Consumer welfare beyond GDP

Antoine Germain*

May 28, 2025

Abstract

In a general consumption space where agents may differ in their preferences, endowments and prices, this paper builds a theory of consumer welfare on axioms reflecting the ethics of equality of opportunity: unequal budgets create welfare-relevant inequalities but heterogeneous preferences do not. When combined with an appropriate cross-economy robustness condition, these axioms single out a consumer welfare measure. Like aggregate consumption in national accounts, this measure sums up individual expenditure functions. Unlike national accounts, these functions are evaluated at a common price vector for all individuals. I show that standard measures of cost of living, standards of living, and purchasing power parity can be modified to reflect welfare as equality of opportunity without additional data.

*F.R.S.-F.N.R.S. and CORE/LIDAM, UCLouvain. I am grateful to François Maniquet for invaluable guidance on this project. I thank Benoit Decerf, Jan Eeckhout and Malka Guillot, Johannes Johnen, Rigas Oikonomou, Amma Panin and Bruno Van der Linden for helpful comments.

1 Introduction

When consumers have heterogeneous preferences and budgets, measuring individual and collective consumer welfare is riddled with obstacles. Practitioners are routinely facing such dilemmas and have to make choices notably on how to (i) compare consumers' with heterogeneous preferences interpersonally, (ii) deflate their consumption spawned by different prices and (iii) assess collective welfare changes.

For example, the Penn World Table[1] deflates each country's consumption by some "world" prices and compare real income across countries which is assimilated to welfare. Deaton and Heston (2010) criticized this measure of income of nations because (i) it neglects international taste dispersion, (ii) the deflation is plutocratic as world prices are influenced more by richer countries and (iii) it does not account for inequalities.[2]

The present paper proposes an integrated theory that compares the welfare of consumers both collectively and interpersonally while informing on deflation practices. I build axiomatically a measure of consumer welfare in a general consumption space where agents may have heterogeneous preferences while they received heterogeneous endowments and may face heterogeneous prices.

Axioms pinning down welfare shall reflect the ethics of equality of opportunity: one the one hand, inequalities in consumption due to unequal budgets should be reduced but on the other hand, inequalities in consumption spawned by heterogeneous preferences are unproblematic. Such an approach resonates with the debate on building economic indicators *beyond GDP* (Fleurbaey, 2009; Fleurbaey & Blanchet, 2013).

Comparing agents with heterogeneous preferences is notoriously difficult since Arrow (1950) celebrated impossibility theorem. Indeed, there is no obvious way of summing up utilities when utility functions are differing across individuals. The present paper escapes Arrow's impossibility by using the social choice theory of fairness initiated by Fleurbaey and Maniquet (2006, 2011, 2018). It consists in using information on indifference curves[3] while Arrow's theorem only used utility levels. Yet, this fairness theory typically considers parallel budget sets as it assumes homogeneous prices for all agents. However, it is well-known that the law of one price fails in many datasets.[4]

I show that the key axioms of this fairness theory leads to an impossibility in the general environment with multi-dimensional heterogeneity in prices, endowments and preferences. More precisely, I prove that the axioms of equality of opportunity conflict with the axiom of Fleming

[1]See Feenstra, Inklaar, and Timmer (2015) and Summers and Heston (1991) for the construction of the Penn World Table.

[2]Another popular criticism is that many dimensions of well-being are not accounted for by GDP (Fleurbaey, 2009; Jones & Klenow, 2016). Although the present paper will only focus on the consumption dimension of overall welfare, its results could be used for more dimensions given the generality of the consumption space.

[3]This allows to characterize welfare measures in the realm of *equivalent income*, such as willingness to pay, expenditure functions or money-metric utility functions. All of these are amounts of money that renders the individual indifferent between her situation and a reference situation. For example, the Hicksian compensating and equivalent variations belongs to this class.

[4]For example, practitioners building purchasing power parities across countries have to face the Balassa-Samuelson effect whereby non-tradeable goods are more expensive in richer countries.

(1952) Separability[5] whereby indifferent agents do not matter in the welfare assessment. In some sense, this result highlights an ethical tension between a democratic principle of neutrality of indifferent agents and equality of opportunity.

Next, I argue that one should escape this impossibility by weakening Fleming (1952) Separability rather than the axioms of equality of opportunity. Intuitively, when welfare is based on a resourcist tradition of justice, indifferent agents should matter in welfare assessments because, conditional on their consumption, they are endowed with resources that influence the overall amount of redistribution that can be done in the economy. The weakening strategy consists in applying Separability only between economies where there is no aggregate growth of resources.

The main results of the paper is that this weaker Separability axiom allows to pin down a social welfare function that can be used to compare the welfare of consumers interpersonally and collectively. On the one hand, the interpersonal consumer welfare measure should be a money-metric utility function[6] whose reference situation is the economy's average purchasing power.

On the other hand, the measure of collective consumer welfare aggregates these money-metric utility functions. I show that widely-used aggregators, the sum and the maximin, can be axiomatized from these axioms. Their difference lies in how to planner evaluates inequalities spawned by heterogeneous preferences. If one wishes to treat every agent with different preferences in the same fashion, then one should use the maximin criterion. By contrast, if one disregards individuals' tastes, one should use the utilitarian summation.

An interesting parallel can be drawn with GDP-like measures here. The standard aggregate consumption in national accounts sums up individuals' expenditures using the prices they actually faced, i.e. $\sum_i e_i(p_i, z_i)$ where p_i and z_i are the individual i prices and consumption vectors and $e_i(\cdot)$ is the expenditure function. By contrast, the main results of this paper suggest that a consumer welfare measure that reflects the ethics of equality of opportunity is $\sum_i e_i(\tilde{p}, z_i)$, where $\tilde{p}$ is the price reflecting the average purchasing power in the economy. This suggests that equality of opportunity requires to change the measuring rod, not necessarily the method.

This result also provides a normative basis for deflation techniques from I derive applications to applied welfare problems. First, in the context of measuring real income across countries, I show that this paper's consumer welfare consists in changing the purchasing power parity from what is typically done in the Penn World Table. In particular, world prices for a good should be based on each countries' income and prices but not on countries' consumption of that good. Fortunately, this does not require more data than standard measures. Second, I show that this consumer welfare measure allows to reconcile inflation and growth measures with welfare provided that the base period on which inflation and growth indices are computed are carefully constructed.

[5] Fleming (1952) proved that this Separability axiom combined by Strong Pareto leads to a utilitarian social welfare function.

[6] A money-metric utility is the amount of money that renders an individual indifferent between her situation and a reference scenario (McKenzie, 1957).

Literature

First, the paper contributes to the literature on social choice and fairness. The fact that money-metric utility function is a useful tool to measure welfare interpersonally and collectively comes at no surprise in this literature. Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011, 2017), and Piacquadio (2017) all characterized similar social welfare functions. A lasting[7] critique of money-metric utility[8] function is that they depend on a reference scenario that is let at the researcher's discretion. This motivated the study of reference-independent welfare assessment (Blackorby, Laisney, & Schmachtenberg, 1993; Eden & Freitas, 2024; Roberts, 1980). The contribution of the present paper with respect to that literature is that reference situation is endogenized by the equality of opportunity axioms. More precisely, I show that the axioms used in that literature lead to an impossibility if there is price dispersion and that one way out of the impossibility implies a precise reference situation.

Second, the paper contributes to the index number theory as initiated by Fisher (1922). The index number problem consists in building price and quantity indices based on dataset of observed prices and expenditures for each consumer. It is customary to divide the field into to approaches (van Veelen & van der Weide, 2008). The axiomatic approach imposes series of desirable properties on the indices but is typically not linked with classical demand theory (Samuelson & Swamy, 1974; Van Veelen, 2002). The economic approach rationalizes observed differences from a representative consumer demand system (Diewert, 1976; Neary, 2004). The present paper rejects a representative consumer unlike the latter but has links with classical demand theory unlike the former. Moreover, the paper does not impose domain restrictions to individual preferences such that its results are valid whether preferences are homothetic (Diewert & Nakamura, 1993) or not (Jaravel & Lashkari, 2024). Finally, the indices constructed in this paper reflect ethical views whose foundations are transparently carried over by the axioms.

