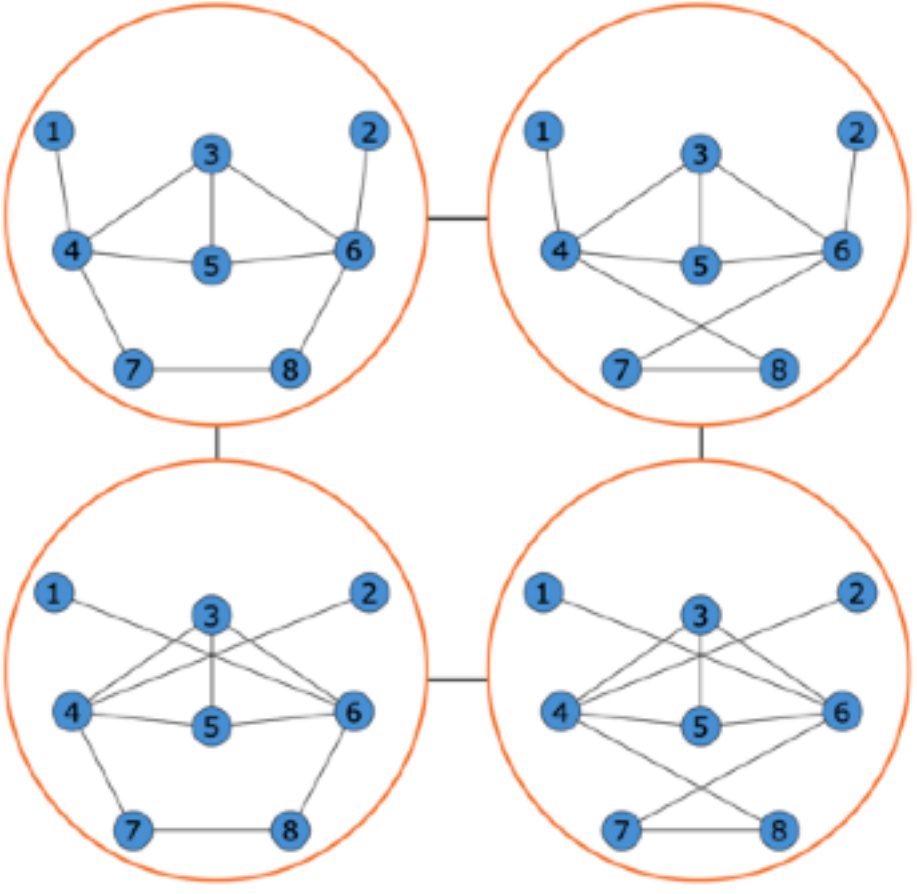
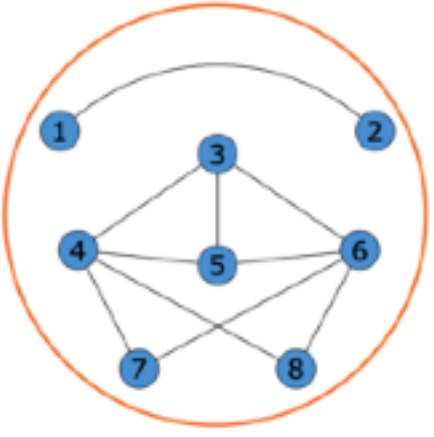
