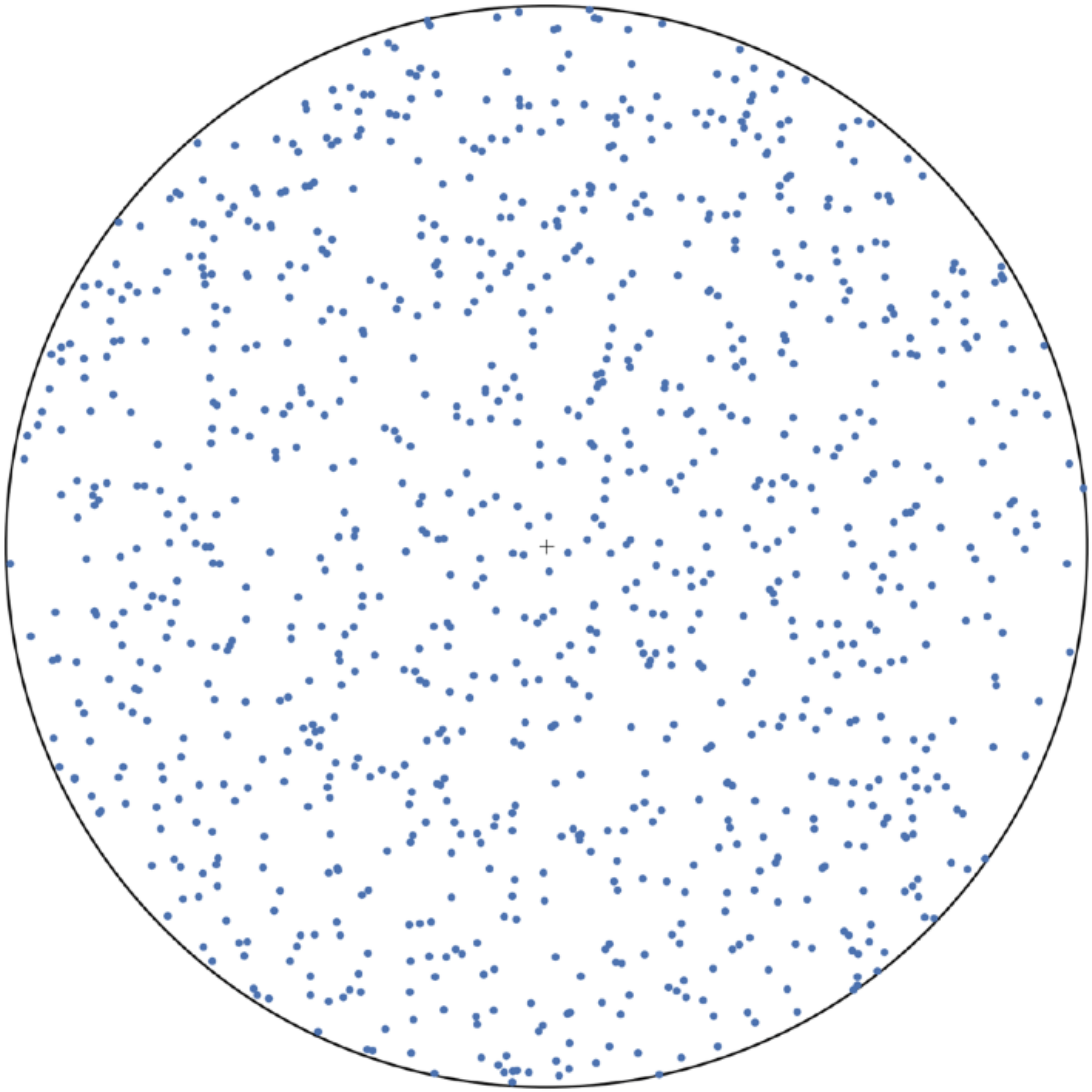
4

A geometric approach to clustering

Identity of indiscernibles	$d(x,y) = 0 \quad \Leftrightarrow \quad x = y$
Non-negativity	$d(x,y) \geq 0$
Symmetry	$d(x,y) = d(y,x)$
Triangle inequality	$d(x,y) \leq d(x,z) + d(z,y)$

