

Three challenges in modeling directed networks

Adapted from Holland & Leinhardt. Local Structure in Social Networks. Sociol. Methodol., 7, 1-45 (1976)

Identity of indiscernibles

$$d(x, y) = 0 \quad \Leftrightarrow \quad x = y$$

Non-negativity

$$d(x, y) \geq 0$$

Symmetry

$$d(x, y) = d(y, x)$$

Triangle inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

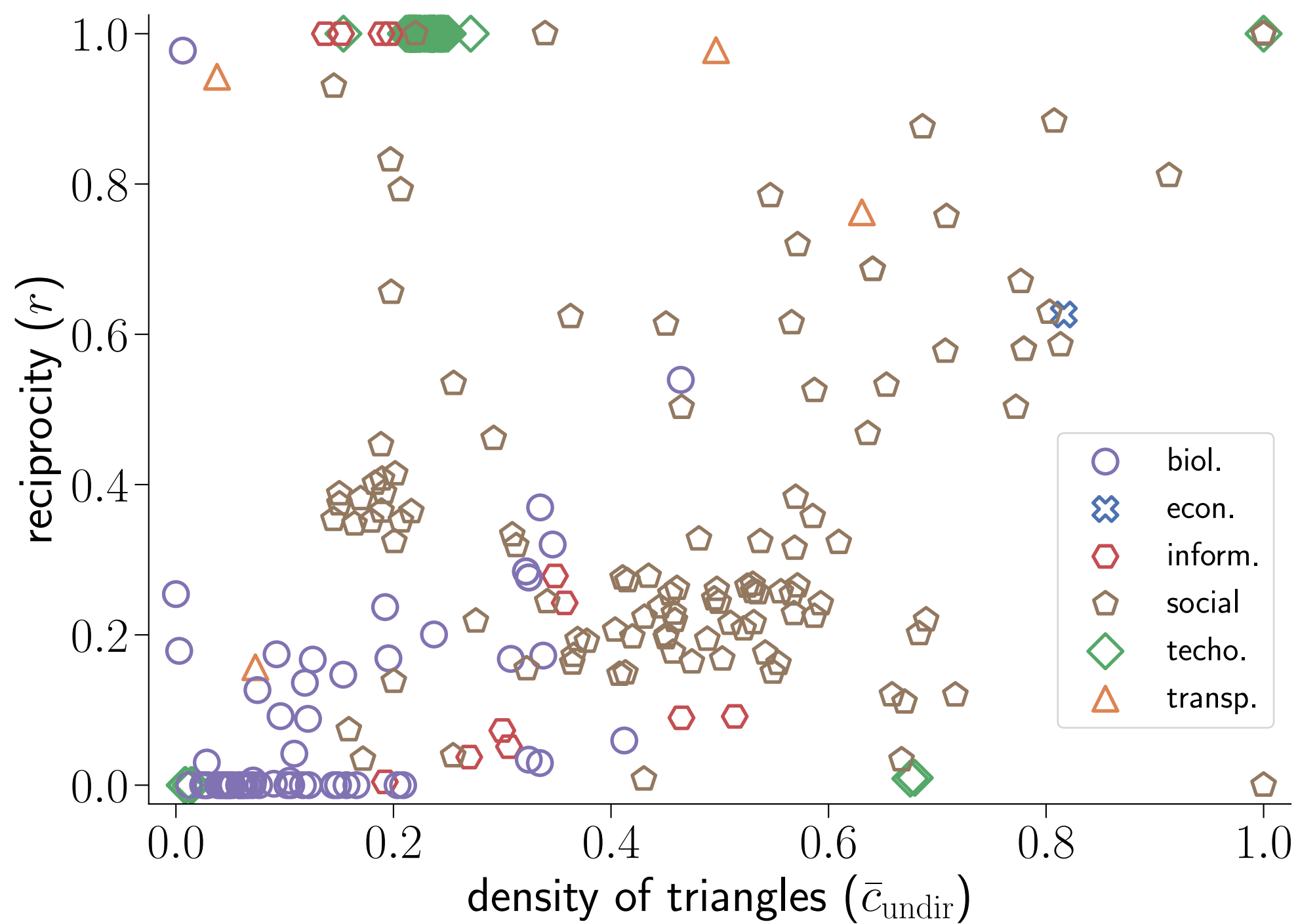
292 network datasets downloaded from Netzhelder (networks.skewed.de).