The paper is structured as follows. In section 2, I present the consumption space with multi-dimensional heterogeneity in prices, endowments and preferences. In section 3, I present the key axioms and prove their incompatibilities. In section 4, I show that weakening Separability allows possibilities and characterize the maximin and the summation of the money-metric utility with average purchasing power reference. In section 5, I derive implications for various applications. In section 6, I conclude.

2 Model

A society $\mathcal{N} = \{1, ..., N\}$ is composed of a finite number $N \geq 5$ of agents. There are $L \geq 2$ goods and an agent i's bundle is denoted by $z_i \in \mathbb{R}^L_+$. Agents are heterogeneous in their preferences $\succsim_i$ that are complete, transitive, locally non-satiated and convex.

Agents receive some endowments in goods[9] $\omega_i \in \mathbb{R}^L_+$ and face a vector personalized prices

[7] A classical critique by Blackorby and Donaldson (1988) was that money-metric utility function may lead to inegalitarian assessment. However, Fleurbaey and Maniquet (2011) and Schlee and Khan (2022, 2023) that the Blackorby and Donaldson (1988) showed that this can be escaped rather easily.

[8] The fact that no theory guides the choice of the reference price is well-known and discussed recently in Bosmans, Decancq, and Ooghe (2018), Capéau, Decoster, and De Sadeleer (2023), and Fleurbaey and Blanchet (2013).

[9] Observe that this model nests a model where one good is set as the numéraire and endowments are only in that good, that is a monetary endowment.

$p_i \in \mathbb{R}_+^L$. Personalized prices may include the standard case of no price dispersion in a Walrasian equilibrium, as well as price dispersion due to e.g. frictions, transport costs, discrimination, heterogeneous needs, comparative advantages in production, or intra-household bargaining *à la* Chiappori (1988, 1992). For brevity, I will sometimes refer to the scalar product $p_i\omega_i$ as income and denoted it by $y_i = p_i\omega_i \geq 0$.

An allocation z is a list of bundles for each agent $z = (z_1, z_2, ..., z_N) \in \mathbb{R}_+^{LN}$ and it may or may not be rationalizable by agents' constrained utility maximization. An economy e is composed of N agents which are heterogeneous in preferences, endowments and prices $e = \{\mathcal{N}, (\succsim_i, \omega_i, p_i)_{i\in\mathcal{N}}\}$. The set of all economies is denoted by $\mathcal{E}$.

The main object of interest will be the ethical observer's ordering function that associates to each economy e a complete and transitive ordering of allocations. I denote this social preference relation by **R(e)**, **P(e)**, **I(e)** for weak preference, strict preference and indifference, respectively.

I now define two useful objects. First, an allocation $\hat{z}$ is feasible by lump-sum transfers from e if there exists a vector of individual lump-sum transfers $(t_1, t_2, ..., t_N) \in \mathbb{R}^N$ with $\sum_{i\in\mathcal{N}} t_i \leq 0$ such that bundles $\hat{z}_i$ are part of individuals' budgets $B(t_i, y_i, p_i)$:

$$\hat{z}_i \in B(t_i, \omega_i, p_i) = \{z_i \in \mathbb{R}_+^L : p_i\ z_i \leq p_i\omega_i + t_i\}$$

Second, the money-metric welfare of agent i consuming the bundle z_i and reference situation (ω_R, p_R) is the transfer that renders the agent indifferent between her bundle z_i and the bundle she would have chosen in the budget determined by that reference situation and the transfer. Formally, we define money-metric welfare as $W_i(z_i, p_R)$ with

$$W_i(z_i, \omega_R, p_R) = \min t \text{ s.t. } z_i \succsim z_0 = \arg\max_{\succsim_i} B(t, \omega_R, p_R)$$

3 Impossibilities

The axiomatic construction of the social ordering **R(e)** consists in imposing desirable properties, i.e. axioms, restricting the scope of possibilities among the universe of transitive and complete **R(e)**. This section introduces the key axioms of equality of opportunity and shows that they lead to an impossibility.

The first one, **Laissez-faire**, embodies the view that inequalities spawned by heterogeneous preferences should not be reduced by the ethical observer: agents should be held responsible for their preferences and the planner should not discriminate along different tastes.[10] Formally, it requires that when there is no cross-sectional heterogeneity in prices and endowments, the Laissez-faire allocation is strictly preferred to any other allocations feasible by lump-sum transfers from the economy.

Laissez-faire: $\forall e \in \mathcal{E}$ such that $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$ and z feasible by lump-sum transfer for e,

$$z^* \mathbf{P(e)}\, z$$

[10] Political philosophers such as Arneson (1990), Cohen (1989), Dworkin (1981), and Rawls (1971) all defended variants of this principle, as discussed in Fleurbaey (2008) and Fleurbaey and Maniquet (2006, 2011).

where z^* is such that $z_i^* \in \max_{\succsim_i} B(0, \omega, p)$ for all $i \in \mathcal{N}$.

Conversely, the second axiom, **Compensation**, reflects the principle that inequalities spawned by heterogeneous prices and income should be fought against. In particular, the axiom imposes that when a pair of agents have identical preferences, a permutation of their bundle should be socially indifferent.[11] In other words, the social ranking is anonymous among agents with identical preferences.

Compensation: $\forall e \in \mathcal{E}$ where agents i, j are such that $\succsim_i = \succsim_j$ then

$$(z_i, z_j, z)\mathbf{I(e)}(z_j, z_i, z).$$

Taken together, these two axioms reflect the ethics of equality of opportunity: heterogeneous tastes should not be materially relevant for welfare inequalities but unequal budgets are creating welfare-relevant inequalities in the eyes of the planner. Observe that both axioms only apply under some specific premises, i.e. for some but not all e in $\mathcal{E}$. In order to obtain a complete ordering **R(e)** for all economies, one must use a cross-economy robustness condition.

Since the characterization of utilitarianism by Fleming (1952), the standard cross-economy condition is Separability which requires that indifferent agents should not have a say in the social evaluation between two alternatives. This is sometimes defended on democratic grounds: only those effectively affected by the alternatives should be heard. More formally, **Fleming (1952) Separability** requires that if some agents are receiving the same bundle in two allocations, then the social ordering between these two allocations is unaffected if these agents change their preferences, endowments, prices, or receive a different bundle.

Fleming (1952) Separability: $\forall e, e' \in \mathcal{E}$ and the partitions $\mathcal{K} + \mathcal{M} = \mathcal{N}$ with $e_{\mathcal{K}} = \{\mathcal{K}, (\succsim_i, \omega_i, p_i)_{i \in \mathcal{K}}\}$, $e'_{\mathcal{K}} = \{\mathcal{K}, (\succsim'_i, \omega'_i, p'_i)_{i \in \mathcal{K}}\}$, $e_{\mathcal{M}} = \{\mathcal{M}, (\succsim_i, \omega_i, p_i)_{i \in \mathcal{M}}\}$, and z, z' then,

$$(z_{\mathcal{K}}, z_{\mathcal{M}})\mathbf{R}(\mathbf{e}_{\mathcal{K}},\mathbf{e}_{\mathcal{M}})(z_{\mathcal{K}}, z'_{\mathcal{M}}) \iff (z'_{\mathcal{K}}, z_{\mathcal{M}})\mathbf{R}(\mathbf{e}'_{\mathcal{K}},\mathbf{e}_{\mathcal{M}})(z'_{\mathcal{K}}, z'_{\mathcal{M}})$$

I note that the version presented here is logically weaker than the axioms of separability in subpopulations used in d'Aspremont and Gevers (1977), Fleurbaey (2003), and Maniquet (2004). The next theorem shows that the three axioms outlined above lead to an impossibility when combined together.

