



▷ There are 5 models of hyperbolic geometry in  $\mathbb{R}^{D,1}$ :

$H$ : the half-space model;

$I$ : the interior of the disk model;

$J$ : the hemisphere model;

$K$ : the Klein model;

$L$ : the hyperboloid model.

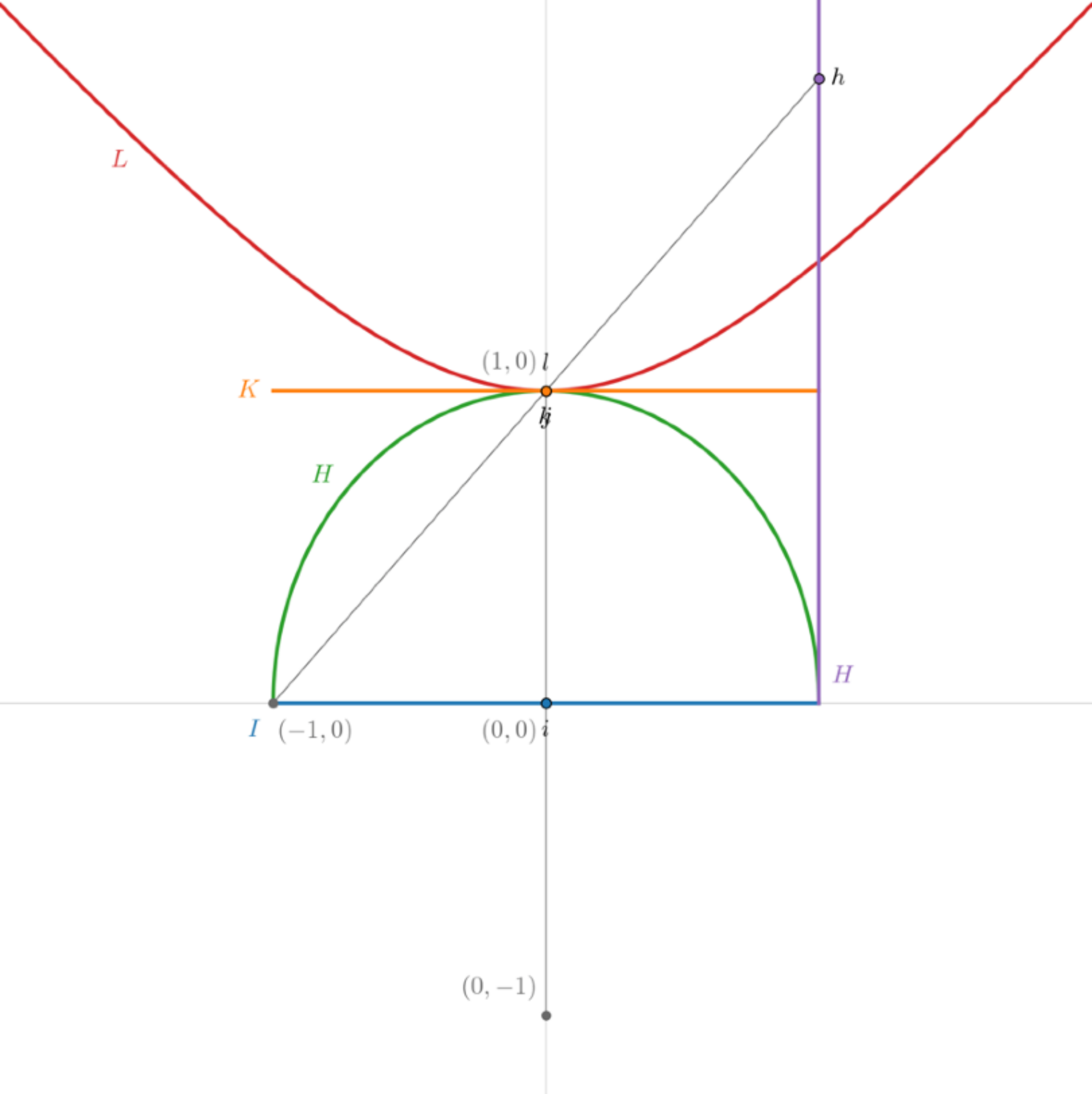
▷ They are isometrically equivalent.

