Physics Multiple Correct (Maximum Marks: 32)

Question No. 1
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

X rays from a tube with a target A of atomic number Z shows strong K lines for target A and weak K lines for impurities. The wavelength of K_α lines is λ_z for target A and λ_1 and λ_2 for two impurities. $\frac{\lambda_z}{\lambda_1} = 4$ and $\frac{\lambda_z}{\lambda_2} = \frac{1}{4}$. Screening constant of K_α lines to be unity. Select the correct statement(s)

A. the atomic number of first impurity is $2z - 1$
B. the atomic number of first impurity is $2z + 1$
C. the atomic number of second impurity is $\frac{(z+1)}{2}$
D. the atomic number of second impurity is $\frac{z}{2} + 1$

Question No. 2
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

The energy, the magnitude of linear momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number n are E, P and r respectively. Then according to Bohr's theory for hydrogen atom

A. EPr is proportional to $\frac{1}{n}$
B. $\frac{P}{E}$ is proportional to n
C. Er is constant for all orbits
D. Pr is proportional to n

Question No. 3
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

In a cylindrical region of radius R, there exists a time varying magnetic field B such that $\frac{dB}{dt} = k(> 0)$. A charged particle having charge q is placed at the point P at a distance $d(> R)$ from its centre O. Now, the particle is moved in the direction perpendicular to OP (see figure) by an external agent upto infinity so that there is no gain in kinetic energy of the charged particle. Choose the correct statement/s.
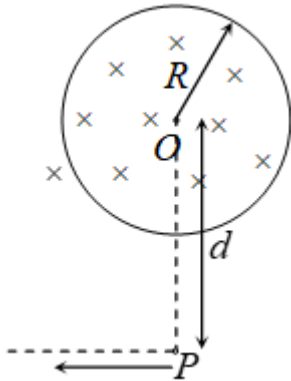
A. Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 2R$

B. Work done by external agent is $\frac{q\pi R^2}{8} k$ if $d = 4R$

C. Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 4R$

D. Work done by external agent is $\frac{q\pi R^2}{4} k$ if $d = 6R$

Question No. 4
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Two moles of a monatomic ideal gas undergoes a thermodynamic process $\frac{V^3}{T^2} =$ constant, if the temperature is raised by 300 K then

A. work done by the gas is $400R$
B. change in internal energy is $900R$
C. molar heat capacity of the gas for the process is $13/6R$
D. molar heat capacity of the gas for the process is $3/2R$

Question No. 5
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

In the given circuit the value of m is varying. The correct statements about the circuit are
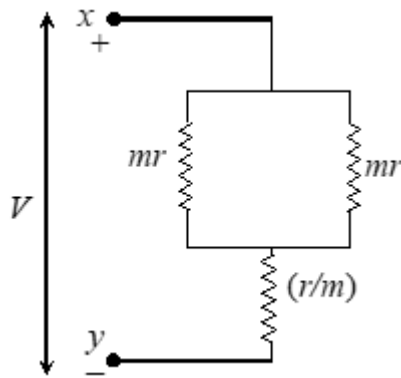
A. The condition for maximum current flowing from x is $m = 2$.
B. The maximum current is $\frac{V}{\sqrt{2}r}$.
C. The condition for maximum current flowing from x is $m = \sqrt{2}$
D. The maximum current is $\frac{3V}{2r}$.

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A particle of mass m moves along a straight line under the action of a force f varying with time as $f = f_0[1 - (\frac{t-T}{T})^2]$ where f_0 and T are positive constants. Then,

A. the speed of the particle after a time $2T$ is $\frac{4f_0T}{3m}$
B. after time interval of $3T$, the particle starts moving backwards
C. between time instants 0 and $2T$, the acceleration first increases and then decreases
D. the particle stops at $t = 3T$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A ring of mass $M = 5$ kg and radius $R = 1$ m is released from the top of an inclined plane with inclination angle $37°$ undergoes pure rolling. The ring collides elastically with a smooth rigid wall perpendicular to the inclined plane and rebounds back, if the speed of the center of mass of the ring is 46 m/s just before collision then (the co-efficient of friction between the ring and the incline plane $\mu = 1.5$)

A. the ring again starts pure rolling 10 s after its collision with the walls
   the speed of the center of mass of the ring when it starts pure rolling again is
B. 30 m/s

after collision the ring starts pure rolling again while coming down the
C. inclined plane
D. after collision the ring reaches the maximum height after nearly 6 seconds

Question No. 8
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Three right angled prism of refractive index μ_1, μ_2 and μ_3 are joined together so that the faces of the middle prism are in contact each with one of the outside prisms. Choose the wrong option(s).

A. If i is the incident angle and e is the emergent angle of the ray then for no net deviation e must be equal to $90 + i$.

B. If no net deviation of light is taking place through the combination then relation between μ_1, μ_2 and μ_3 must be $\mu_1{}^2 + \mu_3{}^2 - \mu_2{}^2 = 1$.

C. If $\mu_2 > \mu_1$, no light will emerge out of the system.

D. Net deviation of light through this combination can not be zero.

Physics Numerical (Maximum Marks: 24)

Question No. 1
Numerical Type
The answer has to be filled into the input box provided below.

