$$\begin{array}{cl}
\begin{bmatrix}
a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\
a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn}
\end{bmatrix}
&
\begin{array}{l}
\\ \\ \\
\longleftarrow i\text{-ésimo renglón} \\
\\ \\
\end{array}
\\
\qquad\qquad \uparrow\; j\text{-ésima columna} &
\end{array}$$

$$A = \begin{bmatrix} 2 & 0 & 3 \\ -2 & 1 & 8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & \ddots & \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbb{u} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \quad \text{y} \quad \mathbb{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}.$$

$$
\begin{aligned}
\mathbb{u} \bullet \mathbb{0} &= \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \\
&= u_1 \cdot 0 + u_2 \cdot 0 + \cdots + u_n \cdot 0 \\
&= 0 + 0 + \cdots + 0 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{u} \bullet \mathbb{v} &= \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \bullet \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \\
&= u_1 v_1 + u_2 v_2 + \cdots + u_n v_n \\
&= v_1 u_1 + v_2 u_2 + \cdots + v_n u_n \\
&= \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \bullet \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \\
&= \mathbb{v} \bullet \mathbb{u}
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{u} \bullet (\mathbb{v} + \mathbb{w}) &= \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} \bullet \begin{pmatrix} v_1 + w_1 \\ v_2 + w_2 \\ \vdots \\ v_n + w_n \end{pmatrix} \\
&= u_1(v_1 + w_1) + u_2(v_2 + w_2) + \cdots + u_n(v_n + w_n) \\
&= u_1 v_1 + u_1 w_1 + u_2 v_2 + u_2 w_2 + \cdots + u_n v_n + u_n w_n \\
&= u_1 v_1 + u_2 v_2 + \cdots + u_n v_n + u_1 w_1 + u_2 w_2 + \cdots + u_n w_n \\
&= \mathbb{u} \bullet \mathbb{v} + \mathbb{u} \bullet \mathbb{w}
\end{aligned}
$$

$$
\begin{aligned}
(\alpha \cdot \mathbb{u}) \bullet \mathbb{v} &= \begin{pmatrix} \alpha u_1 \\ \alpha u_2 \\ \vdots \\ \alpha u_n \end{pmatrix} \bullet \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \\
&= (\alpha u_1)v_1 + (\alpha u_2)v_2 + \cdots + (\alpha u_n)v_n \\
&= \alpha u_1 v_1 + \alpha u_2 v_2 + \cdots + \alpha u_n v_n \\
&= u_1 \alpha v_1 + u_2 \alpha v_2 + \cdots + u_n \alpha v_n \\
&= u_1(\alpha v_1) + u_2(\alpha v_2) + \cdots + u_n(\alpha v_n) \\
&= \mathbb{u} \bullet (\alpha \cdot \mathbb{v}) \\
&= u_1(\alpha v_1) + u_2(\alpha v_2) + \cdots + u_n(\alpha v_n) \\
&= u_1 \alpha v_1 + u_2 \alpha v_2 + \cdots + u_n \alpha v_n \\
&= \alpha u_1 v_1 + \alpha u_2 v_2 + \cdots + \alpha u_n v_n \\
&= \alpha(u_1 v_1) + \alpha(u_2 v_2) + \cdots + \alpha(u_n v_n) \\
&= \alpha\left(u_1 v_1 + u_2 v_2 + \cdots + u_n v_n\right) \\
&= \alpha \cdot (\mathbb{u} \bullet \mathbb{v})
\end{aligned}
$$

$$\mathbb{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \text{y} \quad \mathbb{v} = \begin{pmatrix} 4 \\ 5 \\ -2 \end{pmatrix}.$$

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 1 & 5 \end{bmatrix} \quad \text{y} \quad B = \begin{bmatrix} 7 & -1 & 4 & 7 \\ 2 & 5 & 0 & -4 \\ -3 & 1 & 2 & 3 \end{bmatrix}.$$

$$
AB = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} 7 & -1 & 4 & 7 \\ 2 & 5 & 0 & -4 \\ -3 & 1 & 2 & 3 \end{bmatrix}
$$

$$
= \begin{bmatrix} 23 & -5 & 2 & 5 \\ 15 & 6 & 26 & 39 \end{bmatrix}
$$

$$
\begin{aligned}
AB &= [a_{ij}][b_{jk}] \\
&= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & \ddots & \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{q=1}^{p} a_{1q}b_{q1} & \sum_{q=1}^{p} a_{1q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}b_{qn} \\ \sum_{q=1}^{p} a_{2q}b_{q1} & \sum_{q=1}^{p} a_{2q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}b_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}b_{q1} & \sum_{q=1}^{p} a_{mq}b_{q2} & \cdots & \sum_{q=1}^{p} a_{mq}b_{qn} \end{bmatrix} \\
&= [c_{ik}]
\end{aligned}
$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$B = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

