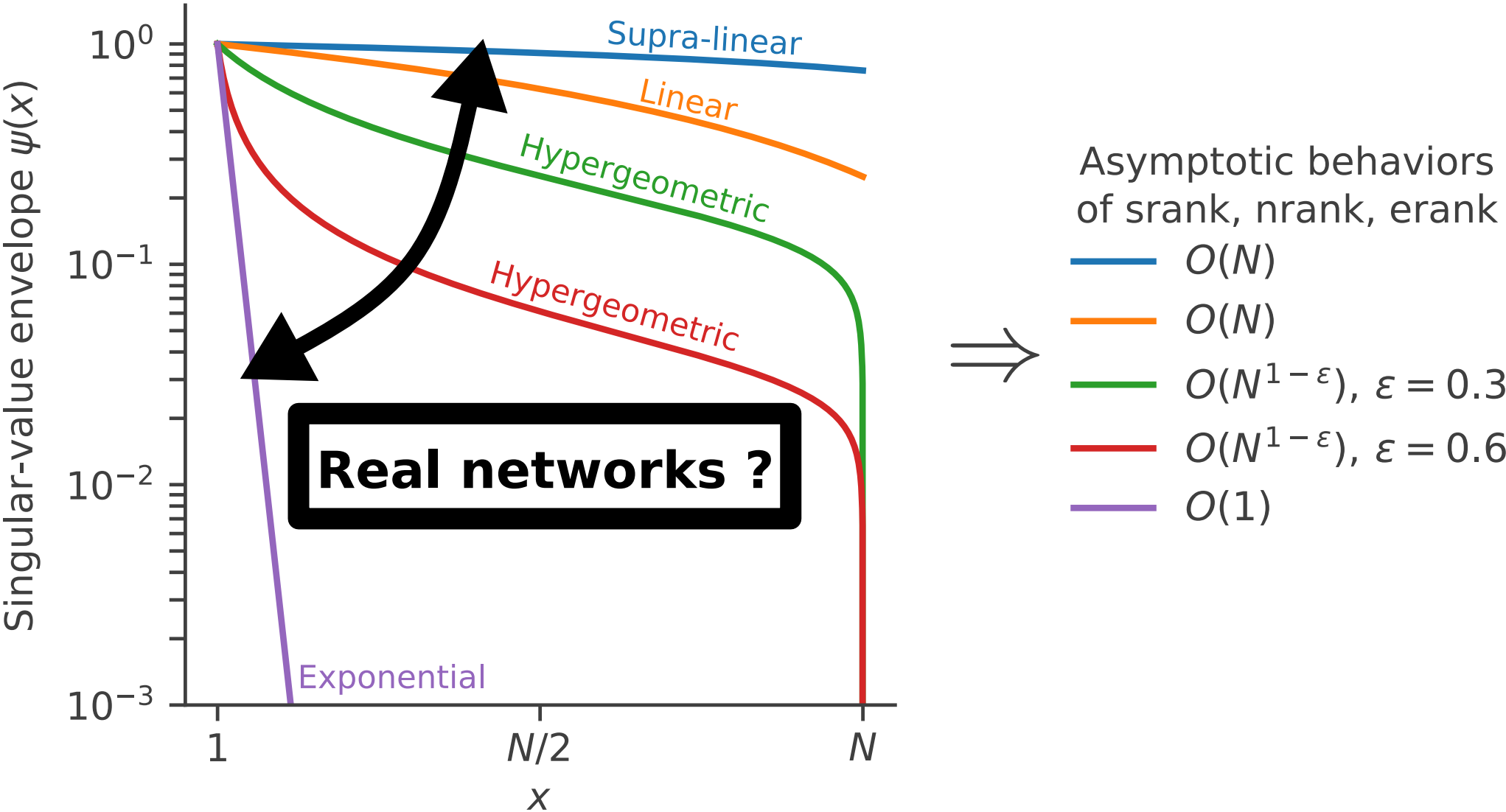


Asymptotic behaviors  
