

Question Paper

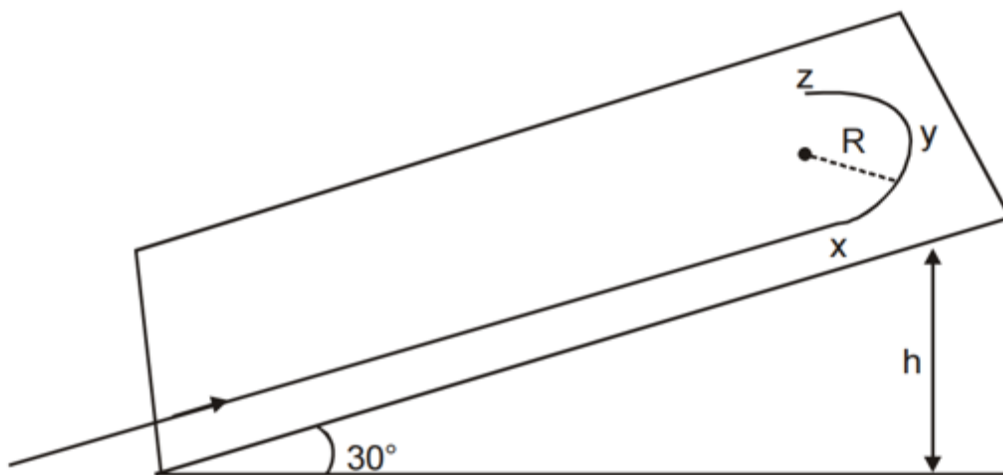
Physics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity)



- A. $v_0^2 - 2gh = \frac{1}{2} gR$
- B. $v_0^2 - 2gh = \frac{\sqrt{3}}{2} gR$
- C. the centripetal force required at points x and z is zero
- D. the centripetal force required is maximum at points x and z

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The de-broglie wavelength of 2 elementary particles in centre of mass frame is same and is equal to $5/8$ times de-broglie wavelength of the lighter particle in lab frame and $5/2$ times the de-broglie wavelength of the heavier particle in lab frame. Both particles have either parallel or anti parallel velocities. The possible combinations of particles are :

- A. ${}^{13}_6\text{C}$ and ${}^{12}_6\text{C}$
- B. ${}^{13}_6\text{C}$ and ${}^4_2\text{He}$

- C. ${}^1_1\text{H}$ and ${}^4_2\text{He}$
 D. ${}^7_3\text{Li}$ and ${}^{13}_6\text{C}$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A car has side window made of glass (shear strength 40MPa) having size $40\text{ cm} \times 40\text{ cm}$. The car is travelling with a velocity of 40 m/s in still air. Density of air can be assumed to be 1.25 kg/m^3 . The windows are closed.

- A. If the glass is very thin, it may break and fall into the car.
 B. If the glass is very thin, it may break & fall out of the car.
 If we want that glass should not break, it's thickness should be greater than
 C. $2.5\mu\text{m}$.
 If we want that glass should not break, it's thickness should be greater than
 D. $8.75\mu\text{m}$.

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two sound waves propagate in a medium. Their equation for displacement are given by

$$S_1 = 1\mu\text{m} \sin(100\pi t - \frac{\pi}{4} x + \frac{\pi}{3});$$

$$S_2 = 1\mu\text{m} \sin(\frac{\pi}{4} x - 100\pi t)$$

Which of the following statement is/are correct?

- A. The velocity of both waves is the same.
 B. Both the waves are equally intense.
 C. They superimpose to produce a wave of amplitude $1\mu\text{m}$.
 D. Both waves are plane waves, propagating in +ve x direction.

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In a meter bridge experiment, when we measure the resistance of two resistors connected in series, null point is obtained at 80 cm. When they are in parallel, null point is at 40 cm. What are the approximate null points when each of the resistances are measured ? Assume that resistance of resistance box remains the same.

- A. 76 cm
 B. 60 cm

- C. 46 cm
- D. 28 cm

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A shiny spoon is held in front of our eye at a distance of 25 cm such that the stem of the spoon is vertical. If the concave side of the spoon is observed, then an inverted image of our head can be seen, whilst if the convex part of the spoon is observed, then the image is upright. The radius of curvature of the vertical section of the spoon is 5 cm.

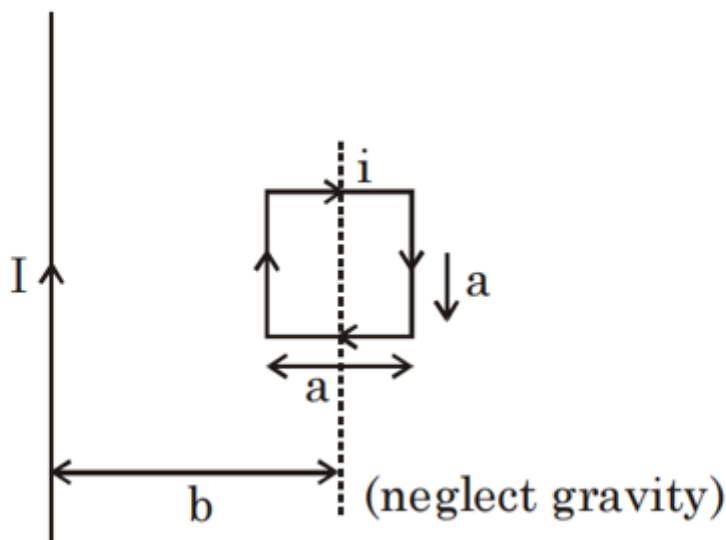
- A. The upright image is shorter than the inverted image.
- B. The inverted image is shorter than the upright image.
- C. The inverted image is at a distance of $\frac{200}{9}$ cm from us.
- D. The upright image is at a distance of $\frac{250}{11}$ cm from us.

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

An infinite current carrying wire is placed at a distance b from a very small square loop of side a . The current in loop is i . Neglect any induced emf and assume these currents remain constant.



- A. The loop is attracted to the wire by a force $F = \frac{\mu_0 I i}{2\pi b^2} a^2$.

The loop must be projected away from wire with a kinetic energy of $\frac{\mu_0 I i a^2}{3\pi b}$,

- B. so that it does not return back.

If the loop is projected into the plane with a kinetic energy of $\frac{\mu_0 I i a^2}{4\pi b}$, it

- C. moves in a circular path.

If the loop is projected into the plane with a kinetic energy of $\frac{\mu_0 I a^2}{2\pi b}$, it
 D. moves in a circular path.

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A simple pendulum time period at night 12' O clock is 2 sec if it is on equator. In which of the following cases will it's time period be greater than 2 sec?

- A. It is taken to poles.
- B. The temperature is increased.
- C. It is taken on a geostationary satellite.
- D. It is taken to a high hill on equator.

