$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix} \longleftarrow i\text{-ésimo renglón}$$

$$A = \begin{bmatrix} 2 & 0 & 3 \\ -2 & 1 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$



$$\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \quad y \quad v =$$

 v_1

 v_{γ}

 v_n

$$\mathbf{u} \bullet \mathbf{0} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$
$$= u_1 \cdot 0 + u_2 \cdot 0 + \dots + u_n \cdot 0$$
$$= 0 + 0 + \dots + 0$$
$$= 0$$

$$\mathbf{u} \bullet \mathbf{v} = \begin{bmatrix} u_2 \\ \vdots \\ u_n \end{bmatrix} \bullet \begin{bmatrix} v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$= u_1 v_1 + u_2 v_2 + \dots + u_n v_n$$

$$= v_1 u_1 + v_2 u_2 + \dots + v_n u_n$$

$$= \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \bullet \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ v_n \end{pmatrix}$$

$$\begin{split} \mathbf{u} \bullet (\mathbf{v} + \mathbf{w}) &= \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \bullet \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{pmatrix} \\ &= u_1(v_1 + w_1) + u_2(v_2 + w_2) + \dots + u_n(v_n + w_n) \\ &= u_1v_1 + u_1w_1 + u_2v_2 + u_2w_2 + \dots + u_nv_n + u_nw_n \\ &= u_1v_1 + u_2v_2 + \dots + u_nv_n + u_1w_1 + u_2w_2 + \dots + u_nw_n \\ &= \mathbf{u} \bullet \mathbf{v} + \mathbf{u} \bullet \mathbf{w} \end{split}$$

$$\begin{split} (\alpha \cdot \mathbf{w}) \bullet \mathbf{v} &= \begin{pmatrix} \alpha u_1 \\ \alpha u_2 \\ \vdots \\ \alpha u_n \end{pmatrix} \bullet \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \\ &= (\alpha u_1) v_1 + (\alpha u_2) v_2 + \dots + (\alpha u_n) v_n \\ &= \alpha u_1 v_1 + \alpha u_2 v_2 + \dots + \alpha u_n v_n \\ &= u_1 \alpha v_1 + u_2 \alpha v_2 + \dots + u_n \alpha v_n \\ &= u_1 (\alpha v_1) + u_2 (\alpha v_2) + \dots + u_n (\alpha v_n) \\ &= \mathbf{w} \bullet (\alpha \cdot \mathbf{v}) \\ &= u_1 (\alpha v_1) + u_2 (\alpha v_2) + \dots + u_n (\alpha v_n) \\ &= u_1 \alpha v_1 + u_2 \alpha v_2 + \dots + u_n \alpha v_n \\ &= \alpha u_1 v_1 + \alpha u_2 v_2 + \dots + \alpha u_n v_n \\ &= \alpha (u_1 v_1) + \alpha (u_2 v_2) + \dots + \alpha (u_n v_n) \\ &= \alpha (u_1 v_1 + u_2 v_2 + \dots + u_n v_n) \\ &= \alpha \cdot (\mathbf{w} \bullet \mathbf{v}) \end{split}$$

$$\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad y \quad v = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 1 & 5 \end{bmatrix} \quad \text{y} \quad B = \begin{bmatrix} 7 & -1 & 4 & 7 \\ 2 & 5 & 0 & -4 \\ -3 & 1 & 2 & 3 \end{bmatrix}.$$

$$AB = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} 7 & -1 & 4 & 7 \\ 2 & 5 & 0 & -4 \\ -3 & 1 & 2 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 23 & -5 & 2 & 5 \\ 15 & 6 & 26 & 39 \end{bmatrix}$$

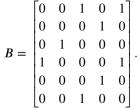
$$=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & \ddots & \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix}$$

$$=\begin{bmatrix} \sum_{q=1}^{p} a_{1q}b_{q1} & \sum_{q=1}^{p} a_{1q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}b_{qn} \\ \sum_{q=1}^{p} a_{2q}b_{q1} & \sum_{q=1}^{p} a_{2q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}b_{qn} \\ \vdots & & \ddots & \end{bmatrix}$$

