

$$\mathbb{R}^n = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \mid x_i \in \mathbb{R}, \text{ con } i = 1, 2, \dots, n \right\}$$

$$+:\quad \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$(\mathbf{x}, \mathbf{y}) \longmapsto \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

$$\cdot:\quad \mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$$

$$(\alpha, \mathbf{x}) \longmapsto \begin{pmatrix} \alpha x_1 \\ \alpha x_2 \\ \vdots \\ \alpha x_n \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \dots, \quad e_n = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}.$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = ae_1 + be_2 + ce_3.$$

$$W =_{\text{gen}} \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$$

$$\begin{aligned}
 W &= \left\{ \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\} \\
 &= \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \alpha \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \mid \alpha, \beta \in \mathbb{R} \right\} \\
 &= \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha + \beta \\ \beta \end{pmatrix} \text{ donde } \alpha, \beta \in \mathbb{R} \right\}
 \end{aligned}$$

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$



$$W = \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \\ \sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 3\mathbf{e}_1 + 2\mathbf{e}_2 + 0\mathbf{w}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 2\mathbf{e}_1 + 1\mathbf{e}_2 + \sqrt{2}\mathbf{w}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 4 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 1 \\ \frac{1}{\sqrt{2}} \end{pmatrix} = 4\mathbf{e}_1 + 3\mathbf{e}_2 - \sqrt{2}\mathbf{w}$$

$$w = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}e_1 + \frac{1}{\sqrt{2}}e_2$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{para cualquier } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad y \quad v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$$



$$v_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \text{para cualquier } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 7 \\ 3 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix},$$

$$\begin{pmatrix} 7 \\ 3 \\ 9 \end{pmatrix},$$

$$\begin{pmatrix} 7 \\ 0 \\ 3 \end{pmatrix},$$

$$\begin{pmatrix} 9 \\ 6 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ 4 \\ -6 \end{pmatrix}, \begin{pmatrix} -8 \\ 4 \\ -24 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ -6 \end{pmatrix}$$



$$\begin{pmatrix} 20 \\ -23 \\ -8 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix}, \begin{pmatrix} 8 \\ -3 \\ -4 \end{pmatrix}, \begin{pmatrix} -2 \\ 24 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ -3 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ 4 \\ -8 \end{pmatrix}, \begin{pmatrix} 6 \\ -6 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} -9 \\ 8 \\ -4 \end{pmatrix}, \begin{pmatrix} 39 \\ 20 \\ 38 \end{pmatrix}, \begin{pmatrix} -34 \\ 12 \\ -22 \end{pmatrix}, \begin{pmatrix} 7 \\ 12 \\ 10 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -3 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 1 \\ 3 \\ -4 \end{pmatrix}, \begin{pmatrix} 9 \\ 12 \\ -18 \\ 6 \end{pmatrix}, \begin{pmatrix} -23 \\ 25 \\ 25 \\ -56 \end{pmatrix}, \begin{pmatrix} -1 \\ 6 \\ 0 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$



$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 8 \\ 7 \\ -8 \end{pmatrix}, \begin{pmatrix} -11 \\ -12 \\ -7 \end{pmatrix}, \begin{pmatrix} 12 \\ -3 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 7 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -1 \\ 0 \\ 11 \end{pmatrix}, \begin{pmatrix} 7 \\ -20 \\ -29 \end{pmatrix}, \begin{pmatrix} 1 \\ -5 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ \alpha \\ 4 \end{pmatrix}$$



$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix}, \begin{pmatrix} \alpha \\ 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} \alpha \\ 5 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \\ \beta \end{pmatrix}, \begin{pmatrix} \alpha \\ 5 \\ 2 \end{pmatrix}$$

$$W = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \mid a_1x_1 + a_2x_2 + \cdots + a_nx_n = 0 \right\}$$