

Question Paper

Physics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A wave front is represented by the plane $y = 3 - x$. The propagation of wave takes place at

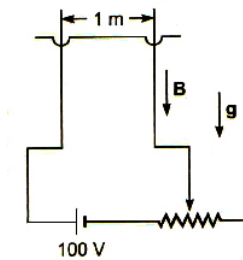
- A. 45° with the positive x-direction
- B. 135° with the positive x-direction
- C. 60° with the positive x-direction
- D. No sufficient data

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A horizontal rod of mass 2 kg is kept touching two vertical parallel rough rails, carrying current. There is a magnetic field $B = 2\text{ T}$ present vertically downward. The rails are connected to battery of 100 V at $t = 0$. The resistance of the circuit is $5\ \Omega$ and starts to increase at constant rate $0.5\ \Omega/\text{s}$. The coefficient of friction between the rails and rod is $\mu = \frac{3}{4}$ ($g = 10\ \text{m/s}^2$ and separation between the rails in 1 m). Then,



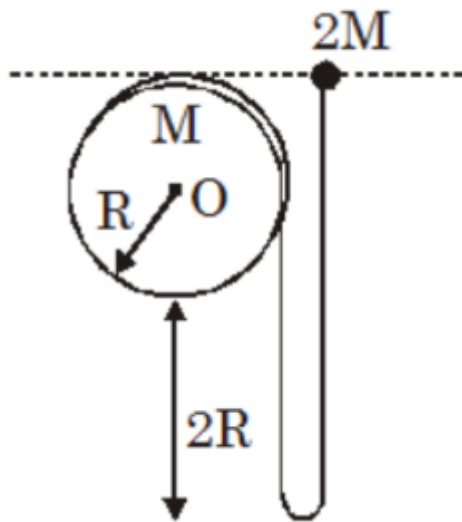
- A. The friction force acting on the rod at $t = 0$ is 20 N
- B. The friction force acting on the rod at $t = 0$ is 30 N
- C. Acceleration of rod at $t = 0$ is $5\ \text{m/s}^2$
- D. At $t = 5\text{ s}$, rod is about to start moving

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A light string is wrapped on a cylindrical shell and a fraction of length of string is unwrapped. A particle of mass $2M$ is attached on another end of string as shown. The system is kept in a vertical plane and cylinder can freely rotate about the axis of cylinder. Particle is released as shown in figure. Assuming there is no slipping between cylinder and string. Choose the correct option(s).



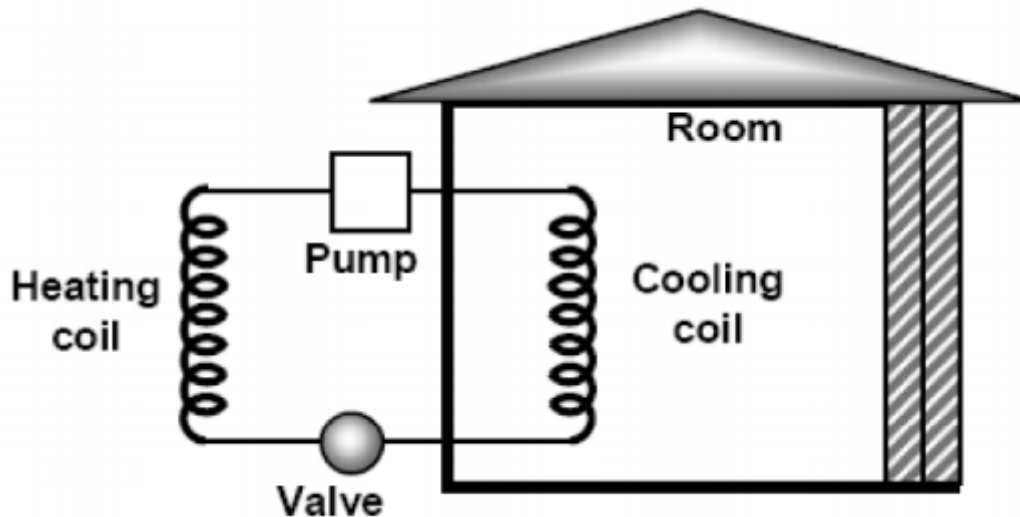
- A. The angular velocity of cylinder just after string taut is $\frac{8}{3} \sqrt{\frac{g}{R}}$.
- B. The angular velocity of cylinder just after string taut is $\frac{16}{3} \sqrt{\frac{g}{R}}$.
- C. Velocity of particle just after string taut is $\frac{8}{3} \sqrt{gR}$.
- D. Velocity of particle just after string taut is $\frac{16}{3} \sqrt{gR}$.

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A room air conditioner is modelled as a heat engine run in reverse. An amount of heat Q_{in} is absorbed from the room at a temperature T_L by the cooling coils which contains a working gas; this gas is compressed adiabatically to a temperature T_H ; then the gas is compressed isothermally in a coil outside of the house, giving off an amount of heat Q_{out} then the gas expands adiabatically back to a temperature T_L and finally expanded isothermally to original state and the cycle is repeated. An amount of energy W is input into the system in every cycle through an electric pump. This model describes the air conditioner with the best possible efficiency.



The outside air temperature is T_H and the inside air temperature is T_L . The air conditioner unit consumes electric power P . Assume that the air is dry sufficiently so that no condensation of water occurs in the cooling coils of the air conditioner. Which of the following may be the rate at which heat is removed from the room?

