mathbf{Z^2}$.
- a = (a_1, a_2) is an element of A. $a \in A$
- a is not an element of A $a \notin A$
- Null (empty) set: $\varnothing$

Set theory (cont.)

- Explicit expression of a set

1 $A = \{a_1, a_2, \ldots, a_n\}$

2 $A = \{ element \,|\, \text{condition for set elements} \}$

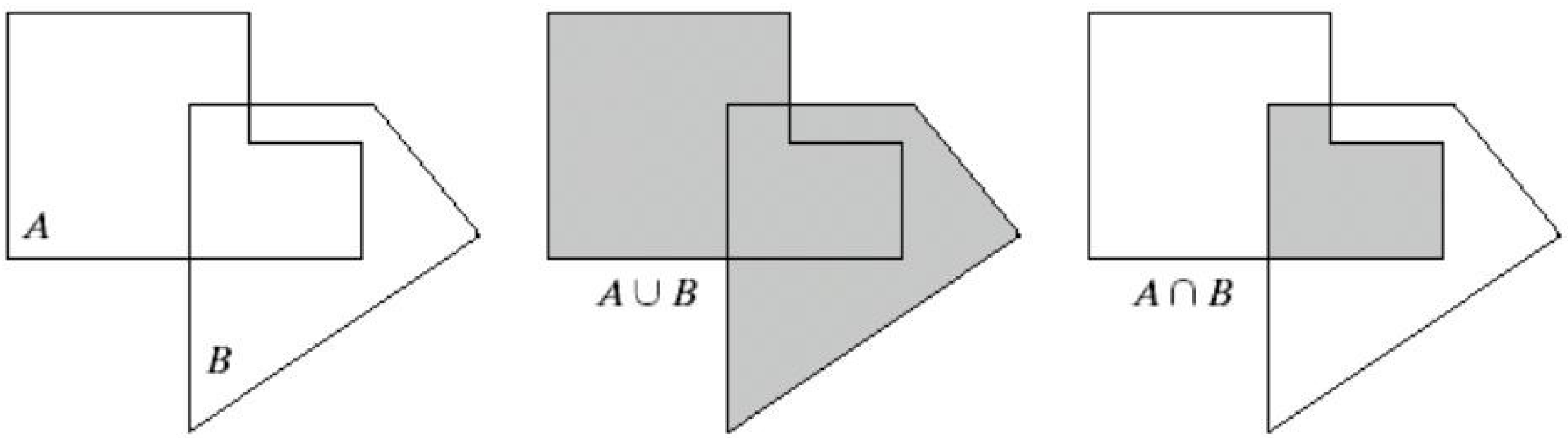
  - Example:

$$C = \{w \,|\, w = -d, \ \text{for } d \in D\}$$

Set operations

- A is a subset of B: every element of A is an element of another set B $A \subseteq B$
- Union $C = A \cup B$
- Intersection $C = A \cap B$
- Mutually exclusive $A \cap B = \varnothing$

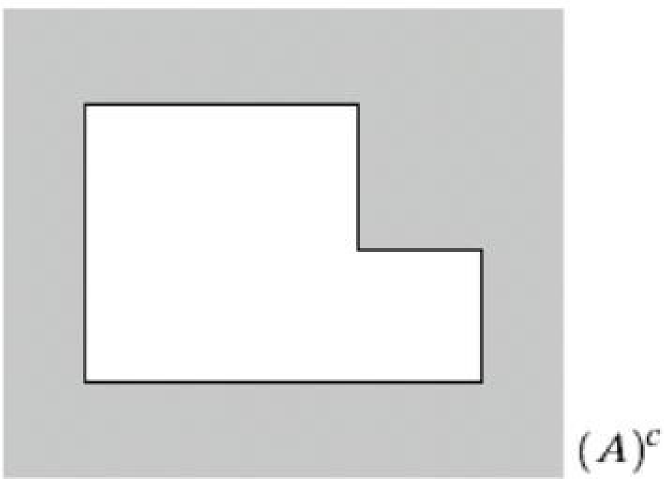
Graphical examples

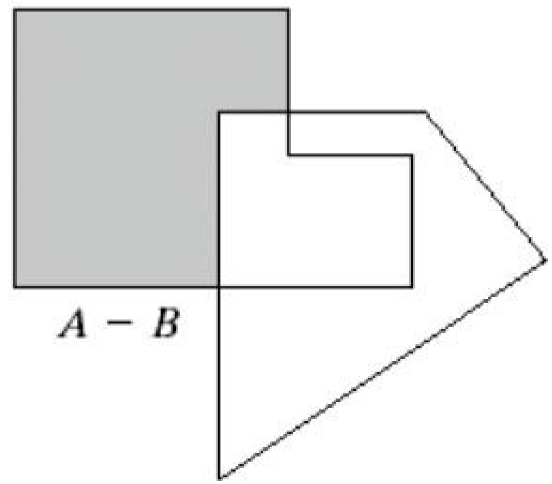


Graphical examples (cont.)

$$A^c = \{w \mid w \notin A\}$$

$$A - B = \{w \mid w \in A, w \notin B\}$$

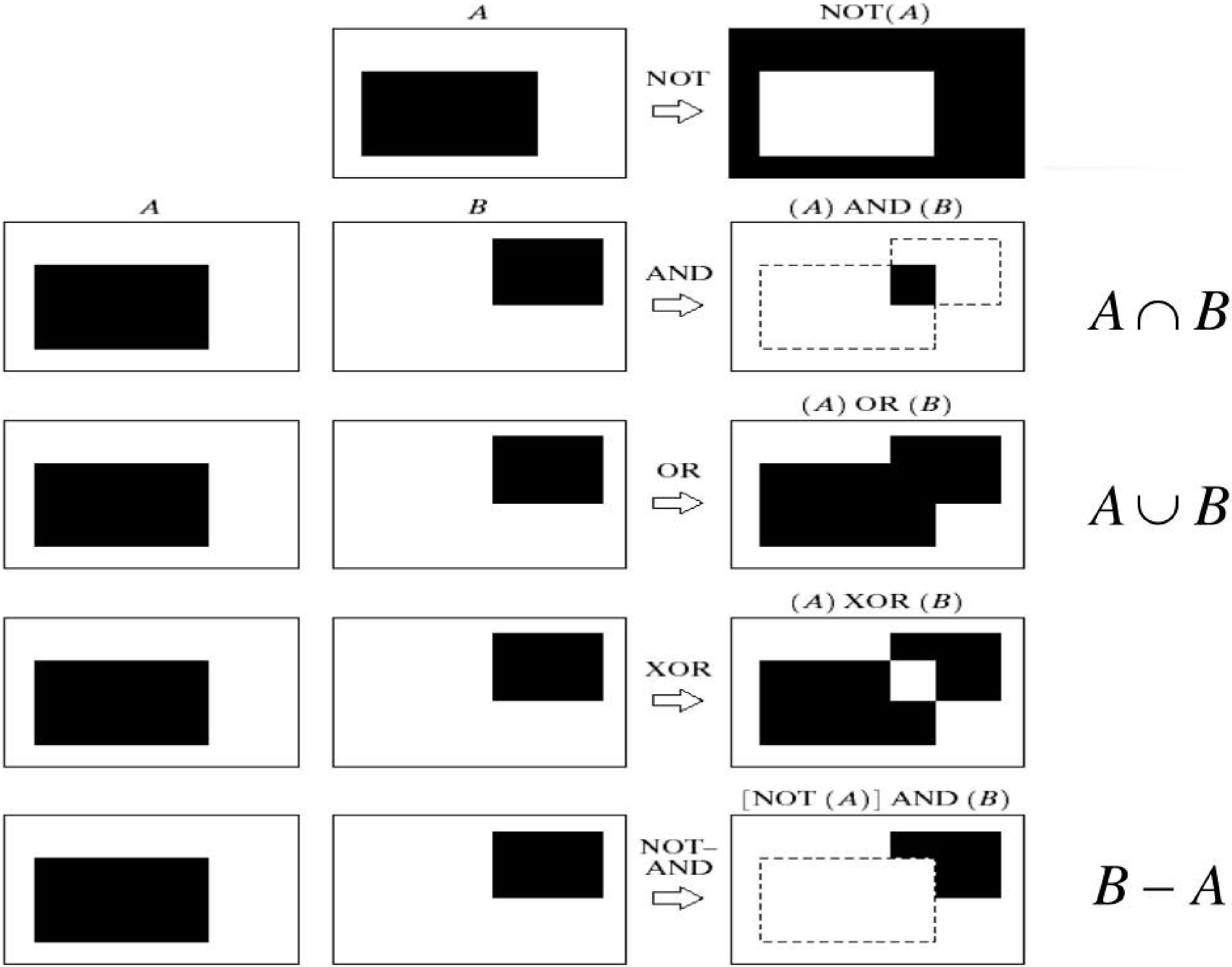




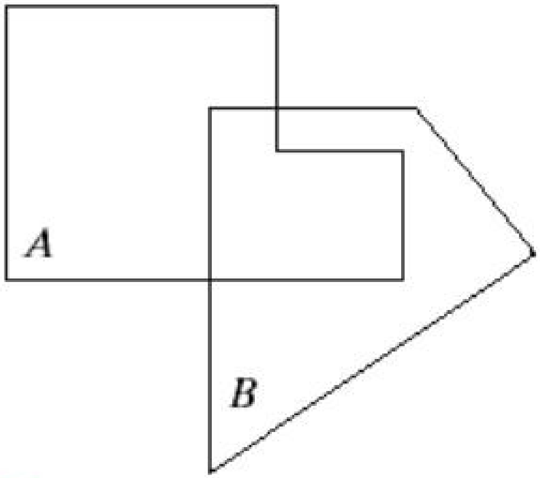
Logic operations on binary images

- Functionally complete operations
  - AND, OR, NOT

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT (**p**) (also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

