

# Statistics Assessment

ENSAE ParisTech

December 2015

Let  $(X_1, \dots, X_n)$   $\mathcal{Be}(\alpha, \beta)$  iid,  $(\alpha, \beta) \in \mathbb{R}_+^{*2}$  unknown. The goal is to numerically estimate  $\alpha$  and  $\beta$ .

1. Suggest an estimator, using the method of moments.
2. Explain why the maximum likelihood estimator can only be obtained numerically. How one should utilize the Newton-Raphson algorithm to get this estimator? Implement it in R on simulated data.
3. Graphically, compare the two estimators, for different values of  $n$ .
4. Explain how to adapt the previous algorithm to perform a Wald test on the hypothesis  $H_0 : \alpha = \beta$ . Implement it for a given asymptotic level (for instance 5%) on simulated data (under  $H_0$ , and for an arbitrary value of  $\alpha = \beta$ ). For different values of  $n$ , compute the non-asymptotic threshold of the test.