$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_1^2 + y_1^2 & x_1 & y_1 & 1 \\ x_2^2 + y_2^2 & x_2 & y_2 & 1 \\ x_3^2 + y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 50 & 1 & 7 & 1 \\ 40 & 6 & 2 & 1 \\ 52 & 4 & 6 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2 & xy & y^2 & x & y & 1 \\ x_1^2 & x_1y_1 & y_1^2 & x_1 & y_1 & 1 \\ x_2^2 & x_2y_2 & y_2^2 & x_2 & y_2 & 1 \\ x_3^2 & x_3y_3 & y_3^2 & x_3 & y_3 & 1 \\ x_4^2 & x_4y_4 & y_4^2 & x_4 & y_4 & 1 \\ x_5^2 & x_5y_5 & y_5^2 & x_5 & y_5 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2 & xy & y^2 & x & y & 1 \\ 64.401 & 66.688 & 69.056 & 8.025 & 8.310 & 1 \\ 103.429 & 64.630 & 40.386 & 10.170 & 6.355 & 1 \\ 125.485 & 35.981 & 10.317 & 11.202 & 3.212 & 1 \\ 115.262 & 4.026 & 0.141 & 10.736 & 0.375 & 1 \\ 82.664 & -20.612 & 5.139 & 9.092 & -2.267 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x & y & z & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 0 & -1 & 1 \\ 2 & 9 & 2 & 1 \end{vmatrix} = 0,$$

$$\begin{vmatrix} x^2+y^2+z^2 & x & y & z & 1 \\ x_1^2+y_1^2+z_1^2 & x_1 & y_1 & z_1 & 1 \\ x_2^2+y_2^2+z_2^2 & x_2 & y_2 & z_2 & 1 \\ x_3^2+y_3^2+z_3^2 & x_3 & y_3 & z_3 & 1 \\ x_4^2+y_4^2+z_4^2 & x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2+y^2+z^2 & x & y & z & 1 \\ 13 & 0 & 3 & 2 & 1 \\ 3 & 1 & -1 & 1 & 1 \\ 5 & 2 & 1 & 0 & 1 \\ 35 & 5 & 1 & 3 & 1 \end{vmatrix} = 0,$$

$$
\begin{bmatrix}
1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\
0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 4 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & & \ddots & \ddots & \ddots & & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 4 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 & 4 & 1
\end{bmatrix}
\begin{bmatrix}
M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_{n-2} \\ M_{n-1} \\ M_n
\end{bmatrix}
= \frac{6}{h^2}
\begin{bmatrix}
y_1 - 2y_2 + y_3 \\
y_2 - 2y_3 + y_4 \\
y_3 - 2y_4 + y_5 \\
\vdots \\
y_{n-4} - 2y_{n-3} + y_{n-2} \\
y_{n-3} - 2y_{n-2} + y_{n-1} \\
y_{n-2} - 2y_{n-1} + y_n
\end{bmatrix}
$$

$$\begin{bmatrix} 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ \vdots \\ M_{n-2} \\ M_{n-1} \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ \vdots \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ \vdots \\ M_{n-2} \\ M_{n-1} \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ \vdots \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ \vdots \\ M_{n-2} \\ M_{n-1} \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ \vdots \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_{n-1} \\ M_n \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} 0 \\ y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ \vdots \\ y_{n-2} - 2y_{n-1} + y_n \\ 0 \end{bmatrix}$$

$$
\begin{bmatrix}
4 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & & \ddots & \ddots & \ddots & & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 & 4
\end{bmatrix}
\begin{bmatrix}
M_2 \\ M_3 \\ M_4 \\ \vdots \\ M_{n-2} \\ M_{n-1}
\end{bmatrix}
= \frac{6}{h^2}
\begin{bmatrix}
y_1 - 2y_2 + y_3 \\
y_2 - 2y_3 + y_4 \\
y_3 - 2y_4 + y_5 \\
\vdots \\
y_{n-3} - 2y_{n-2} + y_{n-1} \\
y_{n-2} - 2y_{n-1} + y_n
\end{bmatrix}
$$