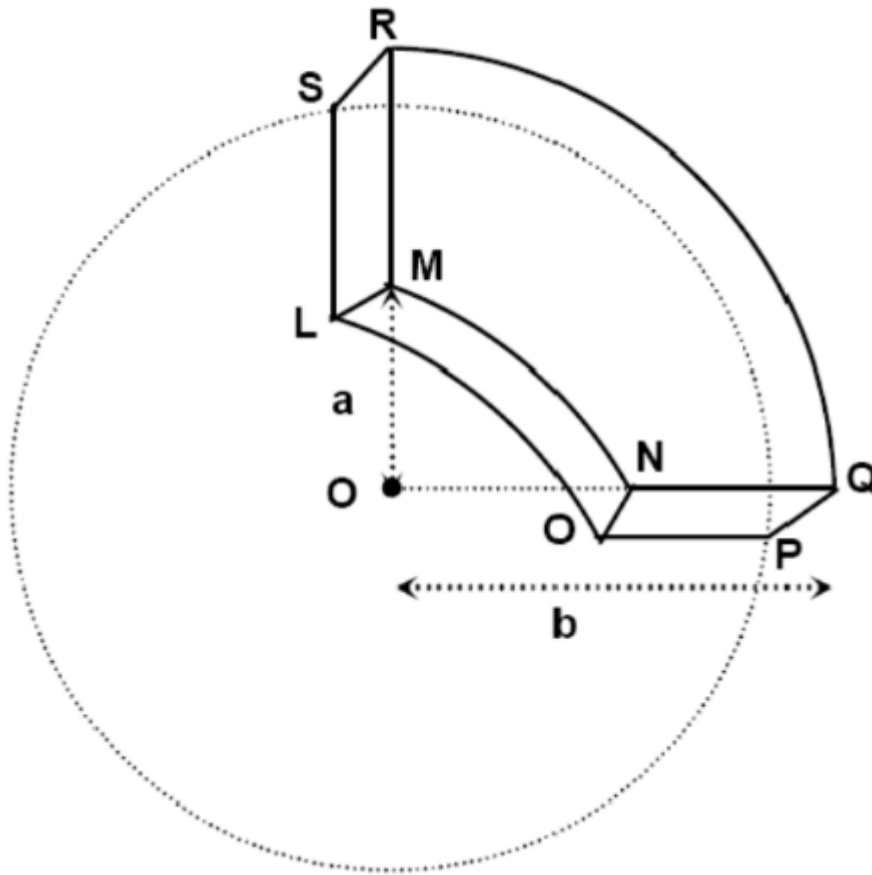
- A. $\frac{P}{2} \left(\frac{T_L}{T_H - T_L} \right)$
- B. $\frac{3P}{4} \left(\frac{T_L}{T_H - T_L} \right)$
- C. $P \left(\frac{T_L}{T_H - T_L} \right)$
- D. $\frac{3P}{2} \left(\frac{T_L}{T_H - T_L} \right)$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A conducting medium is shaped in a form of a quarter of an annulus of radii b and a ($b > a$) and thickness t . Specific resistance of the medium is ρ . The correct option(s) is/are



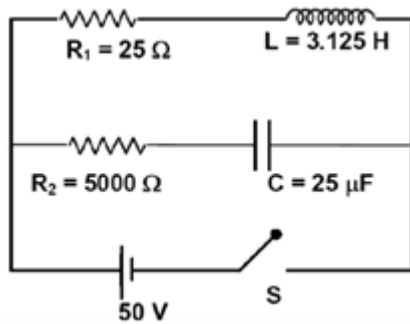
- A. Resistance between faces LMNO and PQRS is $\frac{2\rho}{\pi t} \ln \frac{b}{a}$.
- B. Resistance between faces LMRS and ONQP is $\frac{\pi\rho}{2t \ln(\frac{b}{a})}$.
- C. Resistance between faces MRQN and LSPO is $\frac{4t\rho}{b^2 - a^2}$.
- D. Between faces LMNO & PQRS and LMRS & ONQP, resistance is same.

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two resistors, one inductor and a capacitor are connected in a circuit with a battery and switch as shown in figure. The charge on capacitor and current in inductor is zero at $t = 0$. Now switch is closed at $t = 0$, then choose the correct option(s).



The ratio of maximum current in the branch of inductor to the branch of
A. capacitor is 200 .

At $t = \frac{\ln 2}{8}$, the current in the branch of capacitor is 200 times the current in
B. the branch of inductor.

At $t = \frac{1}{8} \ln\left(\frac{5}{4}\right)$, the potential drop across inductor is 4 times the potential
C. drop across capacitor.

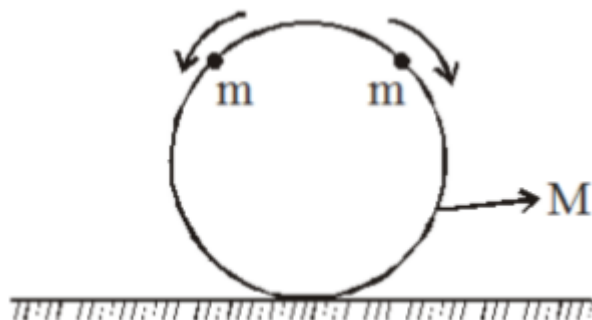
D. The potential drop across inductor is maximum at $t = 0$.

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two beads of mass m are positioned at the top of a frictionless hoop of mass M and radius R , which stands vertically on ground. The beads are now given tiny kicks, and they slide down the hoop, one to the right and one to the left, as shown in figure. Then choose the correct options



The contact force from the ground immediately after the tiny kicks on the
A. hoop is $(M + 2m)g$

The contact force from the ground on the hoop is $(M + 2m)g$ at the instant
B. when radius vector makes an angle of $\cos^{-1}(2/3)$ with the upward vertical.

The smallest value of $\frac{m}{M}$ for which the hoop will rise off the ground at some
C. time during the motion is $\frac{3}{2}$.

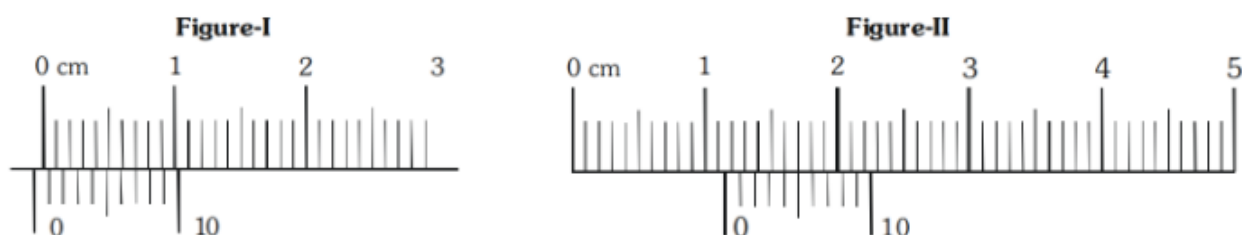
The hoop will never rise off the ground irrespective of the values of m and D . M .

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In ordinary Vernier calipers, 10^{th} division of the Vernier scale coincides with 9^{th} division of the main scale. In a specially designed Vernier calipers the Vernier scale is so constructed that 10^{th} division on it coincides with 11^{th} division on the main scale. Each division on the main scale equals to 1 mm. The calipers have a zero error as shown in the figure-I. When the Vernier caliper is used to measure a length, the concerned portion of its scale is shown in figure-II.



- A. Zero error in the calipers has magnitude 0.7 mm.
- B. The length being measured is 1.08 cm.
- C. The length being measured is 1.22 cm.

Though the given Vernier scale does not follow the principle of Vernier scale, yet can be used satisfactorily.

