Lecture on

Image Filtering

Credit:
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Image filtering

- The word *filter* comes from frequency-domain processing, where "filtering" refers to the process of accepting or rejecting certain frequency components
- We distinguish between low-pass and high-pass filtering
  - A **low-pass filter** smooths an image (retains low-frequency components)
  - A **high-pass filter** retains the contours (also called edges) of an image (high frequency)

Low-pass filtered image

High-pass filtered image

Low-pass filtering

Low-pass filtering
Motivation: noise reduction

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian distribution

Original

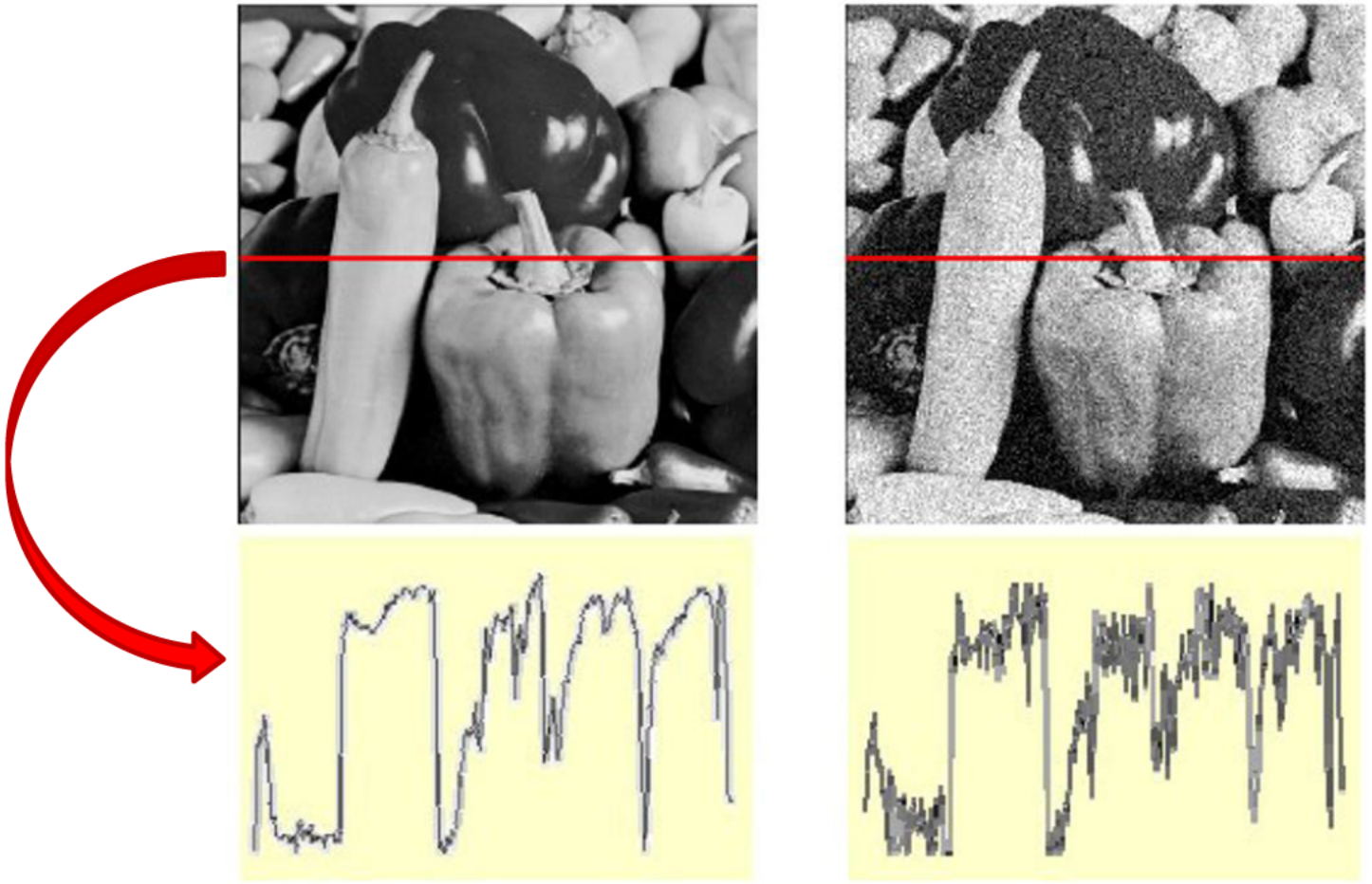
Salt and pepper noise

Impulse noise

Gaussian noise

Source: S. Seitz

Gaussian noise



Ideal image Noise process Gaussian i.i.d. (white) noise

$$I(x,y) = \hat{I}(x,y) + \eta(x,y) \qquad \eta(x,y) \sim \mathrm{N}(\mu,\sigma)$$

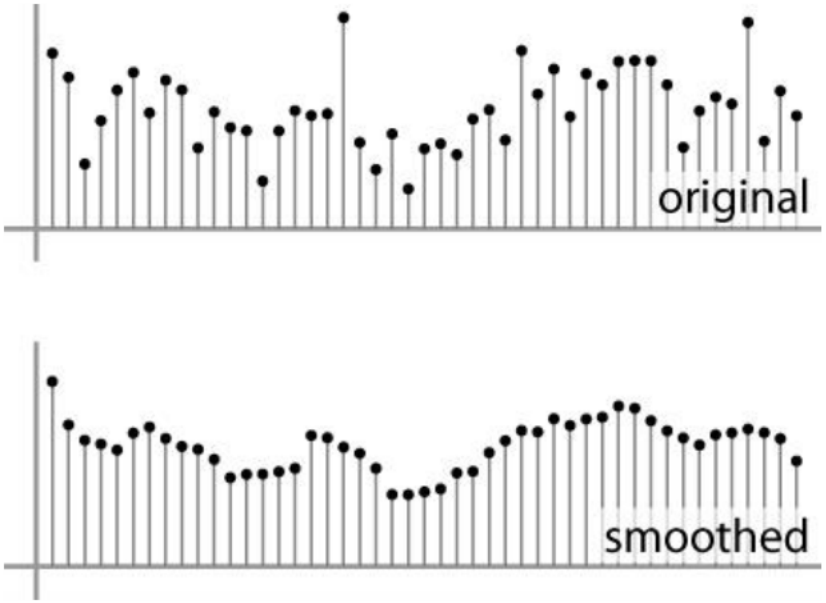
How could we reduce the noise to try to recover the "ideal image"?

Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise process to be independent from pixel to pixel

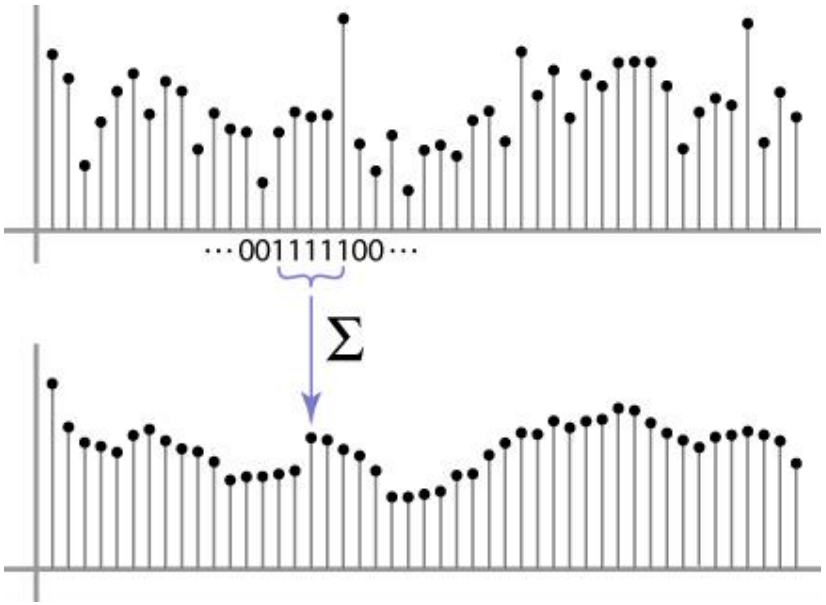
Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



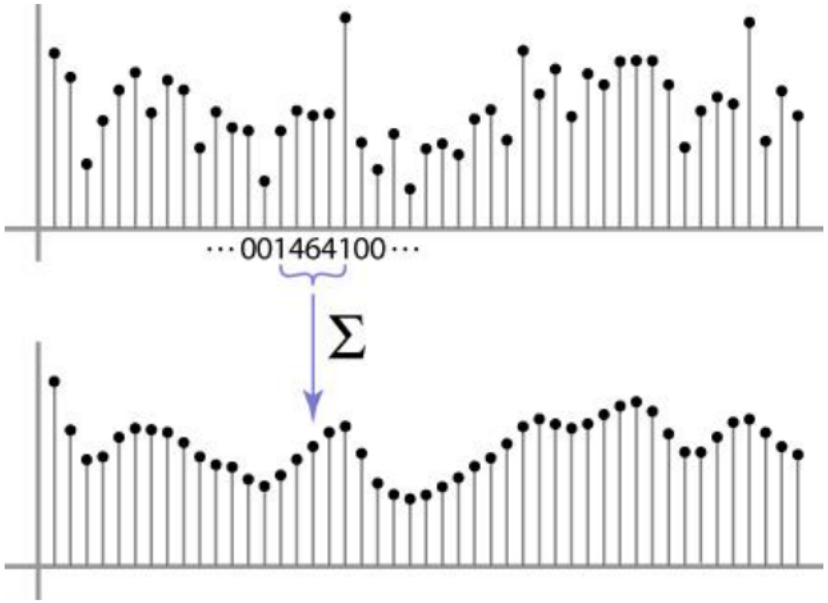
Weighted Moving Average

- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Weighted Moving Average

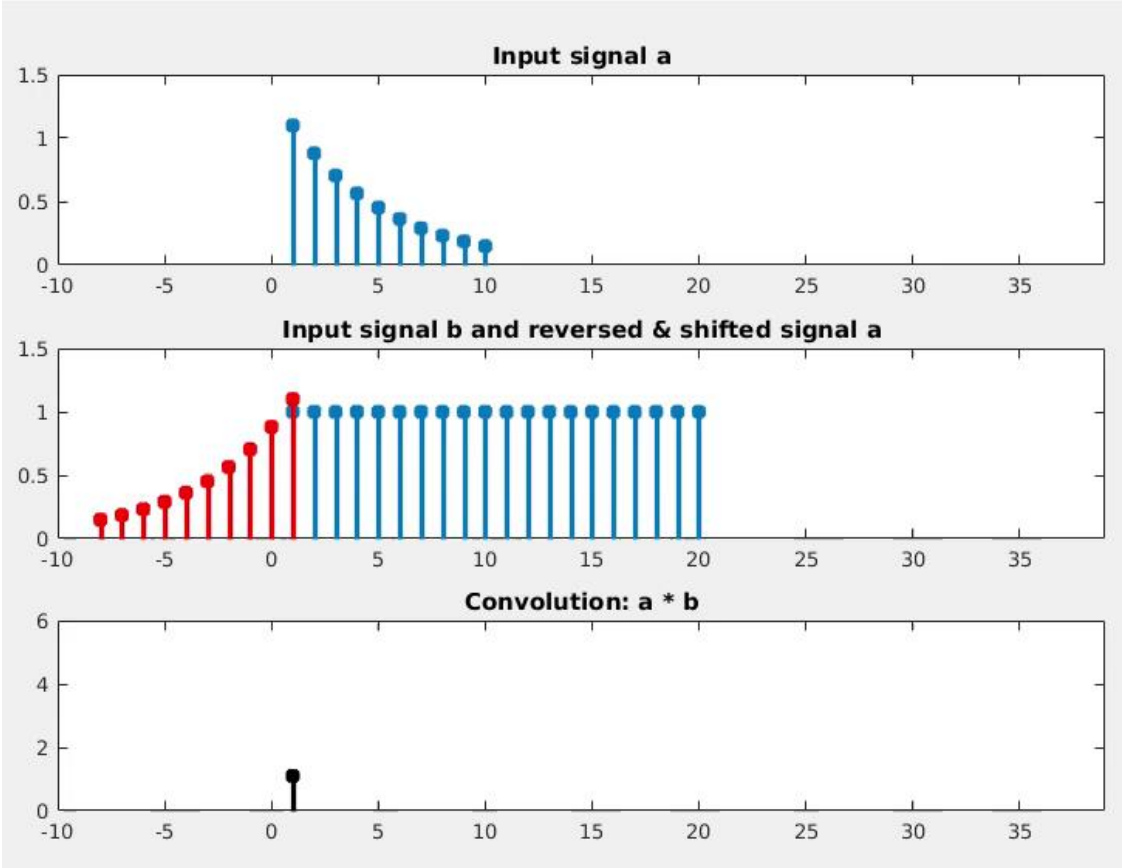
- Non-uniform weights [1, 4, 6, 4, 1] / 16



This operation is called *convolution*

Example of convolution of two sequences (or “signals”)

- One of the sequences is flipped (right to left) before sliding over the other
- Notation: $a * b$
- Nice properties: linearity, associativity, commutativity, etc.