Theorem 1. *There does not exists any **R(e)** that satisfies **Laissez-faire**, **Compensation** and **Fleming (1952) Separability***

Proof. By contradiction, assume that the statement does not hold. Consider the following four-agent economies constructed with two preference orderings and two budgets

$$e_1 = \left\{(\succsim_i, \bar{\omega}, \bar{p}), (\succsim_i, \bar{\omega}, \bar{p}), (\succsim_j, \bar{\omega}, \bar{p}), (\succsim_j, \bar{\omega}, \bar{p})\right\}$$

$$e_2 = \left\{(\succsim_i, \underline{\omega}, \underline{p}), (\succsim_i, \underline{\omega}, \underline{p}), (\succsim_j, \underline{\omega}, \underline{p}), (\succsim_j, \underline{\omega}, \underline{p})\right\}$$

$$e_3 = \left\{(\succsim_i, \bar{\omega}, \bar{p}), (\succsim_i, \underline{\omega}, \underline{p}), (\succsim_j, \underline{\omega}, \underline{p}), (\succsim_j, \bar{\omega}, \bar{p})\right\}$$

[11] This type of axioms also received support in political philosophy among liberal egalitarians (Dworkin, 1981). It is reminiscent to Suppes (1966)'s grading principle. See Hammond (1976) for an early treatment and Maniquet (2004) for its relationship to equality of opportunity.

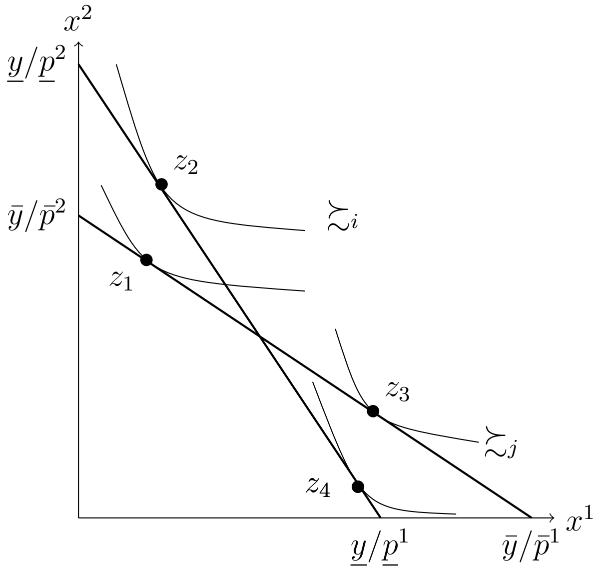


Figure 1: Illustration of the proof of Theorem 1 where $\bar{y} = \bar{p}\bar{\omega}$ and $\underline{y} = \underline{p}\underline{\omega}$.

There are four bundles of interest that maximize either $\succsim_i$ or $\succsim_j$ over the budgets $B(0, \bar{\omega}, \bar{p})$ and $B(0, \underline{\omega}, \underline{p})$ and we denote them by z_1, z_2, z_3, z_4 respectively. Moreover, the values of $\bar{\omega}, \underline{\omega}, \bar{p}, \underline{p}$ are chosen such that z_1, z_2, z_3, z_4 are the vertices of a parallelogram. Because the diagonals of a parallelogram bisect each other, we know that (z_1, z_1, z_3, z_3) is feasible as lump-sum transfers from e_2 and (z_2, z_2, z_4, z_4) is feasible as lump-sum transfers from e_1 . The situation is depicted in Figure 1.

By **Laisser-faire**,

$$(z_1, z_1, z_3, z_3)\mathbf{P(e}_1\mathbf{)}(z_2, z_1, z_3, z_4)$$

By **Fleming (1952) Separability**,

$$(z_1, z_2, z_4, z_3)\mathbf{P(e}_3\mathbf{)}(z_2, z_2, z_4, z_4)$$

By **Compensation**,

$$(z_2, z_1, z_3, z_4)\mathbf{I(e}_3\mathbf{)}(z_1, z_2, z_4, z_3)$$

By Transitivity,

$$(z_2, z_1, z_3, z_4)\mathbf{P(e}_3\mathbf{)}(z_2, z_2, z_4, z_4)$$

By **Fleming (1952) Separability**,

$$(z_2, z_1, z_3, z_4)\mathbf{P(e}_2\mathbf{)}(z_2, z_2, z_4, z_4)$$

But this contradicts **Laissez-faire**, proving the statement for $N = 4$. The statement for $N \geq 5$ follows by duplicating this reasoning. ■

A natural attempt to escape the impossibility would consist in weakening **Laissez-faire** to a weak social preference for the Laissez-faire allocation rather than a strict preference. This is what the next axiom, **Laissez-faire**^{W}, achieves.

Laissez-faire^W: $\forall e \in \mathcal{E}$ such that $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$ and z feasible by lump-sum transfers from e, then $z^* \mathbf{R(e)} z$

Unfortunately, the next theorem shows that the impossibility persists if one looks for orderings that satisfy the widely-used **Weak Pareto** principle.[12]

Theorem 2. *There does not exist any **R(e)** that satisfies **Weak Pareto, Laissez-Faire***^{W}*, **Compensation** and **Fleming Separability***

Proof. I proceed again by contradiction. Consider the same economies as in the proof of Theorem 1 and z_5, z_6 such that (z_5, z_5, z_6, z_6) is feasible as lump-sum transfers from e_2 while $z_5 \succ_i z_1$ while $z_6 \succ_j z_3$.
Using the exact same steps as in Theorem 1 but with a weak preference relation rather than a strict one, we get that by ***Laissez-Faire***^{W}, ***Compensation***, Transitivity and **Fleming (1952) Separability**,

$$(z_2, z_1, z_3, z_4)\mathbf{R(e_2)}(z_2, z_2, z_4, z_4) \tag{1}$$

By **Fleming (1952) Separability**,

$$(z_1, z_1, z_3, z_3)\mathbf{R(e_2)}(z_1, z_2, z_4, z_3)$$

By **Compensation** and Transitivity,

$$(z_1, z_1, z_3, z_3)\mathbf{R(e_2)}(z_2, z_1, z_3, z_4)$$

By Transitivity with equation 1,

$$(z_1, z_1, z_3, z_3)\mathbf{R(e_2)}(z_2, z_2, z_4, z_4)$$

By **Laissez-Faire**^{W},

$$(z_2, z_2, z_4, z_4)\mathbf{R(e_2)}(z_5, z_5, z_6, z_6)$$

By Transitivity,

$$(z_1, z_1, z_3, z_3)\mathbf{R(e_2)}(z_5, z_5, z_6, z_6)$$

which contradicts Weak Pareto and completes the proof. ■

These impossibilities are surprising[13] because Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011, 2017), and Piacquadio (2017) all characterized possibilities with similar axioms.[14] The present paper only differs by allowing price dispersion in the cross-section

[12] Weak Pareto imposes that $\forall e \in \mathcal{E}$ and z, z' such that $z_i \succ z'_i$ for all $i \in \mathcal{N}$, then $z\mathbf{P(e)}z'$.

[13] I note that Bosmans and Öztürk (2022) studied the impossibility between a laissez-faire principle and the Pareto principle. Yet, their laissez-faire principle is much stronger than ours as it holds under any initial distribution of endowments while ours restrict it to economies with $(\omega_i, p_i) = (\omega, p)$ for all $i \in \mathcal{N}$. As such, the ethical view reflecting in their paper is closer to libertarianism while the present paper builds on the ethics of equality of opportunity.

[14] I note that the impossibility holds for a version of Compensation that use anonymity rather than transfers as in Bosmans, Decancq, and Ooghe (2018). It is not difficult to see from the proof of Theorem 1 that the impossibilities would persist with the transfer-based axiom of Compensation considered in their paper.

and this engenders impossibilities.

Impossibilities are sometimes discouraging for the unfamiliar reader. However, they show to way forward to the welfare economist: one needs to weaken one of these axioms to obtain a possibility. Moreover, these impossibilities have normative content: they suggests that the ethics of equality of opportunity are incompatible with the democratic ethics of **Fleming (1952) Separability**. In the next section, I will argue that the culprit is not to be found in the axioms of equality of opportunity but rather in **Fleming (1952) Separbility**.

4 Possibilities

In the previous section, impossibilities arised because repeated applications of **Fleming (1952) Separability** allows us to move from e_1 to e_2 and compare an allocation derived from the Laissez-faire in e_1 with the Laissez-faire allocation in e_2. More precisely, economies e_1 and e_2 have a different amount of aggregate income as $4\bar{y} > 4\underline{y}$. Yet, prices are such that the Laissez-faire allocation of economy e_1 is feasible as lump-sum transfers from economy e_2 and *vice versa*. To see this, observe that because the bundles in Figure 1 are vertices of a parallelogram the following identity holds

$$z_1 + z_1 + z_3 + z_3 = \tilde{Z} = z_2 + z_2 + z_4 + z_4$$

I depict aggregate budgets for e_1, e_2 and e_3 in Figure 2.

