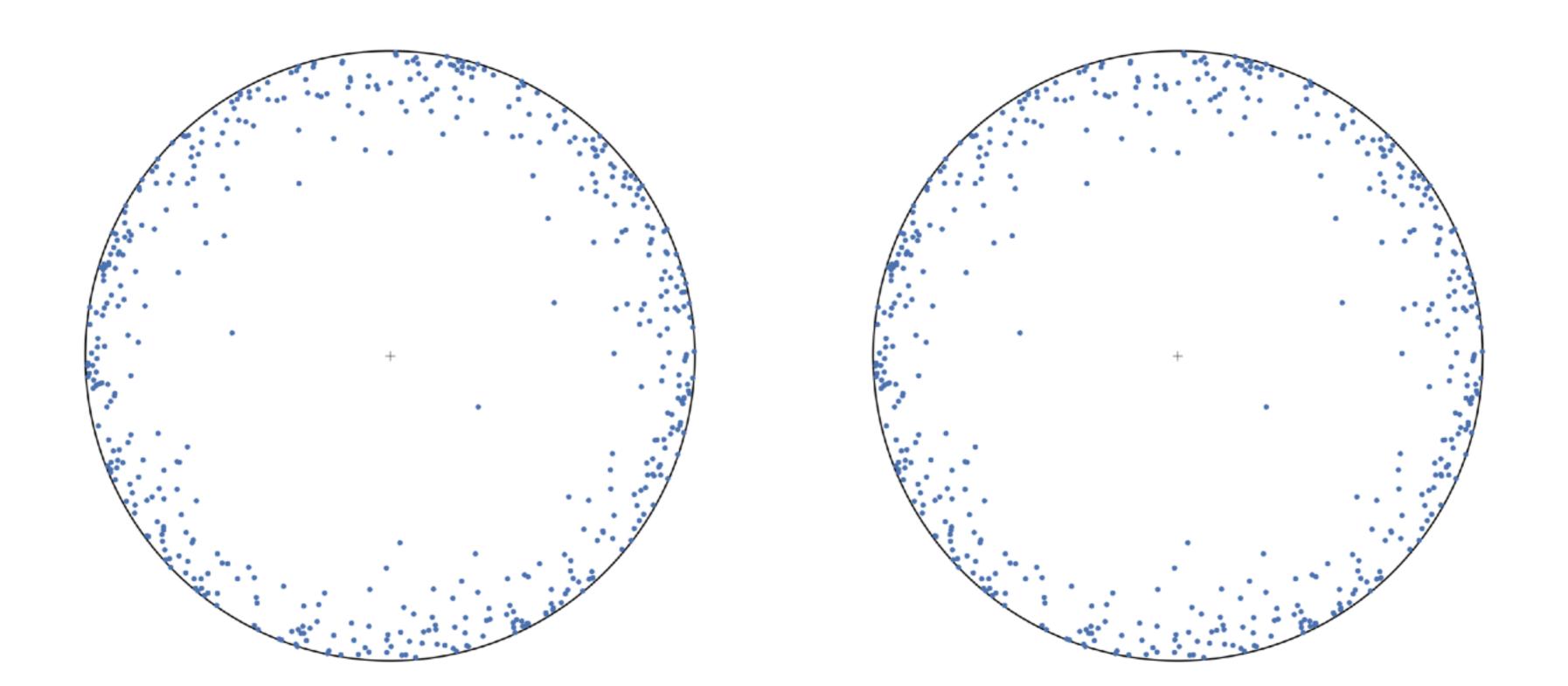