Power supplied to a particle of mass 2 kg varies with time as $P = \frac{3t^2}{2}$ W . Here t is in second. Velocity of particle at $t = 0$ is $v = 0$. The velocity of particle (in m/s) at time $t = 2s$ will be

Question No. 2
Numerical Type
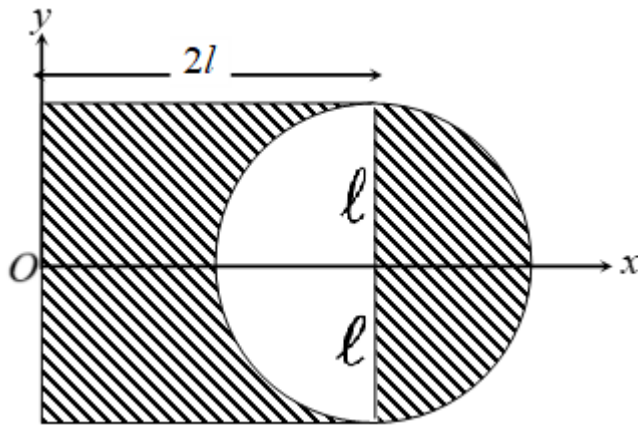The answer has to be filled into the input box provided below.

The Schrodinger equation for a free electron of mass m and energy E written in terms of the wavefunction ψ is $\frac{d^2\psi}{dx^2} + \frac{8\pi^2 mE}{h^2} \psi = 0$. The dimensions of the coefficient of ψ in the second term must be $[M^a L^b T^c]$, find a + b + c ?

Question No. 3
Numerical Type
The answer has to be filled into the input box provided below.

Consider a uniform square plate of side $2l$ made of wood. A semicircular portion is cut and attached to the right as shown. The coordinates of centre of mass of the redesigned plate is $(\frac{l}{n}, 0)$. Find the value of n.
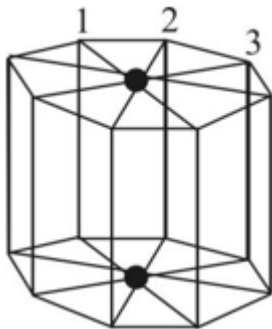
Numerical Type
The answer has to be filled into the input box provided below.

In the diagram shown, all the wires have resistance 8Ω. Find the equivalent resistance between the upper and lower dots shown in the diagram.

Numerical Type
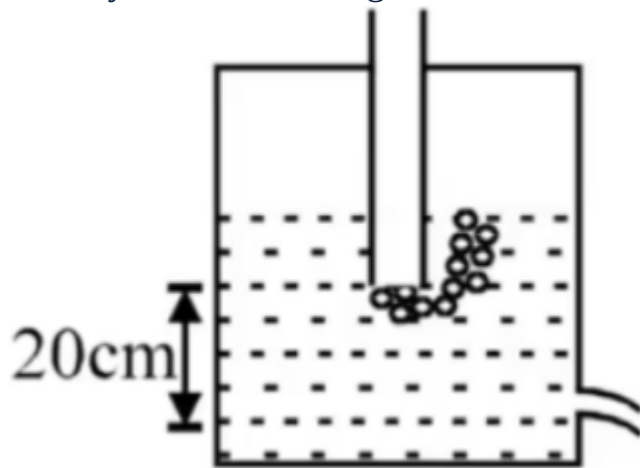The answer has to be filled into the input box provided below.

A double convex lens forms a real image of an object on a screen which is fixed. Now the lens is given a constant velocity v_0 along its axis and away from the screen. For the purpose of forming the image always on the screen, the object is also required to be given an appropriate velocity. Find the velocity of the object at the instant its size is n times the size of image. (Take $n = 1/2$ and $v_0 = 1$ m/s) (mark absolute value of velocity as answer)

Numerical Type
The answer has to be filled into the input box provided below.

A tube is attached as shown in closed vessel (wide) containing water. The velocity of water coming out from a small hole is (in m/sec)
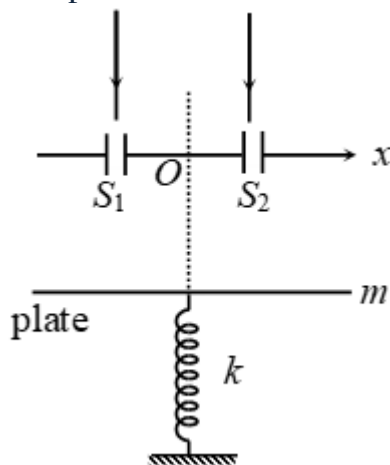

Physics Paragraph Type (Maximum Marks: 12)

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Two slits S_1 and S_2 lie on the x-axis and symmetric with respect to y-axis are illuminated by a parallel monochromatic light beam of wavelength λ as shown. The distance between slits is $d(\gg \lambda)$. Point O is the mid point of the line $S_1 S_2$ and this point is considered as the origin. The slits are in horizontal plane. The interference pattern is observed on a horizontal plate (acting as screen) of mass m which is connected to one end of a vertical massless spring of spring constant k. The other end of the spring is fixed to ground. At $t = 0$, the plate is a a distance $D(\gg> d)$ below the plane of slits and spring is in its natural length. The plate is released from rest from its initial position.

Question

The rate by which fringe, width will increase when acceleration of plate is zero, is

A. $\dfrac{\lambda g}{d} \sqrt{\dfrac{m}{k}}$

B. $\frac{\lambda g}{3d} \sqrt{\frac{m}{k}}$

C. $\frac{\lambda g}{4d} \sqrt{\frac{m}{k}}$

D. $\frac{\lambda g}{2d} \sqrt{\frac{m}{k}}$

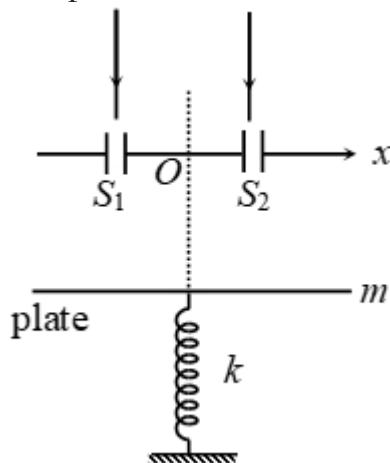Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Two slits S_1 and S_2 lie on the x-axis and symmetric with respect to y-axis are illuminated by a parallel monochromatic light beam of wavelength λ as shown. The distance between slits is $d(\gg \lambda)$. Point O is the mid point of the line $S_1 S_2$ and this point is considered as the origin. The slits are in horizontal plane. The interference pattern is observed on a horizontal plate (acting as screen) of mass m which is connected to one end of a vertical massless spring of spring constant k. The other end of the spring is fixed to ground. At $t = 0$, the plate is a a distance $D(\gg d)$ below the plane of slits and spring is in its natural length. The plate is released from rest from its initial position.