$$\begin{bmatrix} 5 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ M_4 \\ \vdots \\ M_{n-2} \\ M_{n-1} \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} y_1 - 2y_2 + y_3 \\ y_2 - 2y_3 + y_4 \\ y_3 - 2y_4 + y_5 \\ \vdots \\ y_{n-3} - 2y_{n-2} + y_{n-1} \\ y_{n-2} - 2y_{n-1} + y_n \end{bmatrix}$$

$$
\begin{bmatrix}
6 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & & \ddots & \ddots & \ddots & & \vdots \\
0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 6
\end{bmatrix}
\begin{bmatrix}
M_2 \\ M_3 \\ M_4 \\ \vdots \\ M_{n-2} \\ M_{n-1}
\end{bmatrix}
= \frac{6}{h^2}
\begin{bmatrix}
y_1 - 2y_2 + y_3 \\
y_2 - 2y_3 + y_4 \\
y_3 - 2y_4 + y_5 \\
\vdots \\
y_{n-3} - 2y_{n-2} + y_{n-1} \\
y_{n-2} - 2y_{n-1} + y_n
\end{bmatrix}
$$

$$\begin{bmatrix} 5 & 1 & 4 \\ 1 & 4 & 1 \\ 0 & 1 & 5 \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ M_4 \end{bmatrix} = \begin{bmatrix} -0.0001116 \\ -0.0000816 \\ -0.0000636 \end{bmatrix}$$

$$\begin{array}{ccc} & & \text{Estado precedente} \\ & & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \text{Nuevo estado} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} \end{array}$$

$$
\begin{array}{ccc}
 & & \text{Alquilado desde la ubicación} \\
 & & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{array}{c}\text{Regresado} \\ \text{a la} \\ \text{ubicación}\end{array} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}
\end{array}
$$

$$\mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$$

$$\mathbb{x}^{(0)} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$
\begin{array}{cc}
 & \text{Antigua intersección} \\
\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \\ \text{Nueva} \\ \text{intersección} \end{array} &
\begin{array}{c@{\quad}c}
 & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} &
\begin{bmatrix}
\frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{5} & 0 & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & \frac{1}{5} & 0 & \frac{1}{3} & 0 & 0 \\
\frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{5} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\
0 & \frac{1}{3} & 0 & \frac{1}{5} & \frac{1}{4} & 0 & 0 & \frac{1}{3} \\
0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{3} & \frac{1}{4} & 0 \\
0 & 0 & 0 & \frac{1}{5} & 0 & \frac{1}{3} & \frac{1}{4} & \frac{1}{3} \\
0 & 0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{3}
\end{bmatrix}
\end{array}
\end{array}
$$

$$P^n \to \begin{bmatrix} q_1 & q_1 & \cdots & q_1 \\ q_2 & q_2 & \cdots & q_2 \\ \vdots & & \ddots & \vdots \\ q_k & q_k & \cdots & q_k \end{bmatrix}$$

$$Q = \begin{bmatrix} q_1 & q_1 & \cdots & q_1 \\ q_2 & q_2 & \cdots & q_2 \\ \vdots & & \ddots & \vdots \\ q_k & q_k & \cdots & q_k \end{bmatrix} \quad \text{y} \quad \mathbb{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_k \end{bmatrix}$$

$$
\begin{aligned}
Q\mathbb{x} &= \begin{bmatrix} q_1 & q_1 & \cdots & q_1 \\ q_2 & q_2 & \cdots & q_2 \\ \vdots & & \ddots & \vdots \\ q_k & q_k & \cdots & q_k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} \\
&= \begin{bmatrix} q_1 x_1 + q_1 x_2 + \cdots + q_1 x_k \\ q_2 x_1 + q_2 x_2 + \cdots + q_2 x_k \\ \vdots \\ q_k x_1 + q_k x_2 + \cdots + q_k x_k \end{bmatrix} \\
&= (x_1 + x_2 + \cdots + x_k) \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_k \end{bmatrix} \\
&= (1)\mathbb{q} \\
&= \mathbb{q}
\end{aligned}
$$

$$P^n \mathbb{x} \rightarrow \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_k \end{bmatrix} = \mathbb{q}$$

