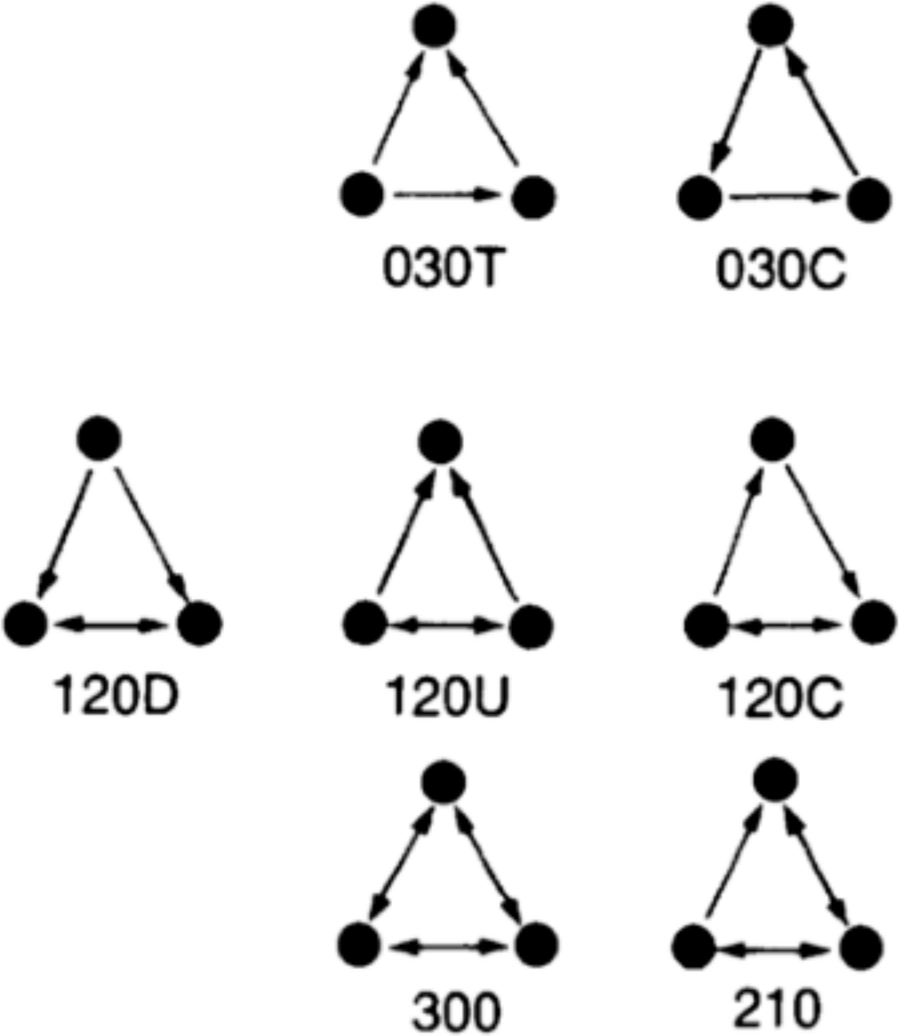
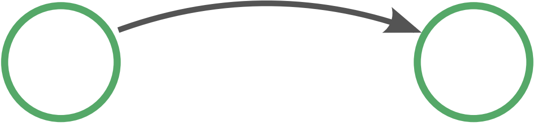
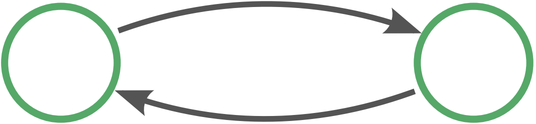
12


030T
030C
120D
120U
120C
300
210





Three challenges in modeling directed networks

Adapted from Holland & Leinhardt. Local Structure in Social Networks. Sociol. Methodol., 7, 1–45 (1976)

Identity of indiscernibles	$d(x, y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x, y) \geq 0$
Symmetry	$d(x, y) = d(y, x)$
Triangle inequality	$d(x, y) \leq d(x, z) + d(z, y)$