Question

The difference between two fringe widths when plate is at rest for a moment is

A. $\frac{2\lambda}{d}$

B. $\frac{\lambda mg}{dk}$

C. $\frac{2\lambda mg}{dk}$

D. $\frac{mg}{k} \frac{d}{\lambda}$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

In YDSE experiment two slits S_1 and S_2 are kept at $(0, \frac{d}{2}, 0)$ and $(0, -\frac{d}{2}, 0)$. A screen is kept in $y - z$ plane at $x = D$ and a source of light is placed at $(-\frac{D}{4}, 0, 0)$. The central bright fringe is found to be at a point P with coordinate $(D, 0, 0)$ on the screen.

Question

Another student now fills the region $0 \leq x \leq D, -\infty < y, z < +\infty$ with another medium of refractive index $\mu_2 > \mu_1$. Now he found the central bright fringe is formed at point Q. Where y coordinate of Q is

    A. same as that of P
    B. less than that of P
    C. more than that of P
    D. zero

Question No. 4
Only One Option Correct Type
Each question has multiple options out of which ONLY ONE is correct.

Paragraph

In YDSE experiment two slits S_1 and S_2 are kept at $(0, \frac{d}{2}, 0)$ and $(0, -\frac{d}{2}, 0)$. A screen is kept in $y - z$ plane at $x = D$ and a source of light is placed at $(-\frac{D}{4}, 0, 0)$. The central bright fringe is found to be at a point P with coordinate $(D, 0, 0)$ on the screen.

Question

Another student now removes both the medium and rearrange them such that medium of refractive index μ_1 is kept in a region $-\frac{D}{8} \leq x \leq 0$ and $0 < y < \infty$ whereas medium of refractive index $\mu_2 > \mu_1$ is kept in a region $-\frac{D}{8} \leq x \leq 0$ and $-\infty < y \leq 0$. He also kept a convex lens of focal length $f = \frac{D}{8}$ at point $(-\frac{D}{8}, 0, 0)$. Now when he repeated the experiment he found the central bright fringe at a point R whose y co-ordinate is

    A. $y < 0$
    B. $y > 0$
    C. $y = 0$
    D. $y = \frac{\mu_1 D}{\mu_2}$

Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Which out of the following statement(s) is/are correct?

A. KO_2 is an orange coloured compound.
B. RbO_2 is diamagnetic in nature.
C. K_{sp} of $BeSO_4$ is larger than that of $MgSO_4$.
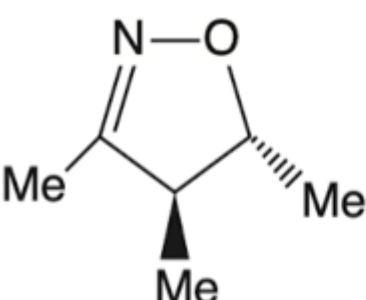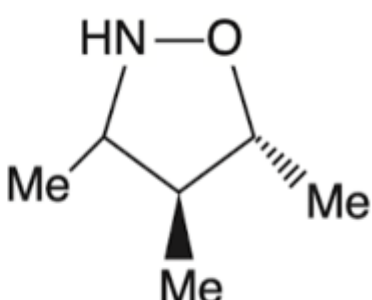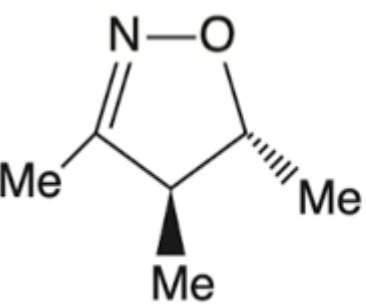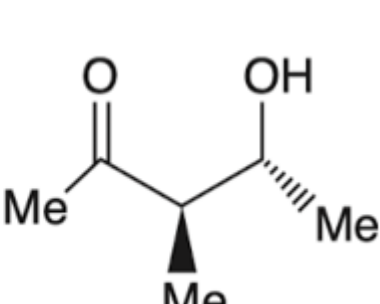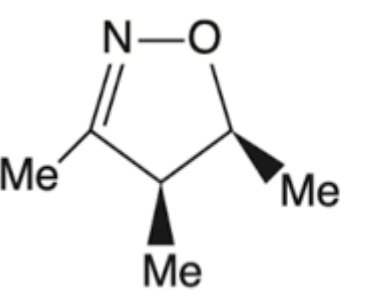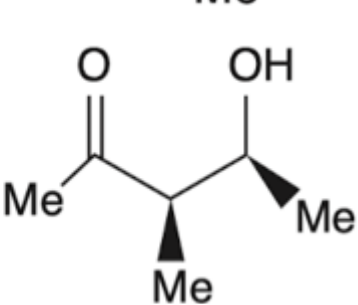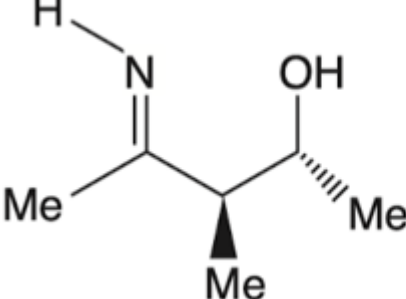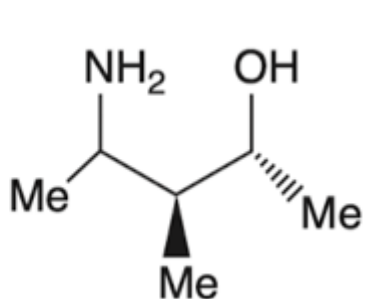D. Ca and Sr crystallise in ccp while Ba in bcc structure.

