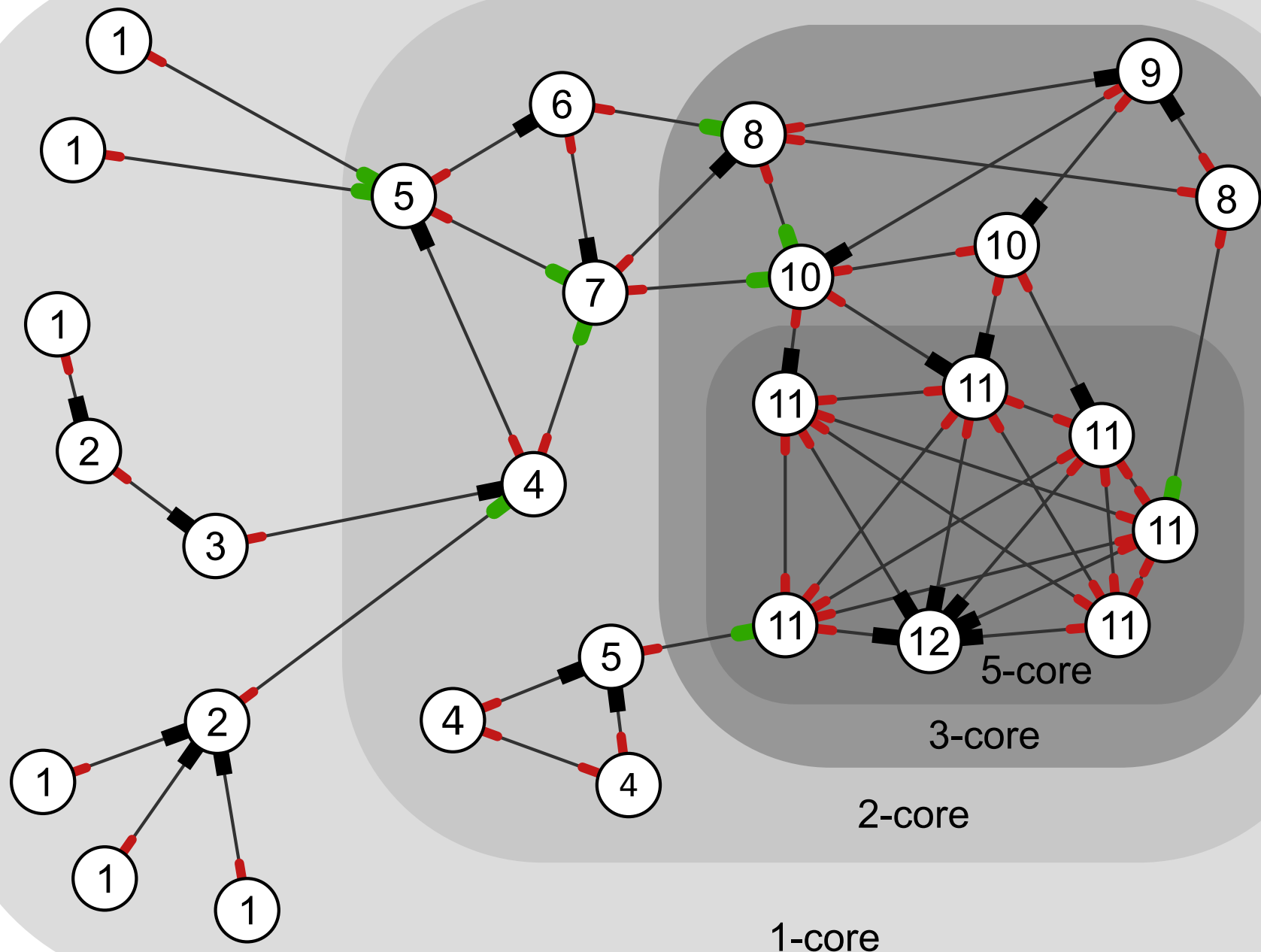


2

6



Message level: The k -core/union decomposition

Onion decomposition: k -core decomposition with additional information about the positions of nodes within every k -shell (layers).

Information about layers is obtained from the k -core decomposition with minimal additional computational cost.