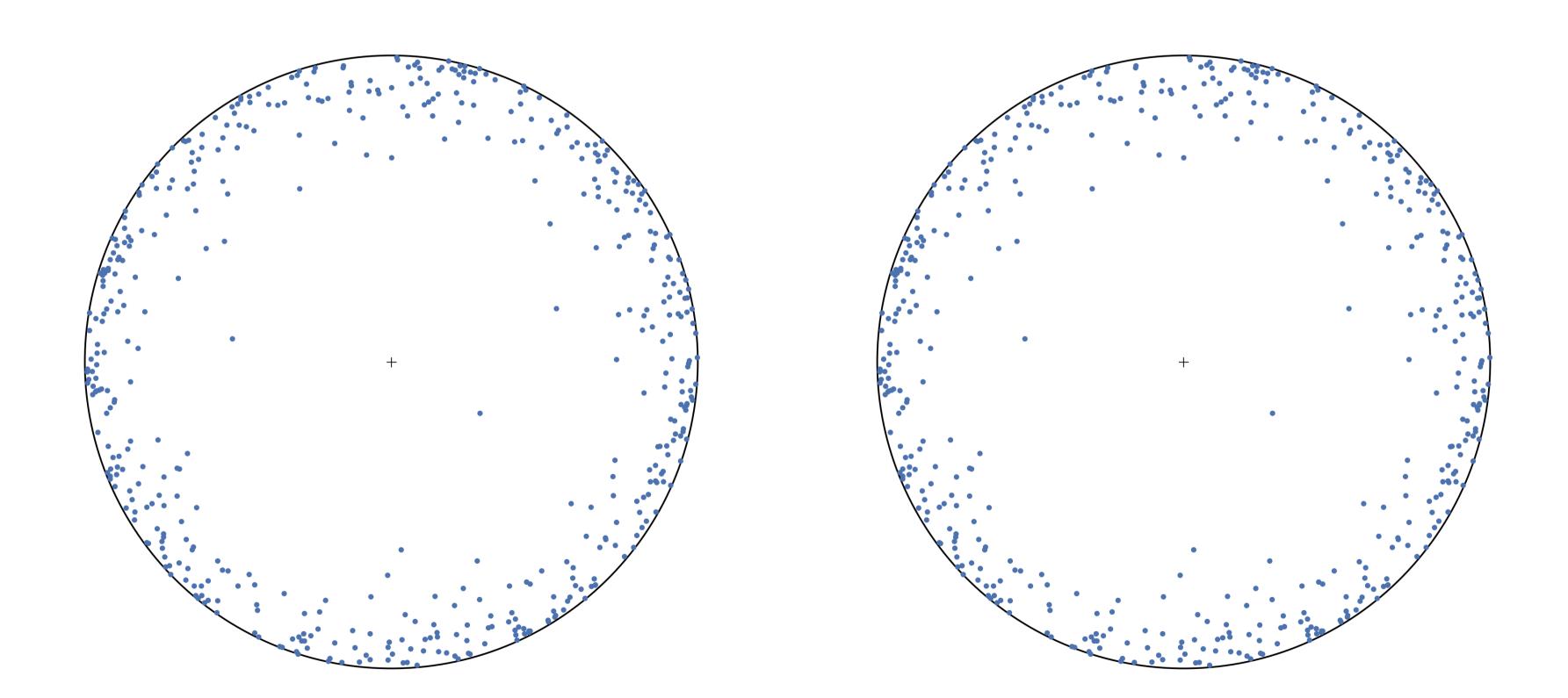