Question No. 2
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

The major products P_1 and P_2, respectively, in the following reaction sequence are
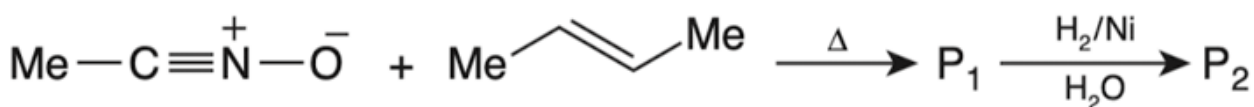
Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The acid dissociation constant (K_a) for HCOOH , CH_3COOH_2, $CH_2CICOOH$ and HCN at 25°C are 1.8×10^{-4} , 1.8×10^{-5} , 1.4×10^{-3} and 4.8×10^{-10} , respectively. The acid that gives highest pH at the equivalence point when 0.2M solution of each acid is titrated with a 0.2M solution of sodium hydroxide is
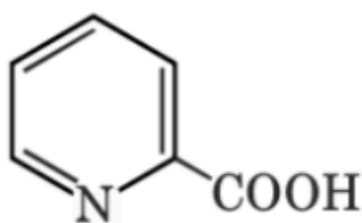
A. HCOOH
B. CH_3COOH
C. $CH_2ClCOOH$
D. HCN

Question No. 4

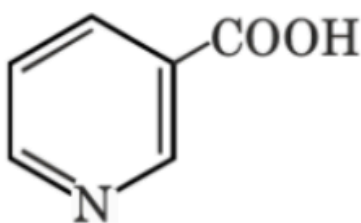**One or More Options Correct Type**

The question has multiple options out of which ONE or MORE is/are correct.

Consider the following compounds
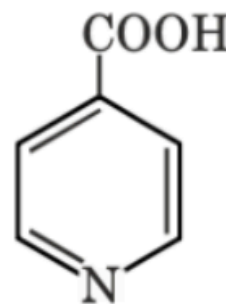
| I | II | III |

Select the correct statements

A. (I) undergoes decarboxylation faster than II or III on heating
   (I) when heated in the presence of acetophenone produces a compound
B. having tertiary alcoholic group
C. Compound (I) acts as bidentate ligand
D. The compound (III) has the highest melting point.

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The free energy change (ΔG) of 1 mol of an ideal gas that is compressed isothermally from 1 atm to 2 atm is

A. $RT \ln 2$
B. $-2RT$

C. $-RT \ln 2$
D. $2RT$

Question No. 6
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Which of the following statement(s) is/are correct about group 2 elements salt?

A. Thermal stability of sulphates and carbonates increases from Be^{2+} to Ba^{2+}
B. $Mg(NO_3)_2$ and $LiNO_3$ differ in the nature of decomposition products
C. The lattice energies of group 2 salts are much higher than those of group 1 salts
D. Hydration energies of group 2 salts are higher than those of group 1 salts

Question No. 7
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Which of the following is/are correct?

A. $[CrCl_2(CN)_2(NH_3)_2]$ and $[CrCl_3(NH_3)_3]$, both have d^2sp^3 hybridisation
B. Magnetic moment of $[PdCl_4]^{2-}$ is zero
C. $[Cu(NH_3)_4]^{2+}$, $[Pt(NH_3)_4]^{2+}$ and $[Ni(CN)_4]^{2-}$, all have dsp^2 hybridization of central metal
D. It is difficult to substitute chelating ligands compared to similar monodentate ligands
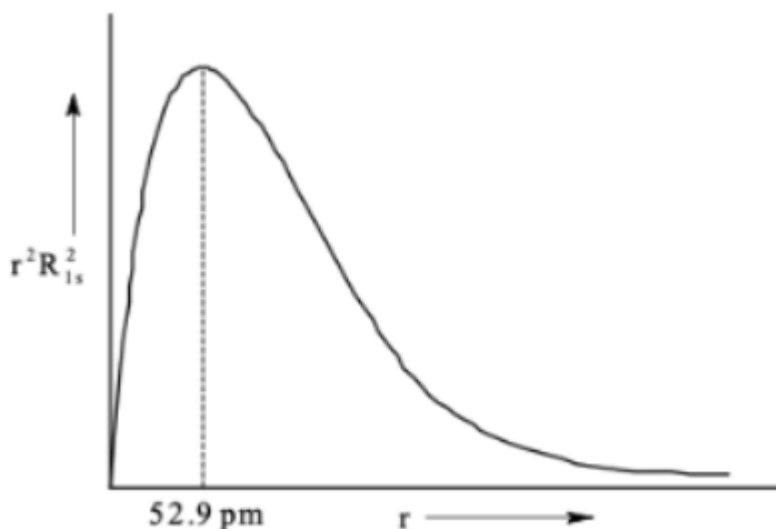
Question No. 8
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Which of the following option is/are correct?