Properties of any metric space



Simple random geometric graph

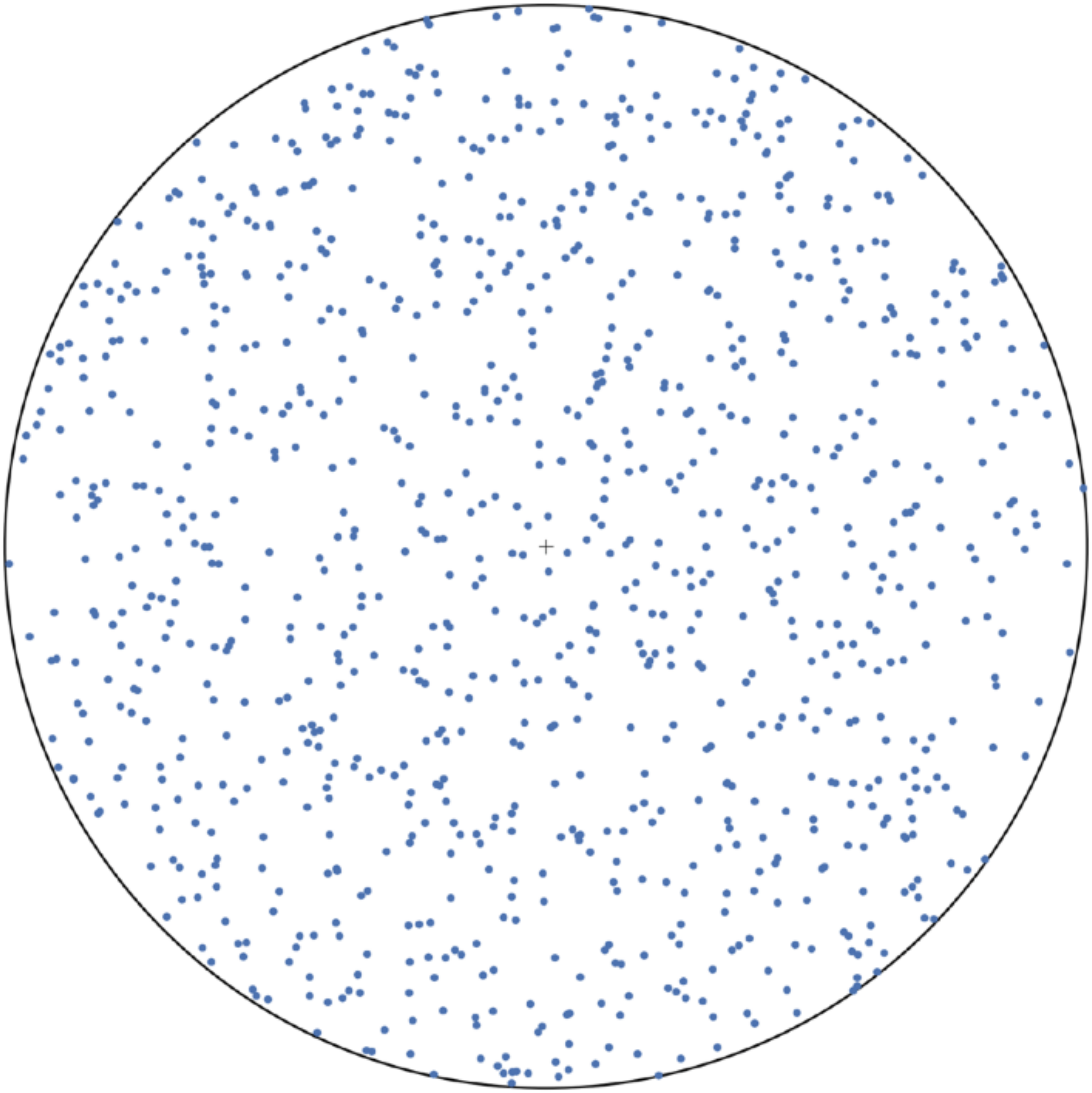
1. Sprinkle N nodes uniformly on a disk of radius R.
2. Connect any nodes separated by a distance less than r.