$$C = AB = \begin{bmatrix} 0 & 0 & 0 & 2 & 0 \\ 1 & 0 & 2 & 0 & 2 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 1 \end{bmatrix}.$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = [r_{ij}].$$

$$R^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}.$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad W = \begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{24} \\ w_{31} & w_{32} & w_{33} & w_{34} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

$$Y = \begin{bmatrix} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 + w_{14}x_4 + b_1 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 + w_{24}x_4 + b_2 \\ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 + w_{34}x_4 + b_3 \end{bmatrix}.$$

$$
\begin{aligned}
B + C &= \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & & \ddots & \\ b_{p1} & b_{p2} & \cdots & b_{pn} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & & \ddots & \\ c_{p1} & c_{p2} & \cdots & c_{pn} \end{bmatrix} \\
&= \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} & \cdots & b_{1n} + c_{1n} \\ b_{21} + c_{21} & b_{22} + c_{22} & \cdots & b_{2n} + c_{2n} \\ \vdots & & \ddots & \\ b_{p1} + c_{p1} & b_{p2} + c_{p2} & \cdots & b_{pn} + c_{pn} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
A(B+C) &= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mp} \end{bmatrix} \begin{bmatrix} b_{11}+c_{11} & b_{12}+c_{12} & \cdots & b_{1n}+c_{1n} \\ b_{21}+c_{21} & b_{22}+c_{22} & \cdots & b_{2n}+c_{2n} \\ \vdots & & \ddots & \\ b_{p1}+c_{p1} & b_{p2}+c_{p2} & \cdots & b_{pn}+c_{pn} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{q=1}^{p} a_{1q}(b_{q1}+c_{q1}) & \sum_{q=1}^{p} a_{1q}(b_{q2}+c_{q2}) & \cdots & \sum_{q=1}^{p} a_{1q}(b_{qn}+c_{qn}) \\ \sum_{q=1}^{p} a_{2q}(b_{q1}+c_{q1}) & \sum_{q=1}^{p} a_{2q}(b_{q2}+c_{q2}) & \cdots & \sum_{q=1}^{p} a_{2q}(b_{qn}+c_{qn}) \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}(b_{q1}+c_{q1}) & \sum_{q=1}^{p} a_{mq}(b_{q2}+c_{q2}) & \cdots & \sum_{q=1}^{p} a_{mq}(b_{qn}+c_{qn}) \end{bmatrix}
\end{aligned}
$$

$$
A(B+C) = \begin{bmatrix} \sum_{q=1}^{p} a_{1q}b_{q1} & \sum_{q=1}^{p} a_{1q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}b_{qn} \\ \sum_{q=1}^{p} a_{2q}b_{q1} & \sum_{q=1}^{p} a_{2q}b_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}b_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}b_{q1} & \sum_{q=1}^{p} a_{mq}b_{q2} & \cdots & \sum_{q=1}^{p} a_{mq}b_{qn} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \sum_{q=1}^{p} a_{1q}c_{q1} & \sum_{q=1}^{p} a_{1q}c_{q2} & \cdots & \sum_{q=1}^{p} a_{1q}c_{qn} \\ \sum_{q=1}^{p} a_{2q}c_{q1} & \sum_{q=1}^{p} a_{2q}c_{q2} & \cdots & \sum_{q=1}^{p} a_{2q}c_{qn} \\ \vdots & & \ddots & \\ \sum_{q=1}^{p} a_{mq}c_{q1} & \sum_{q=1}^{p} a_{mq}c_{q2} & \cdots & \sum_{q=1}^{p} a_{mq}c_{qn} \end{bmatrix}
$$

$$
= AB + AC
$$

$$A^2 = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A.$$

$$AI_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = A$$

$$A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -9 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 6 & 1 \\ 0 & 2 & 7 \\ 0 & 0 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 4 & 3 & 0 \\ 8 & 5 & 1 \end{bmatrix}$$

$$A\mathbb{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}.$$

$$\mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \text{ en vez de } \mathbb{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}.$$

