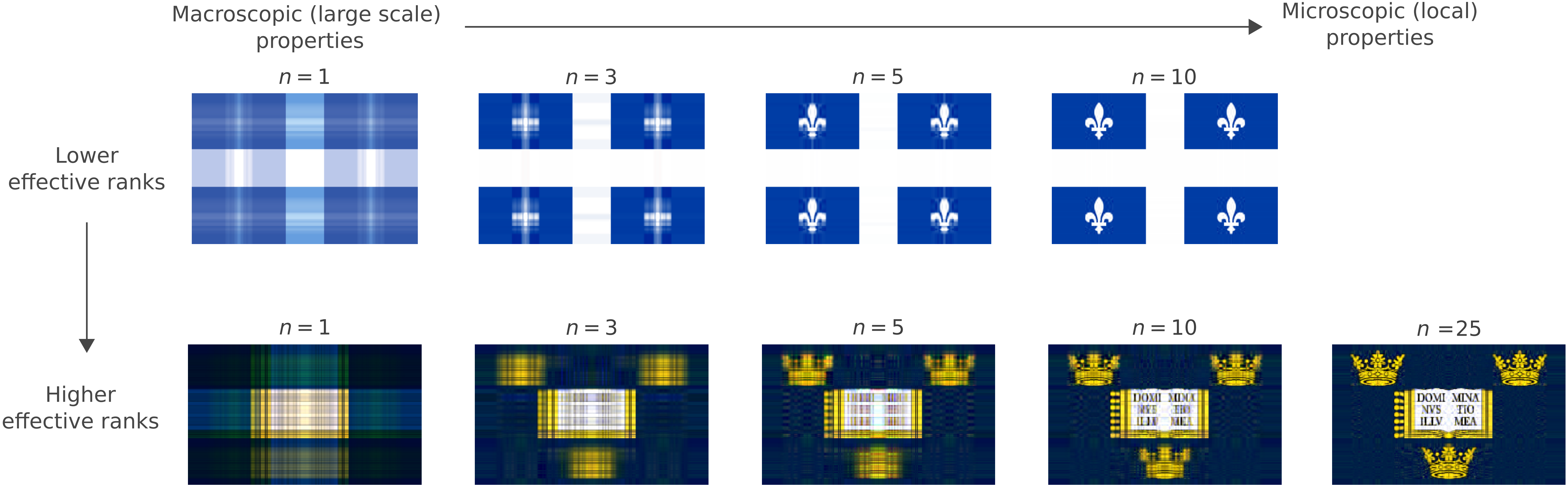
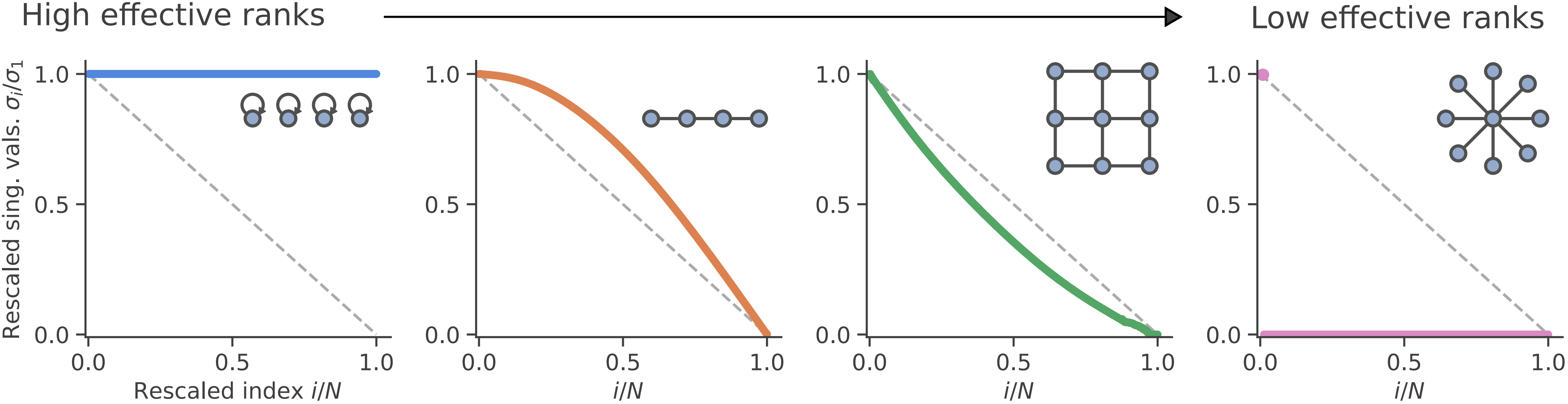

