```
% function Frequency_Peters
% Floquet Frequency Plot for Mathieu's Equation (Figure 2 Appearance)
% Separates branches according to the integer multiple 'm' used.
%
% Based on:
% David A. Peters, Sydnie M. Lieb, Loren A. Ahaus
% "Interpretation of Floquet Eigenvalues and Eigenvectors for Periodic
Systems"
% JOURNAL OF THE AMERICAN HELICOPTER SOCIETY 56, 032001 (2011)
clear; clc; close all;

% --- Setup for Figure Saving ---
fDir = 'figureFolder'; % Folder for figures
if ~isdir(fDir) %#ok<ISDIR>
    mkdir(fDir)
end
fDirPeters = fullfile(fDir,'figureFolderPeters'); % Subfolder specific to
Peters' plots
if ~isdir(fDirPeters) %#ok<ISDIR>
    mkdir(fDirPeters)
end
% Plotting style selection
K = 'ColoredLines';
% K = 'BlackLines';
useK = strcmp(K,'BlackLines');

% -----------------------------------------------------------------------
% --- Parameters and Initialization ---
% -----------------------------------------------------------------------
Omega = 1;       % Fundamental frequency (Omega = 1 rad/s)
T = 2*pi/Omega; % Period
% Outer loop for different unperturbed frequencies (w)
w_values = [0.3, 0.5, 0.7];
for w = w_values
    w_sq = w^2;
    % --- Basis Frequency omega0 Calculation (Peters' Convention) ---
    % The basis frequency, omega0, must satisfy 0 <= omega0 <= Omega/2.
    basis_freq = mod(w, Omega);
    if basis_freq > Omega/2
        basis_freq = Omega - basis_freq;
    end
    omega0 = basis_freq;

    % Filename generation for saving the plot
    pngname = strrep(sprintf('PetersFrequency%s_w%1.1f',K,w),'.','dot');
    pngfile = fullfile(fDirPeters,[pngname,'.png']);

    % Determine the range of epsilon (x-axis limit)
    if abs(w - 0.3) < 0.001
        eps_end = 5;
        eps_no = 1000; % High resolution for w=0.3
    elseif abs(w - 0.7) < 0.001
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        eps_end = 3.5; % Axis limit for w=0.7 plot
        eps_no = 150;
    else % Case for w=0.5 and others
        eps_end = 3.5;
        eps_no = 150;
    end
    eps_vals = linspace(0, eps_end, eps_no);
    m_range = (-4:4); % Integer multiple range for plotting branches
(m*Omega)

    % Structure to hold results, organized by branch 'm' (e.g., m_0, m_neg_1)
    results_by_branch = struct();
    for m = m_range
        % Create a valid field name (MATLAB field names cannot start with a
sign)
        if m < 0
            field_name = ['m_neg_', num2str(abs(m))];
        else
            field_name = ['m_', num2str(m)];
        end
        results_by_branch.(field_name) = []; % Initialize with empty array
    end

    % ------------------------------------------------------------------------
    % --- Floquet Exponent Calculation and Branch Separation ---
    % ------------------------------------------------------------------------
    x0 = eye(2); % Initial state matrix for Phi(0) = I
    for k = 1:length(eps_vals)

        epsilon = eps_vals(k);
        % The state matrix D(t) for the ODE (Eq. 1):
        % dot(x) + (w^2 + epsilon*sin(Omega*t)) * x = 0
        % State-space form (Eq. 2): d{x}/dt = [D(t)]{x}
        D_func = @(t) [0, 1; -(w_sq + epsilon*sin(t)), 0];
        % Solve for the Transition Matrix Phi(t): {x(t)}=[Phi(t)]{x(0)} (Eq.
7)
        [~, Phi_t] = ode45(@(t, x) reshape(D_func(t) * reshape(x, 2, 2), 4,
1), [0, T], reshape(x0, 4, 1));
        Phi_T = reshape(Phi_t(end, :), 2, 2); % Monodromy Matrix at Phi(T)
```

