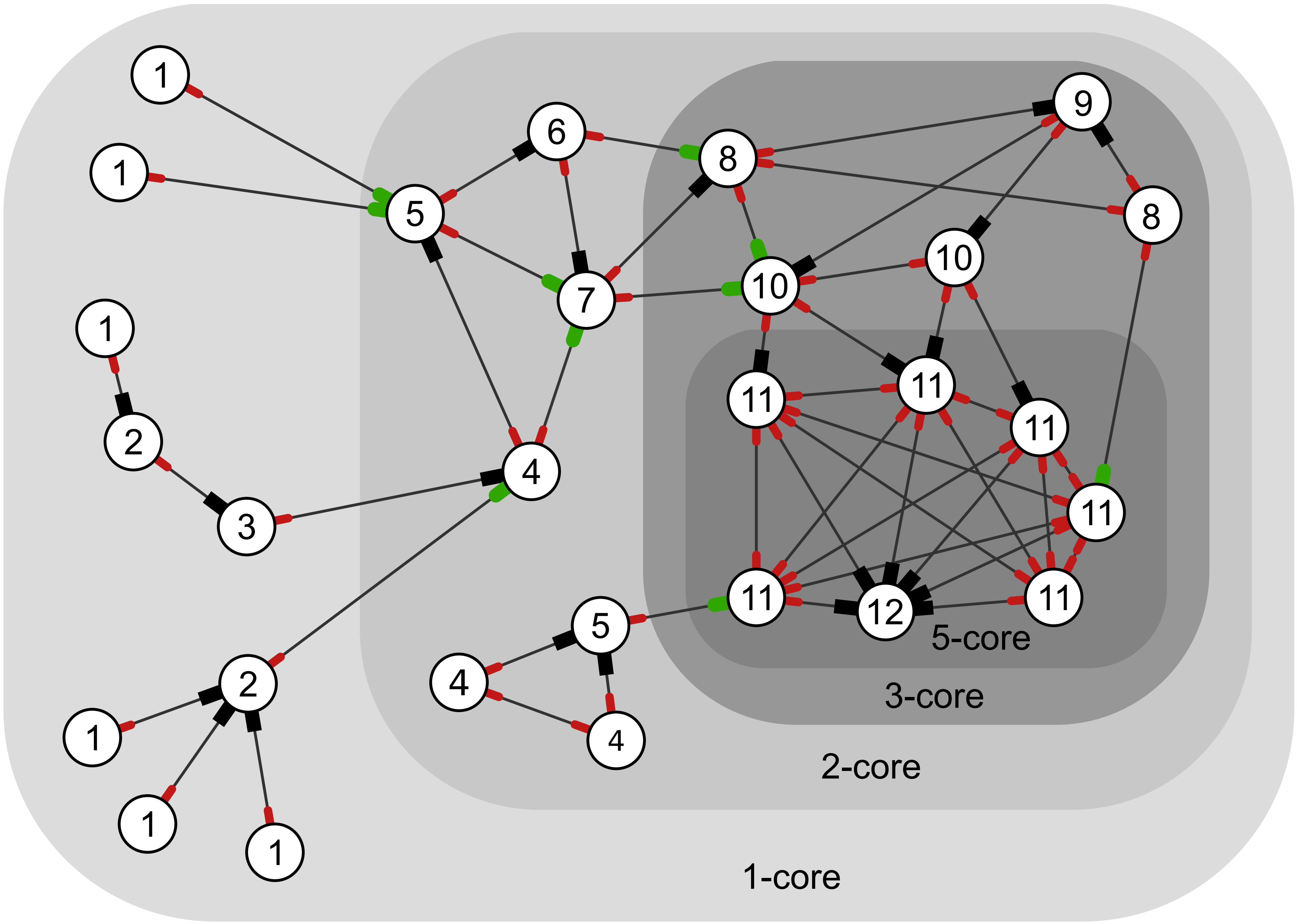
26



1
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6
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11
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12
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5
4
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2
1
1
1
5-core
3-core
2-core
1-core

Mesoscopic level: The k-core/onion decomposition

Onion decomposition: k-core decomposition with additional information about the positions of nodes within every k-shell (layers).

Information about layers is obtained from the k-core decomposition with minimal additional computational cost.

Stub matching scheme

- One type of nodes per layer
- Three types of stubs (red, green, black)
- Rules:

  1. Allowed links: red–red, red–green, red–black
  2. Nodes in layer ℓ and shell k must
     (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k-shell).
     (b) have at least $k+1$ links to nodes in layers $\ell' \geq \ell - 1$ and at most k links to nodes in layers $\ell' \geq \ell$ (if it is not in the first layer of the k-shell).

