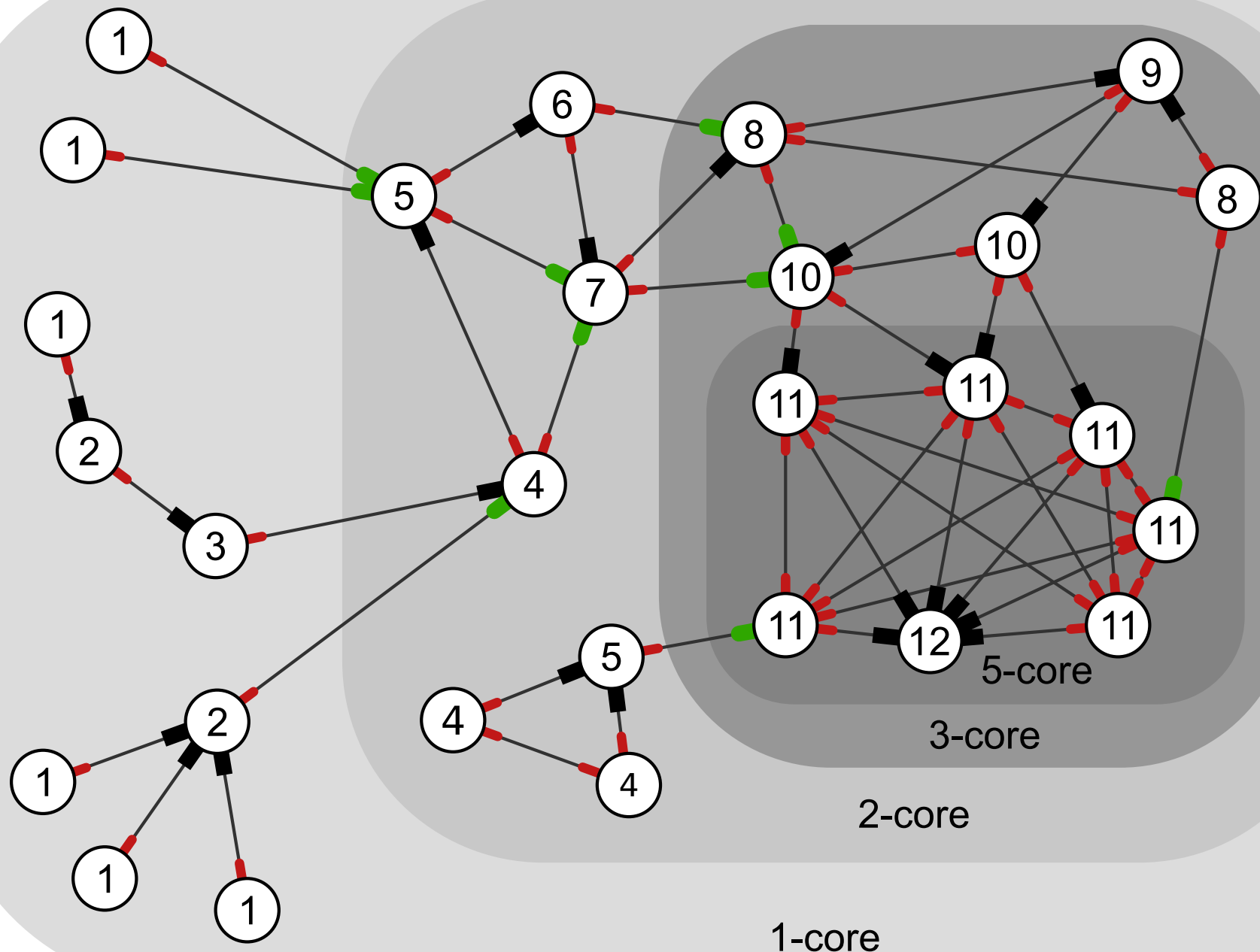