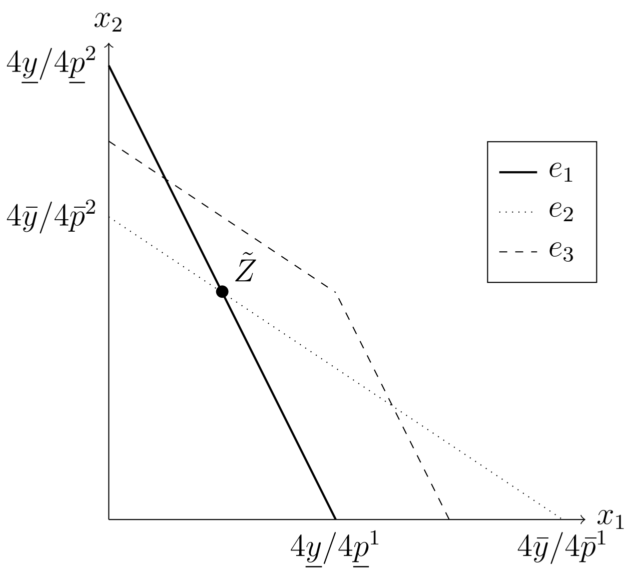


Figure 2: Aggregate budgets in the proof of Theorem 1.

In other words, the cross-economy robustness condition does not take into account the fact that the aggregate trade-off between goods is completely different in e_1 and e_2: aggregate budgets are crossing at $\tilde{Z}$. I argue that this shows that Separability should be weakened: when studying equality of opportunity, indifferent agents matter for the social evaluation not because of their bundles but rather because of their endowments and prices.

In order to escape the impossibility, we should make sure to apply Separability on a partition of economies whose elements do not contain two intersecting simplices as depicted in Figure

2. The most natural way is to define a partition by its intersection with the axis. The next axiom, **No-Growth Separability**, applies **Fleming (1952) Separability** only between economies where the maximal amount of each good that could be consumed is identical. This avoids the comparisons of non-nested aggregate budgets depicted in Figure 2.

No-Growth Separability: $\forall e, e' \in \mathcal{E}$ and the partition $\mathcal{K}+\mathcal{M}=\mathcal{N}$ with $e_{\mathcal{K}}=\{\mathcal{K},(\succsim_i,\omega_i,p_i)_{i\in\mathcal{K}}\}$, $e'_{\mathcal{K}}=\{\mathcal{K},(\succsim'_i,\omega'_i,p'_i)_{i\in\mathcal{K}}\}$, $e_{\mathcal{M}}=\{\mathcal{M},(\succsim_i,\omega_i,p_i)_{i\in\mathcal{M}}\}$ such that

$$\forall l=\{1,...,L\}:\sum_{i\in\mathcal{K}}\frac{y_i}{p_i^l}=\sum_{i\in\mathcal{K}}\frac{y'_i}{p_i'^l}$$

and for z, z', then,

$$(z_{\mathcal{K}},z_{\mathcal{M}})\mathbf{R}(\mathbf{e}_{\mathcal{K}},\mathbf{e}_{\mathcal{M}})(z_{\mathcal{K}},z'_{\mathcal{M}}) \implies (z'_{\mathcal{K}},z_{\mathcal{M}})\mathbf{R}(\mathbf{e}'_{\mathcal{K}},\mathbf{e}_{\mathcal{M}})(z'_{\mathcal{K}},z'_{\mathcal{M}})$$

Why does this 'no-growth' requirement pop up? Intuitively, starting from any arbitrary economy e, we should only be able to compare it through the separability axiom with one and only one equal-price economy. When the maximal total amount of each good is identical in both economies, there is no way one can construct a parallelogram as in Theorem 1, thereby escaping the impossibility.

The two main theorems of this paper will make use of **No-Growth Separability** as well as **Compensation**. However, they will feature two stronger versions of the *laissez-faire* principle which I now define in turn.

First, **Egalitarian Responsibility** imposes that when all agents have equal endowments and prices, any reduction in lump-sum transfers inequality is a social improvement. Of course, the Laissez-faire allocation z^*, where no lump-sum transfers inequality remains, is still the social optimum. Second, by contrast, **Utilitarian Responsibility** imposes under the same premise that there should no be lump-sum transfers waste: all resources at the planner's disposal should be used.

More formally, $\forall e \in \mathcal{E}$ such that $(\omega_i,p_i)=(\omega,p)$ $\forall i \in \mathcal{N}$, and z, z' such that $z_k=z'_k$ $\forall k \in \mathcal{N}\backslash\{i,j\}$, and

$$z_i \in \max_{\succsim_i} B(t_i,\omega,p) \qquad z'_i \in \max_{\succsim_i} B(t'_i,\omega,p)$$
$$z_j \in \max_{\succsim_j} B(t_j,\omega,p) \qquad z'_j \in \max_{\succsim_j} B(t_j,\omega,p)$$

If **R(e)** satisfies **Egalitarian Responsibility**, then $|t'_i-t'_j|\geq|t_i-t_j|$ implies that $z\mathbf{R(e)}z'$
If **R(e)** satisfies **Utilitarian Responsibility**, then $t_i+t_j\geq t'_i+t'_j$ implies that $z\mathbf{R(e)}z'$.

It is useful to observe that both **Egalitarian Responsibility** and **Utilitarian Responsibility** imply **Laissez-faire**^{W}. Indeed, the *laissez-faire* allocation is such that there is no lump-sum transfer inequality among agents, which **Egalitarian Responsibility** finds most desirable. However, the *laissez-faire* allocation is also the one where the sum of lump-sum transfers is 0, which is preferred to all other feasible allocations (i.e. $\sum t_i \leq 0$) by **Utilitarian Responsibility**. Both axioms confirm the supremacy of *laissez-faire* allocation in equal-budget economies, but for different reasons. They also have bite beyond that as any reform toward the *laissez-faire* allocation is desirable, thus being logically stronger than **Laissez-faire**^{W}.

I now present the main theorems of the paper. They characterize precisely the social welfare function that abides by the axioms.

Theorem 3 shows that **Egalitarian Responsibility**, combined with **Compensation** and **No-Growth Separability** leads to a social welfare function that has an infinite inequality aversion in a particular money-metric utility function whose reference situation is the cross-sectional average of purchasing powers.

Theorem 3. ***R(e)*** *satisfies* ***No-Growth Separability, Compensation*** *and* ***Egalitarian Responsibility*** *if and only if for z, z' and e such that*

$$W_i(z_i', \tilde{\omega}, \tilde{p}) \leq W_i(z_i, \tilde{\omega}, \tilde{p}) \leq W_j(z_j, \tilde{\omega}, \tilde{p}) \leq W_j(z_j', \tilde{\omega}, \tilde{p})$$

with

$$\tilde{p}\tilde{\omega} = \tilde{y} = \frac{1}{N}\sum_{i\in\mathcal{N}} y_i = \frac{1}{N}\sum_{i\in\mathcal{N}} p_i\omega_i$$
$$\tilde{p}^l = \frac{\sum_{i\in\mathcal{N}} y_i}{\sum_{i\in\mathcal{N}} \frac{y_i}{p_i^l}} \quad \text{for } l = \{1, ..., L\}$$

while $z_k' = z_k$ for all $k \in \mathcal{N}\backslash\{i,j\}$,
*Then, z**R(e)**z'*

The proof is relegated to the appendix. I provide a graphical illustration of the construction of $W_i(z_i, \tilde{\omega}, \tilde{p})$ for a two-agent case with two goods in Figure 3. In panel (a), I show that $\tilde{y}$ and $\tilde{p}^l$ pin down the budget of the average purchasing power of buying l in the economy : the ratio $\frac{\tilde{y}}{\tilde{p}^l}$ gives the per-capita quantity of good l that could be bought if everyone was spending one's income in buying good l. In panel (b), I show that the welfare of agent i consuming z_i is the transfer that renders that agent i indifferent between z_i and the reference budget $B(0, \tilde{\omega}, \tilde{p})$.