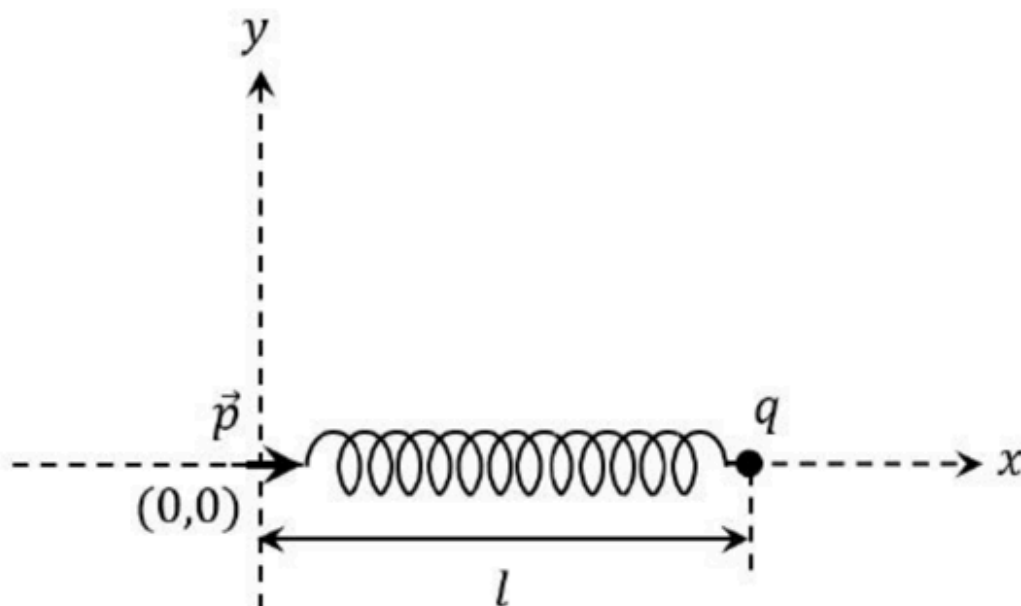
Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

One end of a spring of negligible unstretched length and spring constant k is fixed at the origin $(0, 0)$. A point particle of mass m carrying a positive charge q is attached at its other end. The entire system is kept on a smooth horizontal surface. When a point dipole \vec{p} pointing towards the charge q is fixed at the origin, the spring gets stretched to a length ℓ and attains a new equilibrium position (see figure below). If the point mass is now displaced slightly by $\Delta\ell \ll \ell$ from its equilibrium position and released, it is found to oscillate at frequency $\frac{1}{\delta} \sqrt{\frac{k}{m}}$. The value of δ is.



Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

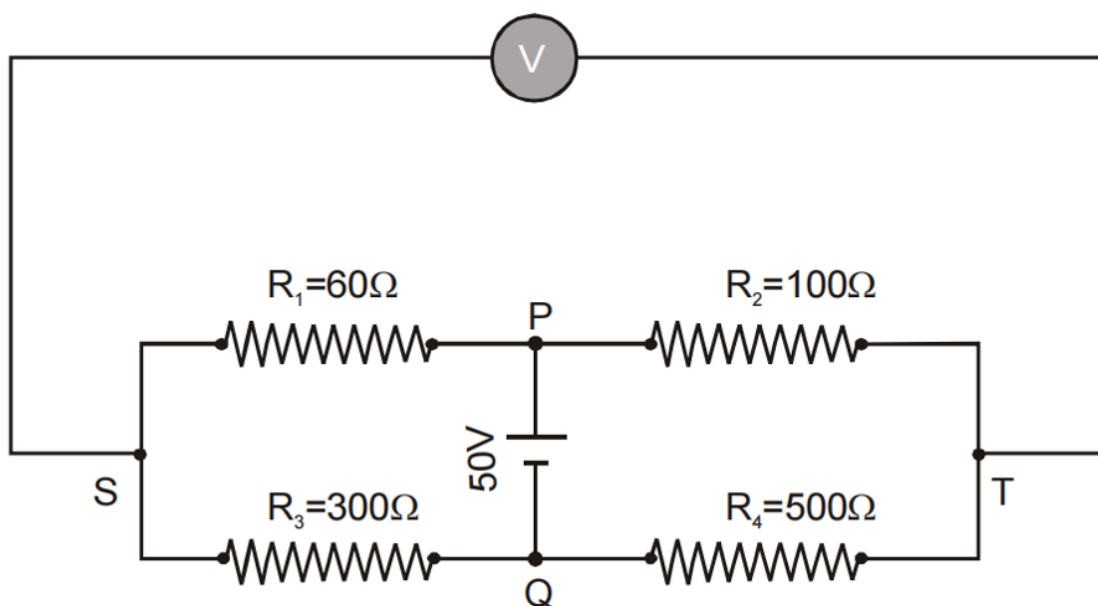
A stationary tuning fork is in resonance with an air column in a pipe. If the tuning fork is moved with a speed of 2 ms^{-1} in front of the open end of the pipe and parallel to it, the length of the pipe should be changed for the resonance to occur with the moving tuning fork. If the speed of sound in air is 320 ms^{-1} , the smallest value of the percentage change required in the length of the pipe is

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

In the balanced condition, the values of the resistances of the four arms of a Wheatstone bridge are shown in the figure below. The resistance R_3 has temperature coefficient $0.0004^\circ\text{C}^{-1}$. If the temperature of R_3 is increased by 100°C , the voltage developed between S and T will be volt.



Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 W m^{-2} and it is absorbed by the water over an effective area of 0.05 m^2 . Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in $^\circ\text{C}$) in the temperature of water and the surroundings after a long time will be (Ignore effect of the container, and take constant for Newton's law of cooling $= 0.001 \text{ s}^{-1}$, Heat capacity of water $= 4200 \text{ J kg}^{-1} \text{ K}^{-1}$)

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

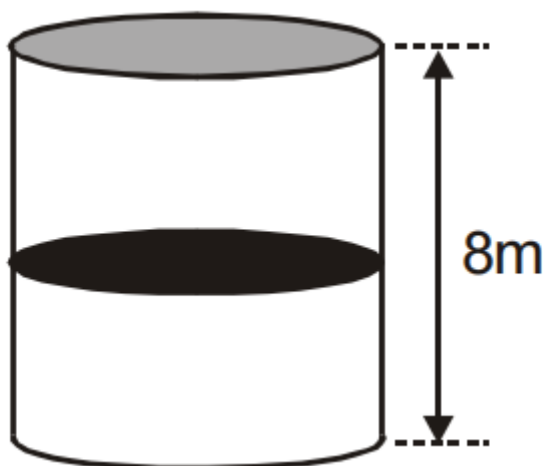
A spherical bubble inside water has radius R . Take the pressure inside the bubble and the water pressure to be p_0 . The bubble now gets compressed radially in an adiabatic manner so that its radius becomes $(R - a)$. For $a \ll R$ the magnitude of the work done in the process is given by $(4\pi p_0 R a^2)X$, where X is a constant and $\gamma = \frac{C_p}{C_v} = \frac{41}{30}$. The value of X is

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

A thermally isolated cylindrical closed vessel of height 8 m is kept vertically. It is divided into two equal parts by a diathermic (perfect thermal conductor) frictionless partition of mass 8.3 kg. Thus the partition is held initially at a distance of 4 m from the top, as shown in the schematic figure below. Each of the two parts of the vessel contains 0.1 mole of an ideal gas at temperature 300 K. The partition is now taken slowly to equilibrium position and moves without any gas leaking from one part of the vessel to the other. When equilibrium is reached, the distance of the partition from the top (in m) will be (take the acceleration due to gravity = 10 ms^{-2} and the universal gas constant = $8.3 \text{ J mol}^{-1} \text{ K}^{-1}$).