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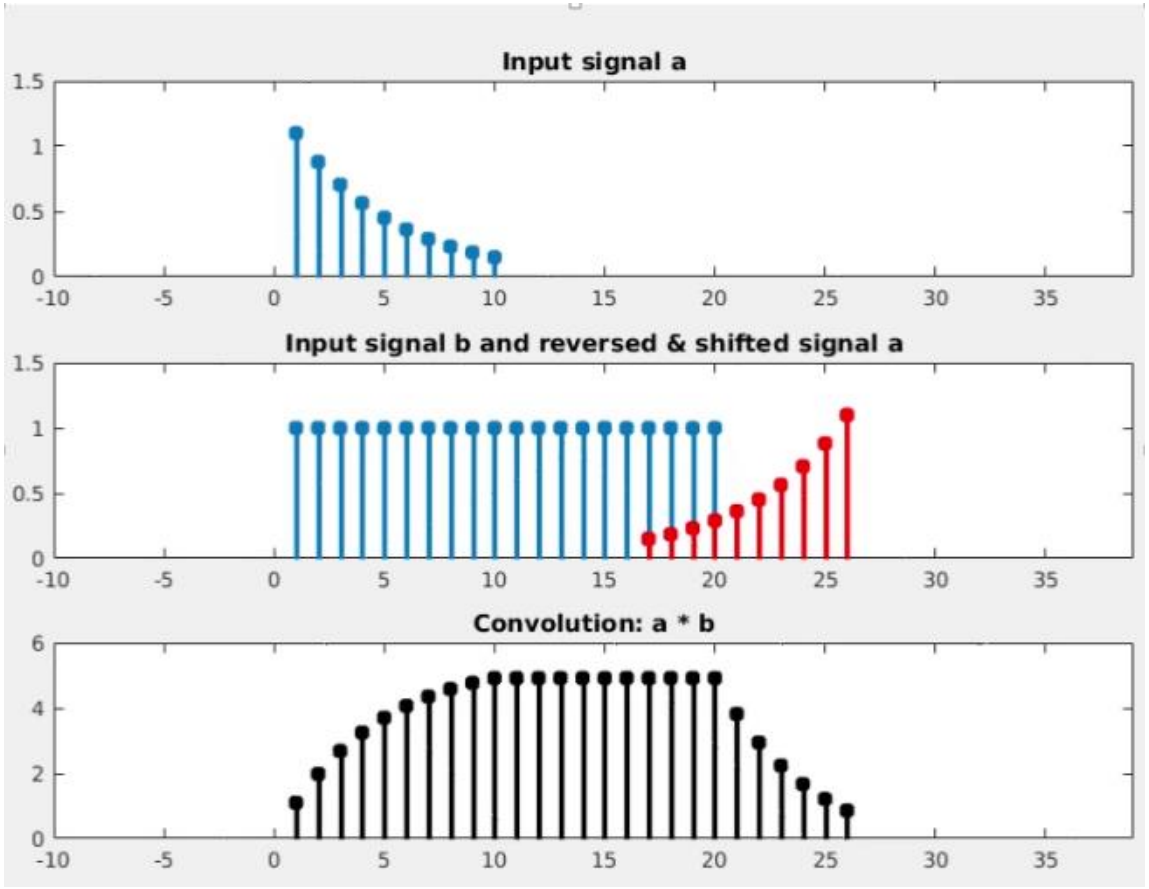
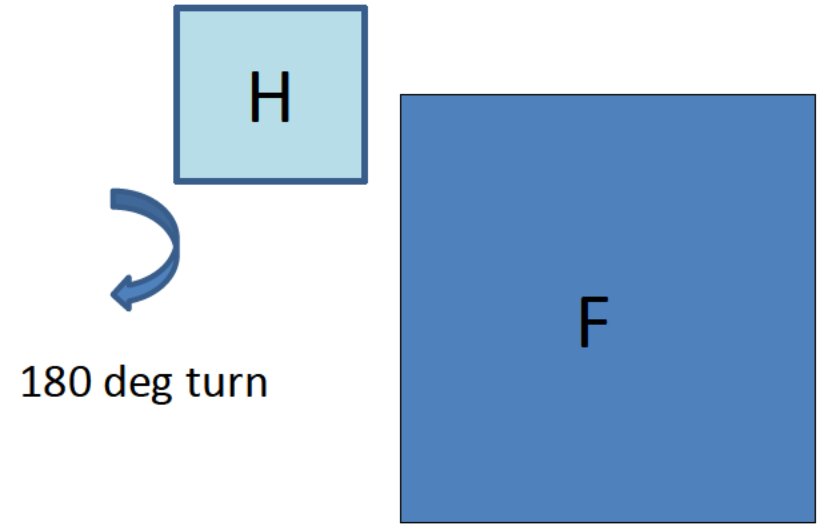


2D Filtering

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left) (=180 deg turn)
  - Then slide the filter over the image and compute sum of products

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x-u, y-v] H[u,v]$$

$$G = F * H$$



Filtering an image: replace each pixel
with a linear combination of its neighbors.

The **filter H** is also called "**kernel**" or "**mask**".

Review: Convolution vs. Cross-correlation

Convolution $G = F * H$

$$G[x, y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x-u, y-v]H[u, v]$$

Properties: linearity, associativity, commutativity

Cross-correlation $G = F \otimes H$

$$G[x, y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x+u, y+v]H[u, v]$$

Properties: linearity, but not associativity and commutativity

For a Gaussian or box filter, will the output of convolution and correlation be different?

Example: Moving Average in 2D

Input image

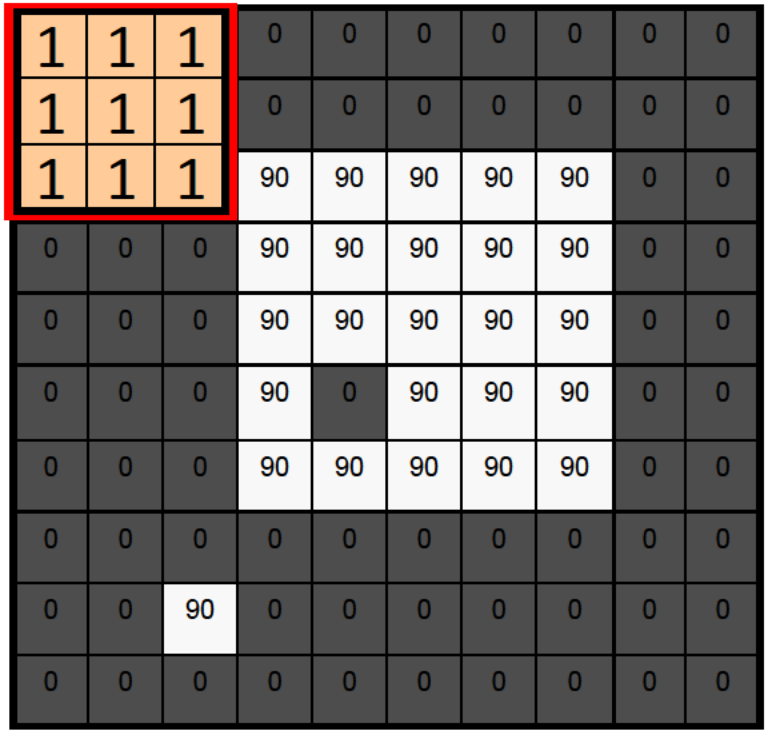
$F[x, y]$

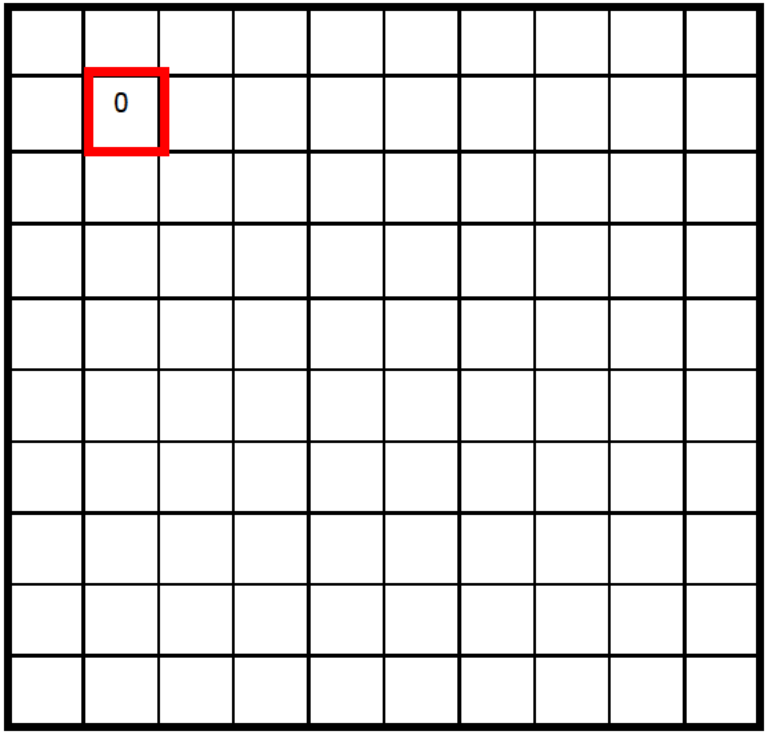
Filtered image

$G[x, y]$

"box filter"