▷ They have their own metric, geodesics, isometries, and so on.

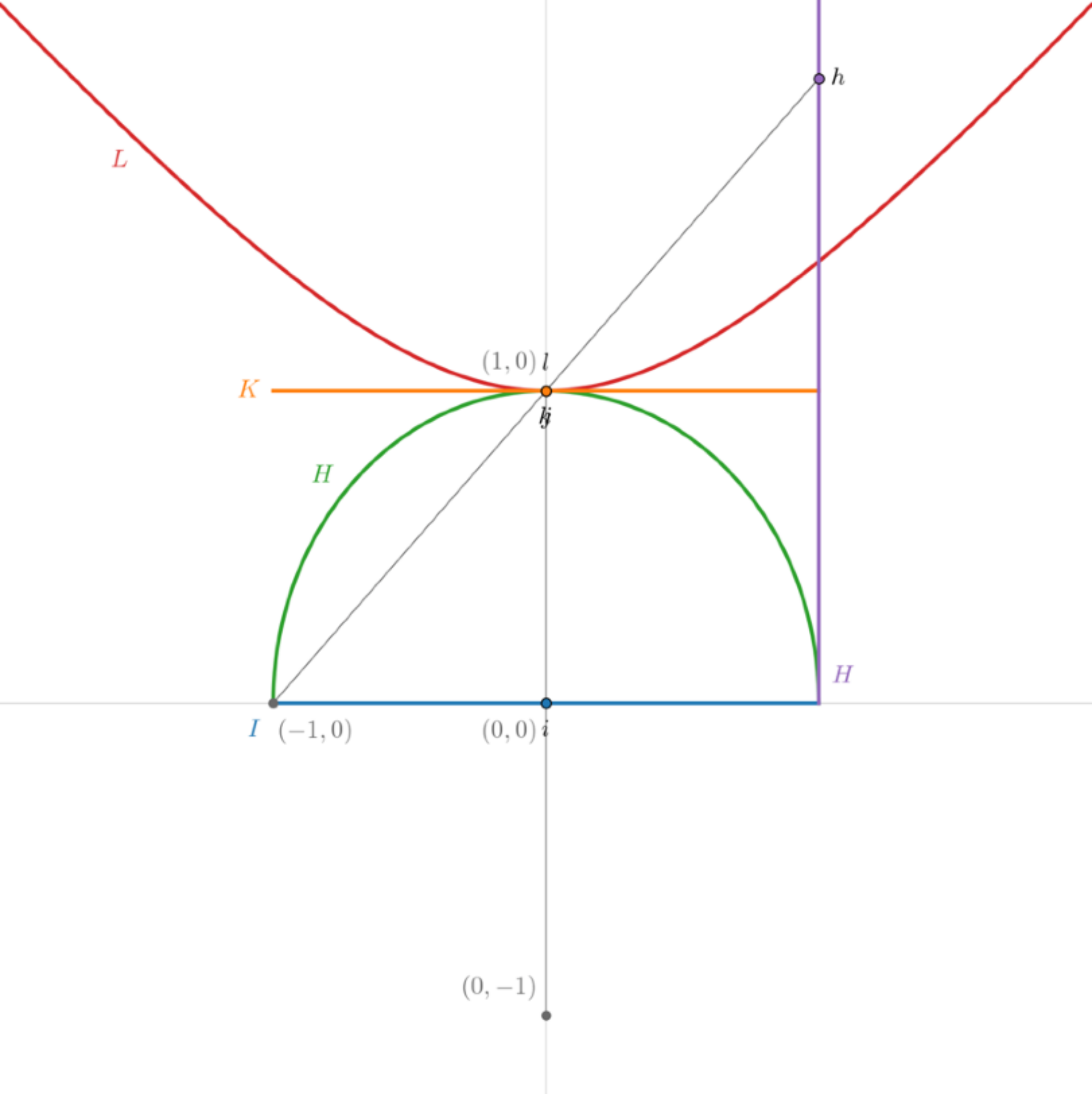
▷ Each model supplies its own natural intuitions.

