Physics Single Correct (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in from of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement (s) is(are) true?

   A. The refractive index of the lens is 2.5
   B. The radius of curvature of the convex surface is 45 cm
   C. The faint image is erect and real
   D. The focal length of the lens is 2 cm

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Nuclei of radioactive element A are being produced at rate t^2 at time t. The element A has decay constant equal to λ. If N is the number of active nuclei of element A at any time then $\frac{dN}{dt}$ was found to be minimum at $t = t_0$. N at time $t = t_0$ is

   A. $\dfrac{t_0 - xt_0^2}{x^2}$
   B. $\dfrac{t_0 - 2\lambda t^2}{\lambda^2}$
   C. $\dfrac{4t_0 - xt_0^2}{\lambda^2}$
   D. $\dfrac{2t_0 - \lambda t_0^2}{\lambda^2}$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Which of the following statements is/are correct for an x-ray tube ?

   A. On increasing potential difference between filament and target, photon flux of x-rays increases
   B. On increasing potential difference between filament and target, frequency of x-rays increases
   C. On increasing filament current, cut-off wavelength increases
   D. None of the above.

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A semicircular ring of mass m and radius R is fixed as shown. A point mass m_0 is moved by an external agent from O to O'. The work done by gravitational field in moving m_0 from O to O' is :
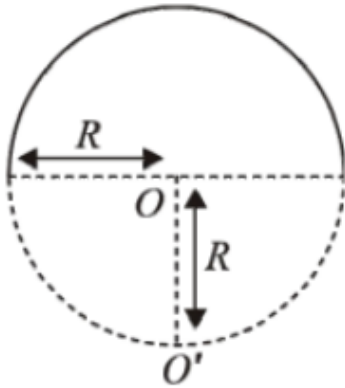
A. $\dfrac{Gmm_0}{R}[\{\frac{2}{\pi}\ln(\sqrt{2}+1)\}-1]$

B. $\dfrac{Gmm_0}{R}[\{\frac{1}{\pi}\ln(\sqrt{2}+1)\}-1]$

C. $\dfrac{Gmm_0}{R}[\{-\frac{2}{\pi}\ln(\sqrt{2}+1)\}-1]$

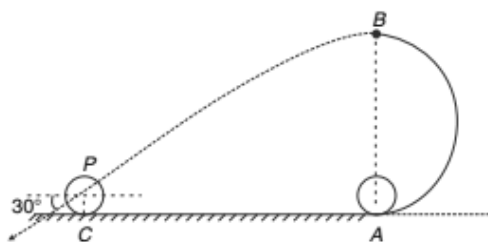D. $\dfrac{Gmm_0}{R}[\{-\frac{1}{\pi}\ln(\sqrt{2}+1)\}-1]$

Physics Question Stem (Maximum Marks: 8)

Question No. 1
Numerical Type
The answer has to be filled into the input box provided below.

A disc of radius $r = 0.1$ m is rolled from a point A on a track as shown in the figure. The part AB of the track is a semi-circle of radius R in a vertical plane. The disc rolls without sliding and leaves contact with the track at its highest point B. Flying through the air it strikes the ground at point C. The velocity of the center of mass of the disc makes an angle of $30°$ below the horizontal at the time of striking the ground. At the same instant, velocity of the topmost point P of the disc is found to be 6 m/s . (Take $g = 10$ m/s^2).
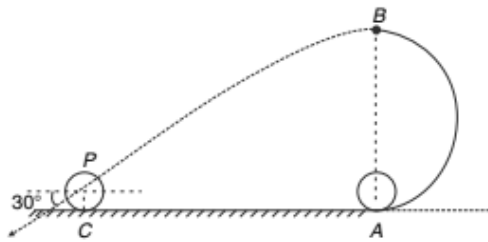

Find the value of R.

Question No. 2
Numerical Type
The answer has to be filled into the input box provided below.

A disc of radius $r = 0.1$ m is rolled from a point A on a track as shown in the figure. The part AB of the track is a semi-circle of radius R in a vertical plane. The disc rolls without sliding and leaves contact with the track at its highest point B. Flying through the air it strikes the ground at point C. The velocity of the center of mass of the disc makes an angle of $30°$ below the horizontal at the time of striking the ground. At the same instant, velocity of the topmost point P of the disc is found to be 6 m/s . (Take $g = 10$ m/s^2).

Find the velocity of the center of mass of the disc when it strikes the ground.
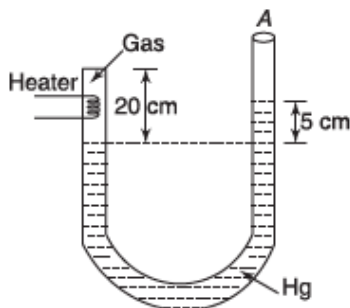
Question No. 3
Numerical Type
The answer has to be filled into the input box provided below.