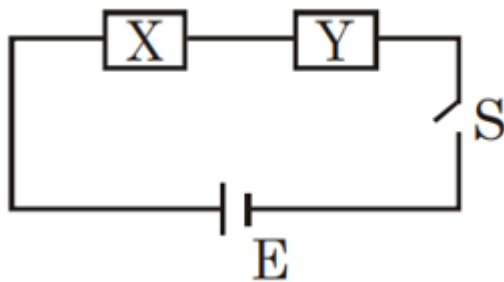
Physics Paragraph Type (Maximum Marks: 12)

Question No. 1

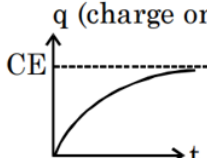
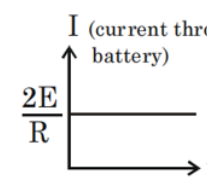
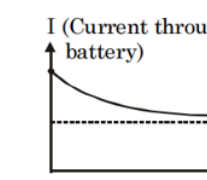
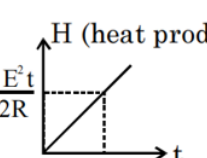
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Following is a circuit containing an ideal battery of emf E in which the elements X & Y are unknown & may be a capacitor C , a resistor R , a battery of emf E , or may be their combination as mentioned in each case. The connecting wires are perfectly conducting.



The switch S is initially open & it is closed at $t = 0$.

| Column-1 | Column-2 | Column-3 |
|--|---|---|
| (I) Element X is R Element Y is C | (i) Entire energy (non-zero) supplied by batteries is dissipated as heat | (P)  |
| (II) Both elements X & Y are R each | (ii) Total charge drawn from the battery(s) after long time is $\frac{CE}{2}$ | (Q)  |
| (III) Element X is R , element Y is a parallel combination of R and C | (iii) Total heat produced in circuit is less than the energy supplied by the battery(s) | (R)  |
| (IV) Element X is R element Y is battery E joined with same polarity | (iv) $I = 0$ just after S is closed as well as after long time | (S)  |

Of the given 4 situations, which one has maximum value of current just after closing the switch?

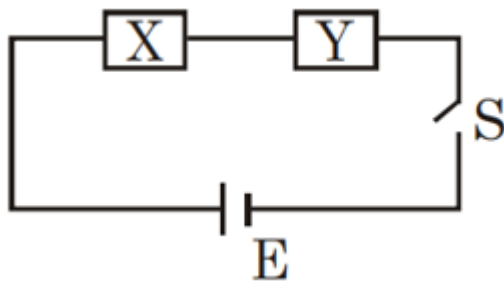
- A. I, iii, P
- B. IV, i, Q
- C. II, iii, P
- D. III, i, S

Question No. 2

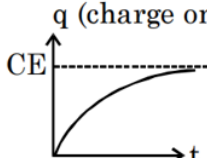
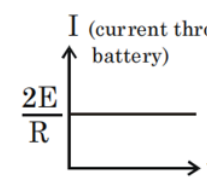
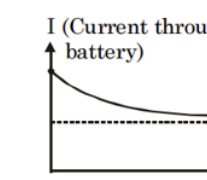
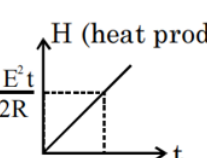
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Following is a circuit containing an ideal battery of emf E in which the elements X & Y are unknown & may be a capacitor C , a resistor R , a battery of emf E , or may be their combination as mentioned in each case. The connecting wires are perfectly conducting.



The switch S is initially open & it is closed at $t = 0$.

| Column-1 | Column-2 | Column-3 |
|--|---|---|
| (I) Element X is R Element Y is C | (i) Entire energy (non-zero) supplied by batteries is dissipated as heat | (P)  |
| (II) Both elements X & Y are R each | (ii) Total charge drawn from the battery(s) after long time is $\frac{CE}{2}$ | (Q)  |
| (III) Element X is R , element Y is a parallel combination of R and C | (iii) Total heat produced in circuit is less than the energy supplied by the battery(s) | (R)  |
| (IV) Element X is R element Y is battery E joined with same polarity | (iv) $I = 0$ just after S is closed as well as after long time | (S)  |

In which case the charge drawn from the battery (s) during $t = 0$ to $t = RC$ is the least?

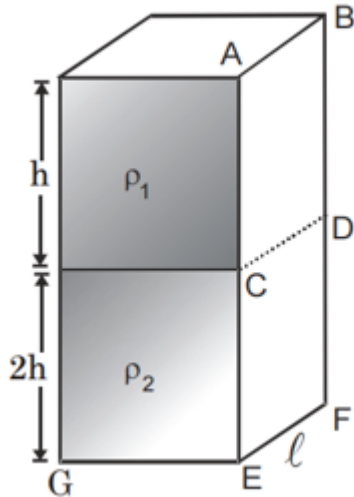
- A. II, i, S
- B. I, iii, P
- C. III, ii, Q
- D. IV, i, Q

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A cuboid is filled with liquid of density ρ_2 upto height $2h$ & with liquid of density ρ_1 , up height h as shown in the figure and assume pressure at top to be zero.



| Column-1 | Column-2 | Column-3 |
|---|------------------------------------|--|
| (I) Force on face ABCD due to liquid of density ρ_1 | (i) zero | (P) Effectively acts at centre of that face |
| (II) Force on face ABCD due to liquid of density ρ_2 | (ii) $\frac{\rho_1 g h^2 \ell}{2}$ | (Q) Effectively acts at $\frac{h}{3}$ above the bottom of that face |
| (III) Magnitude of increase in force on face CDEF due to filling liquid of density ρ_1 | (iii) $2\rho_1 g h^2 \ell$ | (R) Effectively acts of at $\frac{4h}{3}$ below the top of that face |
| (IV) Force on face CDEF if liquid of density ρ_2 only is present | (iv) $2\rho_2 g h^2 \ell$ | (S) Cannot produce a net torque about bottom edge of that face |

Which force has the second highest magnitude (given $\rho_1 < \rho_2$)?

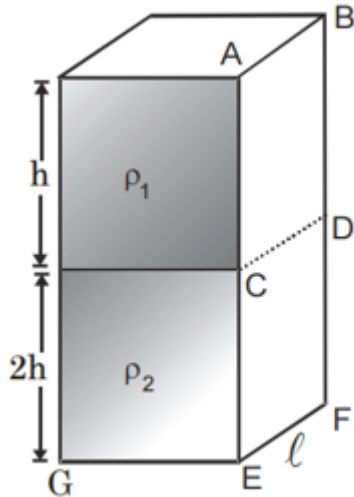
- A. I,ii, Q
- B. IV, i, Q
- C. II,i, S
- D. III,iii, P

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

A cuboid is filled with liquid of density ρ_2 upto height $2h$ & with liquid of density ρ_1 , up height h as shown in the figure and assume pressure at top to be zero.



| Column-1 | Column-2 | Column-3 |
|---|------------------------------------|--|
| (I) Force on face ABCD due to liquid of density ρ_1 | (i) zero | (P) Effectively acts at centre of that face |
| (II) Force on face ABCD due to liquid of density ρ_2 | (ii) $\frac{\rho_1 g h^2 \ell}{2}$ | (Q) Effectively acts at $\frac{h}{3}$ above the bottom of that face |
| (III) Magnitude of increase in force on face CDEF due to filling liquid of density ρ_1 | (iii) $2\rho_1 g h^2 \ell$ | (R) Effectively acts of at $\frac{4h}{3}$ below the top of that face |
| (IV) Force on face CDEF if liquid of density ρ_2 only is present | (iv) $2\rho_2 g h^2 \ell$ | (S) Cannot produce a net torque about bottom edge of that face |

Which force produces minimum (Non zero) torque about side EF (given $\rho_1 = \rho_2$)?