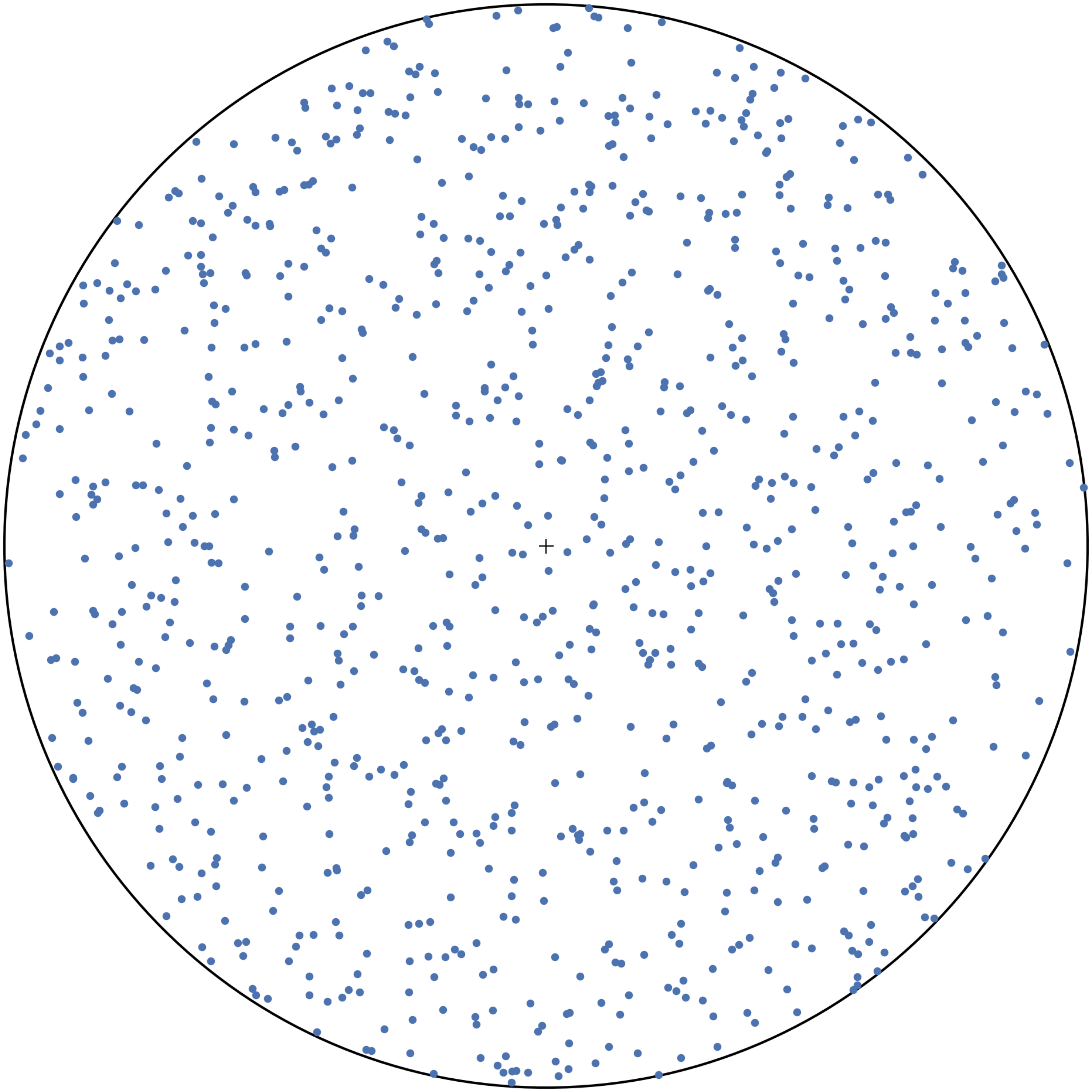
✓ high clustering

✗ binomial/Poisson degree distribution

Assume that the nodes are embedded in a metric space and that any two nodes are connected with a probability that is a decreasing function of the distance between them.

