# Hydrodynamics

*Tutorial 4: lava flows and viscous slumping*

## 1 The spreading of a viscous lava flow (Huppert, 1982)

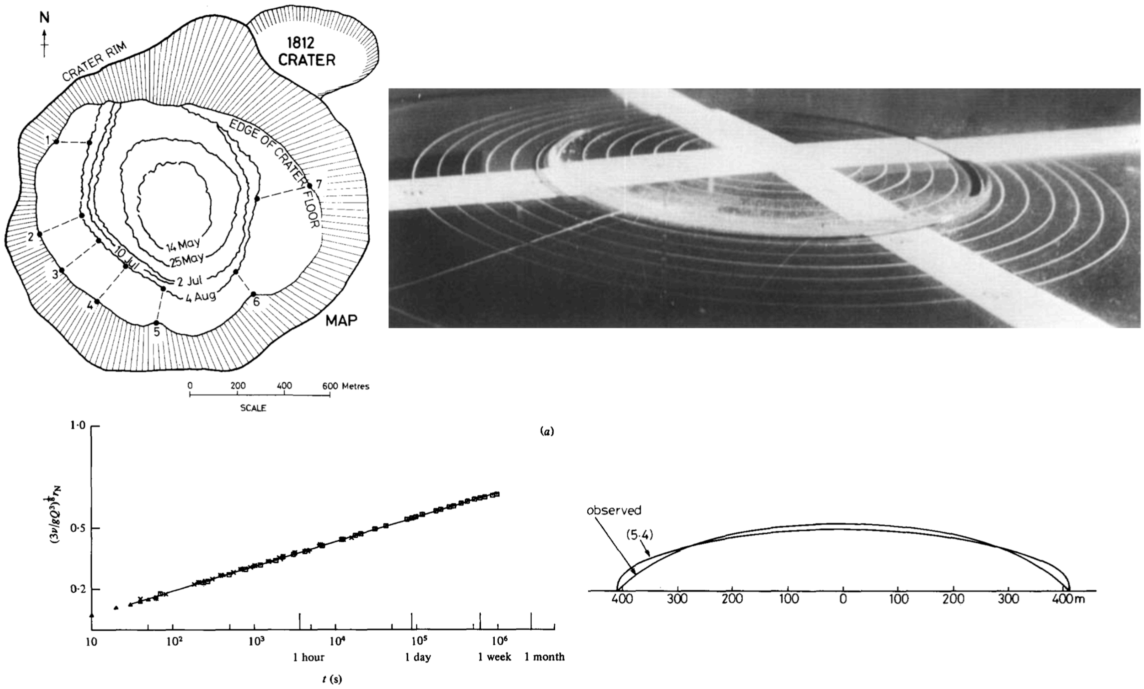


Figure 1: **Lava flow modelling.** Top left: evolution of lava spreading during the 1979 Soufrière's eruption. Top right: spreading of a 400 cm$^3$ volume of silicone soil with $\nu = 13.2$ cm$^2$s$^{-1}$ (photograph taken approximately 12 min after release). Bottom left: temporal evolution of the silicone oil spread. The best fit line is $r = (0.887 \pm 0;002)(gV^3/3\nu)^{1/8} t^{0.122\pm0.002}$. Bottom right: comparison between the actual lave shape and the one from the model. From Huppert (1982) and Huppert *et al.* (1982).

1. The axisymmetric thin film equation in presence of gravity alone reads:

$$\frac{\partial h}{\partial t} = \frac{1}{3}\frac{g}{\nu}\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial h}{\partial r}\right), \tag{1}$$

   where $h(r, t)$ stands for the local thickness of the liquid layer. Furthermore, regularity requirements on the axis and finite volume impose $\frac{\partial h}{\partial r}(0, t) = 0$ and $\lim_{r\to\infty} h = 0$. Express the global conservation of mass (or equivalently, in the framework of an incompressible evolution, global conservation of volume $V$). Is this problem well posed?
2. Search for a scale invariant solution for the spreading and explain the scaling $t^{0.122}$ observed in experiments.

## References

Huppert, Herbert E. 1982 The propagation of two-dimensional and axisymmetric viscous gravity currents over a rigid horizontal surface. *J. Fluid Mech.* **121**, 43.

Huppert, Herbert E., Shepherd, John B., Sigurdsson, R. Haraldur & Sparks, Stephen J. 1982 On lava dome growth, with application to the 1979 lava extrusion of the soufrière of st. vincent. *Journal of Volcanology and Geothermal Research* **14** (3-4), 199–222.