

# Maximally random geometric graph ensembles

Example 3: fixing the expected degree sequence and the expected total energy

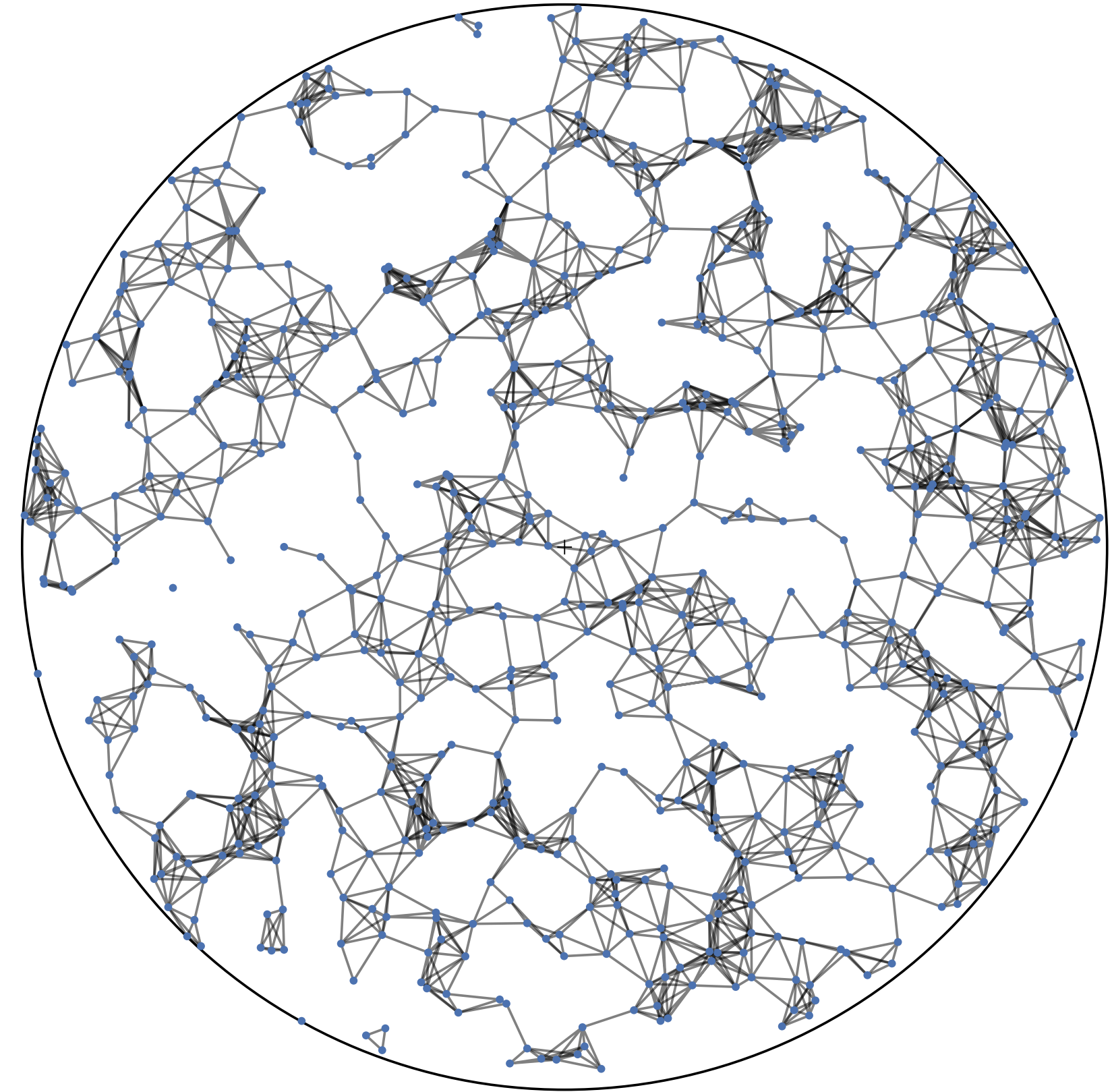
$$\bar{F}_1 = \sum_{i=1}^N \sum_{j=i+1}^N a_{ij} = M$$

$$\bar{F}_2 = \sum_{i=1}^N \sum_{j=i+1}^N \varepsilon_{ij} a_{ij} = \sum_{i=1}^N \sum_{j=i+1}^N f(x_{ij}) a_{ij} = E$$

yields the homogeneous random geometric graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^N \prod_{j=i+1}^N p_{ij}^{a_{ij}} (1 - p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1}.$$

The graphs will be sparse, highly clustered and small-world iif  $f(x_{ij}) \sim \ln x_{ij}$  and  $\beta \in [D, D + 2]$ .



# Maximally random geometric graph ensembles

Example 4: fixing the expected number of edges and the expected total energy

$$\bar{F}_l = \sum_{j=1}^N a_{lj} = k_l \quad (l = 1, \dots, N)$$

$$\bar{F}_{N+1} = \sum_{i=1}^N \sum_{j=i+1}^N \varepsilon_{ij} a_{ij} = \sum_{i=1}^N \sum_{j=i+1}^N f(x_{ij}) a_{ij} = E$$

yields the heterogeneous random geometric graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^N \prod_{j=i+1}^N p_{ij}^{a_{ij}} (1 - p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta \varepsilon_{ij} - \alpha_i - \alpha_j} + 1} .$$

The graphs will be sparse, highly clustered, small-world and devoid of non-structural degree-degree correlation iff  $f(x_{ij}) = \ln x_{ij}$  and  $\beta \in [D, D + 2]^a$ . Redefining  $\alpha_l = -(\beta/D) \ln(\sqrt{\mu} \kappa_l)$  yields

$$p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1} \quad \text{with} \quad \varepsilon_{ij} = \ln \left( \frac{x_{ij}}{(\kappa_i \kappa_j)^{\frac{1}{D}}} \right) .$$

<sup>a</sup> No upper bound if expected degree sequence is scale-free.