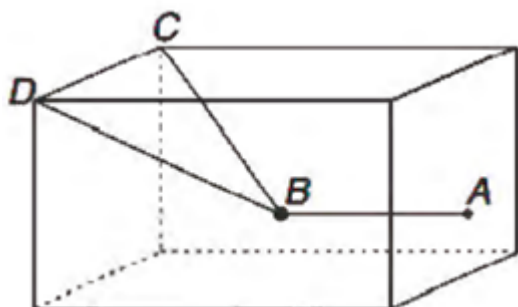
Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

A room is in shape of a cube. A heavy ball (B) is suspended at the centre of the room tied to three inextensible strings as shown. String BA is horizontal with A being the centre point of the wall. Find the ratio of tension in the string BA and BC .



Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

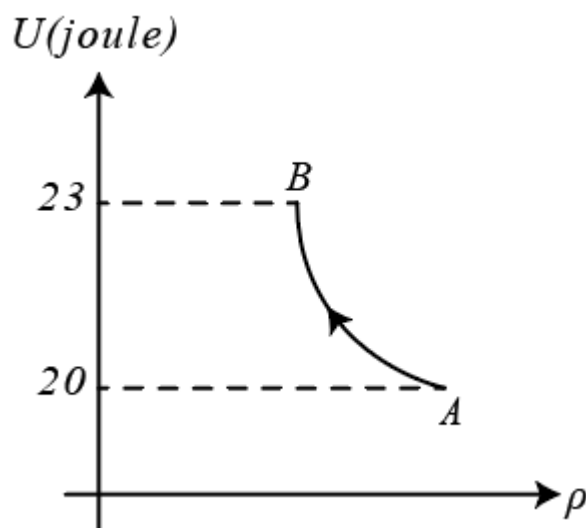
The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B , respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Graph shows variation of internal energy U with density ρ of one mole of an ideal monoatomic gas. Process AB is a part of rectangular hyperbola. Find work done in the process (in Joules)



Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

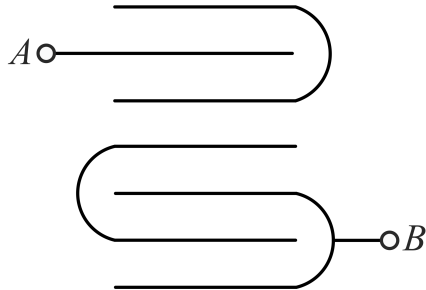
A vessel of volume V_0 is evacuated by means of a piston air pump. One piston stroke captures the volume $\Delta V = 0.2 V_0$. If process is assumed to be isothermal then find the minimum number of strokes after which pressure in the vessel becomes $\left(\frac{1}{1.728}\right)(P_{\text{initial}})$.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Seven identical large conducting plates are placed parallel to each other. The separation between any two consecutive plates is a . Area of each plate is S . The effective capacitance of the system between A and B is $\frac{x}{y} \frac{\epsilon_0 S}{a}$. Find the value of y if x is a single digit integer. Then represent it as the sum of digits of y .



Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Two radioactive samples A_1 and A_2 having half life 3 years and 2 years respectively have been decaying for many years. Today the number of atoms in the sample A_1 is twice the number of atoms in the sample A_2 . Both the samples had same number of atoms X years ago, calculate X .

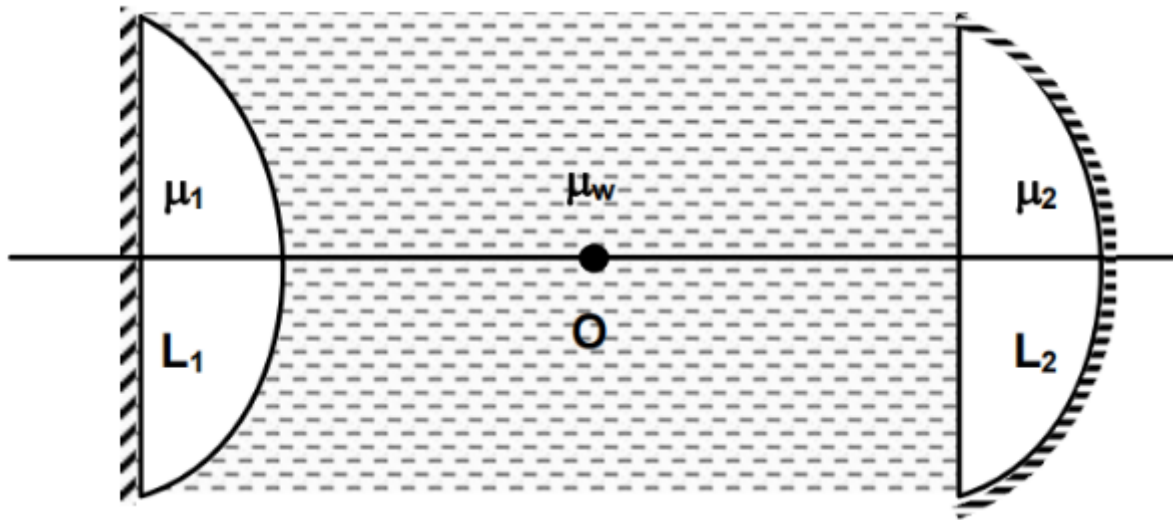
Physics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A cylindrical tube filled with water ($\mu_w = 4/3$) is closed at its both ends by two silvered plano convex lenses as shown in the figure. Refractive index of lenses L_1 and L_2 are 2.0 and 1.5 while their radii of curvature are 5 cm and 9 cm respectively. A point object is placed somewhere at a point O on the axis of cylindrical tube. It is found that the object and image coincide each other.



The position of object w.r.t. lens L_1 is

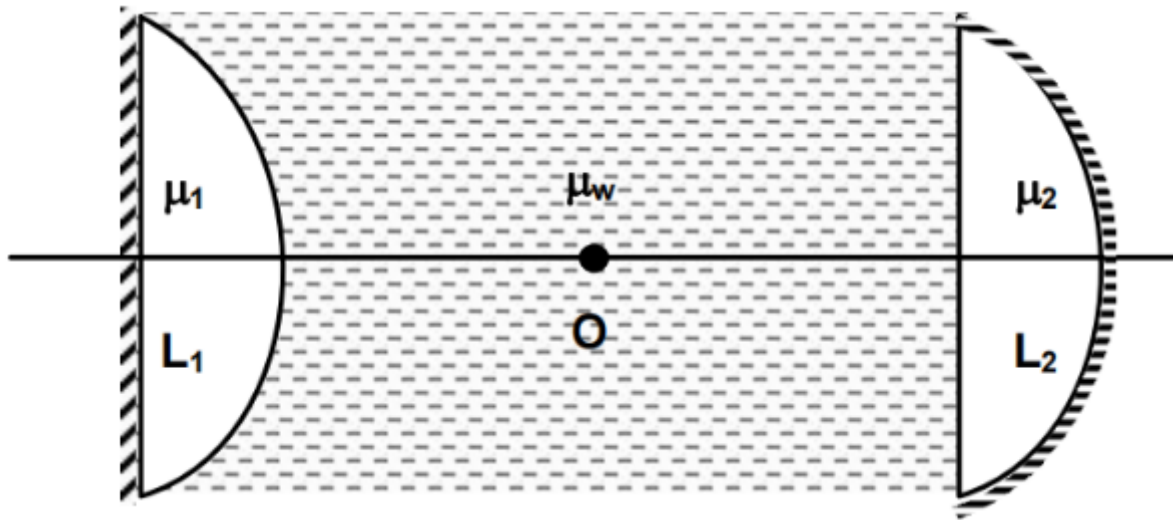
- A. 8 cm
- B. 10 cm
- C. 12 cm
- D. 14 cm

