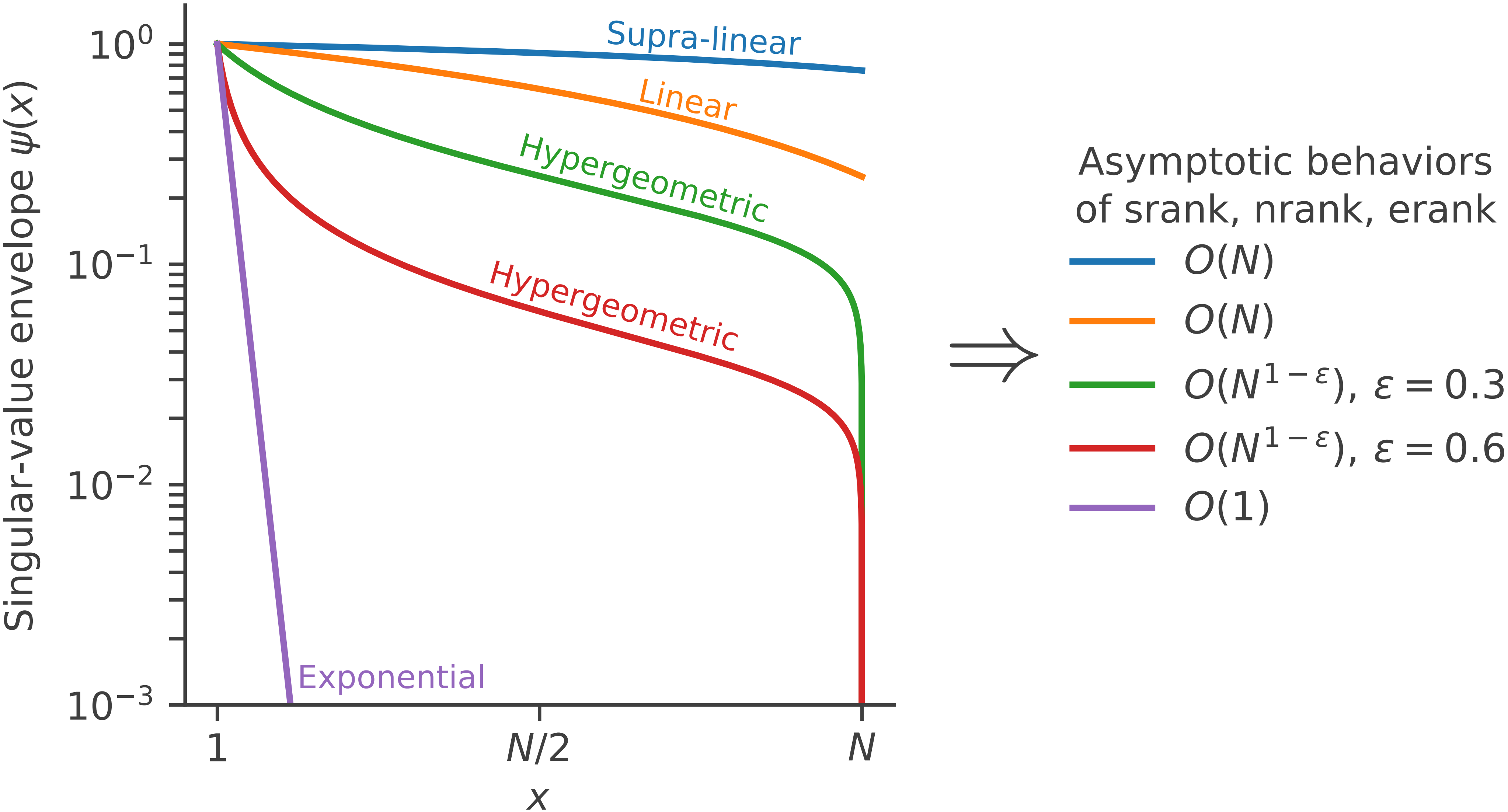