- A. I,ii, Q
- B. II, i, S
- C. III,iii, P
- D. IV, iii, S

Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Consider the following values of I.E.(eV) for elements W and X :

| Element | I.E.. ₁ | I.E.. ₂ | I.E.. ₃ | I.E.. ₄ |
|---------|--------------------|--------------------|--------------------|--------------------|
| W | 10.5 | 15.5 | 24.9 | 79.8 |
| X | 8 | 14.8 | 78.9 | 105.8 |

Other two element Y and Z have outer electronic configuration ns^2np^4 and ns^2np^5 respectively. Then according to given information which of the following compound(s) is/are not possible.

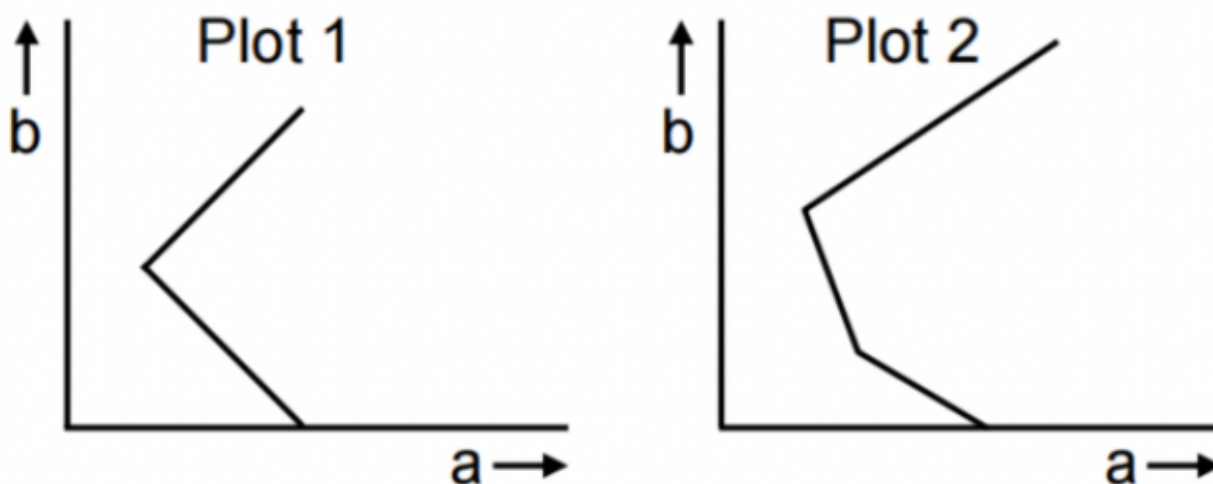
- A. W_2Y_3
- B. X_2Y_3
- C. WZ_2
- D. XZ_2

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Given below are 2 conductometric titration plots. In each plot conductance of a solution in flask is plotted on x-axis (shown as a) and the volume of the titrant added from burette is plotted on y-axis (shown as b). Which of the following option(s) is/are correct ?



- A. Plot 1 corresponds to titration of NaOH (in flask) versus HCl (in burette)
- B. Plot 2 corresponds to titration of NaOH (in flask) versus HCl (in burette)
- C. Plot 1 corresponds to titration of NaOH (in flask) versus mixed acid HCl + CH_3COOH (in burette)
- D. Plot 2 corresponds to titration of NaOH (in flask) versus mixed acid HCl + CH_3COOH (in burette)

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Choose the correct statement(s) among the following :

- A. $[\text{FeCl}_4]^-$ has tetrahedral geometry.
- B. $[\text{Co}(\text{en})(\text{NH}_3)_2\text{Cl}_2]^+$ has 2 geometrical isomers.
 $[\text{FeCl}_4]^-$ has higher spin-only magnetic moment than
- C. $[\text{Co}(\text{en})(\text{NH}_3)_2\text{Cl}_2]^+$.
- D. The cobalt ion in $[\text{Co}(\text{en})(\text{NH}_3)_2\text{Cl}_2]^+$ has $\text{sp}^3 \text{d}^2$ hybridization.

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Select correct statement(s).

- A. $(\text{CH}_3)_3\text{COH}$ is less acidic as compared to $(\text{CH}_3)_3\text{SiOH}$
- B. $(\text{CH}_3)_2\text{N}(\text{SiH}_3)$ is more basic than $(\text{CH}_3)\text{N}(\text{SiH}_3)_2$
- C. NO_2 more easily dimerizes as compared to NO
- D. BCl_3 is better L.P. acceptor (Lewis acid) as compared to BF_3 & AlCl_3

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

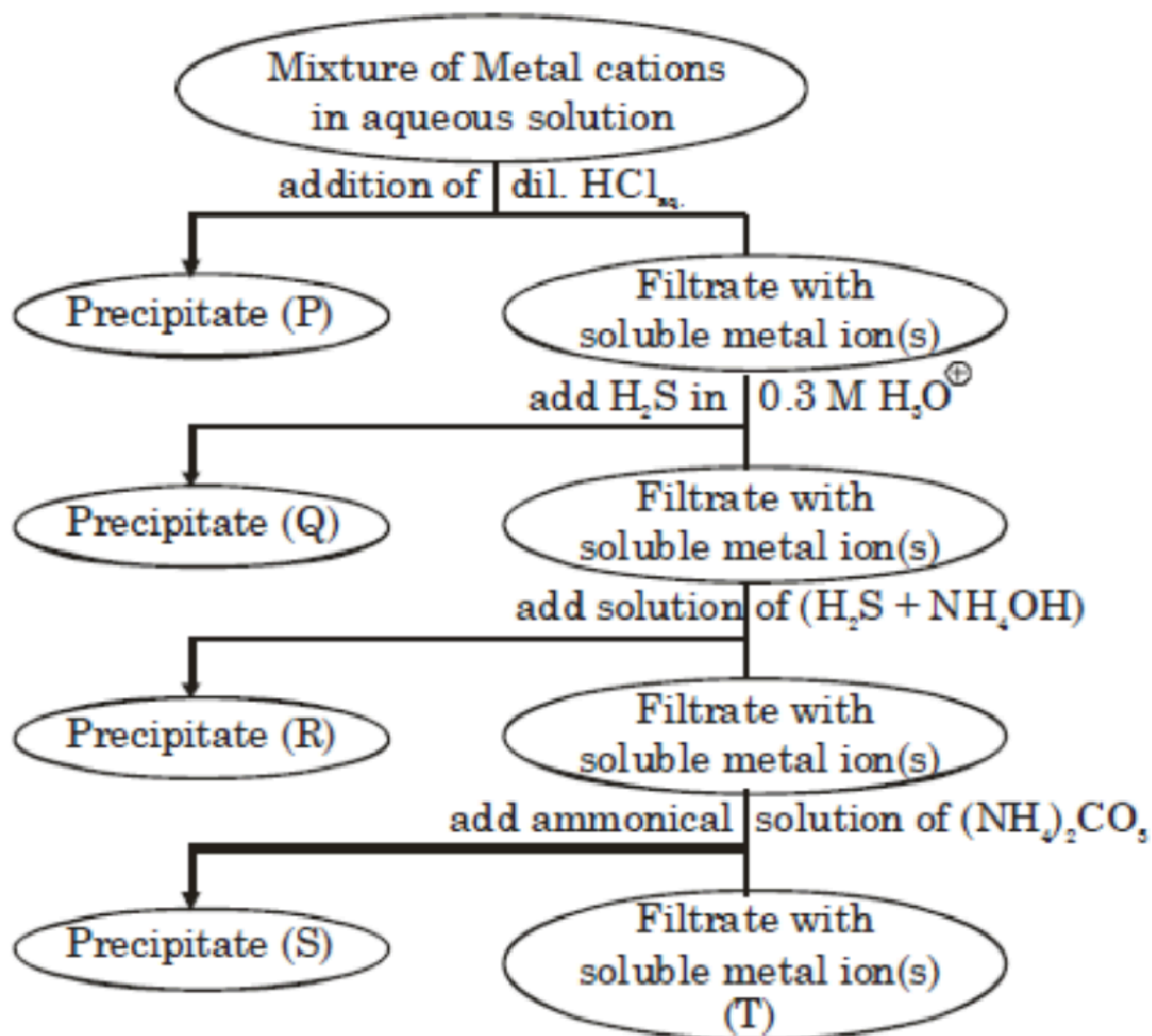
Two gases X and Y, one being the dimer of other are at equilibrium. Increase of volume at constant temperature or increase of temperature at constant pressure favours the formation of more Y. The reaction could not be represented by $[Q > 0]$

- A. $2\text{X} \rightleftharpoons \text{Y} + Q$ calories
- B. $2\text{X} \rightleftharpoons \text{Y} - Q$ calories
- C. $2\text{Y} \rightleftharpoons \text{X} - Q$ calories
- D. $2\text{Y} \rightleftharpoons \text{X} + Q$ calories

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.



