$$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \mid x_i \in \mathbb{R}, \ \text{con } i = 1,\ 2,\ \ldots,\ n \right\}$$

$$
\begin{aligned}
+\colon \quad \mathbb{R}^n \times \mathbb{R}^n &\longrightarrow \mathbb{R}^n \\
(\mathbf{x},\ \mathbf{y}) &\longmapsto \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
\cdot\colon \quad \mathbb{R} \times \mathbb{R}^n &\longrightarrow \mathbb{R}^n \\
(\alpha,\ \mathbf{x}) &\longmapsto \begin{pmatrix} \alpha x_1 \\ \alpha x_2 \\ \vdots \\ \alpha x_n \end{pmatrix}
\end{aligned}
$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ \ldots, \ \mathbf{e}_n = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = a\mathbf{e}_1 + b\mathbf{e}_2 + c\mathbf{e}_3.$$

$$W = \operatorname{gen}\left(\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}\right)$$

$$\begin{aligned}
W &= \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \mid \alpha,\ \beta \in \mathbb{R} \right\} \\
&= \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \mid \alpha,\ \beta \in \mathbb{R} \right\} \\
&= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha + \beta \\ \beta \end{pmatrix} \text{ donde } \alpha,\ \beta \in \mathbb{R} \right\}
\end{aligned}$$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix},\ \mathbf{v}_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix},\ \mathbf{v}_3 = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix},\ \mathbf{v}_4 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{w} = \begin{pmatrix} \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 3\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} + 0\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix} = 3\mathbf{e}_1 + 2\mathbf{e}_2 + 0\mathbf{w}$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 2\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} + \sqrt{2}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix} = 2\mathbf{e}_1 + 1\mathbf{e}_2 + \sqrt{2}\mathbf{w}$$

$$\begin{pmatrix}3\\2\end{pmatrix} = 4\begin{pmatrix}1\\0\end{pmatrix} + 3\begin{pmatrix}0\\1\end{pmatrix} - \sqrt{2}\begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix} = 4\mathbf{e}_1 + 3\mathbf{e}_2 - \sqrt{2}\mathbf{w}$$

$$\mathbf{w} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\mathbf{e}_1 + \frac{1}{\sqrt{2}}\mathbf{e}_2$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \text{para cualquier} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{y} \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$

$$\mathbf{v}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{para cualquier } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \\ 9 \end{pmatrix}, \begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 9 \\ 6 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 4 \\ -6 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \\ -24 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 20 \\ -23 \\ -8 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} 8 \\ -3 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ 24 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ -3 \\ -2 \end{pmatrix}, \; \begin{pmatrix} -2 \\ 4 \\ -8 \end{pmatrix}, \; \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} -9 \\ 8 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} 39 \\ 20 \\ 38 \end{pmatrix}, \quad \begin{pmatrix} -34 \\ 12 \\ -22 \end{pmatrix}, \quad \begin{pmatrix} 7 \\ 12 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 1 \\ 3 \\ -4 \end{pmatrix}, \quad \begin{pmatrix} 9 \\ 12 \\ -18 \\ 6 \end{pmatrix}, \quad \begin{pmatrix} -23 \\ 25 \\ 25 \\ -56 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 6 \\ 0 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \; \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \; \begin{pmatrix} 4 \\ -2 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 7 \\ -8 \end{pmatrix}, \quad \begin{pmatrix} -11 \\ -12 \\ -7 \end{pmatrix}, \quad \begin{pmatrix} 12 \\ -3 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 0 \\ 11 \end{pmatrix}, \ \begin{pmatrix} 7 \\ -20 \\ -29 \end{pmatrix}, \ \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}, \ \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}, \ \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \begin{pmatrix} 3 \\ \alpha \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} \alpha \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} \alpha \\ 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ -1 \\ \beta \end{pmatrix}, \quad \begin{pmatrix} \alpha \\ 5 \\ 2 \end{pmatrix}$$

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \mid a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = 0 \right\}$$