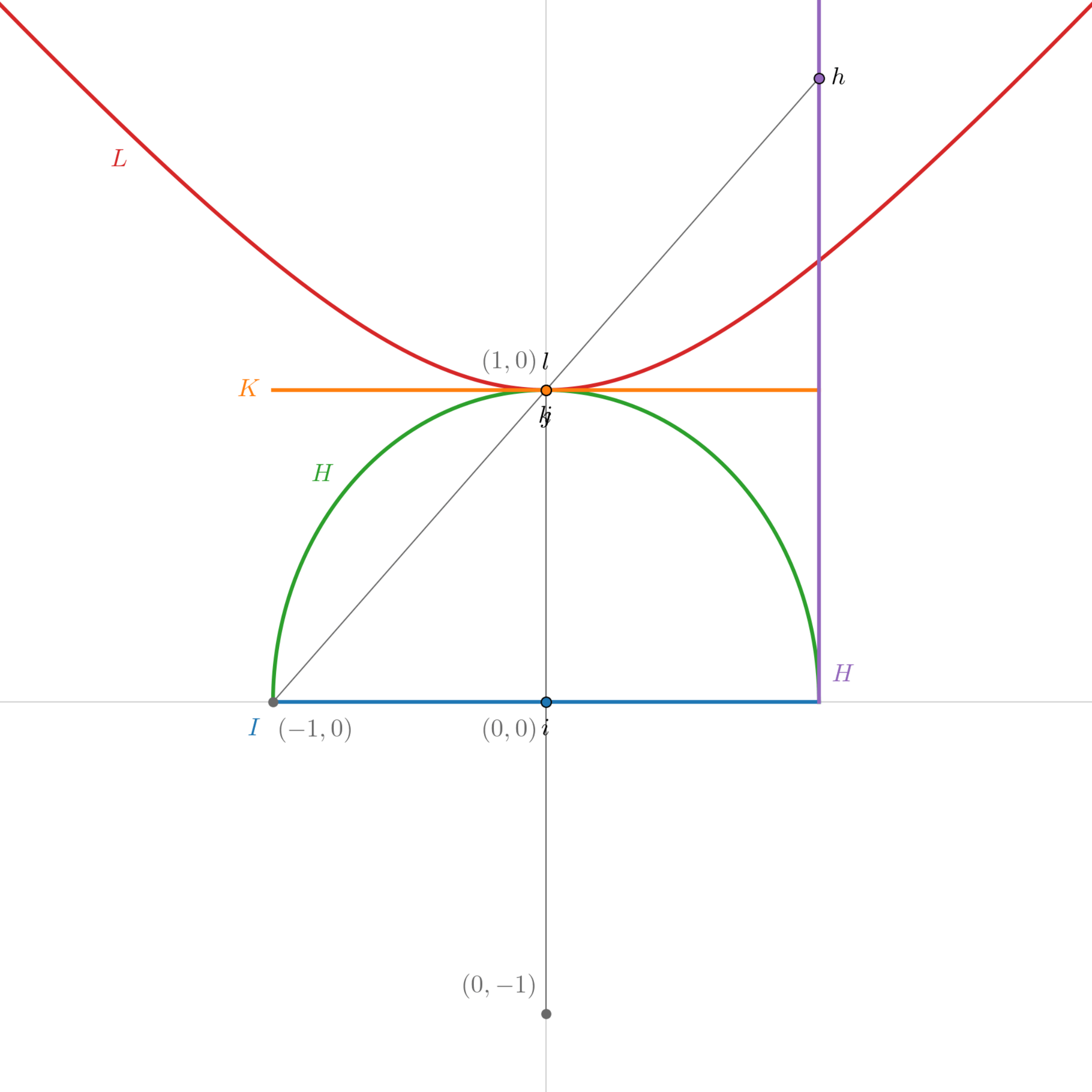


Hyperbolic geometry



The five analytic models and their connecting isometrics in  $D=1$ .



