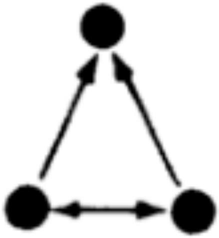
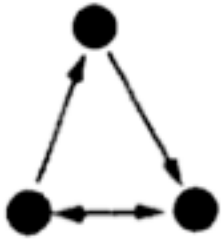
030T

030C

120D



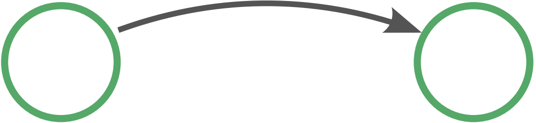
120U

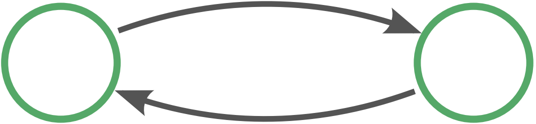


120C

300

210





Three challenges in modeling directed networks

Adapted from Holland & Leinhardt. Local Structure in Social Networks. Sociol. Methodol., 7, 1–45 (1976)

Identity of indiscernibles	$d(x, y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x, y) \geq 0$
Symmetry	$d(x, y) = d(y, x)$
Triangle inequality	$d(x, y) \leq d(x, z) + d(z, y)$

Properties of any metric space

Clustering : 7 cycles of length 3

Reciprocity : cycles of length 2

$$r = \frac{L^{\leftrightarrow}}{L} = \frac{[\text{number of reciprocal links}]}{[\text{number of links}]}$$

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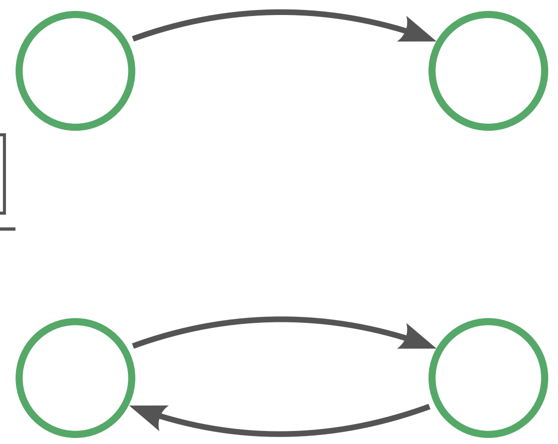

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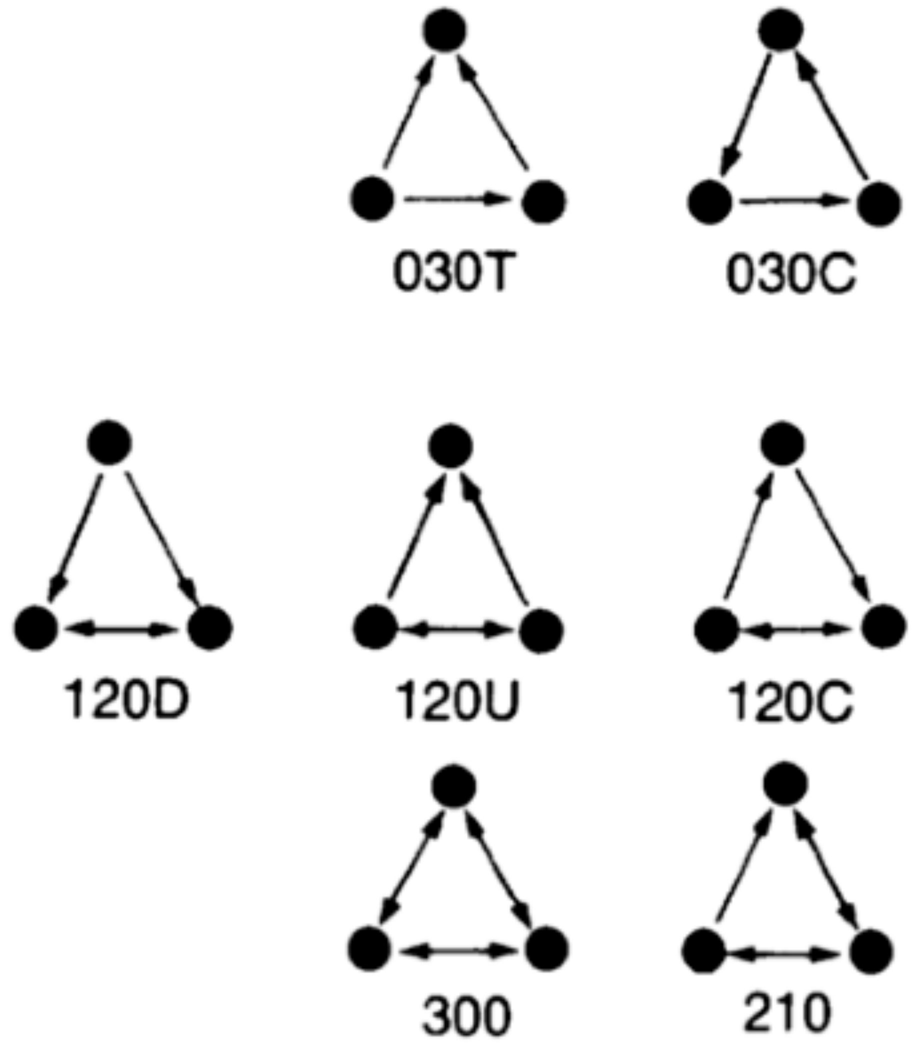