$\frac{1}{9}$

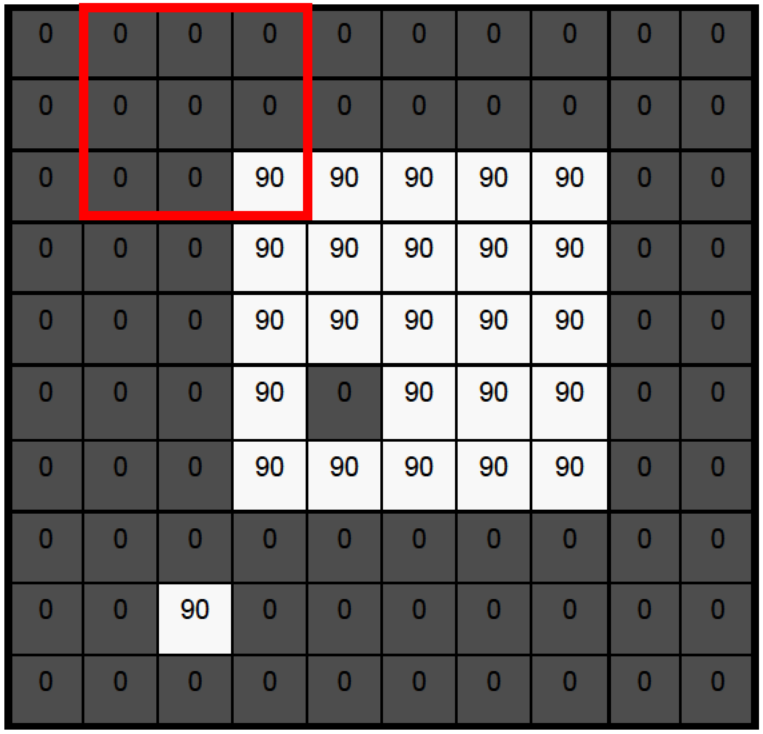




Example: Moving Average in 2D

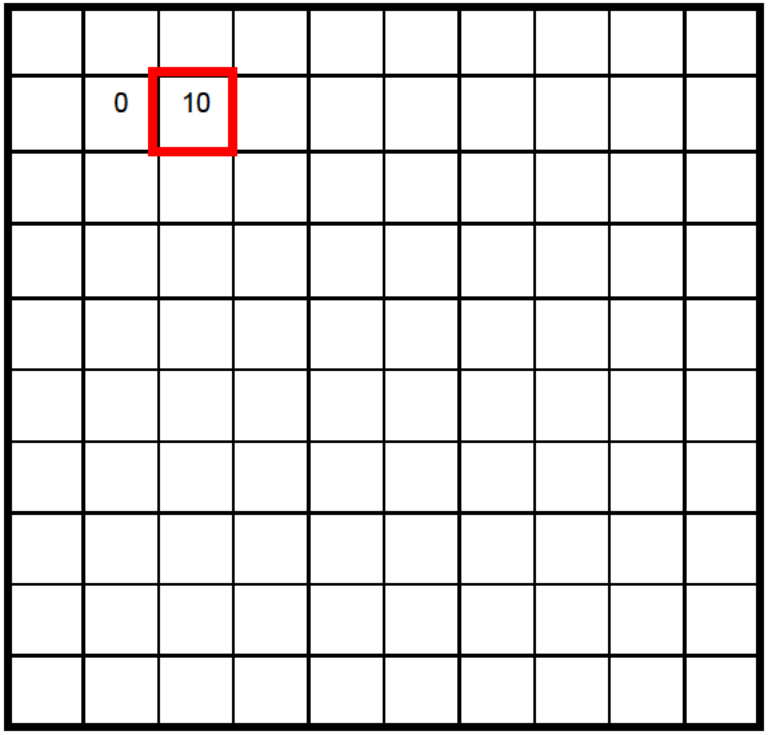
Input image

$F[x, y]$



Filtered image

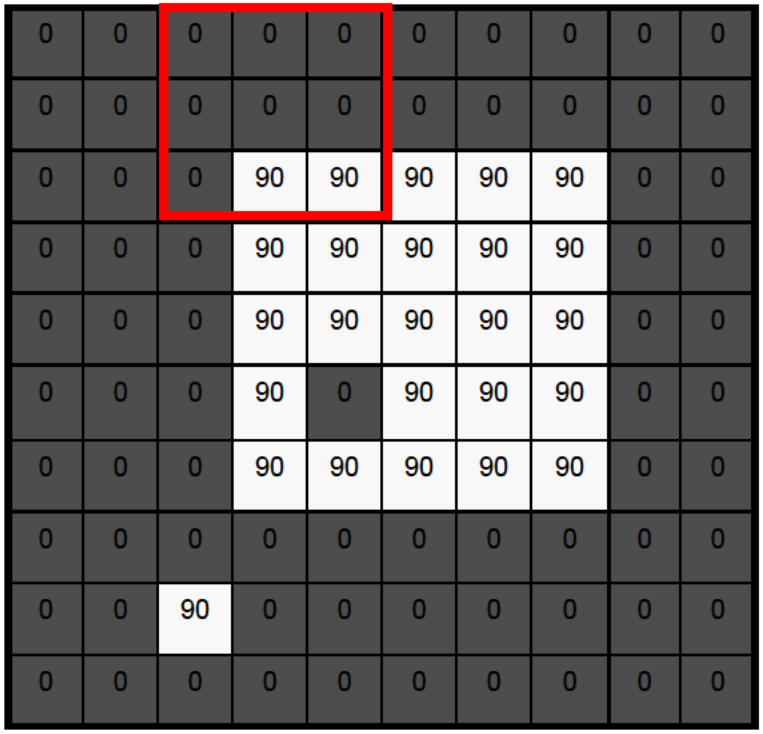
$G[x, y]$



Example: Moving Average in 2D

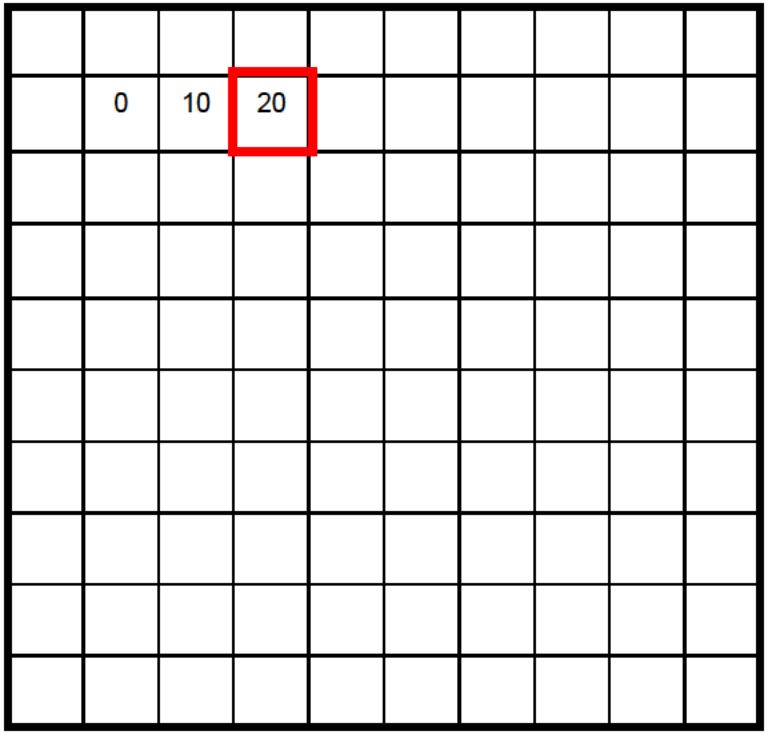
Input image

$F[x, y]$



Filtered image

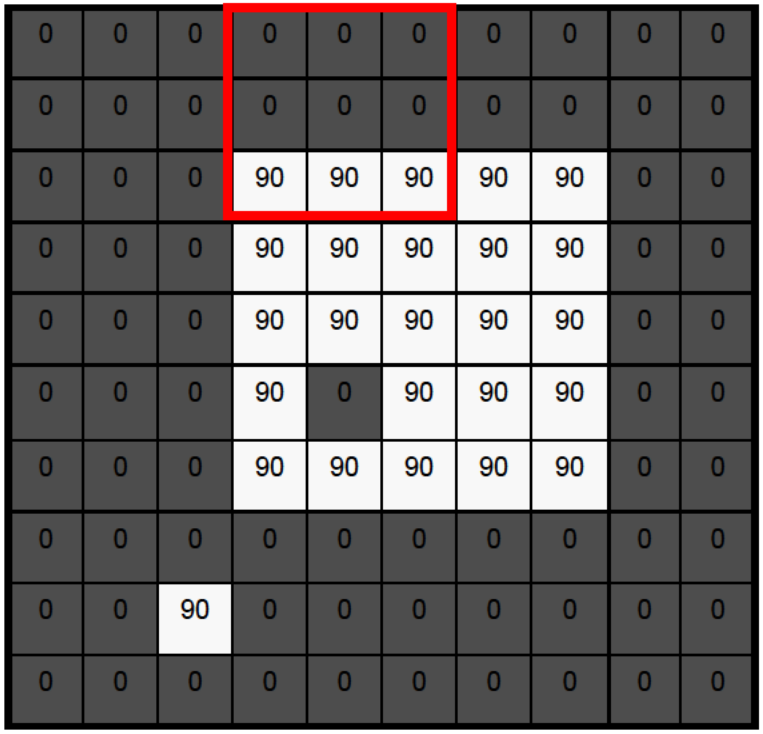
$G[x, y]$



Example: Moving Average in 2D

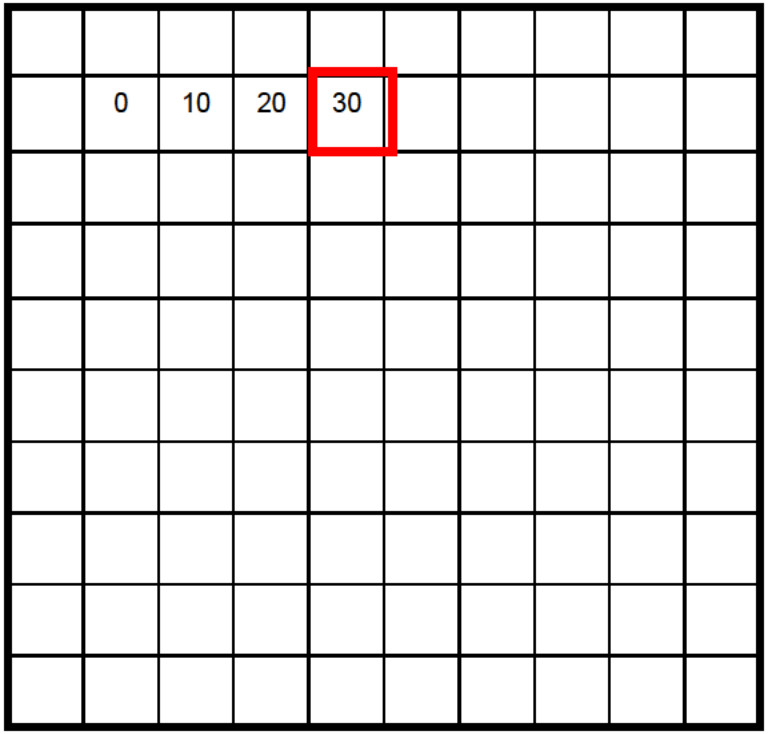
Input image

$F[x, y]$



Filtered image

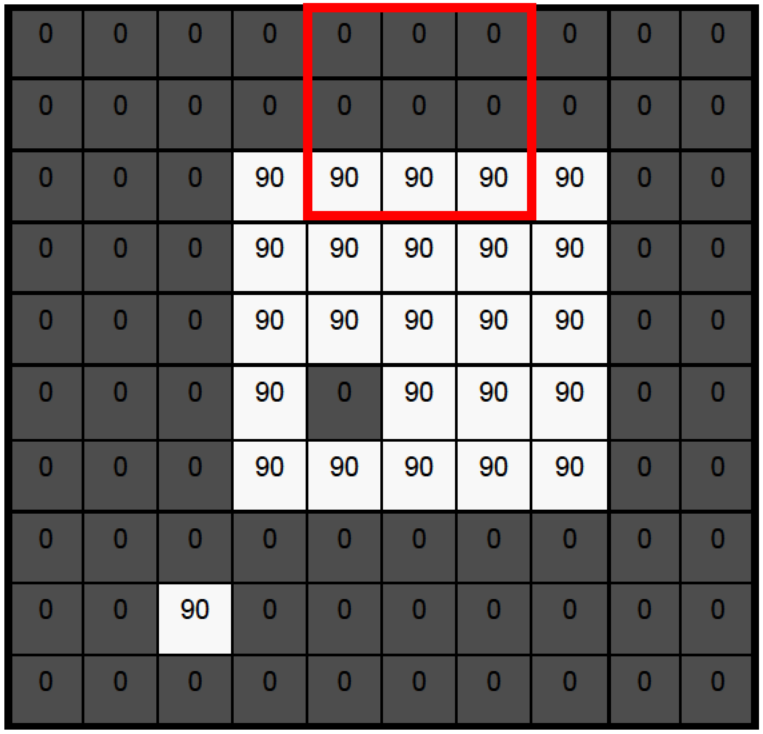
$G[x, y]$



Example: Moving Average in 2D

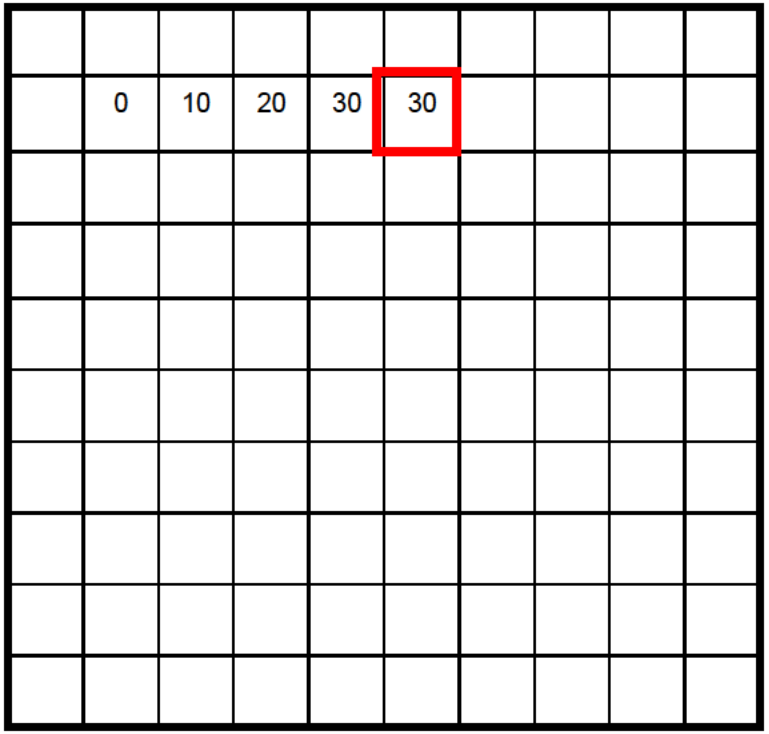
Input image

$F[x, y]$



Filtered image

$G[x, y]$



Example: Moving Average in 2D

Input image

$F[x, y]$

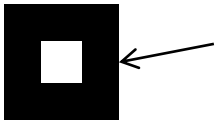
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$G[x, y]$

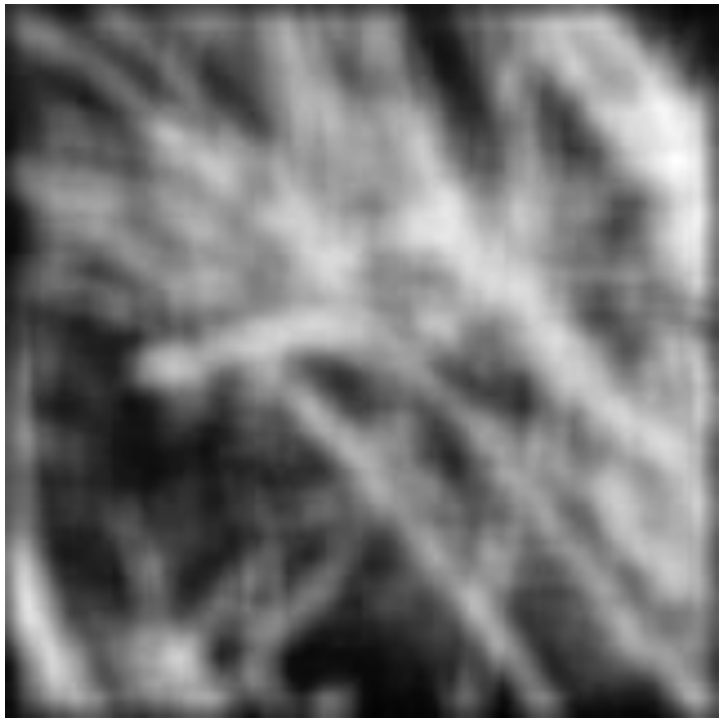
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Example: Moving Average in 2D



Box filter:
white = high value, black = zero value

original



filtered

Gaussian filter

- What if we want the closest pixels to have higher influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

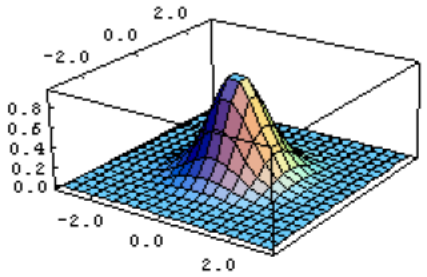
$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

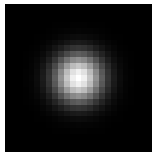
$H[u, v]$

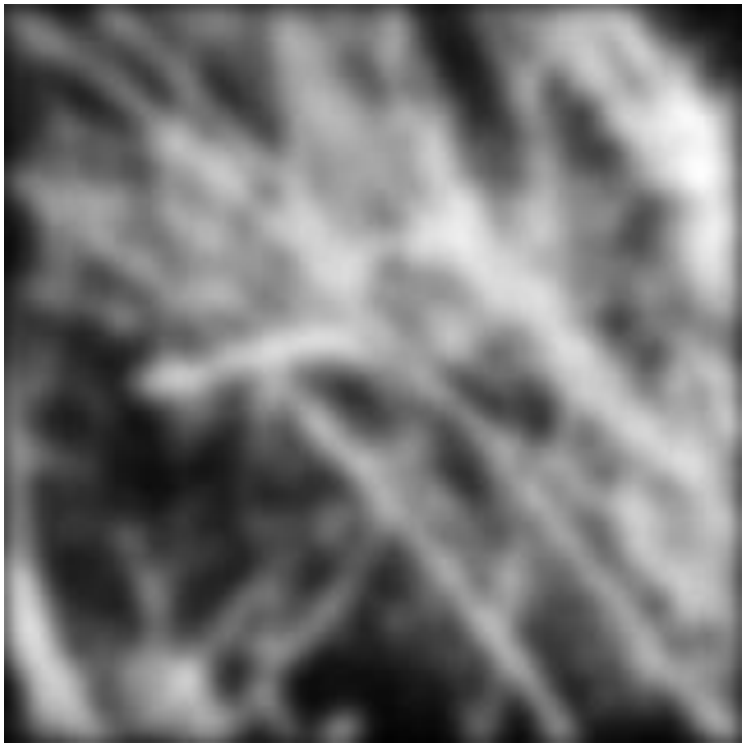
This kernel is the approximation of a Gaussian function:

$$H[u, v] \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

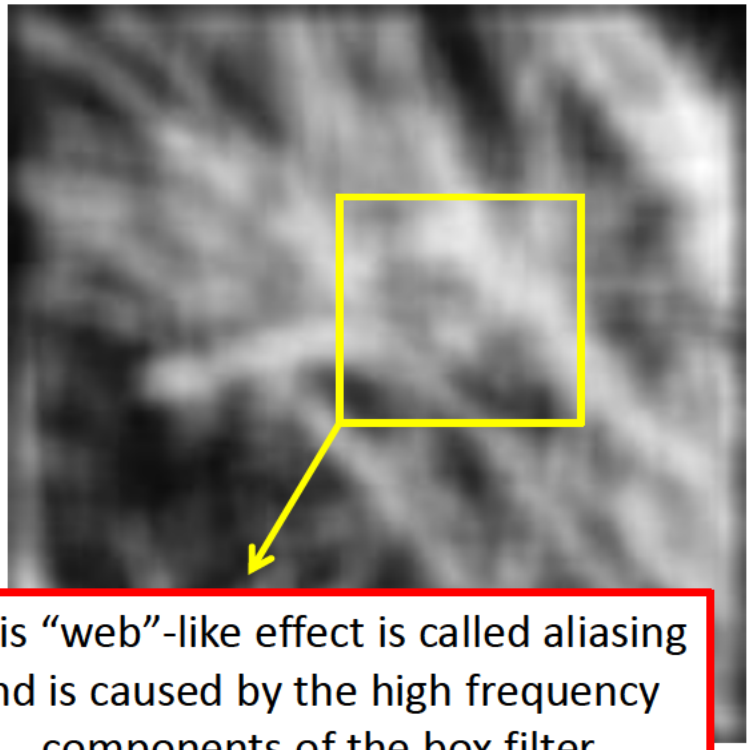


Smoothing with a Gaussian





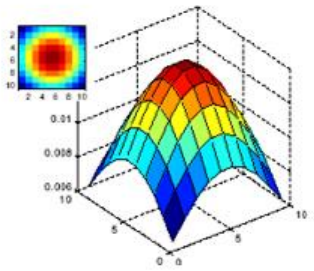
Compare the result with a box filter



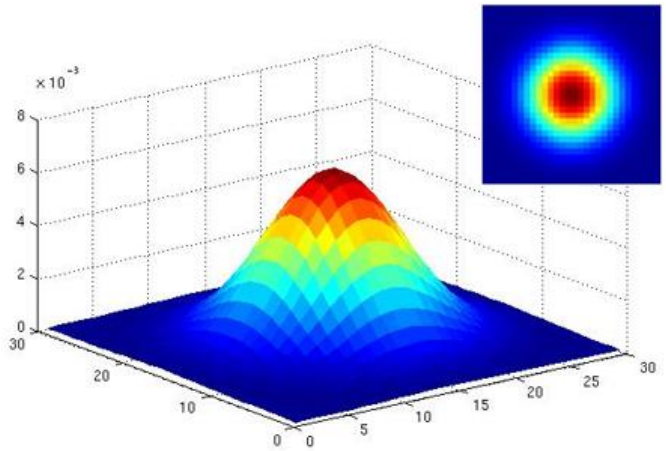
This “web”-like effect is called aliasing and is caused by the high frequency components of the box filter

Gaussian filters

- What parameters matter?
- **Size** of the kernel
  - NB: a Gaussian function has **infinite support**, but discrete filters use finite kernels



σ = 5 pixels
with 10 × 10 pixel kernel

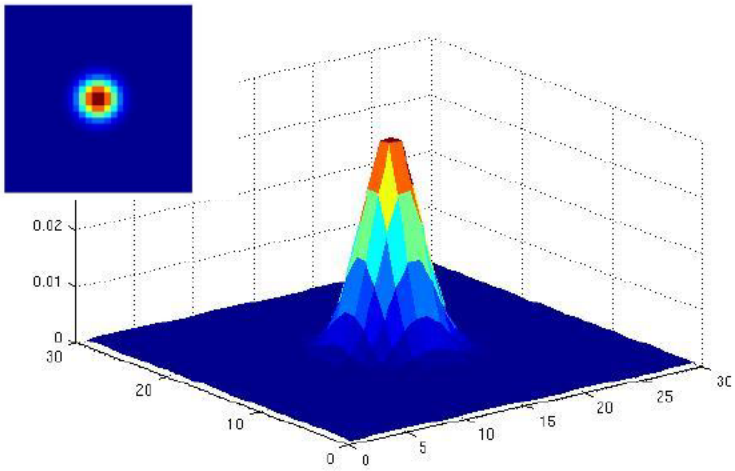


σ = 5 pixels
with 30 × 30 pixel kernel

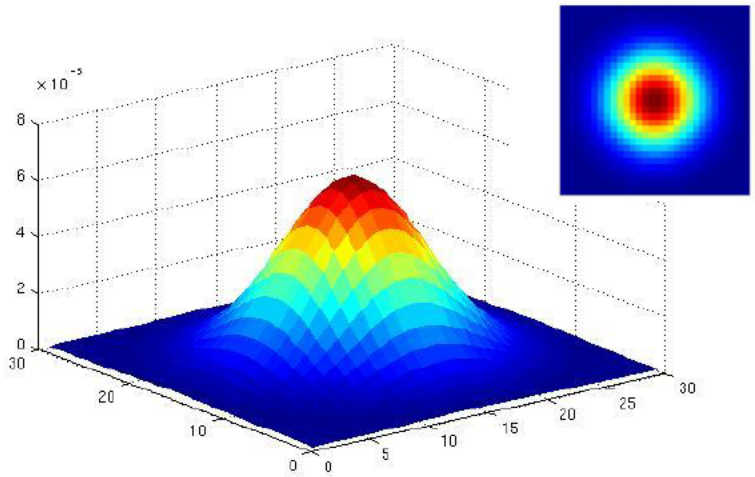
Which one approximates better the ideal Gaussian filter, the left or the right one?

Gaussian filters

- What parameters matter?
- **Variance** of Gaussian: control the amount of smoothing



σ = 2 pixels
with 30 × 30 pixel kernel

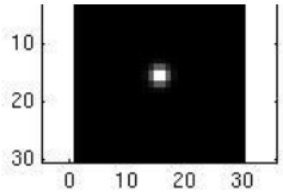


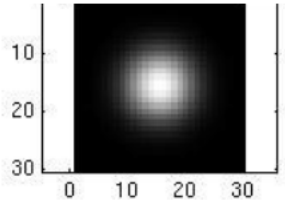
σ = 5 pixels
with 30 × 30 pixel kernel

Recall: standard deviation = σ [pixels], variance = σ^2 [pixels^2]

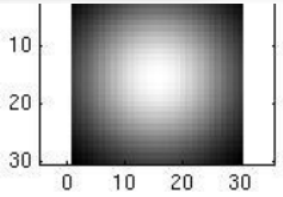
Smoothing with a Gaussian

σ is called “scale” of the Gaussian kernel, and controls the amount of smoothing.





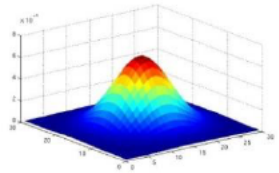
…



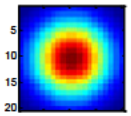
Sample Matlab code

```
>> hsize = 20;
>> sigma = 5;
>> h = fspecial('gaussian', hsize, sigma);
```

```
>> mesh(h);
```



```
>> imagesc(h);
```




```
>> im = imread('panda.jpg');
>> outim = imfilter(im, h);
>> imshow(outim);
```

im

outim

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)
    - wrap around
    - copy edge
    - reflect across edge

Summary on (linear) smoothing filters

- Smoothing filter
  - has positive values (also called coefficients)
  - **sums to 1** → preserve brightness of constant regions
  - removes “high-frequency” components; “low-pass” filter

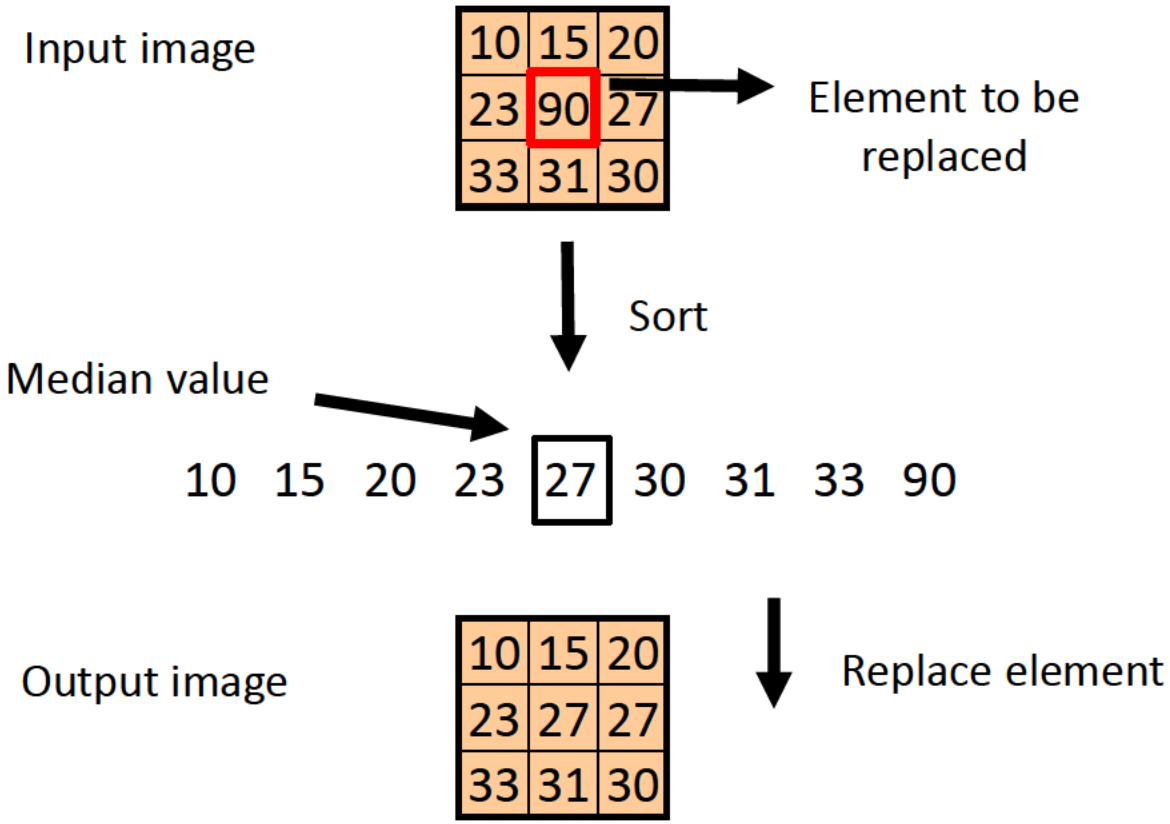
Non-linear filtering

Effect of smoothing filters

Linear smoothing filters do not alleviate salt and pepper noise!

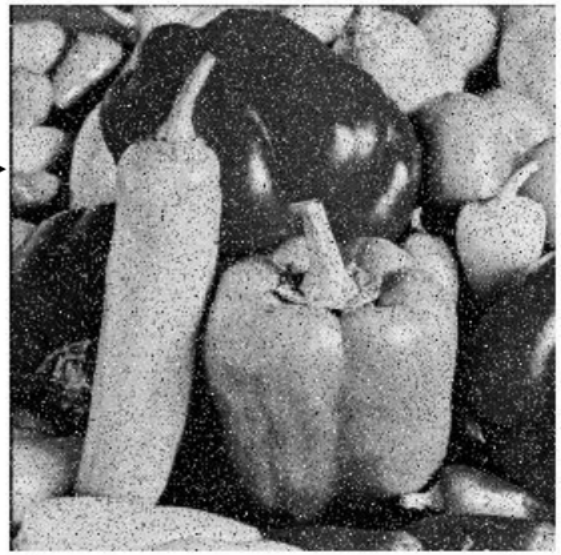
Median filter

- It is a non-linear filter
- Removes spikes: good for "*impulse noise*" and "*salt & pepper noise*"