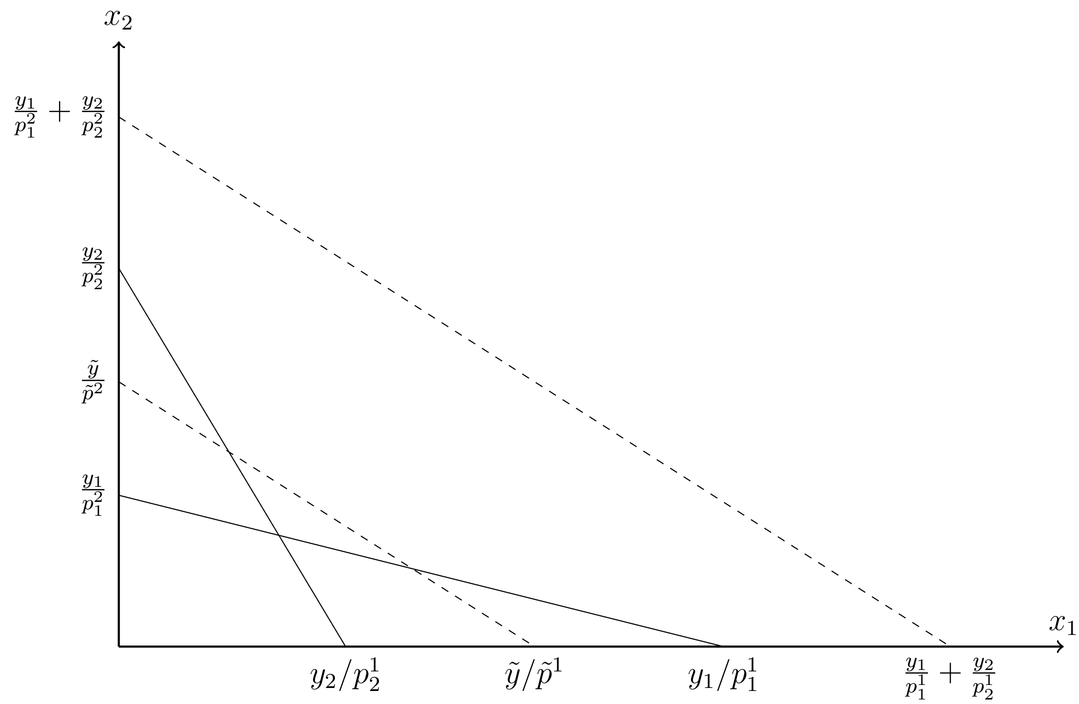
The previous theorem has an infinite aversion to inequality in the particular money-metric utility $W_i(z_i, \tilde{\omega}, \tilde{p})$. However, observe that not all consumption inequalities are deemed unfair. Indeed, unequal preferences imply that agents consume different bundles in the Laissez-faire allocation even if they have the same budgets. Yet, this allocation is the optimal one with respect to Theorem 3. This is why these axioms reflect the ethics of equality of *opportunity* and not of *outcomes* : budget sets should be equalized but not consumption vectors.

The next theorem shows the consequences of replacing **Egalitarian Responsibility** by **Utilitarian Responsibility**. Its proof is relegated to the appendix as well.

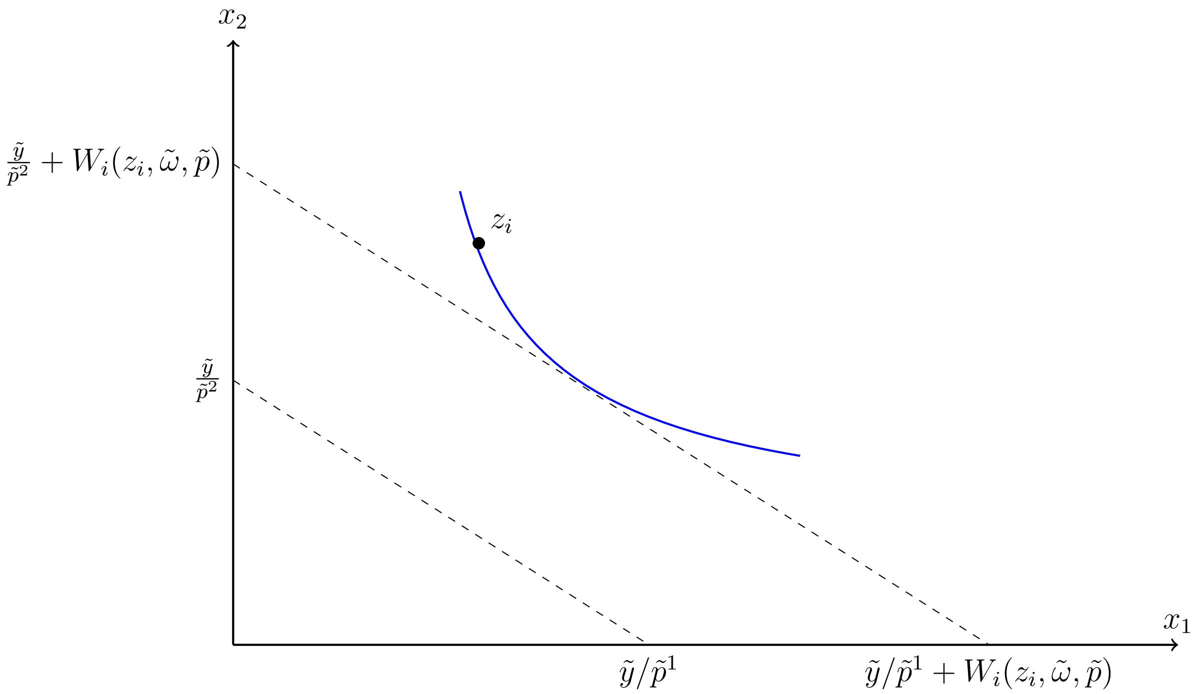
Theorem 4. ***R(e)*** *satisfies* ***No-Growth Separability, Compensation*** *and* ***Utilitarian Responsibility*** *if and only if for z, z' and e such that*

$$W_i(z_i, \tilde{\omega}, \tilde{p}) + W_j(z_j, \tilde{\omega}, \tilde{p}) \geq W_i(z_i', \tilde{\omega}, \tilde{p}) + W_j(z_j', \tilde{\omega}, \tilde{p})$$

while $z_k = z_k'$ $\forall k \in \mathcal{N}\backslash\{i,j\}$.
*Then, z**R(e)**z'*



(a) Illustration of the construction of $\tilde{y} = \tilde{p}\tilde{\omega}$ and $\tilde{p}$.



(b) Illustration of the construction of $W_i(z_i, \tilde{\omega}, \tilde{p})$.

Figure 3: Illustrations of the construction of the interpersonal consumer welfare measure.

These results deserve some comments. First, these two theorems characterize the social welfare function that are endogenized by the axioms. They differ in the aggregator used to compute social welfare (the *maximin* and the *sum*, respectively), but they use the very same measure of interpersonal welfare, i.e. $W_i(z_i, \tilde{\omega}, \tilde{p})$. It suggests that this money-metric utility function should be used to compare the welfare of heterogeneous consumers if one abides by **Compensation**, **Laissez-faire**^{W} and **No-Growth Separability**, while the choice of the aggregator depends on which logical strengthening the ethical observer prefers between **Utilitarian Responsibility** and **Egalitarian Responsibility**.

Second, this welfare measure only uses ordinal and non-comparable elements of preferences to build a welfare measure. Indeed, the only component of preferences needed to compute $W_i(z_i, \tilde{y}, \tilde{p})$ is the indifference curve at z_i, not the utility level. Despite that, this measure allows interpersonal comparisons of welfare between individuals. Thus, this measure is in line with the long-standing tradition since Robbins (1938) and Samuelson (1947) of not comparing cardinal utility levels across individuals.[15] Besides, both measures also satisfy the Pareto principle as is widely accepted by the literature.