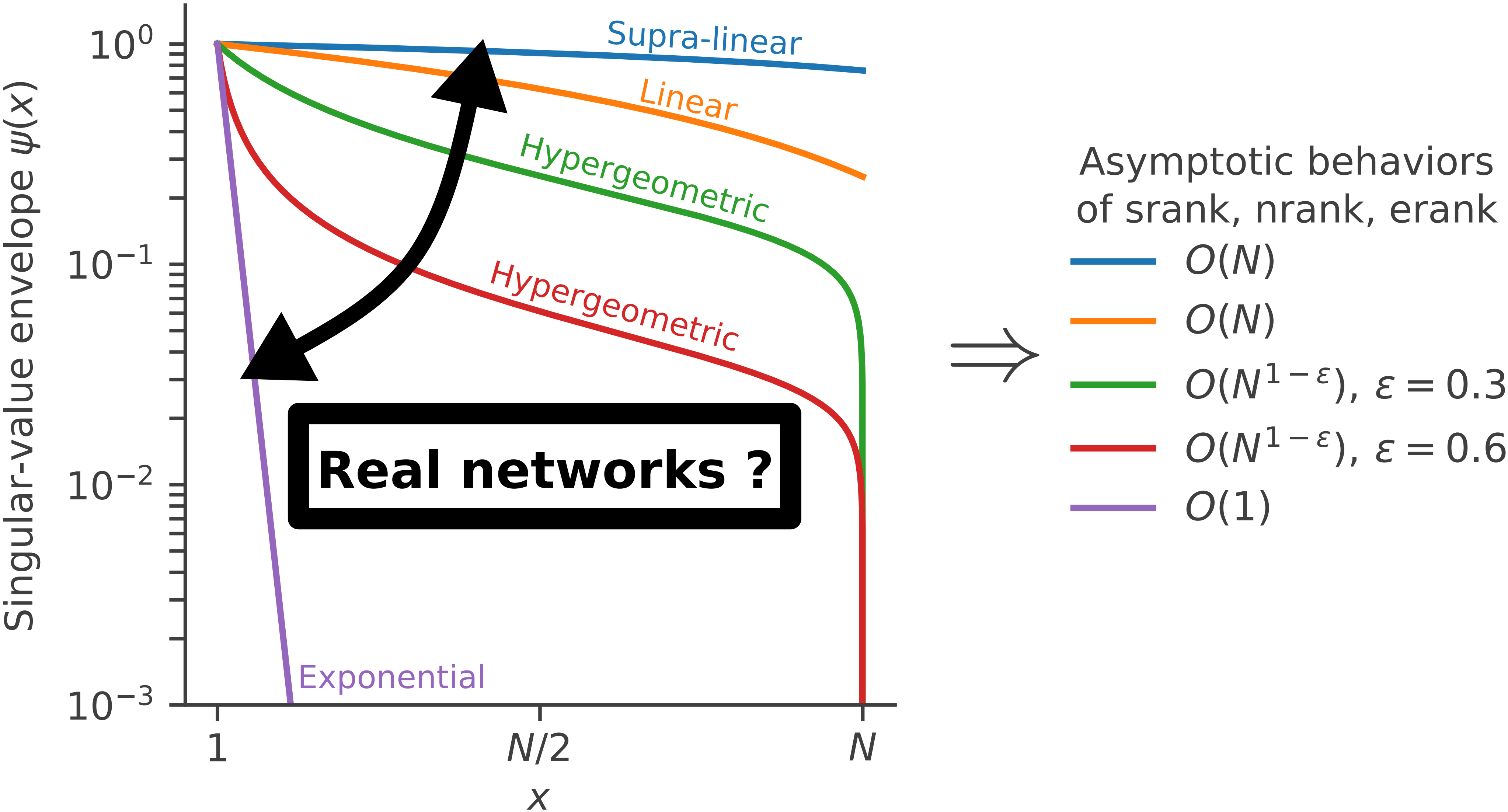