### Hyperbolic geometry

| Simple random geometric graph                                    |   |
|--|---|
| 1. Sprinkle $N$ nodes uniformly on the hyperbolic disk of radius | R |
| 2. Connect any nodes separated by a distance less than $r=R$     | • |

✓ high clustering ✓ power-law degree distribution with exponent -3 Phys. Rev. E 82, 036106 (2010)

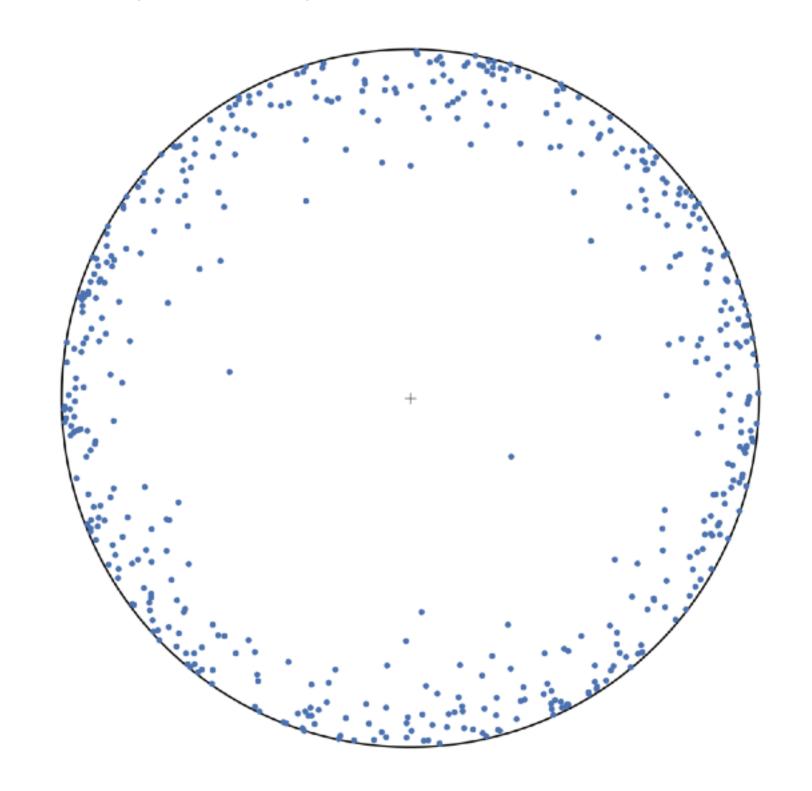


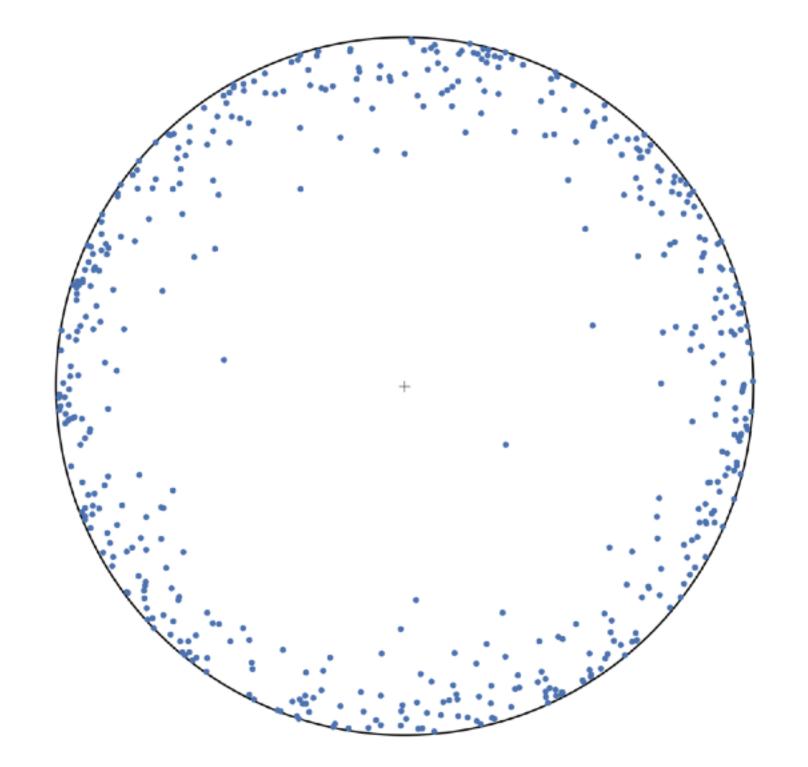


# Hyperbolic geometry

### Simple random geometric graph

- 1. Sprinkle N nodes uniformly on the hyperbolic disk of radius R.
- 2. Connect any nodes separated by a distance less than r = R.