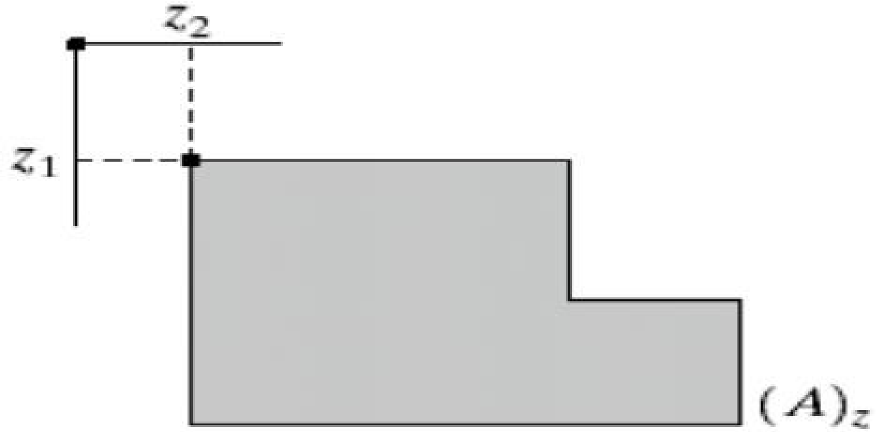

A
NOT(A)
NOT
A
B
(A) AND (B)
AND
A ∩ B
(A) OR (B)
OR
A ∪ B
(A) XOR (B)
XOR
[NOT (A)] AND (B)
NOT–AND
B − A

Special set operations for morphology



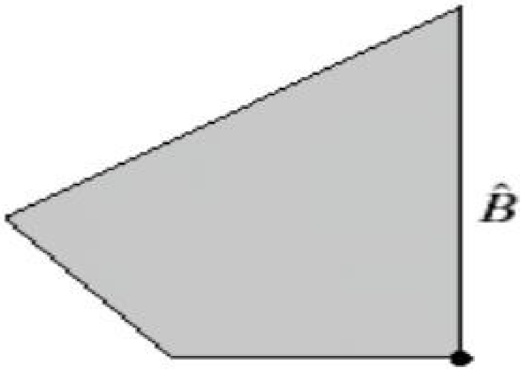
translation

$$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$$



reflection

$$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$$

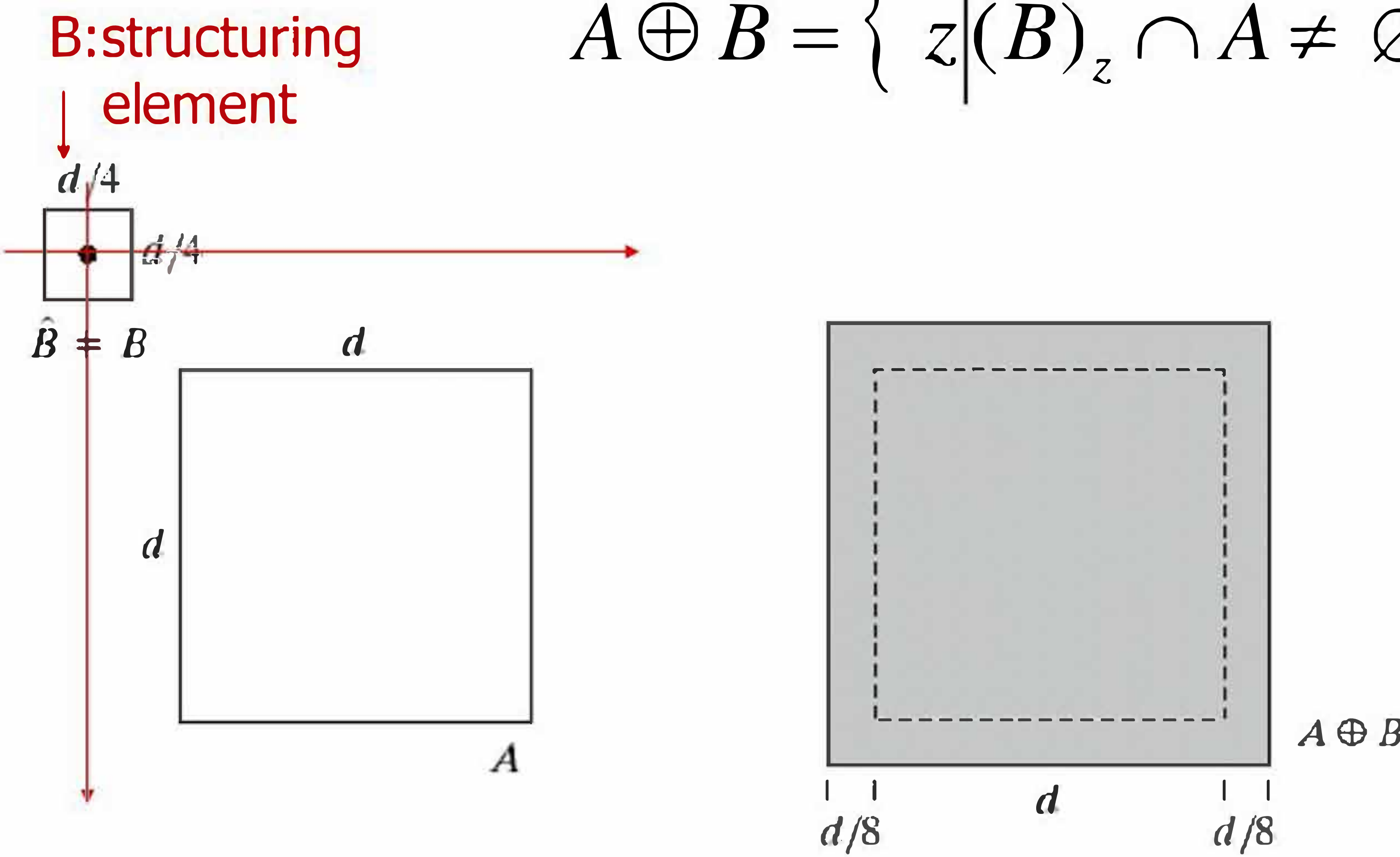


Outline

- Preliminaries
- Dilation and erosion
- Opening and closing
- Hit-or-miss transformation
- Some basic morphological algorithms
- Extensions to gray-scale images

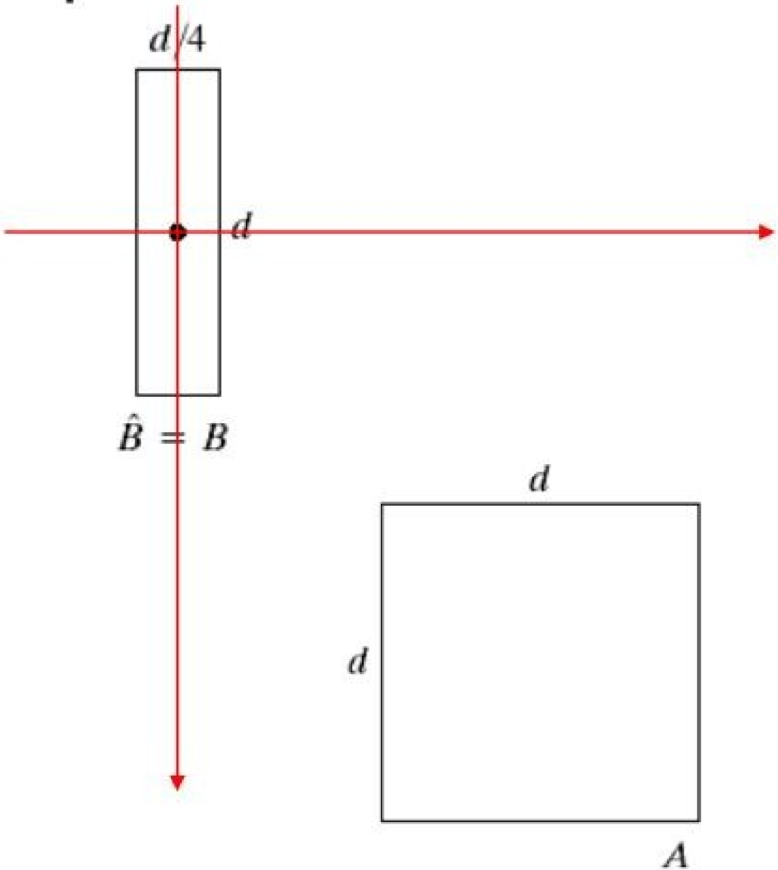
Dilation

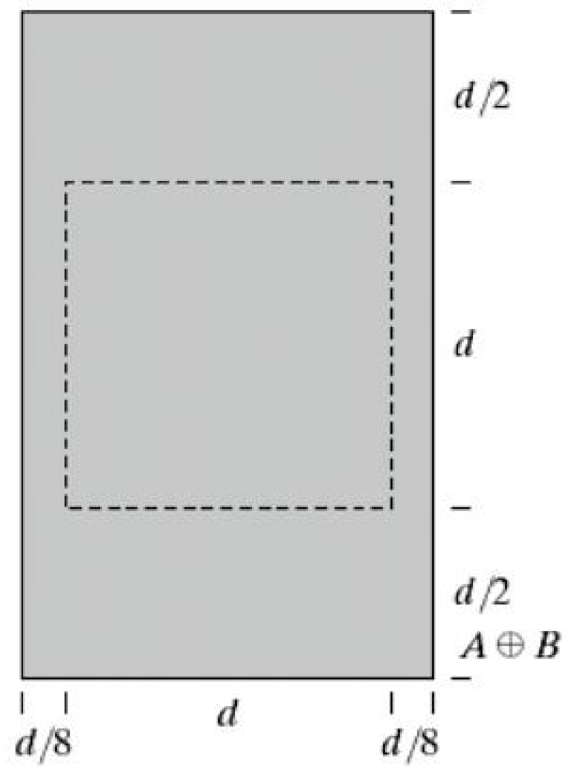
$$A \oplus B = \left\{ z \,\middle|\, (\hat{B})_z \cap A \neq \varnothing \right\}$$



Dilation: another formulation

$$A \oplus B = \left\{ z \middle| \left[(\hat{B})_z \cap A \right] \subseteq A \right\}$$