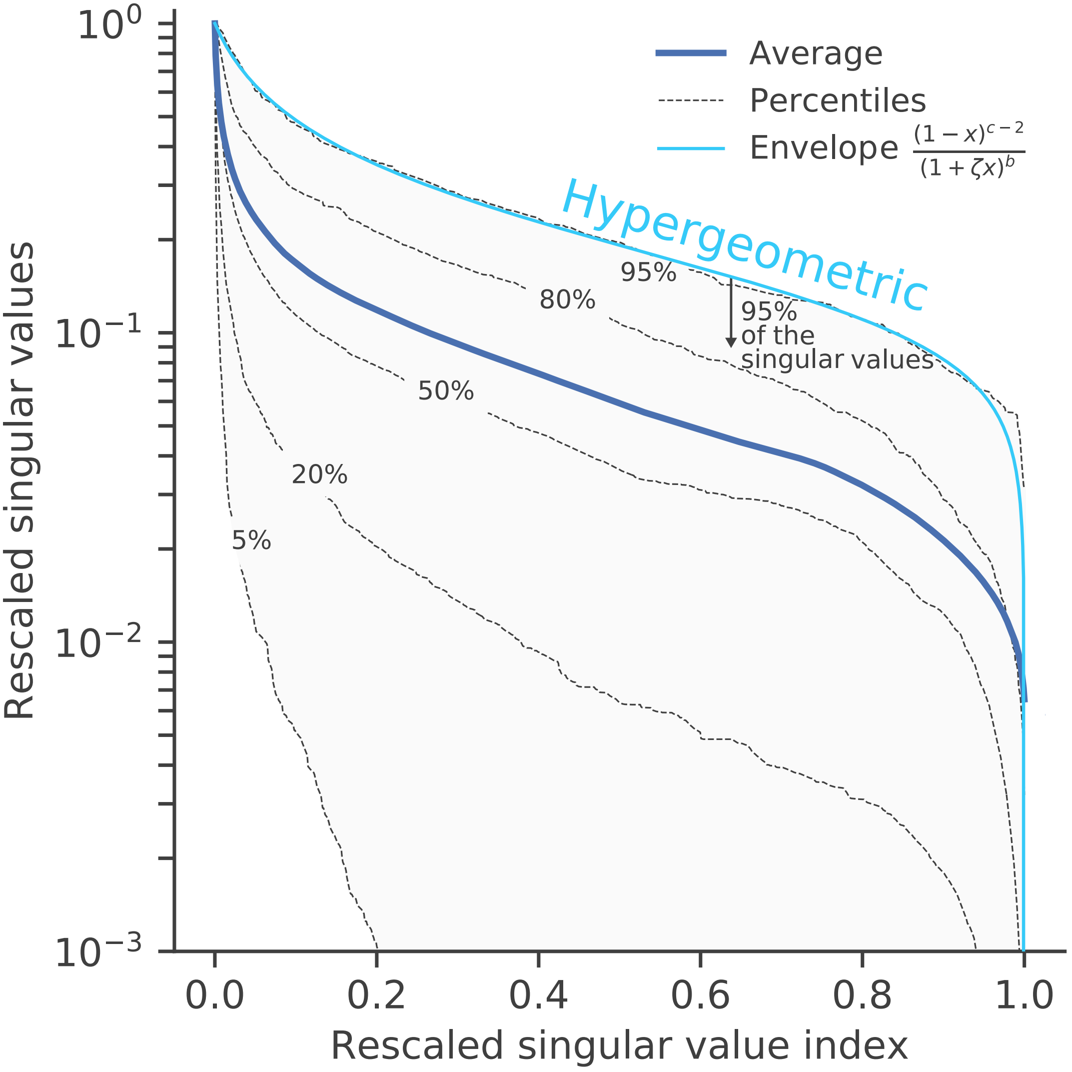
Singular-value envelope $\psi(x)$
10^0
10^{-1}
10^{-2}
10^{-3}
1
$N/2$
N
x
Supra-linear
Linear
Hypergeometric
Hypergeometric
Exponential
$\Rightarrow$
Asymptotic behaviors of srank, nrank, erank
$O(N)$
$O(N)$
$O(N^{1-\varepsilon})$, $\varepsilon = 0.3$
$O(N^{1-\varepsilon})$, $\varepsilon = 0.6$
$O(1)$



