





A geometric approach to clustering

Identity of indiscernibles

$$d(x, y) = 0 \quad \Leftrightarrow \quad x = y$$

Non-negativity

$$d(x, y) \geq 0$$

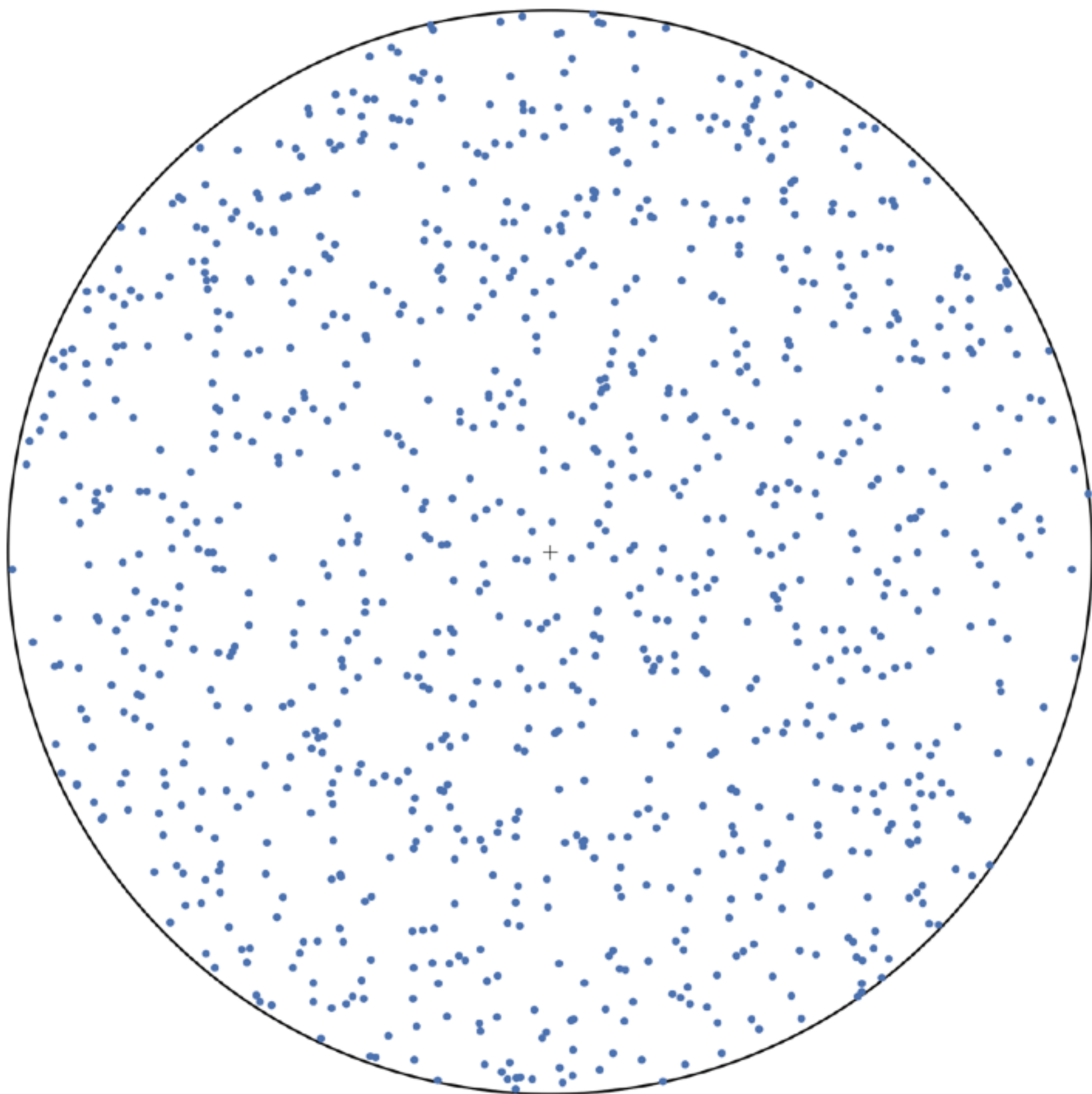
Symmetry

$$d(x, y) = d(y, x)$$

Triangle inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

Properties of any metric space



# Simple random geometric graph

1. Sprinkle  $N$  nodes uniformly on a disk of radius  $R$ .
2. Connect any nodes separated by a distance less than  $r$ .