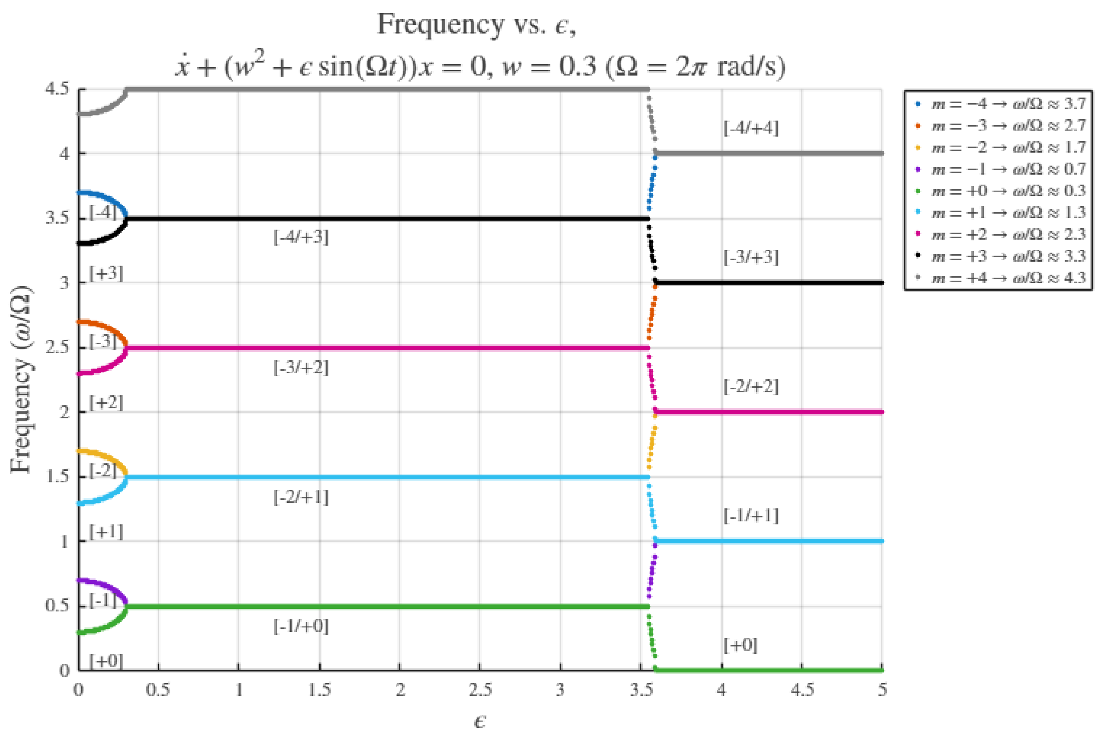

Calculate Floquet Exponents: eta = log(Lambda) / T, see Eq. 8

[Phi(t)]=[A(t)][−exp(eta_j t)−][A(0)]^−1 [A(0)]^−1[Phi(T)][A(0)]=[−Lambda_j−]=[−exp(eta_j T)−]

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        Lambda = eig(Phi_T);
        % Characteristic Exponent (Eq. 9)
        eta = log(Lambda) / T;
        % The **real part** of eta (Re(eta)) determines stability.
        % If Re(eta) > 0, the system is unstable (exponential growth).
        % The **imaginary part** of eta (Im(eta)) determines the frequency
(mu).

        % Extract the Imaginary Part (normalized frequency, mu*Omega =
Im(eta))
        % omega/Omega = Im(eta)/Omega (where Omega=1, so omega = Im(eta))
        normalized_omega = imag(eta) / Omega;

        % --- Separate and Store Branches (m) ---
        % 1. Find the basis frequency (principal value)
        % This step effectively extracts the fractional part of the
frequency,
        % which corresponds to the base frequency component (omega0/Omega)
(Eq. 10).
        basis_freq_r =normalized_omega(1);
        % 2. Map this base frequency to all possible branches 'm'
        % The physical frequency omega is defined by omega/Omega = m + mu.
        % Here, 'm' represents the **integer multiple** of the excitation
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frequency Omega.

        % - **Positive m (m>0):** Frequencies omega approx m*Omega +
omega_0. These branches generally increase with m.
        % - **Negative m (m<0):** Frequencies omega approx |m|*Omega -
omega_0. These branches approach |m|*Omega from below (or above, depending
on convention).

        for m = m_range

            if m==0
                branch_freq = basis_freq_r;
            else
                % This formula reconstructs the full frequency based on the
integer m and basis frequency.
                % For m > 0, it gives branch_freq = m*Omega + basis_freq_r.
                % For m < 0, it gives branch_freq = |m|*Omega - basis_freq_r.
                branch_freq = abs(m) *Omega + sign(m) *basis_freq_r;
            end
            % Determine the valid field name
            if m < 0
                field_name = ['m_neg_', num2str(abs(m))];
            else
                field_name = ['m_', num2str(m)];
            end
            % Store [epsilon, frequency] for this specific branch
            results_by_branch.(field_name) = [results_by_branch.
(field_name); epsilon, branch_freq];
        end