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A cylindrical tube filled with water ($\mu_w = 4/3$) is closed at its both ends by two silvered plano convex lenses as shown in the figure. Refractive index of lenses L_1 and L_2 are 2.0 and 1.5 while their radii of curvature are 5 cm and 9 cm respectively. A point object is placed somewhere at a point O on the axis of cylindrical tube. It is found that the object and image coincide each other.



The position of object w.r.t. lens L_2 is

- A. 8 cm
- B. 10 cm
- C. 12 cm
- D. 14 cm

Question No. 3

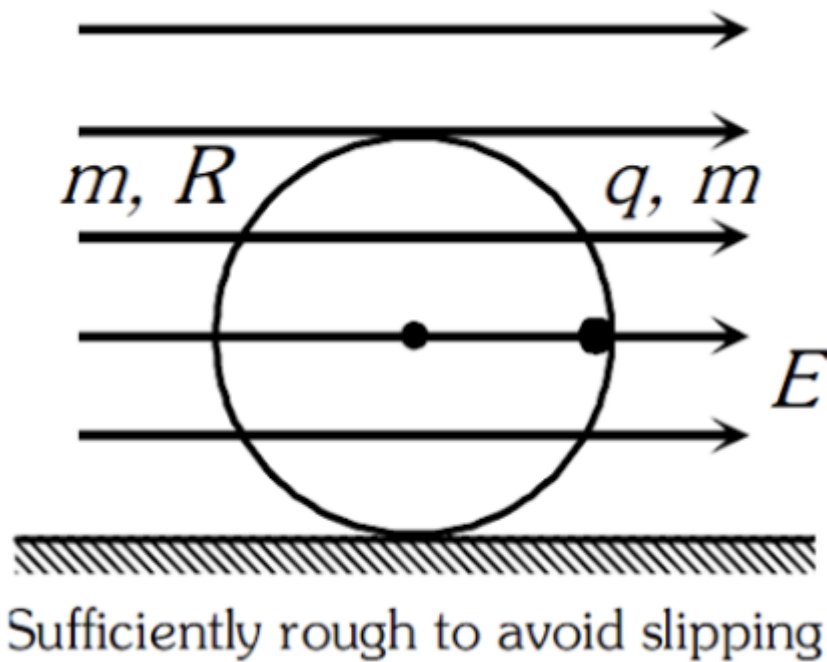
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A particle of charge q and mass m is clamped rigidly at the circumference of a ring with mass m and radius R . Initially ring is in vertical plane resting on a sufficiently rough horizontal surface with charge q at the same horizontal level as that of the centre of the ring. There exists uniform horizontal electric fields as shown.

At $t = 0$ the system is let free. (Given that $qE = mg$, $\pi = \frac{22}{7}$)

Sufficiently rough to avoid slipping



The magnitude of the friction force at $t = 0$ is

- A. mg
- B. $\sqrt{2}mg$
- C. 0
- D. $\frac{mg}{2}$

Question No. 4

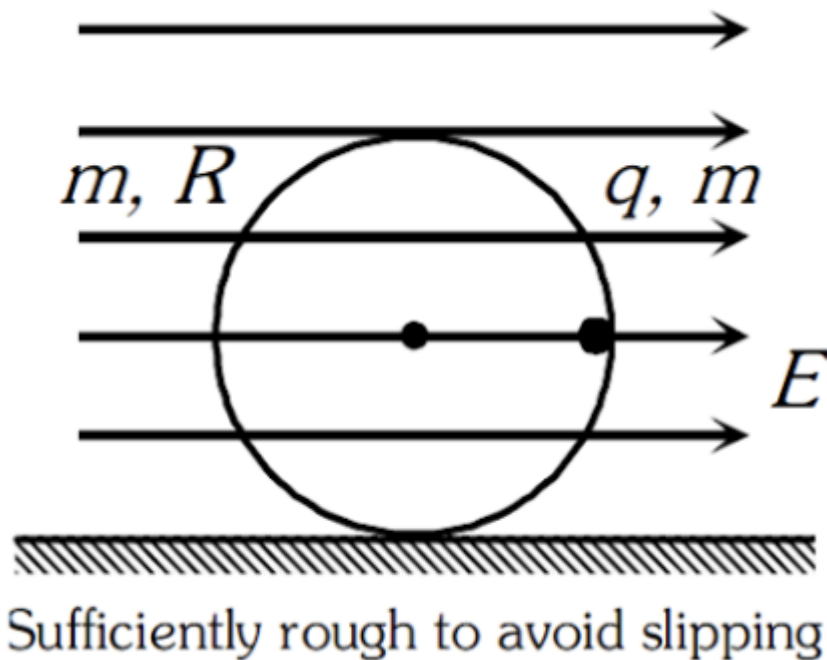
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A particle of charge q and mass m is clamped rigidly at the circumference of a ring with mass m and radius R . Initially ring is in vertical plane resting on a sufficiently rough horizontal surface with charge q at the same horizontal level as that of the centre of the ring. There exists uniform horizontal electric fields as shown.

At $t = 0$ the system is let free. (Given that $qE = mg$, $\pi = \frac{22}{7}$)

Sufficiently rough to avoid slipping



Work done by the electric field when the ring has rotated through 90° is

- A. $\frac{2}{7} mgR$
- B. $\frac{4}{7} mgR$
- C. $-mgR$
- D. $mg\sqrt{2}R$

Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A cubic crystalline ionic solid AB is like rock salt structure in which B^- ion is present at body centre. If all the ions along one axis passing through opposite face centres are removed, then which of the following statements is/are correct resulting crystalline solid.