A. Atomic orbitals are completely described as the regions where the probability of finding the electron is maximum.

$r^2 R_{1s}^2$

52.9 pm   r

The weighted average of large number of observations for measuring the radius of 1s orbital is greater than 52.9pm($r^2 R^2$ dr represents the total

B. probability of finding the electron between r and $r + dr$) .

C. The energy of 4 s is always lower than 3 d for multi electronic atom/ ion. Energy needed to excite an electron from $n = 2$ to $n = 4$ state is $\frac{25}{28}$ times the energy needed to excite an electron from $n = 2$ to $n = 5$ for a single

D. electron atom / ion.

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

A spherical glass bulb of radius 1.0 m contains a concentric rubber balloon that contains some N_2 gas and the remaining space in the flask contains 50 gH_2. In the given conditions, radius of the rubber balloon was found to be 40 cm. Now seal of the glass bulb was opened and 25 g$H_2(g)$ was further added and released. Assuming constant temperature throughout, the radius of the rubber balloon in the new conditions is _____ (nearest integer).

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Compound (A) on treatment with $CH_3CH_2Cl/FeCl_3$ gives (B), which on treating with NBS gives (C). This compound (C) on Frankland reaction gives (D). (D) on treatment with 2 equivalent of NBS producing compound (E) of molecular weight 368, which on treatment with $Zn - Cu/CH_3OH$ gives compound (F) . Compound (F) when treated with Lemieux reagent (alkaline $KMnO_4$ followed by $NaIO_4$) gives compound (G) that can give haloform test. Find the number of moles of (G). (D) and (E) both are meso compounds.

Question No. 3
Numerical Type

The answer has to be filled into the input box provided below.

Two electrochemical cells are assembled in which the following reactions occur.

$$V^{2+} + VO^{2+} + 2H^+ \longrightarrow 2\,V^{3+} + H_2O \quad ; \quad E^{\circ}_{cell} = 0.61\ V$$

$$V^{3+} + Ag^+ + H_2O \longrightarrow VO^{2+} + 2H^+ + Ag(s) \quad ; \quad E^{\circ}_{cell} = 0.209\ V$$
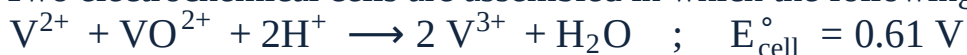
Calculate E° (in terms of _____ $\times 10^{-2}$ volt) for half reaction

$$V^{2+} \longrightarrow V^{3+} + e^- \quad .$$

Given, $E^{\circ}_{Ag^+/Ag} = 0.799$ volt.

Question No. 4
Numerical Type

The answer has to be filled into the input box provided below.

The rate constant of a first order reaction, $X \to Y$, is $1.6 \times 10^{-3}\ s^{-1}$ at 300 K Given that the activation energy of the reaction is $28\ kJ\ mol^{-1}$ and assuming Arrhenius behavior for the temperature dependence, the total time required to obtain 90% of Y at 350 K is _____ s.
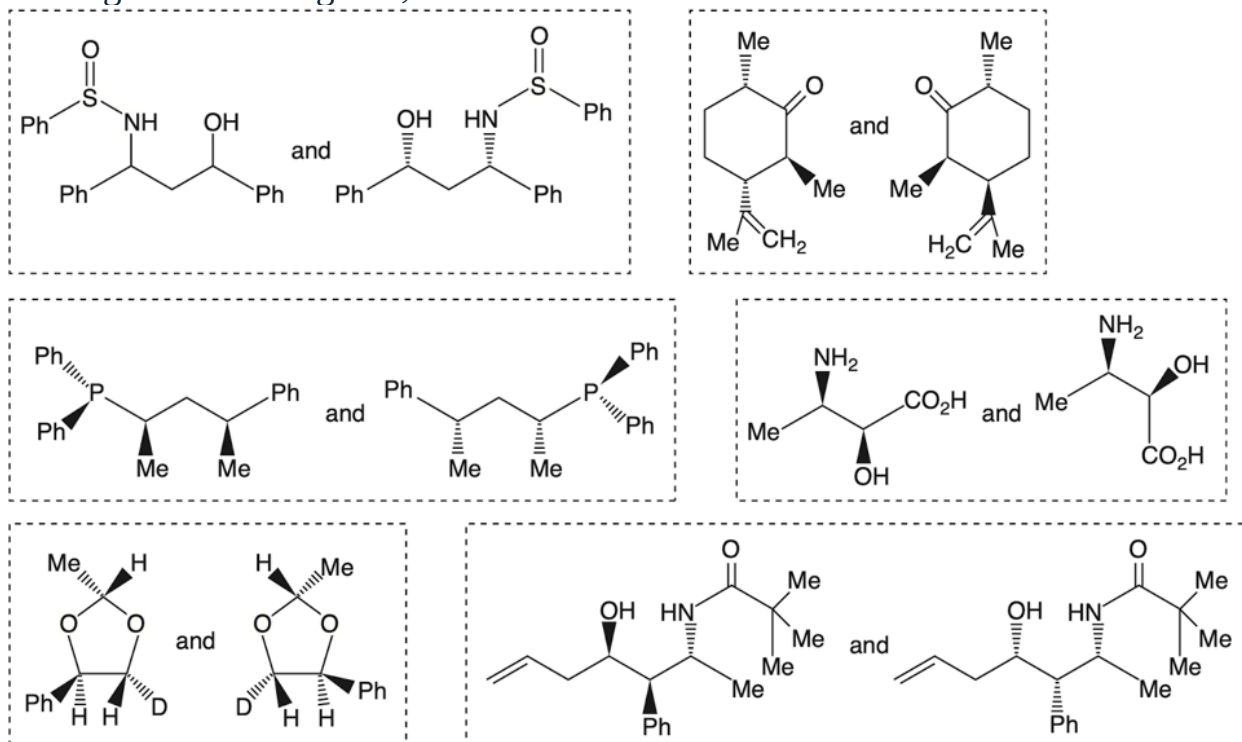(Up to one decimal place. Use $R = 8.31\ J\ K^{-1}\ mol^{-1}$)

Question No. 5
Numerical Type

The answer has to be filled into the input box provided below.