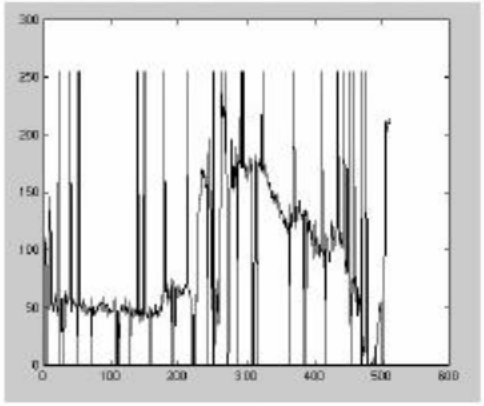


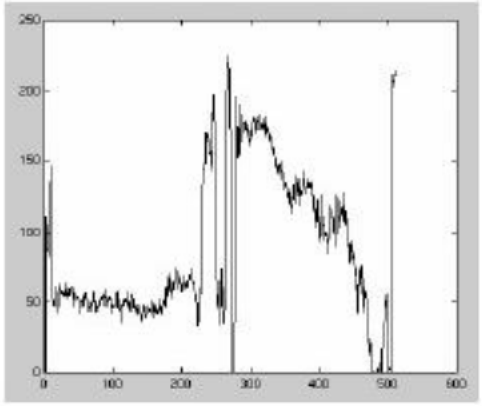
Median filter

Salt and pepper noise



Median filtered

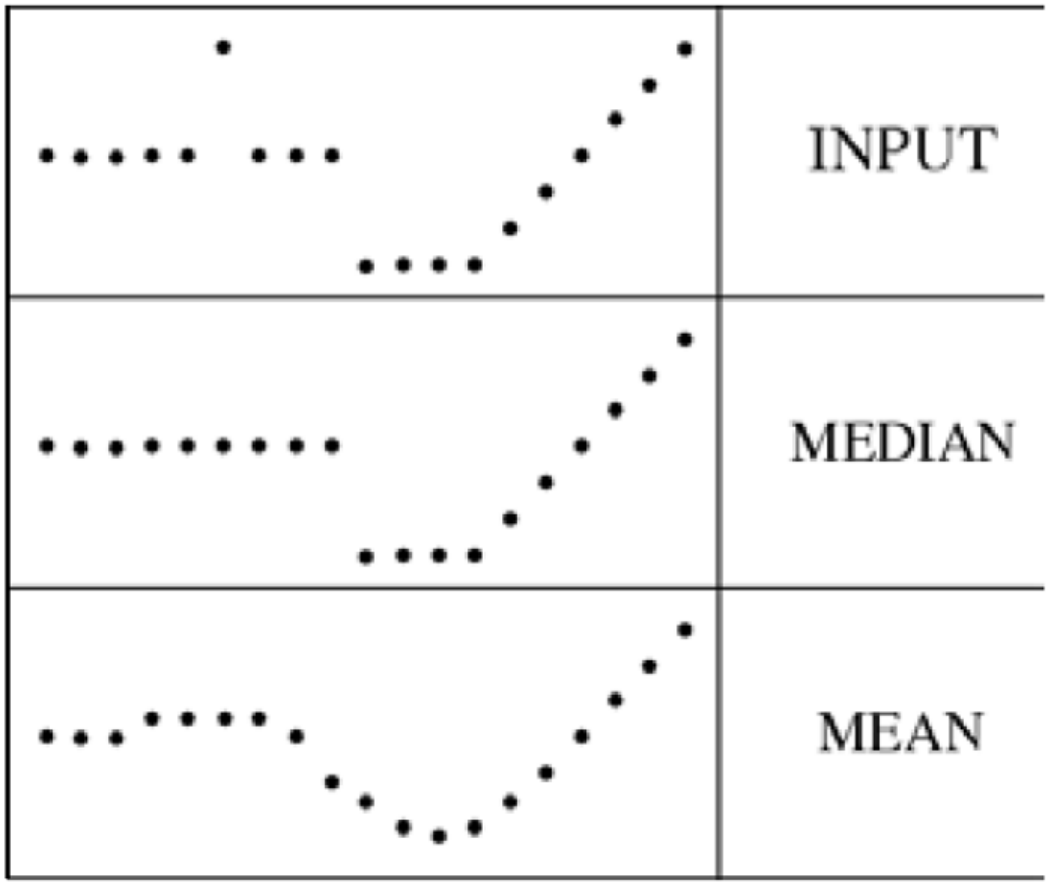




Plots of a row of the image

Median filter

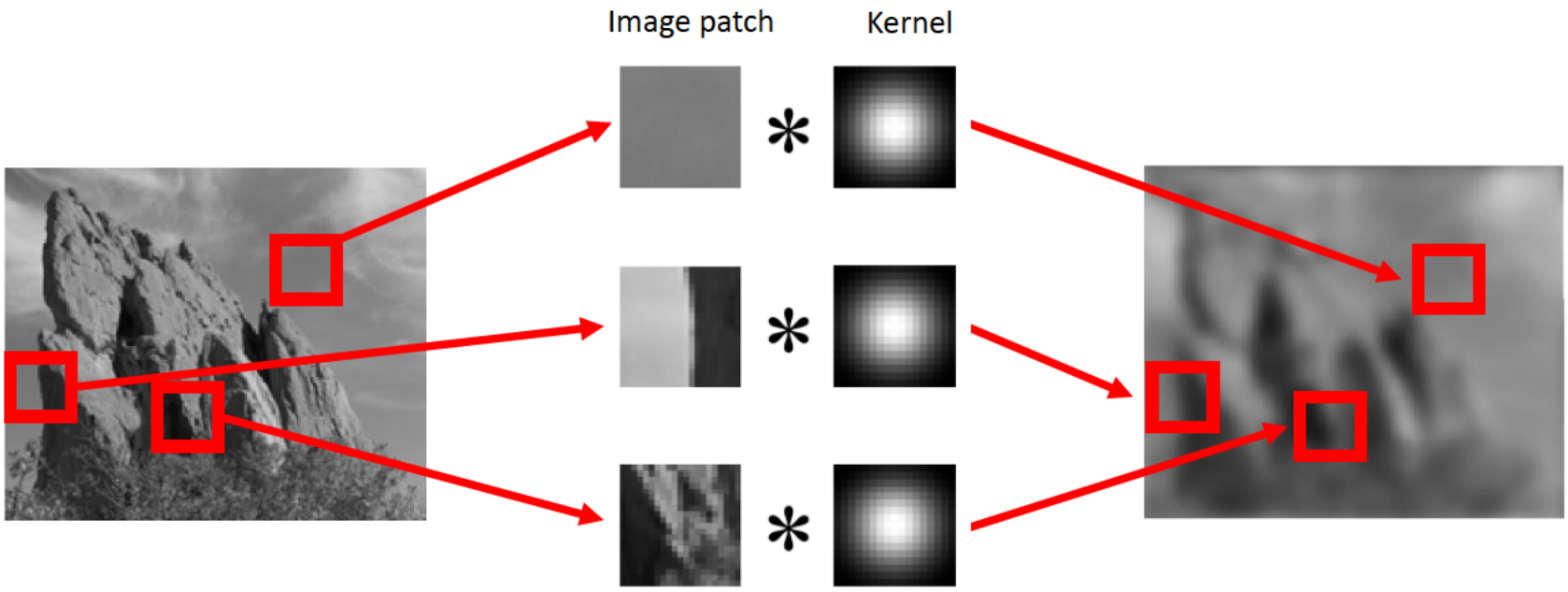
- Median filter preserves strong edges,



… but it removes small edges.

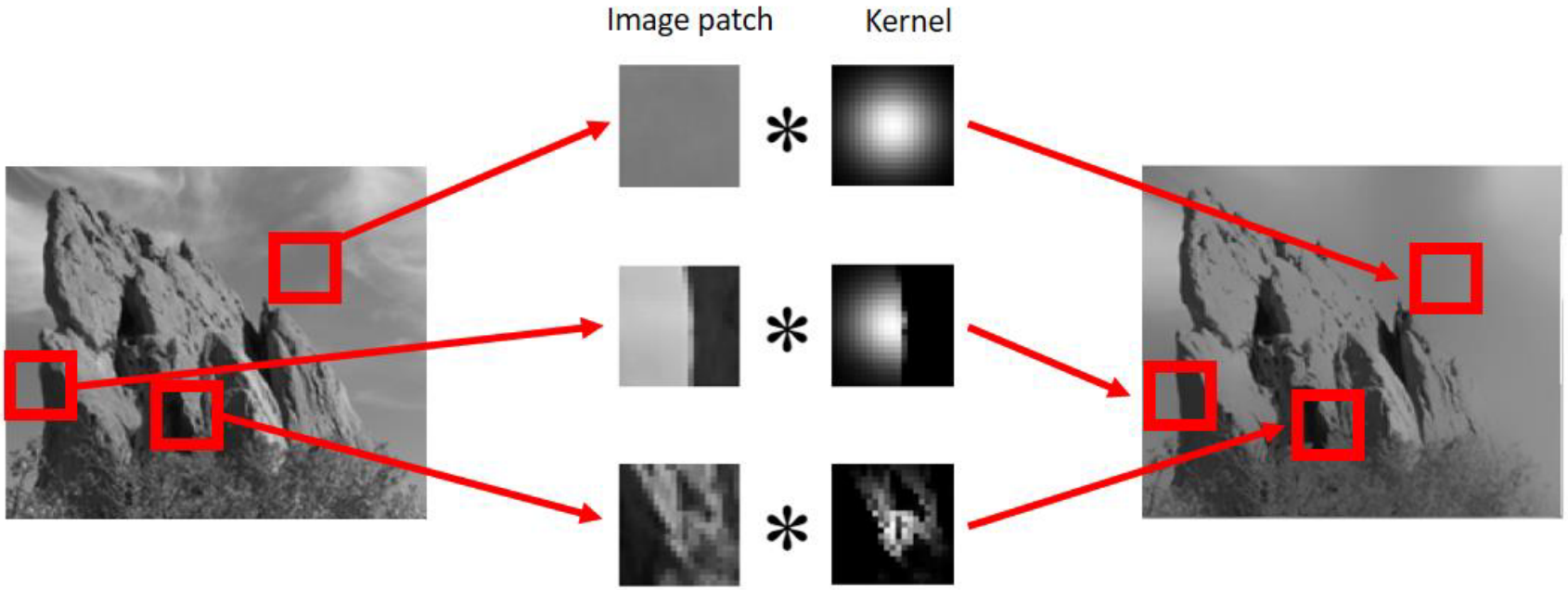
Bilateral filter

- **Gaussian filters do not preserve strong egdes (discontinuites)**. This is because they apply the same kernel everywhere.
- **Median filters** do preserve strong edges but remove small (weak) edges.



Bilateral filter

- **Gaussian filters do not preserve strong egdes (discontinuites)**. This is because they apply the same kernel everywhere.
- **Median filters** do preserve strong edges but remove small (weak) edges.
- **Bilateral filters** solve this by adapting the kernel locally to the intensity profile, so they are patch-content dependent: they only average pixels with similar brightness. Basically, the weighted average discards the influence of pixels with different brightness (across the discontinuity), while all the pixels that are on the same size of the discontinuity are smoothed.

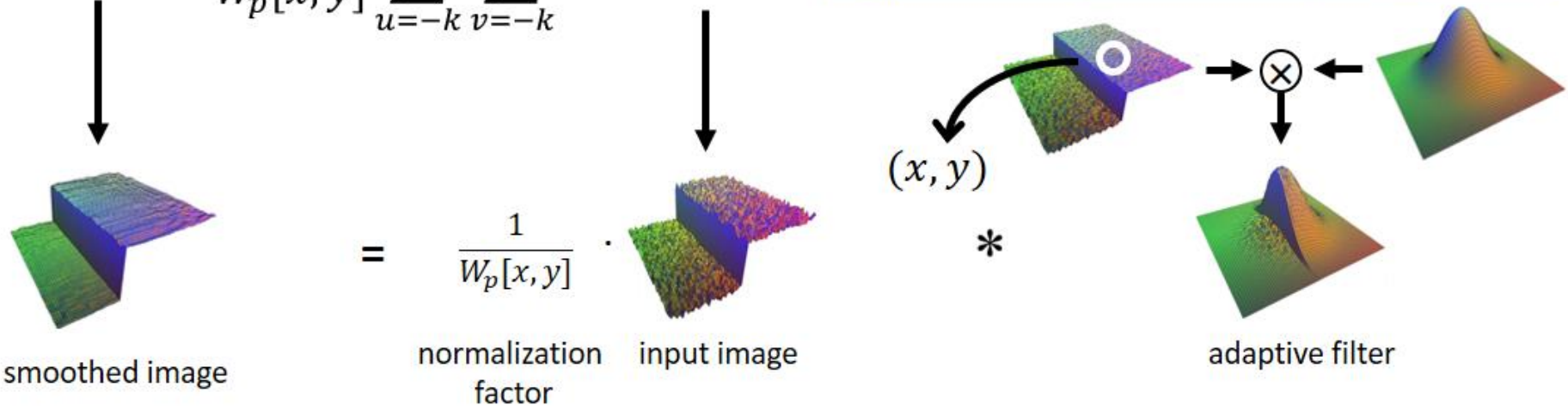


Bilateral filter

$$I'[x,y] = \frac{1}{W_p[x,y]} \sum_{u=-k}^{k} \sum_{v=-k}^{k} I[x-u, y-v] G_{\sigma_r}(I[x-u, y-v] - I[x,y]) G_{\sigma_s}[u,v]$$

1-D Gaussian range component

spatial component



smoothed image

normalization factor

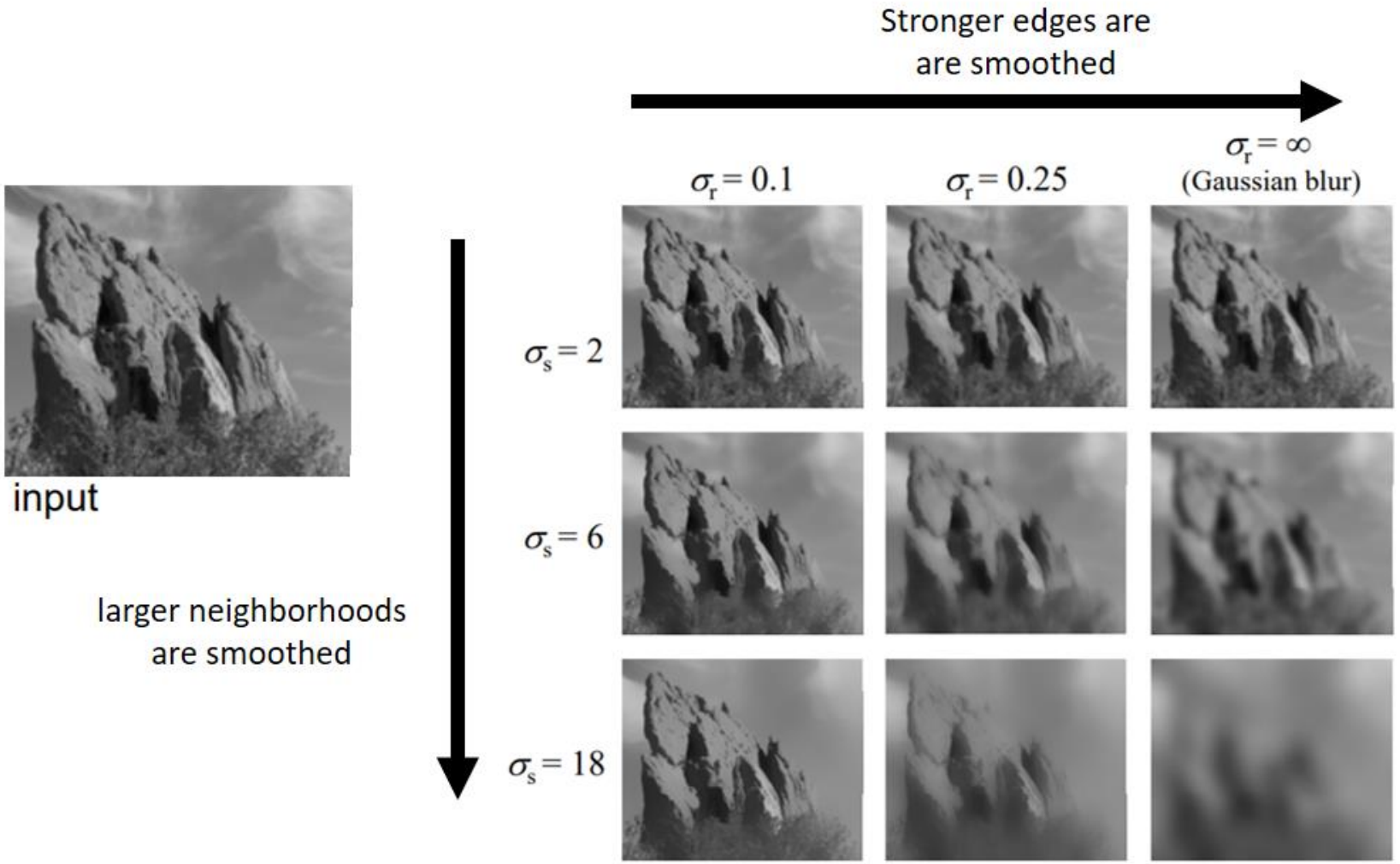
input image

adaptive filter

$$W_p[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} G_{\sigma_r}(I[x-u, y-v] - I[x,y]) G_{\sigma_s}[u,v]$$