One end of an insulating U tube is sealed using insulating material. A mono atomic gas at temperature 300 K occupies 20 cm length of the tube as shown. The level of mercury on two sides of the tube differ by 5 cm. The other end of the tube is open to atmosphere.(Patm $= 75$ cm of Hg column)

Area of cross section of the tube is uniform and is equal to 0.01 m^2. The gas in the tube is heated by an electric heater so as to raise its temperature to 562.5 K. Assume that no heat is conducted to mercury by the gas.


Find the final length of the gas column (in cm).

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

One end of an insulating U tube is sealed using insulating material. A mono atomic gas at temperature 300 K occupies 20 cm length of the tube as shown. The level of mercury on two sides of the tube differ by 5 cm. The other end of the tube is open to atmosphere. (Patm = 75 cm of Hg column)

Area of cross section of the tube is uniform and is equal to 0.01 m^2. The gas in the tube is heated by an electric heater so as to raise its temperature to 562.5 K. Assume that no heat is conducted to mercury by the gas.
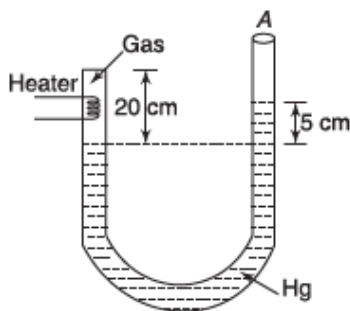
Find the amount of heat supplied by the heater to the gas.

Physics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A point charge having mass m charge $+q$ is revolving around a fixed point charge $-Q$ in elliptical orbit of semi major axis 'a' under the action of their mutual electrostatic interaction force only. Choose the correct option(s).

A. Total energy of the system is $-\dfrac{kQq}{a}$

B. Total energy of the system is $-\dfrac{kQq}{2a}$

C. Time period of one revolution is $2\pi a^{\frac{3}{2}} \sqrt{\dfrac{m}{kQq}}$

D. Time period of one revolution is $\pi a^{\frac{3}{2}} \sqrt{\dfrac{m}{kQq}}$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider a planar body of irregular size. It is kept in equilibrium in vertical plane by using 2 pins A& B lying at same vertical level as shown in figure.
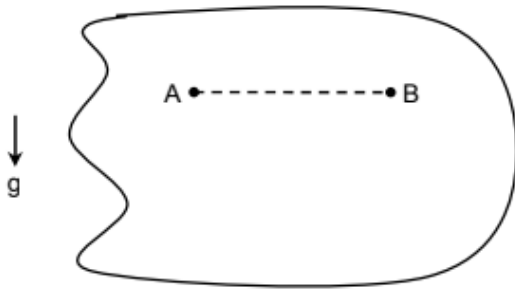
Separation between $A\&B$ is d. Pins offer a small friction to the body. Normal contact offered to the body by pin A is N_A and by pin B is N_B. It is observed that when pin A is removed body rotates $150°$ anti-clockwise w.r.t. B and attains equilibrium. In the same manner if pin B is removed body rotates $120°$ clockwise w.r.t. A and attains equilibrium. Choose the correct option(s) :

A. Value of $\dfrac{N_B}{N_A}$ is 3

B. Value of $\dfrac{N_B}{N_A}$ is 2

C. Separation between centre of mass of body and pin A is $\dfrac{\sqrt{3}d}{2}$

D. Separation between centre of mass of body and pin A is $\dfrac{d}{2}$

Question No. 3
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

A metal rod of length ℓ is suspended to the ceiling by one of it's end. The density of metal is ρ, young module is y. Choose the correct option(s)

A. The stress at mid point is $\dfrac{\ell g \rho}{2}$

B. Extension in rod due to it's own weight $\Delta\ell = \dfrac{\rho g \ell^2}{2y}$

C. Extension in rod due to it's own weight $\Delta\ell = \dfrac{\rho g \ell^2}{4y}$

D. The stress at mid point is $\ell \rho g$

Question No. 4
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Consider 2 identical coherent sources S_1 & S_2 kept as shown in figure. Screen is very large. Wavelength of light emitted by source is λ.
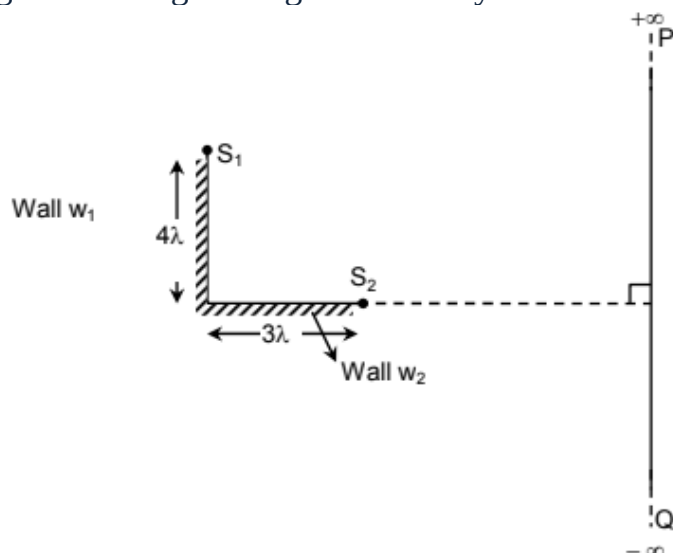
Assume no reflection and transmission from wall w_1 and w_2. Choose correct option(s) regarding the interference pattern observed on straight line PQ.