Properties of any metric space

Clustering: 7 cycles of length 3

Reciprocity: cycles of length 2

$$r = \frac{L^{\leftrightarrow}}{L} = \frac{[\text{number of reciprocal links}]}{[\text{number of links}]}$$

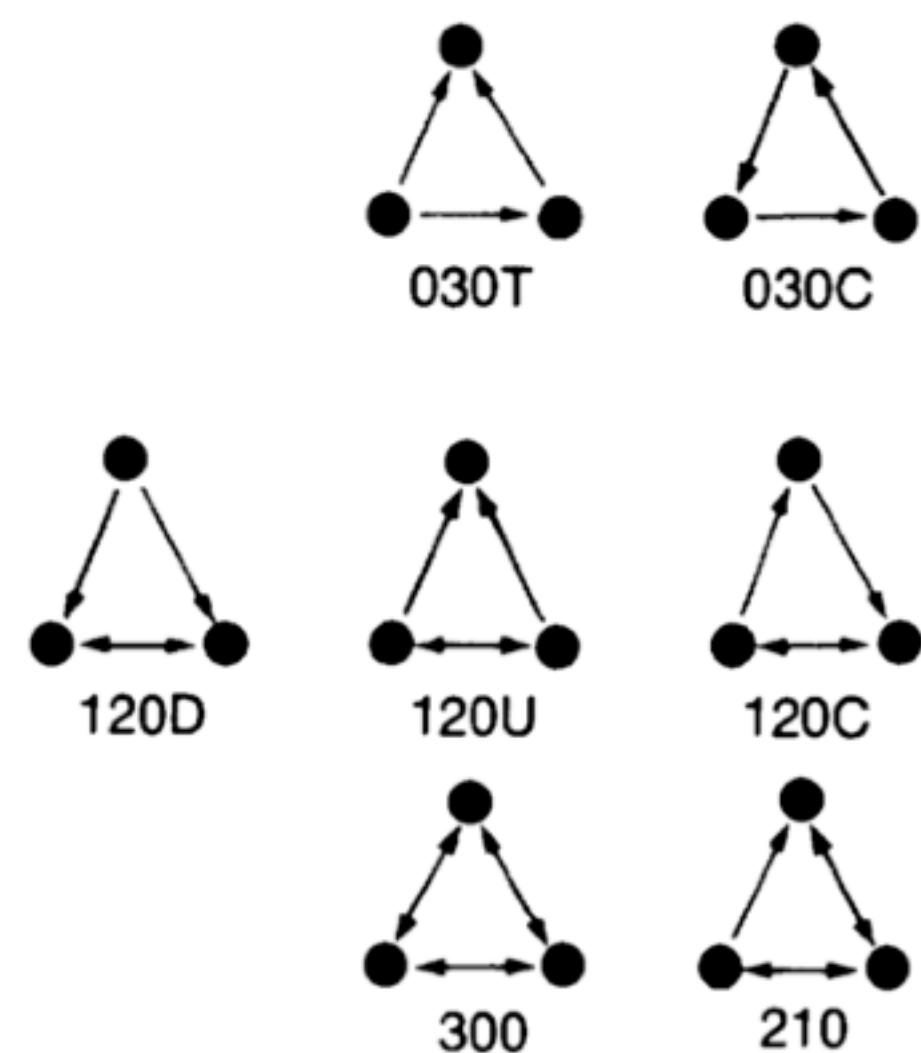


Three challenges in modeling directed networks

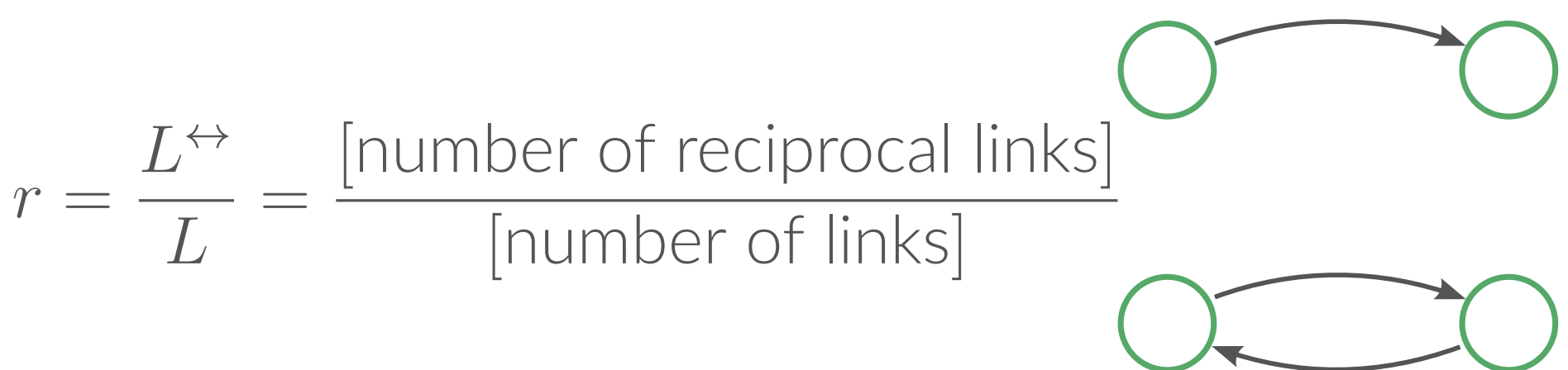
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| | |
|----------------------------|----------------------------------|
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| Symmetry | $d(x, y) = d(y, x)$ |
| Triangle inequality | $d(x, y) \leq d(x, z) + d(z, y)$ |