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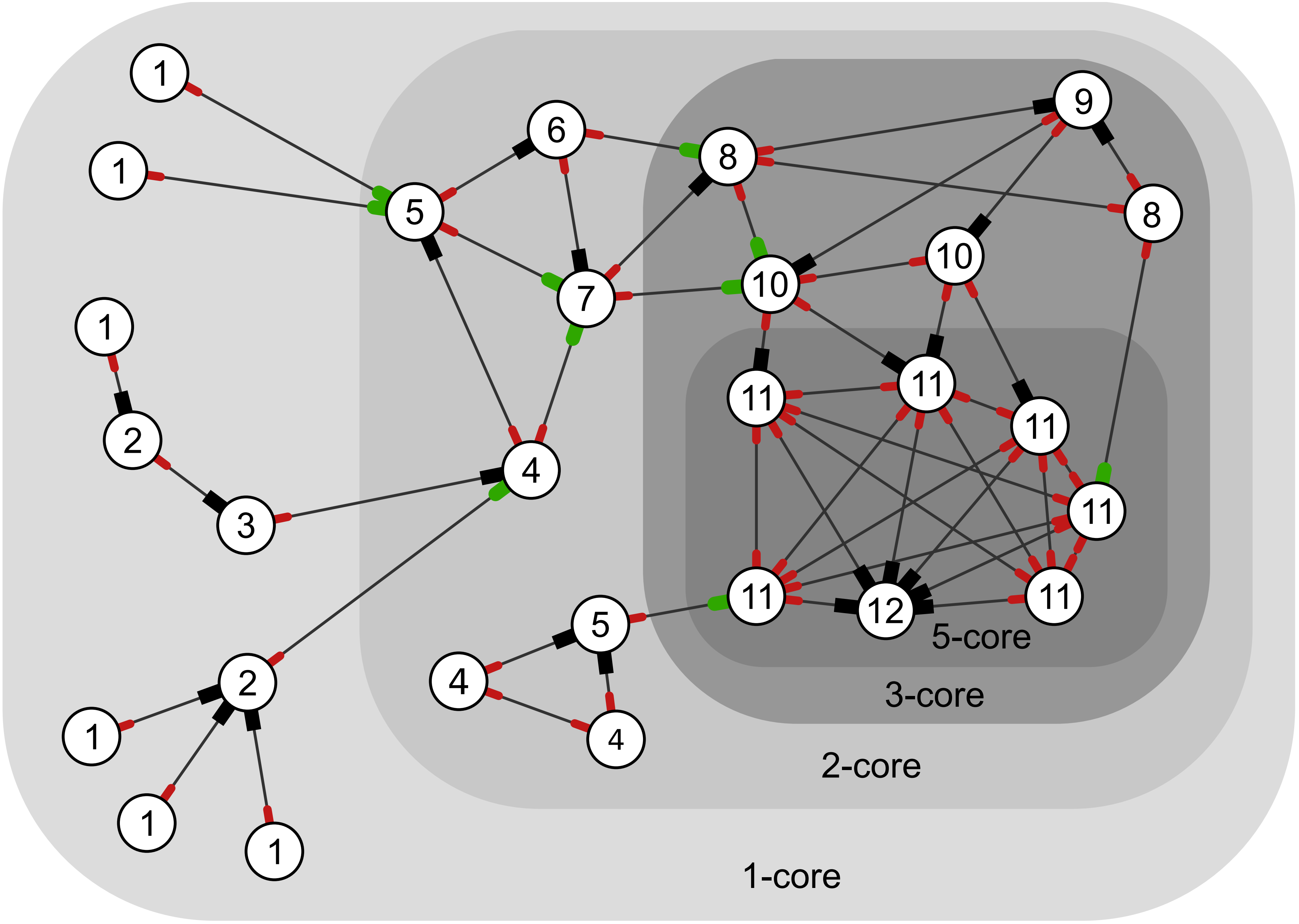

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Challenges

- Many analytical approaches (PGF, ODE) can be adapted to account for stub types, but the graphs considered are infinite in size.
- Sampling naively is easy, sampling right is tricky (ex.: uniform sampling via edge swaps).

The onion decomposition accounts for a lot when it comes to the precision of message-passing description of percolation.

The sample space is not necessarily connected under 2-edge swaps.

Numerical evidence that the ignored graphs could be negligible (using k-edge swaps), but still no formal demonstration.