   A. Total number of maxima observed on line PQ above central maxima is 3.
   B. Total number of maxima observed on line PQ below central maxima is 3 .
   C. Total number of maxima observed on line PQ is 7 .
   D. Total number of minima observed on line is 9

Question No. 5
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Initially switch S is open, no charge on capacitor and no current in inductor coils. Now switch is closed then at steady state.
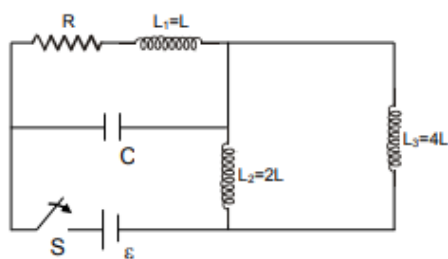
   A. The ratio of energy stored in L_1 and L_2 is $\frac{9}{8}$
   B. The ratio of energy stored in L_1 and L_3 is $\frac{9}{4}$
   C. The ratio of energy stored in L_2 and L_3 is 2
   D. The charge on capacitor is $2C\varepsilon$

Question No. 6
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

A non-relativistic particle of mass m is held in a circular orbit around the origin by an attractive force $F(r) = -kr$ where k is positive constant. Assuming the Bohr quantization of the angular momentum of the particle, Select the correct alternatives, if the radius of the orbit is r and speed of the particle is v.

A. $v^2 = \left(\frac{n\hbar}{m}\right)\left(\frac{k}{m}\right)^{1/2}$

B. $r^2 = \left(\frac{n\hbar}{k}\right)\left(\frac{k}{m}\right)^{1/2}$

C. $v^2 = \left(\frac{n\hbar}{2\,m}\right)\left(\frac{k}{m}\right)^{1/2}$

D. $r^2 = \left(\frac{n\hbar}{2k}\right)\left(\frac{k}{m}\right)^{1/2}$

Question No. 7
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

The current flowing through the resistor in a series RLC $a.c.$ circuit, is $I = \frac{\varepsilon}{R}$. Now the inductor and capacitor are connected in parallel and joined in series with the resistor as shown in figure. The current in the circuit is now. (symbols have their usual meaning)
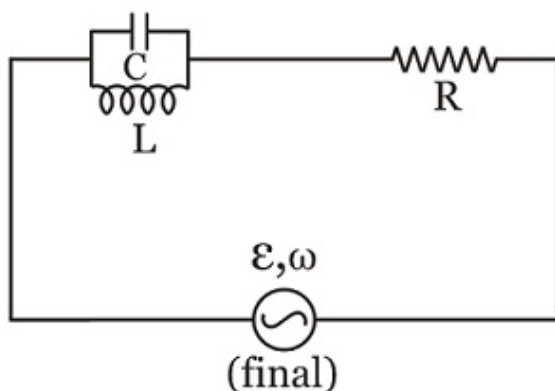
A. Equal to I
B. More than I
C. Less than I
D. Zero

Physics Numerical (Maximum Marks: 12)

Question No. 1
Numerical Type
The answer has to be filled into the input box provided below.

When a soap bubble of radius r is charged to a uniform density σ, its radius becomes thrice. Assume temperature remains constant during expansion and atmospheric pressure is P_0.

Pressure inside the initially uncharged bubble of radius r was found to be $\alpha(\frac{3\sigma^2}{\beta\varepsilon_0} - \gamma P_0)$. Here α, β and γ are integers. Find the value of $\alpha + \beta + \gamma$ (mark answer to nearest integer).

Question No. 2
Numerical Type
The answer has to be filled into the input box provided below.

A glass sphere ($\mu = \frac{5}{3}$) with an inbuilt point source of light P is placed on table. The source P is just inside the surface of glass and lies on the lower end of vertical diameter. The apparatus is placed in a dark room. The surface area of the glowing portion of the glass sphere as observed by the eye is found to be $(\frac{\alpha}{\beta})^2 \pi R^2$. Here α and β are integers. Find minimum value of $\alpha - \beta$.

glass sphere

small point source of light
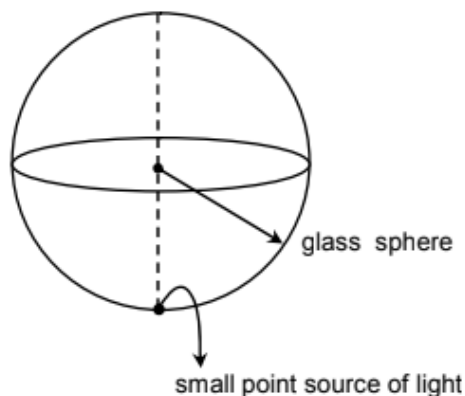
Question No. 3
Numerical Type
The answer has to be filled into the input box provided below.