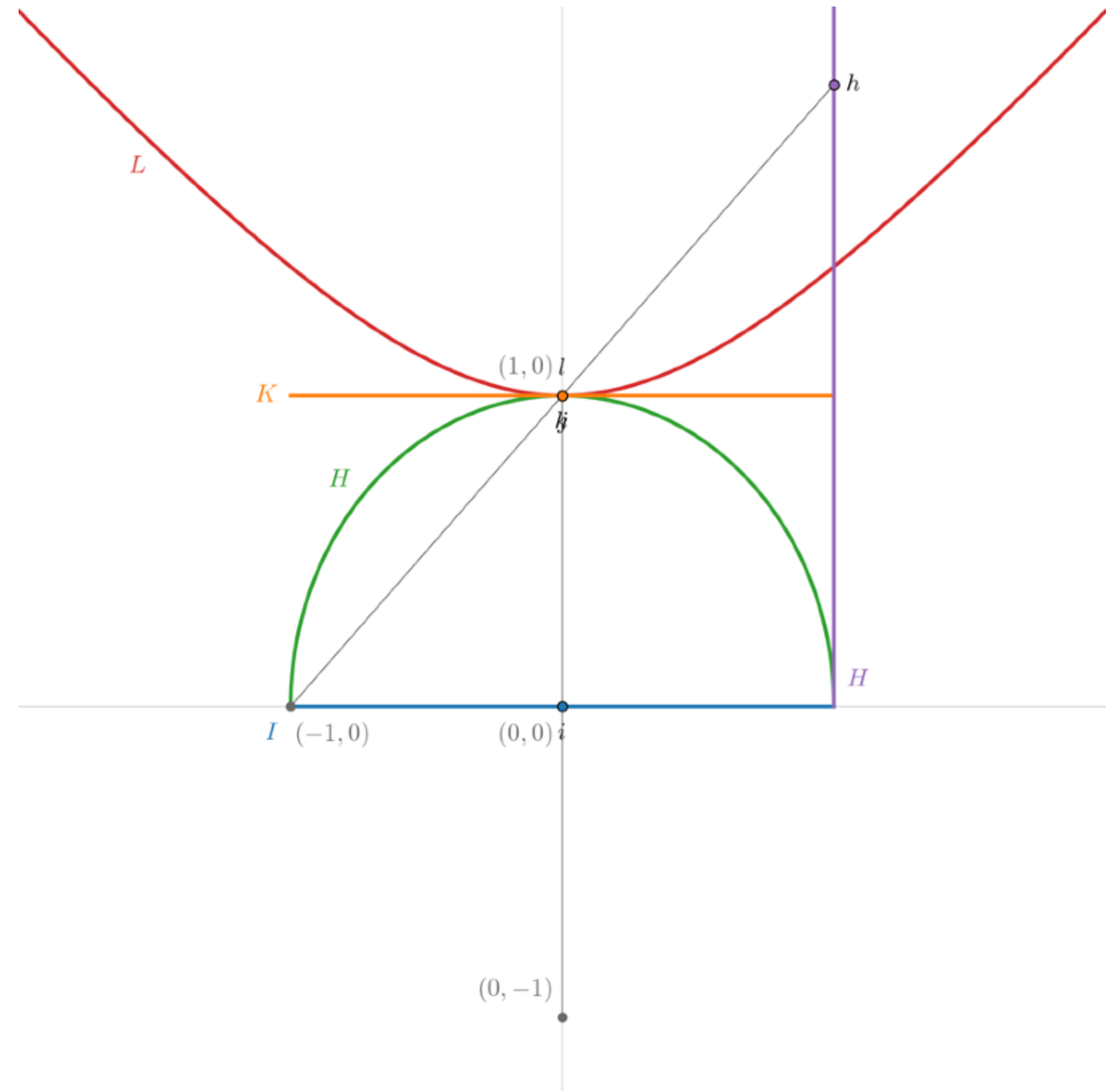




# Hyperbolic geometry

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- ▷ They are isometrically equivalent.
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The five analytic models and their connecting isometries in  $D = 1$ .