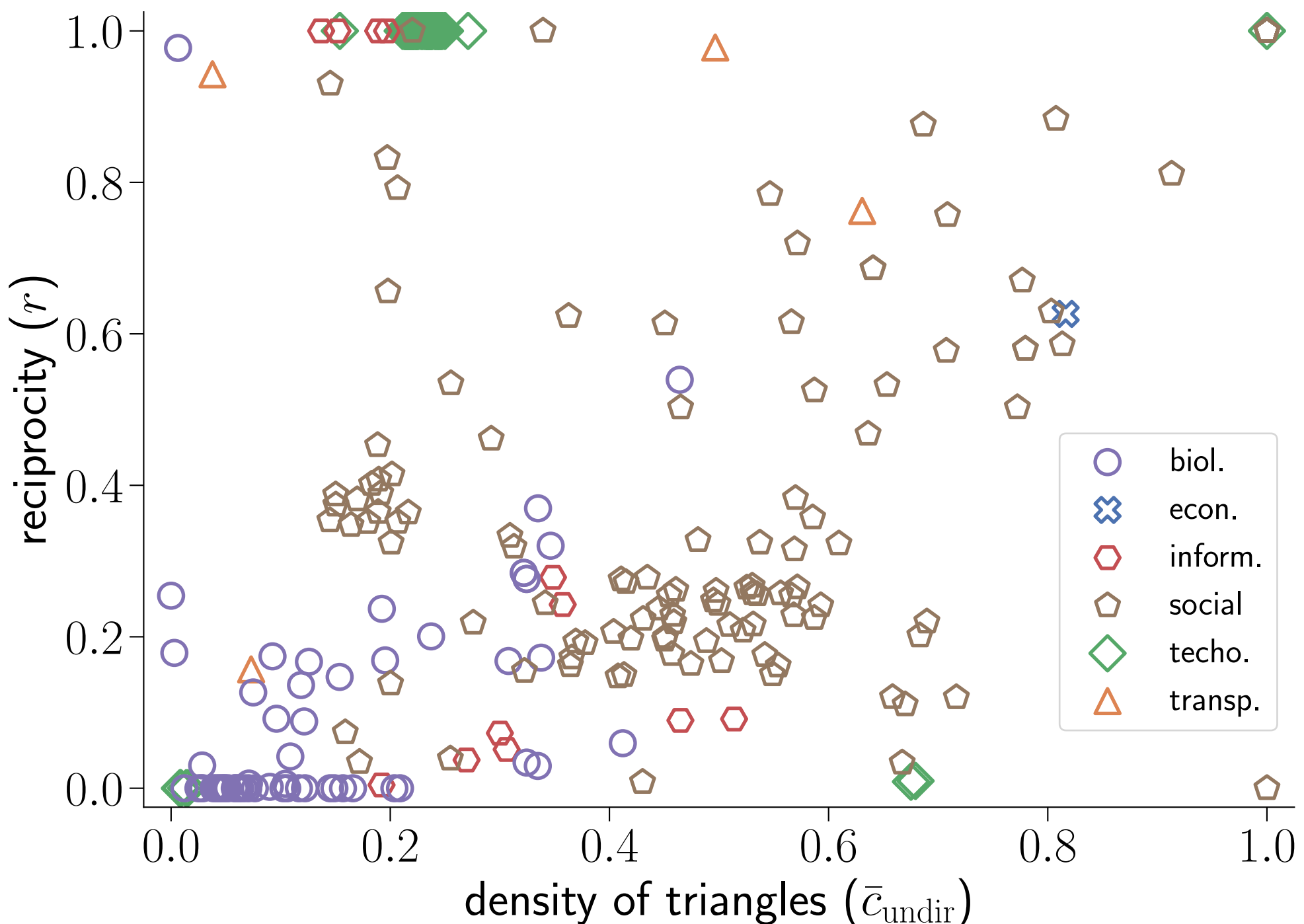
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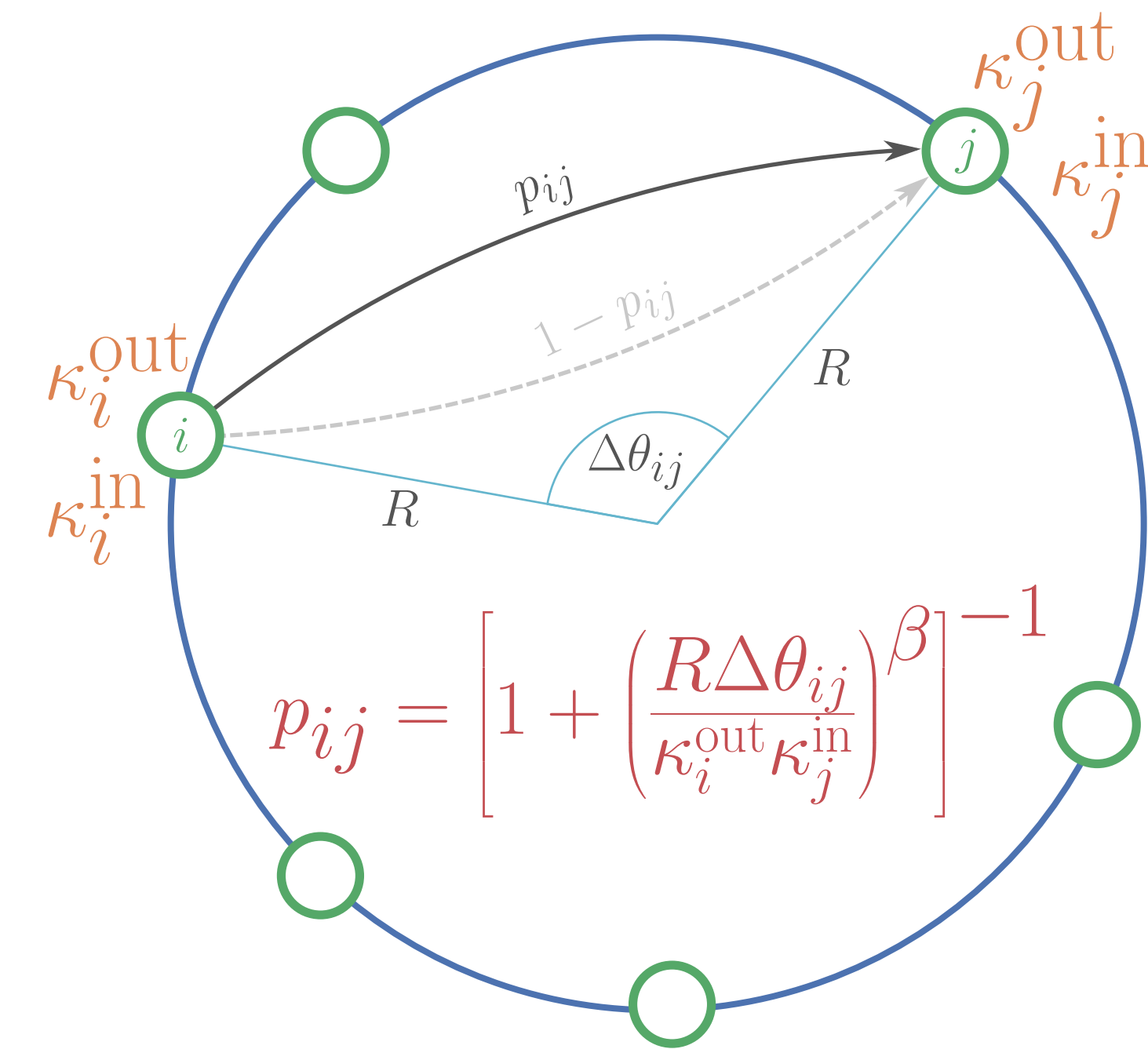
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The directed \mathbb{S}^1 model



The directed \mathbb{S}^1 model

1. Sprinkle N nodes uniformly on a circle of radius R .
2. Assign an expected in-degree κ^{in} and out-degree κ^{out} to each node according to some pdf $\rho(\kappa^{\text{in}}, \kappa^{\text{out}})$.
3. Draw a link from node i to node j with probability p_{ij} .

- ★ fixes the expected in-degree and out-degree of nodes $(\kappa^{\text{in}}, \kappa^{\text{out}}) \rightarrow$ soft directed CM
- ★ triangle inequality of the underlying metric space \rightarrow triangles from pairwise interactions
- ★ level of clustering tuned with parameter β