of srank, nrank, erank

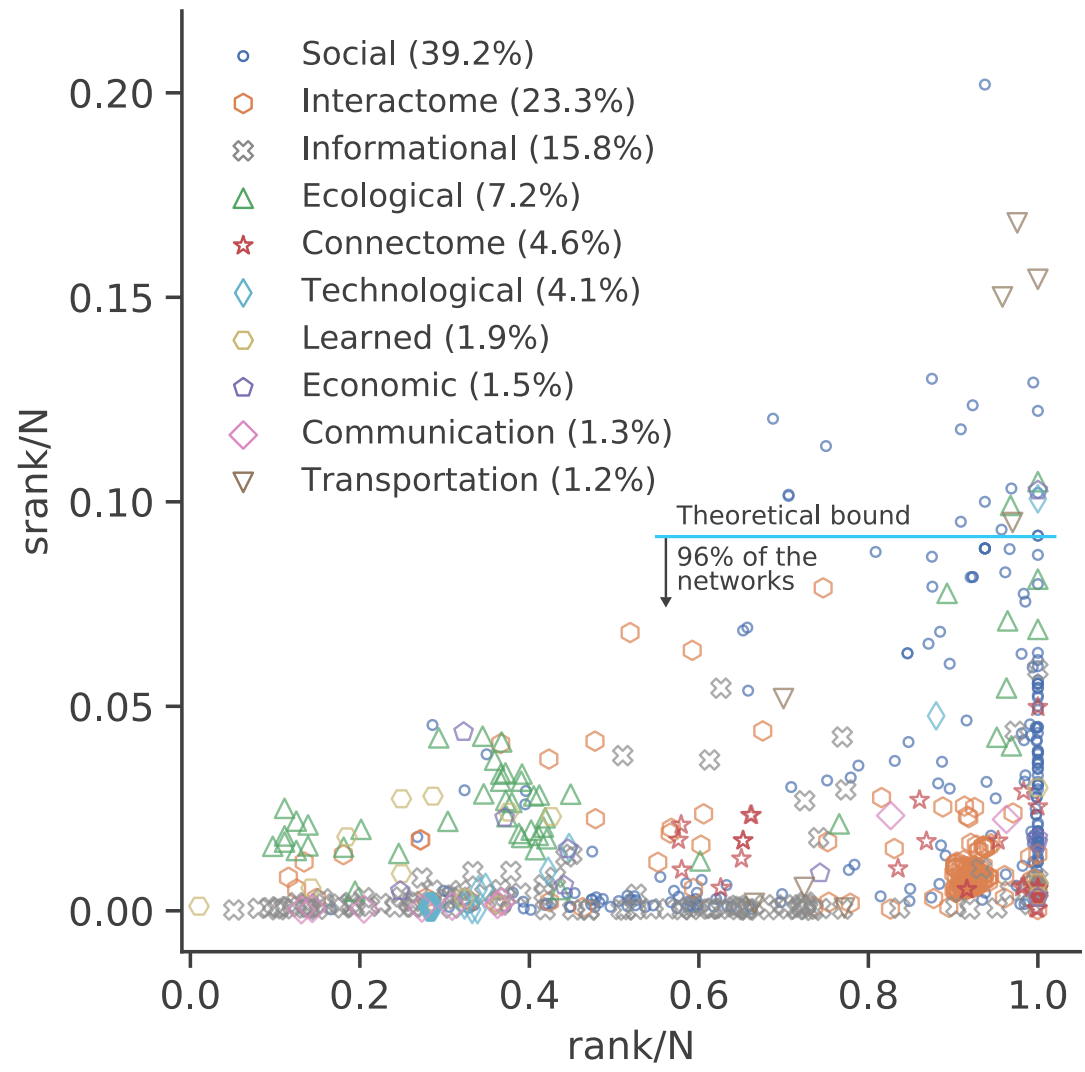