Third, it is interesting to contrast the result of Theorem 4 with the standard aggregate consumption computed in national accounts. Observe that $W_i(z_i, \tilde{\omega}, \tilde{p})$ is cardinally equivalent[16] to the expenditure function at prices $\tilde{p}$, defined as

$$e(\tilde{p}, u_i(z_i)) = \min\{\tilde{p}z_0 : z_i \succsim_i z_0\}$$

Theorem 4 suggests that the social welfare function should be $\sum_i e(\tilde{p}, u_i(z_i))$. Interestingly, aggregate consumption in national accounts simply sums up individuals' expenditures using individuals' prices, i.e. $\sum_i e(p_i, u_i(z_i))$. This suggests that measuring consumer welfare with the ethics of equality of opportunity simply consists in changing the measuring rod, not the method. It also shows the importance of price normalization when building indicators beyond GDP. We come back on this point in the next section.

Fourth, that the sum of expenditure functions is an attractive way of measuring welfare has a long history in welfare economics marked with controversies over the choice of the reference price vector. First, the Hicksian compensating and equivalent variations are expenditure functions using post- and pre-reform prices as reference situation respectively[17], but a social welfare function using their sum may lead to intransitive assessment as is known since Boadway (1974) and Scitovsky (1941).[18] Second, money-metric utility functions have been criticized by Blackorby and Donaldson (1988) because they may yield anti-redistributive consequences for some reference prices[19] even when the social welfare function is egalitarian. Third, Blackorby, Laisney, and Schmachtenberg (1993) and Roberts (1980) underlined the arbitrariness of the choice of the reference price and studied price-independent welfare assessment. What the present paper offers are social welfare functions based on money-metric utility that overcome each of these

[15] This is not a new result in itself as it is characteristic of the fairness literature in social choice. See Bosmans, Decancq, and Ooghe (2018), Fleurbaey (2003), Fleurbaey and Maniquet (2011), and Piacquadio (2017) among others.

[16] Indeed, one has that $W_i(z_i, \tilde{\omega}, \tilde{p}) = e(\tilde{p}, u_i(z_i)) - \tilde{p}\tilde{\omega}$ which is an affine transformation preserving cardinality.

[17] The widely-used consumer surplus is a Hicksian variation when preferences are quasilinear. See Hicks (1939, 1941, 1942, 1943, 1945) for the original contributions.

[18] See Blackorby and Donaldson (1990) for a review.

[19] Bosmans, Decancq, and Ooghe (2018), Fleurbaey and Maniquet (2011), and Schlee and Khan (2022, 2023) discussed several ways to escape this critique.

critiques: they are transitive, inequality-averse, and their reference situation is endogenized by the axioms. Moreover, $\tilde{p}$ is independent of the normalization of the numéraire of the economy. I now present some consequences for applied welfare analysis.

5 Applications

The following applications all use the interpersonal consumer welfare measure $W_i(\cdot)$ and are valid whether Theorems 3 or 4 are used, that is whether the aggregator of social welfare is the minimum or the sum. The fact that the axioms endogenize the same reference price $\tilde{p}$ allows to contribute to various applied welfare problems without taking a stance on which aggregator should be used.

5.1 Wealth of nations

In the index number problem[20] initiated by Fisher (1922), the economist must compare the *real income* of a set of countries index by i based on a dataset of observed prices p_i and quantities consumed z_i and decompose this real income into a price index and a quantity index. Our analysis of welfare above can readily be applied to this problem if each country is considered as an agent and *real income* is considered as $e(\tilde{p}, u_i(z_i))$. As is standard in index number theory, real income can be decomposed into a price index P_i and a quantity index $Q_{i,j}$ as follows

$$\tilde{P}_i = \frac{e(p_i, u_i(z_i))}{e(\tilde{p}, u_i(z_i))}$$
$$\tilde{Q}_{i,j} = \frac{e(\tilde{p}, u_i(z_i))}{e(\tilde{p}, u_j(z_j))}$$

where the price index P_i informs on the deflation needed to compare the prices faced by country i and $Q_{i,j}$ informs on the real income difference between countries i and j.[21]

The key innovations with respect to the index number theory are that the present paper (i) allows for heterogeneous preferences across countries, (ii) does not restrict the set of admissible preferences and (iii) derives the indices from ethical judgments on equality of opportunity. These differences could have lead to completely new indices. Surprisingly, the indices defined here are close to those used by practitioners. Most notably, the Geary (1958) method that underpins the Penn World Table computes a world price $\mathring{p}^l$ for good l, is such that

$$\mathring{p}^l = \frac{\sum_i p_i^l z_i^l \epsilon_i}{\sum_i z_i^l}$$

where ϵ_i is a normalization of countries' income such that $\epsilon_i = \frac{\sum_l \mathring{p}^l z_i^l}{\sum_l p_i^l z_i^l}$. The real income of country i is then computed as the cost of that country's bundle z_i at the world prices $\mathring{p}$. While this approach ignores preferences, Neary (2004) shows that consistency with a representative agent preferences can be obtained by setting ϵ_i as the ratio of expenditure functions at these prices.

[20] That expenditure functions and welfare economics are useful for index number theory is known at least since the classical contributions of Deaton (1979) and Samuelson and Swamy (1974).

[21] In the literature, the former is a true Könus price index while the latter is an Allen true quantity index.

Deaton and Heston (2010) criticized[22] these Geary and Neary measures because countries with larger consumption weight more in the world price, which renders it plutocratic. This is completely true, but the present paper suggests that it does not conflict with social welfare understood as equality of opportunity insofar as actual consumption z_i^l in the denominator is replaced by consumption possibility y_i/p_i^l of good l. In other words, a ranking of countries real income based on $e(\tilde{p}, u_i(z_i))$ takes into account the fact that poor countries have low income and faced lower prices and one wishes to cancel the between-country inequalities in budgets. The fact that the reference price should not depend on actual quantities consumed z_i^l but on consumption possibilities y_i/p_i^l comes from the axioms that imply that only budgets are creating welfare-relevant inequalities, not necessarily consumption vectors.

I note that this modification of the purchasing power parity does not require more data than what is commonly used. Moreover, because $\tilde{p}$ depends only on observed quantities and not on preferences, this paper's index is less data-intensive than Neary (2004)'s.

5.2 Lifetime inflation and growth

Consider the case where each good l is a time period. For clarity, let me substitute the superscript l by $t = \{1, ..., T\}$. The problem for the growth economist consists in measuring inflation and real income growth over the past T periods for a set of countries based on a dataset of country i's prices p_i^t and consumption z_i^t. Preferences are defined over the T periods, i.e. over the stream of consumption z_i, and must be understood as *lifetime* welfare.

The present paper suggests that the welfare-relevant inflation and growth indices between countries i and j over the T periods are

$$\Pi_i = \frac{e(\tilde{p}, u_i(z_i))}{e(p_i, u_i(z_i))}$$
$$Q_{i,j} = \frac{e(\tilde{p}, u_i(z_i))}{e(\tilde{p}, u_j(z_j))}$$

Interestingly, in such case the reference price vector $\tilde{p}$ becomes close to the GDP deflator i.e. $\tilde{p}^t = \frac{\sum_i y_i}{\sum_i y_i/p_i^t}$.[23] Unlike the Consumer Price Index (CPI), the GDP deflator does not depend on the weights of each consumption good in the bundle consumed, but rather only on income and prices. Again, this can be directly traced back to the axioms: only budgets are creating welfare-relevant inequalities, not preferences.

Importantly, this does not mean that preferences are ignored. Indeed, Π_i and $Q_{i,j}$ both depend on preferences as they use expenditure functions as arguments. However, the measuring rod does not: $\tilde{p}$ only depends on consumption possibilities.

[22] Deaton and Heston (2010) consider that the main advantage of the approach is the preservation of integrability. Indeed, because it uses a single world price for each good, a summation across goods does not perturb the ranking. This paper's measure escapes the Van Veelen (2002) impossibility by using third-country information when computing the real income of a pair of countries.

[23] In practice, the GDP deflator is computed on more goods than mere private consumption goods and include the price of investments as well as government expenditures.