Moment of inertia of a disc of mass $M = 2 \pm 0.02$ kg , inner radius a = 5 ± 0.05 cm , outer radius $b = 7 \pm 0.07$ cm , with respect to an axis passing through geometrical centre of the disc and perpendicular to the plane of disc is given by $(A_0 \pm B_0 \times 10^{-2})$kg $-$ cm^2 . Find the value of $\frac{B_0}{A_0}$.

Chemistry Single Correct (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

If 0.5 L of a $1M$ $SnSO_4$ solution is electrolysed for a period of 30 minutes using a current of 9.65 A and inert electrodes, the final concentration of Sn^{2+} in the solution will be (Given:

$E^0_{Sn^{2+}|Sn} = -0.14$ V, $E^0_{Sn^{2+}|Sn^{4+}} = -0.13$ V, $E^{\circ}_{OH^-|O_2} = -0.4$ V, $E^0_{SO_4^{2-}|S_2O_8^{2-}} = -2.0$ V)

A. 0.64 M
B. 0.28 M
C. 1.18 M
D. 0.84 M

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

An optically active substance undergoes first order mutarotation in an aqueous solution. The observed rotation at 0 min, 180 min and ∞ time are $190^{\circ}, 150^{\circ}$ and 30° respectively. The sum of forward and backward rate constants $(k_f + k_r)$ would be ($\log 3 = 0.477, \log 2 = 0.3$)

A. 1.57×10^3 min^{-1}
B. 1.57×10^{-2} min^{-1}
C. 1.57×10^{-3} min^{-1}
D. 15.7 min^{-1}

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Which of the following is a correct statement about s-block compounds?

A. Stability of metal chlorides decreases down the alkali metal group.
B. Lattice energy of M_2O_2 is greater than that of M_2O.
C. Stability of peroxide increases with the increase in size of alkali metal cation.
D. The water of crystallization is greater in alkali metal salts than in alkaline earth metal salts.

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For the irreversible unimolecular type reaction $A \xrightarrow{k}$ products, in a batch reactor, 80% reactant $A(C_{A_0} = 1$ mole / lit.) is converted in a 480 second run and conversion is 90% after 18 minutes. The order of this reaction is:
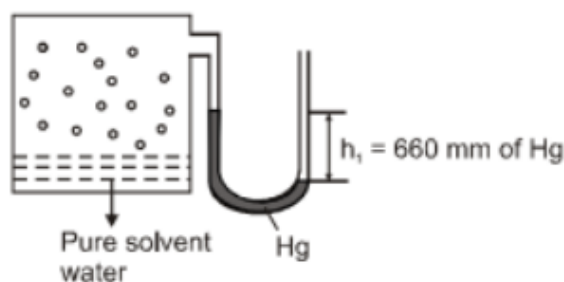
A. 1

B. 2

C. $\dfrac{1}{2}$

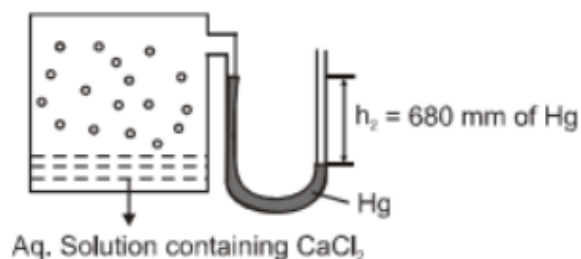D. $\dfrac{3}{2}$

Chemistry Question Stem (Maximum Marks: 8)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

$h_1 = 660$ mm of Hg

Pure solvent water   Hg

Arrangement -I

$h_2 = 680$ mm of Hg

Hg

Aq. Solution containing $CaCl_2$

Arrangement -II

The difference in height of the mercury column in two arms of U tube manometer in arrangement-I is $h_1 = 660$ mm . In another arrangement-II at same temperature, 222gm of $CaCl_2$ is dissolved in 324gm of water and difference in height of mercury column in two arms is found to be $h_2 = 680$ mm . If the value of degree of dissociation for $CaCl_2$ in arrangement-II is α: [Atmospheric pressure $= 1$ atm]