$$
\begin{aligned}
\mathcal{N}(A) &= \left\{ \mathbb{x} \in \mathbb{R}^3 \mid A\mathbb{x} = \mathbb{0} \right\} \\
&= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \\
&= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3 \mid \begin{bmatrix} x_1 + 2x_2 - x_3 \\ 2x_1 - x_2 + 3x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{N}(\boldsymbol{A}) &= \left\{ \begin{pmatrix} x_1 \\ -x_1 \\ -x_1 \end{pmatrix} \mid x_1 \in \mathbb{R} \right\} \\
&= \left\{ x_1 \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \mid x_1 \in \mathbb{R} \right\} \\
&= \operatorname{gen}\left(\left\{ \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \right\} \right)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{R}(A) &= \left\{ \mathbb{y} \in \mathbb{R}^2 \mid A\mathbb{x} = \mathbb{y},\ \text{para algún } \mathbb{x} \in \mathbb{R}^3 \right\} \\
&= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2 \mid \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},\ \text{para algún } \mathbb{x} \in \mathbb{R}^3 \right\} \\
&= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in \mathbb{R}^2 \mid \begin{bmatrix} x_1 + 2x_2 - x_3 \\ 2x_1 - x_2 + 3x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix},\ \text{para algún } \mathbb{x} \in \mathbb{R}^3 \right\} \\
&= \left\{ \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 3 \end{pmatrix} \mid x_1, x_2, x_3 \in \mathbb{R} \right\} \\
&= \operatorname{gen}\left(\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix} \right\} \right)
\end{aligned}
$$

$$
A = \begin{array}{c c}
\begin{array}{cccccc} \mathbb{c}_1 & \mathbb{c}_2 & \cdots & \mathbb{c}_j & \cdots & \mathbb{c}_n \end{array} & \\
\begin{bmatrix}
a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\
a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn}
\end{bmatrix}
&
\begin{array}{c} \mathbb{r}_1 \\ \mathbb{r}_2 \\ \vdots \\ \mathbb{r}_i \\ \vdots \\ \mathbb{r}_m \end{array}
\end{array}
$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \text{y} \quad \mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}.$$

$$A\mathbb{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

$$A\mathbb{x} = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}.$$

$$
\begin{aligned}
a_{1j} &= \alpha_{11}s_{1j} + \alpha_{12}s_{2j} + \cdots + \alpha_{1k}s_{kj} \\
a_{2j} &= \alpha_{21}s_{1j} + \alpha_{22}s_{2j} + \cdots + \alpha_{2k}s_{kj} \\
&\vdots \\
a_{mj} &= \alpha_{m1}s_{1j} + \alpha_{m2}s_{2j} + \cdots + \alpha_{mk}s_{kj}
\end{aligned}
\qquad \text{donde } \mathbb{s}_i = \begin{pmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{in} \end{pmatrix}
$$

$$\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix} = s_{1j} \underbrace{\begin{bmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{bmatrix}}_{\mathbb{u}_1} + s_{2j} \underbrace{\begin{bmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{bmatrix}}_{\mathbb{u}_2} + \cdots + s_{kj} \underbrace{\begin{bmatrix} \alpha_{1k} \\ \alpha_{2k} \\ \vdots \\ \alpha_{mk} \end{bmatrix}}_{\mathbb{u}_k}$$

$$R_A = \operatorname{gen}\left(\left\{\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}\right\}\right),\ C_A = \mathcal{R}(A) = \operatorname{gen}\left(\left\{\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \end{pmatrix}\right\}\right).$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & \ddots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right].$$

$$\begin{aligned} 7x_1 + 2x_2 + 4x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 - 8x_2 - 5x_3 &= 0 \end{aligned} \qquad \text{es} \qquad \left[\begin{array}{rrr|r} 7 & 2 & 4 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & -8 & -5 & 0 \end{array}\right].$$

$$\left[\begin{array}{rrr|r} -1 & -1 & 3 & -1 \\ 5 & -1 & -3 & -7 \\ 4 & 3 & 1 & 2 \end{array}\right].$$

$$\left[\begin{array}{rrr|r} -1 & -1 & 3 & -1 \\ 5 & -1 & -3 & -7 \\ 4 & 3 & 1 & 2 \end{array}\right] \xrightarrow{\mathbb{r}_2 \leftarrow \mathbb{r}_2 + 5\mathbb{r}_1} \left[\begin{array}{rrr|r} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 4 & 3 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 4 & 3 & 1 & 2 \end{array}\right] \xrightarrow{r_3 \leftarrow r_3 + 4r_1} \left[\begin{array}{rrr|r} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 3 & -1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow (-1)\mathbb{r}_1} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & -6 & 12 & -12 \\ 0 & -1 & 13 & -2 \end{array}\right] \xrightarrow{\mathbb{r}_2 \leftarrow \left(-\frac{1}{6}\right)\mathbb{r}_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 13 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & -1 & 13 & -2 \end{array}\right] \xrightarrow{r_3 \leftarrow r_3 + r_2} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 11 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 11 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \left(\frac{1}{11}\right)\mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & 1 & -3 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] = R$$