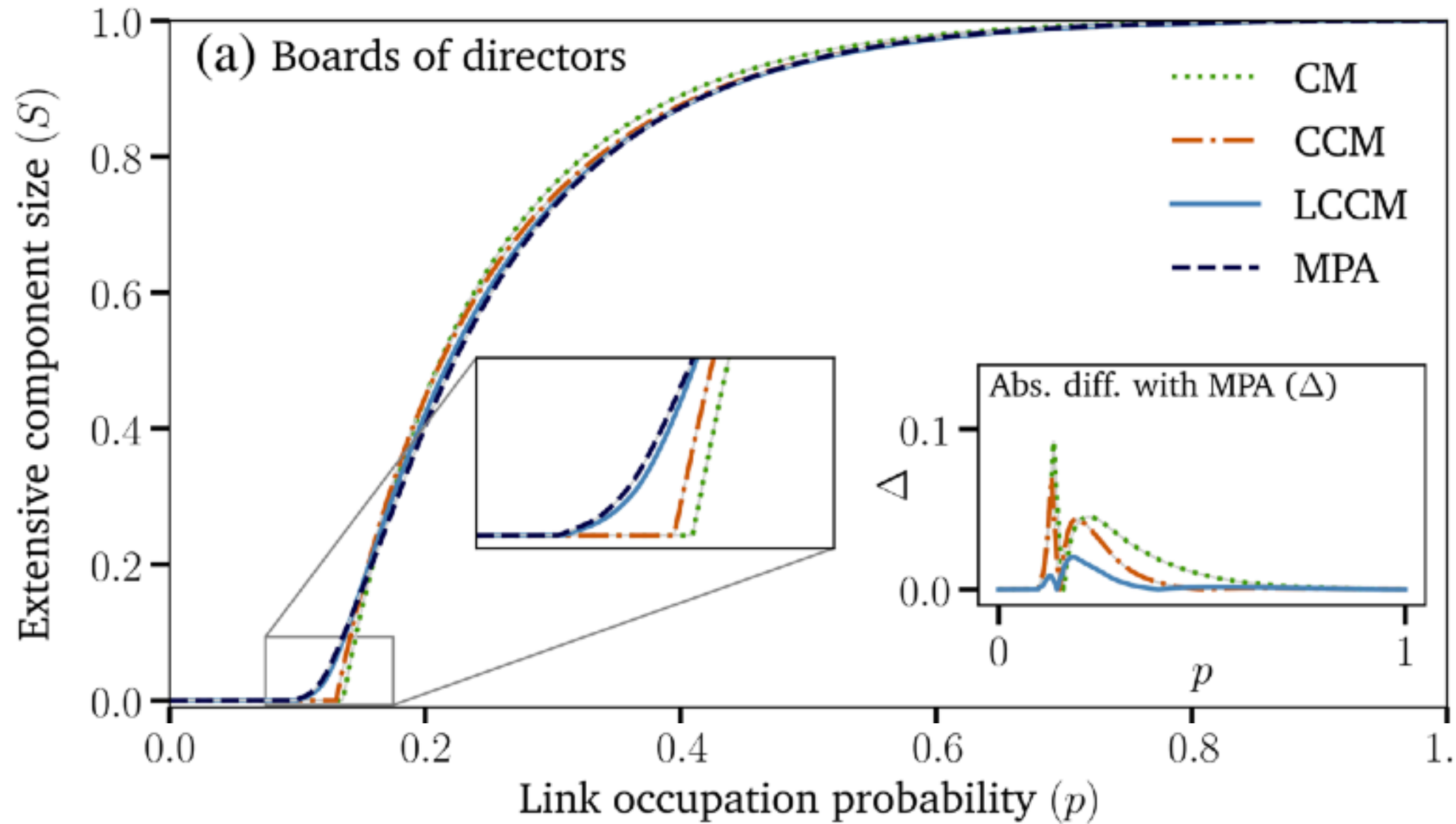