5.3 Year-by-year inflation and growth

Consider the case where each time period the preferences of the agent under study may vary. In such case, we can consider that the index i used above may be substituted by t. We can then see our contribution as a way to measure year-by-year inflation and welfare growth for a single country when prices, endowments, and preferences may change from one year to the next. The relevant inflation and growth indices are

$$\Pi_t = \frac{e(\tilde{p}, u_t(z_t))}{e(p_t, u_t(z_t))}$$
$$Q_{t,t-1} = \frac{e(\tilde{p}, u_t(z_t))}{e(\tilde{p}, u_{t-1}(z_{t-1}))}$$

In this setup, the reference price for a good l $\tilde{p}^l$ becomes the ratio between the sum of expenditures over time over the the sum of maximal quantity of good l that could have been consumed.

$$\tilde{p}^l = \frac{\sum_t y_t}{\sum_t y_t/p_t^l}$$

While traditional inflation and growth measures typically ignore preferences or assumed homothetic preferences, a recent contribution by Jaravel and Lashkari (2024) provides an algorithm to measure inflation and growth for non-homothetic and general preferences. Interestingly, they also rely on expenditure functions with a common, time-invariant reference price vector which they call *base period*.

However, neither their theory nor their algorithm provides guidance on which base period to choose. While their estimates significantly differ from standard estimates derived under homothetic preferences, the magnitude of these differences depends on the *base period*. What the present paper gives is a normative justification for the base period that connects the measure with consumer welfare.

6 Conclusion

The present paper has built a measure of consumer welfare that is transitive and applicable to a wide set of problems. In essence, it builds interpersonal comparison between heterogeneous individuals, and as such may inform several other problems not mentioned so far, such as indexing the preference-based poverty line (Decerf, 2023), solving for optimal consumption taxes with preference heterogeneity (Ferey, Lockwood, & Taubinsky, 2024; Golosov et al., 2013), or the welfare costs of inflation (Adam & Weber, 2023; Blanco, 2021; Burstein & Hellwig, 2008; Craig & Rocheteau, 2008).

Moreover, by no means this paper has solved all issues of comparing heterogeneous consumers. There many practical issues that should be accounted for, for example, the incorporation of quality (Errico & Lashkari, Forthcoming) but also incomplete price information (Atkin et al., 2024) or individual information (Baqaee, Burstein, & Koike-Mori, 2024).

Moreover, individuals derive welfare beyond their mere consumption, notably from health and social relations. The inclusion of such well-being dimensions in the framework of equal opportunity is beyond the scope of the present paper.

References

Adam, K., & Weber, H. (2023). Estimating the optimal inflation target from trends in relative prices. *American Economic Journal: Macroeconomics*, *15*(3), 1–42.

Arneson, R. J. (1990). Liberalism, distributive subjectivism, and equal opportunity for welfare. *Philosophy & public affairs*, 158–194.

Arrow, K. J. (1950). A difficulty in the concept of social welfare. *Journal of Political Economy*, *58*(4), 328–346.

Atkin, D., Faber, B., Fally, T., & Gonzalez-Navarro, M. (2024). Measuring welfare and inequality with incomplete price information. *The Quarterly Journal of Economics*, *139*(1), 419–475.

Baqaee, D. R., Burstein, A. T., & Koike-Mori, Y. (2024). Measuring welfare by matching households across time. *The Quarterly Journal of Economics*, *139*(1), 533–573.

Blackorby, C., & Donaldson, D. (1988). Money metric utility: A harmless normalization? *Journal of Economic Theory*, *46*(1), 120–129.

Blackorby, C., & Donaldson, D. (1990). A review article: The case against the use of the sum of compensating variations in cost-benefit analysis. *Canadian Journal of Economics*, 471–494.

Blackorby, C., Laisney, F., & Schmachtenberg, R. (1993). Reference-price-independent welfare prescriptions. *Journal of Public Economics*, *50*(1), 63–76.

Blanco, A. (2021). Optimal inflation target in an economy with menu costs and a zero lower bound. *American Economic Journal: Macroeconomics*, *13*(3), 108–141.

Boadway, R. W. (1974). The welfare foundations of cost-benefit analysis. *The Economic Journal*, *84*(336), 926–939.

Bosmans, K., Decancq, K., & Ooghe, E. (2018). Who's afraid of aggregating money metrics? *Theoretical Economics*, *13*(2), 467–484.

Bosmans, K., & Öztürk, Z. E. (2022). Laissez-faire versus pareto. *Social Choice and Welfare*, *58*(4), 741–751.

Burstein, A., & Hellwig, C. (2008). Welfare costs of inflation in a menu cost model. *American Economic Review*, *98*(2), 438–443.

Capéau, B., Decoster, A., & De Sadeleer, L. (2023). Interpersonal comparisons by means of money metric utilities: Why one should use the same reference prices for all. *Journal of Income Distribution®*.

Chiappori, P.-A. (1988). Rational household labor supply. *Econometrica*, 63–90.

Chiappori, P.-A. (1992). Collective labor supply and welfare. *Journal of political Economy*, *100*(3), 437–467.

Cohen, G. A. (1989). On the currency of egalitarian justice. *Ethics*, *99*(4), 906–944.

Craig, B., & Rocheteau, G. (2008). Inflation and welfare: A search approach. *Journal of Money, Credit and Banking*, *40*(1), 89–119.

d'Aspremont, C., & Gevers, L. (1977). Equity and the informational basis of collective choice. *The Review of Economic Studies*, *44*(2), 199–209.

Deaton, A. (1979). The distance function in consumer behaviour with applications to index numbers and optimal taxation. *The Review of Economic Studies*, *46*(3), 391–405.

Deaton, A., & Heston, A. (2010). Understanding ppps and ppp-based national accounts. *American Economic Journal: Macroeconomics*, *2*(4), 1–35.

Decerf, B. (2023). A preference-based theory unifying monetary and non-monetary poverty measurement. *Journal of Public Economics*, *222*, 104898.

Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of econometrics*, *4*(2), 115–145.

Diewert, W. E., & Nakamura, A. (1993). *Essays in index number theory*. North-Holland, Amsterdam.

Dworkin, R. (1981). What is equality? part 1: Equality of welfare. *Philosophy & Public Affairs*, *10*(3), 185–246.

Eden, M., & Freitas, L. M. (2024). Income anonymity. *Mimeo*.

Errico, M., & Lashkari, D. (Forthcoming). Aggregation and estimation of quality: Application to the us. import prices. *Quaterly Journal of Economics*.

Feenstra, R. C., Inklaar, R., & Timmer, M. P. (2015). The next generation of the penn world table. *American economic review*, *105*(10), 3150–3182.

Ferey, A., Lockwood, B. B., & Taubinsky, D. (2024). Sufficient statistics for nonlinear tax systems with general across-income heterogeneity. *American Economic Review*, *114*(10), 3206–3249.

Fisher, I. (1922). *The making of index numbers*. Houghton Mifflin.

Fleming, M. (1952). A cardinal concept of welfare. *Quarterly Journal of Economics*, *66*(3), 366–384.

Fleurbaey, M. (2003). On the informational basis of social choice. *Social Choice and Welfare*, *21*(2), 347–384.

Fleurbaey, M. (2008). *Fairness, responsibility, and welfare*. Oxford University Press.

Fleurbaey, M. (2009). Beyond gdp: The quest for a measure of social welfare. *Journal of Economic literature*, *47*(4), 1029–1075.

Fleurbaey, M., & Blanchet, D. (2013). *Beyond gdp: Measuring welfare and assessing sustainability*. Oxford University Press.

Fleurbaey, M., & Maniquet, F. (2006). Fair income tax. *The Review of Economic Studies*, *73*(1), 55–83.

Fleurbaey, M., & Maniquet, F. (2011). *A theory of fairness and social welfare*. Econometric Society Monograph (vol 48). Cambridge University Press.

Fleurbaey, M., & Maniquet, F. (2017). Fairness and well-being measurement. *Mathematical Social Sciences*, *90*, 119–126.

Fleurbaey, M., & Maniquet, F. (2018). Optimal income taxation theory and principles of fairness. *Journal of Economic Literature*, *56*(3), 1029–79.

Geary, R. C. (1958). A note on the comparison of exchange rates and purchasing power between countries. *Journal of the Royal Statistical Society. Series A (General)*, *121*(1), 97–99.

Golosov, M., Troshkin, M., Tsyvinski, A., & Weinzierl, M. (2013). Preference heterogeneity and optimal capital income taxation. *Journal of Public Economics*, *97*, 160–175.