$$S = \left\{ \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \right\}.$$

$$\left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 2 & -9 & 2 & 2 \\ 9 & 4 & 2 & 9 \end{array}\right].$$

$$\left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 2 & -9 & 2 & 2 \\ 9 & 4 & 2 & 9 \end{array}\right] \xrightarrow{\mathbb{r}_2 \leftarrow \mathbb{r}_2 + (-2)\mathbb{r}_1} \left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 9 & 4 & 2 & 9 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 9 & 4 & 2 & 9 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \mathbb{r}_3 + (-9)\mathbb{r}_1} \left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -15 & 6 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_2 \leftarrow \left(\frac{1}{3}\right)\mathbb{r}_2} \left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 1 & 3 & -2 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow \mathbb{r}_1 + \mathbb{r}_2} \left[\begin{array}{rrr|r} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & -23 & 20 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \mathbb{r}_3 + (-10)\mathbb{r}_2} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 27 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 27 & 0 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \left(\frac{1}{27}\right)\mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_2 \leftarrow \mathbb{r}_2 + 5\mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow \mathbb{r}_1 + 2\mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_2 \rightleftarrows \mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \left(\frac{1}{2}\right)\mathbb{r}_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] = R$$

$$S = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}.$$

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}.$$

$$\left[\begin{array}{rrrrrr|r} -1 & 2 & 0 & 4 & 5 & -3 & 0 \\ 3 & -7 & 2 & 0 & 1 & 4 & 0 \\ 2 & -5 & 2 & 4 & 6 & 1 & 0 \\ 4 & -9 & 2 & -4 & -4 & 7 & 0 \end{array}\right].$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & -4 & -28 & -37 & 13 & 0 \\ 0 & 1 & -2 & -12 & -16 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

$$S=\left\{\begin{pmatrix}x_1\\x_2\\x_3\\x_4\\x_5\\x_6\end{pmatrix}=r\begin{pmatrix}4\\2\\1\\0\\0\\0\end{pmatrix}+s\begin{pmatrix}28\\12\\0\\1\\0\\0\end{pmatrix}+t\begin{pmatrix}37\\16\\0\\0\\1\\0\end{pmatrix}+u\begin{pmatrix}-13\\-5\\0\\0\\0\\1\end{pmatrix}\ \middle|\ r,s,t,u\in\mathbb{R}\right\}.$$

$$\mathcal{N}(A) = \text{gen}\left(\left\{\begin{pmatrix}4\\2\\1\\0\\0\\0\end{pmatrix}, \begin{pmatrix}28\\12\\0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}37\\16\\0\\0\\1\\0\end{pmatrix}, \begin{pmatrix}-13\\-5\\0\\0\\0\\1\end{pmatrix}\right\}\right),$$

$$A = \begin{bmatrix} 1 & 1 & 6 & 0 \\ 9 & 7 & 2 & 0 \\ 2 & 9 & 4 & 0 \\ 6 & 3 & 5 & 0 \end{bmatrix}.$$

$$\mathbb{c}_1 = \begin{pmatrix} 1 \\ 9 \\ 2 \\ 6 \end{pmatrix}, \quad \mathbb{c}_2 = \begin{pmatrix} 1 \\ 7 \\ 9 \\ 3 \end{pmatrix}, \quad \mathbb{c}_3 = \begin{pmatrix} 6 \\ 2 \\ 4 \\ 5 \end{pmatrix}, \quad \mathbb{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 1 & 4 & 4 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & -1 & 3 & 6 \\ 4 & 4 & 10 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

$$[A \mid I] = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow \mathbb{r}_1 + (-2)\mathbb{r}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccr|ccr} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{r_2 \leftarrow r_2 + (-1) r_2} \left[\begin{array}{ccr|ccr} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & -2 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow \mathbb{r}_1 + (-1)\mathbb{r}_2} \left[\begin{array}{ccc|ccc} 0 & 0 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbb{r}_1 \leftarrow (-1)\mathbb{r}_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\mathbb{r}_3 \leftarrow \mathbb{r}_3 + (-2)\mathbb{r}_1} \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & -1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{array}\right] \xrightarrow{\mathbb{r}_1 \rightleftarrows \mathbb{r}_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \end{array}\right] \xrightarrow{r_2 \rightleftarrows r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & -1 & 1 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}, \quad \mathbb{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbb{b} = \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix}.$$