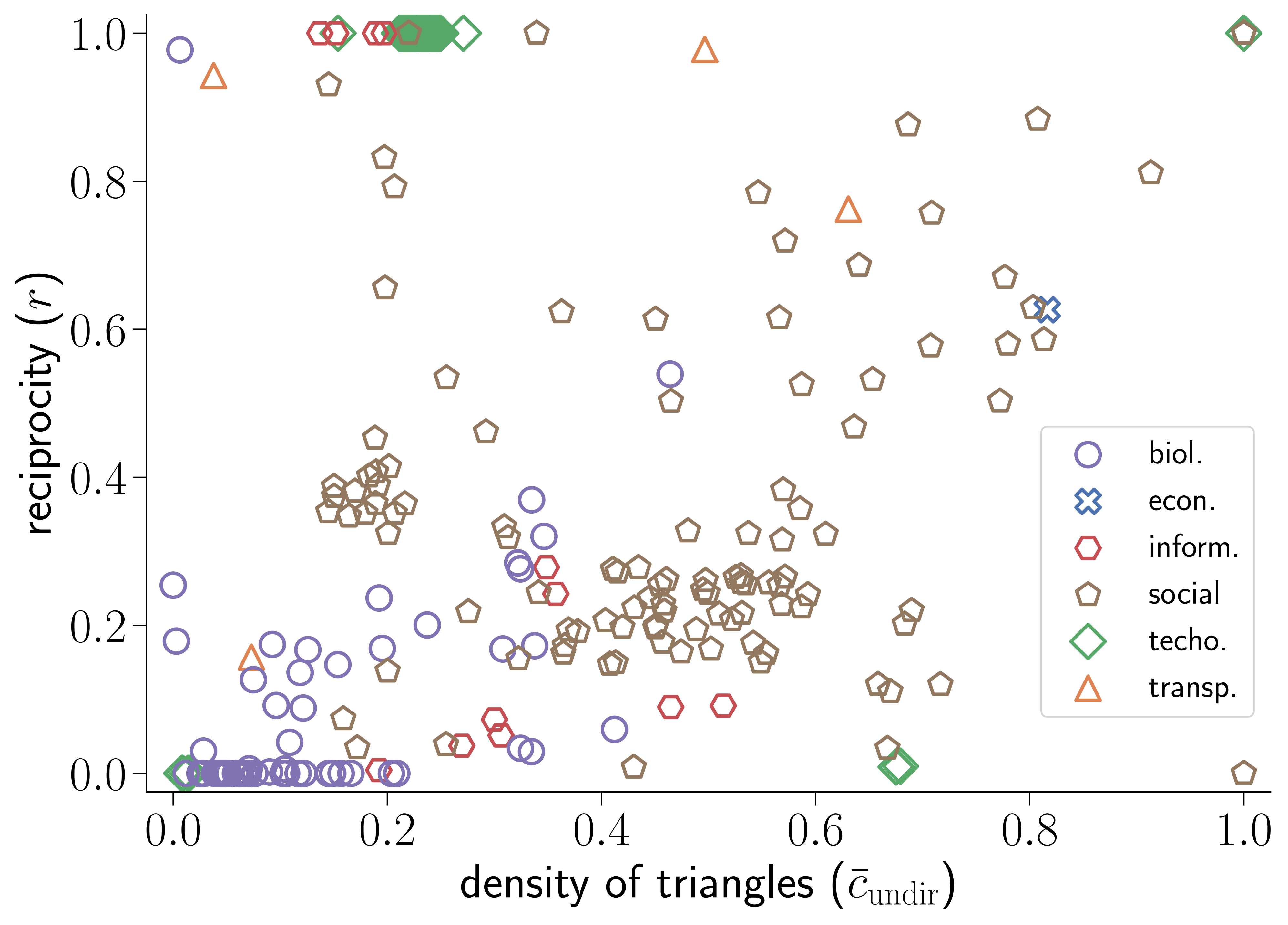
292 network datasets downloaded from Netzschleuder (networks.skewed.de).

Properties of any metric space

Clustering : 7 cycles of length 3

Reciprocity : cycles of length 2

$$r = \frac{L^{\leftrightarrow}}{L} = \frac{\text{[number of reciprocal links]}}{\text{[number of links]}}$$



1.0
0.8
0.6
0.4
0.2
0.0
reciprocity (r)
0.0
0.2
0.4
0.6
0.8
1.0
density of triangles ($\bar{c}_{\mathrm{undir}}$)
biol.
econ.
inform.
social
techo.
transp.