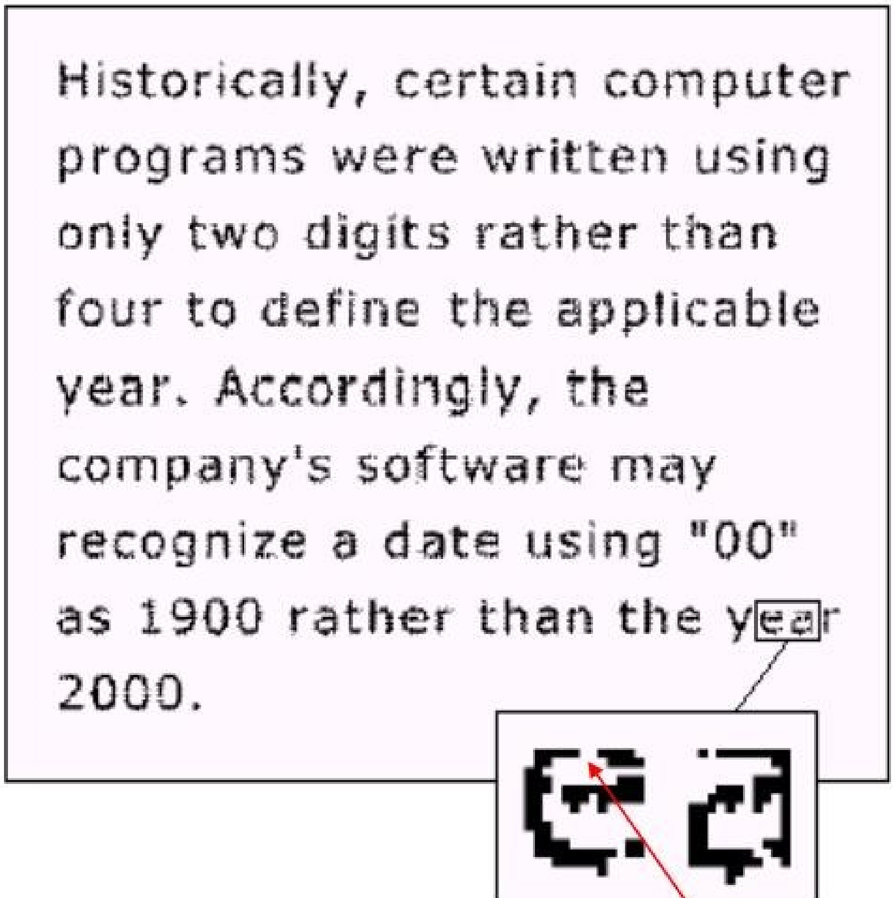


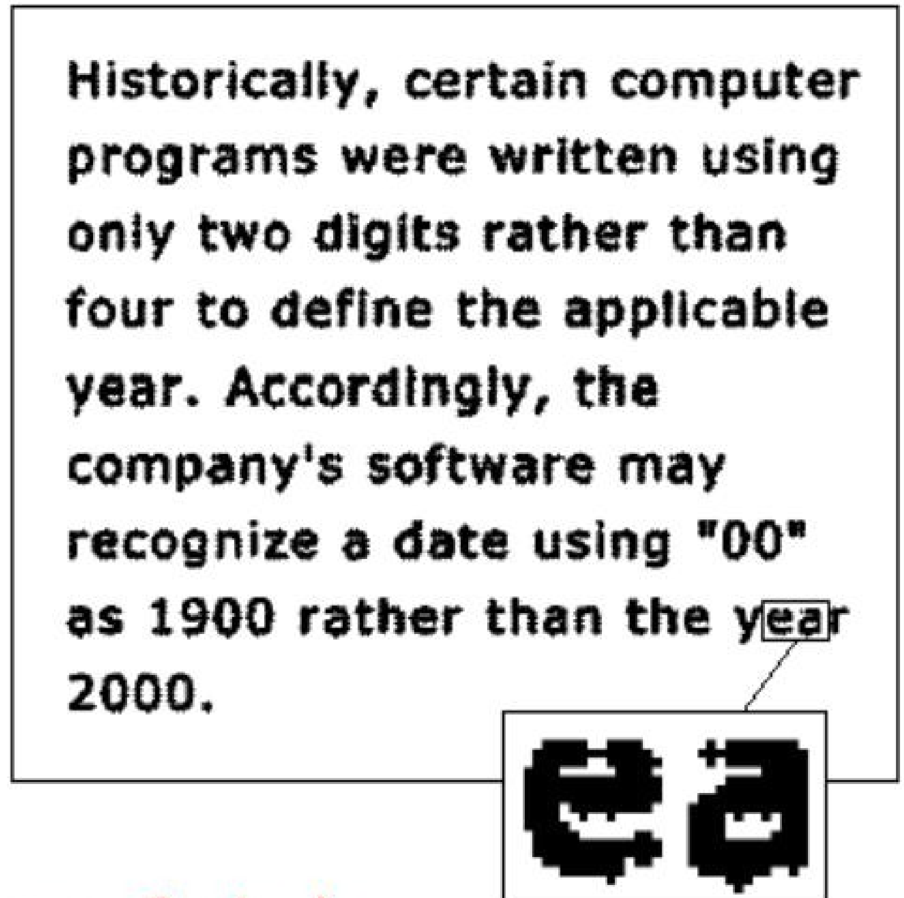
Application of dilation: bridging gaps in images

$$\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

Structuring element

Effects: increase size, fill gap

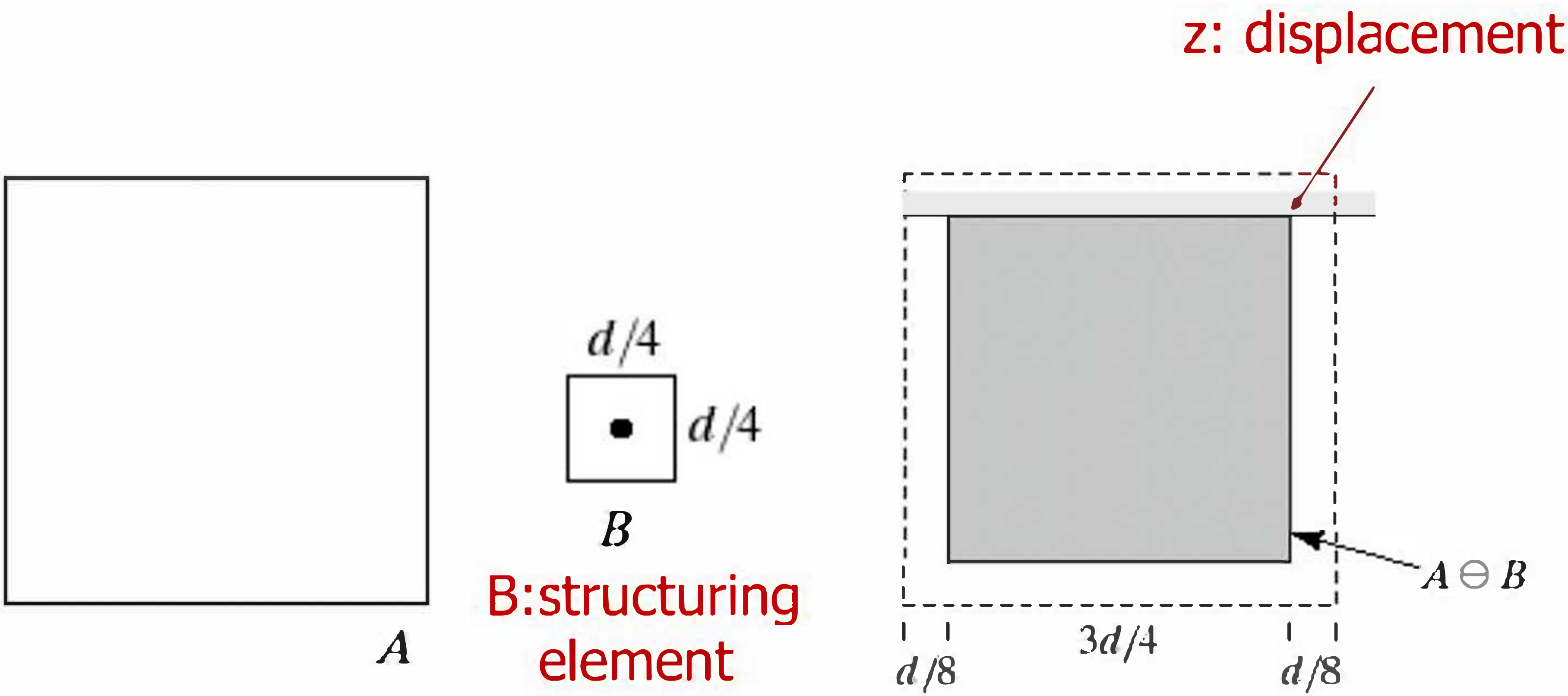




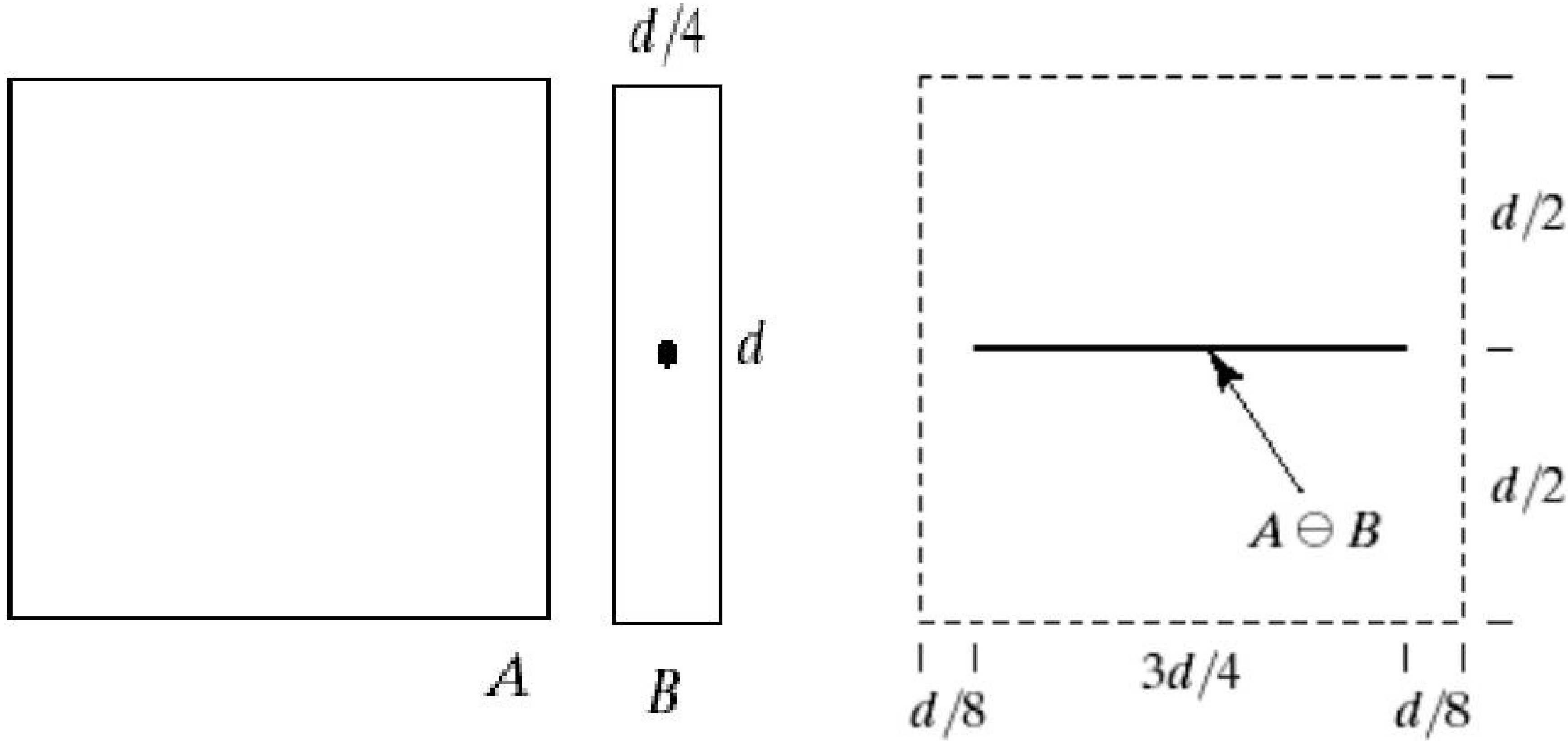
max. gap=2 pixels

Erosion

$$A \ominus B = \left\{ z \,\middle|\, (B)_z \subseteq A \right\}$$



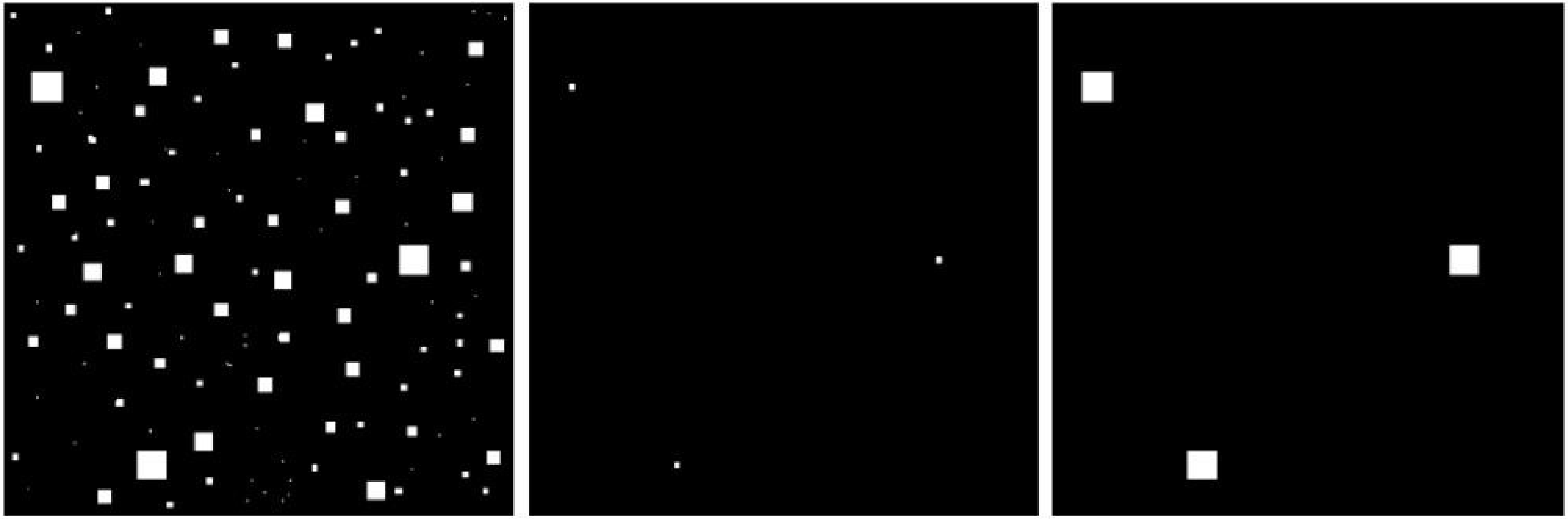
Erosion (cont.)