Stub matching scheme

- ▷ One type of nodes per layer
- ▷ Three types of stubs (red, green, black)
- ▷ Rules:

1. Allowed links: red–red, red–green, red–black

2. Nodes in layer ℓ and shell k must

- (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k -shell).
- (b) have at least $k + 1$ links to nodes in layers $\ell' \geq \ell - 1$ and at most k links to nodes in layers $\ell' \geq \ell$ (if it is not in the first layer of the k -shell).

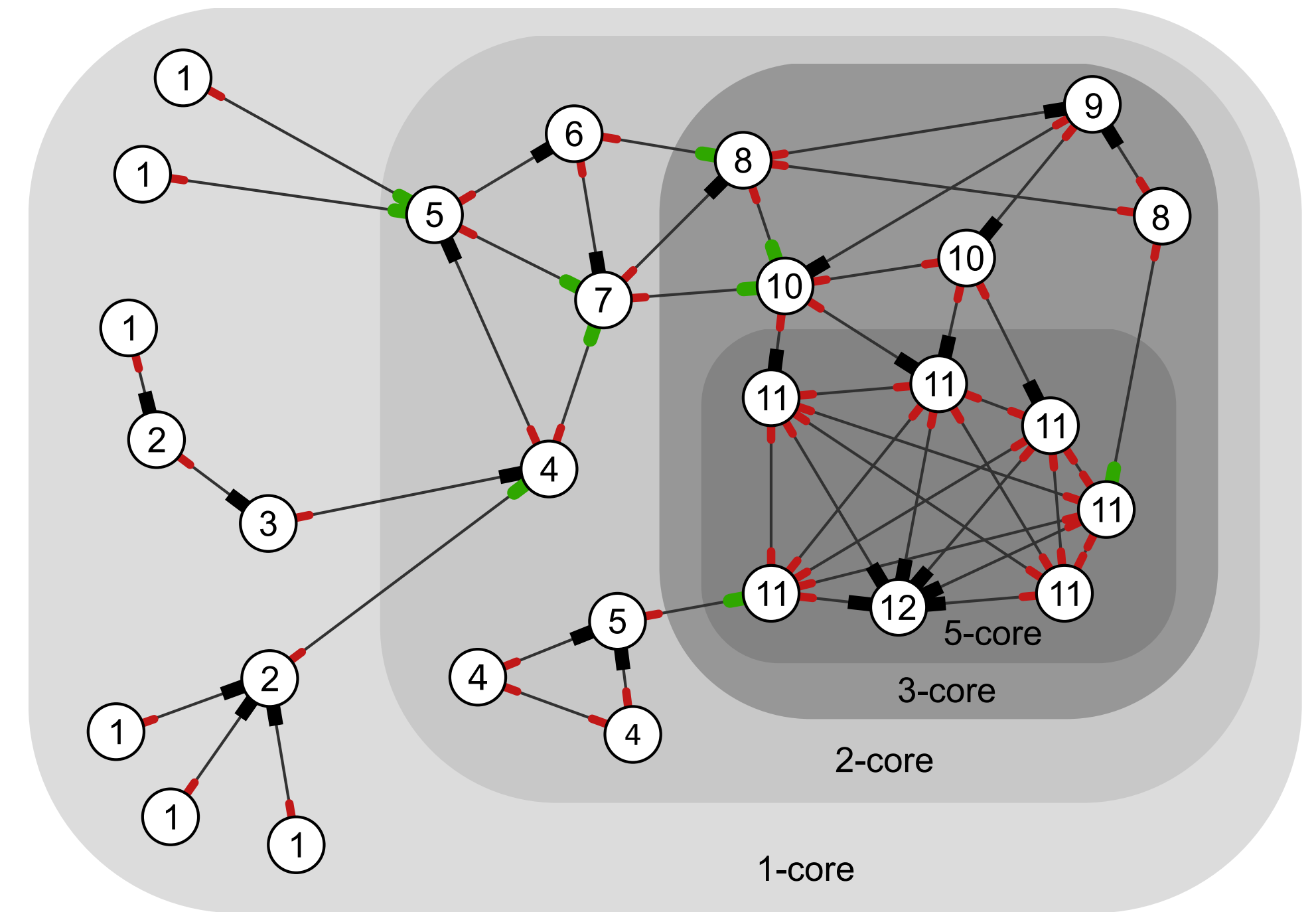
Mesoscopic level: The k -core/onion decomposition

Onion decomposition: k -core decomposition with additional information about the positions of nodes within every k -shell (layers).