2

5



Message level: The k -core/union decomposition



Onion decomposition: k -core decomposition with additional information about the positions of nodes within every k -shell (layers).

Information about layers is obtained from the k -core decomposition with minimal additional computational cost.

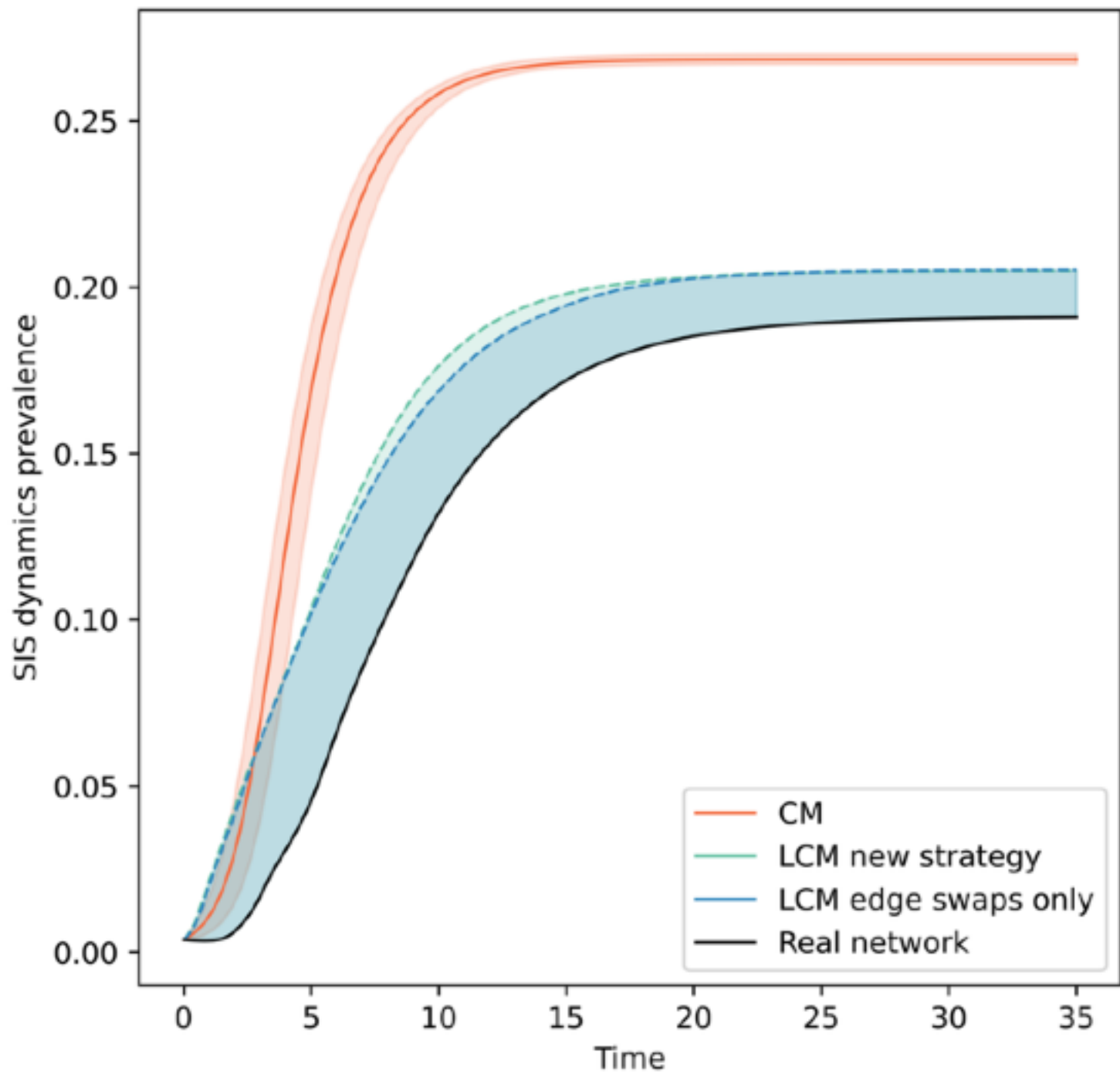
Stub matching scheme

- ▷ One type of nodes per layer
- ▷ Three types of stubs (red, green, black)
- ▷ Rules:

1. Allowed links: red–red, red–green, red–black

2. Nodes in layer ℓ and shell k must

- (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k -shell).
- (b) have at least $k + 1$ links to nodes in layers $\ell' \geq \ell - 1$ and at most k links to nodes in layers $\ell' \geq \ell$ (if it is not in the first layer of the k -shell).