The value of vont hoff factor is

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

h₁ = 660 mm of Hg

Pure solvent water

Hg

Arrangement -I

h_2 = 680 mm of Hg
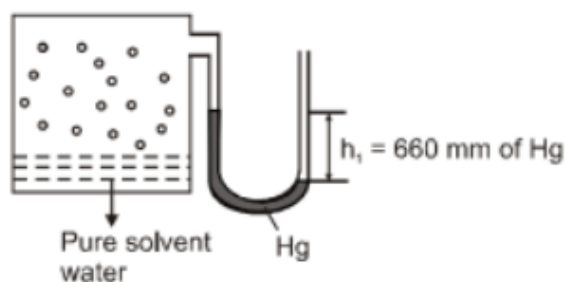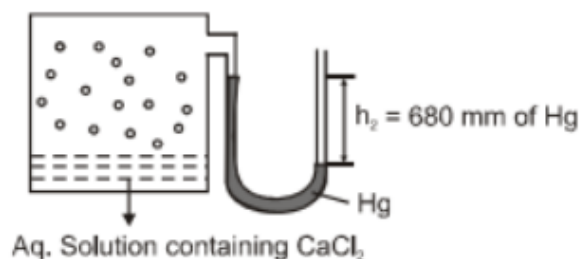
Hg

Aq. Solution containing $CaCl_2$

Arrangement -II

The difference in height of the mercury column in two arms of U tube manometer in arrangement-I is h_1 = 660 mm . In another arrangement-II at same temperature, 222gm of $CaCl_2$ is dissolved in 324gm of water and difference in height of mercury column in two arms is found to be h_2 = 680 mm . If the value of degree of dissociation for $CaCl_2$ in arrangement-II is α : [Atmospheric pressure = 1 atm]
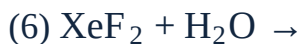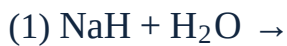
Then the value of 6.4 α is :

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Consider following reactions :

(1) $NaH + H_2O \rightarrow$

(2) $H_2S + Cl_2$ water $\rightarrow$

(3) $AgNO_3 + H_2O + H_3PO_2 \rightarrow$

(4) $(NH_4)_2Cr_2O_{7(s)} \xrightarrow{\Delta}$

(5) $H_2S + SO_{2\,(aq.)} \rightarrow$

(6) $XeF_2 + H_2O \rightarrow$

In how many of the reaction(s) at least one of the product is obtained in elemental state?

Question No. 4
Numerical Type
The answer has to be filled into the input box provided below.

Consider following reactions :

(1) $NaH + H_2O \rightarrow$

(2) $H_2S + Cl_2$ water $\rightarrow$

(3) $AgNO_3 + H_2O + H_3PO_2 \rightarrow$

(4) $(NH_4)_2Cr_2O_{7(s)} \xrightarrow{\Delta}$

(5) $H_2S + SO_{2\,(aq.)} \rightarrow$

(6) $XeF_2 + H_2O \rightarrow$
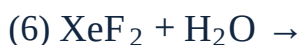
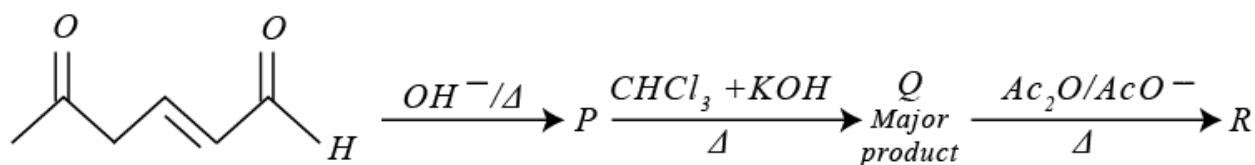In how many of the reaction(s) water is one of the products ?

Chemistry Multiple Correct (Maximum Marks: 28)

Question No. 1
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Observed the following sequence of reaction :

$$\xrightarrow{OH^-/\Delta} P \xrightarrow[\Delta]{CHCl_3 +KOH} \underset{\substack{Major \\ product}}{Q} \xrightarrow[\Delta]{Ac_2O/AcO^-} R$$

Which of the following is/are correct?

A. Product Q is

B. Product R is

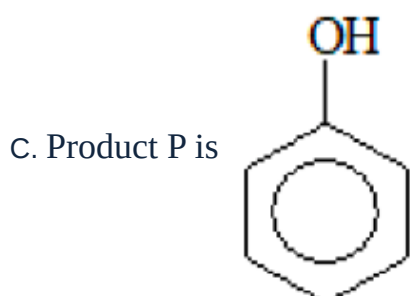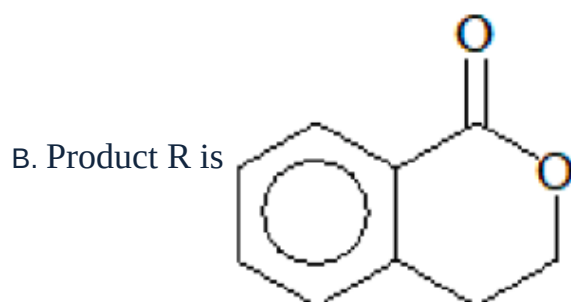C. Product P is

D. Reimer Tiemann reaction is involved

Question No. 2
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Which of the following tests are shown by Glucose

A. $2, 4 -$ DNP

B. Schiff Test

C. $NaHSO_3$

D. Tollen's

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Compound $(X)C_4H_8O$ decolourises Baeyer's reagent. It undergoes hydrolysis on reaction with dil. H_2SO_4 and produces (Y) and (Z). Both (Y) and (Z) give Iodoform test positive. Only (Y) gives Tollen's test positive. Choose the correct statements.