 $AB = [a_{ij}][b_{jk}]$

$$= \begin{bmatrix} \sum_{q=1}^{p} a_{2q} b_{q1} & \sum_{q=1}^{p} a_{2q} b_{q2} & \cdots & \sum_{q=1}^{p} a_{2q} b_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq} b_{q1} & \sum_{q=1}^{p} a_{mq} b_{q2} & \cdots & \sum_{q=1}^{p} a_{mq} b_{qn} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$



$$C = AB = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 1 \end{bmatrix}.$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = [r_{ij}].$$

$$R^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad W = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{24} \\ w_{31} & w_{32} & w_{33} & w_{34} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

$$Y = \begin{bmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + b_1 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + b_2 \\ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + b_3 \end{bmatrix}.$$

$$B+C = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & \ddots & \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & \ddots & \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} & \cdots & b_{1n} + c_{1n} \\ b_{21} + c_{21} & b_{22} + c_{22} & \cdots & b_{2n} + c_{2n} \\ \vdots & & \ddots & \\ b_{p1} + c_{p1} & b_{p2} + c_{p2} & \cdots & b_{pn} + c_{pn} \end{bmatrix}$$

$$\begin{split} A(B+C) &= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} & \cdots & b_{1n} + c_{1n} \\ b_{21} + c_{21} & b_{22} + c_{22} & \cdots & b_{2n} + c_{2n} \\ \vdots & & \ddots & \vdots \\ b_{p1} + c_{p1} & b_{p2} + c_{p2} & \cdots & b_{pn} + c_{pn} \end{bmatrix} \\ &= \begin{bmatrix} \sum_{q=1}^{p} a_{1q}(b_{q1} + c_{q1}) & \sum_{q=1}^{p} a_{1q}(b_{q2} + c_{q2}) & \cdots & \sum_{q=1}^{p} a_{1q}(b_{qn} + c_{qn}) \\ \sum_{q=1}^{p} a_{2q}(b_{q1} + c_{q1}) & \sum_{q=1}^{p} a_{2q}(b_{q2} + c_{q2}) & \cdots & \sum_{q=1}^{p} a_{2q}(b_{qn} + c_{qn}) \\ \vdots & & \ddots & \vdots \\ \sum_{q=1}^{p} a_{mq}(b_{q1} + c_{q1}) & \sum_{q=1}^{p} a_{mq}(b_{q2} + c_{q2}) & \cdots & \sum_{q=1}^{p} a_{mq}(b_{qn} + c_{qn}) \end{bmatrix} \end{split}$$

$$A(B+C) = \begin{bmatrix} \sum_{q=1}^{p} a_{1q}b_{q1} & \sum_{q=1}^{p} a_{1q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}b_{qn} \\ \sum_{q=1}^{p} a_{2q}b_{q1} & \sum_{q=1}^{p} a_{2q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}b_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}b_{q1} & \sum_{q=1}^{p} a_{mq}b_{q2} & \cdots & \sum_{q=1}^{p} a_{mq}b_{qn} \end{bmatrix}$$

$$+ \begin{bmatrix} \sum_{q=1}^{p} a_{1q}c_{q1} & \sum_{q=1}^{p} a_{1q}c_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}c_{qn} \\ \sum_{q=1}^{p} a_{2q}c_{q1} & \sum_{q=1}^{p} a_{2q}c_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}c_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}c_{q1} & \sum_{q=1}^{p} a_{mq}c_{q2} & \cdots & \sum_{q=1}^{p} a_{mq}c_{qn} \end{bmatrix}$$

=AB+AC

$$A^{2} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A.$$

$$AI_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = A$$

$$T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -9 \end{bmatrix}$$





$$\mathbf{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$
 en vez de $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\mathcal{N}(A) = \left\{ \mathbf{x} \in \mathbb{R}^3 \mid A\mathbf{x} = \mathbf{0} \right\}$$