Three challenges in modeling directed networks

Properties of any metric space

Identity of indiscernibles	$d(x,y)=0 \quad \Leftrightarrow \quad x=y$
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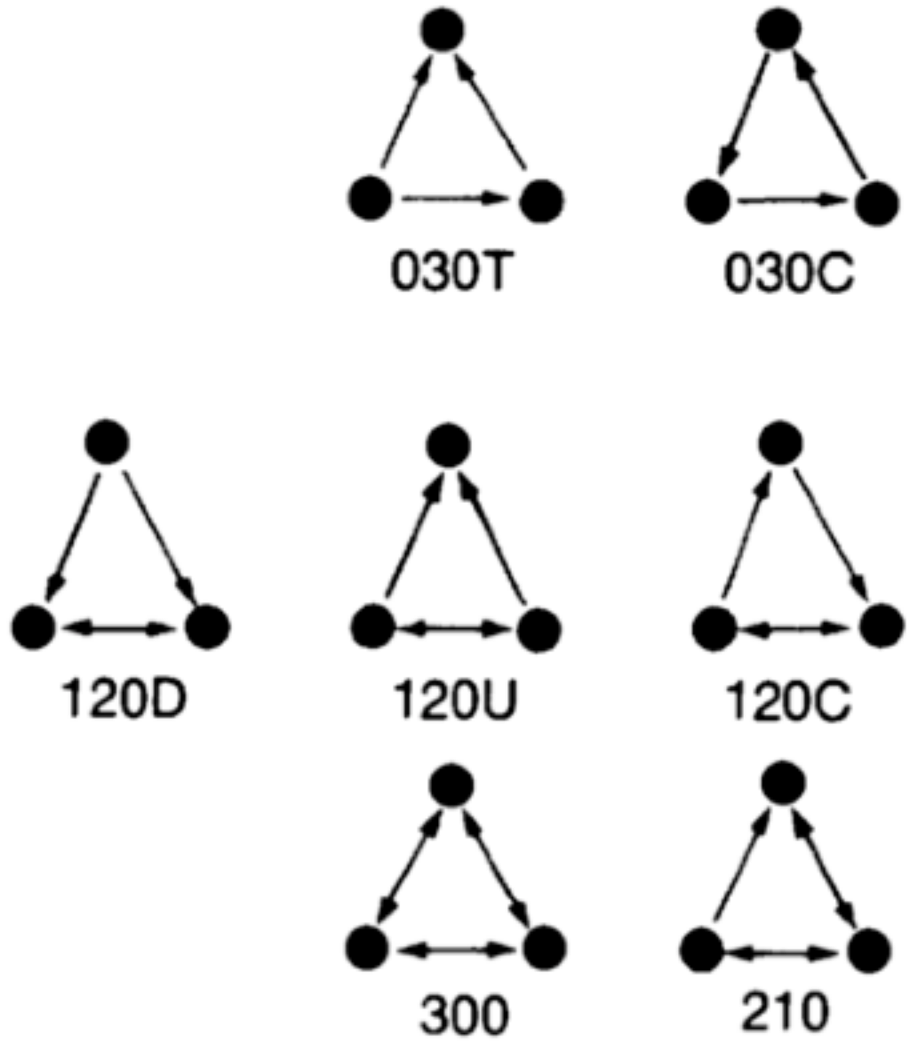
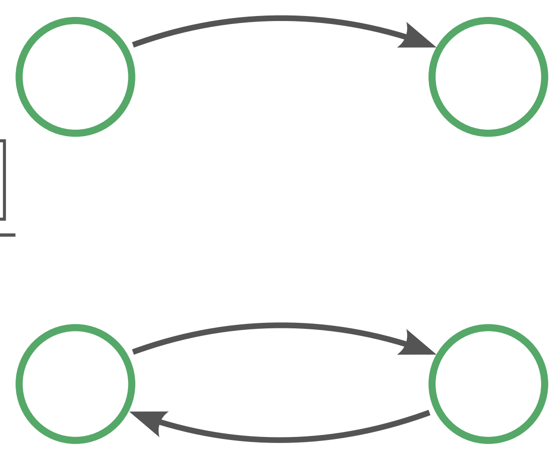
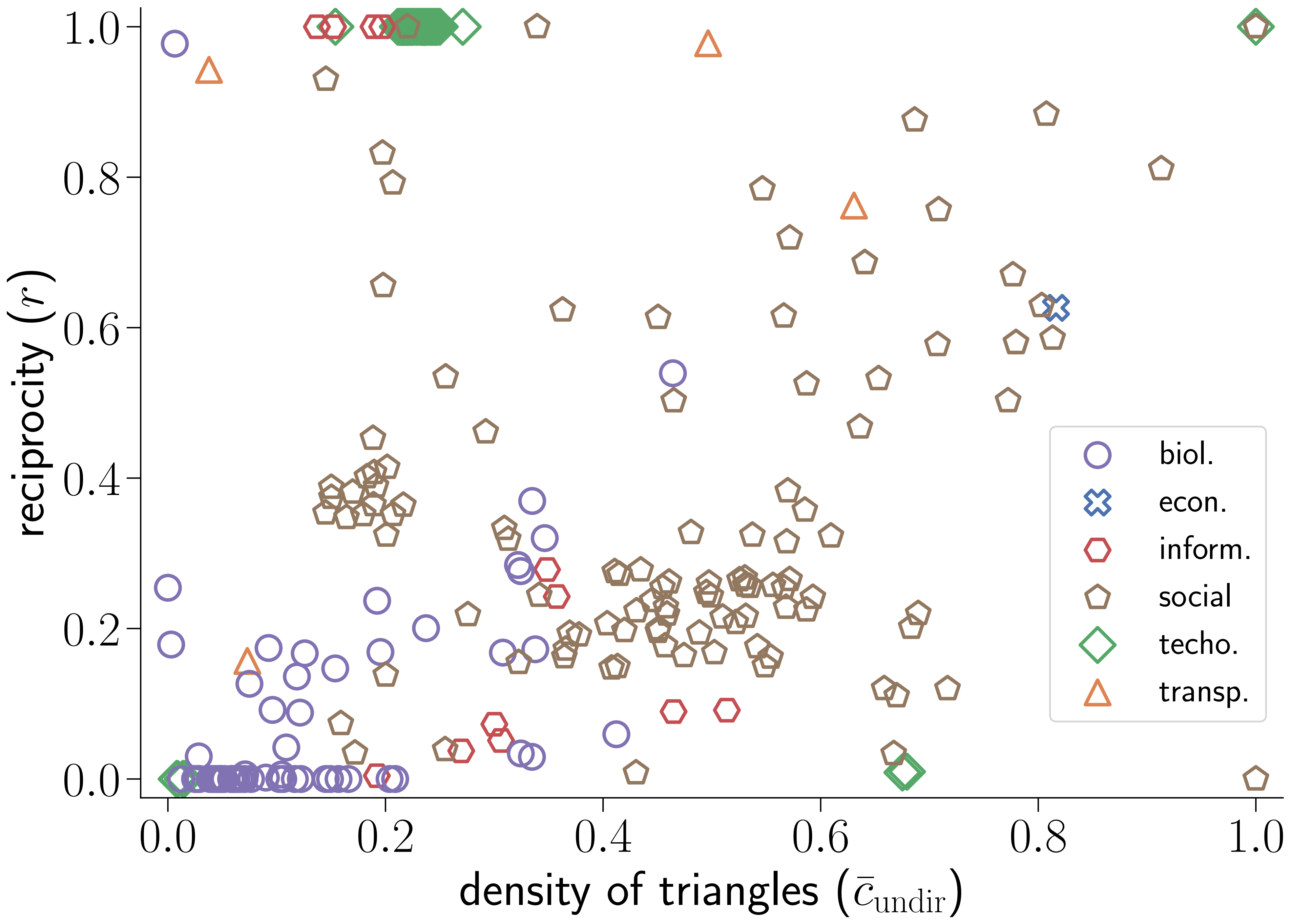