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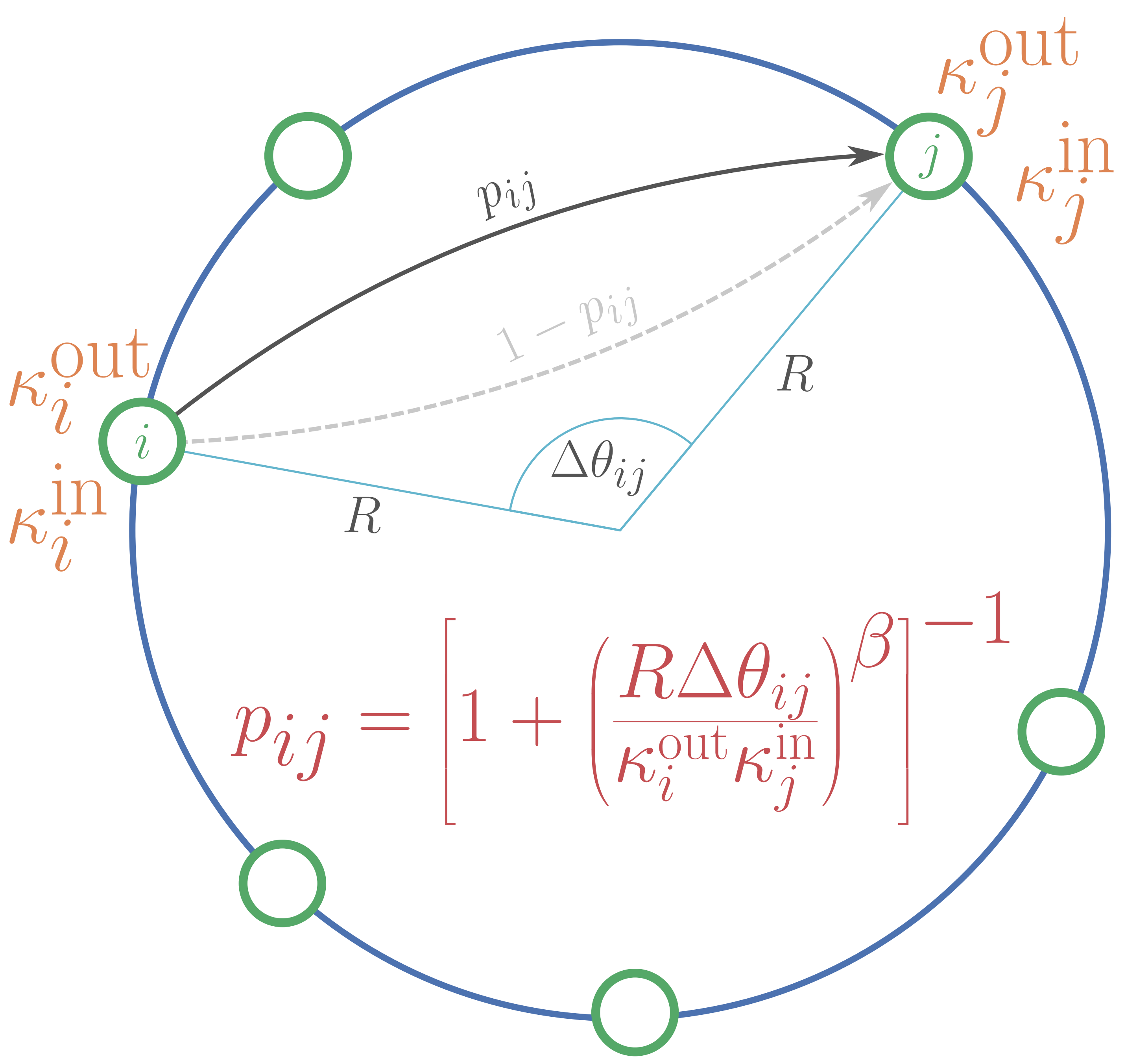
The directed $\mathbb{S}^1$ model: A straightforward generalization



The directed $\mathbb{S}^1$ model

1. Sprinkle N nodes uniformly on a circle of radius R.
2. Assign an expected in-degree κ^{in} and out-degree κ^{out} to each node according to some pdf $\rho(\kappa^{\text{in}}, \kappa^{\text{out}})$.
3. Draw a link from node i to node j with probability p_{ij}.

★ fixes the expected in-degree and out-degree of nodes (κ^{in}, κ^{out}) $\rightarrow$ soft directed CM

★ triangle inequality of the underlying metric space $\rightarrow$ triangles from pairwise interactions

★ level of clustering tuned with parameter β