In above reaction sequence (P)(Q)(R)(S) and (T) can be respectively :-

- A. Hg_2Cl_2 , CuS , Cr_2S_3 , BaCO_3 , K^+
- B. AgCl , Bi_2S_3 , MnS , CaCO_3 , Na^+
- C. HgCl_2 , CdS , CoS , SrCO_3 , Mg^+
- D. PbCl_2 , HgS , Al(OH)_3 , K_2CO_3 , Na^+

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A pale blue diamagnetic gas (Y) is best prepared by flowing paramagnetic gas (X) at 1 atm and 25°C through concentric metallized glass tubes to which low frequency power at 50 – 500 Hz and 10 – 20kV is applied to maintain a silent electric discharge.

Gas (X) is colourless in gas form but liquid is pale blue. Gas (Y) is used for sterilization of foodstuffs and disinfection, and also form during the generation of photochemical smog. (X)&(Y) are the different form of a element which is about 47% by mass of earth crust.

Identify correct statement(s) regarding (X)&(Y)

Concentration of ' Y ' in the mixture of X & Y is determined by

- A. iodometrically using H_3BO_3 & $\text{Na}[\text{B}(\text{OH})_4]$ buffer.
- B. Conversion of (X) to (Y) is an exothermic process
- C. Y ' can oxidise black lead sulphide
- D. Y is used to convert potassium manganate to potassium permanganate

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Which is/are similar for Cr & Mn :-

- A. Both have 13 electrons in (n-1) shell
- B. Both have tetrahedral coloured oxo anion in their highest oxidation state
 MCl_2 salt of both are oxidised to highest oxidation state of metal on fusion
- C. with Na_2CO_3 & air
- D. Both transition metals have $E_{\text{M}+2}^0 / \text{M}$ have +ve value

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

In the saturated aqueous solution of PbCl_2 the freezing point decreases by $(\frac{X}{100})^\circ\text{C}$ then ' X ' is :

(Given k_{sp} of $\text{PbCl}_2 = 4 \times 10^{-6}$, $K_{\text{f, water}} = 2\text{kg/mole}$)

Question No. 2

Numerical Type

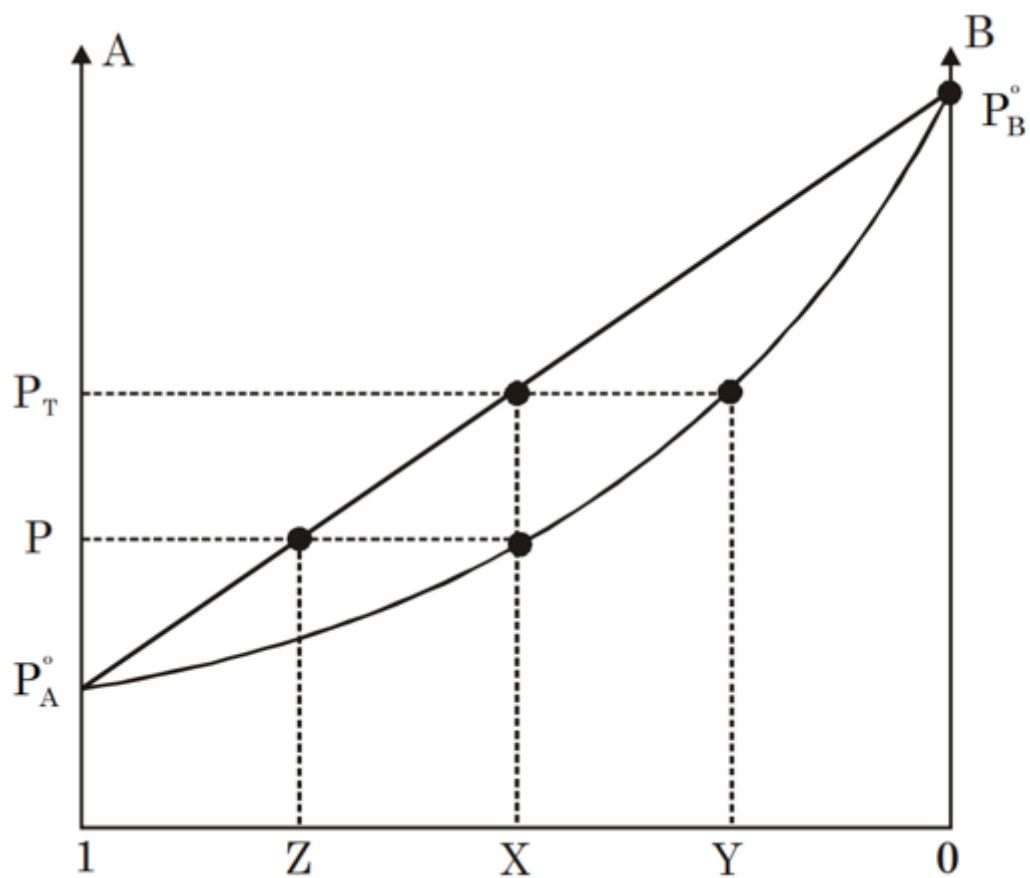
The answer has to be filled into the input box provided below.

A sample of H-atoms containing all the atoms in a particular excited state, absorb radiations of a particular wave length by which the atoms get excited to another excited state. When the atoms finally de-excite to the ground state, they emit radiations of 15 different wavelength. Out of these 15 radiations, 5 have wavelengths shorter than the absorbed radiation and 9 have wavelength longer than the absorbed radiation. What is the initial excited state of atoms?

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.



X = Mole fraction of liquid A in liquid solution.

Y = Mole fraction of vapour A in vapour mixture.

Z is the graphical mid point between "1" & "X".