$$P = \begin{bmatrix} 0.8 & 0.3 & 0.2 \\ 0.1 & 0.2 & 0.6 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & -0.3 & -0.2 \\ -0.1 & 0.8 & -0.6 \\ -0.1 & -0.5 & 0.8 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -\frac{34}{13} \\ 0 & 1 & -\frac{14}{13} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbb{q} = \begin{bmatrix} \frac{34}{13} q_3 \\ \frac{14}{13} q_3 \\ q_3 \end{bmatrix} = q_3 \begin{bmatrix} \frac{34}{13} \\ \frac{14}{13} \\ 1 \end{bmatrix}$$

$$\mathbb{q} = \begin{bmatrix} \frac{34}{61} \\ \frac{14}{61} \\ \frac{13}{61} \end{bmatrix} = \begin{bmatrix} 0.5573\ldots \\ 0.2295\ldots \\ 0.2131\ldots \end{bmatrix}$$

$$\mathbb{q} = \frac{1}{28}\begin{bmatrix} 3 \\ 3 \\ 3 \\ 5 \\ 4 \\ 3 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.1071\ldots \\ 0.1071\ldots \\ 0.1071\ldots \\ 0.1785\ldots \\ 0.1428\ldots \\ 0.1071\ldots \\ 0.1428\ldots \\ 0.1071\ldots \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\begin{bmatrix}
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0
\end{bmatrix}
$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\begin{array}{c c}
 & \begin{array}{ccccc} M & P & H & A & B \end{array} \\
\begin{array}{c} M \\ P \\ H \\ A \\ B \end{array} &
\begin{bmatrix}
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 1 & 1 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 2 & 2 & 3 & 1 \\ 4 & 0 & 2 & 2 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$S^3 = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$S^3 = \begin{bmatrix} 2 & 4 & 0 & 4 & 3 \\ 4 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 3 & 0 & 2 & 1 \\ 3 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A = M + M^2 = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 2 & 0 & 3 & 3 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 2 & 3 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$$\mathbb{p} = \begin{bmatrix} p_1 & p_2 & \cdots & p_m \end{bmatrix} \quad \text{y} \quad \mathbb{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}.$$

$$\mathbb{p} = \begin{bmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \quad \text{y} \quad \mathbb{q} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{3} \\ \frac{1}{6} \end{bmatrix}.$$

$$E(\mathbb{p}, \mathbb{q}) = \begin{bmatrix} p_1 & p_2 & \cdots & p_m \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix} = \mathbb{p} A \mathbb{q}.$$

$$E(\mathbb{p}, \mathbb{q}) = \mathbb{p} A \mathbb{q} = \begin{bmatrix} \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 & 5 & -2 & -1 \\ -2 & 4 & -3 & -4 \\ 6 & -5 & 0 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{3} \\ \frac{1}{6} \end{bmatrix} = \frac{13}{72} \approx 0.1805 \ldots$$

$$\begin{bmatrix} 3 & \boxed{1} \\ -4 & 0 \end{bmatrix}, \qquad \begin{bmatrix} -50 & 30 & -5 \\ 90 & \boxed{60} & 75 \\ 60 & -10 & -30 \end{bmatrix}, \qquad \begin{bmatrix} 0 & -3 & 5 & -9 \\ 15 & -8 & -2 & 10 \\ 7 & 10 & \boxed{6} & 9 \\ 6 & 11 & -3 & 2 \end{bmatrix}.$$

$$\mathbb{p}^* = \begin{bmatrix} 0 & 0 & \cdots & \underset{r\text{-ésima entrada}}{1} & \cdots & 0 \end{bmatrix}, \qquad \mathbb{q}^* = \begin{array}{cl} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} & \begin{array}{l} \\ \\ \\ \leftarrow s\text{-ésima entrada} \\ \\ \\ \end{array} \end{array}$$

$$\begin{bmatrix} 10 & -30 & -20 & 5 \\ 0 & 25 & -5 & 10 \\ 20 & -5 & -15 & -20 \end{bmatrix}$$

$$\mathbb{p}^* = \left[\frac{a_{22} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \quad \frac{a_{11} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \right]$$

$$\mathbb{q}^* = \begin{bmatrix} \dfrac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{12} - a_{21}} \\ \dfrac{a_{11} - a_{21}}{a_{11} + a_{22} - a_{12} - a_{21}} \end{bmatrix}$$

