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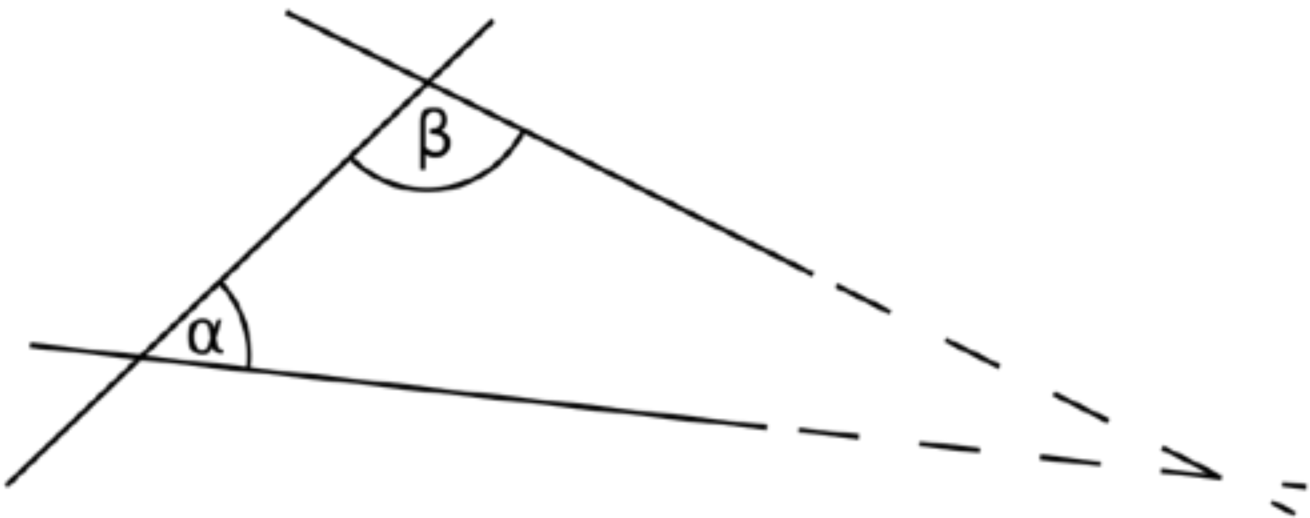
Euclid’s postulates


β
α

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.

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5. (parallel postulate) If a line segment intersects two straight lines forming two interior angles on the same side that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles.

Trying to get rid of the fifth postulate motivated the discovery of non-Euclidean geometries.