Find value of $\left(\frac{P \cdot Y}{100X}\right)$. [Given : $P_T = 450 \text{ mmHg}$]

If $P_A^o = 300 \text{ mm of Hg}$

$P_B^o = 700 \text{ mm of Hg}$

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

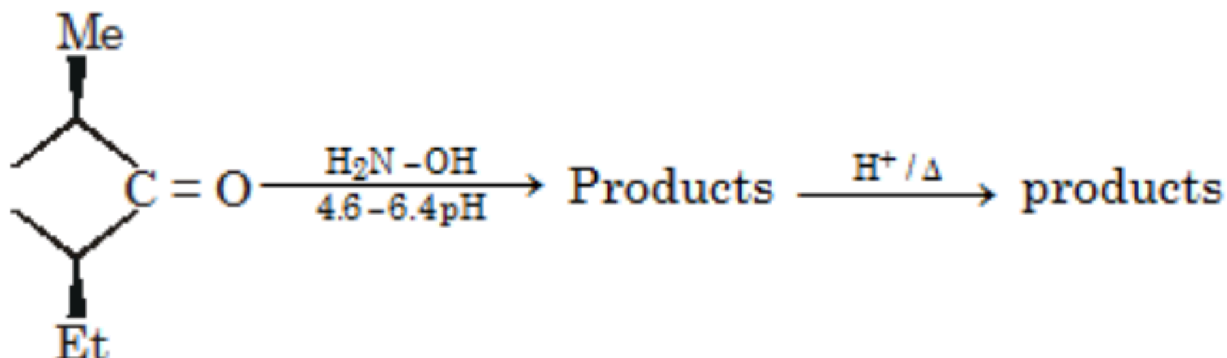


Number of $\text{CH}_3 - \text{I}$ consumed during the following reaction. In which Hoffmann exhaustive elimination take place.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.



Find out total number of final products?

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

The formation of dextrorotatory isomer A from its levorotatory isomer B in water is reversible reaction having 1st order kinetics in both direction. Racemic mixture is the equilibrium product if reaction starts from either A or B. Analysis of polarimeter experiment showed that 10% of B was converted into A in 230.3 sec at 25°C.

($\log 5 = 0.7$, $\log 3 = 0.48$ and $\log 2 = 0.3$)

Based on given information select number of correct statement(s) among following.

- (A) Rate constant of forward reaction at 25°C is $5 \times 10^{-4} \text{ s}^{-1}$
- (B) Rate constant of backward reaction at 25°C is $5 \times 10^{-4} \text{ s}^{-1}$
- (C) Activation energy of forward reaction is equal to activation energy of backward reaction.
- (D) Equilibrium constant (K) at 25°C of $B \rightleftharpoons A$ is 1.
- (E) Equilibrium constant increases by increasing temperature.
- (F) Molar entropy of A and B at 25°C must be same.
- (G) $\Delta_r H$ of the process is zero.
- (H) $\Delta_r G^\circ$ of the process is zero.

(I) $\Delta_r G$ of the process $\underset{(1\text{-form})}{B} \xrightleftharpoons[k_b]{k_f} \underset{(d\text{-form})}{A}$ is zero.

Chemistry Paragraph Type (Maximum Marks: 12)

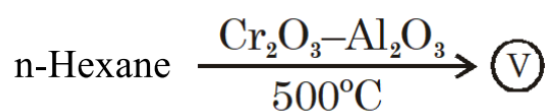
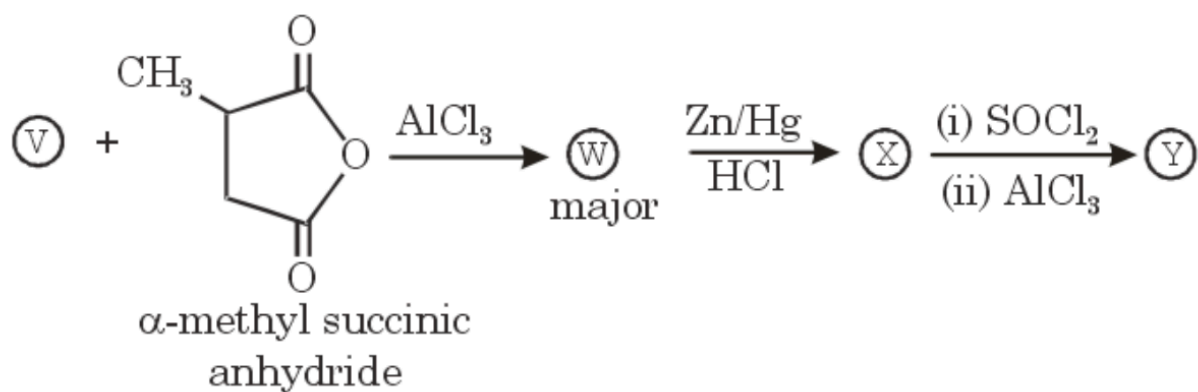
Question No. 1

Only One Option Correct Type

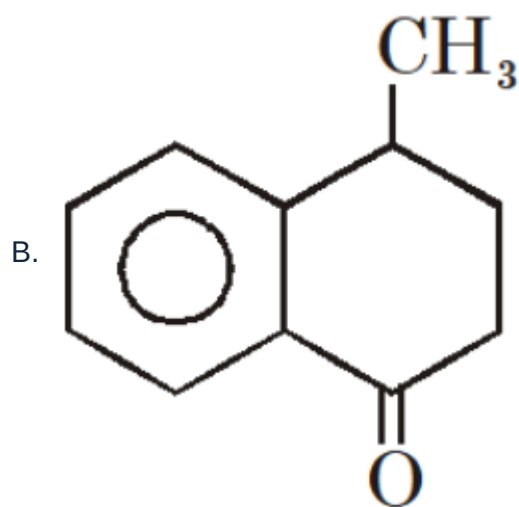
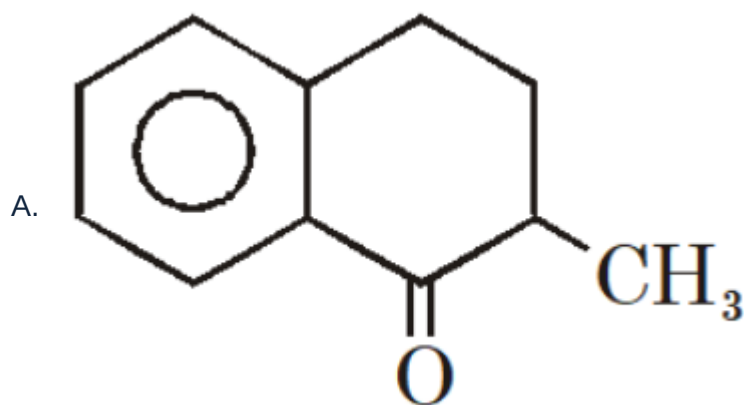
Each question has multiple options out of which ONLY ONE is correct.

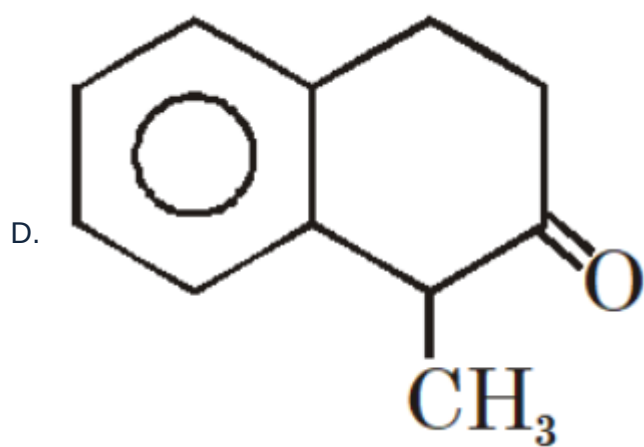
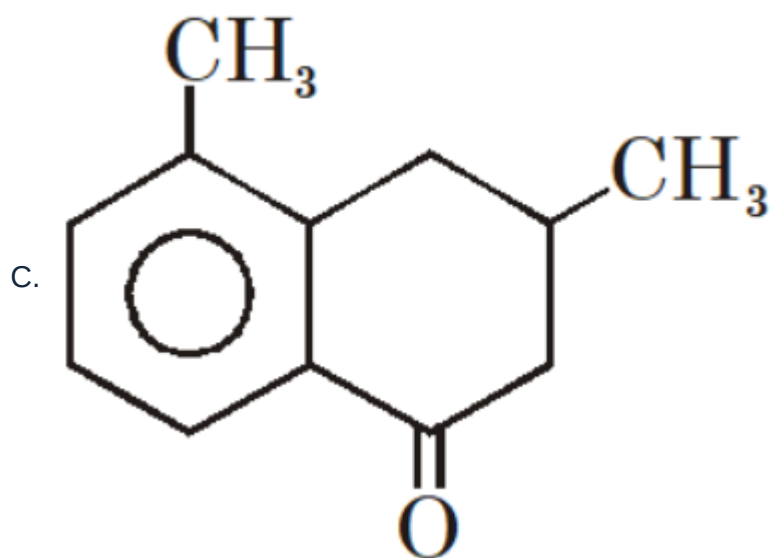
Paragraph:

Observe the following reaction sequence and give answers to questions given below :

**Question:**

Identify (Y) :





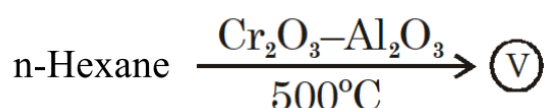
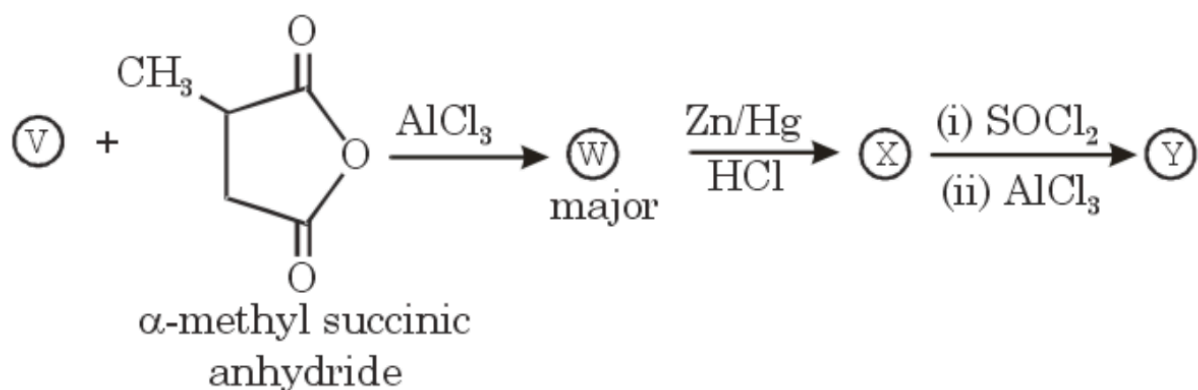
Question No. 2

Only One Option Correct Type

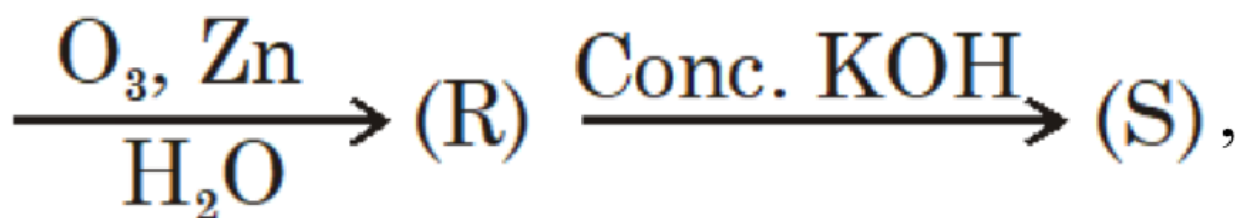
Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

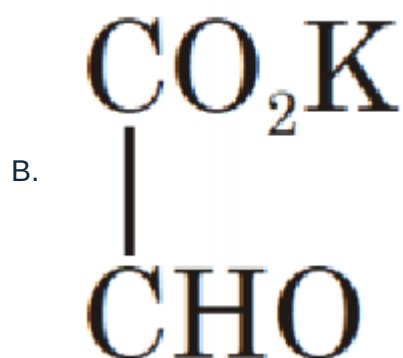
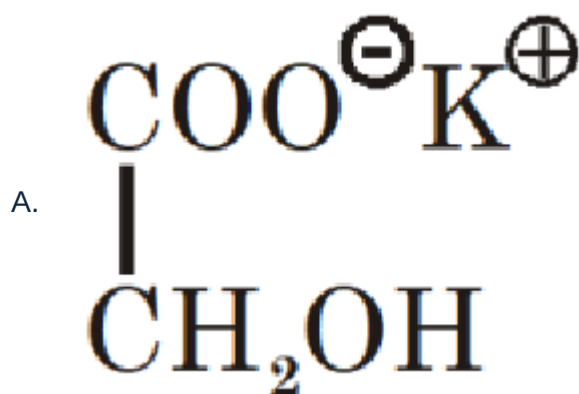
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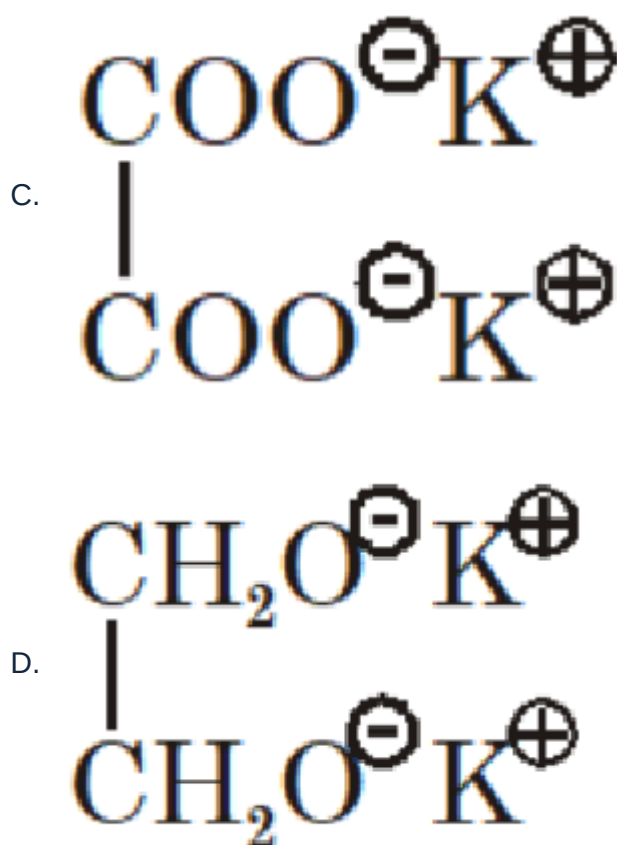
**Question:**

If



Identify (S)





Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The atomic orbitals combine to form new set of equivalent orbitals known as hybrid orbitals. Unlike pure orbitals, the hybrid orbitals are used in bond formation. The phenomenon is known as hybridisation which can be defined as the process of intermixing of the orbitals of slightly different energies in the formation of new set of orbitals of equivalent energies.

Question:

Which molecule is non planar as well as polar

- A. IF
- B. IF₃
- C. IF₅
- D. IF₇

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The atomic orbitals combine to form new set of equivalent orbitals known as hybrid orbitals. Unlike pure orbitals, the hybrid orbitals are used in bond formation. The phenomenon is known as hybridisation which can be defined as the process of intermixing of the orbitals of slightly different energies in the formation of new set of orbitals of equivalent energies.