Among the following sets,

the total number of set(s) of diastereomeric pair(s) is _____.

Question No. 6
Numerical Type
The answer has to be filled into the input box provided below.

The normal boiling point of a compound (X) is 350 K (heat of vaporization, $\Delta_{vap} H = 30$ kJ mol^{-1}). The pressure required to boil ' X ' at 300 K is _____ Torr. (Round off to two decimal places)
(Ignore the temperature variation of $\Delta_{vap} H$; Gas constant $R = 8.31$ J mol^{-1} K^{-1} and 1 atm = 760 Torr)

Chemistry Paragraph Type (Maximum Marks: 12)

Question No. 1
Only One Option Correct Type
Each question has multiple options out of which ONLY ONE is correct.

Paragraph

In the presence of excess base and excess halogen a methyl ketone is converted first into a trihalo substituted ketone and then into a carboxylic acid. After the trihalo substituted ketone is formed hydroxide ion attacks the carboxyl carbon because the trihalomethyl ion is the group more easily expelled from the tetrahedral intermediate. The conversion of a methyl ketone to a carboxylic acid is called a haloform reaction because one of the product is haloform $CHCl_3$ or CHI_3 or $CHBr_3$.
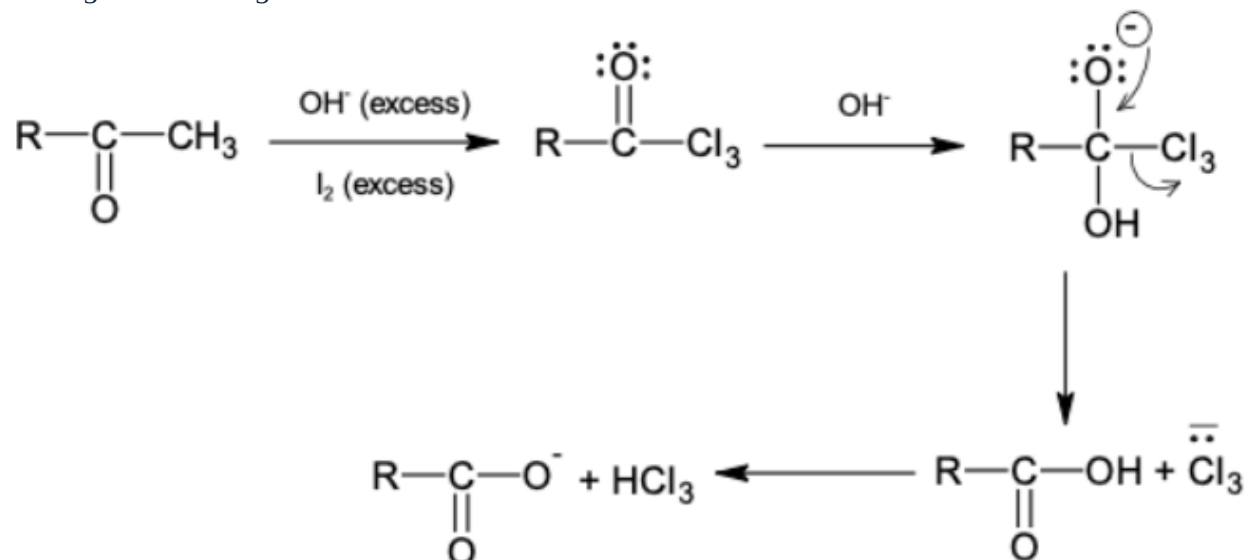

Question

Product is

Question No. 2

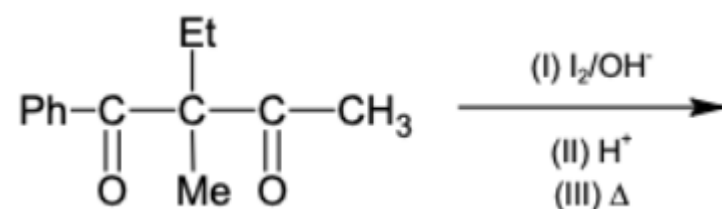**Only One Option Correct Type**

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

In the presence of excess base and excess halogen a methyl ketone is converted first into a trihalo substituted ketone and then into a carboxylic acid. After the trihalo substituted ketone is formed hydroxide ion attacks the carboxyl carbon because the trihalomethyl ion is the group more easily expelled from the tetrahedral intermediate. The conversion of a methyl ketone to a carboxylic acid is called a haloform reaction because one of the product is haloform $CHCl_3$ or CHI_3 or $CHBr_3$.

Question

X is

A.

B.

C. (structure with O, ONa, COCH$_3$)

D. (structure with O, COCH$_3$, COCH$_3$)

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

A Colorless solid A liberates a brown gas B on acidification, a colorless alkaline gas (C) on treatment with NaOH and a colorless non-reactive gas D on heating. If heating of the solid is continued, it completely disappears.