Macroscopic (large scale) properties
Microscopic (local) properties
Lower effective ranks
Higher effective ranks
$n=1$
$n=3$
$n=5$
$n=10$
$n=1$
$n=3$
$n=5$
$n=10$
$n=25$

The effective ranks of adjacency matrices

$$\mathbf{W} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top \simeq \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top$$

Effective rank n of a matrix

- number of "significant" nonzero singular values
- term after which it is "reasonable" to truncate the sum



High effective ranks
Low effective ranks
Rescaled sing. vals. σ_i/σ_1
1.0
0.5
0.0
0.0
0.5
1.0
Rescaled index i/N
i/N
i/N
i/N

Abbreviation	Expression
srank	$\sum_{i=1}^{r} \sigma_i^2 / \sigma_1^2$
nrank	$\sum_{i=1}^{r} \sigma_i / \sigma_1$
energy	$\min\left[\arg\max_{\ell \in \{1,\ldots,N\}} \left(\sum_{i=1}^{\ell} \sigma_i^2 / \sum_{j=1}^{r} \sigma_j^2 > \tau\right)\right]$
elbow	$\frac{1}{\sqrt{2}} \arg\max_{i \in \{1,\ldots,N\}} \left\lvert \frac{i-1}{N-1} + \frac{\sigma_i - \sigma_N}{\sigma_1 - \sigma_N} - 1 \right\rvert - 1$
erank	$\exp\left[-\sum_{i=1}^{r} \frac{\sigma_i}{\sum_{j=1}^{r} \sigma_j} \log \frac{\sigma_i}{\sum_{j=1}^{r} \sigma_j}\right]$
thrank	$\#\left\{\sigma_i \mid i \in \{1, \ldots, N\} \text{ and } \sigma_i > \frac{4\sigma_{\text{med}}}{\sqrt{3\mu_{\text{med}}}}\right\}$
shrank	$\#\{s^*(\sigma_i) \mid i \in \{1, \ldots, N\} \text{ and } s^*(\sigma_i) > 0\}$

The singular values (and the effective rank) encode information about the network's topology.