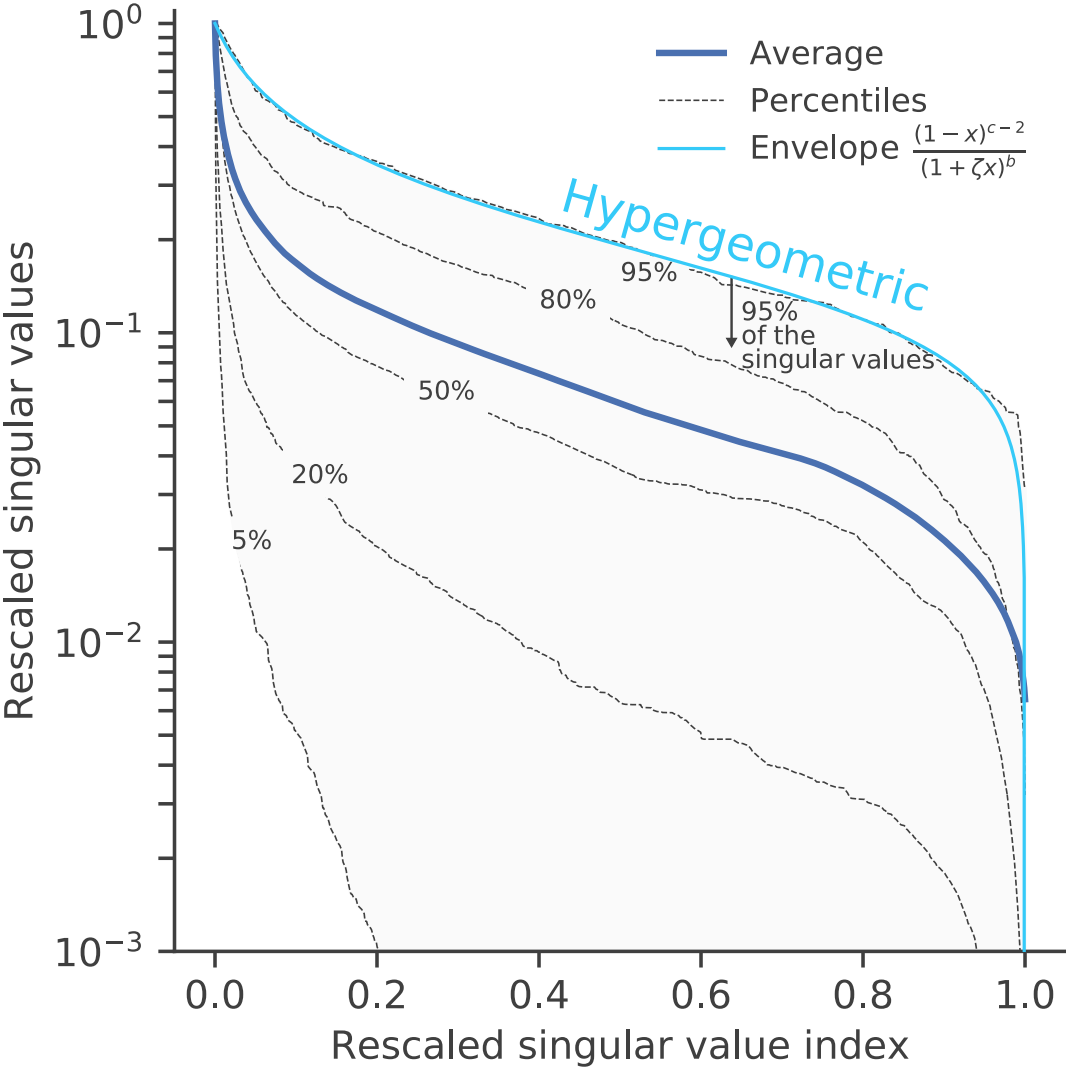
$\Rightarrow$