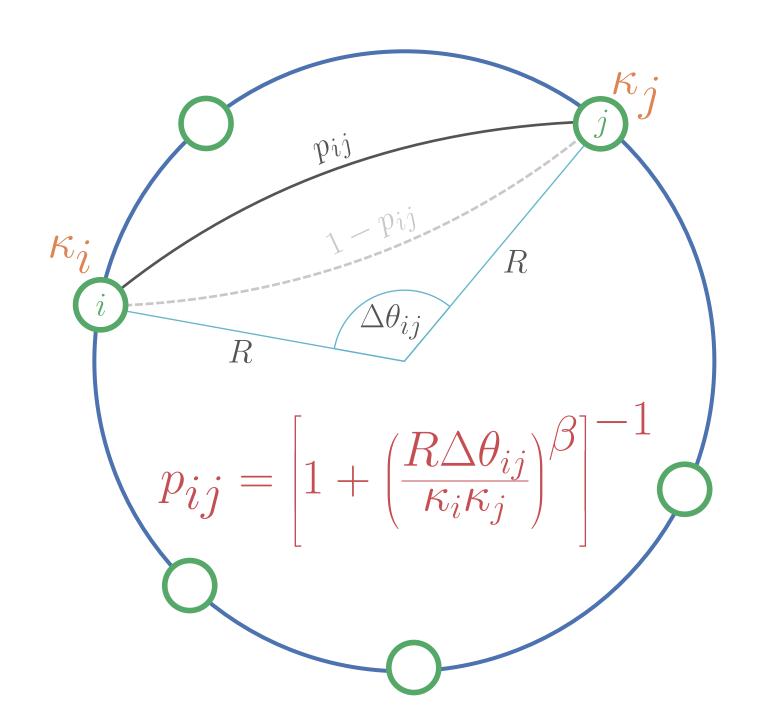




- ✓ high clustering
- ✓ power-law degree distribution with exponent -3

Phys. Rev. E 82, 036106 (2010)

# A geometric approach to clustering: the $\mathbb{S}^1/\mathbb{H}^2$ model



#### The S<sup>1</sup> model

- 1. Sprinkle N nodes uniformly on a circle of radius R.
- 2. Assign an expected degree  $\kappa$  to each node according to some pdf  $\rho(\kappa)$ .
- 3. Draw a link between node i and node j with probability  $p_{ij}$ .
- $\star$  fixes the expected degree of nodes ( $\kappa$ )  $\to$  soft configuration model (CM)
- $\star$  triangle inequality of the underlying metric space  $\to$  triangles from pairwise interactions
- $\star$  level of clustering tuned with parameter  $\beta$

[12] Sci. Rep. 5, 9421 (2015)

[13] J. Stat. Phys. 173, 775 (2018)

[14] New J. Phys. 20, 052002 (2018)

[15] New J. Phys. 21, 123033 (2019)

<sup>[1]</sup> Phys. Rev. E 80, 035101 (2009) [2] Phys. Rev. E 82, 036106 (2010)

<sup>[3]</sup> Phys. Rev. Lett. 100, 078701 (2008)

<sup>[4]</sup> Nat. Rev. Phys. 3, 114 (2021)

<sup>[5]</sup> Nat. Commun. 8, 14103 (2017)

<sup>[6]</sup> Phys. Rev. E 84, 026114 (2011)

<sup>[7]</sup> Phys. Rev. E 95, 032309 (2017)

<sup>[8]</sup> Mol. Biosyst. 8, 843 (2012)

<sup>[9]</sup> Nat. Phys. 12, 1076 (2016)

<sup>[10]</sup> Phys. Rev. Lett. 118, 218301 (2017) [11] Nature 489, 537 (2012)

<sup>[16]</sup> Nat. Commun. 8, 1615 (2017)

<sup>[17]</sup> Nat. Commun. 1, 62 (2010) [18] PNAS 117, 20244 (2020)