6

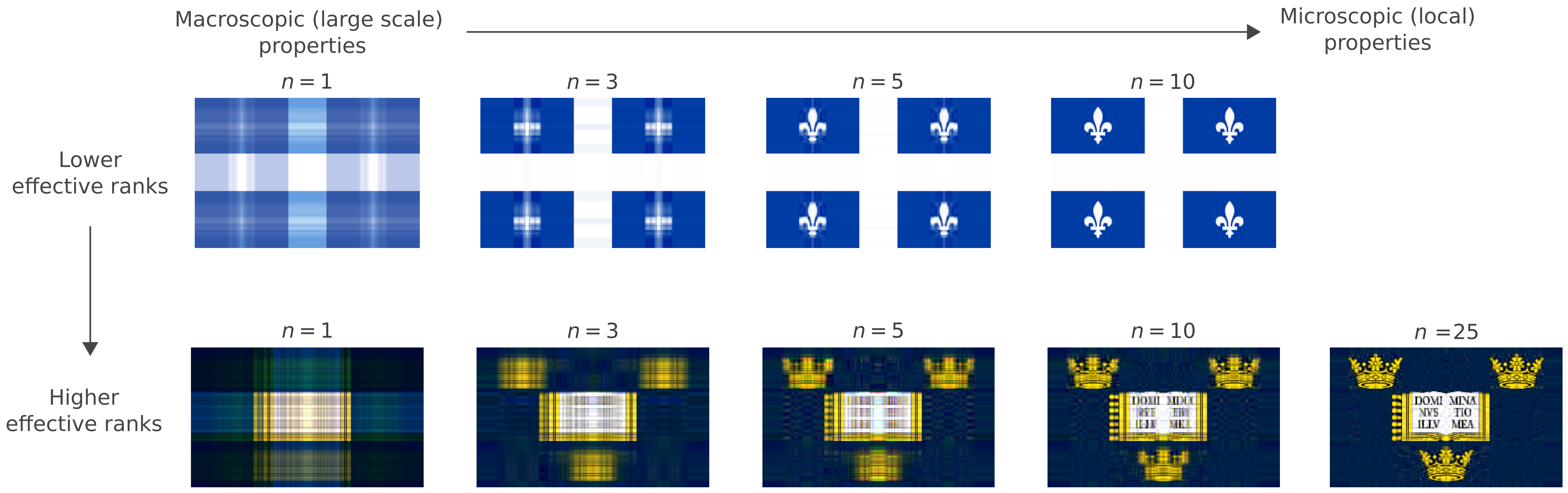
The effective ranks of adjacency matrices

$$\mathbf{W} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top \simeq \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top$$