Normalization factor
(so that the filter values sum up to 1)

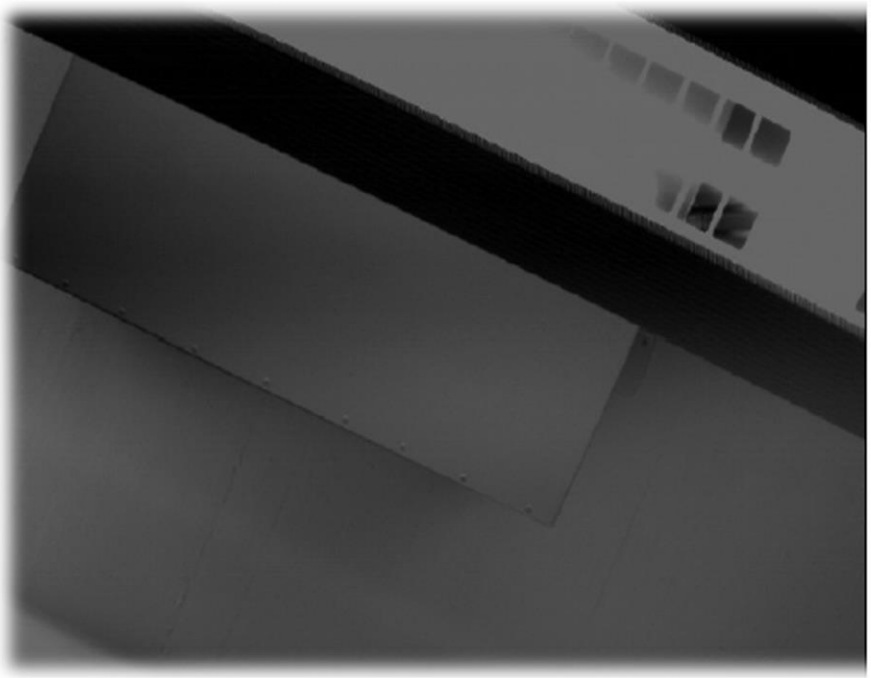
Bilateral filter

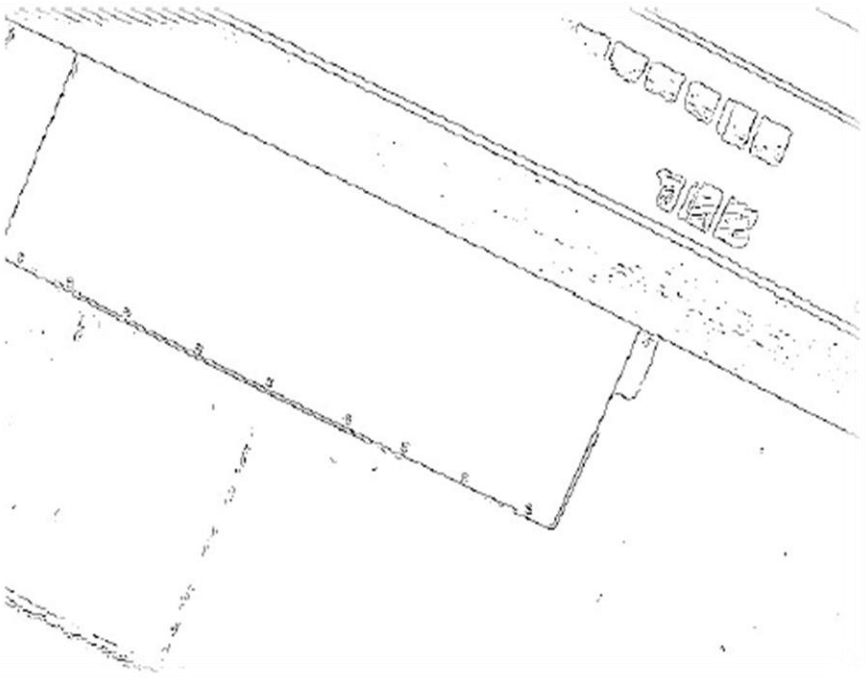


High-pass filtering
(edge detection)

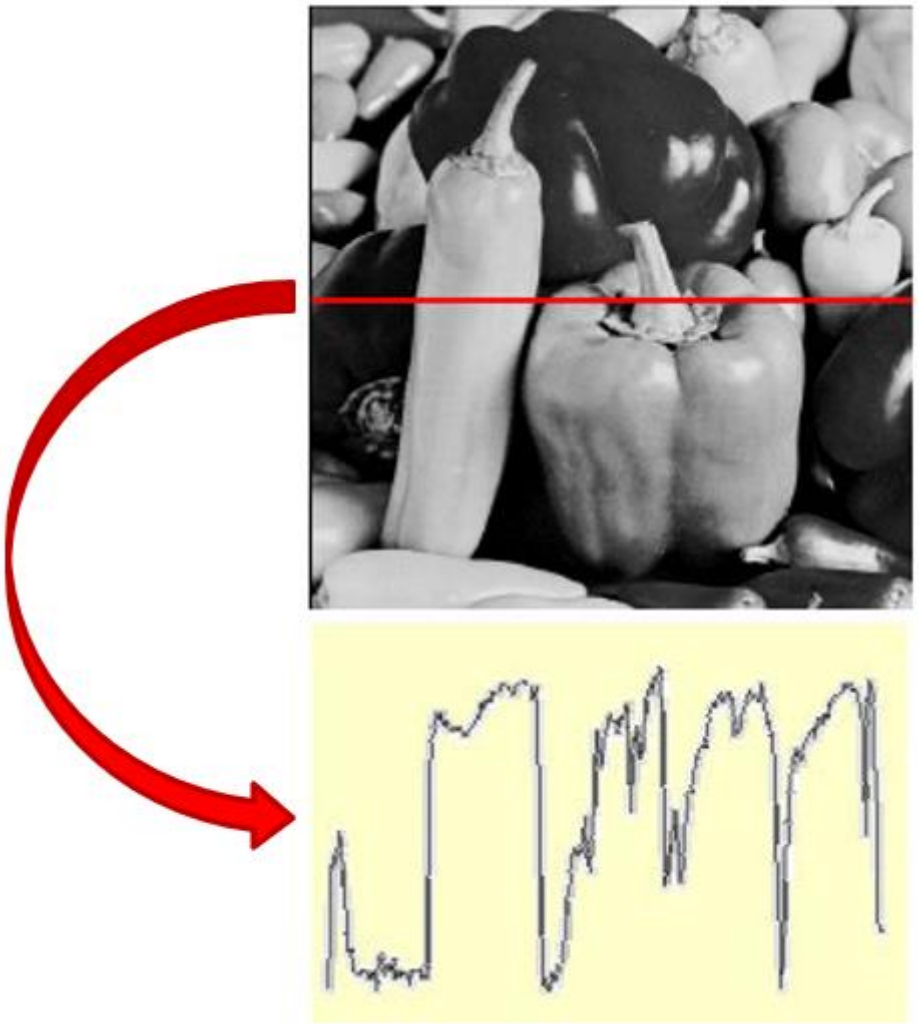
Edge detection

- Ultimate goal of edge detection: an idealized line drawing.
- Edge contours in the image correspond to important scene contours.

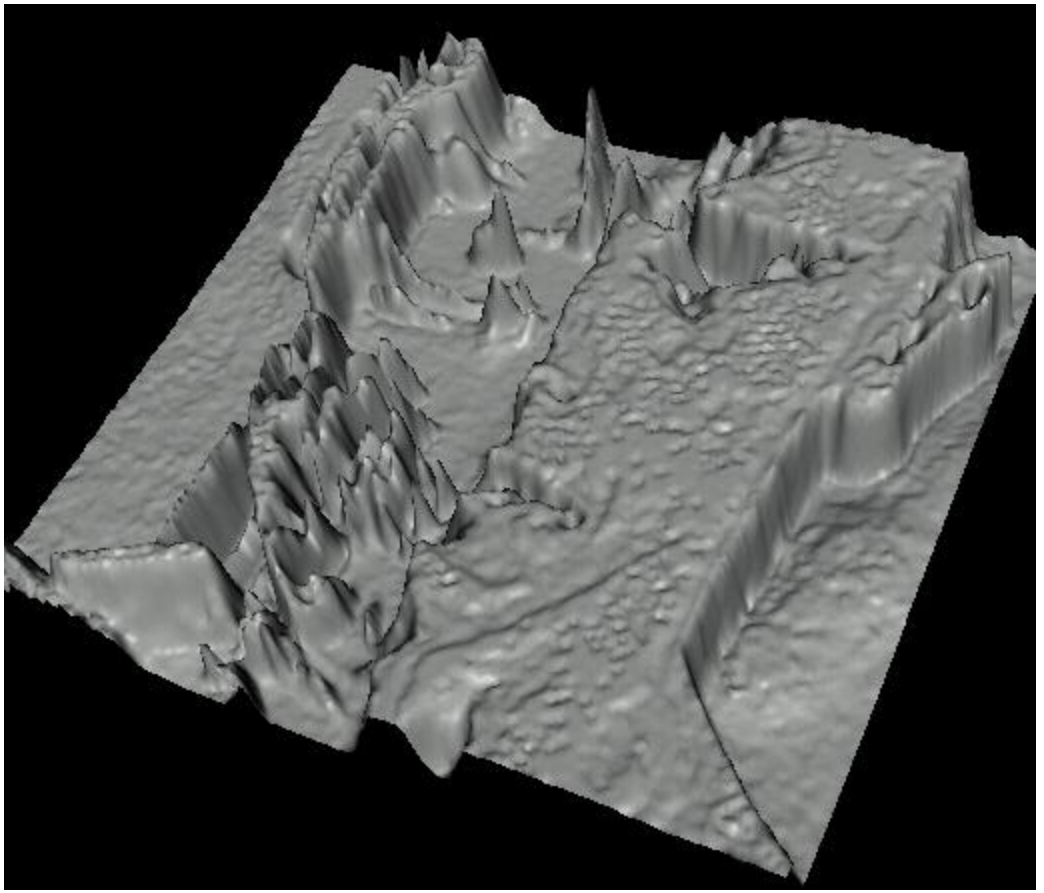




Edges are sharp intensity changes



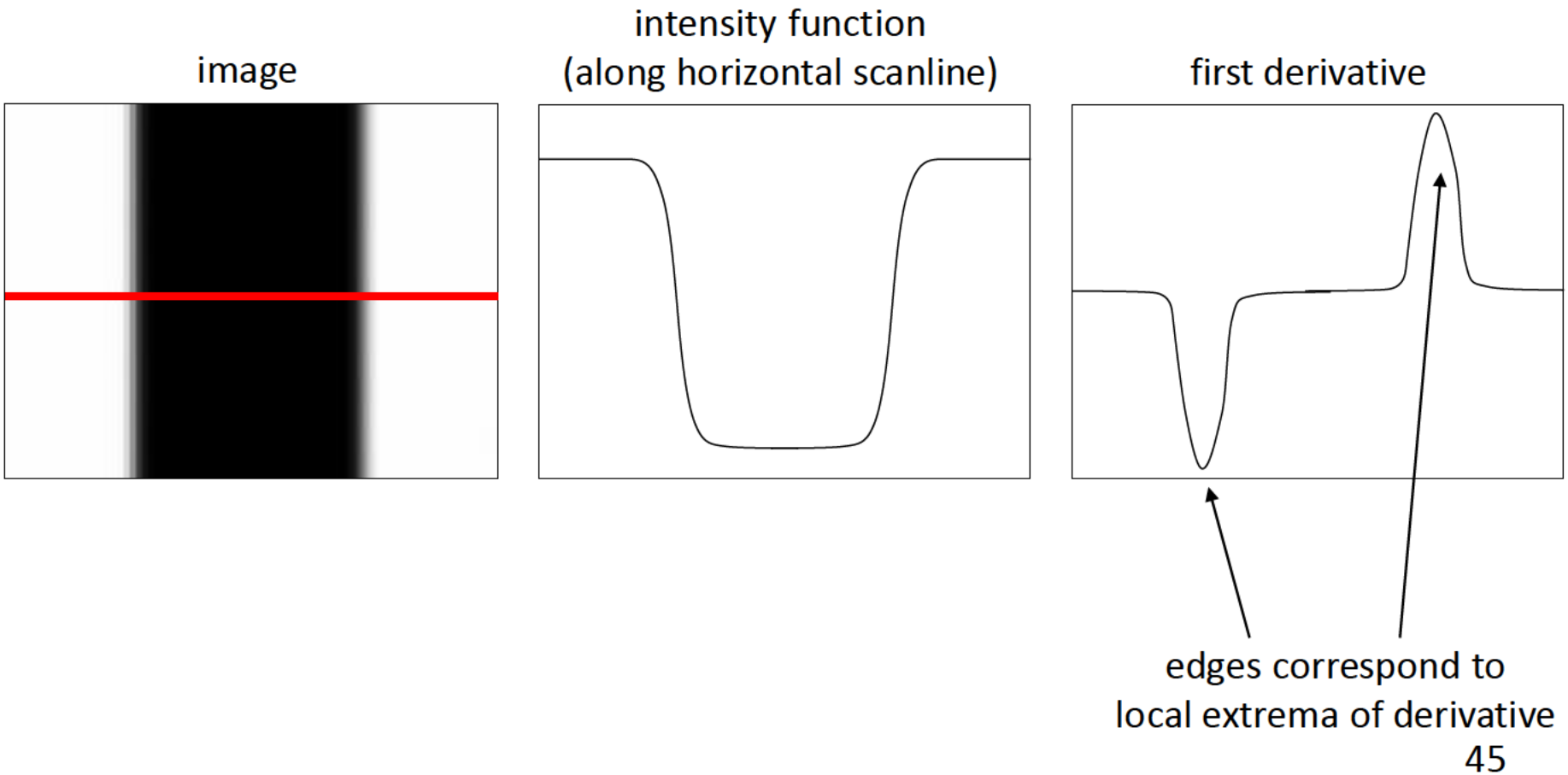
Images as functions $\mathrm{F}(x, y)$



Edges look like steep cliffs

Derivatives and edges

An edge is a place of rapid change in the image intensity function.