Application of erosion: eliminate irrelevant detail

Squares of size 1,3,5,7,9,15 pels

Erode with 13x13 square



original image → erosion → dilation

Dilation and erosion are duals

$$(A \ominus B)^c = \left\{ z \middle| (B)_z \subseteq A \right\}^c$$

$$= \left\{ z \middle| (B)_z \cap A^c = \varnothing \right\}^c$$

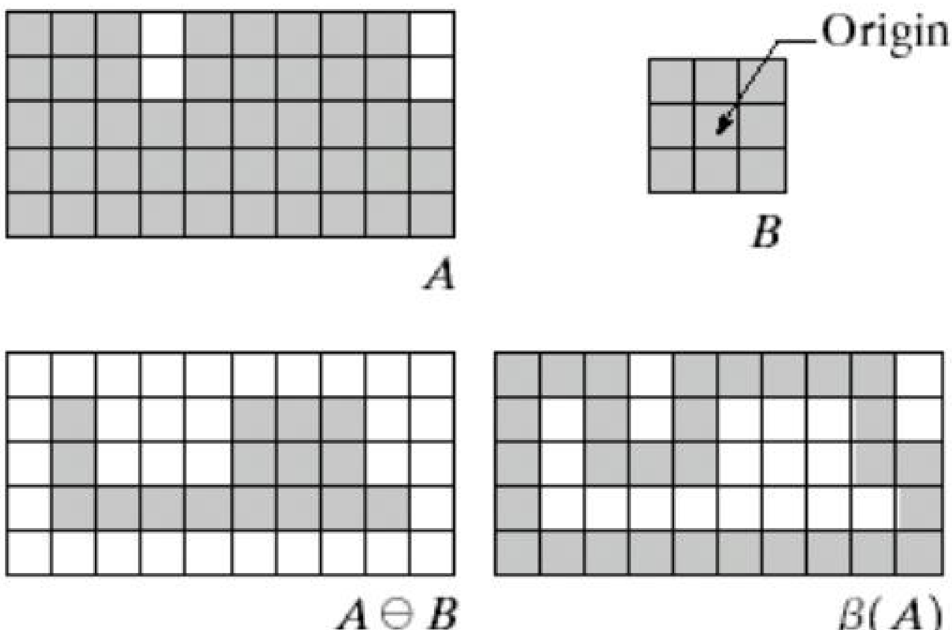
$$= \left\{ z \middle| (B)_z \cap A^c \neq \varnothing \right\}$$

$$A \oplus B = \left\{ z \middle| (\hat{B})_z \cap A \neq \varnothing \right\}$$

$$= A^c \oplus \hat{B}$$

Application: Boundary extraction

- Extract boundary of a set A:
  - First erode A (make A smaller)
  - A – erode(A)



$$\beta(A) = A - (A \ominus B)$$

Application: boundary extraction

original image

Using 5x5 structuring element

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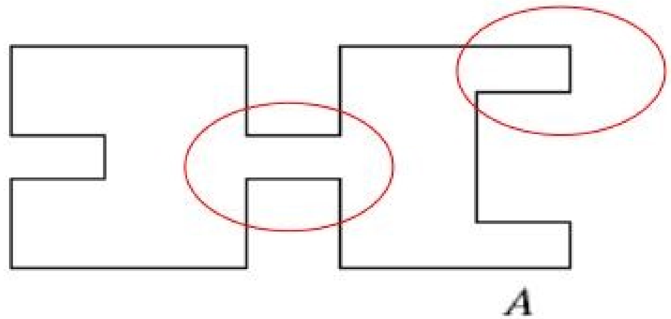
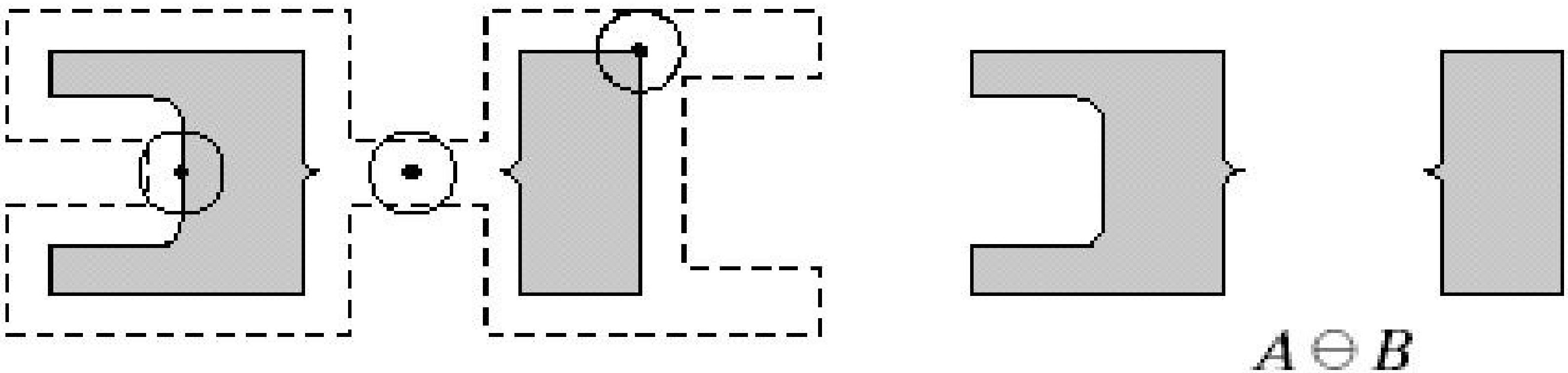
Opening

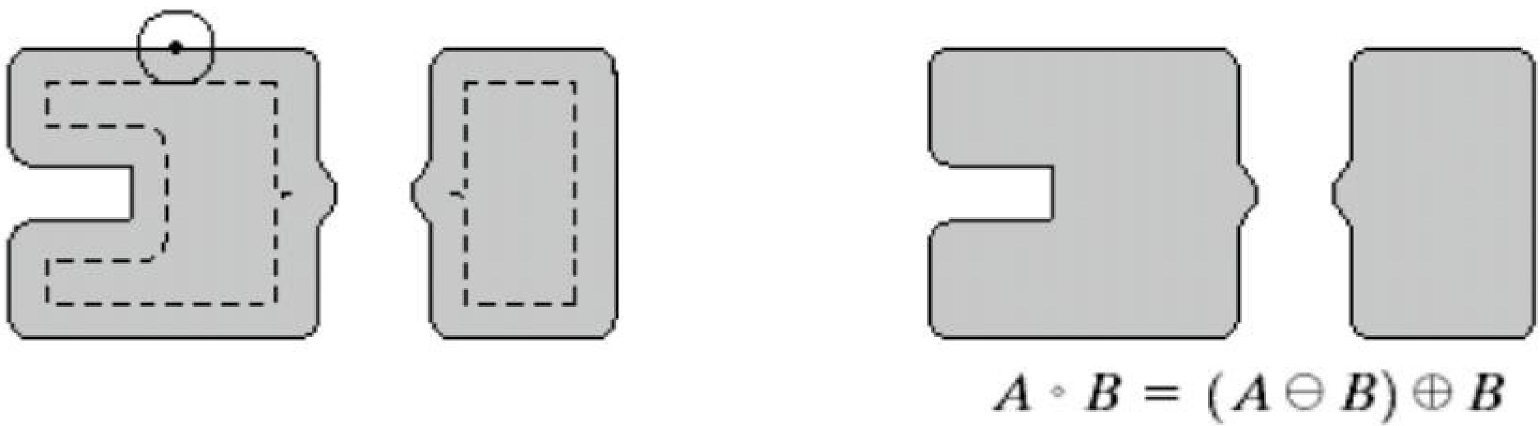
- Dilation: expands image w.r.t structuring elements
- Erosion: shrink image
- erosion+dilation = original image ?
- Opening= erosion + dilation

$$A \circ B = (A \ominus B) \oplus B$$

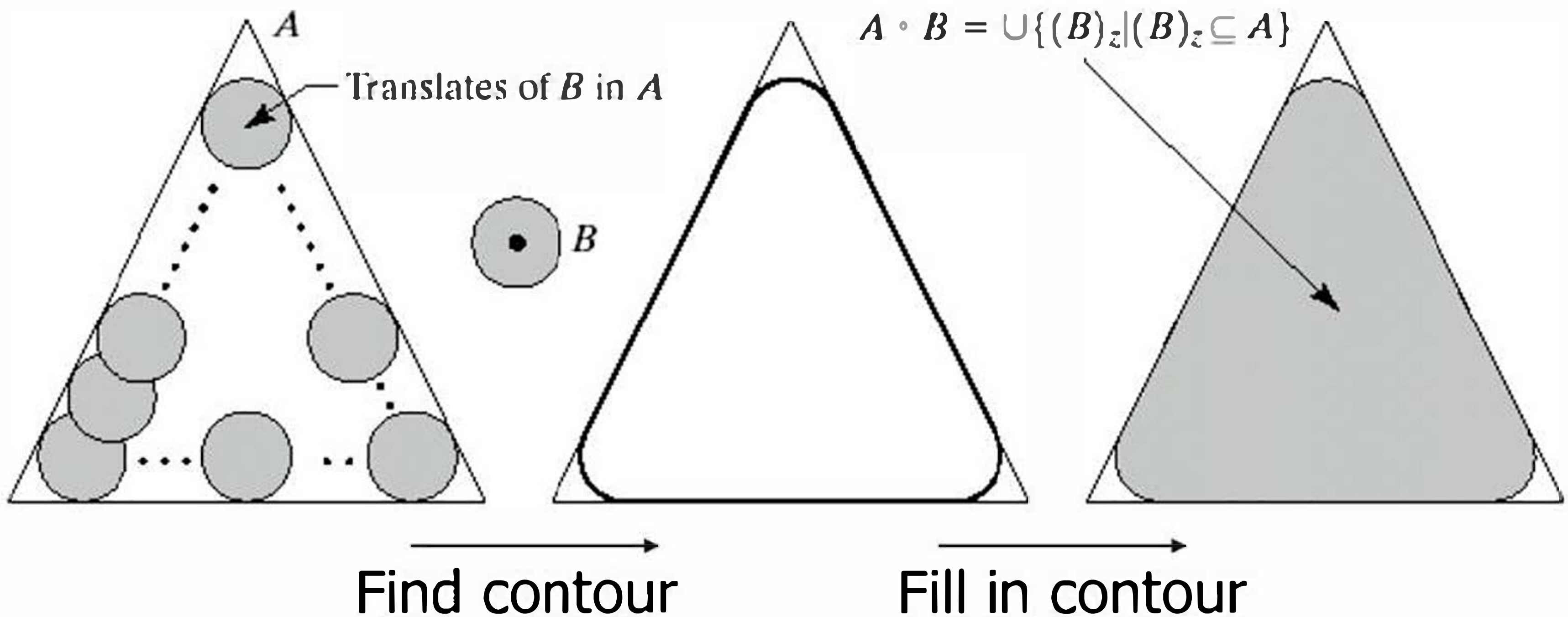
Opening (cont.)