Question

The compound A is

   A. NH_4NO_2
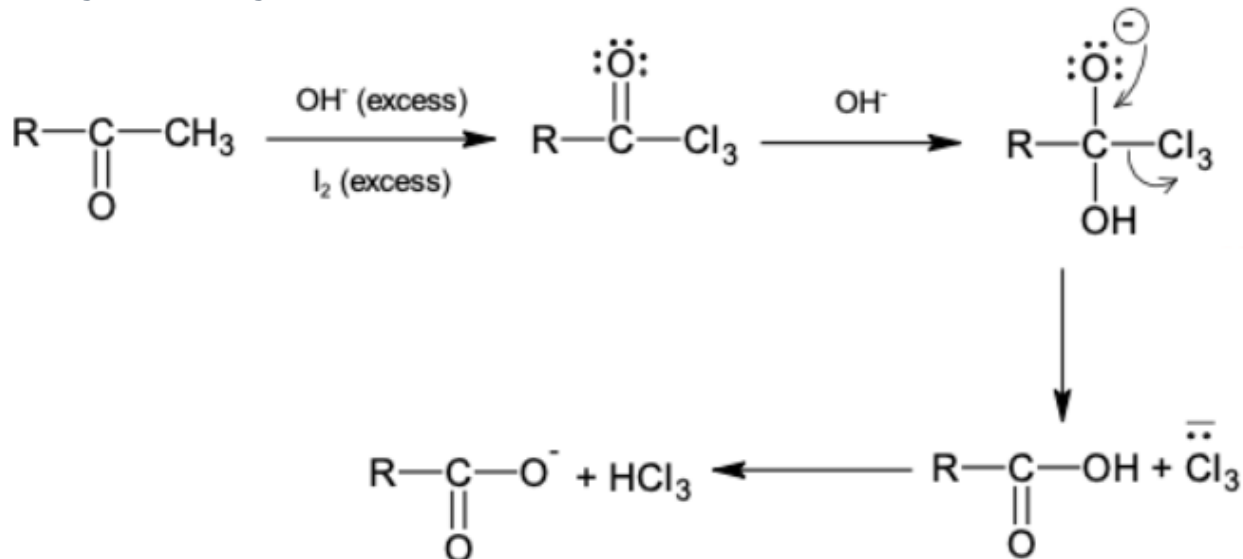   B. PH_4NO_3
   C. NH_4NO_3
   D. none of these

Question No. 4

Only One Option Correct Type

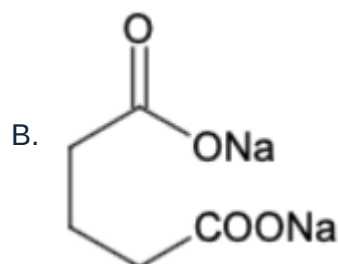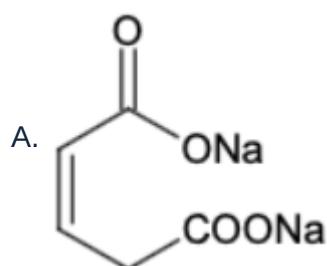Each question has multiple options out of which ONLY ONE is correct.

Paragraph

A Colorless solid A liberates a brown gas B on acidification, a colorless alkaline gas (C) on treatment with NaOH and a colorless non-reactive gas D on heating. If heating of the solid is continued, it completely disappears.

Question

Which one of the following is correct?

   A. Bond order of D is 2.5
   B. D is a paramagnetic gas
   C. Smell of gas c is like rotten fish
   D. lonization energy of gas D is more than N atom

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1
One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $f(x) = ||x - 1| + |x - 3| - |2x - 1|| + ||x - 1| + |x - 3| + |2x - 1||$,
then

    A. $f(x)$ is non-differentiable at $x = 1$
    B. $f(x)$ is non-differentiable at $x = \frac{3}{2}$
    C. $f(x)$ has minimum value 2
    D. $x = 1$ is point of local minima

Question No. 2
One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the following is (are) correct?

    If the coefficient of term independent of x in $(\sqrt{x} - \frac{k}{x^2})^{10}$ is 405, then
    A. product of possible values of k is -9.
    The value of r for which $S = {}^{20}C_r\,{}^{10}C_0 + {}^{20}C_{r-1}\,{}^{10}C_1 + \ldots + {}^{20}C_0\,{}^{10}C_r$
    B. is maximum is equal to 15.
    C. The value of $\sum_{k=1}^{\infty} \sum_{r=0}^{k} \frac{{}^kC_r}{3^k}$ is equal to 2.
    D. The digit at unit's place of $N = 17^{50} + 7^{50}$ is 8

Question No. 3
One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A circle $(x - 3)^2 + (y - 6)^2 = r^2$ touches parabola $y^2 = 4x$ at $P(a, b)$. If the slope of common tangent at P is m, then (where b, r > 0) -

    A. $r = 5$
    B. $\frac{r^2}{m} = 10$
    C. $(a + b)m = 4$
    D. $a\sqrt{b} = 8$

Question No. 4
One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $A = \{z_1; z_1^{16} = 1, z_1 \in C\}, B = \{z_2; z_2^{72} = 1, z_2 \in C\}$ and
$P = \{z_1 z_2; z_1 \in A, z_2 \in B\}$ are three sets of complex roots of unity (where C denotes set of complex numbers), then

A. $n(A \cap B) = 8$
B. $n(A \cap B) = 4$
C. $n(P) = 144$
D. $n(P) = 72$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $\int_a^b |\sin x| dx = 8$ & $\int_0^{a+b} |\cos x| dx = 9$ then;

A. $a + b = 9\pi$
B. $b - a = 4\pi$
C. $\int_a^b x \sin x dx = 2\sqrt{2}\pi$
D. $\int_a^b x \sin x dx = -2\sqrt{2}\pi$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x)$ be a continuous function which takes positive values for $x \geq 0$ and satisfy $\int_0^x f(t)dt = x\sqrt{f(x)}$ with $f(1) = \frac{1}{2}$. Then

A. $f(x)$ is increasing function in domain
B. $f(x)$ is decreasing function in domain
C. $f(\sqrt{2} + 1) = \frac{1}{2\sqrt{2}}$
D. $f(\sqrt{2} + 1) = \frac{1}{4}$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The length of two opposite edges of a tetrahedron are 12 and 15 units and the shortest distance between them 10 units. If the volume of the tetrahedron is 200 cubic units, then the angle between the 2 given opposite edges is

