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Level of reciprocity controlled with parameter $-1 \leq \nu \leq 1$

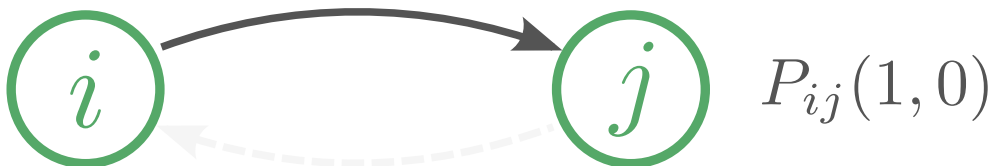
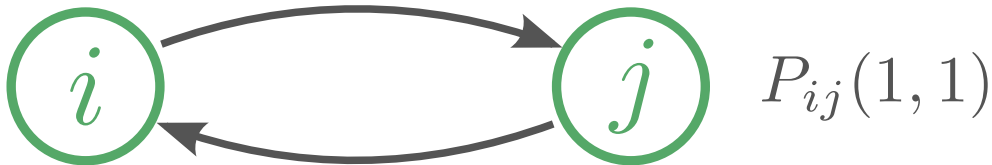
$$P_{ij}(1, 1) = \begin{cases} (1 - \nu)p_{ij}p_{ji} + \nu \min\{p_{ij}, p_{ji}\} & 0 \leq \nu \leq 1 \\ (1 + \nu)p_{ij}p_{ji} + \nu(1 - p_{ij} - p_{ji})H(p_{ij} + p_{ji} - 1) & -1 \leq \nu \leq 0 \end{cases}$$

$\nu = 1$: maximal reciprocity

$\nu = 0$: fortuitous reciprocity

$\nu = -1$: minimal reciprocity

Deliberate reciprocity



Condition 1: Preserves marginal probabilities

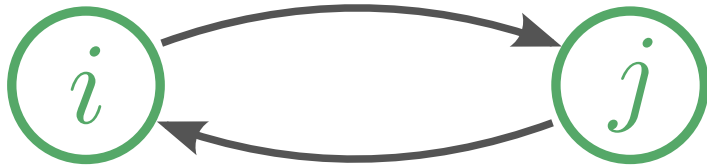
$$P_{ij}(1, 0) + P_{ij}(1, 1) = p_{ij}$$