$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \begin{bmatrix} x_1 + 2x_2 - x_3 \\ 2x_1 - x_2 + 3x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$$

$$\mathcal{N}(A) = \left\{ \begin{pmatrix} x_1 \\ -x_1 \\ -x_1 \end{pmatrix} \mid x_1 \in \mathbb{R} \right\}$$
$$= \left\{ x_1 \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \mid x_1 \in \mathbb{R} \right\}$$
$$= \operatorname{gen} \left(\left\{ \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \right\} \right)$$

$$\begin{split} &=\left\{\begin{pmatrix}y_1\\y_2\end{pmatrix}\in\mathbb{R}^2\mid\begin{bmatrix}1&2&-1\\2&-1&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}y_1\\y_2\end{bmatrix}, \text{ para algún }\mathbf{x}\in\mathbb{R}^3\right\}\\ &=\left\{\begin{pmatrix}y_1\\y_2\end{pmatrix}\in\mathbb{R}^2\mid\begin{bmatrix}x_1+2x_2-x_3\\2x_1-x_2+3x_3\end{bmatrix}=\begin{bmatrix}y_1\\y_2\end{bmatrix}, \text{ para algún }\mathbf{x}\in\mathbb{R}^3\right\}\\ &=\left\{\begin{pmatrix}x_1\\y_1\end{pmatrix}=x_1\begin{pmatrix}1\\2\end{pmatrix}+x_2\begin{pmatrix}2\\-1\end{pmatrix}+x_3\begin{pmatrix}-1\\3\end{pmatrix}\mid x_1,x_2,x_3\in\mathbb{R}\right\}\\ &=\text{gen}\left(\left\{\begin{pmatrix}1\\2\end{pmatrix},\begin{pmatrix}2\\-1\end{pmatrix},\begin{pmatrix}-1\\3\end{pmatrix}\right\}\right) \end{split}$$

 $\mathcal{R}(A) = \{ y \in \mathbb{R}^2 \mid Ax = y, \text{ para algún } x \in \mathbb{R}^3 \}$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & c_{j} & \cdots & c_{n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix} \quad \begin{matrix} \mathbf{r}_{1} \\ \mathbf{r}_{2} \\ \vdots \\ \mathbf{r}_{m} \end{matrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \mathbf{y} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

$$=\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{bmatrix}$$

$$\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots + x$$

 $a_{1n} \\ a_{2n}$

 a_{mn}

$$\begin{aligned} a_{1j} &= \alpha_{11} s_{1j} + \alpha_{12} s_{2j} + \dots + \alpha_{1k} s_{kj} \\ a_{2j} &= \alpha_{21} s_{1j} + \alpha_{22} s_{2j} + \dots + \alpha_{2k} s_{kj} \\ &\vdots \\ a_{mj} &= \alpha_{m1} s_{1j} + \alpha_{m2} s_{2j} + \dots + \alpha_{mk} s_{kj} \end{aligned} \qquad \text{donde } s_i = \begin{pmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{in} \end{pmatrix}$$

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix} = s_{1j} \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix} + s_{2j} \begin{bmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{bmatrix} + \dots + s_{kj} \begin{bmatrix} \alpha_{1k} \\ \alpha_{2k} \\ \vdots \\ \alpha_{mk} \end{bmatrix}$$

$$R_A = \operatorname{gen}\left(\left\{\begin{pmatrix}1\\2\\-1\end{pmatrix}, \begin{pmatrix}2\\-1\\3\end{pmatrix}\right\}\right), \ C_A = \mathcal{R}(A) = \operatorname{gen}\left(\left\{\begin{pmatrix}2\\-1\end{pmatrix}, \begin{pmatrix}-1\\3\end{pmatrix}\right\}\right).$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

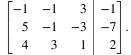
$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & \ddots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{bmatrix}.$$

$$7x_1 + 2x_2 + 4x_3 = 9$$

$$2x_1 + 4x_2 - 3x_3 = 1$$

$$3x_1 - 8x_2 - 5x_3 = 0$$
es
$$\begin{bmatrix} 7 & 2 & 4 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & -8 & -5 & 0 \end{bmatrix}.$$



$$\begin{bmatrix} -1 & -1 & 3 & -1 \\ 5 & -1 & -3 & -7 \\ 4 & 3 & 1 & 2 \end{bmatrix} \xrightarrow{r_2 \leftarrow r_2 + 5r_1} \begin{bmatrix} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 4 & 3 & 1 & 2 \end{bmatrix}$$