Hammond, P. J. (1976). Equity, arrow's conditions, and rawls' difference principle. *Econometrica*, 793–804.

Hicks, J. R. (1939). Value and capital: An inquiry into some fundamental principles of economic theory.

Hicks, J. R. (1941). The rehabilitation of consumers' surplus. *The Review of Economic Studies*, *8*(2), 108–116.

Hicks, J. R. (1942). Consumers' surplus and index-numbers. *The Review of Economic Studies*, *9*(2), 126–137.

Hicks, J. R. (1943). The four consumer's surpluses. *The review of economic studies*, *11*(1), 31–41.

Hicks, J. R. (1945). The generalised theory of consumer's surplus. *The Review of Economic Studies*, *13*(2), 68–74.

Jaravel, X., & Lashkari, D. (2024). Measuring growth in consumer welfare with income-dependent preferences: Nonparametric methods and estimates for the united states. *The Quarterly Journal of Economics*, *139*(1), 477–532.

Jones, C. I., & Klenow, P. J. (2016). Beyond gdp? welfare across countries and time. *American Economic Review*, *106*(9), 2426–2457.
Maniquet, F. (2004). On the equivalence between welfarism and equality of opportunity. *Social Choice and Welfare*, *23*, 127–147.
McKenzie, L. (1957). Demand theory without a utility index. *The Review of Economic Studies*, *24*(3), 185–189.
Neary, J. P. (2004). Rationalizing the penn world table: True multilateral indices for international comparisons of real income. *American Economic Review*, *94*(5), 1411–1428.
Piacquadio, P. G. (2017). A fairness justification of utilitarianism. *Econometrica*, *85*(4), 1261–1276.
Rawls, J. (1971). *A theory of justice*. Belknap Press.
Robbins, L. (1938). Interpersonal comparisons of utility: A comment. *The economic journal*, *48*(192), 635–641.
Roberts, K. (1980). Price-independent welfare prescriptions. *Journal of Public Economics*, *13*(3), 277–297.
Samuelson, P. A. (1947). *Foundations of economic analysis*. Harvard University Press.
Samuelson, P. A., & Swamy, S. (1974). Invariant economic index numbers and canonical duality: Survey and synthesis. *American Economic Review*, *64*(4), 566–593.
Schlee, E. E., & Khan, M. A. (2022). Money metrics in applied welfare analysis: A saddlepoint rehabilitation. *International Economic Review*, *63*(1), 189–210.
Schlee, E. E., & Khan, M. A. (2023). Money-metrics in local welfare analysis: Pareto improvements and equity considerations. *Journal of Economic Theory*, *213*, 105717.
Scitovsky, T. (1941). A note on welfare propositions in economics. *The Review of Economic Studies*, *9*(1), 77–88.
Summers, R., & Heston, A. (1991). The penn world table (mark 5): An expanded set of international comparisons, 1950–1988. *The Quarterly Journal of Economics*, *106*(2), 327–368.
Suppes, P. (1966). Some formal models of grading principles. *Synthese*, *16*(3/4), 284–306.
Van Veelen, M. (2002). An impossibility theorem concerning multilateral international comparison of volumes. *Econometrica*, *70*(1), 369–375.
van Veelen, M., & van der Weide, R. (2008). A note on different approaches to index number theory. *American Economic Review*, *98*(4), 1722–1730.

A Proofs

Proof of Theorem 3. The proof of the "if" proceeds by construction. Take a generic economy $e \in \mathcal{E}$ and two allocations z, z' such that the premise holds. Without loss of generality, set $i = 3$ and $j = 4$.

By **Egalitarian *Responsibility***, we have

$$z \ \ \mathbf{R}\Bigg((\succsim_1, \tilde{\omega}, \tilde{p})(\succsim_2, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{5, \ldots N\}}\Bigg)(z_1, z_2, z_3', z_4', \ldots z_N)$$

By ***No-Growth Separability***, we can write

$$z \ \ \mathbf{R}\Bigg((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i \in \{5, \ldots N\}}\Bigg)(z_1, z_2, z_3', z_4' \ldots z_N)$$

By ***Compensation*** applied twice,

$$(z'_3, z'_4, z_1, z_2..., z_N)\mathbf{I}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{5,...N\}}\Big)(z_1, z_2, z'_3, z'_4..., z_N)$$

By transitivity,

$$z\ \ \mathbf{R}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{5,...N\}}\Big)(z'_3, z'_4, z_1, z_2...z_N)$$

By ***Compensation*** applied twice,

$$z\ \ \mathbf{I}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{5,...N\}}\Big)(z_3, z_4, z_1, z_2..., z_N)$$

By transitivity,

$$(z_3, z_4, z_1, z_2...z_N)\mathbf{R}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{5,...N\}}\Big)(z'_3, z'_4, z_1, z_2...z_N)$$

By ***No-Growth Separability***,

$$(z_3, z_4, z_1, z_2..., z_N)\mathbf{R}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_k, y_k, p_k)_{k\in\{3,4\}}(\succsim_5, \hat{\omega}_5, \hat{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{6,...N\}}\Big)(z'_3, z'_4, z_1, z_2..., z_N)$$

only if we can find $\hat{\omega}_5$ and $\hat{p}$ such that

$$\text{for each } l = \{1, ..., L\} : 3\frac{\tilde{y}}{\tilde{p}^l} = \frac{y_3}{p_3^l} + \frac{y_4}{p_4^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} \tag{2}$$

By ***Compensation*** applied four times and transitivity,

$$z\ \ \mathbf{R}\Big((\succsim_3, \tilde{\omega}, \tilde{p})(\succsim_4, \tilde{\omega}, \tilde{p})(\succsim_k, \omega_k, p_k)_{k\in\{3,4\}}(\succsim_5, \hat{\omega}_5, \hat{p})(\succsim_i, \tilde{\omega}, \tilde{p})_{i\in\{6,...N\}}\Big)(z_1, z_2, z'_3, z'_4...z_N)$$

By ***No-Growth Separability***,

$$z\ \ \mathbf{R(e)}(z_1, z_2, z'_3, z'_4..., z_N)$$

if and only if

$$\text{for each } l = \{1, ..., L\} : (N-3)\frac{\tilde{y}}{\tilde{p}^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} = \sum_{i\in\mathcal{N}\setminus\{3,4\}} \frac{y_i}{p_i^l} \tag{3}$$

Summing over equations (2) and (3), we get

$$\text{for each } l = \{1, ..., L\} : N\frac{\tilde{y}}{\tilde{p}^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l} = \sum_{i\in\mathcal{N}} \frac{y_i}{p_i^l} + \frac{\hat{p}\hat{\omega}_5}{\hat{p}^l}$$

which holds by construction of $\tilde{\omega}$ and $\tilde{p}$. The result is obtained by observing that $(z_1, z_2, z'_3, z'_4..., z_N)$ is the same allocation as z', proving $z\mathbf{R(e)}z'$, the required result.

To prove the "only if", observe that $W_i(z_i, \tilde{\omega}, \tilde{p})$ does not depend on the individual's endowments (ω_i, p_i) such that a permutation between equal-preference individuals leads the ordering unchanged, proving that **R(e)** satisfies **Compensation**.
As for **No-Growth Separability**, observe that any change of endowments such that $\sum_i \frac{y_i}{p_i^l}$ is unchanged for all l does not modify $\tilde{\omega}$ nor $\tilde{p}$. The proof for **Egalitarian Responsibility** is trivial and left to the reader. ■

Proof of Theorem 4. Without loss of generality, consider that agents 3 and 4 are i and j. Then, by ***Utilitarian Responsibility***, we have

$$z \;\; \mathbf{R}\left((\succsim_1, \tilde{y}, \tilde{p})(\succsim_2, \tilde{y}, \tilde{p})(\succsim_3, \tilde{y}, \tilde{p})(\succsim_4, \tilde{y}, \tilde{p})(\succsim_i, \tilde{y}, \tilde{p})_{i\in\{5,\ldots N\}}\right)(z_1, z_2, z_3', z_4', \ldots z_N)$$

The remaining steps of the proof follow exactly the proof of the previous theorem. ■