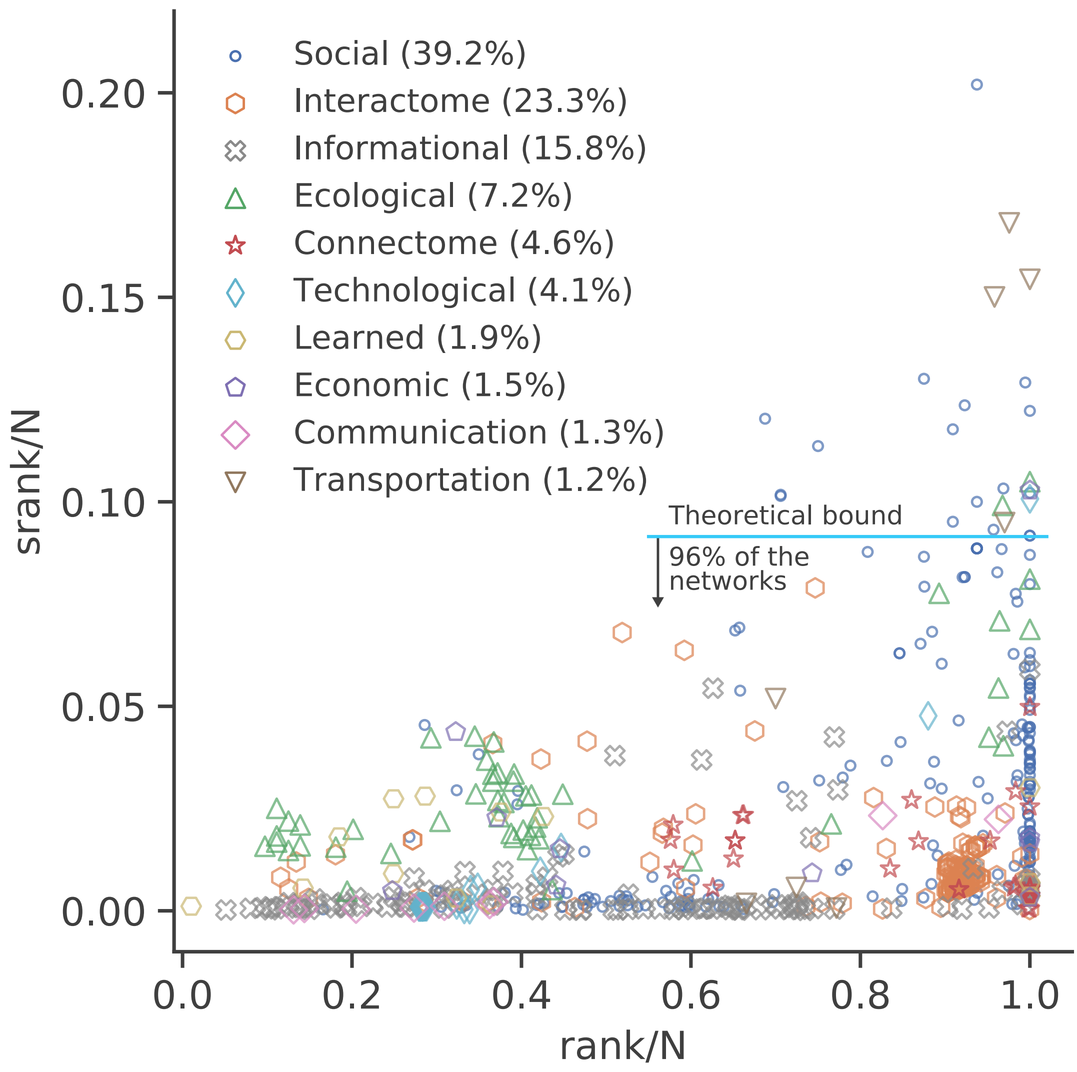
Supra-linear
Linear
Hypergeometric
Hypergeometric
Exponential
Real networks ?
Singular-value envelope $\psi(x)$
10^0
10^{-1}
10^{-2}
10^{-3}
1
$N/2$
N
x
$\Rightarrow$
Asymptotic behaviors
of srank, nrank, erank
$O(N)$
$O(N)$
$O(N^{1-\varepsilon})$, $\varepsilon = 0.3$
$O(N^{1-\varepsilon})$, $\varepsilon = 0.6$
$O(1)$