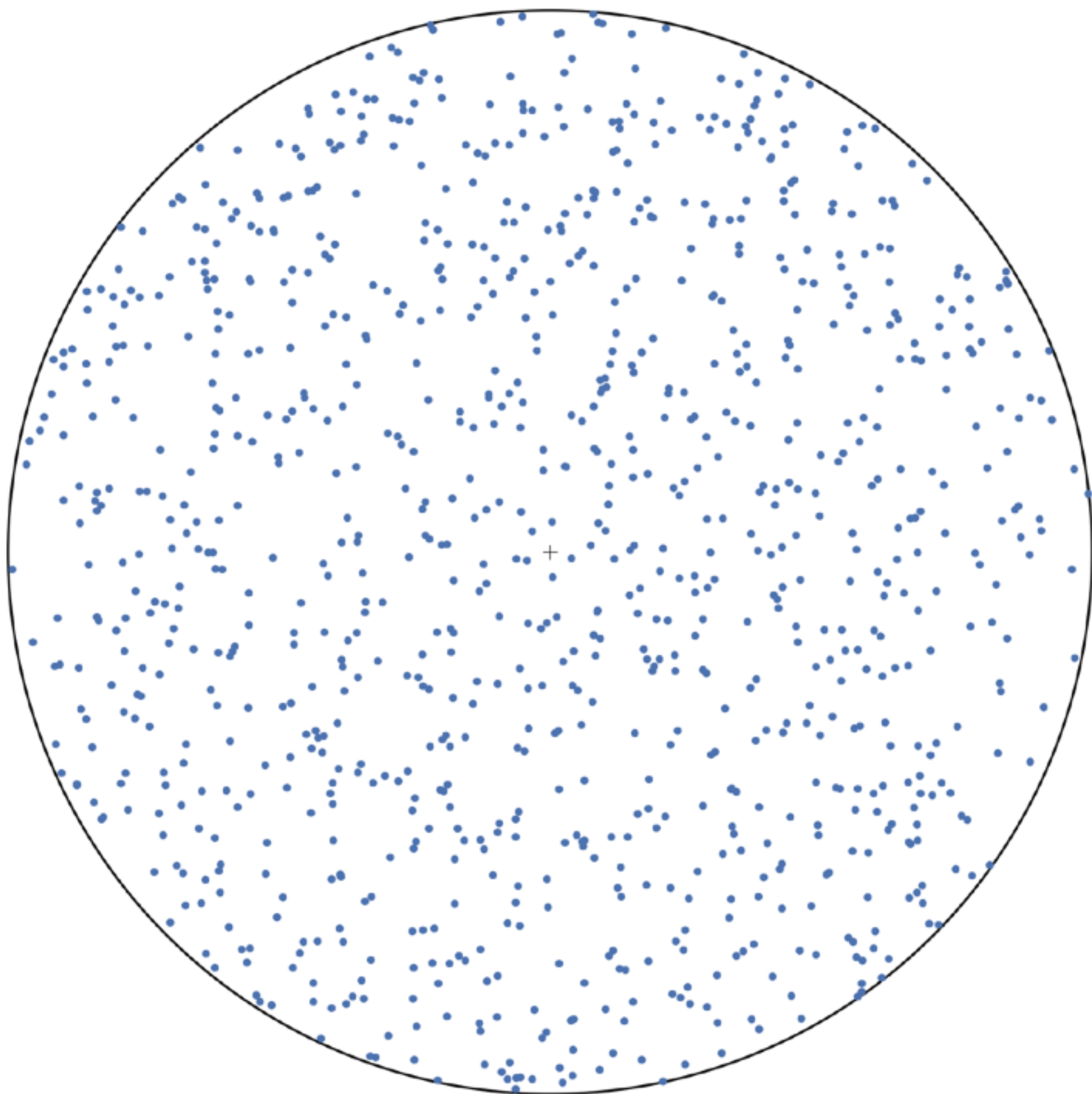
✓ high clustering

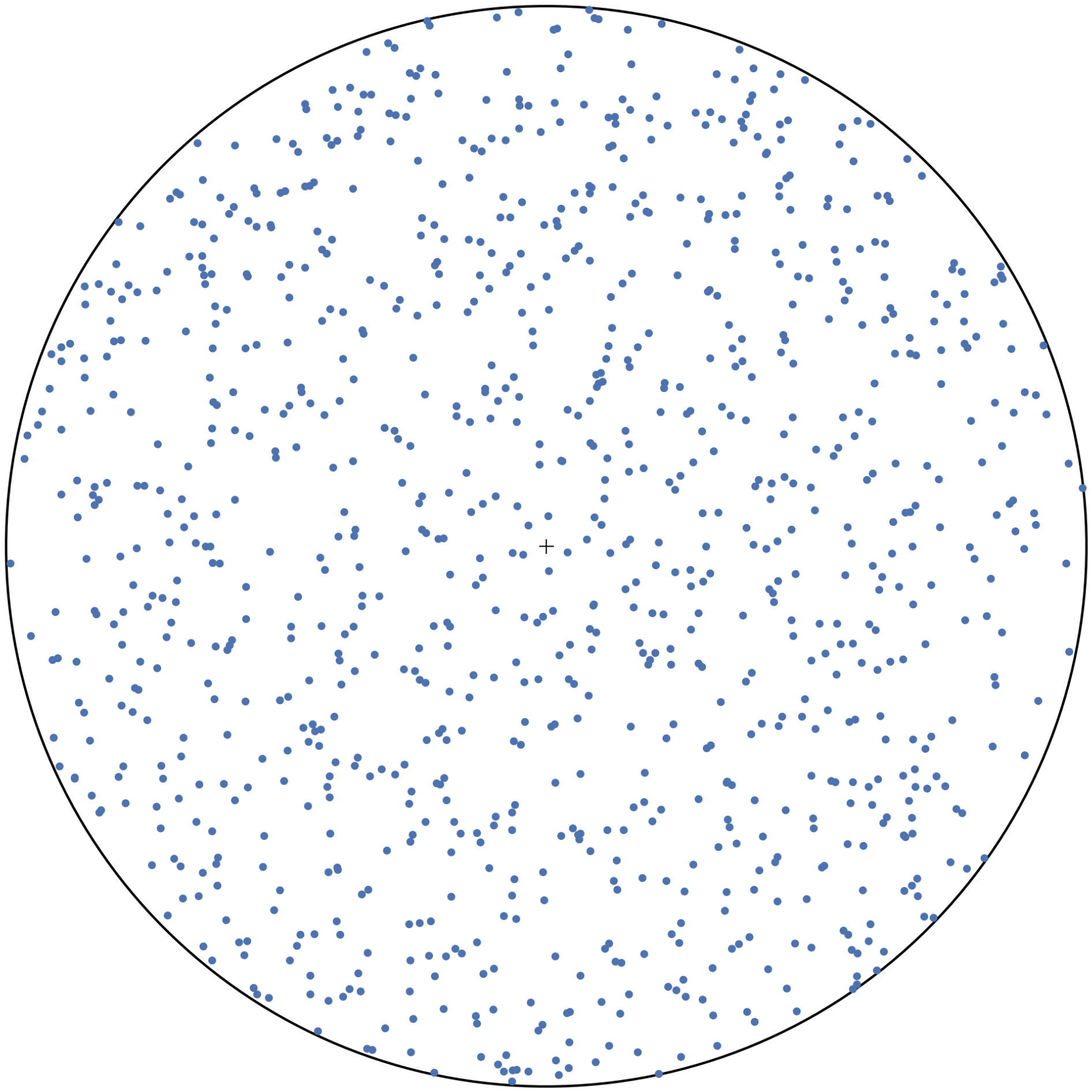
✗ binomial/Poisson degree distribution

Assume that the nodes are embedded in a metric space and that any two nodes are connected with a probability that is a decreasing function of the distance between them.



For further info, see Phys. Rep. 499, 1-101 (2011)





# A geometric approach to clustering

Assume that the nodes are **embedded in a metric space** and that any two nodes are connected with a probability that is a **decreasing function of the distance** between them.