Opening (cont.)

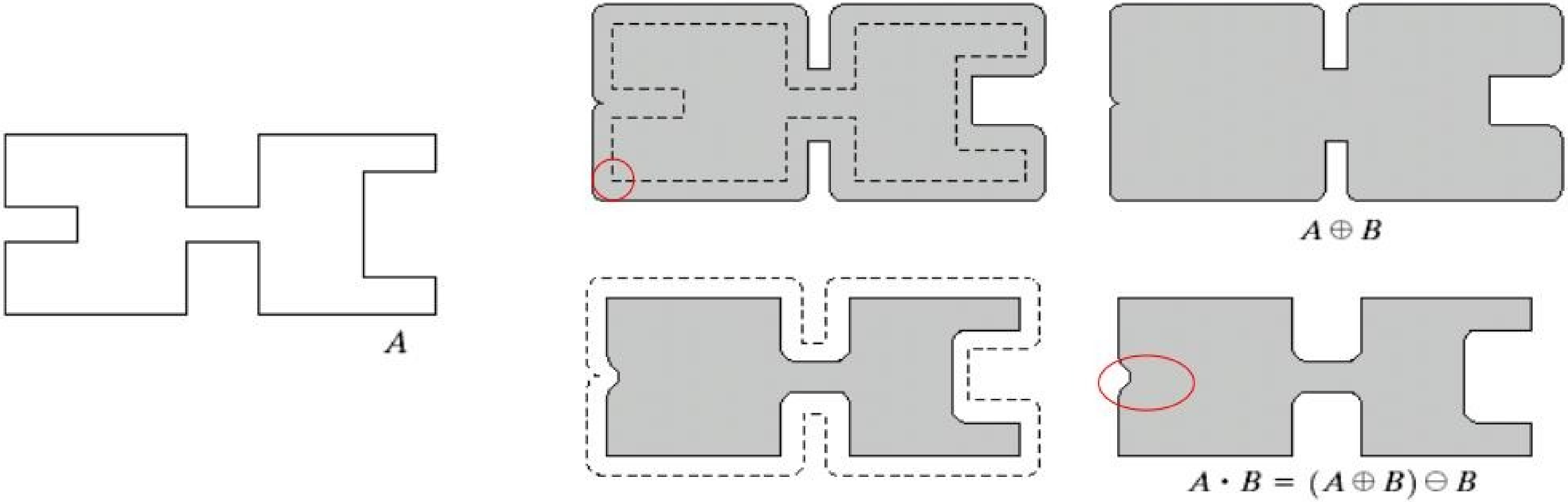


Smooth the contour of an image, breaks narrow isthmuses, eliminates thin protrusions

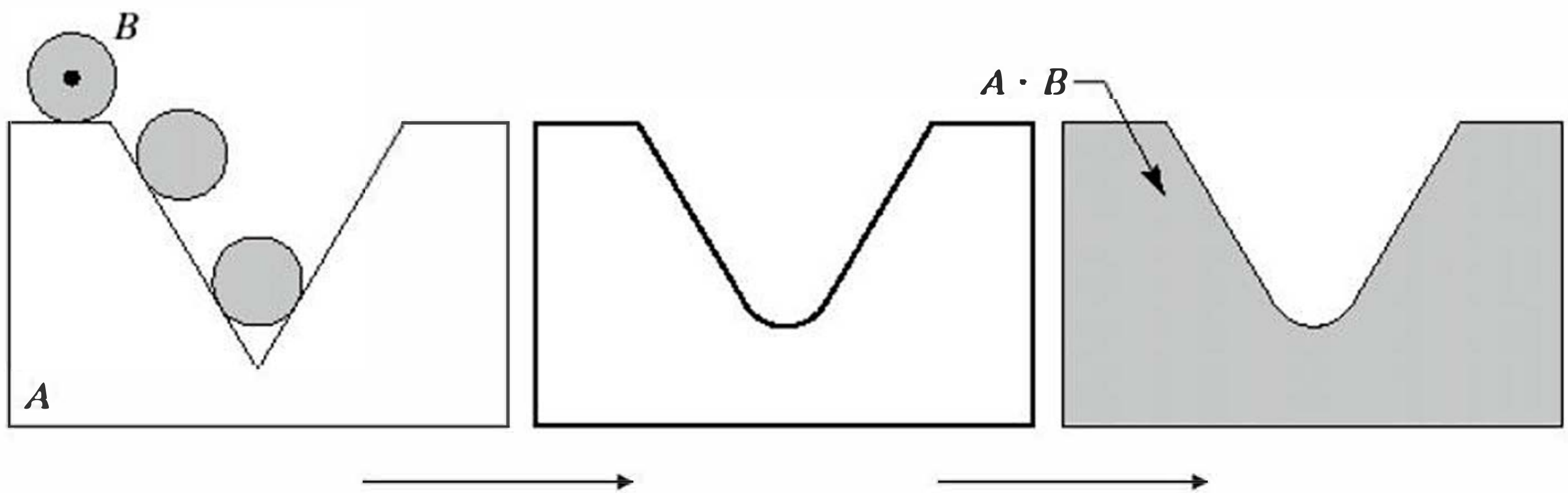
Closing

- Dilation+erosion = erosion + dilation ?
- Closing = dilation + erosion

$$A \bullet B = (A \oplus B) \ominus B$$



Closing (cont.)



Find contour

Fill in contour

Smooth the object contour, fuse narrow breaks and long thin gulfs, eliminate small holes, and fill in gaps

Properties of opening and closing

- Opening

(i) $A \circ B$ is a subset (subimage) of A

(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

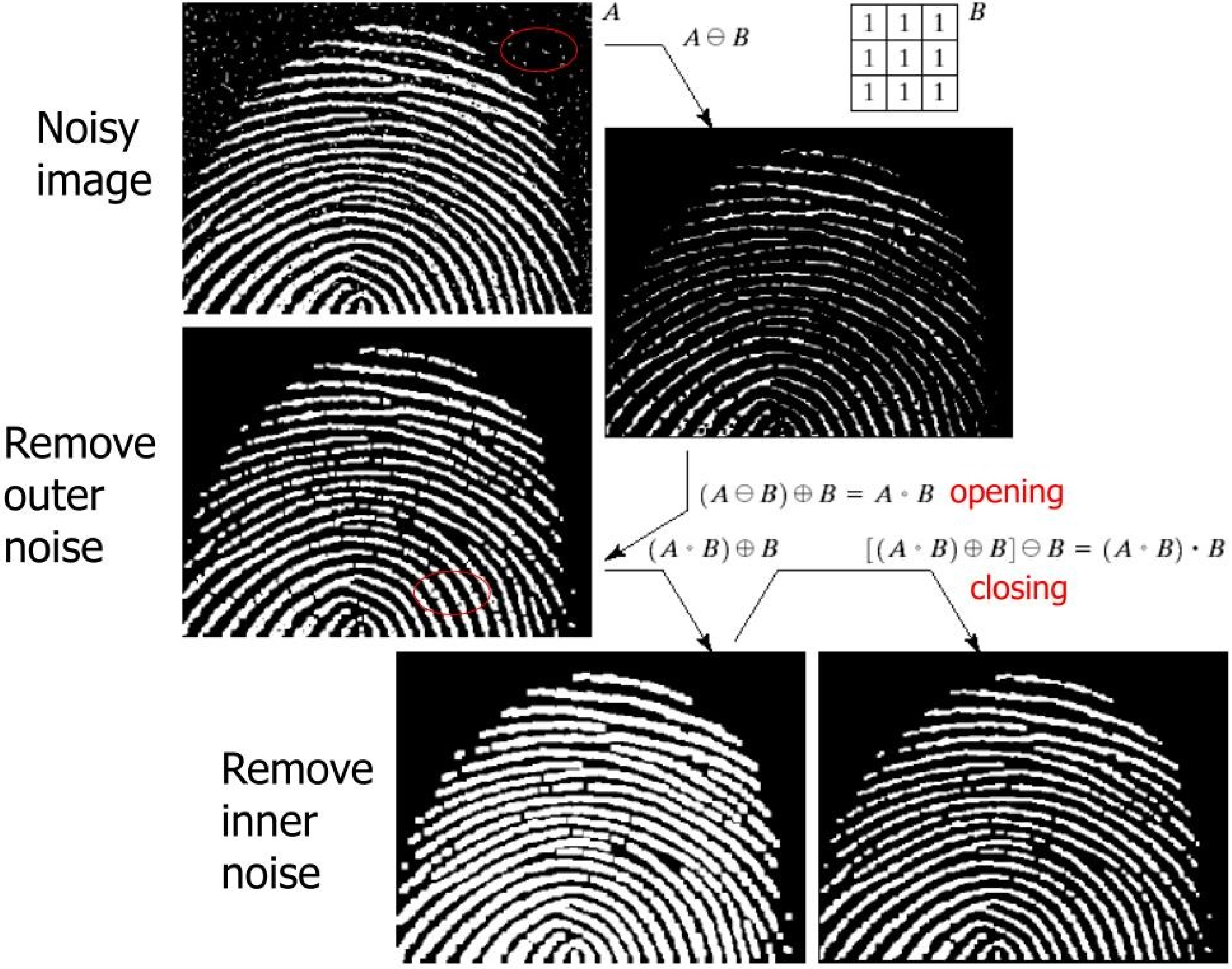
(iii) $(A \circ B) \circ B = A \circ B$

- Closing

(i) A is a subset (subimage) of $A \bullet B$ close

(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(iii) $(A \bullet B) \bullet B = A \bullet B$


Noisy image
A
$A \ominus B$
B
1 1 1
1 1 1
1 1 1
Remove outer noise
$(A \ominus B) \oplus B = A \circ B$ opening
$(A \circ B) \oplus B$
$[(A \circ B) \oplus B] \ominus B = (A \circ B) \bullet B$
closing
Remove inner noise