A *workable* definition of “low” effective rank

9



10^0
10^{-1}
10^{-2}
10^{-3}
Rescaled singular values
Average
Percentiles
Envelope $\frac{(1-x)^{c-2}}{(1+\zeta x)^b}$
Hypergeometric
95%
80%
95% of the singular values
50%
20%
5%
0.0
0.2
0.4
0.6
0.8
1.0
Rescaled singular value index



0.20
0.15
0.10
0.05
0.00
srank/N
Social (39.2%)
Interactome (23.3%)
Informational (15.8%)
Ecological (7.2%)
Connectome (4.6%)
Technological (4.1%)
Learned (1.9%)
Economic (1.5%)
Communication (1.3%)
Transportation (1.2%)
Theoretical bound
96% of the networks
0.0
0.2
0.4
0.6
0.8
1.0
rank/N

The singular values of approx. 96% of considered networks are bounded from above by an hypergeometric envelope $\Rightarrow$ sublinear effective ranks!

Workable definition of low effective rank: $\sim$10% of the number of nodes N

Approx. 96% of the 679 networks qualify for having a low effective rank!

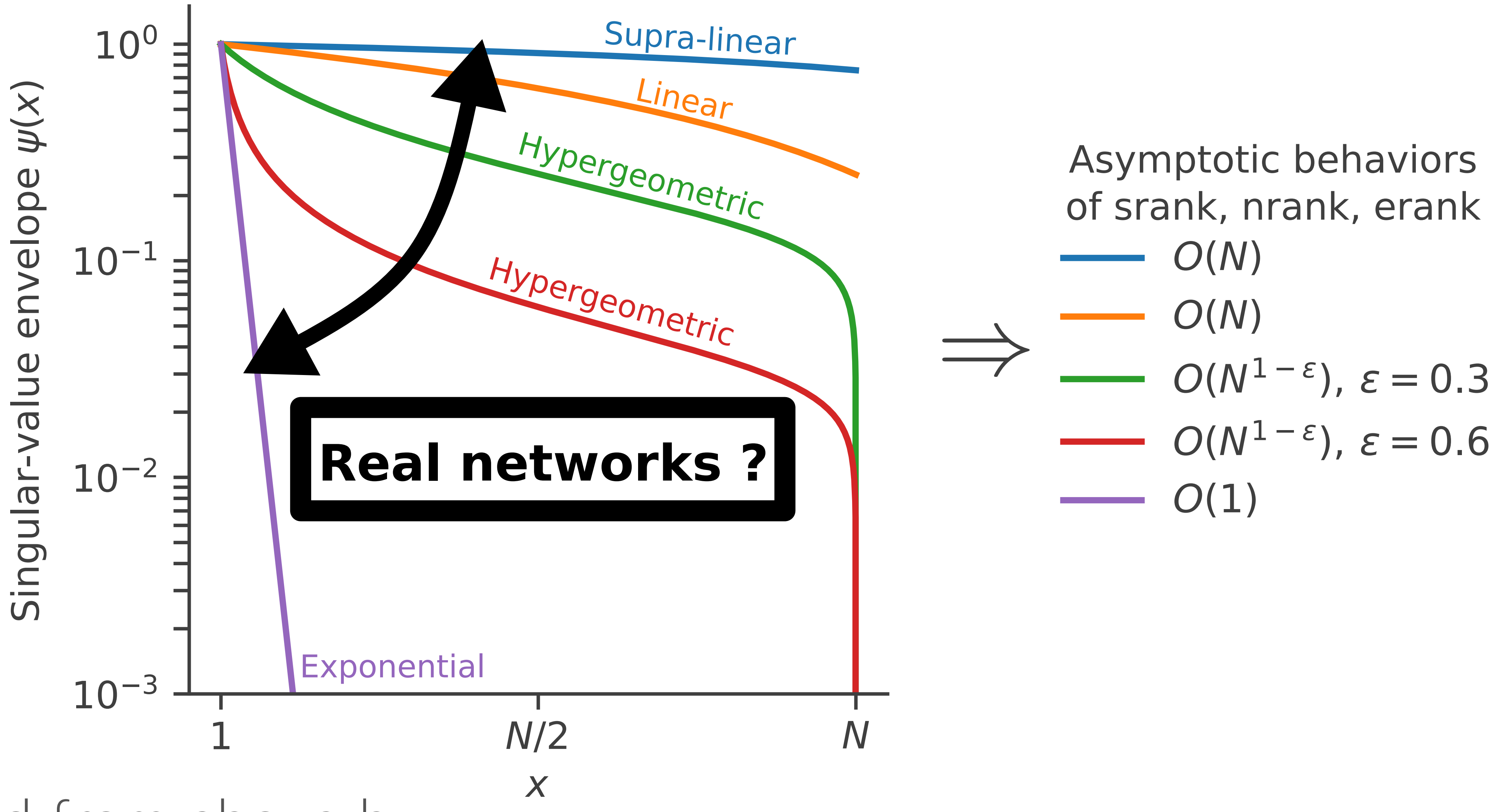
Hint: the rapid decrease of the dominant singular values of the adjacency matrix implies a low effective rank