    end

    % -----------------------------------------------------------------------
    % --- Plotting ---
    % -----------------------------------------------------------------------
    figure;
    hold on;
    color_map = lines;
    color_map = [color_map(1:7,:);0*ones(1,3);0.5*ones(1,3)];
    % Title Update: Includes ODE, w, and Omega definition
    ode_str = '$\dot{x} + (w^2 + \epsilon\sin(\Omega t)) x = 0$';
    w_str = num2str(w, '%1.1f');
    new_title = {'Frequency vs. $\epsilon$, ', [ode_str, ', $w = ', w_str,
'$ ($\Omega = 2\pi$ rad/s)']};
    title(new_title, 'FontSize', 16, 'Interpreter', 'latex');
    xlabel('$\epsilon$', 'FontSize', 14, 'Interpreter', 'latex');
    ylabel('Frequency ($\omega/\Omega$)', 'FontSize', 14, 'Interpreter',
'latex');
    idx = 1;
    for m = m_range
        % Determine field name
        if m < 0
            field_name = ['m_neg_', num2str(abs(m))];
        else
```

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            field_name = ['m_', num2str(m)];
        end
        % --- Legend Calculation: Peters' Frequency Convention (FIXED
\omega) ---
        if m >= 0
            % omega/Omega approx omega0/Omega + m
            freq_normalized = omega0/Omega + m;
            % Use \\omega to represent \omega to the LaTeX interpreter
            freq_str = sprintf('$m=%+d \\rightarrow \\omega/\\Omega \\approx
%.1f$', m, freq_normalized);
        elseif m < 0
            % omega/Omega approx |m| - omega0/Omega
            m_abs = abs(m);
            freq_normalized = m_abs - omega0/Omega;
            %  Use \\omega to represent \omega to the LaTeX interpreter
            freq_str = sprintf('$m=%+d \\rightarrow \\omega/\\Omega \\approx
%.1f$', m, freq_normalized);
        end
        % Get data for plotting
        data = results_by_branch.(field_name);
        % The initial color based on sequential index 'idx'
        current_color = color_map(idx, :);
        % Plotting using dots/scatters to form the continuous curves
        if useK == 1
            plot(data(:, 1), data(:, 2), '.', 'Color','k', 'MarkerSize', 8,
'DisplayName', freq_str);
        else
            % Pass the calculated 'current_color' to the plot function.
            % This ensures both the line and the legend swatch use the
swapped color.
             plot(data(:, 1), data(:, 2), '.', 'Color', current_color,
'MarkerSize', 8, 'DisplayName', freq_str);
        end

        idx = idx + 1;
    end
    % Set Axis limits
    grid on;
    set(gca, 'TickLabelInterpreter', 'latex');
    % Add Legend if colors are used
    if ~useK
        legend('Location', 'northeastoutside', 'Interpreter', 'latex');
    end

    % ----------------------------------------------------------------------
    % --- Branch Label Annotations ---
    % ----------------------------------------------------------------------
    % These labels mark the primary frequency branches at different regions
    % of the stability chart.
    hold on;
    % Left-side labels (ε -> 0)
    text(0.05, 0.08, '[+0]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
w/Omega = 0.7 - 0.5 = 0.2 (using basis freq)
    text(0.05, 0.55, '[-1]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
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1 - 0.3 = 0.7
    text(0.05, 1.08, '[+1]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
1 + 0.3 = 1.3
    text(0.05, 1.55, '[-2]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
2 - 0.3 = 1.7
    text(0.05, 2.08, '[+2]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
2 + 0.3 = 2.3
    text(0.05, 2.55, '[-3]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
3 - 0.3 = 2.7
    text(0.05, 3.08, '[+3]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
3 + 0.3 = 3.3
    text(0.05, 3.55, '[-4]', 'FontSize', 10, 'Interpreter', 'latex'); % Near
4 - 0.3 = 3.7
    % Middle labels (Near the primary instability boundaries)
    text(1.2, 0.35, '[-1/+0]', 'FontSize', 10, 'Interpreter', 'latex');
    text(1.2, 1.35, '[-2/+1]', 'FontSize', 10, 'Interpreter', 'latex');
    text(1.2, 2.35, '[-3/+2]', 'FontSize', 10, 'Interpreter', 'latex');
    text(1.2, 3.35, '[-4/+3]', 'FontSize', 10, 'Interpreter', 'latex');
    % --- RIGHT-SIDE LABELS (Secondary Instability Boundaries) ---
    % The labels are positioned based on the value of 'w' used in the
analysis.
    if abs(w - 0.3) < 0.001
        % Labels for w = 0.3
        text(4.0, 0.20, '[+0]', 'FontSize', 10, 'Interpreter', 'latex');
        text(4.0, 1.20, '[-1/+1]', 'FontSize', 10, 'Interpreter', 'latex');
        text(4.0, 2.20, '[-2/+2]', 'FontSize', 10, 'Interpreter', 'latex');
        text(4.0, 3.20, '[-3/+3]', 'FontSize', 10, 'Interpreter', 'latex');
        text(4.0, 4.20, '[-4/+4]', 'FontSize', 10, 'Interpreter', 'latex');
    elseif abs(w - 0.7) < 0.001
        % Labels for w = 0.7
        text(3.0, 0.20, '[+0]', 'FontSize', 10, 'Interpreter', 'latex');
        text(3.0, 1.20, '[-1/+1]', 'FontSize', 10, 'Interpreter', 'latex');
        text(3.0, 2.20, '[-2/+2]', 'FontSize', 10, 'Interpreter', 'latex');
        text(3.0, 3.20, '[-3/+3]', 'FontSize', 10, 'Interpreter', 'latex');
        text(3.0, 4.20, '[-4/+4]', 'FontSize', 10, 'Interpreter', 'latex');
    end
    % Print to Png file
    print(pngfile, '-dpng')
end
```

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