Differentiation and convolution

For a 2D function $I(x, y)$ the partial derivative along x is:

$$\frac{\partial I(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{I(x+\varepsilon, y) - I(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

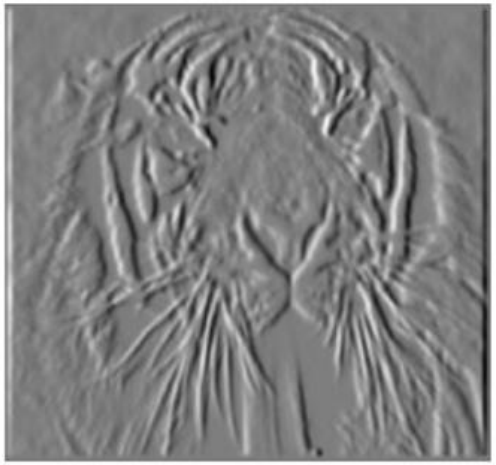
$$\frac{\partial I(x,y)}{\partial x} \approx \frac{I(x+1, y) - I(x,y)}{1}$$

What would be the respective filters along x and y to implement the partial derivatives as a convolution?

Partial derivatives of an image

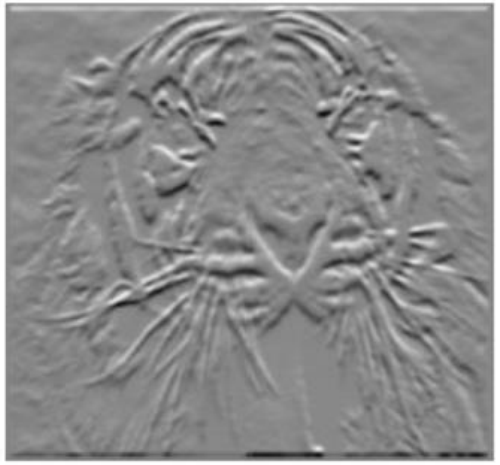
$$\frac{\partial I(x,y)}{\partial x}$$

-1	1



$$\frac{\partial I(x,y)}{\partial y}$$

-1
1



Alternative Finite-difference filters

Prewitt filter $G_x = \begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix}$ and $G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix}$

Sobel filter $G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$ and $G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$

Sample Matlab code

```
>> im = imread('lion.jpg');
>> h = fspecial('sobel');
>> outim = imfilter(double(im), h);
>> imagesc(outim);
>> colormap gray;
```

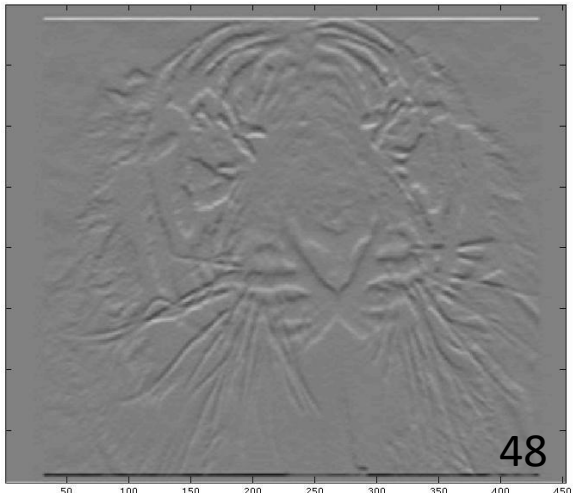
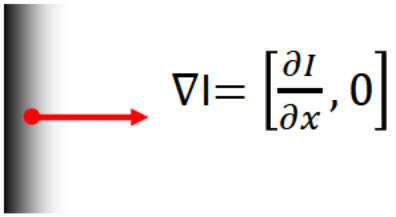
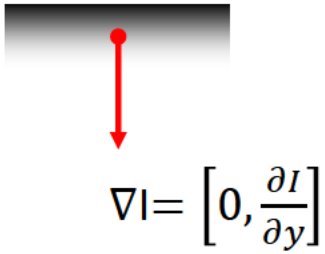


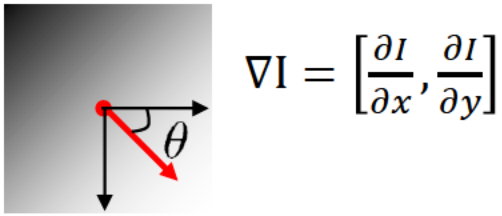
Image gradient

The gradient of an image: $$\nabla \mathrm{I} = \left[\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right]$$

The gradient points in the direction of fastest intensity change







The gradient direction (orientation of edge normal) is given by:

$$\theta = atan2\left(\frac{\partial I}{\partial y}, \frac{\partial I}{\partial x}\right)$$

The *edge strength* is given by the gradient magnitude

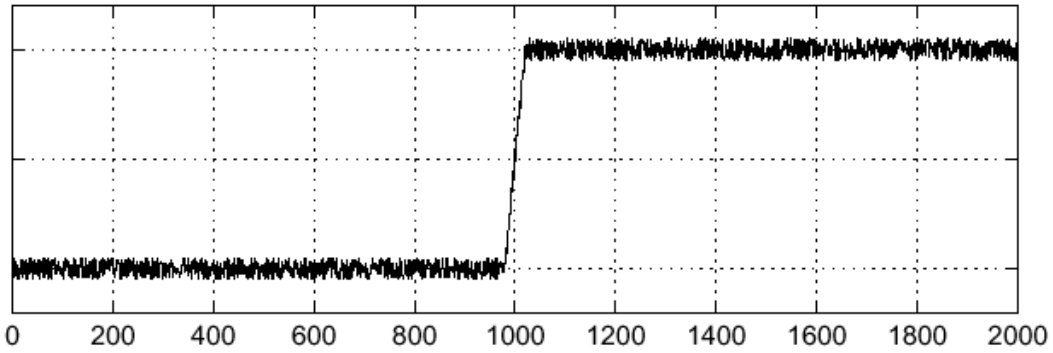
$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$

Effects of noise

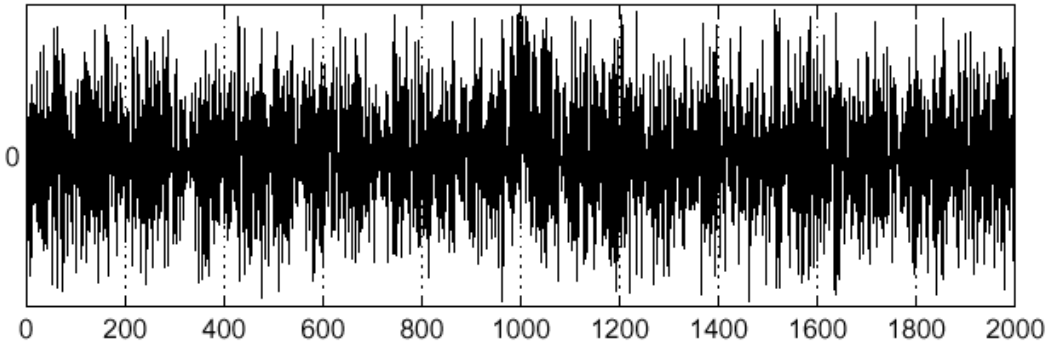
Consider a single row or column of the image

– Plotting intensity as a function of position gives a signal

$I(x)$



$\frac{\partial I(x)}{\partial x}$



Where is the edge?

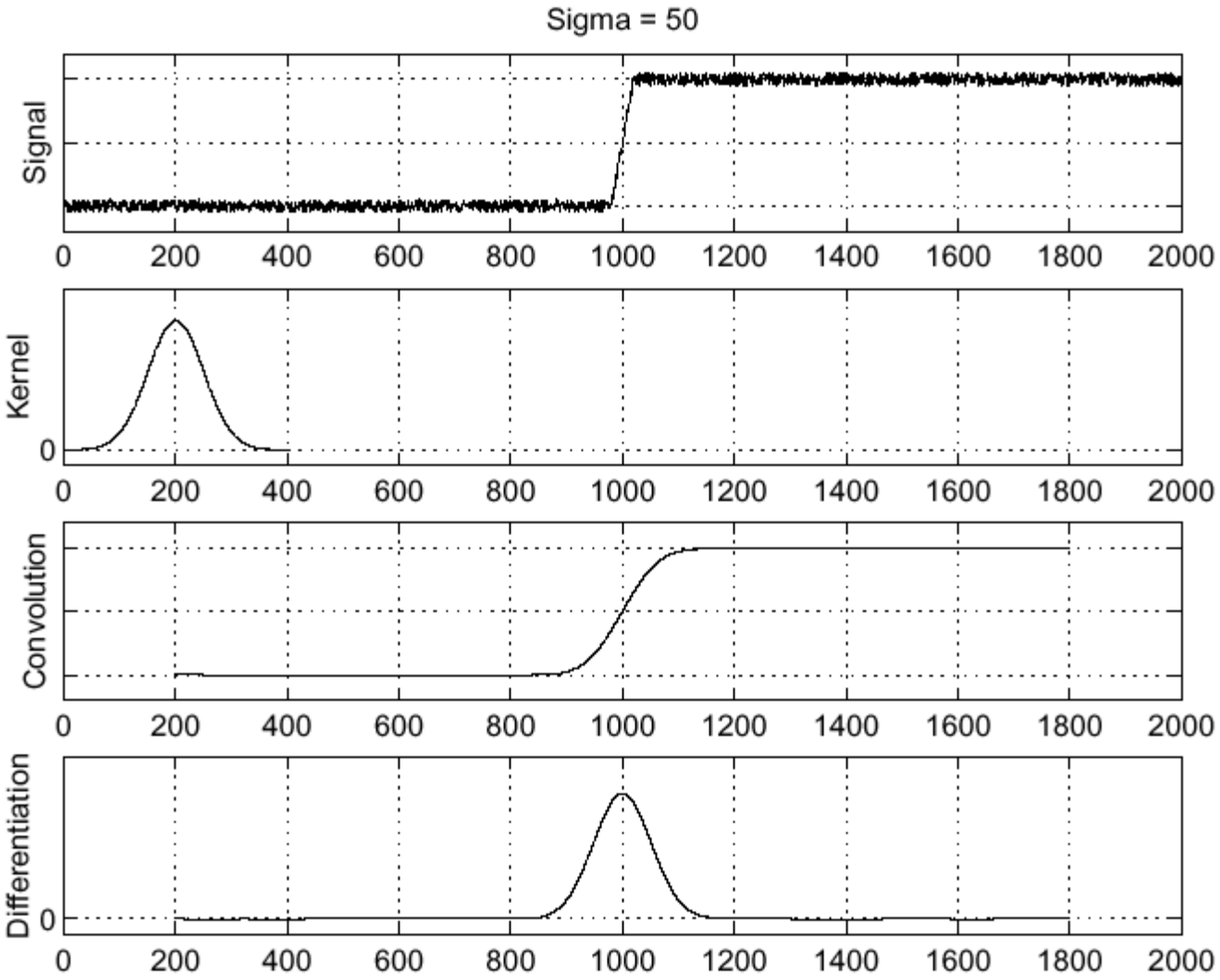
Solution: smooth first

I

H

$I * H$

$\frac{\partial}{\partial x}(I * H)$



Where is the edge?

Look for peaks in $\frac{\partial}{\partial x}(I * H)$

Alternative: combine derivative and smoothing filter

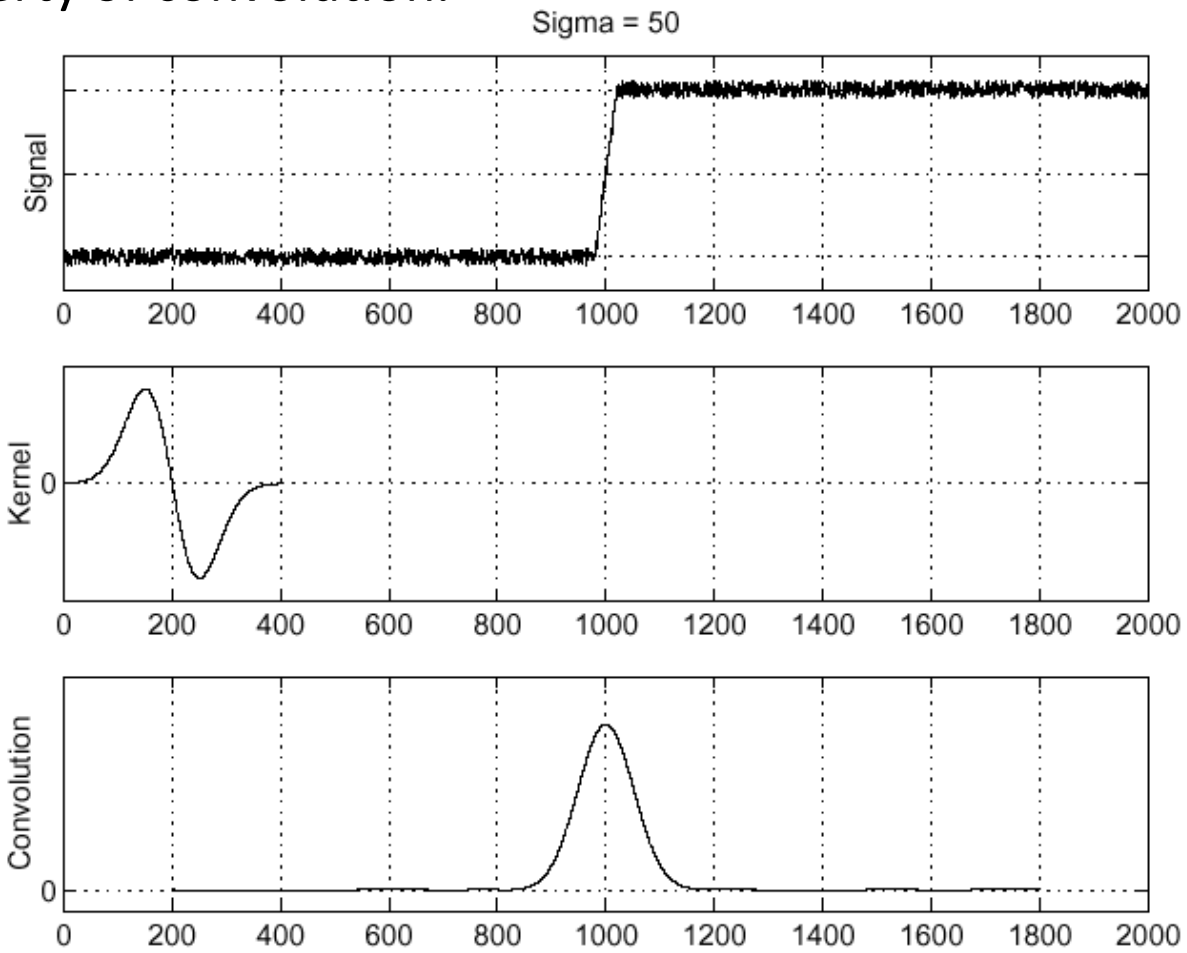
$$\frac{\partial}{\partial x}(I * H) = I * \frac{\partial H}{\partial x}$$

Differentiation property of convolution.