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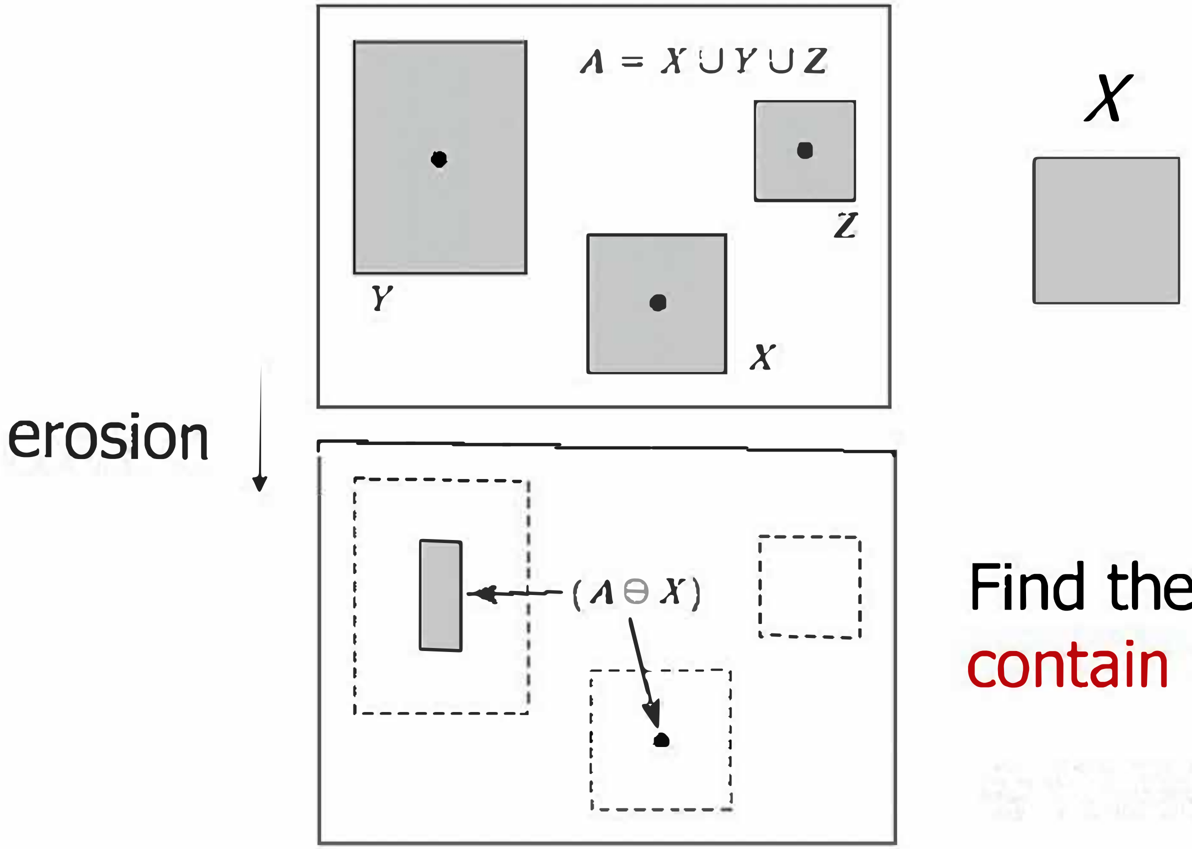
Hit-or-miss transformation

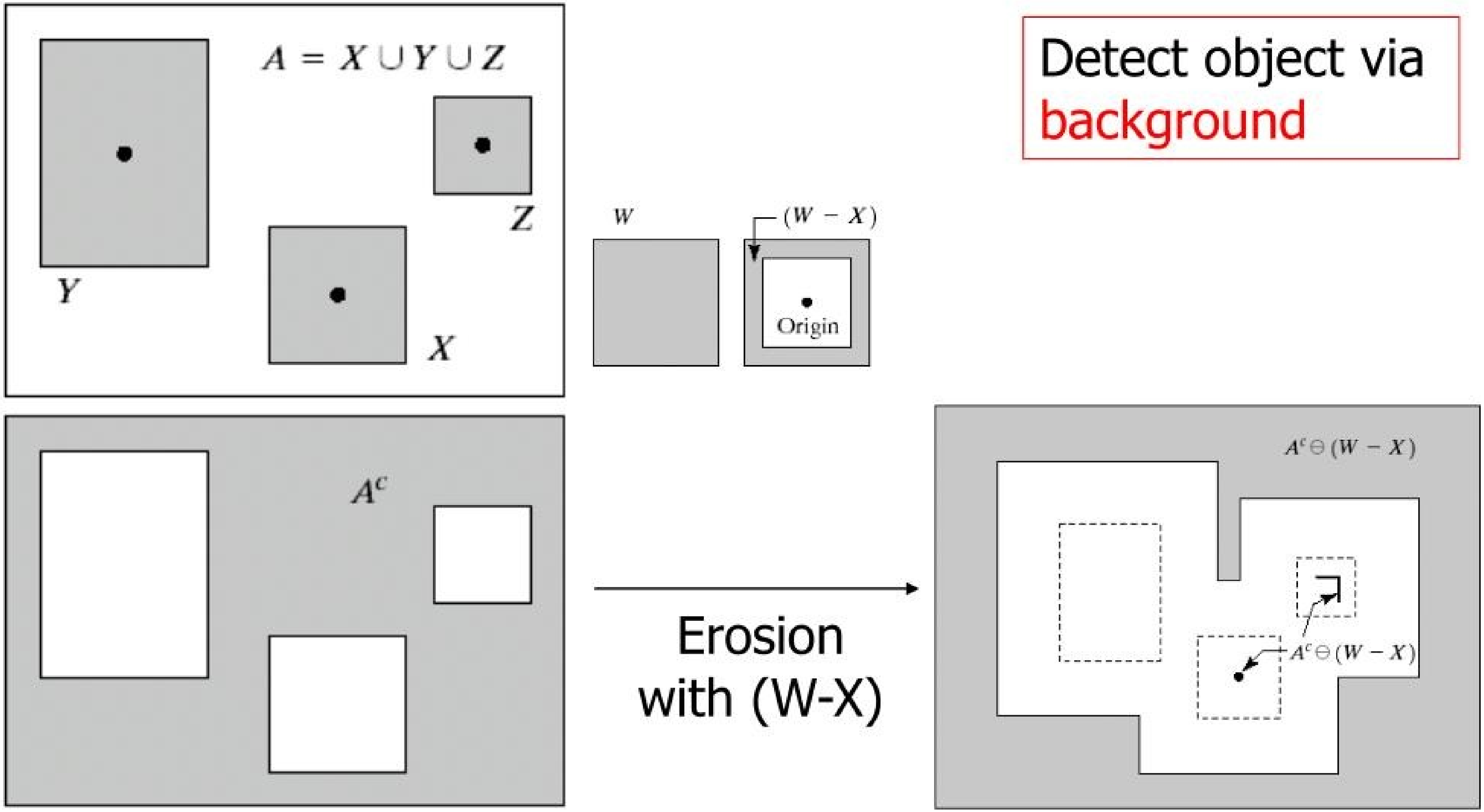
- Find the location of certain shape



Find the set of pixels that contain shape X

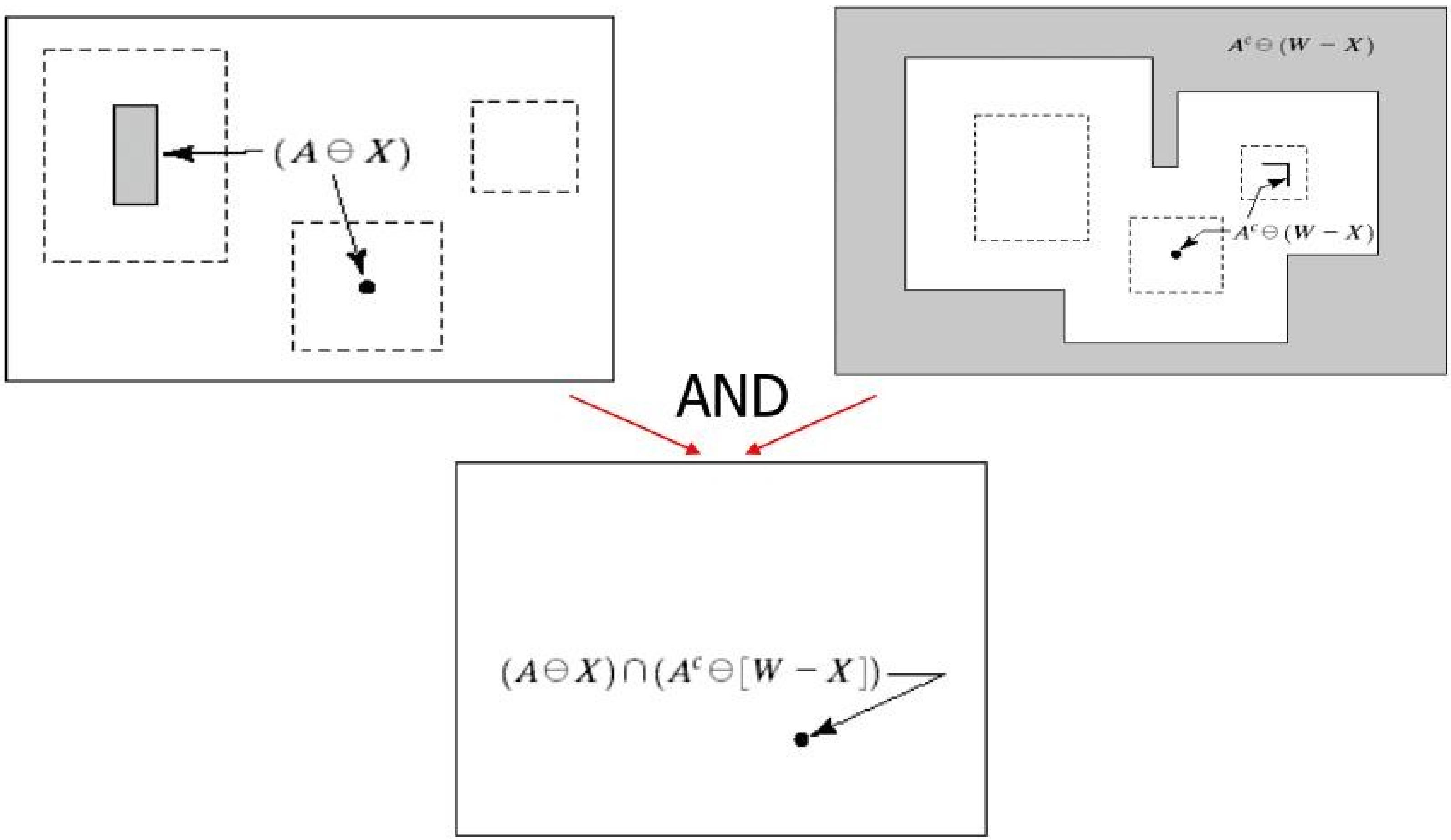
Hit-or-miss transformation

Detect object via background



Hit-or-miss transformation

- Eliminate un-necessary parts



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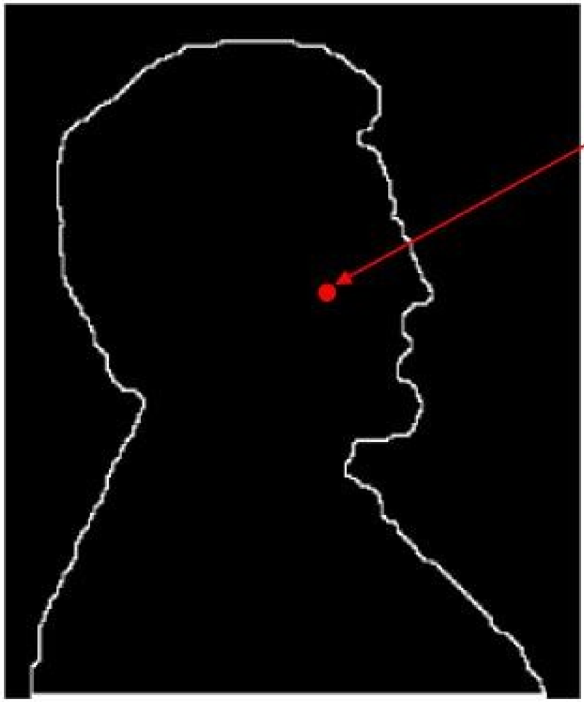
Basic morphological algorithms