Adapted from Holland & Leinhardt. Local Structure in Social Networks. Sociol. Methodol., 7, 1–45 (1976)

Reciprocity : cycles of length 2

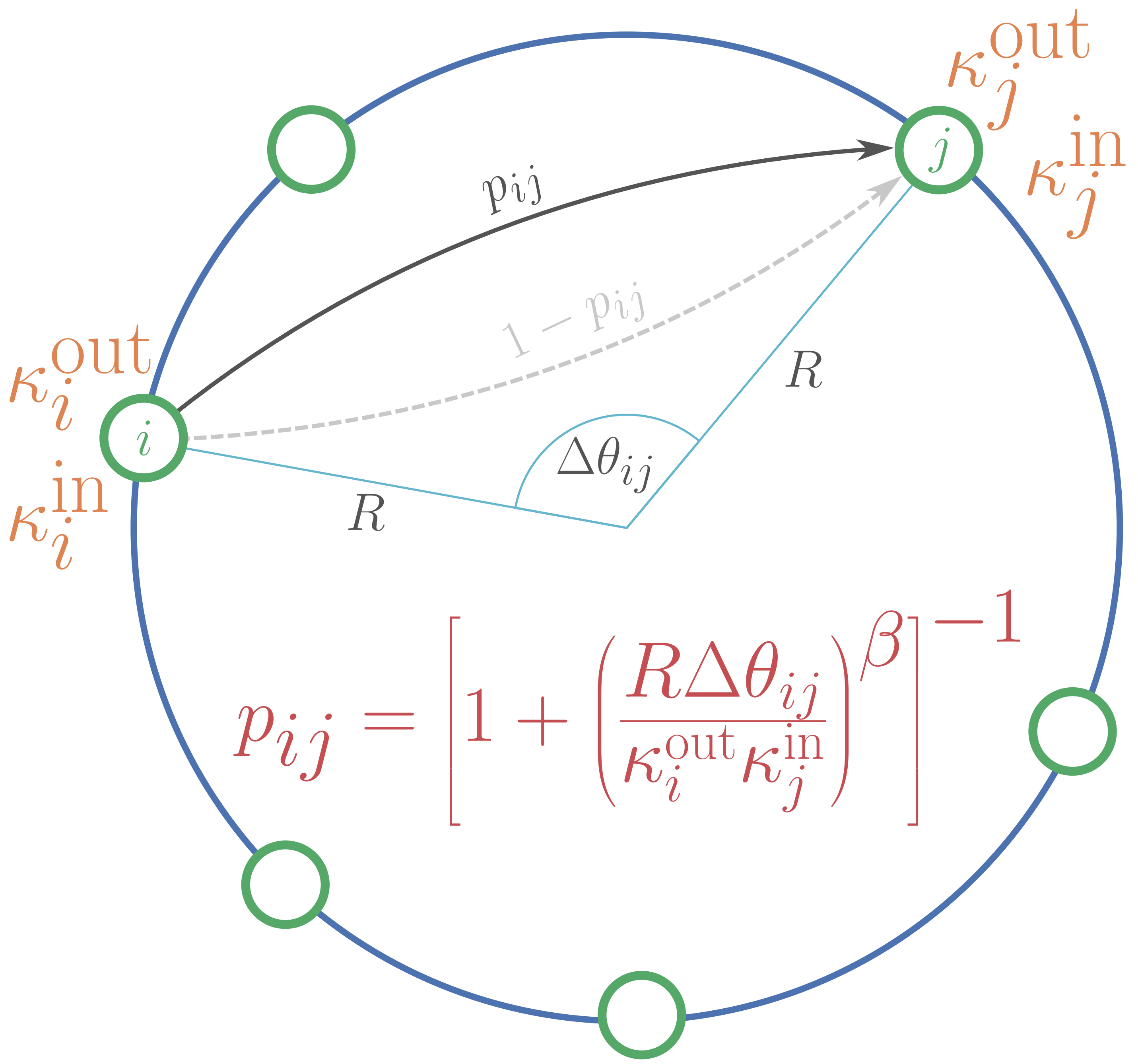
$$r=\frac{L^{\leftrightarrow}}{L}=\frac{\text{[number of reciprocal links]}}{\text{[number of links]}}$$





292 network datasets downloaded from Netzschleuder (networks.skewed.de).

The directed $\mathbb{S}^1$ model



The directed $\mathbb{S}^1$ model

1. Sprinkle N nodes uniformly on a circle of radius R.
2. Assign an expected in-degree κ^{in} and out-degree κ^{out} to each node according to some pdf $\rho(\kappa^{\text{in}}, \kappa^{\text{out}})$.
3. Draw a link from node i to node j with probability p_{ij}.

★ fixes the expected in-degree and out-degree of nodes (κ^{in}, κ^{out}) $\rightarrow$ soft directed CM

★ triangle inequality of the underlying metric space $\rightarrow$ triangles from pairwise interactions

★ level of clustering tuned with parameter β