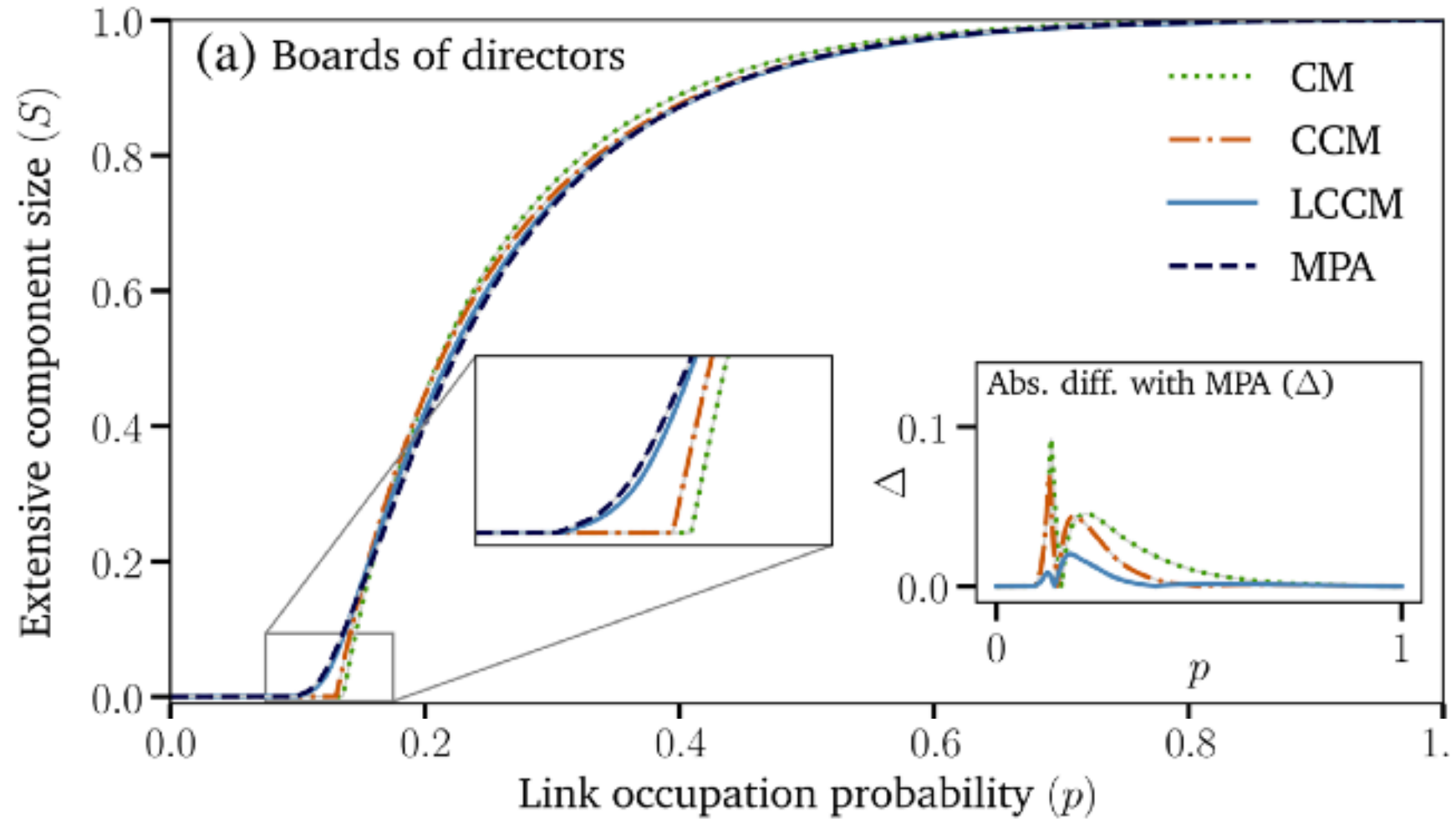
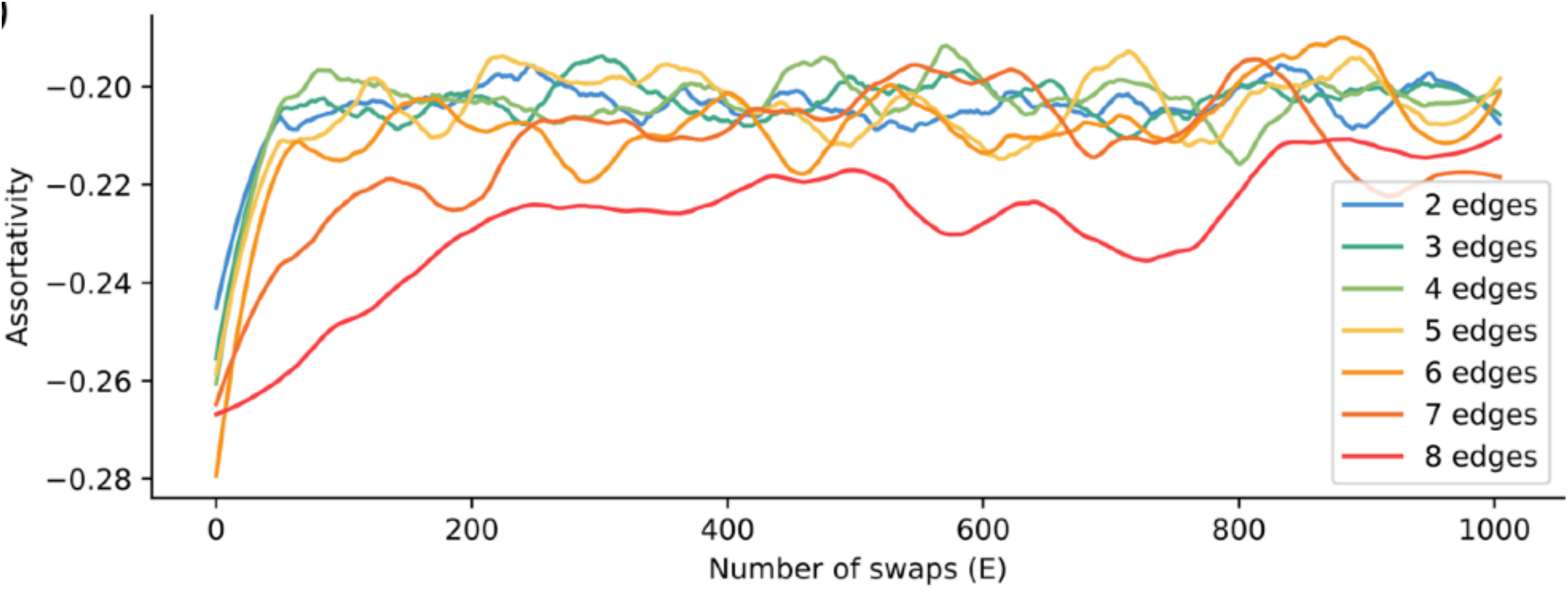
27