## Properties of any metric space

Identity of indiscernibles  $d(x, y) = 0 \Leftrightarrow x = y$

Non-negativity  $d(x, y) \geq 0$

Symmetry  $d(x, y) = d(y, x)$

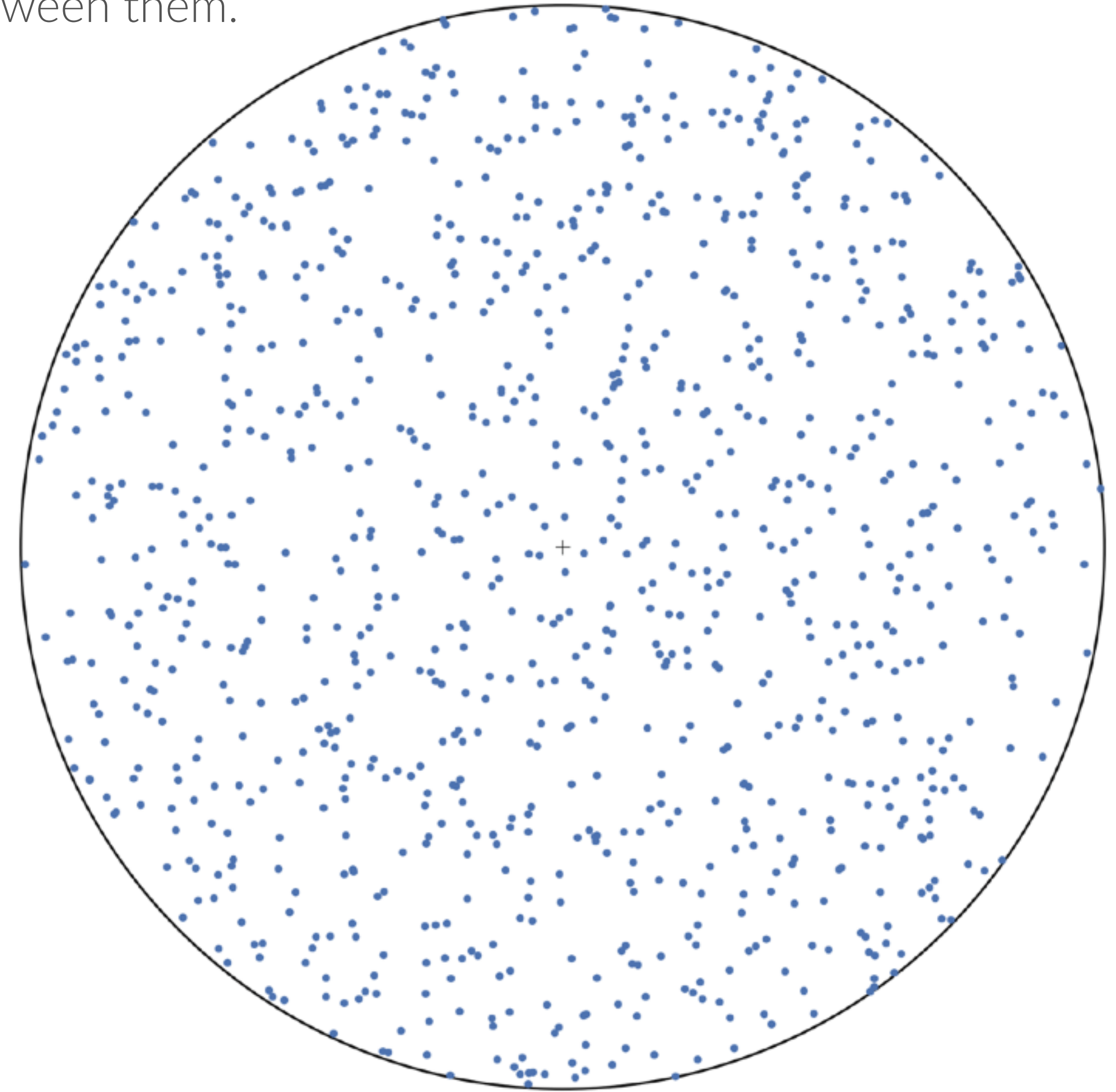
Triangle inequality  $d(x, y) \leq d(x, z) + d(z, y)$

## Simple random geometric graph

1. Sprinkle  $N$  nodes uniformly on a disk of radius  $R$ .
2. Connect any nodes separated by a distance less than  $r$ .

✓ high clustering

✗ binomial/Poisson degree distribution



# Euclid's postulates

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.