$$\mathbb{x} = A^{-1}\mathbb{b} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 17 \end{bmatrix} = \begin{bmatrix} -14 \\ -13 \\ 15 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c|r} 1 & 2 & 3 & 4 & 1 \\ 2 & 5 & 3 & 5 & 6 \\ 1 & 0 & 8 & 9 & -6 \end{array}\right] \longrightarrow \cdots \longrightarrow \left[\begin{array}{ccc|c|r} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{array}\right]$$

$$\begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \end{pmatrix}; \begin{pmatrix} 3 \\ -7 \\ 4 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix};\ \begin{pmatrix} 1 \\ 6 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{2} \\ -\sqrt{2} \\ 2 \end{pmatrix};\ \begin{pmatrix} \sqrt{18} \\ \sqrt{32} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \pi \\ 3\pi \\ 3 \end{pmatrix};\ \begin{pmatrix} \pi^2 \\ -9\pi \\ \pi^3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix};\ \begin{pmatrix} y \\ z \\ x \end{pmatrix}$$

$$\mathbb{u} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \quad \mathbb{v} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}, \quad \mathbb{w} = \begin{pmatrix} -6 \\ 8 \\ 0 \end{pmatrix}.$$

$$\begin{bmatrix} 7 & 1 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 6 \\ 0 & 4 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 5 & 1 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 6 \\ 0 & 4 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 7 & 1 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 6 \\ -2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -3 & 5 \\ 1 & 0 & 6 \\ 2 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ 2 & 4 \\ 1 & 0 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ 1 & 0 & 6 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \\ -2 & 3 & 5 \\ 1 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 0 & 6 \\ 5 & 1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 & -2 \\ -6 & 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & -2 \\ -1 & 3 & 2 \\ 1 & 1 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 4 & 0 & 6 \\ 5 & 1 & 9 \end{bmatrix}$$

$$AB = \begin{bmatrix} \mathbb{a}_1 \\ \mathbb{a}_2 \\ \vdots \\ \mathbb{a}_m \end{bmatrix} B = \begin{bmatrix} \mathbb{a}_1 B \\ \mathbb{a}_2 B \\ \vdots \\ \mathbb{a}_m B \end{bmatrix}.$$

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}.$$

$$A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 3 & -2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 6 \\ -2 & 4 \\ 0 & 5 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{y} \quad B = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} \quad \text{y} \quad B = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$

$$\begin{pmatrix} 7 \\ -5 \\ 4 \\ 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 4 \\ -3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix};\ \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}; \ \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} a \\ 0 \\ b \\ 0 \\ c \end{pmatrix}; \quad \begin{pmatrix} 0 \\ d \\ 0 \\ e \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ -1 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 5 & -1 & 8 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 & 1 \\ 2 & -4 & -2 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & -3 \\ 0 & 0 & -1 \\ -3 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & 4 & 3 \\ 1 & 0 & 6 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & 2 & 0 \\ 0 & 0 & 1 & 6 \\ 1 & 0 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 & 3 \\ -3 & 1 & -1 \\ 0 & 0 & -2 \\ -1 & -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 0 & -1 & -1 \\ -1 & 4 & 4 & -1 \\ 0 & 2 & 3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 & 3 \\ -2 & 2 & -4 & -6 \\ 2 & -2 & 4 & 6 \\ 3 & -3 & 6 & 9 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 4 & 0 & -2 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 4 & -3 & -1 \\ 4 & 4 & -2 & -2 \\ -1 & -1 & 1 & -1 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 3 \\ 3 & 4 & -2 \\ -1 & 5 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 24 & 48 \\ 0 & -3 & 12 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 8 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -3 \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \alpha & \beta & \gamma \\ \delta & \varepsilon & \zeta \\ 3\alpha - 2\delta & 3\beta - 2\varepsilon & 3\delta - 2\zeta \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 1 & 3 & 3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 1 & 0 & 4 \\ 2 & 1 & -1 & 3 \\ -1 & 0 & 5 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & -3 & 0 \\ 0 & -3 & -2 & 4 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & -2 \\ 3 & 12 & -2 & -6 \\ 2 & 10 & 2 & 5 \\ 1 & 6 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 3 & 4 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 0 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -\pi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 2 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 5 & 1 & 0 & -10 \\ -4 & 2 & 1 & 8 \\ 3 & 6 & 0 & -5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 4 & 5 & 1 \\ 0 & 3 & -1 \end{bmatrix}, \quad \mathbb{b}_1 = \begin{bmatrix} 0 \\ 1 \\ 7 \end{bmatrix}, \quad \mathbb{b}_2 = \begin{bmatrix} 11 \\ 5 \\ 3 \end{bmatrix}, \quad \mathbb{b}_3 = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}.$$