$F(x) = y \qquad MA = B$

$$\underset{\text{Func}}{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}} \underset{\text{input}}{\begin{bmatrix} 5 & 3 \\ 2 & 9 \\ 1 & 9 \end{bmatrix}} = \underset{\text{output}}{\begin{bmatrix} 5+4+3 & 3+8+27 \\ 1 & 9 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 & 4 & 7 \\ 5 & 2 & 8 \\ 6 & 1 & 2 \\ 9 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3+5+6+9 & 4+2+1+3 & 7+8+2+5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 8 \\ 5 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2+0+0 & 8+0+0 \\ 0+5+0 & 0+3+0 \\ 0+0+6 & 0+0+2 \end{bmatrix}$$

$$\boxed{a \; b} \quad \boxed{\begin{matrix} a \\ b \end{matrix}} \quad \begin{bmatrix} a & c \\ b & d \end{bmatrix} \text{ Data}$$

$$\underset{\text{Func}}{\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}}$$

Food Price

Person 1, 2, 3

$$\begin{array}{l} \text{Apple} \\ \text{Orange} \\ \text{Total} \end{array} \begin{bmatrix} 2 & 0 \\ 0 & 1{,}5 \\ 2 & 1{,}5 \end{bmatrix} \begin{bmatrix} 6 & 2 & 10 \\ 3 & 5 & 1 \end{bmatrix} \begin{array}{l} \text{Apple Count} \\ \text{Orange Count} \end{array}$$

$$= \begin{bmatrix} 12 & 4 & 20 \\ 4{,}5 & 7{,}5 & 1{,}5 \\ 16{,}5 & 11{,}5 & 21{,}5 \end{bmatrix}$$

$$x + y + z = 3$$
$$x + 2y + 3z = 0$$
$$x + 3y + 4z = -2$$

$$x = 5$$
$$y = -1$$
$$z = -1$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 4 & -2 \end{array}\right] \quad \begin{array}{l} R_2 - R_1 \to R_2 \\ \\ R_3 - R_1 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & -3 \\ 0 & 2 & 3 & -5 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 6 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & +1 & -1 \end{array}\right] \quad \begin{array}{l} R_1 - R_2 \to R_1 \\ R_3 - 2R_2 \to R_3 \\ -R_3 \to R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} \underline{1} & 0 & 0 & 5 \\ 0 & \underline{1} & 0 & -1 \\ 0 & 0 & \underline{1} & -1 \end{array}\right] \quad \begin{array}{l} R_1 + R_3 \to R \\ R_2 - 2R_3 \to R_2 \end{array}$$

$$-3 - (-1 \cdot 2) = -3 + 2 = -1$$

Pivelot entries?

Reduced row echelon form

Inverses of Matrices

$A = \begin{bmatrix} 4 & -3 \\ 18 & -9 \end{bmatrix} \quad |A| = -36 + 54 = 18$

$A = \begin{bmatrix} -9 & 3 \\ -18 & 4 \end{bmatrix} \frac{1}{18} = \begin{bmatrix} \frac{-9}{18} & \frac{3}{18} \\ -1 & \frac{4}{18} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{6} \\ -1 & \frac{2}{9} \end{bmatrix}$

$B = \begin{bmatrix} 3 & -1 \\ -1 & 9 \end{bmatrix} \quad |B| = 27 - 1 = 26$

$B = \frac{1}{26} \begin{bmatrix} 9 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{9}{26} & \frac{1}{26} \\ \frac{1}{26} & \frac{3}{26} \end{bmatrix} = \begin{bmatrix} \frac{9}{26} & \frac{1}{26} \\ \frac{1}{26} & \frac{3}{26} \end{bmatrix}$

$C = \begin{bmatrix} 8 & 7 \\ 2 & 6 \end{bmatrix} \quad |C| = 48 - 14 = 34$

$C = \begin{bmatrix} 6 & -7 \\ -2 & 8 \end{bmatrix} \frac{1}{34} = \begin{bmatrix} \frac{6}{34} & -\frac{7}{34} \\ -\frac{2}{34} & \frac{8}{34} \end{bmatrix} = \begin{bmatrix} \frac{3}{17} & -\frac{7}{34} \\ -\frac{1}{17} & \frac{4}{17} \end{bmatrix}$

$D = \begin{bmatrix} 12 & 8 \\ 9 & 2 \end{bmatrix} \quad |D| = 24 - 72 = -48$

$D = -\frac{1}{48} \begin{bmatrix} 2 & -8 \\ -9 & 12 \end{bmatrix} = \begin{bmatrix} -\frac{2}{48} & +\frac{8}{48} \\ +\frac{9}{48} & -\frac{12}{48} \end{bmatrix} = \begin{bmatrix} -\frac{1}{24} & \frac{1}{6} \\ \frac{3}{16} & -\frac{1}{4} \end{bmatrix}$

Adjoint
of D

Determine if inversible

$A = \begin{bmatrix} -7 & 5 \\ 12 & 2 \end{bmatrix}$ $|A| = 14 - 60 = -46$ ✓

$B = \begin{bmatrix} -8 & -3 \\ 24 & 2 \end{bmatrix}$ $|B| = -16 - -72 = 56$ ✓

$C = \begin{bmatrix} 13 & 7 \\ 15 & 1 \end{bmatrix}$ $|C| = 13 - 95 = 32$ ✓

$D = \begin{bmatrix} 33 & 21 \\ 1 & -2 \end{bmatrix}$ $|D| = -66 - 21 = -87$ ✓

$$E = \begin{bmatrix} 3 & 5 \\ -7 & 2 \end{bmatrix} \quad \frac{1}{\det(E)} \cdot \operatorname{Adj}(E) = \frac{1}{6+35} \begin{bmatrix} 2 & -5 \\ 7 & 3 \end{bmatrix}$$

$$= \frac{1}{41} \begin{bmatrix} \frac{2}{41} & \frac{-5}{41} \\ \frac{7}{41} & \frac{3}{41} \end{bmatrix}$$

Adj = Adjugate / Adjoint

= transpose of cofactor C^T