Question:

Which molecule is having same electronic geometry as well as molecular geometry

- A. NCl_3
- B. PCl_5
- C. SF_4
- D. XeF_4

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let \vec{a}, \vec{b} and \vec{c} are three unit vectors such that $|\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$ and let $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = \lambda$

- A. the maximum value of λ is 0
when λ is maximum then the volume of parallelepiped determined by \vec{a}, \vec{b}
- B. and \vec{c} is $\sqrt{3}$
when λ is maximum then the value of
- C. $(2\vec{a} + 3\vec{b} + 4\vec{c}) \cdot (\vec{a} \times \vec{b} + 5\vec{b} \times \vec{c} + 6\vec{c} \times \vec{a})$ is 32 .
- D. the maximum value of λ is 2

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let $f(x) = \sin x + ax + b$. Then $f(x) = 0$ has

- A. only one real root which is positive if $a > 1, b < 0$
- B. only one real root which is negative if $a > 1, b > 0$
- C. only one real root which is negative if $a < -1, b < 0$
- D. only one real root which is positive if $a < -1, b < 0$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let for $n > 1, n \in I, \lim_{n \rightarrow \infty} \int_0^{(n!+1)\frac{\pi}{2}} \left(\frac{\cos x + x \sin x}{x^2 + \cos^2 x} \right) dx = \ell$ and $\lim_{x \rightarrow \infty} (x^2 \ln(x \cot^{-1} x)) = m$, then

- A. $\ell \in Q, m \notin Q$
- B. $\ell \notin Q, m \in Q$
- C. $[\ell] + [m] = 0$; ($[\cdot]$ denotes G.I.F.)
- D. $\ell < 0 < m$

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The range of the function

- A. $\frac{1}{\pi^2} \{(\cos^{-1} x)^2 - (\sin^{-1} x)^2\}$ is $[-\frac{1}{4}, \frac{3}{4}]$
- B. $\frac{1}{\pi^2} \{(\cos^{-1} x)^2 + (\sin^{-1} x)^2\}$ is $[\frac{1}{8}, \frac{5}{4}]$
- C. $\frac{1}{\pi^3} \{(\cos^{-1} x)^3 - (\sin^{-1} x)^3\}$ is $[-\frac{1}{8}, \frac{9}{8}]$
- D. $\frac{1}{\pi^3} \{(\cos^{-1} x)^3 + (\sin^{-1} x)^3\}$ is $[\frac{1}{32}, \frac{7}{8}]$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Suppose m boys and m girls take their seats randomly around a circle. The probability of their sitting is $({}^{2m-1}C_m)^{-1}$ when

- A. no two boys sit together
- B. no two girls sit together
- C. boys and girls sit alternative
- D. all the boys sit together

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If C_r stands for nC_r , then the sum of the series $\frac{2(\frac{n}{2})!(\frac{n}{2})!}{n!}$ $[C_0^2 - 2C_1^2 + 3C_2^2 - \dots + (-1)^n (n+1)C_n^2]$, where n is an even positive integer, is equal to

- A. $(-1)^n (n + 1)$
- B. $(-1)^{\frac{n}{2}} (n + 1)$
- C. $(-1)^n (n + 2)$
- D. $(-1)^{\frac{n}{2}} (n + 2)$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $\sum_{r=1}^n T_r = \frac{n(n+1)(n+2)(n+3)}{8}$, then the value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{4016}{T_r}$ must be

- A. 1004
- B. 4016
- C. 502
- D. 2008

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A square is inscribed in a circle, if p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square, then

- A. $p_1 < p_2$
- B. $p_1 = p_2$
- C. $p_1 > p_2$
- D. $p_1^2 - p_2^2 < \frac{1}{3}$

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If area bounded by curve $xy^2 = (2 - x)$ with $x = 0$ is $k\pi$ sq. units, then the value of k is equal to

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

The complete solution set of the inequality $\frac{1}{\log_4\left(\frac{x+1}{x+2}\right)} > \frac{1}{\log_4(x+3)}$ is $\left(\frac{-a}{20}, \infty\right)$,
then determine a

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that its derivative f' is continuous and $f(\pi) = -6$. If $F : [0, \pi] \rightarrow \mathbb{R}$ is defined by $F(x) = \int_0^x f(t)dt$ and $\int_0^\pi (f'(x) + F(x))\cos x \, dx = 2$, then the value of $f(0)$ is equal to

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A quadratic polynomial $f(x)$ with positive leading coefficient such that $g(x) = f(\ln x) \forall x > 0$. Also the curve $y = g(x)$ satisfy the following condition.

(a) There is exactly one value for a positive number p such that $(p, g(p))$ is its extremum point and $(p^2, g(p^2))$ is its inflection point.

(b) Exactly one tangent line can be drawn from the point $(0, 0)$ to the curve $y = g(x)$.

The value of definite integral of $\int_1^2 \frac{f(0)}{f(x)} \, dx$ equals:

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Find the mode of the following distribution:

| | | | | | | | |
|----------------|--------|---------|---------|---------|---------|---------|---------|
| Class Interval | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
| f | 5 | 8 | 7 | 12 | 28 | 20 | 10 |

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

If α, β, γ are different from 1 and are the roots of $ax^3 + bx^2 + cx + d = 0$ and $(\beta - \gamma)(\gamma - \alpha)(\alpha - \beta) = \frac{25}{2}$ then $\sqrt{-\frac{2(a+b+c+d)\Delta}{d}} = \dots\dots\dots$

$$\text{if } \Delta = \begin{vmatrix} \frac{\alpha}{1-\alpha} & \frac{\beta}{1-\beta} & \frac{\gamma}{1-\gamma} \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}$$

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Many times the methods of coordinates becomes effective in solving problems of properties of triangles. We may choose one vertex of the triangle as origin and one side passing through this vertex as x-axis. Thus, without loss of generality, we can assume that every triangle ABC has a vertex B situated at $B(0, 0)$, vertex C at $(a, 0)$ and A as (h, k) .

Question: If in a triangle ABC , $AC = 3$, $BC = 4$, medians AD and BE are perpendicular, then find the area of triangle ABC .

- A. $\sqrt{11}$
- B. $2\sqrt{11}$
- C. $\sqrt{3}/2$
- D. $\sqrt{3}$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Many times the methods of coordinates becomes effective in solving problems of properties of triangles. We may choose one vertex of the triangle as origin and one side passing through this vertex as x-axis. Thus, without loss of generality, we can assume that every triangle ABC has a vertex B situated at $B(0, 0)$, vertex C at $(a, 0)$ and A as (h, k) .

Question: If the median to side BC has length p and it divides A into angles 30° and 45° , then a must be equal to

- A. $2p(11 - 6\sqrt{3})^{1/2}$
- B. $2p(13 - 6\sqrt{3})^{1/2}$
- C. $2p$

D. none of these

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Suppose that f is defined on R by the rule $f(x) = (1 - x)(1 + x^2)$. The function is invertible and its inverse is denoted by f^{-1} .

Question: If $h = f^{-1}(\ln(f(x)))$, $x < 1$, then the value of $(3 + \frac{1}{h(0)})$ is:

- A. 2
- B. 5
- C. 3
- D. 6

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Suppose that f is defined on R by the rule $f(x) = (1 - x)(1 + x^2)$. The function is invertible and its inverse is denoted by f^{-1} .

Question: The value of $\int_0^1 \frac{(1-x) \ln(1+x)}{f(x)} dx$ is equal to:

- A. $\frac{\pi \ln 2}{2}$
- B. $\pi \ln 2$
- C. $\frac{\pi \ln 2}{8}$
- D. $\frac{\pi \ln 2}{4}$