For further info, see Phys. Rep. 499, 1–101 (2011)





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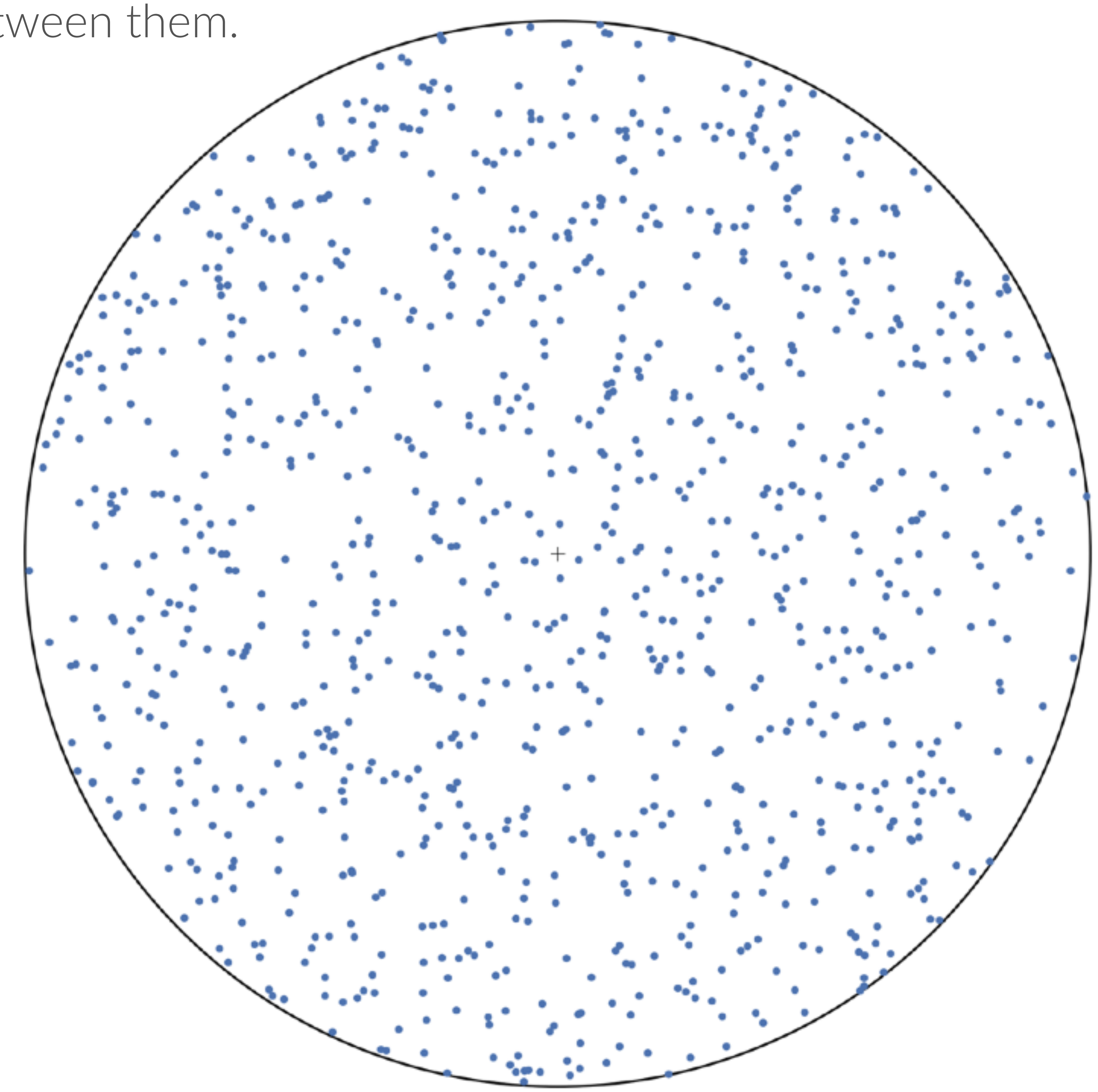