# Hyperbolic geometry

- ▷ Space of constant **negative curvature** (as opposed to flat or Euclidean space, or spherical space)
- ▷ Model for the  $D = 2$  hyperbolic space : positive sheet of the **hyperboloid** defined by

$$x^2 + y^2 - z^2 = -1$$

- ▷ Distance between points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  is

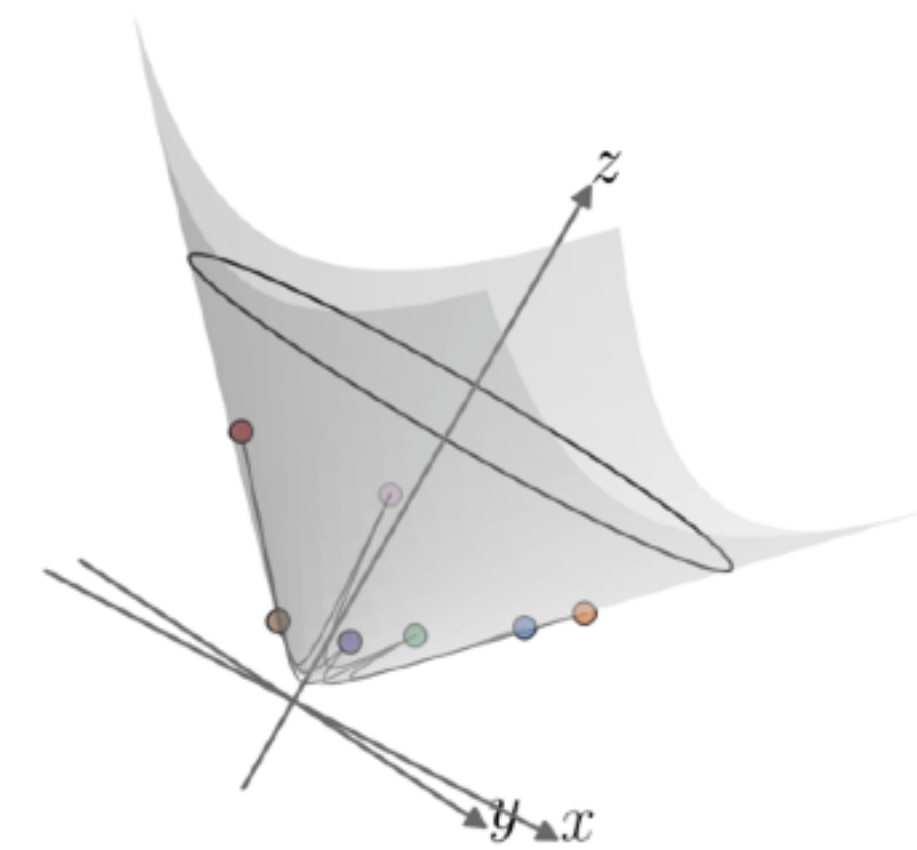
$$d(1, 2) = \operatorname{arccosh}(z_1 z_2 - x_1 x_2 - y_1 y_2)$$