1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8


1
2
3
4
5
6
7
8


(a) Boards of directors
CM
CCM
LCCM
MPA
Extensive component size (S)
Link occupation probability (p)
Abs. diff. with MPA (Δ)
Δ
p
0.0
0.2
0.4
0.6
0.8
1.0
0.1


Assortativity
−0.20
−0.22
−0.24
−0.26
−0.28
0
200
400
600
800
1000
Number of swaps (E)
2 edges
3 edges
4 edges
5 edges
6 edges
7 edges
8 edges

Challenges

- Many analytical approaches (PGF, ODE) can be adapted to account for stub types, but the graphs considered are infinite in size.

- Sampling naively is easy, sampling right is tricky (ex.: uniform sampling via edge swaps).

The onion decomposition accounts for a lot when it comes to the precision of message-passing description of percolation.

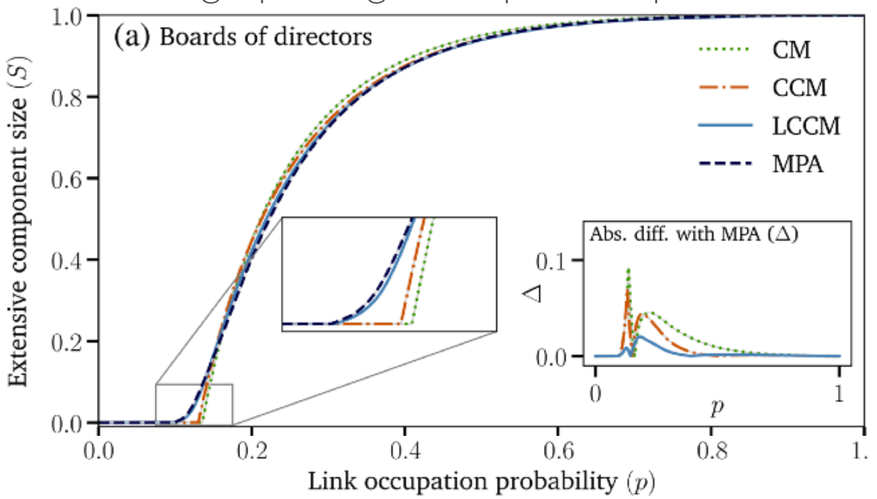
The sample space is not necessarily connected under 2-edge swaps.

Numerical evidence that the ignored graphs could be negligible (using k-edge swaps), but still no formal demonstration.

