

Contact network epidemiology

Probability generating functions (PGFs) formalism

- probability u that an outbreak eventually dies out

$$u = \text{[diagram of a node with a red cross]} = \text{[diagram of a black node]} + \text{[diagram of a black node with one red cross]} + \text{[diagram of a black node with two red crosses]} + \text{[diagram of a black node with three red crosses]} + \dots = \sum_{k \geq 0} \frac{(k + 1)p_{k+1}}{\langle k \rangle} u^k = G_1(u)$$

- the fraction of the population infected in an epidemic wave (and the probability of such wave) is

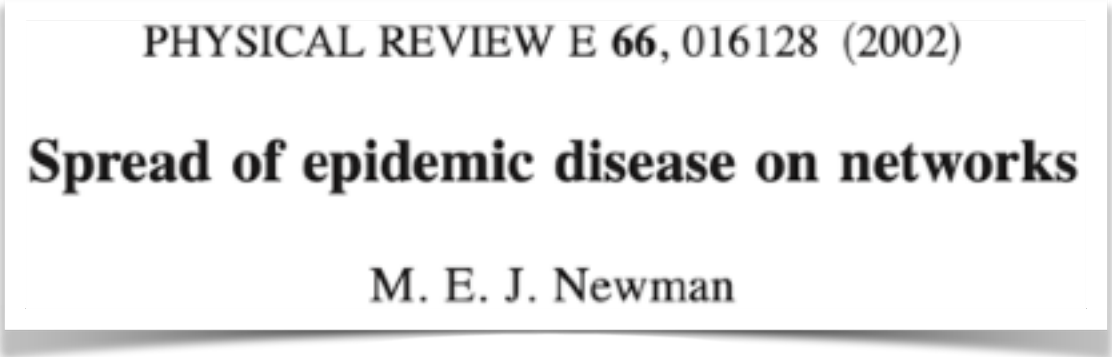
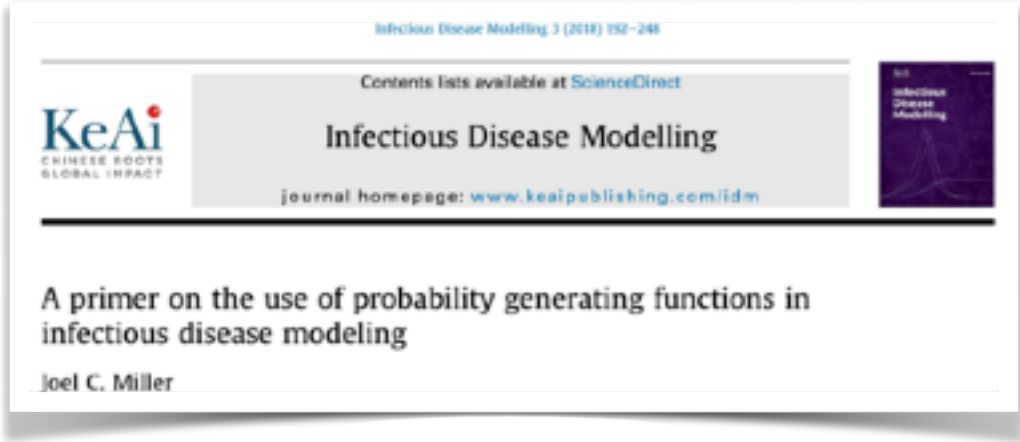
$$R(\infty) = \sum_{k \geq 0} p_k (1 - u^k) = 1 - G_0(u)$$

- $H_0(x)$: PGF of the distribution of the size of outbreaks that will eventually die out

$$H_1(x) = \text{[diagram of a white node]} = \text{[diagram of a black node]} + \text{[diagram of a black node with one white node]} + \text{[diagram of a black node with two white nodes]} + \text{[diagram of a black node with three white nodes]} + \dots = x \sum_{k \geq 0} \frac{(k + 1)p_{k+1}}{\langle k \rangle} [H_1(x)]^k = xG_1(H_1(x))$$

- the distribution of the size of outbreaks that will eventually die out can be extracted from

$$H_0(x) = xG_0(H_1(x))$$



Message #1 : the friendship paradox

- on average, your friends have more friends than you do
 - a random individual has k friends with probability p_k
 - however, their friends have k friends with probability $\propto kp_k$
- by spreading on a contact network, the disease naturally oversamples individuals more likely to cause a larger number of secondary infections
- ignoring this effect leads back to the mass-action assumption