- Extract image components that are useful in the representation and description of shape
- Boundary extraction
- Region filling
- Extract of connected components
- Convex hull
- Thinning
- Thickening
- Skeleton
- Pruning

Region filling

- How?
- Idea: place a point inside the region, then dilate that point iteratively



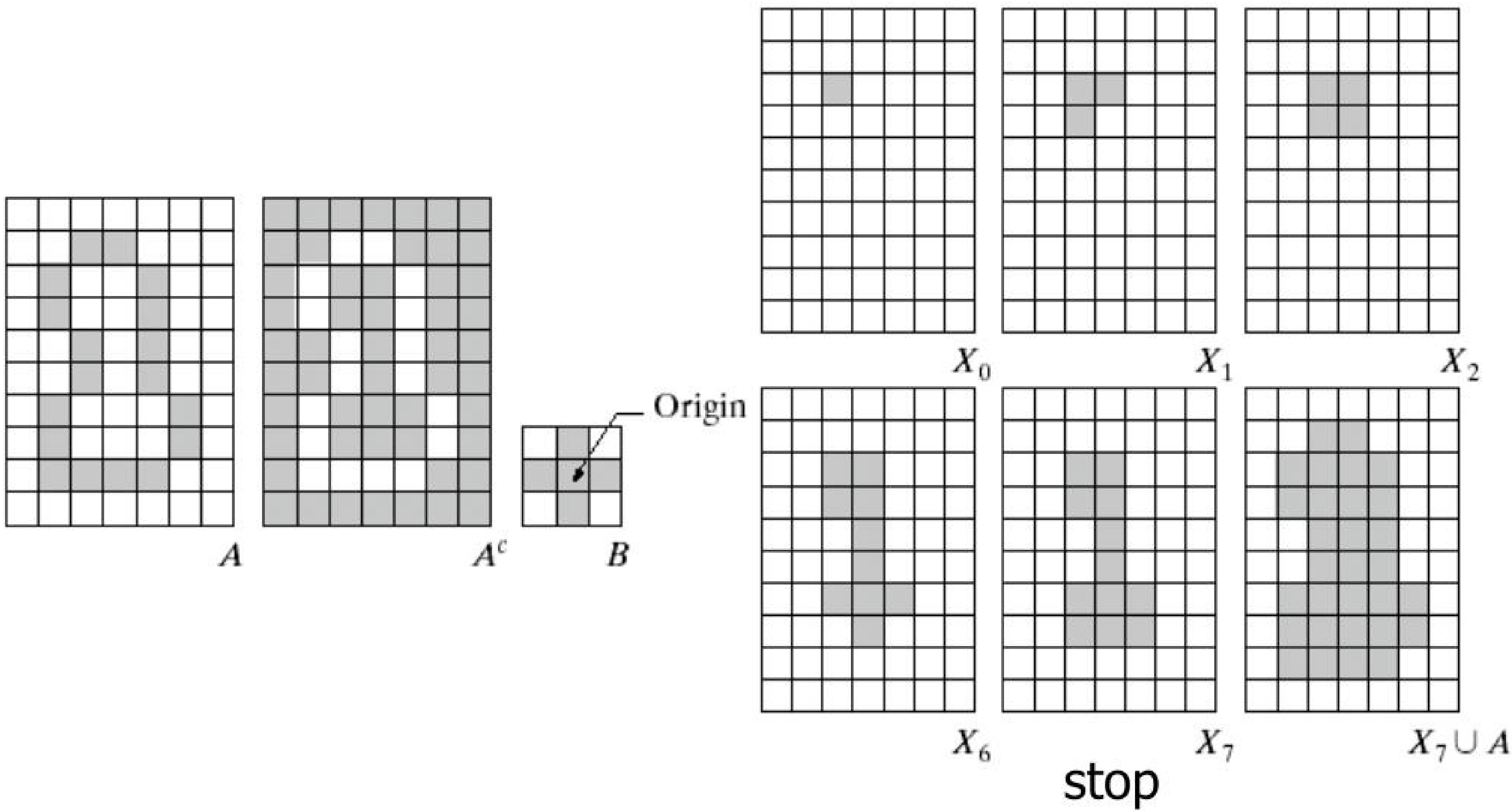
$$X_0 = p$$

$$X_k = (X_{k-1} \oplus B) \cap A^c, k = 1,2,3,...$$

Until $X_k = X_{k-1}$

Bound the growth

Region filling (cont.)

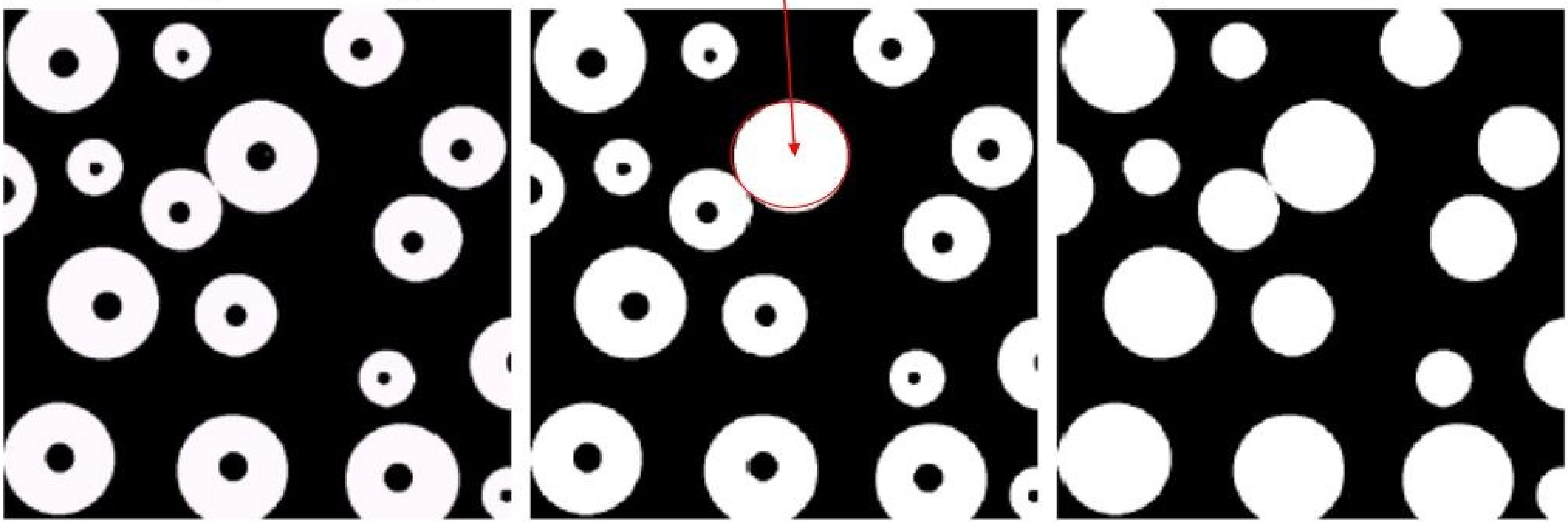


Application: region filling

Original image

The first filled region

Fill all regions



Extraction of connected components

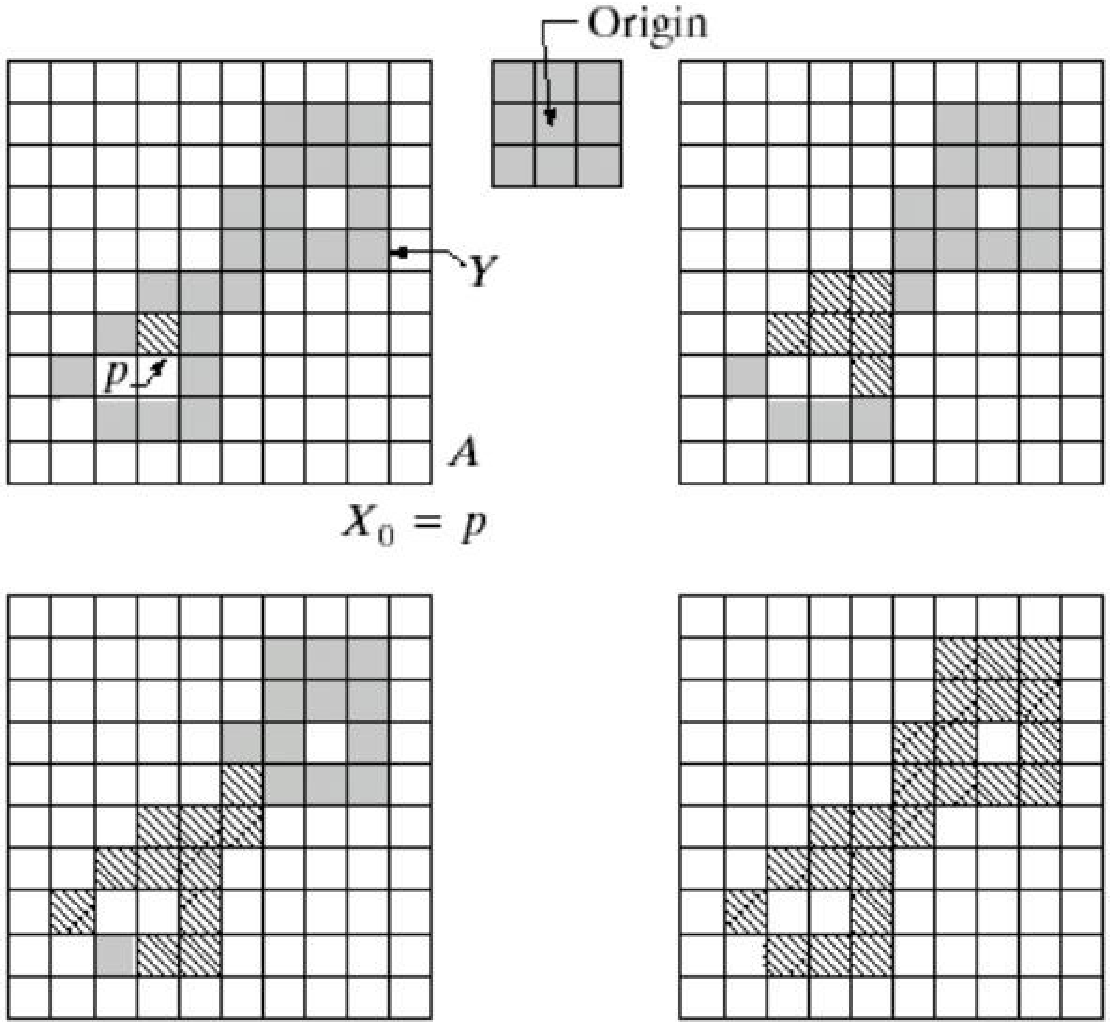
- Idea: start from a point in the connected component, and dilate it iteratively

$$X_0 = p$$

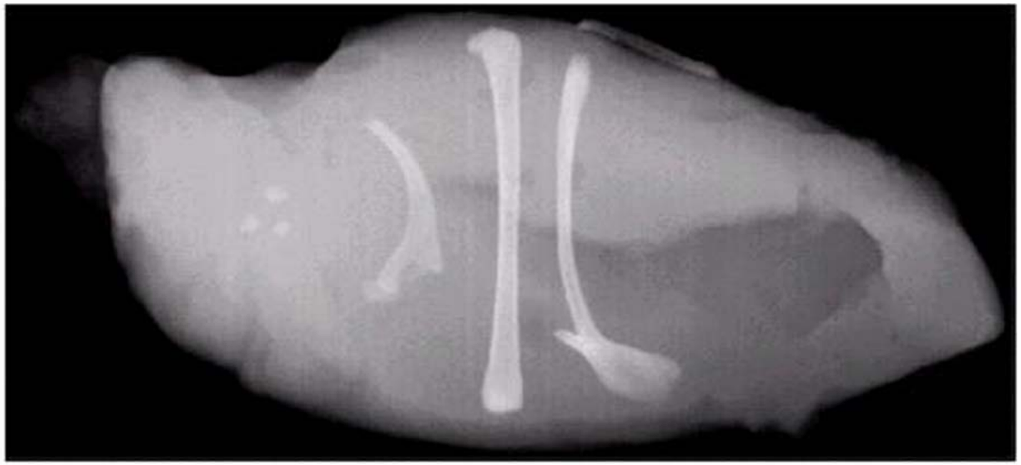
$$X_k = (X_{k-1} \oplus B) \cap A, \;\; k = 1,2,3,\ldots$$