A. $(Y) \xrightarrow{\text{aq. NaOH(10\%) } \Delta} CH_3-CH=CH-CHO$

B. $(Z) \xrightarrow{\text{Pyridinium chlorochromate (PCC)}} (Y)$

C. $(X) \xrightarrow{Br_2/CCl_4}$ $CH_2-CH-O-CH_2-CH_3$ with Br and Br substituents

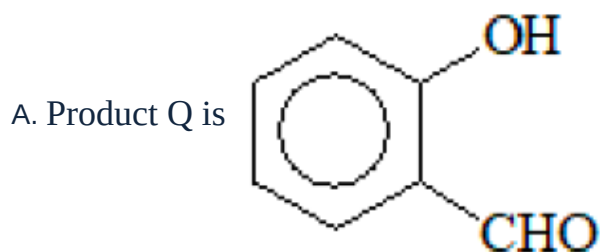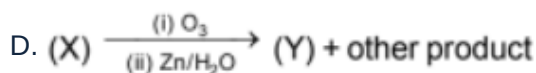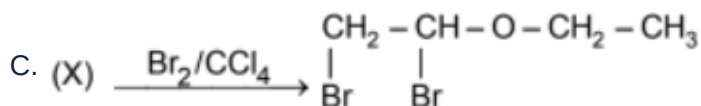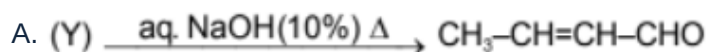D. $(X) \xrightarrow[\text{(ii) Zn/H}_2\text{O}]{\text{(i) O}_3} (Y) + \text{other product}$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

5 moles of NO_2 and 10 moles of N_2O_4 are taken in a vessel having total pressure of 2 atm . Which of the following statement is correct about the reaction,
$2NO_2(g) \rightleftharpoons N_2O_4(g)$

[Given: $\Delta G_f^{\circ}(N_2O_4) = 97.3$ kJ/ mole and $\Delta G_f^{\circ}(NO_2) = 51.5$ kJ/mole] ?

A. The reaction moves in forward direction.

B. The reaction moves in reverse direction.

C. K_P of the equilibrium reaction is 10.

D. Q_P of the reaction is 6.

Question No. 5

One or More Options Correct Type

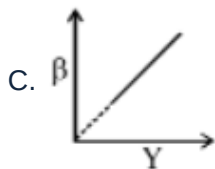The question has multiple options out of which ONE or MORE is/are correct.

Select the correct statement(s) regarding buffer solution :

$$CH_3COOH(X \text{ molar }) + CH_3COO^- Na^+ (Y \text{ molar })$$

A. At constant buffer concentration, buffer capacity increases to its maximum value if $\frac{Y}{X}$ approaches 1.

B. At constant buffer concentration, variation of index of buffer with Y is parabolic in nature.

C. Keeping $\frac{Y}{X} = 1$,β (buffer capacity) vs Y can be represented by

D. On dilution, pH of the buffer solution remains constant whereas its buffer capacity increases.

One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Consider following reactions/statements :

1. $Ca^{2+} + Na_2[H_2EDTA] \longrightarrow$ complex I

2. $Xe + PtF_6 \longrightarrow$ complex II

3. Anti-cancer drug cis-platin (complex III)

Which of the following statement(s) is/are CORRECT regarding above complexes ?

A. Complex I exists in only two stereoisomeric forms
B. Complex II has central atom in +4 oxidation state.
C. Complex III shows both geometrical and optical isomerism
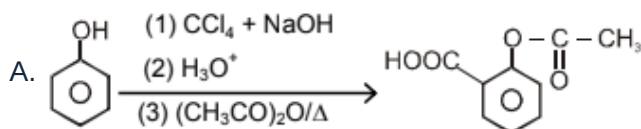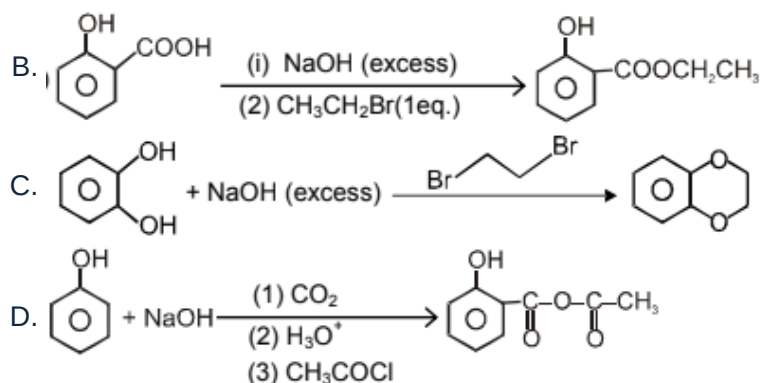D. I & III are diamagnetic and II is paramagnetic

One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

In which of the following reaction (s), reactant and products are correctly matched ?