I

$\frac{\partial H}{\partial x}$

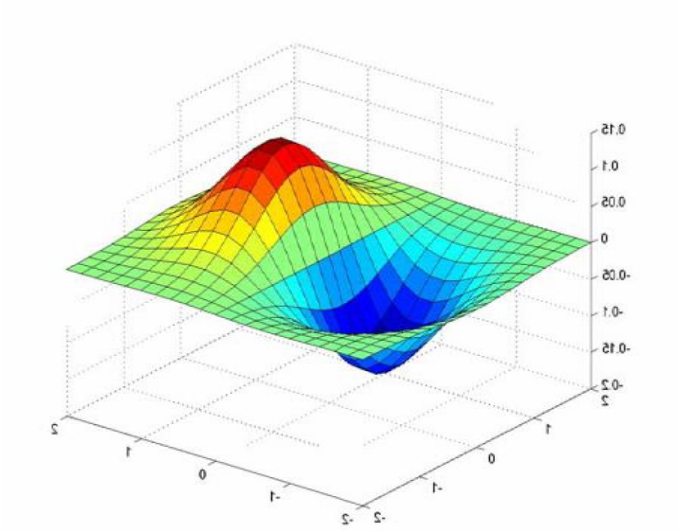
$I * \frac{\partial H}{\partial x}$



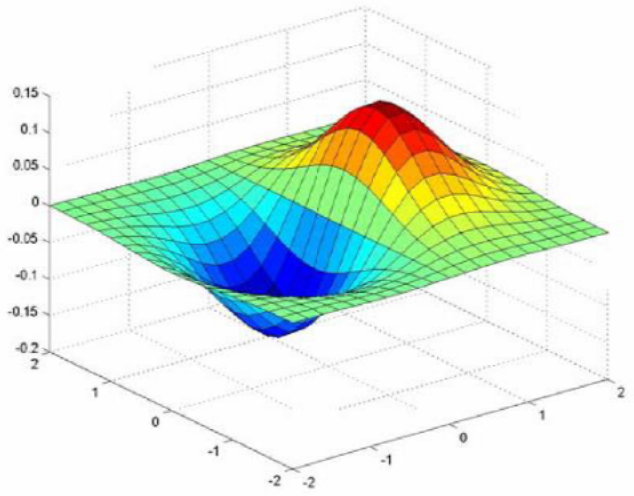
Derivative of Gaussian filter (along x)

$$(I * G) * H = I * (G * H)$$

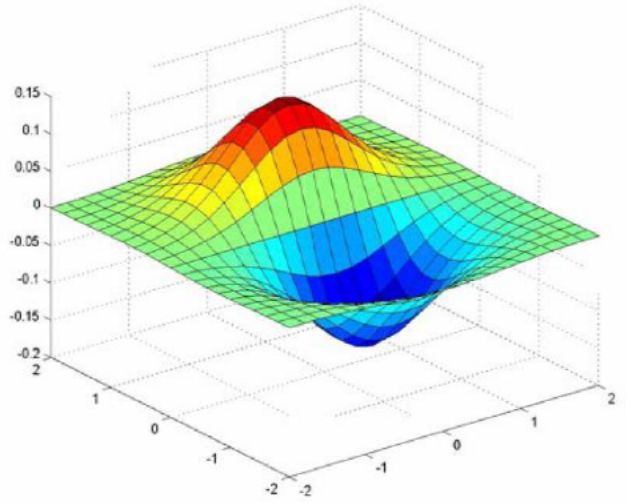
$$\begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$$



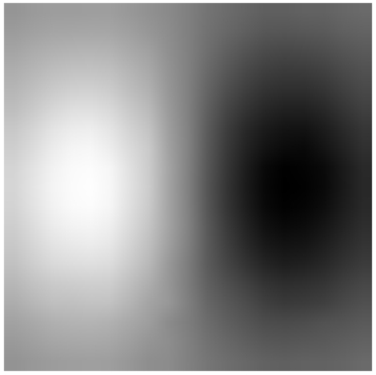
Derivative of Gaussian filters

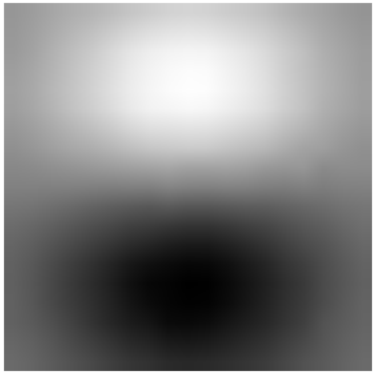


x-direction



y-direction





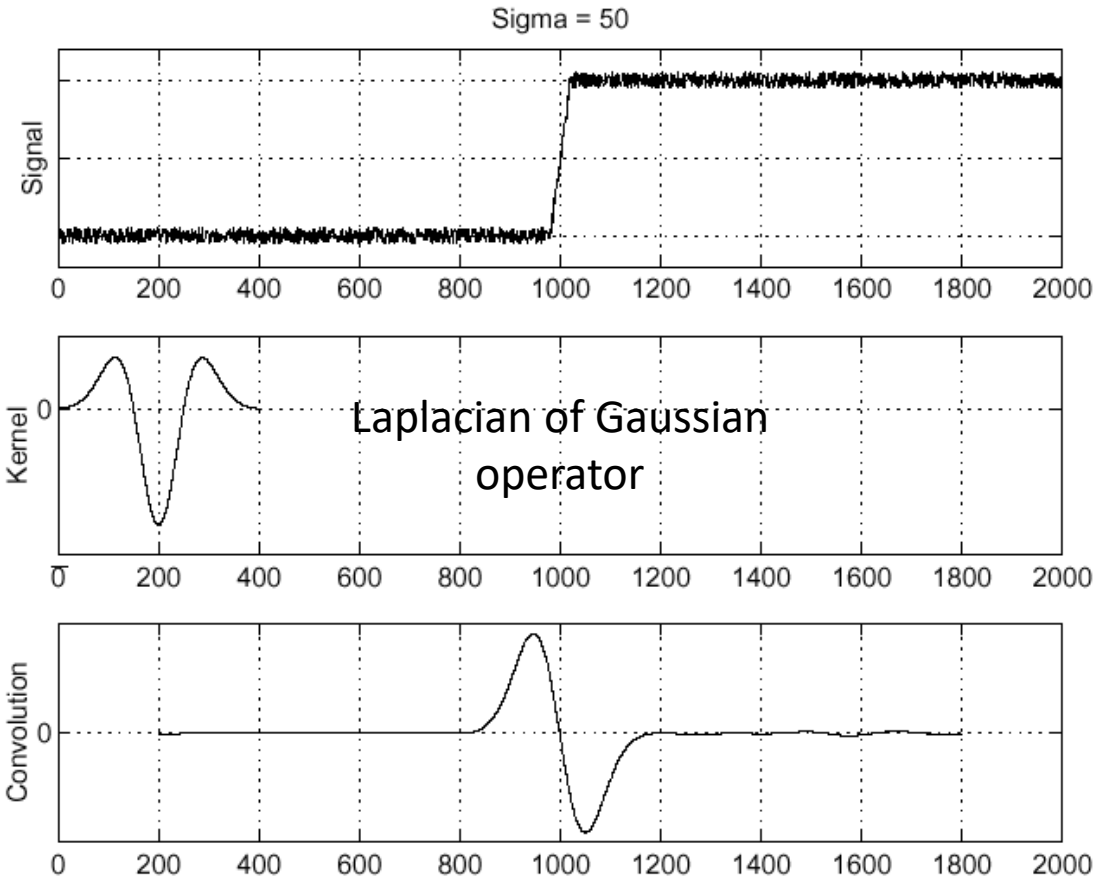
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(I * H) = I * \frac{\partial^2 H}{\partial x^2}$

$I(x)$

$\frac{\partial^2 H}{\partial x^2}$

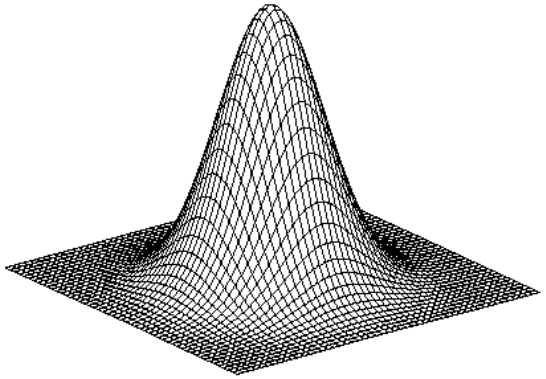
$I * \frac{\partial^2 H}{\partial x^2}$



Where is the edge?

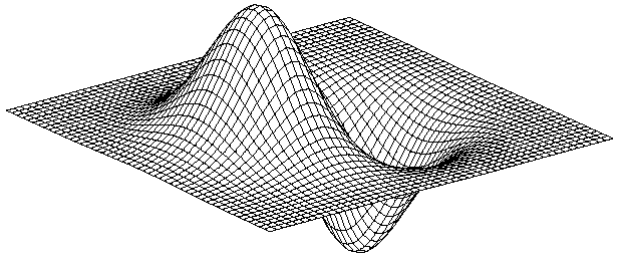
Zero-crossings of bottom graph

2D edge detection filters



Gaussian

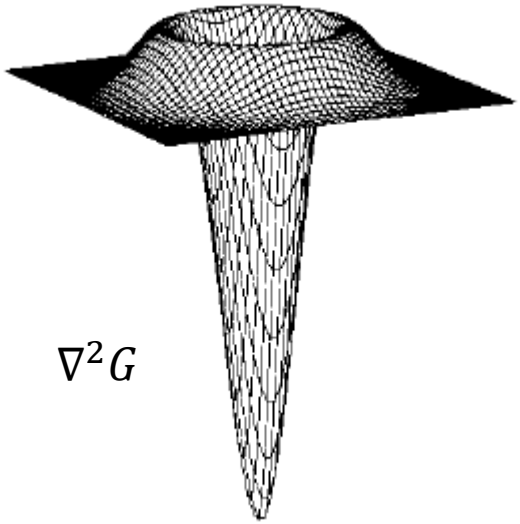
$$G = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial G}{\partial x}$$

Laplacian of Gaussian



$$\nabla^2 G$$

- ∇^2 is the Laplacian operator: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Summary on (linear) filters

- Smoothing filter:
  - **has positive values**
  - **sums to 1** → preserve brightness of constant regions
  - **removes "high-frequency"** components: "low-pass" filter
- Derivative filter:
  - **has opposite signs** used to get high response in regions of high contrast
  - **sums to 0** → no response in constant regions
  - **highlights "high-frequency"** components: "high-pass" filter

The Canny edge-detection algorithm (1986)

Take a grayscale image. If not grayscale (i.e., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

Original image (Lenna image: https://en.wikipedia.org/wiki/Lenna)

The Canny edge-detection algorithm (1986)

Take a grayscale image. If not grayscale (i.e., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

Original image (Lenna image: https://en.wikipedia.org/wiki/Lenna)

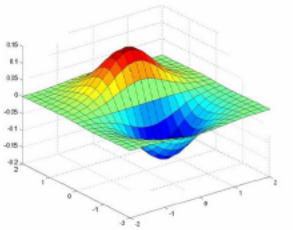
The Canny edge-detection algorithm (1986)

Convolve the image I with x and y derivatives of Gaussian filter

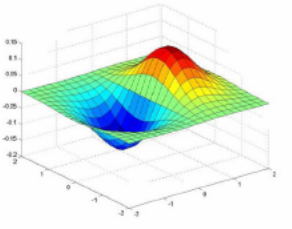
$$\frac{\partial I}{\partial x} = I * \frac{\partial G}{\partial x}$$

$$\frac{\partial I}{\partial y} = I * \frac{\partial G}{\partial y}$$

$\frac{\partial G}{\partial x}$



$\frac{\partial G}{\partial y}$



$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2}$$: Edge strength

The Canny edge-detection algorithm (1986)

Thresholded $\|\nabla I\|$

Threshold it (i.e., set to 0 all pixels whose value is below a given threshold)

The Canny edge-detection algorithm (1986)

Take local maximum
along gradient direction

Thinning: non-maxima suppression (local-maxima detection)
along edge direction

Summary

- Image filtering (definition, motivation, applications)
- Moving average
- Linear filters and formulation: box filter, Gaussian filter
- Boundary issues
- Non-linear filters
  - Median & bilateral filters
- Edge detection
  - Derivating filters (Prewitt, Sobel)
  - Combined derivative and smoothing filters (deriv. of Gaussian)
  - Laplacian of Gaussian
  - Canny edge detector
- Readings: Ch. 3.2, 4.2.1 of Szeliski book