A. Formula of resulting crystalline solid is $A_2 B$

Minimum distance between B^- ion in the resulting crystal is equal to $\frac{1}{\sqrt{2}}$

B. times of edge length of unit cell

C. Number of nearest B ion of B^- ion is ' 8 ' in the resulting crystal

D. Percentage decrease in density is 25%

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which of the following combination will liberate hydrogen gas?

A. Pb metal and dil. HNO_3

B. Cu metal and dil. H_2SO_4

C. Al metal and aq. NaOH

D. Zn metal and conc. H_2SO_4

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An ammonia-ammonium chloride buffer has a pH value of 9 with $[NH_3] = 0.25M$. What will be the new pH if 500ml 0.1MKOH is added to 200ml buffer solution ($K_b = 2 \times 10^{-5}$)

$[\log 2 = 0.3]$

A. 8.4

B. 9.6

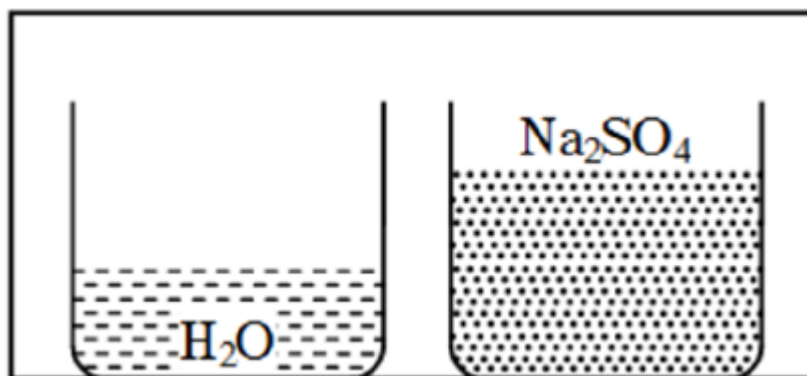
C. 5.6

D. 4.4

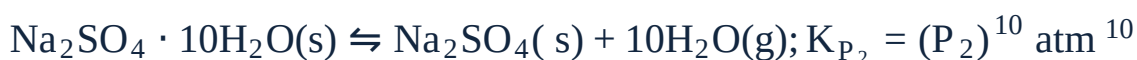
Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.



Consider an previously evacuated vessel containing two separate vessels with $\text{H}_2\text{O}(\text{l})$ and $\text{Na}_2\text{SO}_4(\text{s})$ as shown in the figure. Given that :



Then: (Consider constant temperature throughout)

- A. P_1 must be equal to P_2
- B. If $P_1 < P_2$, no Na_2SO_4 would be converted to $\text{Na}_2\text{SO}_4 \cdot 10\text{H}_2\text{O}$ at all
- C. If $P_1 > P_2$, all the $\text{H}_2\text{O}(\text{l})$ must dry up eventually
- Partial pressure of water vapour in the vessel finally (P) must lie in the
- D. range $P_2 \leq P \leq P_1$ if $P_1 > P_2$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An aq. solution containing excess of CuSO_4 is electrolysed for 10 minutes with a current of 1.5 amperes. Loss in mass of the solution is [$\text{Cu} = 64, \text{O} = 16$]

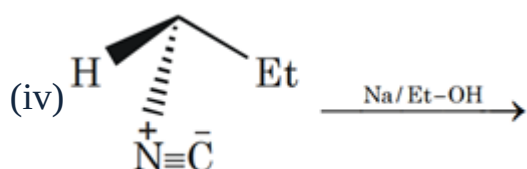
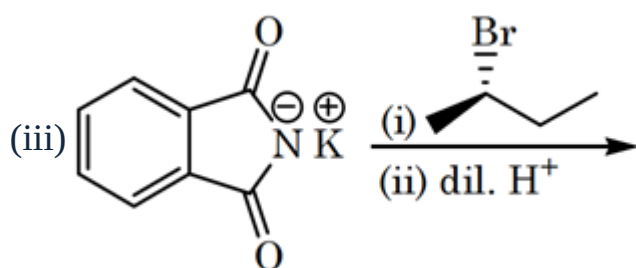
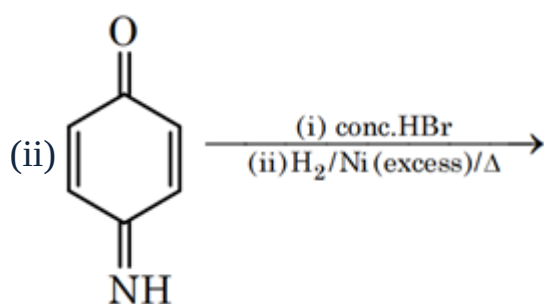
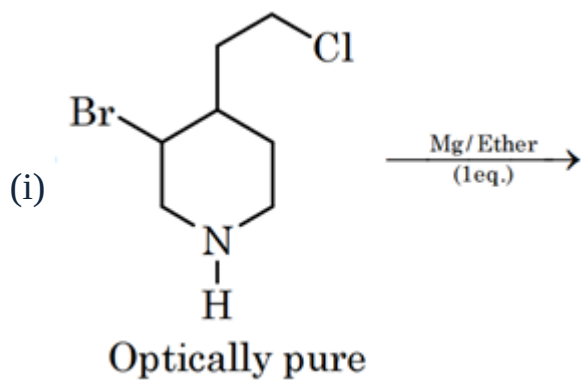
- A. 0.294gm
- B. 0.373gm
- C. 1.073gm
- D. 1.443gm

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Identify which reaction(s) produce(s) single optically active Amine molecule as product



- A. ii, iii, iv
 B. iii only
 C. iii , iv
 D. none of these

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Assuming all non-hydrogenic atoms undergo hybridization & following are various categories of orbital overlapping.

$sp^3 - sp^3$ axial overlap (σ) \longrightarrow Type - A

$sp^2 - sp^2$ axial overlap (σ) \longrightarrow Type - B

$sp^3 - sp^2$ axial overlap (σ) \longrightarrow Type - C

p - p lateral overlap (π) \longrightarrow Type - D

p - d lateral overlap (π) \longrightarrow Type - E

Identify the species which contain only 2 of these types (from A to E) of overlapping of orbitals.

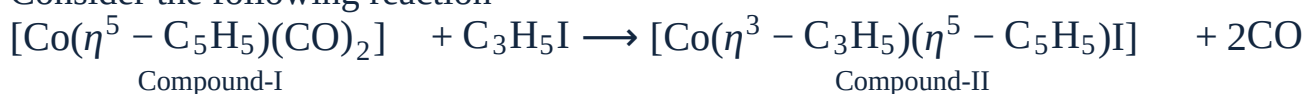
- A. H_2CO_3
- B. $B_3N_3H_3F_3$
- C. SNF_3
- D. $H_2S_4O_6$

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider the following reaction



Which of the following is/are incorrect regarding the above reaction.