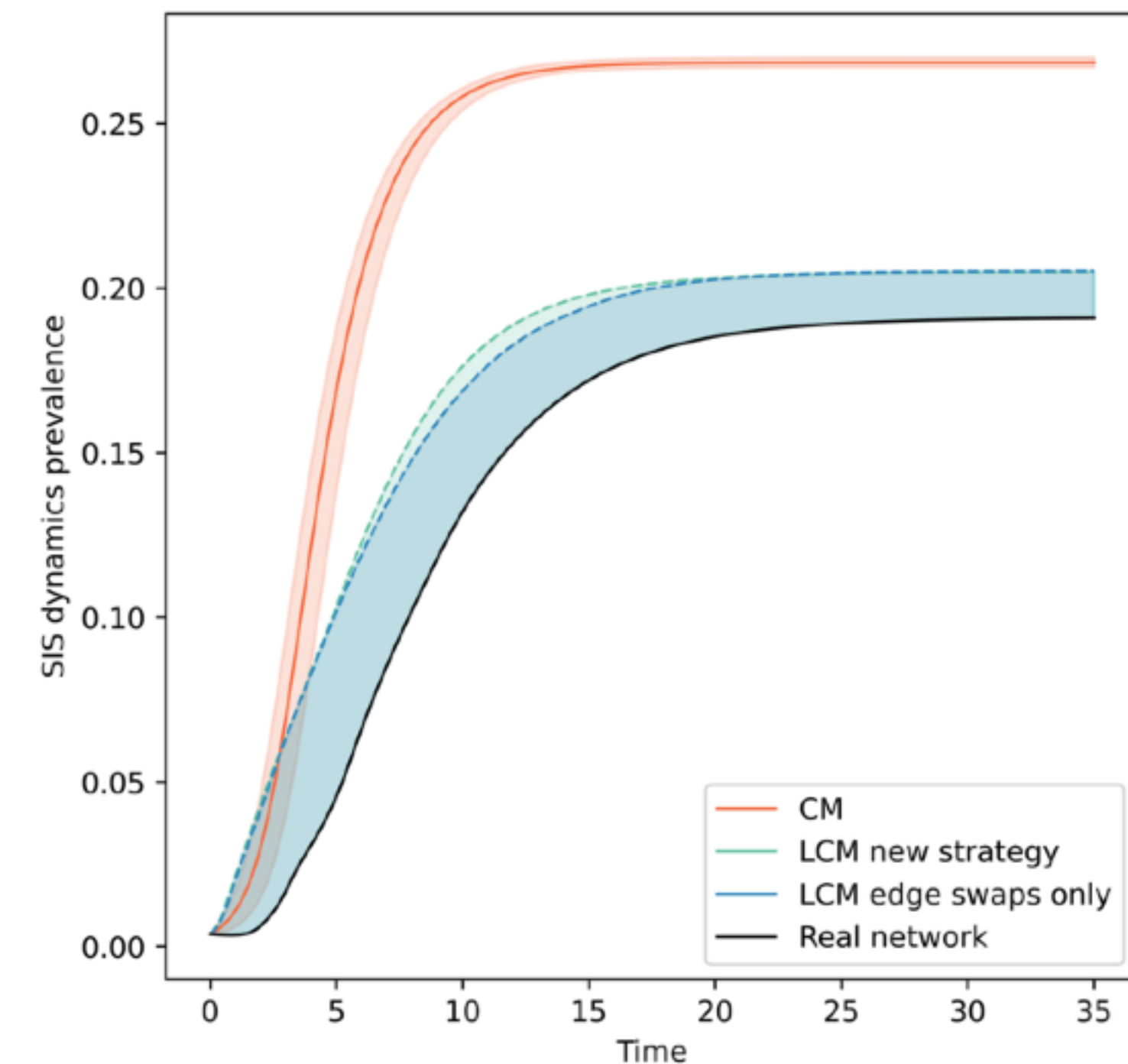
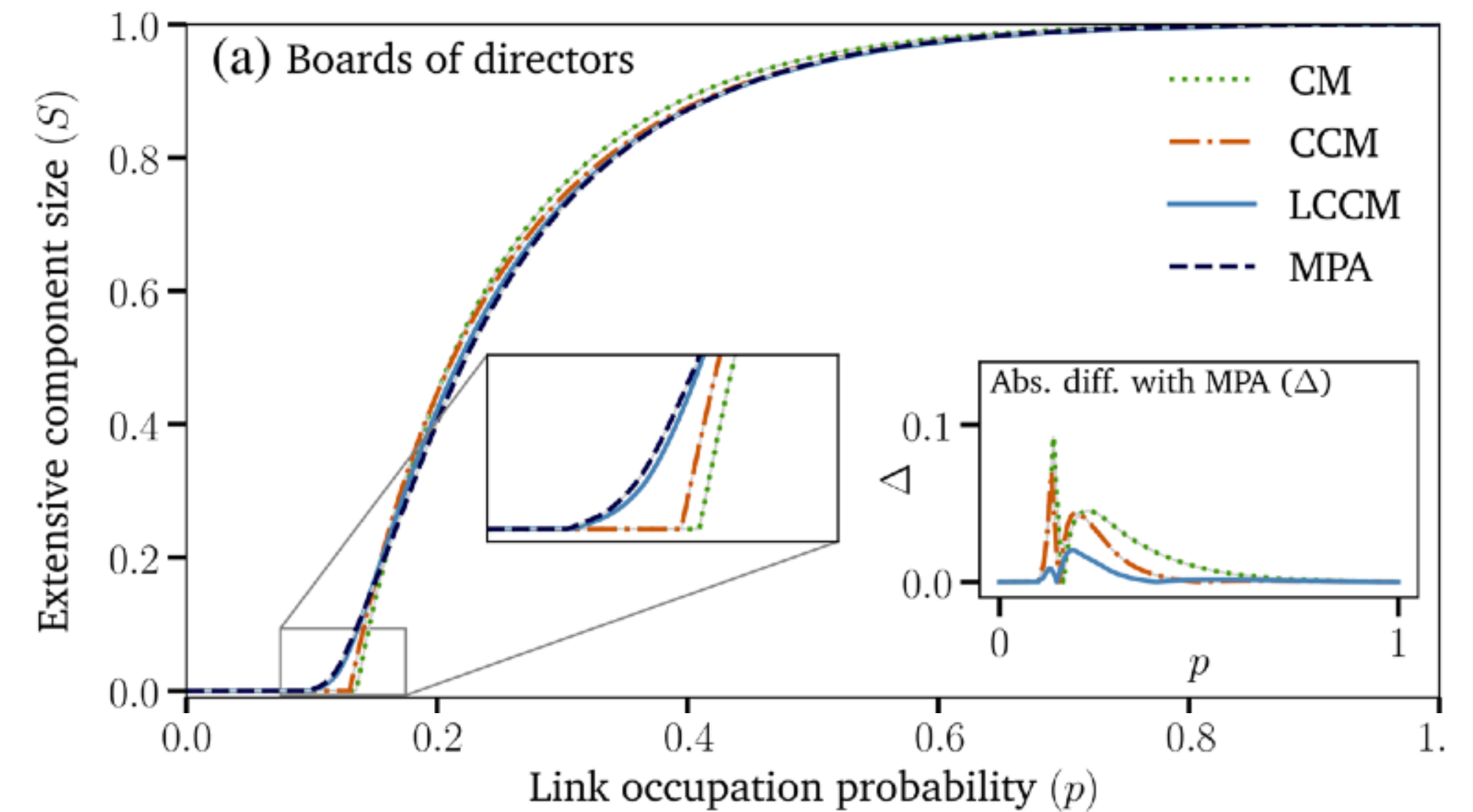
Mesoscopic level: The k -core/onion decomposition

Onion decomposition: k -core decomposition with additional information about the positions of nodes within every k -shell (layers).

Information about layers is obtained from the k -core decomposition with minimal additional computational cost.

Stub matching scheme

- ▷ One type of nodes per layer
- ▷ Three types of stubs (red, green, black)
- ▷ Rules:
 1. Allowed links: red–red, red–green, red–black
 2. Nodes in layer ℓ and shell k must
 - (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k -shell).
 - (b) have at least $k + 1$ links to nodes in layers $\ell' \geq \ell - 1$ and at most k links to nodes in layers $\ell' \geq \ell$ (if it is not in the first layer of the k -shell).



Macroscopic level: Connectivity

Preprocessing: Identify links that lead to the largest connected component and tag them as red; all remaining stubs are tags as black.

Stub matching scheme

- ▷ One type of nodes
- ▷ Two types of stubs (red, black)
- ▷ Rules:
 1. Allowed links: red-red, red-black
 2. Black stubs connect to any other stubs, but only towards nodes with excess red degree 0.
 3. Red stubs connect to any other stubs, but only towards nodes with excess red degree at least 1.