Until $X_k = X_{k-1}$

Extraction of connected components (cont.)



original



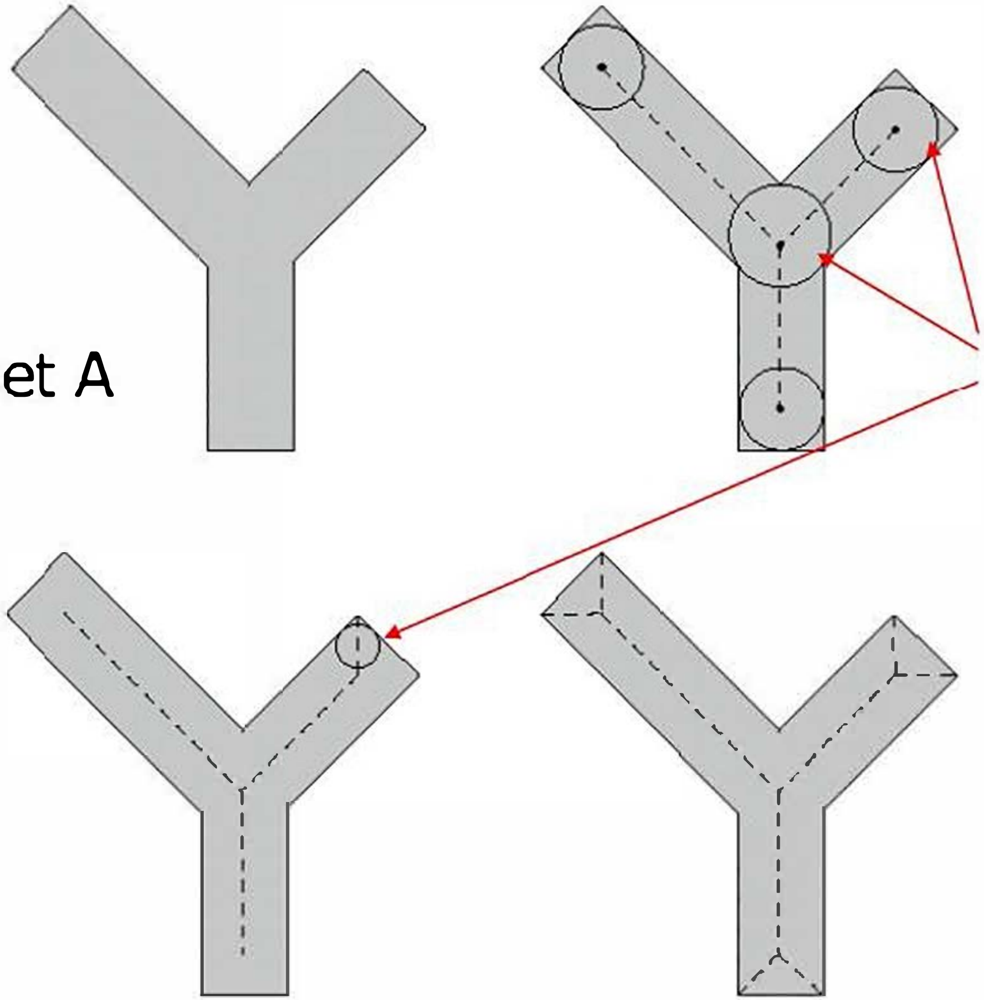
thresholding

erosion

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Skeletons

Set A



How to define a Skeletons?

Maximum disk

1. The largest disk Centered at a pixel
2. Touch the boundary of A at two or more places

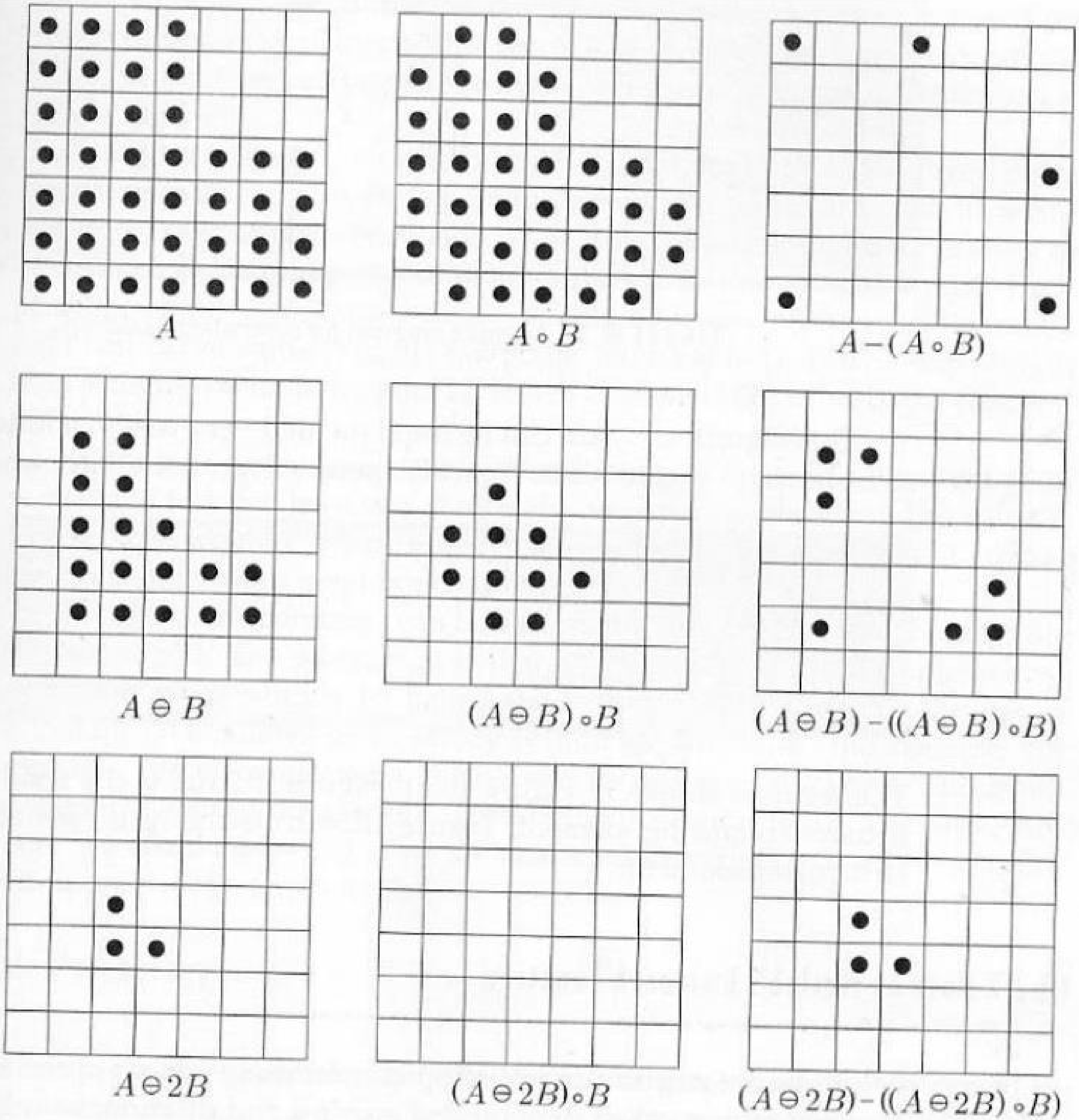
Recall: Balls of erosion!

Skeleton

- Idea: erosion

Erosions	Openings	Set differences
A	$A \circ B$	$A - (A \circ B)$
$A \ominus B$	$(A \ominus B) \circ B$	$(A \ominus B) - ((A \ominus B) \circ B)$
$A \ominus 2B$	$(A \ominus 2B) \circ B$	$(A \ominus 2B) - ((A \ominus 2B) \circ B)$
$A \ominus 3B$	$(A \ominus 3B) \circ B$	$(A \ominus 3B) - ((A \ominus 3B) \circ B)$
$\vdots$	$\vdots$	$\vdots$
$A \ominus kB$	$(A \ominus kB) \circ B$	$(A \ominus kB) - ((A \ominus kB) \circ B)$

Erosion k



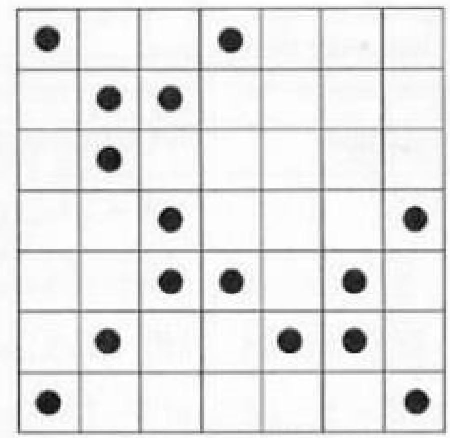
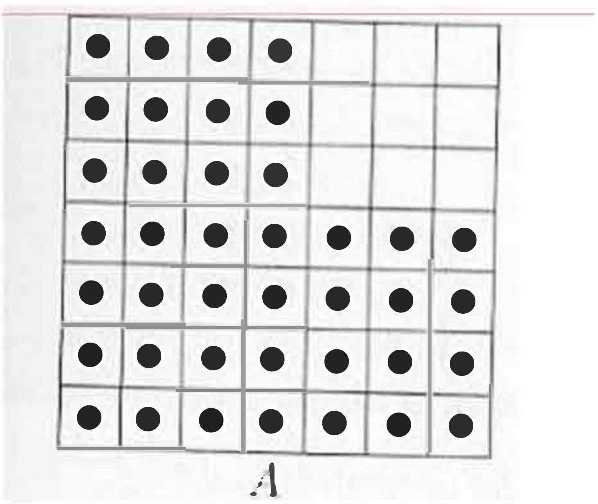


FIGURE 10.29 *The final skeleton.*

FIGURE 10.28 Skeleton

Problem

- The scanned image is not adjusted well



- How to detect the direction of lines?
- How to rotate?