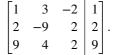
$$\begin{bmatrix} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 4 & 3 & 1 & 2 \end{bmatrix} \xrightarrow{\mathfrak{r}_3 \leftarrow \mathfrak{r}_3 + 4\mathfrak{r}_1} \begin{bmatrix} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{bmatrix} \xrightarrow{\mathfrak{r}_1 \leftarrow (-1)\mathfrak{r}_1} \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{bmatrix} \xrightarrow{\mathfrak{r}_2 \leftarrow \left(-\frac{1}{6}\right)\mathfrak{r}_2} \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 13 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 13 & -2 \end{bmatrix} \xrightarrow{\mathfrak{r}_3 \leftarrow \mathfrak{r}_3 + \mathfrak{r}_2} \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 11 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 11 & 0 \end{bmatrix} \xrightarrow{r_3 \leftarrow \left(\frac{1}{11}\right)r_3} \begin{bmatrix} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} = R$$



$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 2 & -9 & 2 & 2 \\ 9 & 4 & 2 & 9 \end{bmatrix} \xrightarrow{\mathbb{F}_2 \leftarrow \mathbb{F}_2 + (-2)\mathbb{F}_1} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 9 & 4 & 2 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 9 & 4 & 2 & 9 \end{bmatrix} \xrightarrow{\mathbb{F}_3 \leftarrow \mathbb{F}_3 + (-9)\mathbb{F}_1} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix} \xrightarrow{r_2 \leftarrow \left(\frac{1}{3}\right) r_2} \begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -2 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix} \xrightarrow{\mathfrak{r}_1 \leftarrow \mathfrak{r}_1 + \mathfrak{r}_2} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{bmatrix} \xrightarrow{r_3 \leftarrow r_3 + (-10)r_2} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 27 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 27 & 0 & 0 \end{bmatrix} \xrightarrow{r_3 \leftarrow \left(\frac{1}{27}\right) r_3} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{\mathbb{F}_2 \leftarrow \mathbb{F}_2 + 5\mathbb{F}_3} \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

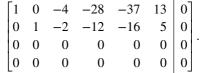
$$\begin{bmatrix} 1 & -2 & 0 & | & 1 \\ 0 & 0 & 2 & | & 0 \\ 0 & 1 & 0 & | & 0 \end{bmatrix} \xrightarrow{r_1 \leftarrow r_1 + 2r_3} \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 0 & 2 & | & 0 \\ 0 & 1 & 0 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \xrightarrow{r_2 \neq r_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \xrightarrow{\mathfrak{r}_3 \leftarrow \left(\frac{1}{2}\right)\mathfrak{r}_3} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = R$$

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$





$$S = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = r \begin{pmatrix} 4 \\ 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 28 \\ 12 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 37 \\ 16 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u \begin{pmatrix} -13 \\ -5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \mid r, s, t, u \in \mathbb{R} \right\}.$$

$$\mathcal{N}(A) = \operatorname{gen} \left\{ \begin{cases} \binom{4}{2} \\ 1 \\ 0 \\ 0 \\ 0 \end{cases}, \begin{pmatrix} 28 \\ 12 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 37 \\ 16 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -13 \\ -5 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\},$$

$$A = \begin{bmatrix} 1 & 1 & 6 & 0 \\ 9 & 7 & 2 & 0 \\ 2 & 9 & 4 & 0 \\ 6 & 3 & 5 & 0 \end{bmatrix}.$$

$$c_1 = \begin{pmatrix} 1 \\ 9 \\ 2 \\ 6 \end{pmatrix}, \quad c_2 = \begin{pmatrix} 1 \\ 7 \\ 9 \\ 3 \end{pmatrix}, \quad c_3 = \begin{pmatrix} 6 \\ 2 \\ 4 \\ 5 \end{pmatrix}, \quad 0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$





