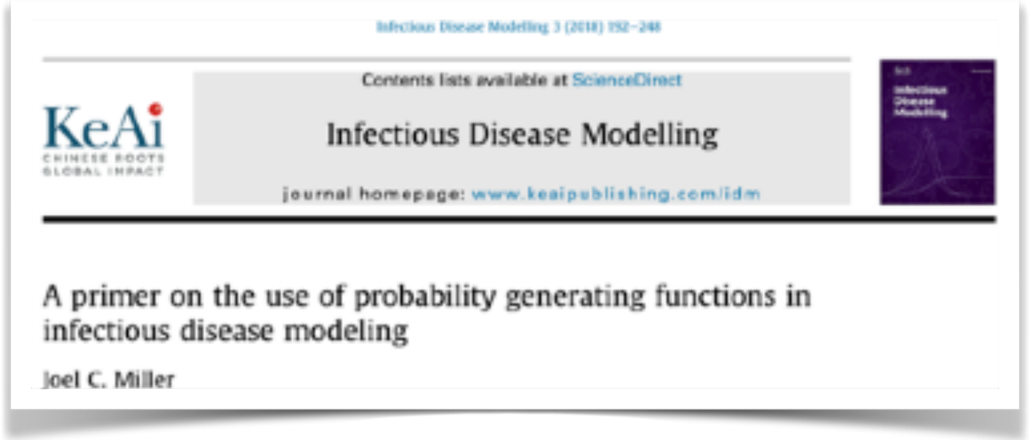
Contact network epidemiology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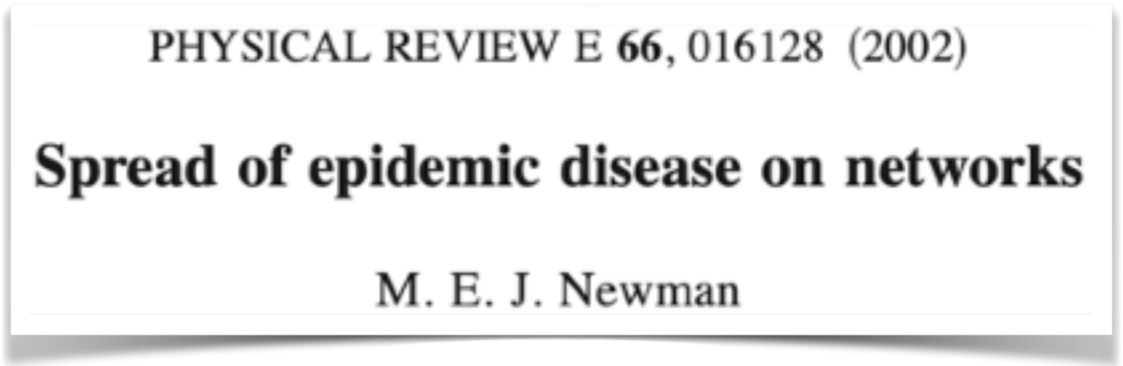
A primer on the use of probability generating functions in infectious disease modeling
Joel C. Miller


PHYSICAL REVIEW E **66**, 016128 (2002)
Spread of epidemic disease on networks
M. E. J. Newman

Probability generating functions (PGFs) formalism

– assuming a very, very large population (i.e. neglecting finite-size effects)
– patient zero causes k secondary infections with probability p_k (degree distribution of thfie network)

$$G_0(x) = \; + \; x + \; x^2 + \; x^3 + \ldots = \sum_{k\geq 0}^{\infty} p_k x^k \; ; \qquad \langle k \rangle = \sum_{k\geq 0}^{\infty} k\, p_k = G_0'(1) \; ; \qquad \langle k^2 \rangle = \sum_{k\geq 0}^{\infty} k^2\, p_k$$

– a newly infected individual causes k *new* infections with probability $(k+1)p_{k+1}/\langle k \rangle$ (*excess* degree distribution of the network)

$$G_1(x) = \; + \; x + \; x^2 + \; x^3 + \ldots = \sum_{k\geq 0}^{\infty} \frac{(k+1)p_{k+1}}{\langle k \rangle} x^k = \frac{G_0'(x)}{G_0'(1)}$$

– average number of secondary infections a newly infected individual causes

$$G_1'(1) = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} \equiv R_0$$

– *all* outbreaks will eventually die out when $R_0 < 1$
– *some* outbreaks will eventually die out when $R_0 > 1$

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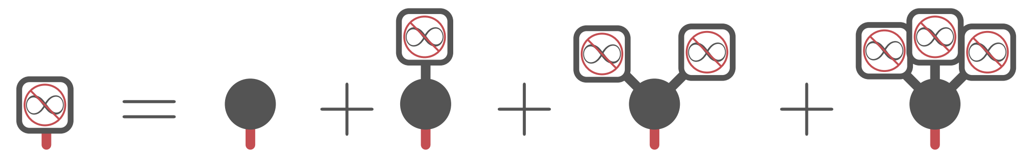
Probability generating functions (PGFs) formalism

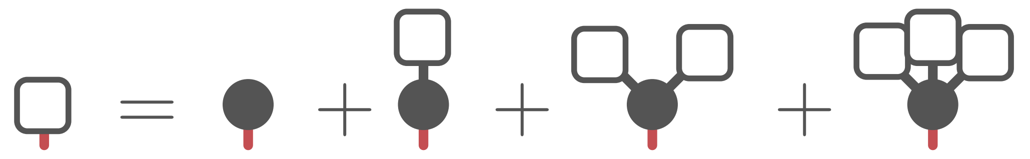
- probability u that an outbreak eventually dies out

$u =$  $+ \ldots = \sum_{k\geq 0}^{\infty} \frac{(k+1)p_{k+1}}{\langle k \rangle} u^k = G_1(u)$

- the fraction of the population infected in an epidemic wave (and the probability of such wave) is

$$R(\infty) = \sum_{k\geq 0}^{\infty} p_k (1 - u^k) = 1 - G_0(u)$$

- $H_0(x)$: PGF of the distribution of the size of outbreaks that will eventually die out

$H_1(x) =$  $+ \ldots = x \sum_{k\geq 0}^{\infty} \frac{(k+1)p_{k+1}}{\langle k \rangle} \left[H_1(x)\right]^k = xG_1\big(H_1(x)\big)$

- the distribution of the size of outbreaks that will eventually die out can be extracted from

$$H_0(x) = xG_0\big(H_1(x)\big)$$