- low effective rank? $\Rightarrow$ effective rank scales at most sublinearly as the number of nodes, N, goes to infitnity ($N^{1-\varepsilon}$ with $\varepsilon \in (0, 1]$)

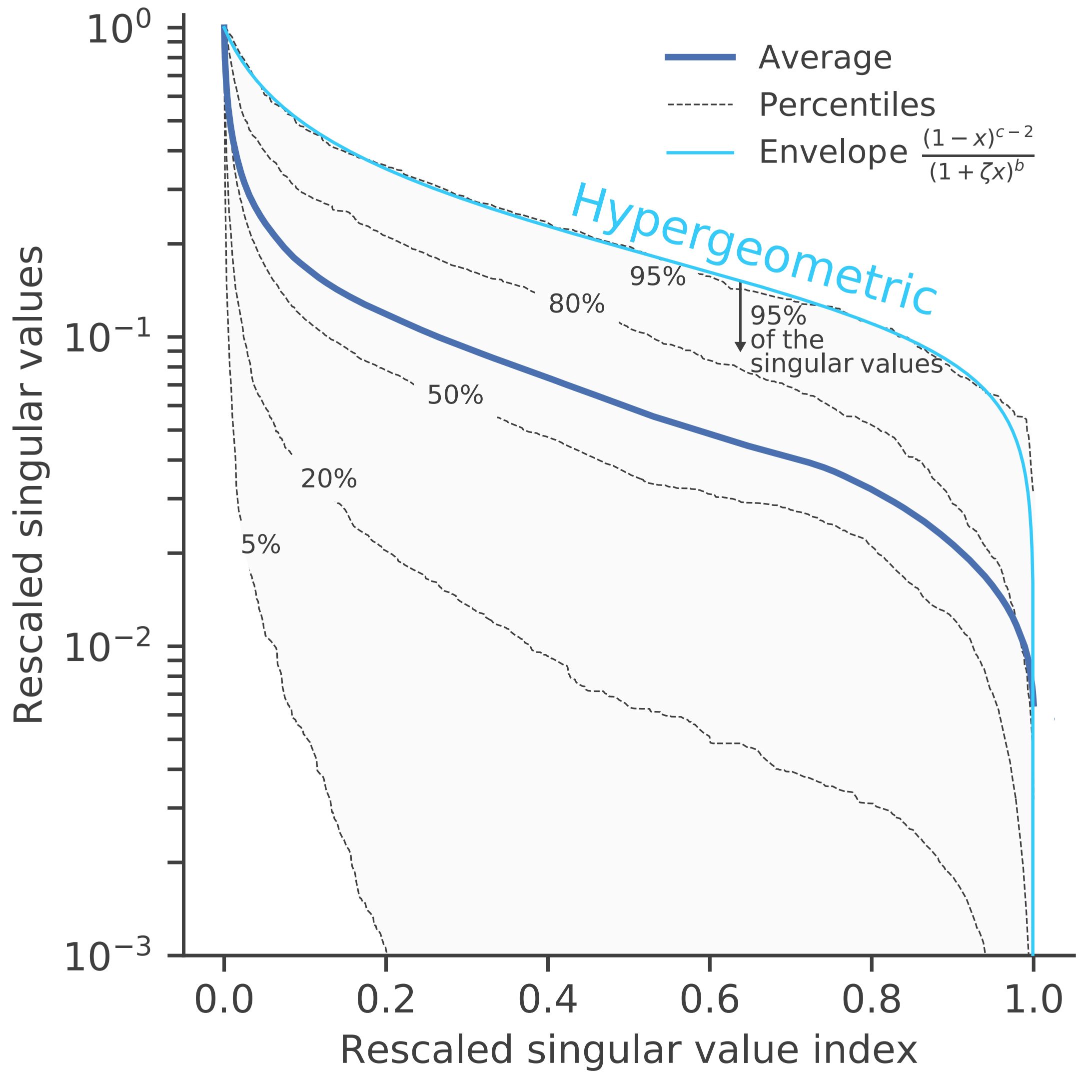
A *workable* definition of “low” effective rank

Hint: the rapid decrease of the dominant singular values of the adjacency matrix implies a low effective rank