spherical geometry
positive curvature

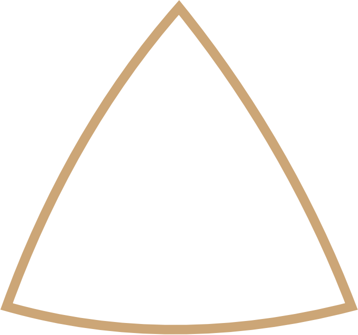
$$\alpha + \beta + \gamma > \pi$$

Euclidean geometry
no curvature

$$\alpha + \beta + \gamma = \pi$$

hyperbolic geometry
negative curvature

$$\alpha + \beta + \gamma < \pi$$



α

β

γ

α

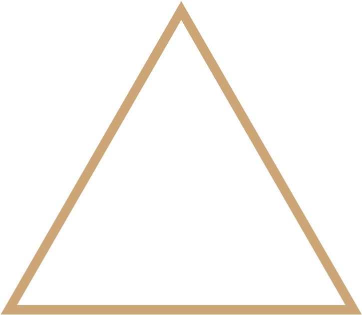
β

γ

α

β

γ



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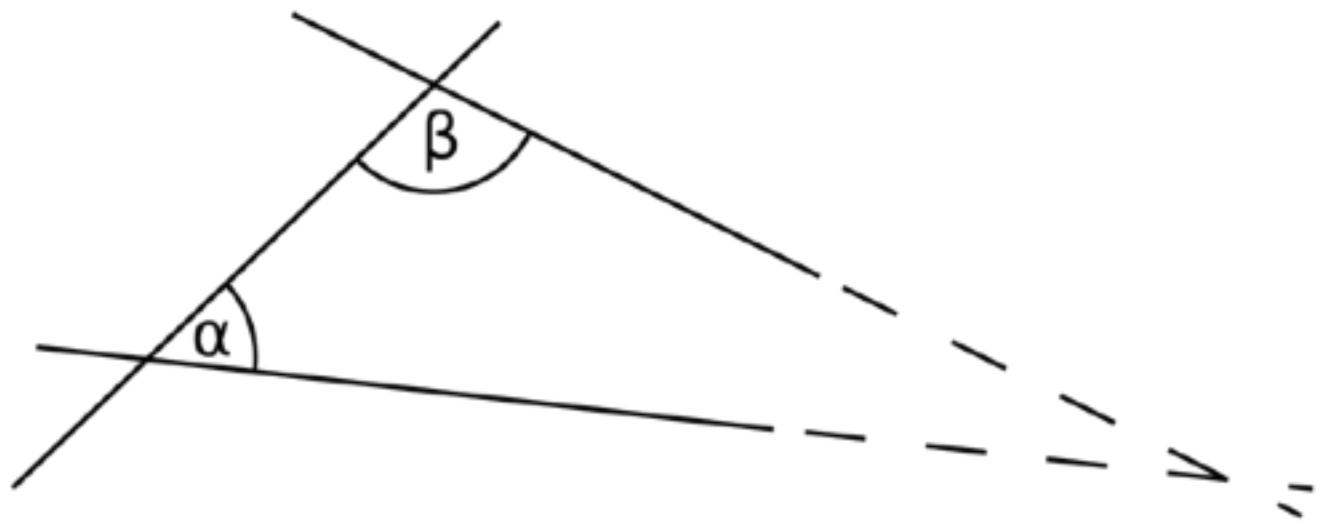

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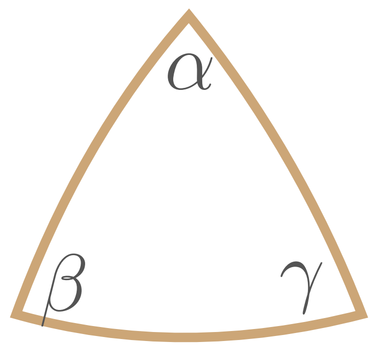
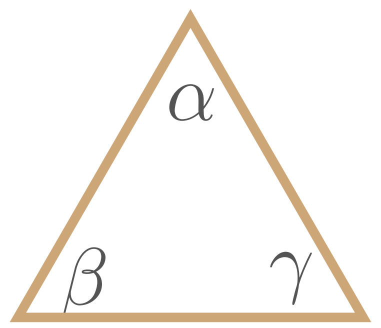


spherical geometry
positive curvature
$\alpha + \beta + \gamma > \pi$



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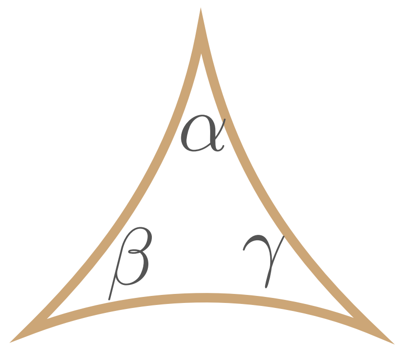

hyperbolic geometry
negative curvature
$\alpha + \beta + \gamma < \pi$

Hyperbolic geometry

- There are 5 models of hyperbolic geometry in $\mathbb{R}^{D,1}$:

  H: the half-space model;
  I: the interior of the disk model;
  J: the hemisphere model;
  K: the Klein model;
  L: the hyperboloid model.

- They are isometrically equivalent.

- They have their own metric, geodesics, isometries, and so on.

- Each model supplies its own natural intuitions.

The five analytic models and their connecting isometries in $D = 1$.