- A. I and II have same value of EAN
- CO formed in the reaction can be used to increase rate of reaction in
- B. Haber's process.
- C. I and II both consist of trihapto ligands.
- D. The above reaction is redox reaction

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Total number of compounds which are soluble in a hot aqueous NaOH are:

- (i) Salicyclic acid
- (ii) Aspirine
- (iii) Formic acid
- (iv) Acetic acid
- (v) Succinic anhydride
- (vi) Cyclohexanone
- (vii) Benzene sulphonamide
- (viii) Cyclohexene

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

The resistance of a solution A is 50Ω and that of solution B is 100Ω . Both the solutions are taken in same conductivity cell. If equal volumes of solution A and B are mixed, what is the resistance of the mixture using the same cell? (Assume there is no change or increase in the α of A and B on mixing.) (mark answer to nearest integer).

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

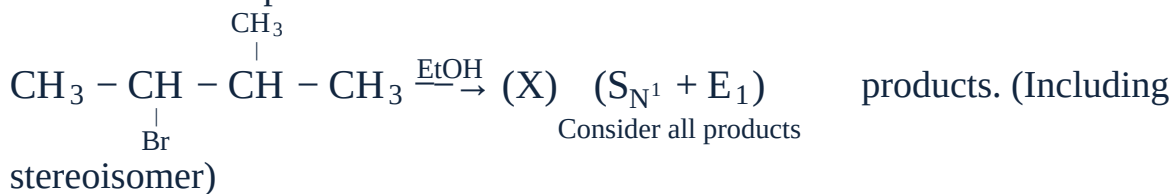
The amino acid glycine (HG) $\text{H}_2\text{N}-\text{CH}_2-\text{COOH}$ can acquire an additional proton to form $\text{NH}_3^+-\text{CH}_2-\text{COOH}$ (H_2G^+) and resulting cation is diprotic $\text{p}K_{a1} = 3$ and $\text{p}K_{a2} = 9$ for successive proton losses at a time. Percentage of glycine that exists in cationic form at equilibrium in $[\text{HG}] = 10^{-2} \text{ M}$, $[\text{H}_2\text{G}^+]$ is 10^{-x} , the value of x will be:

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

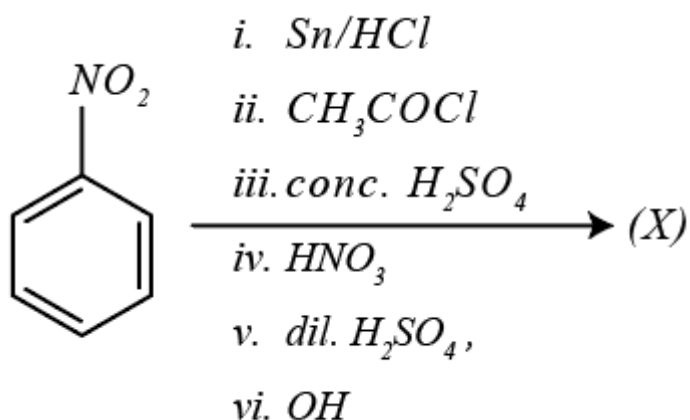
Total number of products are :



Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.



Find the percent of nitrogen in product : [Atomic mass: H = 1, C = 12, N = 14, O = 16, S = 32]

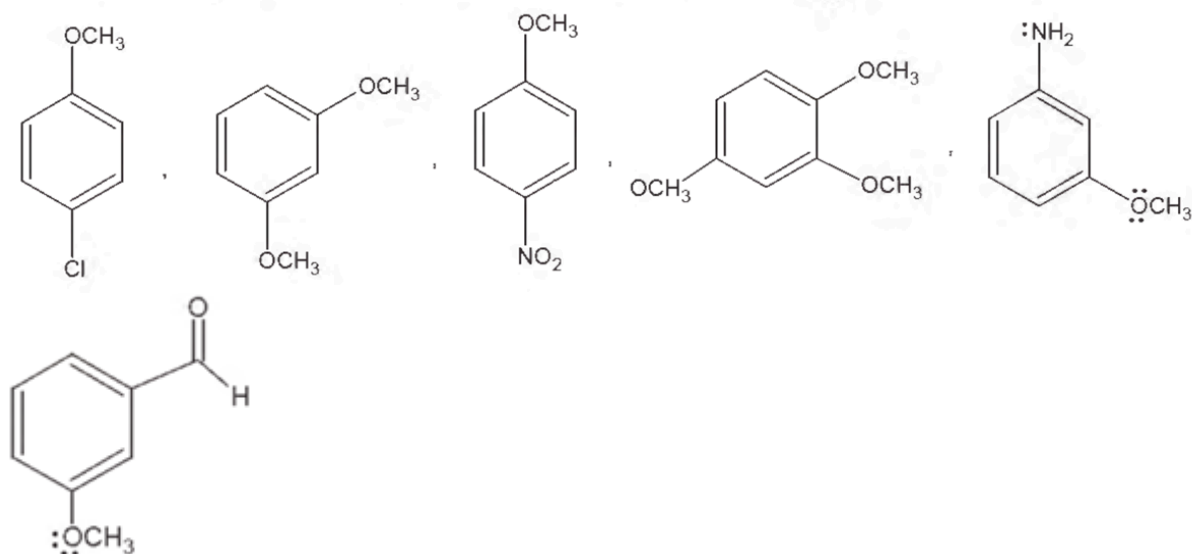
Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Examine the structural formulas shown below and identify how many compounds will show coupling reaction with diazonium salts faster than anisole

(Ph – $\ddot{\text{O}}$ – CH₃)



Chemistry Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Passage:

The shape of transition metal complexes is determined by the tendency of electron pairs to occupy positions as far as possible. In addition the shapes are also affected by whether d-orbitals are symmetrically filled or unsymmetrically filled. In an octahedral complex ligands split the d-orbitals into t_{2g} & e_g levels. There is a repulsion between d-electrons & the ligands. If the electrons are symmetrically arranged they will repel all six ligands equally giving a completely regular octahedron. And in case electrons are asymmetrically arranged they will repel some ligands more causing structure to be distorted. Based on the above paragraph, answer the questions that follow.

Question:

Which of the following configurations/combinations are expected to be distorted in octahedral complexes.