Information about layers is obtained from the k -core decomposition with minimal additional computational cost.

Stub matching scheme

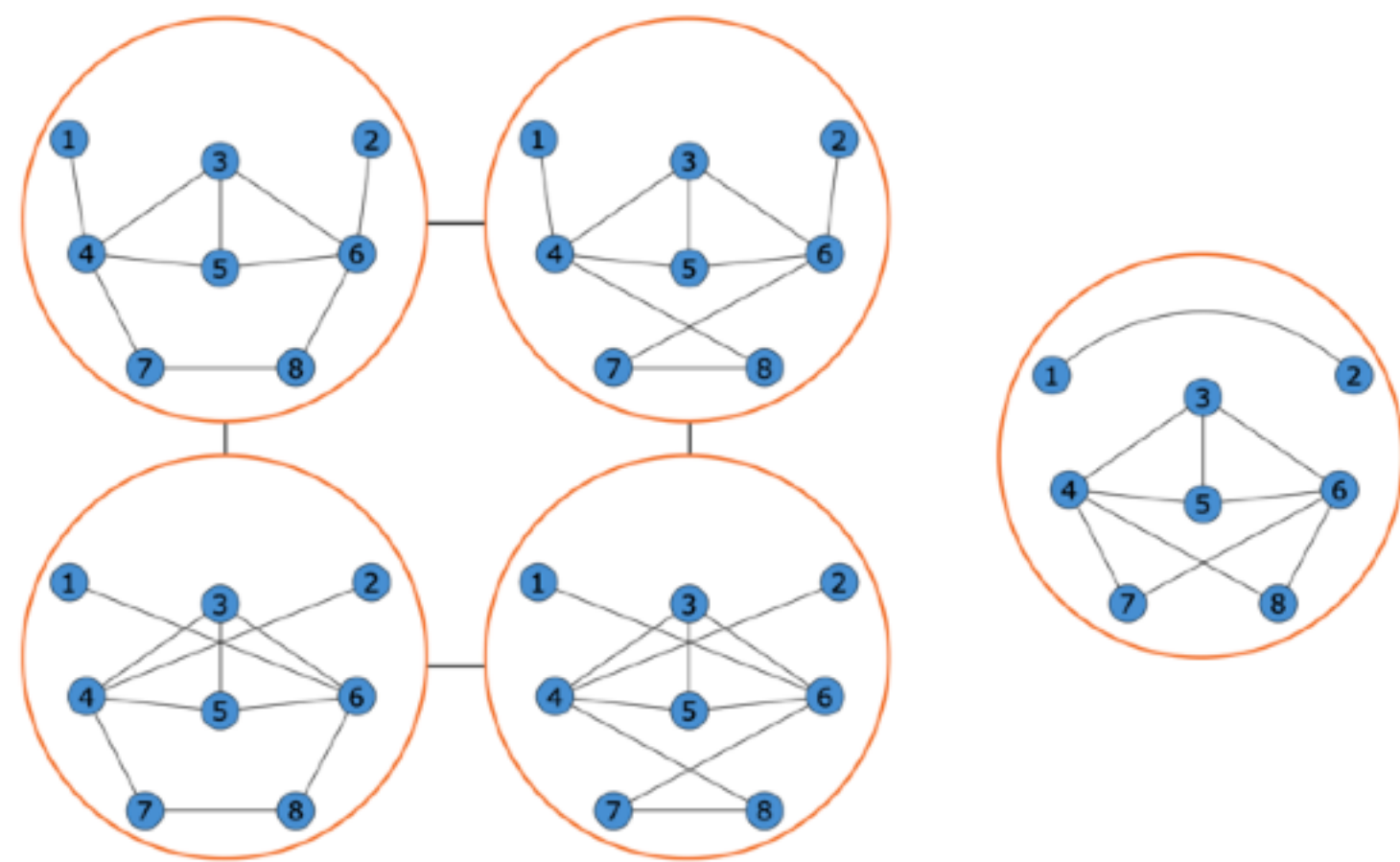
- ▷ One type of nodes per layer
- ▷ Three types of stubs (red, green, black)
- ▷ Rules:
 1. Allowed links: red-red, red-green, red-black
 2. Nodes in layer ℓ and shell k must
 - (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k -shell).
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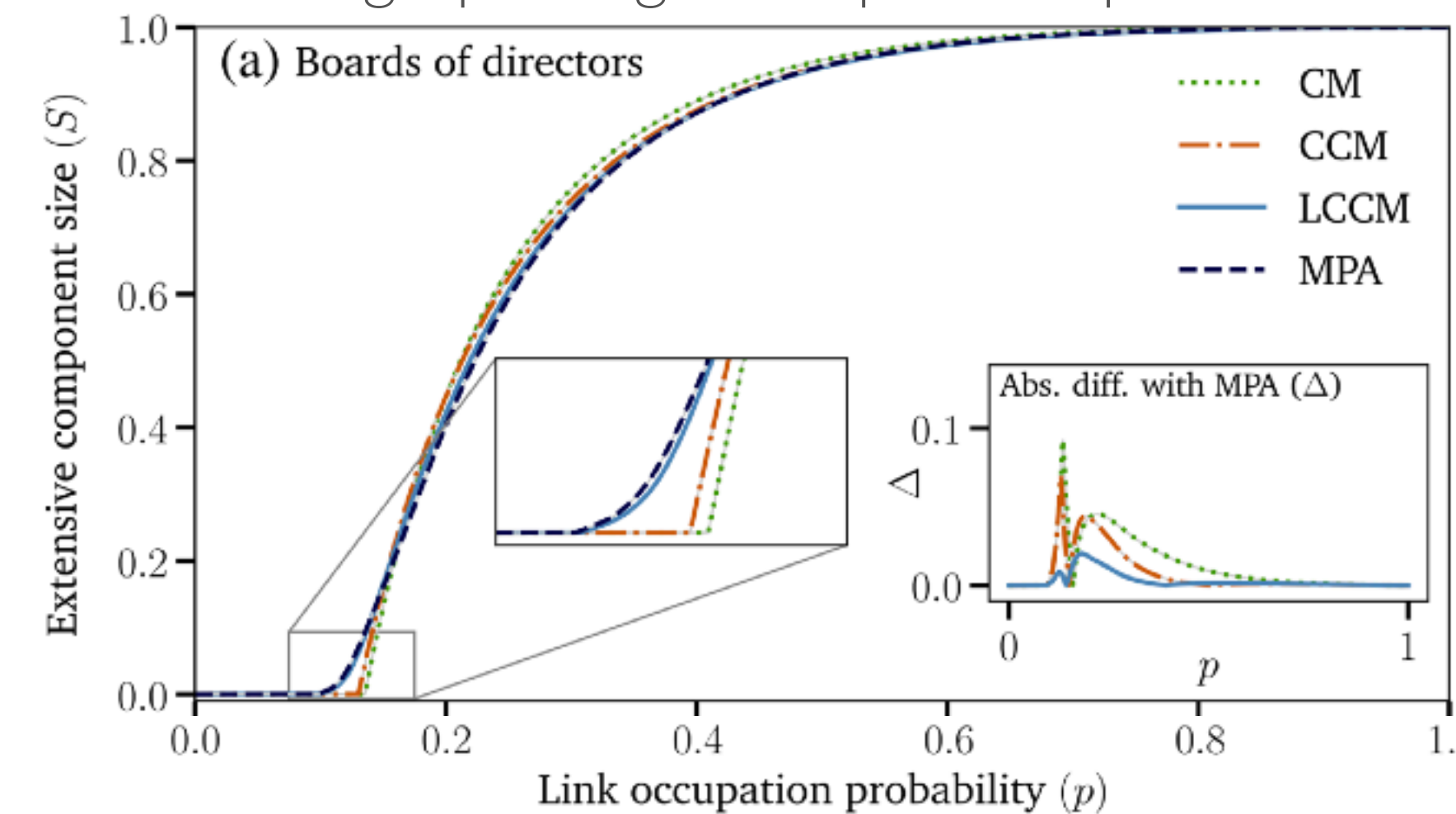
Challenges

- ▶ Many analytical approaches (PGF, ODE) can be adapted to account for stub types, but the graphs considered are infinite in size.
- ▶ Sampling naively is easy, sampling right is tricky (ex.: uniform sampling via edge swaps).

The sample space is not necessarily connected under 2-edge swaps.



The onion decomposition accounts for a lot when it comes to the precision of message-passing description of percolation.



Numerical evidence that the ignored graphs could be negligible (using k -edge swaps), but still no formal demonstration.