$$
\begin{array}{ccc}
 & & \text{Cepa} \\
 & & \begin{array}{cc} 1 & 2 \end{array} \\
\text{Vacuna} & \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.85 & 0.70 \\ 0.60 & 0.90 \end{bmatrix}
\end{array}
$$

$$\begin{bmatrix} 0.2 & 0.1 & 0.6 \\ 0.4 & 0.5 & 0.1 \\ 0.4 & 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

$$\begin{bmatrix} 0.8 & -0.1 & -0.6 \\ -0.4 & 0.5 & -0.1 \\ -0.4 & -0.4 & 0.7 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = s \begin{bmatrix} 31 \\ 32 \\ 36 \end{bmatrix}$$

$$\mathbb{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix}$$

$$E = \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1k} \\ e_{21} & e_{22} & \cdots & e_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ e_{k1} & e_{k2} & \cdots & e_{kk} \end{bmatrix}$$

$$E = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.2 & 0.1 & 0.6 \\ 0.4 & 0.5 & 0.1 \\ 0.4 & 0.4 & 0.3 \end{bmatrix}$$

$$\mathbb{p} = \begin{bmatrix} 31 \\ 32 \\ 36 \end{bmatrix}$$

$$\mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix},$$

$$\mathbb{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_k \end{bmatrix}$$

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1k} \\ c_{21} & c_{22} & \cdots & c_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ c_{k1} & c_{k2} & \cdots & c_{kk} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.65 & 0.55 \\ 0.25 & 0.05 & 0.10 \\ 0.25 & 0.05 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1.00 & -0.65 & -0.55 \\ -0.25 & 0.95 & -0.10 \\ -0.25 & -0.05 & 1.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 50\,000 \\ 25\,000 \\ 0 \end{bmatrix}.$$

$$\mathbb{x} = (I - C)^{-1}\mathbb{d} = \frac{1}{503}\begin{bmatrix} 756 & 542 & 470 \\ 220 & 690 & 190 \\ 200 & 170 & 630 \end{bmatrix}\begin{bmatrix} 50\,000 \\ 25\,000 \\ 0 \end{bmatrix} = \begin{bmatrix} 102\,087 \\ 56\,163 \\ 28\,330 \end{bmatrix}.$$

$$\mathbb{x} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.65 & 0.55 \\ 0.25 & 0.05 & 0.10 \\ 0.25 & 0.05 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} \mathbb{p}_1^T \\ \mathbb{p}_2^T \\ \vdots \\ \mathbb{p}_n^T \end{bmatrix}$$

$$C = \begin{bmatrix} \mathbb{c}_1^T \\ \mathbb{c}_2^T \\ \vdots \\ \mathbb{c}_n^T \end{bmatrix}$$

$$C = \begin{bmatrix} \mathbb{c}_1^T \\ \mathbb{c}_2^T \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 19 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} \mathbb{p}_1^T \\ \mathbb{p}_2^T \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 1 & 18 \end{bmatrix}$$

$$\mathbb{x}^{(n)} = \begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix}, \quad \mathbb{x}^{(n-1)} = \begin{bmatrix} a_{n-1} \\ b_{n-1} \\ c_{n-1} \end{bmatrix}, \quad M = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D^n = \begin{bmatrix} \lambda_1^n & 0 & \cdots & 0 \\ 0 & \lambda_2^n & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_k^n \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} \mathbb{v}_1 & \mathbb{v}_2 & \mathbb{v}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\mathbb{x}^{(n)} = PD^nP^{-1}\mathbb{x}^{(0)} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \left(\dfrac{1}{2}\right)^n & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

$$\mathbb{x}^{(n)} = \begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix} = \begin{bmatrix} 1 & 1 - \left(\frac{1}{2}\right)^n & 1 - \left(\frac{1}{2}\right)^{n-1} \\ 0 & \left(\frac{1}{2}\right)^n & \left(\frac{1}{2}\right)^{n-1} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

$$= \begin{bmatrix} a_0 + b_0 + c_0 - \left(\frac{1}{2}\right)^n b_0 - \left(\frac{1}{2}\right)^{n-1} c_0 \\ \left(\frac{1}{2}\right)^n b_0 + \left(\frac{1}{2}\right)^{n-1} c_0 \\ 0 \end{bmatrix}$$