- $O(N)$
- $O(N)$
- $O(N^{1-\varepsilon}), \varepsilon = 0.3$
- $O(N^{1-\varepsilon}), \varepsilon = 0.6$
- $O(1)$



A workable definition of “low” effective rank





The singular values of approx. 96% of considered networks are bounded from above by an hypergeometric envelope  $\Rightarrow$  **sublinear effective ranks!**

Model definition low effective rank:  $\sim 10\%$  of the number of nodes  $N$



Approx. 96% of the 679 networks qualify for having a low effective rank!

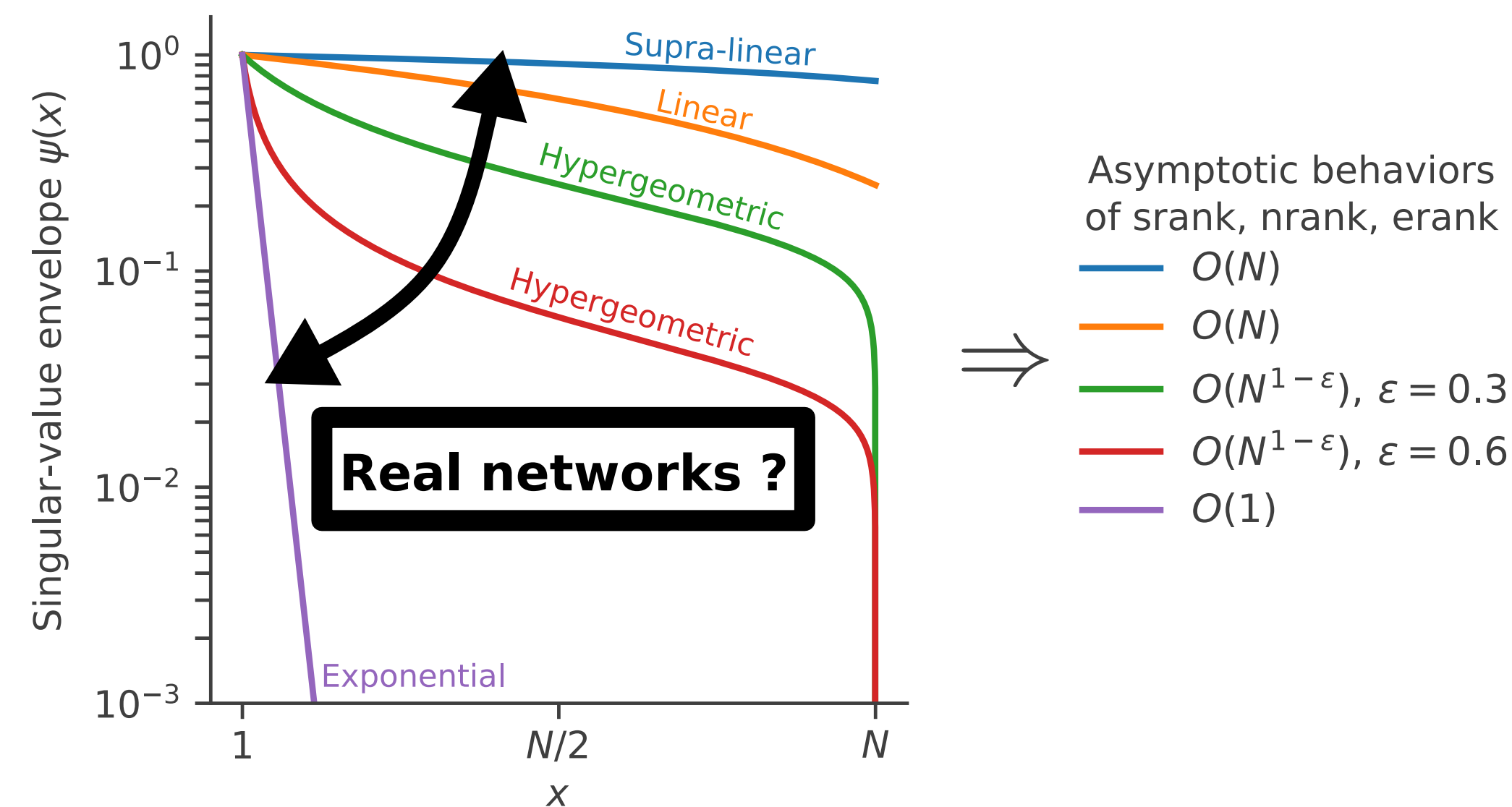
Hint: the rapid decrease of the dominant singular values of the adjacency matrix implies a low effective rank

- ▷ low effective rank?  $\Rightarrow$  effective rank scales at most sublinearly as the number of nodes,  $N$ , goes to infinity ( $N^{1-\varepsilon}$  with  $\varepsilon \in (0, 1]$ )