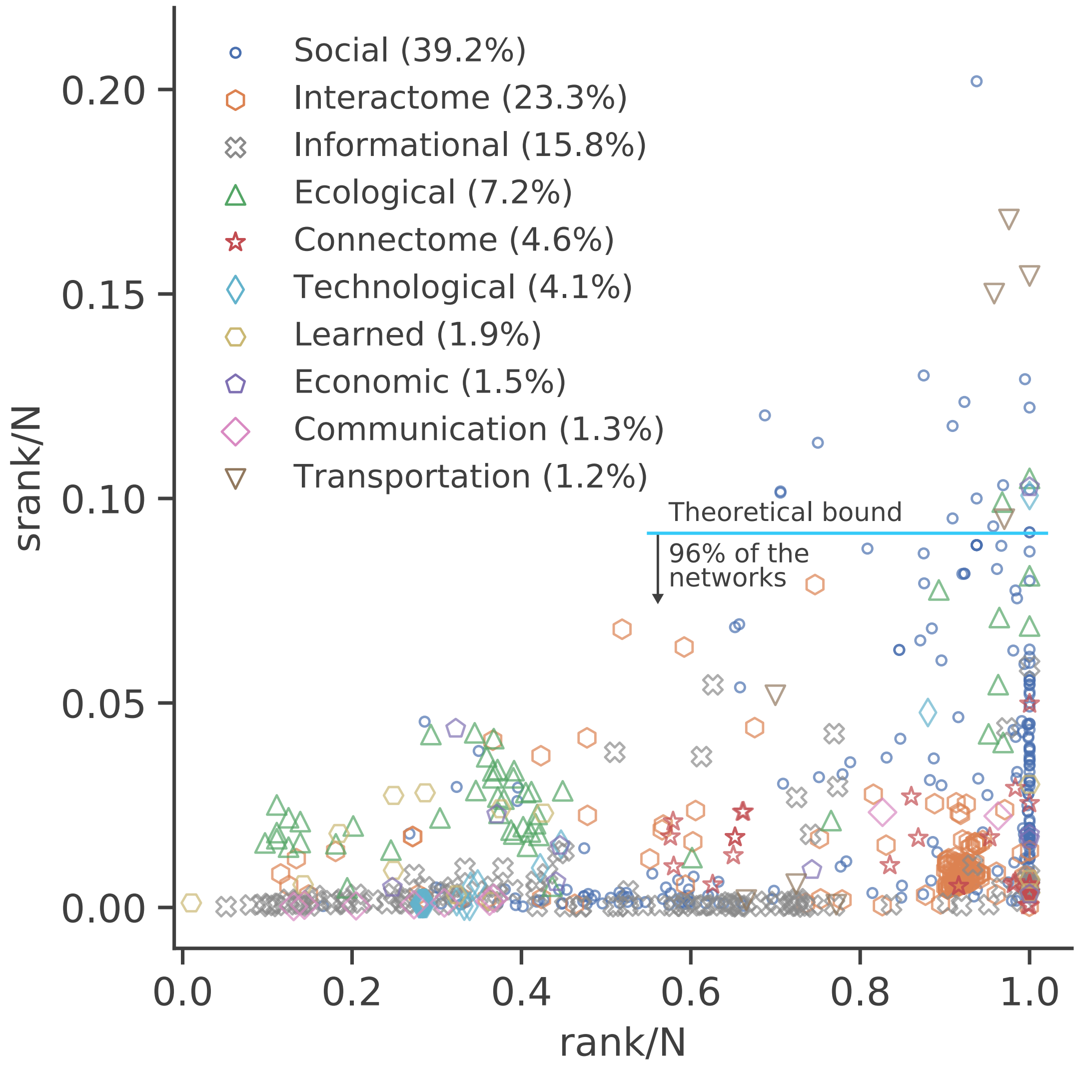
- ▷ low effective rank? ⇒ effective rank scales at most sublinearly as the number of nodes, N, goes to infitnity ($N^{1-\varepsilon}$ with $\varepsilon \in (0, 1]$)



The singular values of approx. 96% of considered networks are bounded from above by an hypergeometric envelope ⇒ sublinear effective ranks!



⇒



Workable definition of low effective rank: ∼10% of the number of nodes N

Approx. 96% of the 679 networks qualify for having a low effective rank!

Reproduction of the dynamics with increasing accuracy