$$\mathbb{x}^{(n)} = M^n \mathbb{x}^{(0)}, \qquad \text{donde } M = \begin{bmatrix} 1 & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{4} & 1 \end{bmatrix}$$

$$\mathbb{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbb{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbb{v}_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\mathbb{x}^{(n)} = M^n \mathbb{x}^{(0)} = PD^nP^{-1}\mathbb{x}^{(0)}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \left(\frac{1}{2}\right)^n \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{1}{2} - \left(\frac{1}{2}\right)^{n+1} & 0 \\ 0 & \left(\frac{1}{2}\right)^n & 0 \\ 0 & \frac{1}{2} - \left(\frac{1}{2}\right)^{n+1} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$\mathbb{x}^{(n)} = PD^n P^{-1}\mathbb{x}^{(0)} = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & \left(\dfrac{1}{2}\right)^n \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1-\left(\dfrac{1}{2}\right)^n \\ 0 & \left(\dfrac{1}{2}\right)^n \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} a_0 + b_0 - \left(\dfrac{1}{2}\right)^n b_0 \\ \left(\dfrac{1}{2}\right)^n b_0 \end{bmatrix}$$

			Genotipos de padres (Padre, Madre)					
			(A, AA)	(A, Aa)	(A, aa)	(a, AA)	(a, Aa)	(a, aa)
Descendiente	Macho	A	1	$\frac{1}{2}$	0	1	$\frac{1}{2}$	0
		a	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	1
	Hembra	AA	1	$\frac{1}{2}$	0	0	0	0
		Aa	0	$\frac{1}{2}$	1	1	$\frac{1}{2}$	0
		aa	0	0	0	0	$\frac{1}{2}$	1

$$\mathbb{x}^{(n)} = \begin{bmatrix} a_n \\ b_n \\ c_n \\ d_n \\ e_n \\ f_n \end{bmatrix}, \quad n = 0, 1, 2, \ldots$$

$$M = \begin{array}{c@{}c}
\begin{array}{cccccc} (A,AA) & (A,Aa) & (A,aa) & (a,AA) & (a,Aa) & (a,aa) \end{array} & \\
\begin{bmatrix}
1 & \frac{1}{4} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{4} & 0 & 1 & \frac{1}{4} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{4} & 0 \\
0 & \frac{1}{4} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{4} & 1 & 0 & \frac{1}{4} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{4} & 1
\end{bmatrix} &
\begin{array}{c} (A,AA) \\ (A,Aa) \\ (A,aa) \\ (a,AA) \\ (a,Aa) \\ (a,aa) \end{array}
\end{array}$$

$$P = \begin{bmatrix} 1 & 0 & -1 & \frac{-3-\sqrt{5}}{4} & \frac{-3+\sqrt{5}}{4} \\ 0 & 0 & 2 & -6 & 1 \\ 0 & 0 & -1 & \frac{-1+\sqrt{5}}{4} & \frac{-1-\sqrt{5}}{4} \\ 0 & 0 & 1 & \frac{-1+\sqrt{5}}{4} & \frac{-1-\sqrt{5}}{4} \\ 0 & -2 & 6 & 1 & 1 \\ 0 & 1 & 1 & \frac{-3-\sqrt{5}}{4} & \frac{-3+\sqrt{5}}{4} \end{bmatrix}$$

$$D^n = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{2}\right)^n & 0 & 0 & 0 \\ 0 & 0 & 0 & \left(-\frac{1}{2}\right)^n & 0 & 0 \\ 0 & 0 & 0 & 0 & \left(\frac{1+\sqrt{5}}{4}\right)^n & 0 \\ 0 & 0 & 0 & 0 & 0 & \left(\frac{1-\sqrt{5}}{4}\right)^n \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 1 \\ 0 & \frac{1}{8} & -\frac{1}{4} & \frac{1}{4} & -\frac{1}{8} & 0 \\ 0 & -\frac{1}{24} & -\frac{1}{12} & \frac{1}{12} & \frac{1}{24} & 0 \\ 0 & \frac{5+\sqrt{5}}{20} & \frac{\sqrt{5}}{5} & \frac{\sqrt{5}}{5} & \frac{5+\sqrt{5}}{20} & 0 \\ 0 & \frac{5-\sqrt{5}}{20} & -\frac{\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} & \frac{5-\sqrt{5}}{20} & 0 \end{bmatrix}$$

