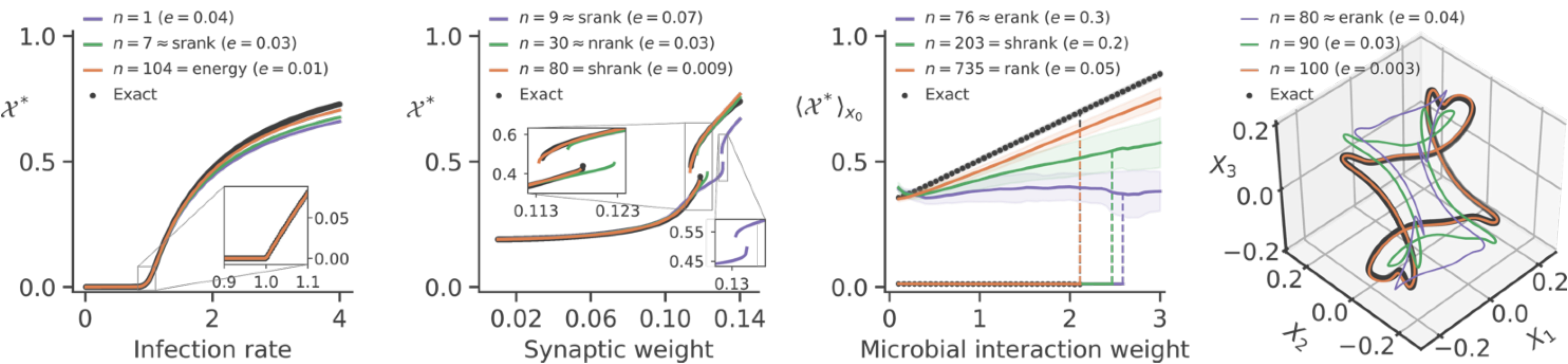


n = 1 (e = 0.04)
n = 7 ≈ srank (e = 0.03)
n = 104 = energy (e = 0.01)
Exact
Infection rate
n = 9 ≈ srank (e = 0.07)
n = 30 ≈ nrank (e = 0.03)
n = 80 = shrank (e = 0.009)
Exact
Synaptic weight
n = 76 ≈ erank (e = 0.3)
n = 203 = shrank (e = 0.2)
n = 735 = rank (e = 0.05)
Exact
Microbial interaction weight
n = 80 ≈ erank (e = 0.04)
n = 90 (e = 0.03)
n = 100 (e = 0.003)
Exact

Emergence of higher-order interactions

Original Kuramoto-Sakaguchi dynamics

$$\dot{z}_j = i\omega_j z_j + \sum_{k=1}^{N} W_{jk}[z_k e^{-i\alpha} - z_j^2 \bar{z}_k e^{i\alpha}], \quad j \in \{1, ..., N\}$$

Reduced dynamics

$$\dot{Z}_\mu = i\sum_{\nu=1}^{n} \Omega_{\mu\nu} Z_\nu + \sum_{\nu=1}^{n} \mathcal{W}_{\mu\nu}^{(2)} Z_\nu e^{-i\alpha} - \sum_{\alpha,\beta,\gamma=1}^{n} \mathcal{W}_{\mu\alpha\beta\gamma}^{(4)} Z_\alpha Z_\beta \bar{Z}_\gamma e^{i\alpha}$$

Original Quenched mean-field SIS dynamics

$$\dot{x}_i = -\alpha x_i + \beta(1 - x_i) \sum_{j=1}^{N} W_{ij} x_j, \quad i \in \{1, ..., N\}$$

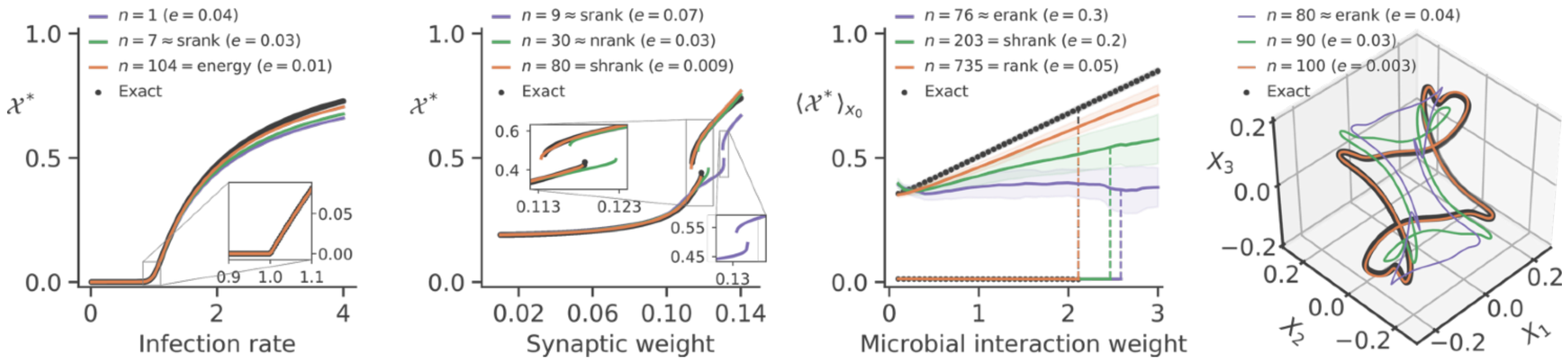
Reduced dynamics

$$\dot{X}_\mu = -\alpha X_\mu + \beta \sum_{\nu=1}^{n} \mathcal{W}_{\mu\nu} X_\nu - \beta \textstyle\sum_{\nu,\tau=1}^{n} \mathcal{W}^{(3)}_{\mu\nu\tau} X_\nu X_\tau, \quad \mu \in \{1, ..., n\}$$

