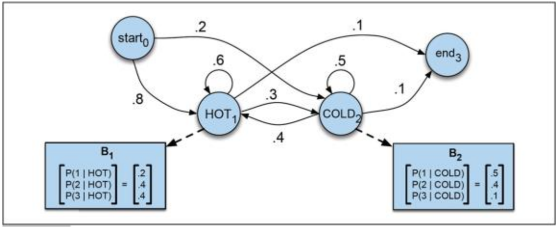
Exercise: The Forward Algorithm & Viterbi Algorithm

Today's exercise is to:

1. Implement the Forward Algorithm for the Hidden Markov Model (shown on the next slide) to compute the probability of the observation sequence *313*.
2. Implement the Viterbi Algorithm to compute the most likely weather sequence for the observation sequence *313*.

Use the file hmm_template.py posted on blackboard, it contains the incomplete functions compute_forward and compute_viterbi.

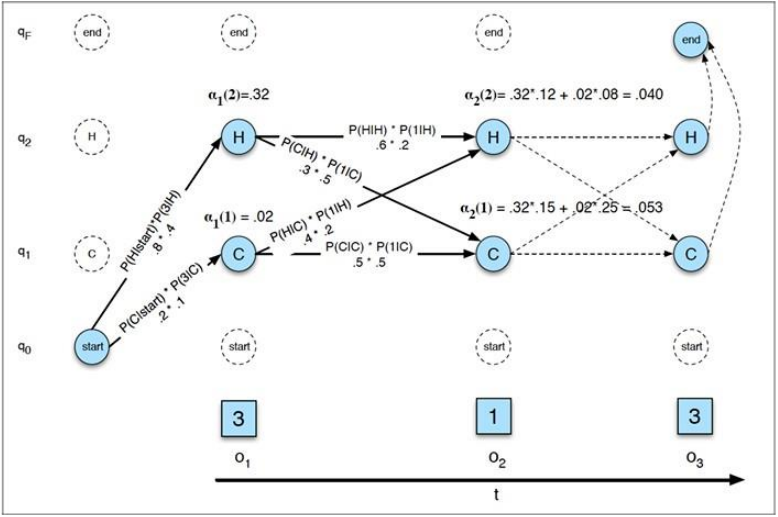
The Hidden Markov Model Used in the Exercise



The above shows a Hidden Markov Model for relating numbers of ice creams eaten by Jason (the observations) to the weather (H or C, the hidden variables).

You can also check https://web.stanford.edu/jurafsky/slp3/9.pdf for further details.

Visual Representation of the Forward Algorithm.



Pseudocode for the Forward Algorithm.

function FORWARD(*observations* of len T, *state-graph* of len N) **returns** *forward-prob*

create a probability matrix *forward[N+2,T]*
for each state s **from** 1 **to** N **do** ; initialization step
$\quad forward[s,1] \leftarrow a_{0,s} * b_s(o_1)$
for each time step t **from** 2 **to** T **do** ; recursion step
$\quad$ **for** each state s **from** 1 **to** N **do**

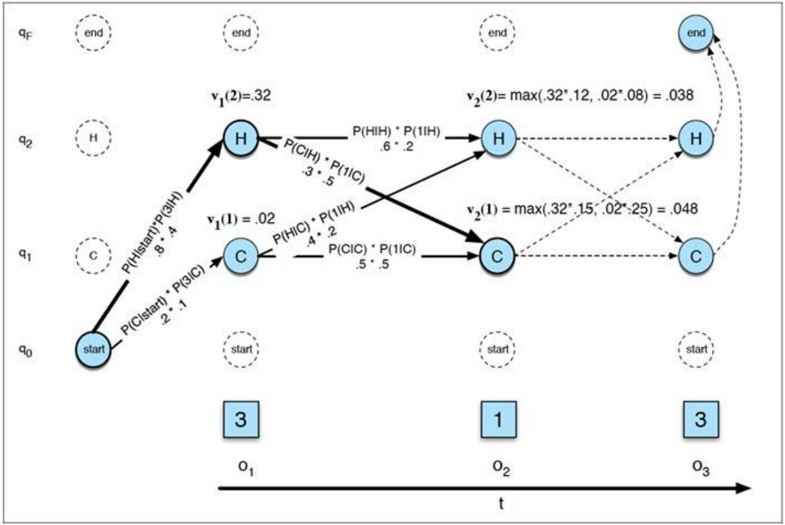
$$forward[s,t] \leftarrow \sum_{s'=1}^{N} forward[s',t-1] * a_{s',s} * b_s(o_t)$$

$$forward[q_F,T] \leftarrow \sum_{s=1}^{N} forward[s,T] * a_{s,q_F} \qquad \text{; termination step}$$

return $forward[q_F,T]$

Note that in the code, the transition matrix corresponds to a, whereas the emissions matrix corresponds to b.

Visual Representation of the Viterbi Algorithm.



Pseudocode for the Viterbi Algorithm.

function VITERBI(*observations* of len T, *state-graph* of len N) **returns** *best-path*

create a path probability matrix $viterbi[N+2,T]$

for each state s **from** 1 **to** N **do** ; initialization step

$viterbi[s,1] \leftarrow a_{0,s} * b_s(o_1)$

$backpointer[s,1] \leftarrow 0$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$$viterbi[s,t] \leftarrow \max_{s'=1}^{N} viterbi[s',t-1] * a_{s',s} * b_s(o_t)$$

$$backpointer[s,t] \leftarrow \operatorname{argmax}_{s'=1}^{N} viterbi[s',t-1] * a_{s',s}$$

$$viterbi[q_F,T] \leftarrow \max_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

$$backpointer[q_F,T] \leftarrow \operatorname{argmax}_{s=1}^{N} viterbi[s,T] * a_{s,q_F}$$; termination step

return the backtrace path by following backpointers to states back in time from $backpointer[q_F,T]$

Note that in the code, the transition matrix corresponds to a, whereas the emissions matrix corresponds to b.

Homework

Must be submitted

Find the probability of the following observation sequences:

- *3, 3, 1, 1, 2, 2, 3, 1, 3*.
- *3, 3, 1, 1, 2, 3, 3, 1, 2*.

Also find the most likely weather sequences for the two observation sequences.