Effective rank n of a matrix

▷ number of “significant” nonzero singular values

▷ term after which it is “reasonable” to truncate the sum



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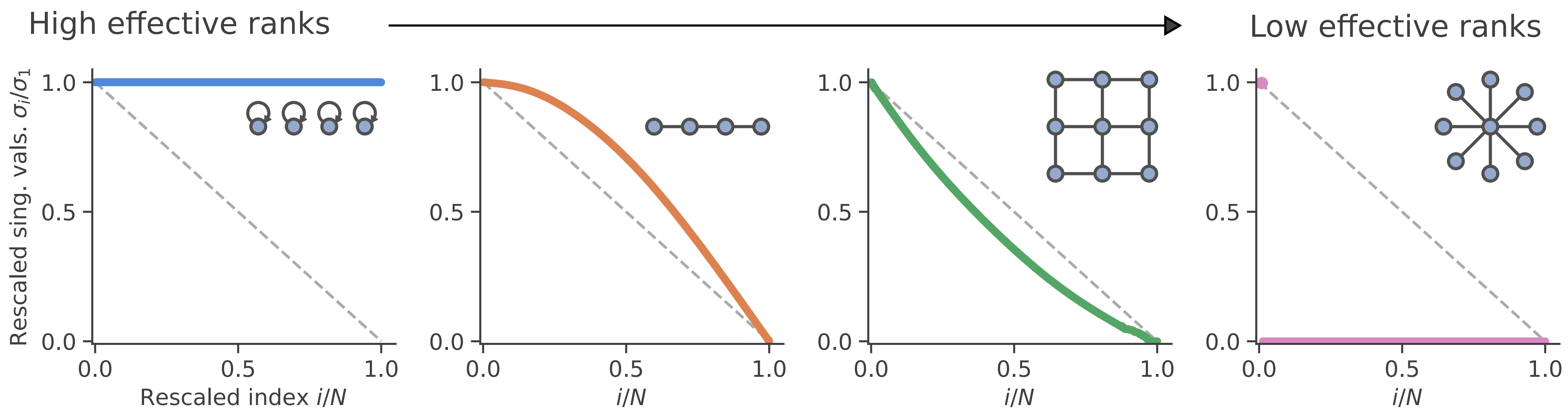
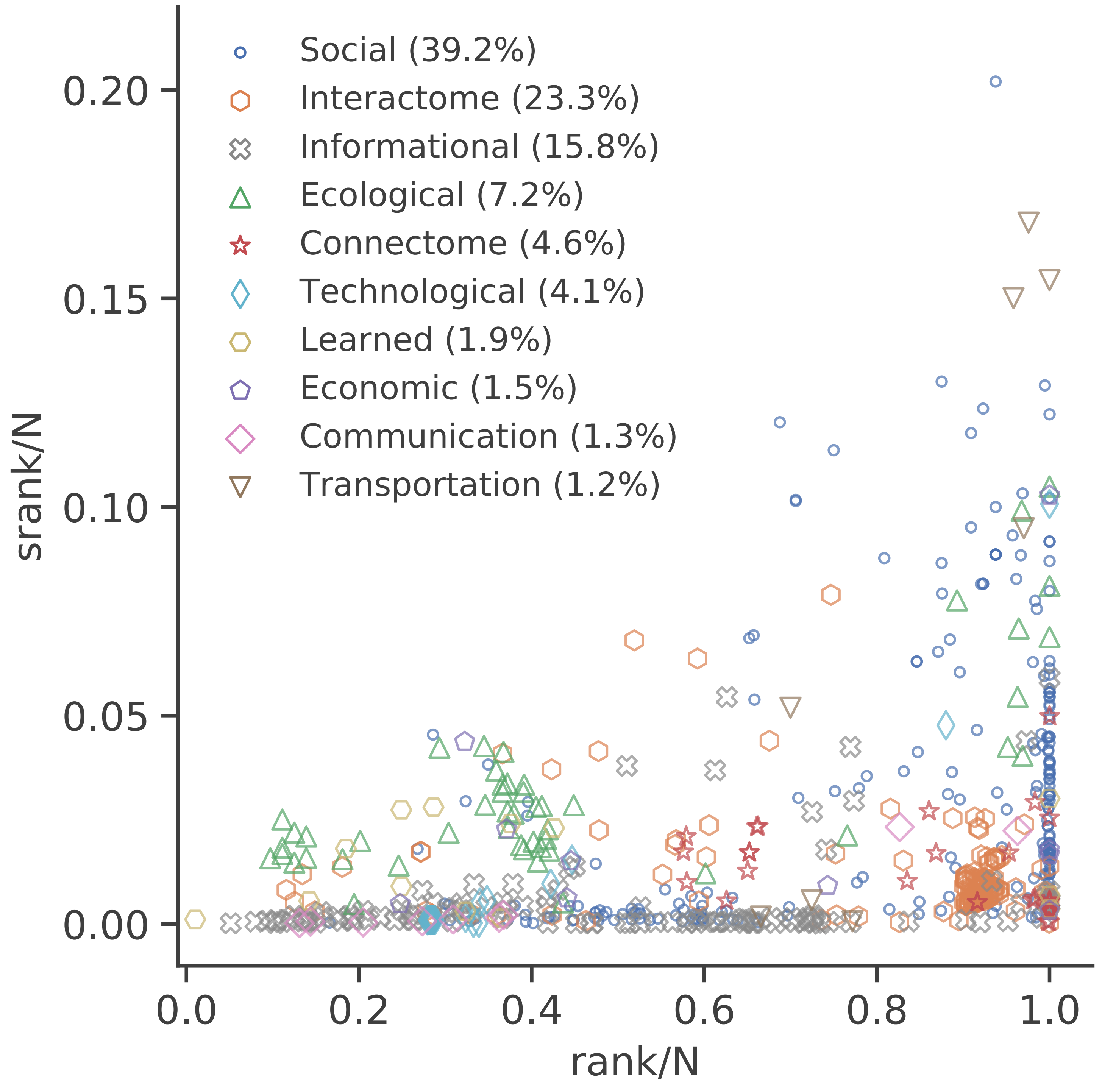
The singular values (and the effective rank) encode information about the network’s topology.



The effective ranks of adjacency matrices

Many empirical networks appear to have a low effective rank!



Results for 679 empirical networks (502 unweighted networks and 177 weighted networks) dowloaded from Netzschleuder.