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Hyperbolic geometry

- Space of constant negative curvature (as opposed to flat or Euclidean space, or spherical space)

- Model for the $D = 2$ hyperbolic space : positive sheet of the hyperboloid defined by

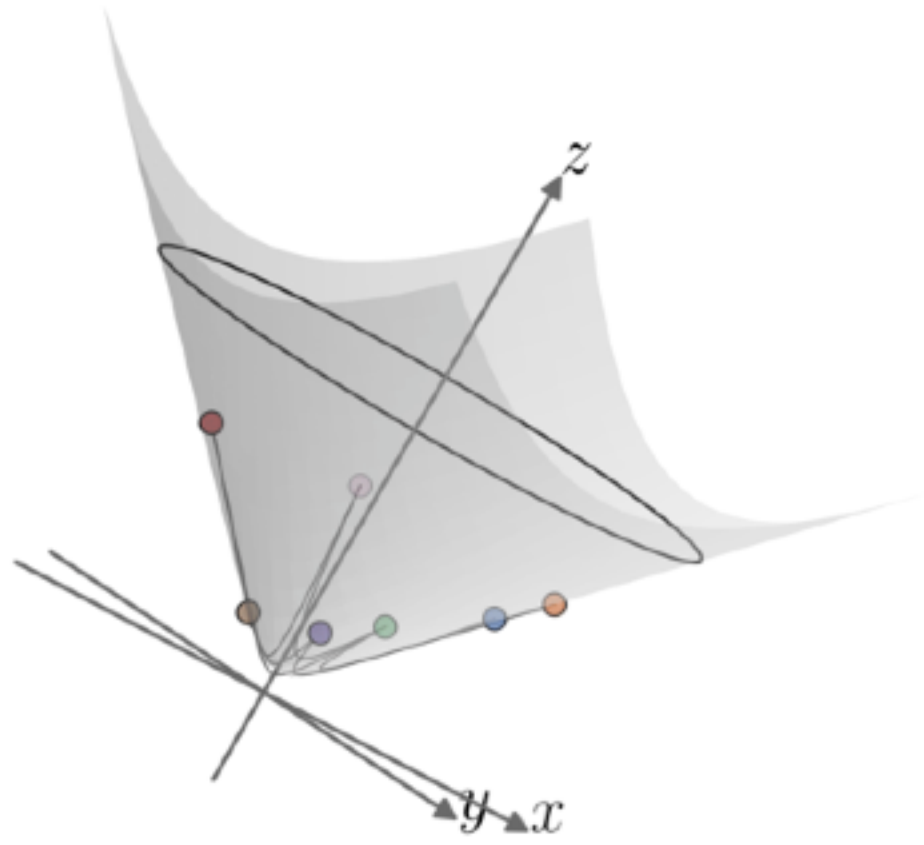
$$x^2 + y^2 - z^2 = -1$$

- Distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

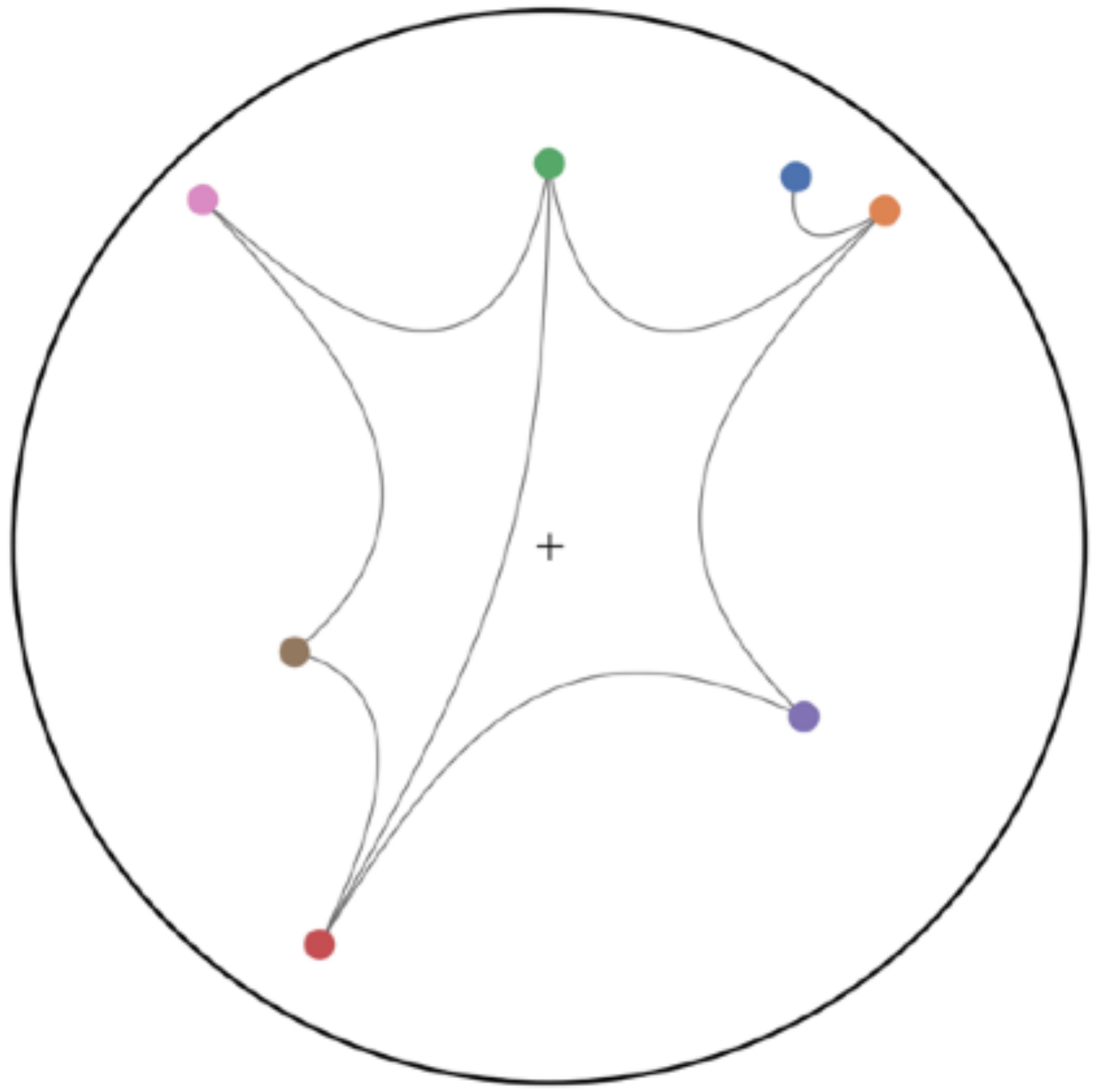
$$d(1,2) = \operatorname{arccosh}\big(z_1 z_2 - x_1 x_2 - y_1 y_2\big)$$

- Polar coordinates

$$\begin{aligned} x &= \sinh(r)\cos(\theta) \\ y &= \sinh(r)\sin(\theta) \\ z &= \cosh(r) \end{aligned}$$



hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

For further info, see Flavors of geometry (Cambridge University Press, 1997) or Foundations of Hyperbolic Manifolds (Springer, 2019)