- A. d^5 configuration with weak ligand
- B. d^8 configuration with weak ligand
- C. d^4 configuration with weak ligand

D. d^3 configuration with weak ligand

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Passage:

The shape of transition metal complexes is determined by the tendency of electron pairs to occupy positions as far as possible. In addition the shapes are also affected by whether d-orbitals are symmetrically filled or unsymmetrically filled. In an octahedral complex ligands split the d-orbitals into t_{2g} & e_g levels. There is a repulsion between d-electrons & the ligands. If the electrons are symmetrically arranged they will repel all six ligands equally giving a completely regular octahedron. And in case electrons are asymmetrically arranged they will repel some ligands more causing structure to be distorted. Based on the above paragraph, answer the questions that follow.

Question:

Which of the following statements is NOT CORRECT.

- A. Octahedral complexes of Cu^{2+} are expected to show these distortions.
In octahedral complexes if the electrons are asymmetrically arranged in t_{2g} then the distortions will be more as compared to when the asymmetric arrangement is in e_g .
- B. Formation of square-planar complexes by $Pt(+2)$ can be attributed to significant distortions.
- C. In CrF_6^{4-} all the Cr – F bond lengths will not be same.

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Passage:

The reaction $2\text{NO}(\text{g}) + \text{H}_2(\text{g}) = \text{N}_2\text{O}(\text{g}) + \text{H}_2\text{O}(\text{g})$ follows the rate law

$$\frac{dp(\text{N}_2\text{O})}{dt} = k(p_{\text{NO}})^2 p_{\text{H}_2}$$

The following data have been obtained.

<i>Expt. No.</i>	$\frac{(p_{\text{NO}})_0}{\text{mmHg}}$	$\frac{(p_{\text{H}_2})_0}{\text{mmHg}}$	$\frac{\text{Half - life time}}{s}$	$\frac{\text{Temperature}}{^\circ\text{C}}$
1.	600	10	19.2	827
2.	600	20	?	827
3.	10	600	830	827
4.	20	600	?	827
5.	600	10	9.6	847

Based on the information, answer the following question:

Question:

The values of half-life time not shown above in the run 2 and 4 , respectively, are

- A. 19.2 s, 8.30 s
- B. 19.2 s, 415 s
- C. 38.4 s, 830 s
- D. 38.4 s, 415 s

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Passage:

The reaction $2\text{NO}(\text{g}) + \text{H}_2(\text{g}) = \text{N}_2\text{O}(\text{g}) + \text{H}_2\text{O}(\text{g})$ follows the rate law

$$\frac{dp(\text{N}_2\text{O})}{dt} = k(p_{\text{NO}})^2 p_{\text{H}_2}$$

The following data have been obtained.

<i>Expt. No.</i>	$\frac{(p_{\text{NO}})_0}{\text{mmHg}}$	$\frac{(p_{\text{H}_2})_0}{\text{mmHg}}$	$\frac{\text{Half - life time}}{s}$	$\frac{\text{Temperature}}{^\circ\text{C}}$
1.	600	10	19.2	827
2.	600	20	?	827
3.	10	600	830	827
4.	20	600	?	827
5.	600	10	9.6	847

Based on the information, answer the following question:

Question:

The rate constant of the reaction is

- A. $10^{-5} \text{ mmHg}^{-2} \text{ s}^{-1}$
- B. $10^{-6} \text{ mmHg}^{-2} \text{ s}^{-1}$
- C. $10^{-7} \text{ mmHg}^{-2} \text{ s}^{-1}$
- D. $10^{-8} \text{ mmHg}^{-2} \text{ s}^{-1}$

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x) = \sin^{-1}\left(\frac{4-x^2}{4+x^2}\right) + \cos^{-1}\left(\frac{4x}{4+x^2}\right)$, then

- A. total number of local extremum of $f(x)$ in $(-4, 4)$ are 1
- B. total number of local maximum of $f(x)$ in $(-4, 4)$ are zero
- C. maximum value of $f(x)$ in $(-4, 4)$ is π
- D. range of $f(x)$ in $(-4, 4)$ is $(-\frac{\pi}{2}, \pi]$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x)$ and $g(x)$ are polynomials of degree 3, where

$$g(\alpha) = g'(\alpha) = 0, g''(\alpha) \neq 0 \quad \text{and} \quad \lim_{x \rightarrow \alpha} \frac{f(x)}{g(x)} = 0.$$

If $h(x) = f(x)g'(x) + f'(x) \cdot g(x)$, then which of the following is/are true ?

Number of solution(s) which are common to all

A. $f(x) = 0, g(x) = 0, h(x) = 0$ is 1 .

Number of solution(s) which are common to all

B. $f(x) = 0, g(x) = 0, h(x) = 0$ is 2 .

C. Number of distinct roots of equation $h(x) = 0$ is 2 .

D. $h'''(\alpha) = 0$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider three unit vectors \vec{a}, \vec{b} and \vec{c} . Let the angle between vectors \vec{a} and \vec{b} is $\frac{\pi}{6}$, between vectors \vec{b} and \vec{c} is $\frac{\pi}{4}$ and between vectors \vec{c} and \vec{a} is $\frac{\pi}{3}$. The angle, the vector \vec{a} makes with the plane containing vectors \vec{b} and \vec{c} , is

A. $\cos^{-1} \sqrt{1 - \sqrt{\frac{2}{3}}}$

B. $\cos^{-1} \sqrt{2 - \sqrt{\frac{3}{2}}}$

C. $\sin^{-1} \sqrt{\sqrt{\frac{3}{2}} - 1}$

D. $\sin^{-1} \sqrt{\sqrt{\frac{2}{3}}}$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let \vec{x} and \vec{y} be unit vectors. If \vec{r} is a vector such that $\vec{r} + (\vec{r} \times \vec{x}) = \vec{y}$, then

A. maximum value of $\vec{r} \cdot \vec{x}$ is 1

B. maximum value of $|\vec{r} \cdot \vec{x} \times \vec{y}|$ is $\frac{1}{2}$

C. maximum value of $|\vec{r}|$ is 1

D. minimum value of $|\vec{r}|$ is $\frac{1}{\sqrt{2}}$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x)$ be a non constant twice derivable function defined on R such that $f(2+x) = f(2-x)$ and $f'(\frac{1}{2}) = 0 = f'(1)$, then minimum number of roots of the equation $f''(x) = 0$ in $(0, 4)$, is

- A. 4
- B. 3
- C. 2
- D. 5

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider $\int_0^{\pi} (\frac{8\sin^2 x}{g(x)} - 1)(g(x))^2 dx = 6\pi$, where $g(x)$ is a continuous positive function in $(0, \pi)$. If the maximum value of $g(x)$ in $(0, \pi)$ is M , then

- A. M is a perfect square number
- B. M is a prime number
- C. M is an even natural number
- D. M is an odd prime number

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The circle $|z| = 2$ intersects the curve whose equation is $z^2 = (\bar{z})^2 + 4i$ in the points A, B, C, D in Argand plane. If z_1, z_2, z_3, z_4 represents the affixes of these points, then which of following is/are incorrect.