A. $\cos^{-1} \frac{\sqrt{7}}{4}$
B. $\sin^{-1} \frac{2}{3}$
C. $\sin^{-1} \frac{3}{4}$
D. $\tan^{-1} \frac{2}{\sqrt{5}}$

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = \vec{c} \cdot \vec{c} = 3$ and
$|\vec{a} + \vec{b} - \vec{c}|^2 + |\vec{b} + \vec{c} - \vec{a}|^2 + |\vec{c} + \vec{a} - \vec{b}|^2 = 36$, then

A. $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-9}{2}$.

B. $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors

C. $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq 0$.

D. $\vec{a}, \vec{b}, \vec{c}$ represent the sides of triangle.

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If $f(2 + x) + f(x) = 2$ for all $x \in [0, 2]$ and $k = \int_0^4 f(x)dx - 4$ such that both roots of $(1 - a^2)x^2 + 2ax - 1 = 0$ lies between $k \& q$ where $q = \frac{\alpha}{\beta}$ such that $\alpha + \beta = 1 \& \alpha^2 + \beta^2 = \frac{1}{2}$ then least positive integral value of a _____.

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A person has 6 friends and during a certain vacation he met them during several dinners. He found that he dined with all 6 exactly on one day, with every 5 of them on 2 days, with every 4 of them on 3 days, with every 3 of them on 4 days, with every 2 on 5 days. Further every friend was present at 7 dinners and every friend was absent at 7 dinners. The number of dinner he had alone is equal to?

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Let $y = f(x)$ be drawn with $f(0) = 2$ and for each real number a, the tangent to $y = f(x)$ at $(a, f(a))$, has x intercept $(a - 2)$. If $f(x)$ is of the form ke^{px} , then $\left(\frac{k}{p}\right)$ has the value equal to

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

If $x = 2 + t^3, y = 3t^2$ and $\left(\frac{d^2y}{dx^2}\right)/\left(\frac{dy}{dx}\right)^n$ is a constant then the value of $343n$ must be

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let $A = [a_{ij}]3 \times 3$ be a matrix such that $AA^\top = 4I$ and $2a_{ij} + c_{ij} = 0$ where c_{ij} is the cofactor of a_{ij} $\forall\ i\ \&\ j, I$ is the unit matrix of order 3 and $A^\top$ is the transpose of the matrix A.

If $\begin{vmatrix} a_{11} + 4 & a_{12} & a_{13} \\ a_{21} & a_{22} + 4 & a_{23} \\ a_{31} & a_{32} & a_{33} + 4 \end{vmatrix} + 5\lambda \begin{vmatrix} a_{11} + 1 & a_{12} & a_{13} \\ a_{21} & a_{22} + 1 & a_{23} \\ a_{31} & a_{32} & a_{33} + 1 \end{vmatrix} = 0$,

then $\lambda = \frac{a}{b}$ where a and b are coprime positive integers then the value of $a + b$ is _____.

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

A die is rolled four times. If the probability that product of first 3 outcomes is equal to fourth outcome is p, then $\left[\frac{1}{\sqrt{p}}\right]$ is equal to (where [.] denotes greatest integer function)

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

$f(x)$ is a differentiable function satisfying the relation
$f^2(x) + f^2(y) + 2(xy - 1) = f^2(x + y)$ $\forall x, y \in R.$
If $f(x) > 0 \forall x, y \in R$ and $f(\sqrt{2}) = 2$ then

Question

Value of $f'(1)$ will be

A. $\dfrac{1}{2\sqrt{6}}$

B. $\dfrac{1}{\sqrt{6}}$

C. $\dfrac{3}{2\sqrt{6}}$

D. none of these

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

$f(x)$ is a differentiable function satisfying the relation
$f^2(x) + f^2(y) + 2(xy - 1) = f^2(x + y)$ $\forall x, y \in R$.
If $f(x) > 0 \forall x, y \in R$ and $f(\sqrt{2}) = 2$ then

Question

Value of $\int_0^1 f(x)dx$ will be

A. $\dfrac{3}{2} + \log_e(\dfrac{1+\sqrt{3}}{\sqrt{2}})$

B. $\dfrac{\sqrt{3}}{2} + \log_e(\dfrac{1+\sqrt{3}}{\sqrt{2}})$

C. $\dfrac{1}{2} + \log_e(\dfrac{1+\sqrt{3}}{2})$

D. none of these

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph

Consider set $S = \{(a, b) : (a + 3)t^2 = 3 - a$ and $bt^2 - 4t + b = 0\}$, where t is are real parameter. Let C is curve which is formed by all elements of set S where (a, b) is a point in R^2. Tangents are drawn from the point $P(3, 4)$ to the curve C touching the curve C at point Q and R.

Question

The circumcentre of triangle PQR is (α, β) , then value of $\alpha + 3\beta$ is-

A. 3

B. 5

C. 7

D. 9

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Consider set $S = \{(a,b) : (a+3)t^2 = 3-a \quad \text{and} \quad bt^2 - 4t + b = 0\}$, where t is are real parameter. Let C is curve which is formed by all elements of set S where (a,b) is a point in R^2. Tangents are drawn from the point $P(3,4)$ to the curve C touching the curve C at point Q and R.

Question

Curve E is equation of ellipse whose foci are Q, R and curve E is passing through point P, then eccentricity of ellipse E is-

A. $\dfrac{2\sqrt{10}}{5+3\sqrt{5}}$

B. $\dfrac{3\sqrt{10}}{5+4\sqrt{5}}$

C. $\dfrac{3\sqrt{10}}{4+5\sqrt{5}}$

D. $\dfrac{2\sqrt{10}}{3+5\sqrt{10}}$