- ▷ Polar coordinates

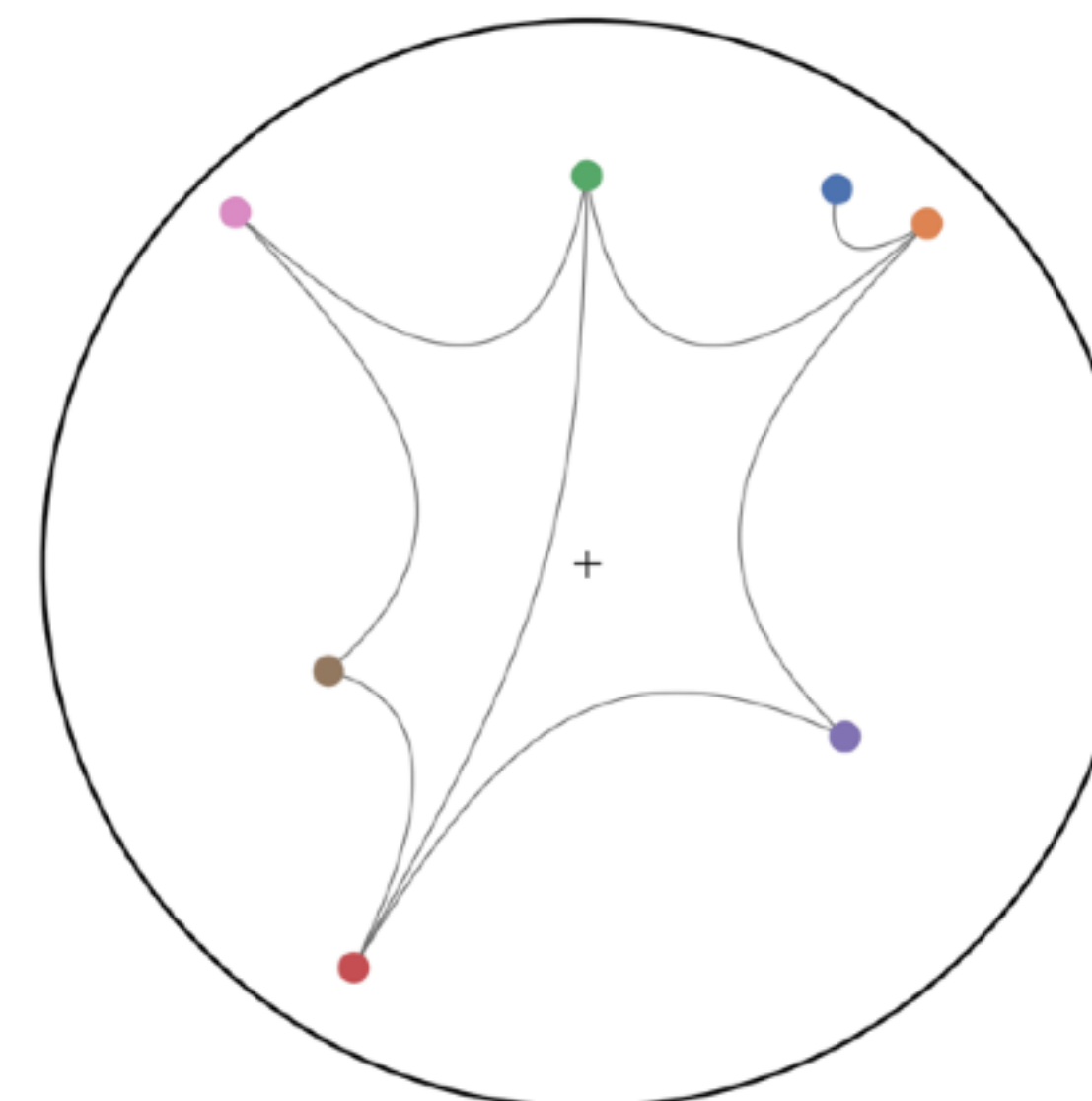
$$x = \sinh(r) \cos(\theta)$$

$$y = \sinh(r) \sin(\theta)$$

$$z = \cosh(r)$$



hyperboloid in  $\mathbb{R}^{2,1}$



hyperbolic disk  $(r, \theta)$