- A. $z_1 z_2 z_3 z_4 = 1$
- B. $\bar{z}_1 + \bar{z}_2 + \bar{z}_3 + \bar{z}_4 = 0$
- C. $z_1 + z_2 + z_3 + z_4 = 2$
- D. $\arg(z_1) + \arg(z_2) + \arg(z_3) + \arg(z_4) = 2k\pi$, where $k = 0, 1$ or -1 .

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A line meets $\frac{x^2}{4a^2} + \frac{y^2}{a^2} = 1$ at A and B and the parabola $y^2 = 4ax$ at C and D. The segment AB subtends 90° at centre of ellipse. If the locus of point of intersection of tangents to the parabola at C and D is $px^2 + qy^2 = 4a^2$ then the value of $p + q$ is

- A. 3
- B. 2
- C. 5
- D. 4

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If the value of θ lying between 0 to $\frac{\pi}{2}$ and satisfying.

The equation $\begin{vmatrix} 1 + \sin^2 \theta & \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & 1 + \cos^2 \theta & 4 \sin 4\theta \\ \sin^2 \theta & \cos^2 \theta & 1 + 4 \sin 4\theta \end{vmatrix} = 0$ is given by $\frac{n\pi}{k}$,

where $\text{HCF}(n, k) = 1$, then the value of $\frac{k}{12}$ is given by

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^6}{1+t^6} dt = 3x - 1$ is

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

The coefficient of x^9 in the expansion $(1+x)(1+x^2)(1+x^3) \dots (1+x^{100})$ is

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Let A, B, C, D are 4 square matrices such that $A^T = BCD, B^T = CDA, C^T = DAB, D^T = ABC$.

If $M = ABCD$ such that $M^{2025} = M^n$ then least positive value of n is

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let $f : (0, \infty) \rightarrow (0, \infty)$ be a differentiable function such that $f'(\frac{a}{x}) = \frac{x}{f(x)}$ where a is positive constant and $f'(1) = 1, f'(2) = 2$, then $f(5)$ is equal to

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Let $f : (0, 1) \rightarrow (0, 1)$ be a differentiable function such that $f'(x) \neq 0 \forall x \in (0, 1)$ and $f(\frac{1}{2}) = \frac{\sqrt{3}}{2}$.

If $f(x) = \lim_{t \rightarrow x} \frac{\int_0^t \sqrt{1-f^2(s)} ds - \int_0^x \sqrt{1-f^2(s)} ds}{f(t)-f(x)}$ then the value of $f(\frac{1}{4}) = \frac{\sqrt{m}}{4}$ and $\int_0^1 f(x) dx = \frac{\pi}{n}$, $m, n \in N$. Find the value of $m - n$.

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Consider, sum of the series $\sum_{0 \leq i < j \leq n} f(i)f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^n \sum_{j=1}^n f(i) f(j) = \sum_{i=1}^n (f(i)(\sum_{j=1}^n f(j)))$ and i and j are independent. In this summation, three types of terms occur, those when $i < j$, $i > j$ and $i = j$.

Also, sum of terms when $i < j$ is equal to the sum of the terms when $i > j$ if $f(i)$ and $f(j)$ are symmetrical.

So, in that case

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n f(i)f(j) &= \sum_{0 \leq i < j \leq n} f(i)f(j) \\ &\quad + \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ &= 2 \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ \Rightarrow \sum_{0 \leq i < j \leq n} f(i)f(j) &= \frac{\sum_{i=0}^n \sum_{j=0}^n f(i)f(j) - \sum_{i=j} f(i)f(j)}{2} \end{aligned}$$

When $f(i)$ and $f(j)$ are not symmetrical, we find the sum by listing all the terms.

When $f(i)$ and $f(j)$ are not symmetrical, we find the sum by listing all the terms.

Question:

$\sum_{0 \leq i < j \leq n} \sum_{i=0}^n C_i^n C_j^n$ is equal to

- A. $\frac{2^{2n} - 2^n C_n}{2}$
- B. $\frac{2^{2n} + 2^n C_n}{2}$
- C. $\frac{2^{2n} - n C_n}{2}$
- D. $\frac{2^{2n} + n C_n}{2}$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Consider, sum of the series $\sum_{0 \leq i < j \leq n} f(i)f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^n \sum_{j=1}^n f(i) f(j) = \sum_{i=1}^n (f(i)(\sum_{j=1}^n f(j)))$ and i and j are independent. In this summation, three types of terms occur, those when $i < j$, $i > j$ and $i = j$.

Also, sum of terms when $i < j$ is equal to the sum of the terms when $i > j$ if $f(i)$ and $f(j)$ are symmetrical.

So, in that case

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n f(i)f(j) &= \sum_{0 \leq i < j \leq n} f(i)f(j) \\ &\quad + \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ &= 2 \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ \Rightarrow \sum_{0 \leq i < j \leq n} f(i)f(j) &= \frac{\sum_{i=0}^n \sum_{j=0}^n f(i)f(j) - \sum_{i=j} f(i)f(j)}{2} \end{aligned}$$

When $f(i)$ and $f(j)$ are not symmetrical, we find the sum by listing all the terms.

Question:

$$\sum_{0 \leq i \leq j \leq n} ({}^n C_i + {}^n C_j)$$

- A. $(n+2)2^n$
- B. $(n)2^n$
- C. $(n-1)2^n$
- D. $(n+1)2^n - 1$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

If $f(x) = \frac{x}{x^2+x+1}$, then

On the basis of above information, answer the following :

Question:

If $\int f(\tan x)dx = x - \frac{2}{\sqrt{3}} \tan^{-1}(g(x)) + c$, then for $x \in (-\frac{\pi}{3}, \frac{\pi}{3})$ range of $g(x)$ is -

- A. $(\frac{1}{\sqrt{3}} - 2, \frac{1}{\sqrt{3}} + 2)$
- B. $(-\sqrt{3}, \sqrt{3})$
- C. $(-\frac{\sqrt{3}}{4-\sqrt{3}}, \frac{\sqrt{3}}{4+\sqrt{3}})$
- D. none of these

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

If $f(x) = \frac{x}{x^2+x+1}$, then

On the basis of above information, answer the following :

Question:

If $\int f(x^2)dx + \int f(\frac{1}{x^2})d(\frac{1}{x}) = \lambda \ln \left| \frac{x^2-x+1}{x^2+x+1} \right| + c$, then λ is -

- A. 1
- B. -1
- C. $\frac{1}{2}$
- D. $-\frac{1}{2}$