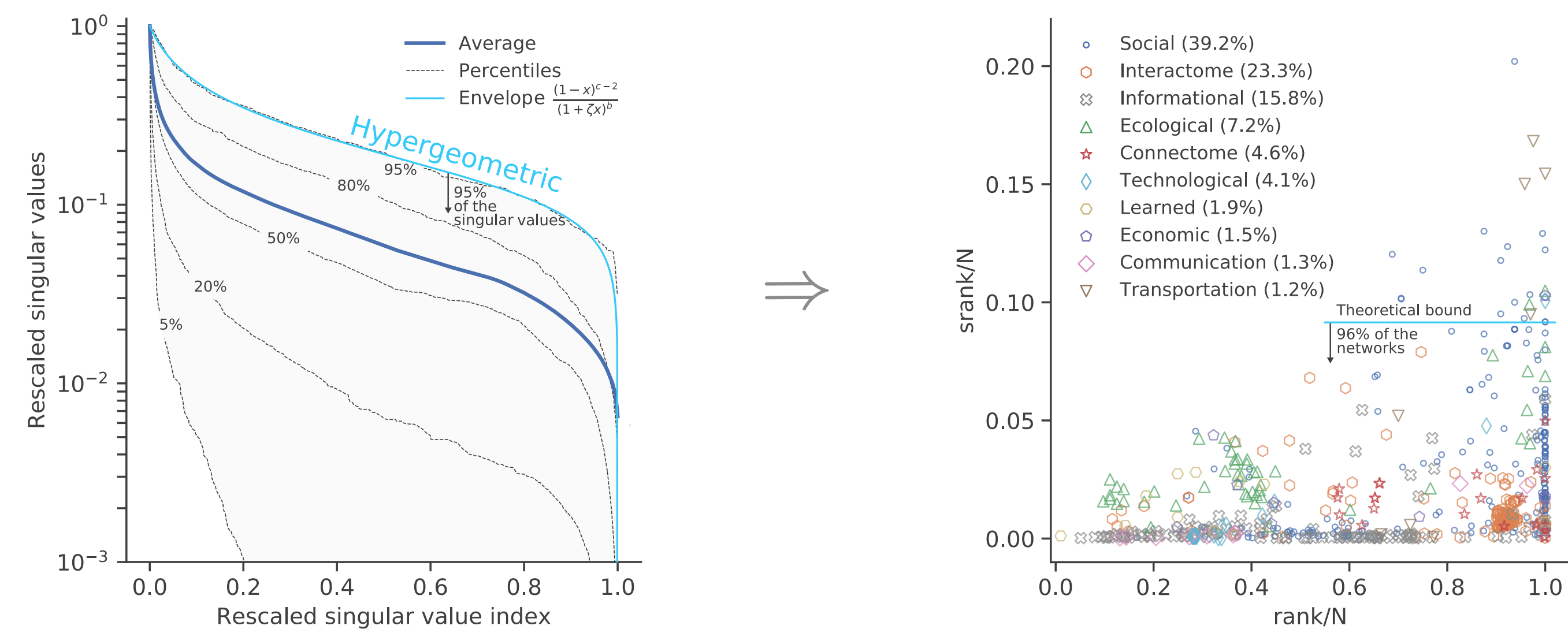
# A workable definition of “low” effective rank

Hint: the rapid decrease of the dominant singular values of the adjacency matrix implies a low effective rank

- low effective rank?  $\Rightarrow$  effective rank scales at most sublinearly as the number of nodes,  $N$ , goes to infinity ( $N^{1-\varepsilon}$  with  $\varepsilon \in (0, 1]$ )



The singular values of approx. 96% of considered networks are bounded from above by an hypergeometric envelope  $\Rightarrow$  sublinear effective ranks!



Workable definition of low effective rank:  $\sim 10\%$  of the number of nodes  $N$

Approx. 96% of the 679 networks qualify for having a low effective rank!

# The impact of effective low rank on the dynamics

Many dynamics on networks have the form

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{g}(\mathbf{x}, \mathbf{W}\mathbf{x}) = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

with  $\mathbf{x} \in \mathbb{R}^N$ .

Examples:

- ▷ SIS (mean-field) :  $\dot{x}_i = -d_i x_i + \gamma(1 - x_i) y_i$
- ▷ Wilson-Cowan:  $\dot{x}_i = -d_i x_i + (1 - ax_i) \frac{1}{1 + e^{-b(\gamma y_i - c)}}$
- ▷ Recurrent Neural Networks (RNN):  $\dot{x}_i = -d_i x_i + \tanh(\gamma y_i + c_i)$
- ▷ Kuramoto-Sakaguchi:  $\dot{z}_j = i\omega_j z_j + \gamma e^{-i\alpha} y_j - \gamma e^{i\alpha} z_j^2 \bar{y}_j$  with  $z_j = e^{i\theta_j}$
- ▷ Population dynamics:  $\dot{x}_i = -dx_i + \gamma x_i y_i$  (Lotka-Volterra)  
 $\dot{x}_i = -dx_i - sx_i^2 + \gamma \frac{x_i y_i}{\alpha + y_i}$   
 $\dot{x}_i = a - dx_i + bx_i^2 - cx_i^3 + \gamma x_i y_i$

for  $i, j \in \{1, \dots, N\}$  and  $y_i = \sum_{j=1}^N W_{ij} x_j$ .