Reproduction of the dynamics with increasing accuracy

10

Reproduction of the dynamics with increasing accuracy



Emergence of higher-order interactions

Original Quenched mean-field SIS dynamics

$$\dot{x}_i = -\alpha x_i + \beta(1 - x_i)\sum_{j=1}^{N} W_{ij}x_j, \quad i \in \{1, ..., N\}$$

Reduced dynamics

$$\dot{X}_\mu = -\alpha X_\mu + \beta\sum_{\nu=1}^{n} \mathcal{W}_{\mu\nu}X_\nu - \beta\sum_{\nu,\tau=1}^{n} \mathcal{W}^{(3)}_{\mu\nu\tau}X_\nu X_\tau, \quad \mu \in \{1, ..., n\}$$

Original Kuramoto-Sakaguchi dynamics

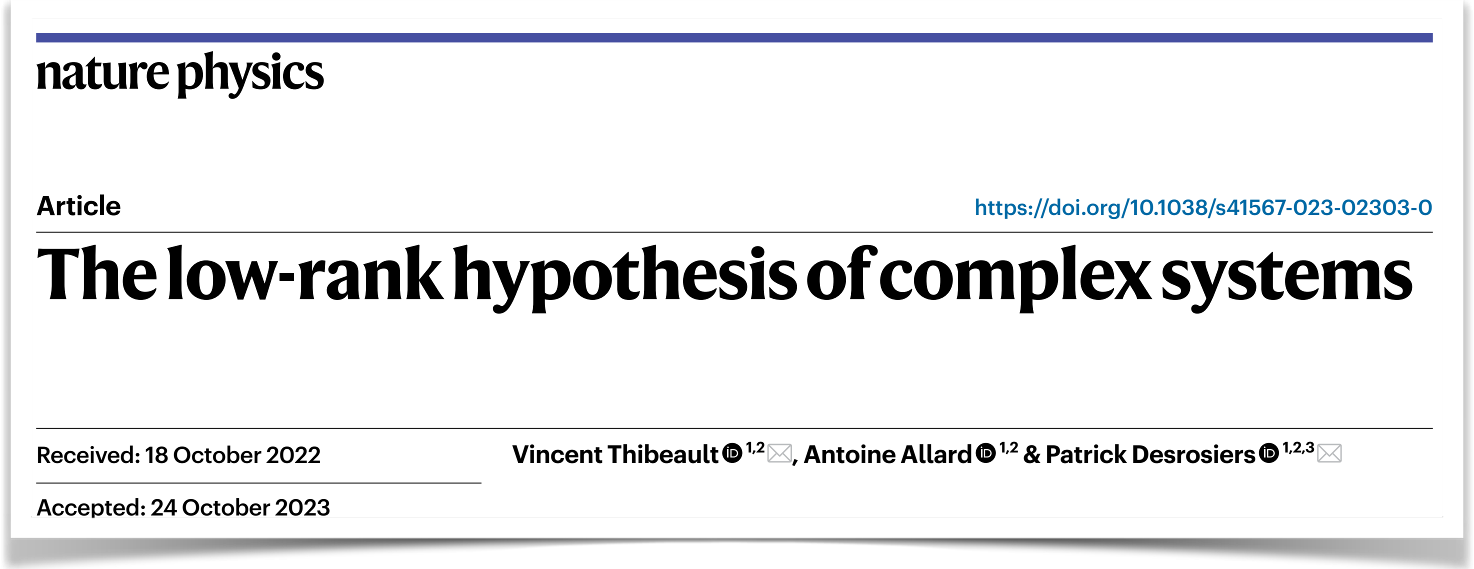
$$\dot{z}_j = i\omega_j z_j + \sum_{k=1}^{N} W_{jk}[z_k e^{-i\alpha} - z_j^2 \bar{z}_k e^{i\alpha}], \quad j \in \{1, ..., N\}$$

Reduced dynamics

$$\dot{Z}_\mu = i\sum_{\nu=1}^{n} \Omega_{\mu\nu}Z_\nu + \sum_{\nu=1}^{n} \mathcal{W}^{(2)}_{\mu\nu}Z_\nu e^{-i\alpha} - \sum_{\alpha,\beta,\gamma=1}^{n} \mathcal{W}^{(4)}_{\mu\alpha\beta\gamma}Z_\alpha Z_\beta \bar{Z}_\gamma e^{i\alpha}$$

Main takeaways

- The rapid decrease of the singular values of adjacency matrices (i.e. low effective rank) offers a *justification* for low-dimensional mathematical models beyond mathematical and/or conceptual convenience.
- A large proportion of real networks can be considered as having a low effective rank.
- The higher-order interactions observed in some systems could be a byproduct of a low-dimensional representation used to analyze them.


nature physics

Article

https://doi.org/10.1038/s41567-023-02303-0

The low-rank hypothesis of complex systems

Received: 18 October 2022

Accepted: 24 October 2023

Vincent Thibeault[1,2], Antoine Allard[1,2] & Patrick Desrosiers[1,2,3]

Challenges and open questions

- Could we measure the effective dimension independently?
- Could we design a random graph model based on observed singular values (singular vectors)?
- Are some of the higher-order interactions inferred from time series artefacts of coarse-grained observations?
- Could we designed more interpretable observables, perhaps nonlinear ones?

Vincent Thibeault
Université Laval

Patrick Desrosiers
Université Laval
CERVO Brain research center