$$D^n \to \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbb{x}^{(n)} \to P \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} P^{-1} \mathbb{x}^{(0)}$$

$$\mathbb{x}^{(n)} \to \begin{bmatrix} a_0 + \frac{2}{3}b_0 + \frac{1}{3}c_0 + \frac{2}{3}d_0 + \frac{1}{3}e_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_0 + \frac{1}{3}b_0 + \frac{2}{3}c_0 + \frac{1}{3}d_0 + \frac{2}{3}e_0 \end{bmatrix}$$

$$\mathbb{x}^{(n)} \to \begin{bmatrix} \frac{2}{3} \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{3} \end{bmatrix}$$

$$\begin{vmatrix} x^2 & y^2 & z^2 & xy & xz & yz & x & y & z & 1 \\ x_1^2 & y_1^2 & z_1^2 & x_1y_1 & x_1z_1 & y_1z_1 & x_1 & y_1 & z_1 & 1 \\ x_2^2 & y_2^2 & z_2^2 & x_2y_2 & x_2z_2 & y_2z_2 & x_2 & y_2 & z_2 & 1 \\ x_3^2 & y_3^2 & z_3^2 & x_3y_3 & x_3z_3 & y_3z_3 & x_3 & y_3 & z_3 & 1 \\ x_4^2 & y_4^2 & z_4^2 & x_4y_4 & x_4z_4 & y_4z_4 & x_4 & y_4 & z_4 & 1 \\ x_5^2 & y_5^2 & z_5^2 & x_5y_5 & x_5z_5 & y_5z_5 & x_5 & y_5 & z_5 & 1 \\ x_6^2 & y_6^2 & z_6^2 & x_6y_6 & x_6z_6 & y_6z_6 & x_6 & y_6 & z_6 & 1 \\ x_7^2 & y_7^2 & z_7^2 & x_7y_7 & x_7z_7 & y_7z_7 & x_7 & y_7 & z_7 & 1 \\ x_8^2 & y_8^2 & z_8^2 & x_8y_8 & x_8z_8 & y_8z_8 & x_8 & y_8 & z_8 & 1 \\ x_9^2 & y_9^2 & z_9^2 & x_9y_9 & x_9z_9 & y_9z_9 & x_9 & y_9 & z_9 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x_1 & x_2 & x_3 & \cdots & x_n & 1 \\ x_{11} & x_{21} & x_{31} & \cdots & x_{n1} & 1 \\ x_{12} & x_{22} & x_{32} & \cdots & x_{n2} & 1 \\ x_{13} & x_{23} & x_{33} & \cdots & x_{n3} & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{1n} & x_{2n} & x_{3n} & \cdots & x_{nn} & 1 \end{vmatrix} = 0$$

$$A_n = \begin{bmatrix} 4 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 4 & 1 & \cdots & 0 & 0 & 0 \\ \vdots & & \ddots & \ddots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 4 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 4 \end{bmatrix}$$

$$\frac{1}{2\sqrt{3}}\begin{bmatrix} \left(2+\sqrt{3}\right)^{n-1}-\left(2-\sqrt{3}\right)^{n-1} & \left(2-\sqrt{3}\right)^{n-2}-\left(2+\sqrt{3}\right)^{n-2} \\ \left(2+\sqrt{3}\right)^{n-2}-\left(2-\sqrt{3}\right)^{n-2} & \left(2-\sqrt{3}\right)^{n-3}-\left(2+\sqrt{3}\right)^{n-3} \end{bmatrix}$$

$$P = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.6 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.1 \end{bmatrix}$$

$$\mathbb{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{4} \end{bmatrix}$$

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

$$P_2 = \begin{bmatrix} 0 & \frac{1}{2} \\ 1 & \frac{1}{2} \end{bmatrix}, \qquad P_3 = \begin{bmatrix} 0 & 0 & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{3} \\ 1 & \frac{1}{2} & \frac{1}{3} \end{bmatrix}$$

$$P_4 = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{3} & \frac{1}{4} \\ 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \end{bmatrix} \qquad P_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{5} \\ 0 & 0 & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 0 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$