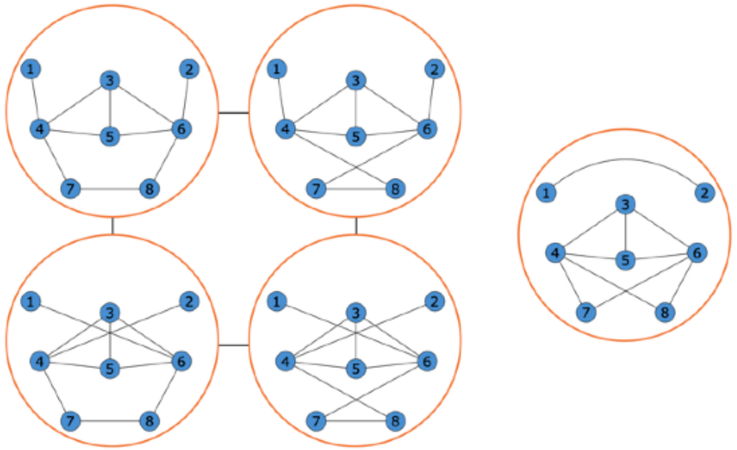
Challenges

- ▷ Many analytical approaches (PGF, ODE) can be adapted to account for stub types, but the graphs considered are infinite in size.
- ▷ Sampling naively is easy, sampling right is tricky (ex.: uniform sampling via edge swaps).

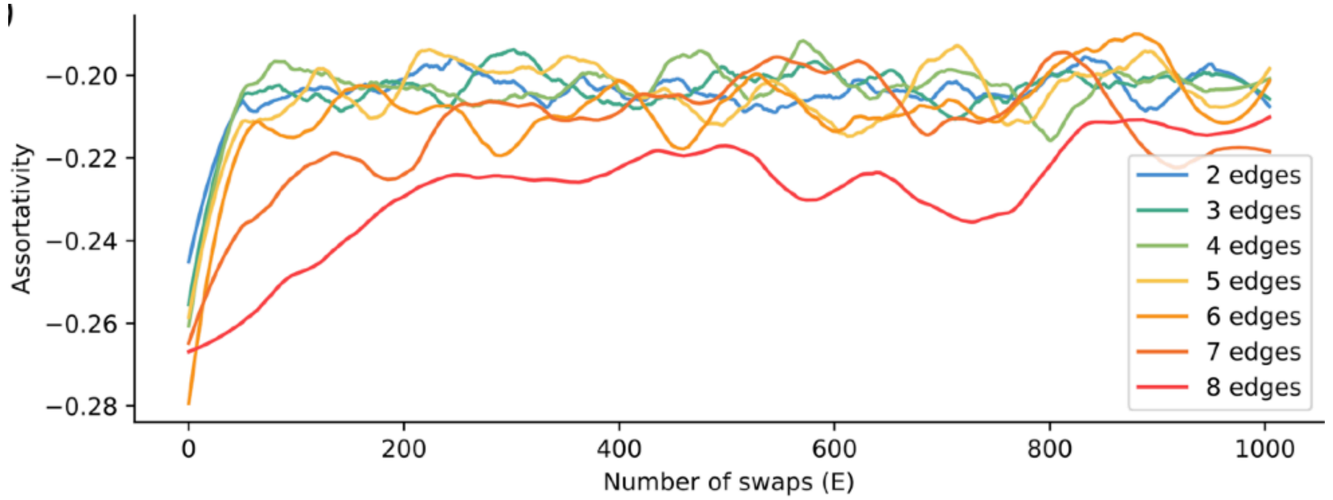
The onion decomposition accounts for a lot when it comes to the precision of message-passing description of percolation.



The sample space is not necessarily connected under 2-edge swaps.



Numerical evidence that the ignored graphs could be negligible (using k-edge swaps), but still no formal demonstration.



Main takeaways

- Local connection rules can enforce strict global topological features.
- These rules can be leveraged to design
  - new mathematical frameworks (ex.: ODEs, PGFs)
  - new sampling algorithms (ex.: edge swapping)

Multi-scale structure and topological anomaly detection via a new network statistic: The onion decomposition

Laurent Hébert-Dufresne[1], Joshua A. Grochow[1] & Antoine Allard[2]

Sci. Rep. 6, 31708 (2016)

Percolation and the Effective Structure of Complex Networks

Antoine Allard[1,2] and Laurent Hébert-Dufresne[3,1]

Phys. Rev. X 9, 011023 (2019)

Modeling critical connectivity constraints in random and empirical networks

Laurent Hébert-Dufresne,[1,2,3] Márton Pósfai,[4] and Antoine Allard[3,5,1]

arXiv:2307.03559

On the Uniform Sampling of the Configuration Model with Centrality Constraints

François Thibault,[1,2] Laurent Hébert-Dufresne,[3,4] and Antoine Allard[1,5]

arXiv:2409.20493

Laurent Hébert-Dufresne
University of Vermont

François Thibault
Université Laval

Márton Pósfai
Central European University

Joshua A. Grochow
University of Colorado, Boulder