A.

B.

$$\text{[salicylic acid derivative]} \xrightarrow[\text{(2) } CH_3CH_2Br(1eq.)]{\text{(i) NaOH (excess)}} \text{[}-COOCH_2CH_3\text{ derivative]}$$

C.

$$\text{[catechol derivative]} + \text{NaOH (excess)} \xrightarrow[\quad]{Br \quad Br} \text{[benzodioxane]}$$

D.

$$\text{[phenol derivative]} + \text{NaOH} \xrightarrow[\substack{\text{(2) } H_3O^+ \\ \text{(3) } CH_3COCl}]{\text{(1) } CO_2} \text{[}-C\text{-}O\text{-}C\text{-}CH_3\text{ derivative]}$$

Chemistry Numerical (Maximum Marks: 12)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

The number of stereoisomer formed by the given compound is
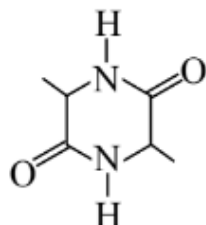
Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

In Industrial extraction, the number of metals produced by reduction of their oxides with carbon or with more active metal among the following is :

$$Fe, Li, Mg, Al, Pb, Ag, Cr, Zn$$

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

140 day old radioactive matter has an activity of 6000 dps. 70 days later it shows an activity of 2000 dps. Then ratio of its initial activity to activity when it was 140 day old is _____.

Mathematics Single Correct (Maximum Marks: 12)

Question No. 1

Let the set of complex numbers $(a_1, b_1), (a_2, b_2), (a_3, b_3) \ldots \ldots$ denoting the points on the complex plane satisfying $(a_{n+1}, b_{n+1}) = (\sqrt{3}a_n - b_n, \sqrt{3}b_n + a_n)$ for $n = 1, 2, 3, \ldots \ldots$ Suppose $(a_{100}, b_{100}) = (2, 4)$, then the value of $(a_1 + b_1)$ is equal to:

    A. $\dfrac{1}{2^{96}}$

    B. $\dfrac{1}{2^{97}}$

    C. $\dfrac{1}{2^{98}}$

    D. $\dfrac{1}{2^{99}}$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

If $10\sin^4\alpha + 15\cos^4\alpha = 6$ and the value of $9\csc^4\alpha + 4\sec^4\alpha$ is S, then the value of $\dfrac{S}{25}$.

    A. 1

    B. 2

    C. 3

    D. 4

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

The sum $S = \dfrac{1}{9!} + \dfrac{1}{3!7!} + \dfrac{1}{5!5!} + \dfrac{1}{7!3!} + \dfrac{1}{9!}$ equals

    A. $\dfrac{2^9}{10!}$

    B. $\dfrac{2^{10}}{8!}$

    C. $\dfrac{2^{11}}{9!}$

    D. $\dfrac{2^{10}}{7!}$

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Let $\{a_n\}$ be a sequence such that $a_0 = 1, a_1 = 0, a_n = 3a_{n-1} - 2a_{n-2}$. The correct statement is:

    A. $a_{45} = 2^{45}$

    B. $a_{51} = 2^{25} - 2$

C. $a_{48} = 2(1 - 2^{47})$

D. $a_{49} = \sqrt{2} - \sqrt{2^{49}}$

Mathematics Question Stem (Maximum Marks: 8)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Paragraph:

Let $f(x)$ be a polynomial of degree 3 such that $f(0) = 1, f(1) = 2$ and zero is a critical point of $f(x)$ having no local extreme.

Question:

If the value of definite integral $\int_{-1}^{1} \frac{f(x)}{\sqrt{x^2+7}} dx$ is equal to $2\ln(\frac{\sqrt{a}+1}{\sqrt{b}+c})$ then the value of $(a + b + c)$, is:

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Paragraph:

Let $f(x)$ be a polynomial of degree 3 such that $f(0) = 1, f(1) = 2$ and zero is a critical point of $f(x)$ having no local extreme.