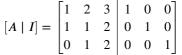


$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$



$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\mathfrak{r}_1 \leftarrow \mathfrak{r}_1 + (-2)\mathfrak{r}_2} \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_2 \leftarrow r_2 + (-1)r_2} \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{r_1 \leftarrow r_1 + (-1)r_2} \begin{bmatrix} 0 & 0 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\mathfrak{r}_1 \leftarrow (-1)\mathfrak{r}_1} \begin{bmatrix} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\mathfrak{r}_3 \leftarrow \mathfrak{r}_3 + (-2)\mathfrak{r}_1} \begin{bmatrix} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{bmatrix} \xrightarrow{\mathfrak{r}_1 \rightleftarrows \mathfrak{r}_2} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{bmatrix} \xrightarrow{r_2 \neq r_3} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}.$$

$$\mathbf{x} = A^{-1} \mathbb{b} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix} = \begin{bmatrix} -14 \\ -13 \\ 15 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 1 \\ 2 & 5 & 3 & 5 & 6 \\ 1 & 0 & 8 & 9 & -6 \end{bmatrix} \longrightarrow \cdots \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -7 \\ 4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix}; \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \\ 2 \end{pmatrix}; \begin{pmatrix} \sqrt{18} \\ \sqrt{32} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \pi \\ 3\pi \\ 3 \end{pmatrix}; \begin{pmatrix} \pi^2 \\ -9\pi \\ \pi^3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}; \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

$$\mathbf{w} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 3 \\ -7 \end{pmatrix}$$

 $\begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix}$.

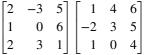
$$\begin{bmatrix} 7 & 1 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 4 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 5 & 1 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 \\ 0 & 4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 7 & 1 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 6 \\ -2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -3 & 5 \\ 1 & 0 & 6 \\ 2 & 3 & 1 \end{bmatrix}$$

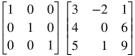
$$\begin{bmatrix} 1 & 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ 2 & 4 \\ 1 & 0 \\ -2 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 0 & 6 \\ 5 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 & -2 \\ -6 & 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & -2 \\ -1 & 3 & 2 \\ 1 & 1 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_m \end{bmatrix} B = \begin{bmatrix} \mathbf{a}_1 B \\ \mathbf{a}_2 B \\ \vdots \\ \mathbf{a}_m B \end{bmatrix}$$

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$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}.$$

$$A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 3 & -2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 6 \\ -2 & 4 \\ 0 & 5 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{y} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad \text{y} \quad B = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$





$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}; \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}; \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$







$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 5 & -1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \\ -3 & 6 & 3 \end{bmatrix}$$

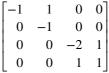






$$\begin{bmatrix} 3 & -2 & 3 \\ -3 & 1 & -1 \\ 0 & 0 & -2 \\ -1 & -3 & 2 \end{bmatrix}$$

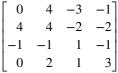




$$\begin{bmatrix} -3 & 0 & -1 & -1 \\ -1 & 4 & 4 & -1 \\ 0 & 2 & 3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ -2 & 2 & -4 & -6 \\ 2 & -2 & 4 & 6 \\ 3 & -3 & 6 & 9 \end{bmatrix}$$























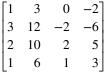


$$\begin{bmatrix} \alpha & \beta & \gamma \\ \delta & \epsilon & \zeta \\ 3\alpha - 2\delta & 3\beta - 2\epsilon & 3\delta - 2\zeta \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 1 & 0 & 4 \\ 2 & 1 & -1 & 3 \\ -1 & 0 & 5 & 7 \end{bmatrix}$$







































$$\begin{bmatrix} -\pi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





























$$A = \begin{bmatrix} 1 & -2 & 2 \\ 4 & 5 & 1 \\ 0 & 3 & -1 \end{bmatrix}, \quad \mathbb{b}_1 = \begin{bmatrix} 0 \\ 1 \\ 7 \end{bmatrix}, \quad \mathbb{b}_2 = \begin{bmatrix} 11 \\ 5 \\ 3 \end{bmatrix}, \quad \mathbb{b}_3 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}.$$