Question:

The value of $\lim_{x \to 0} (f(x))^{\frac{1}{\tan x - x}}$ is equal to: e^a. find the value of a

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Paragraph:

Let $M_n = [M_{ij}]$ be a $n \times n$ matrix such that

$$M_{ij} = \begin{cases} a, i = j \\ 1, |i - j| = 1 \\ 0, \text{ otherwise} \end{cases}$$

Let D_n denotes the determinant value of matrix M_n.

Question:

If $a = 2$ then unit's place digit of D_{2017} is

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Paragraph:

Let $M_n = [M_{ij}]$ be a $n \times n$ matrix such that

$$M_{ij} = \begin{cases} a, i = j \\ 1, |i - j| = 1 \\ 0, \text{ otherwise} \end{cases}$$

Let D_n denotes the determinant value of matrix M_n.

Question:

If $a = 1$ then $\sum_{k=1}^{2017} |D_k|$ is ? (where |.| denotes absolute value function)

Mathematics Multiple Correct (Maximum Marks: 28)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

$\vec{a}, \vec{b}, \vec{c}$ are three coplanar unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$. If three vectors $\vec{p}, \vec{q}$ and $\vec{r}$ are parallel to $\vec{a}, \vec{b}$ and $\vec{c}$ respectively and have integral but different magnitudes from each other, then $|\vec{p} + \vec{q} + \vec{r}|$ can take a value equal to:

A. $\sqrt{3}$

B. 2

C. 1

D. 0

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The equation $a_8 x^8 + a_7 x^7 + a_6 x^6 + \dots + a_0 = 0$ has all its roots positive and real.

$(a_8 = 1, a_7 = -4, a_0 = \frac{1}{2^8})$, then

A. $a_1 = \frac{-1}{2^4}$

B. $a_1 = \frac{-1}{2^8}$

C. $a_2 = \frac{7}{2^4}$

D. $a_3 = \frac{1}{2^2}$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If p, q, r be real, then $f(x) = \begin{vmatrix} x + p^2 & pq & pr \\ pq & x + q^2 & qr \\ pr & qr & x + r^2 \end{vmatrix}$, then $f(x)$

A. increases in interval $(0, \infty)$

B. decreases in interval $(\frac{-2(p^2+q^2+r^2)}{3}, 0)$

C. increases in interval $(-\infty, \frac{-2(p^2+q^2+r^2)}{3})$

D. increases in the interval $(-\frac{2(p^2+q^2+r^2)}{3}, \infty)$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Number of six digit natural numbers made from $1, 2, 3, 4$ whose sum of digits is equal to 10 is equal to

A. number of arrangements of 6 different flowers in a circle.

B. number of ways of distributing 8 oranges among three boys so that each get at least one orange

C. number of positive integral solution of $xyz = 150$

number of four digits numbers formed using the digits $1, 2, 3, 4, 5$ without
D. repetition.

Question No. 5
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

The number of integral values of k for which exactly three roots of the equation
$\sin^2 x + (\cos x - 1) \sin x - \cos x - k \sin x + k = 0, x \in (0, 2\pi)$ are real, is

A. coprime with 15
B. odd
C. even
D. less than 5

Question No. 6
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

Let E_1 and E_2 be two ellipse $\frac{x^2}{a^2} + y^2 = 1$ and $x^2 + \frac{y^2}{a^2} = 1$ (where a is a parameter).
Then, the locus of the points of intersection of the ellipse E_1 and E_2 is a set of curves
comprising

A. Two straight lines
B. One straight line
C. One circle
D. One parabola

Question No. 7
One or More Options Correct Type
The question has multiple options out of which ONE or MORE is/are correct.

A point is moving on the circle $(x - 4)^2 + (y - 8)^2 = 20$, then it broke away from it
and moving along a tangent to the circle cut the x-axis at point $(-2, 0)$. The
coordinates of the point on the circle at which the moving point broke away can be

A. $\left(\frac{42}{5}, \frac{36}{5} \right)$
B. $\left(-\frac{2}{5}, \frac{44}{5} \right)$
C. $(6, 4)$
D. $(2, 4)$

Mathematics Numerical (Maximum Marks: 12)

Question No. 1
Numerical Type
The answer has to be filled into the input box provided below.

Suppose $a, b \in R$ such that all the roots of the equation $(x^2 + 1)x = ax^2 + b$ are real and positive, then the minimum possible value of a^2, is

Numerical Type
The answer has to be filled into the input box provided below.

The solution of the differential equation $(x - 1)dy + ydx = x(x - 1)y^{\frac{1}{3}} dx$ is given by $y^{\frac{2}{3}} = A(x - 1)^{-\frac{2}{3}} + \frac{1}{4}(x - 1)^2 + \frac{2}{\lambda}(x - 1)$, then the value of λ should be

Numerical Type
The answer has to be filled into the input box provided below.

Let $f(x) = \max(|5 - x|, 7^{-|x|})$. If α is the number of points where $f(x)$ is discontinuous and β be the number of points where $f(x)$ is non-differentiable, then find $(\alpha + \beta)$.