$$P_{ij}(0, 1) + P_{ij}(1, 1) = p_{ji}$$

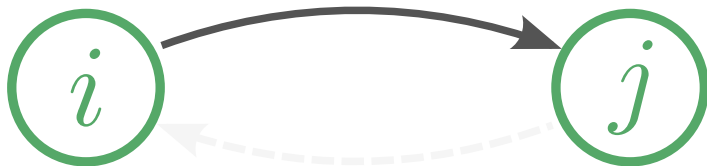
Condition 2: Normalized

$$\sum_{a_{ij}=0}^1 \sum_{a_{ji}=0}^1 P_{ij}(a_{ij}, a_{ji}) = 1$$

Fortuitous reciprocity



$$p_{ij}p_{ji}$$



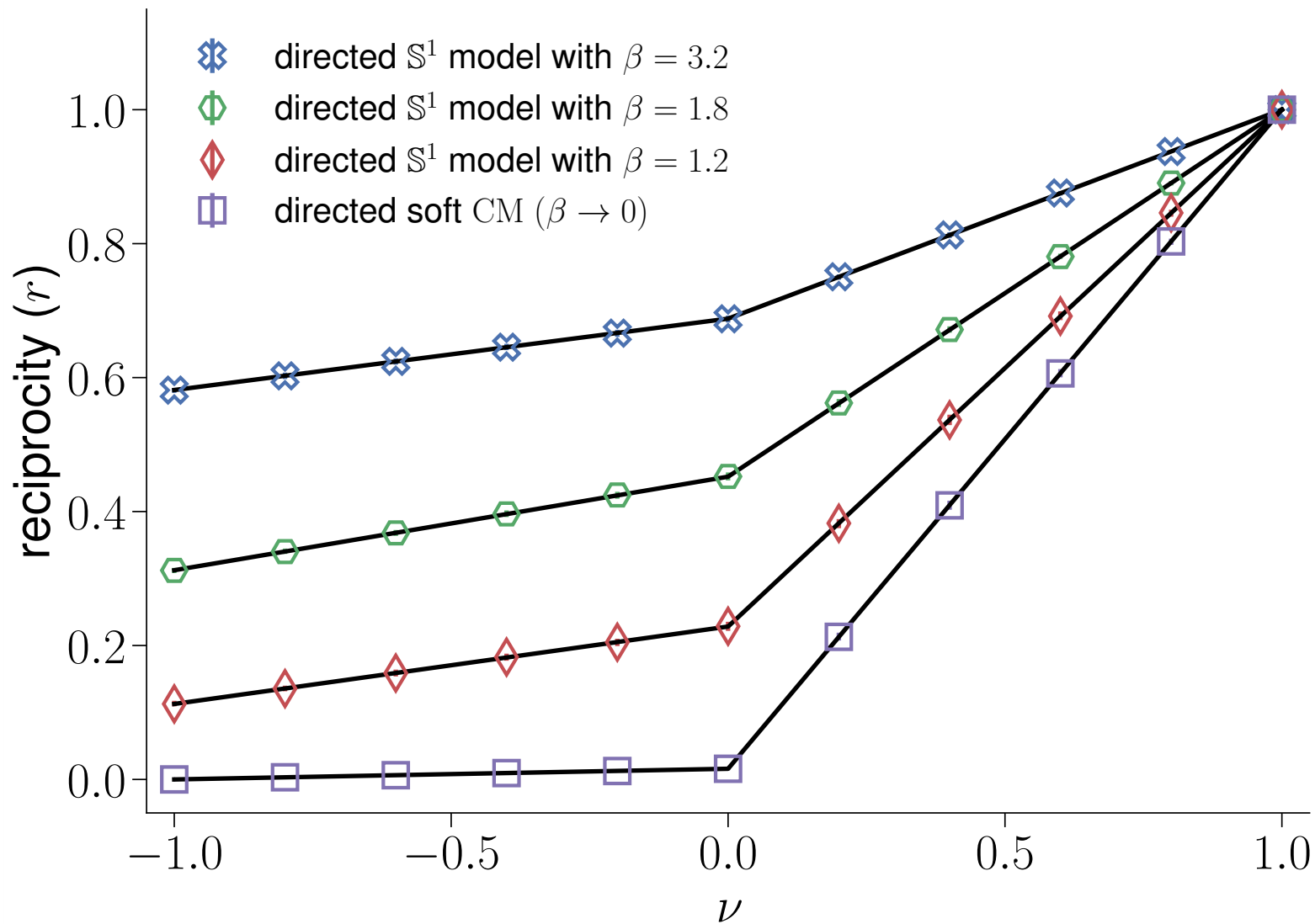
$$p_{ij}(1 - p_{ji})$$



$$(1 - p_{ij})p_{ji}$$



$$(1 - p_{ij})(1 - p_{ji})$$

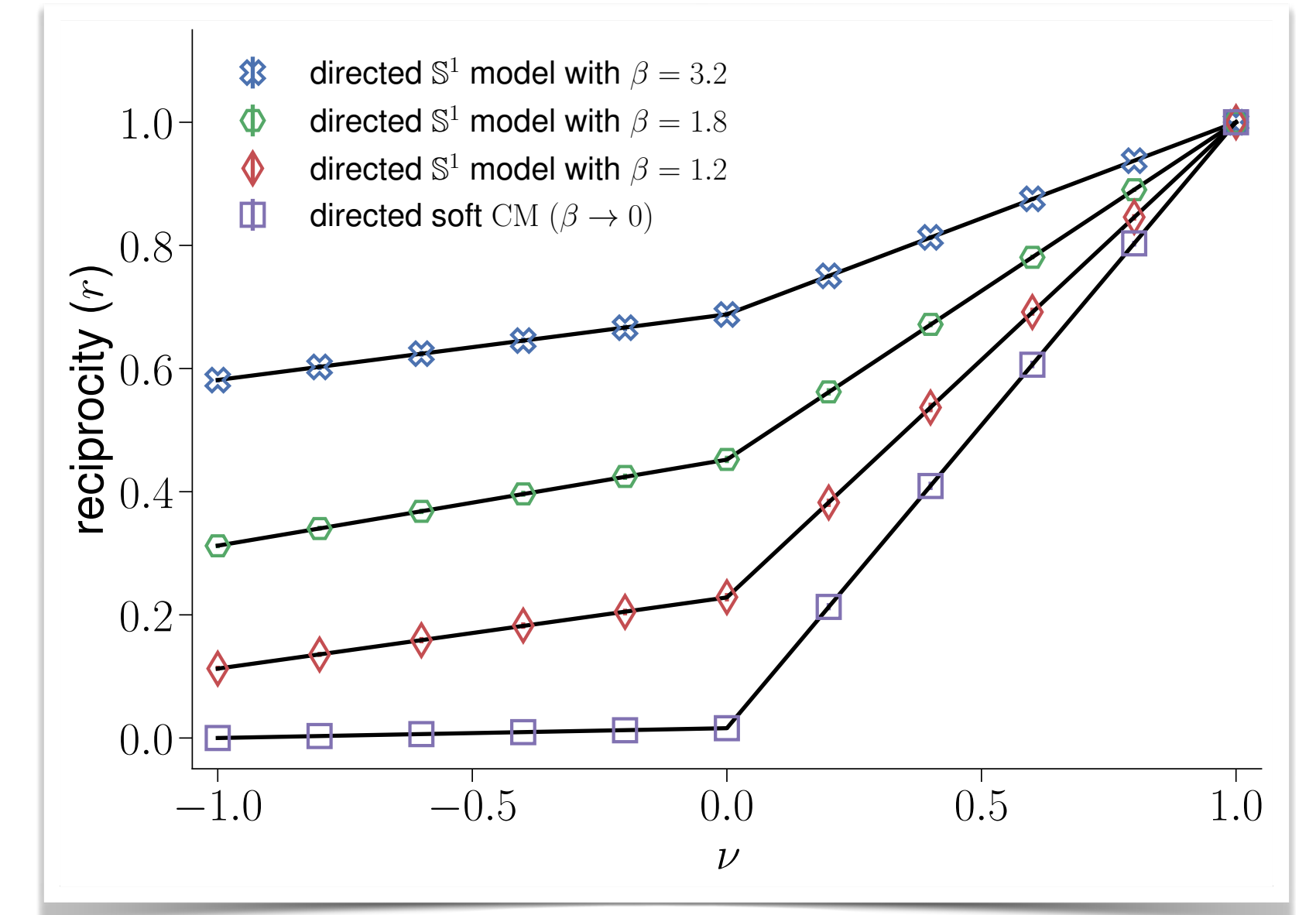
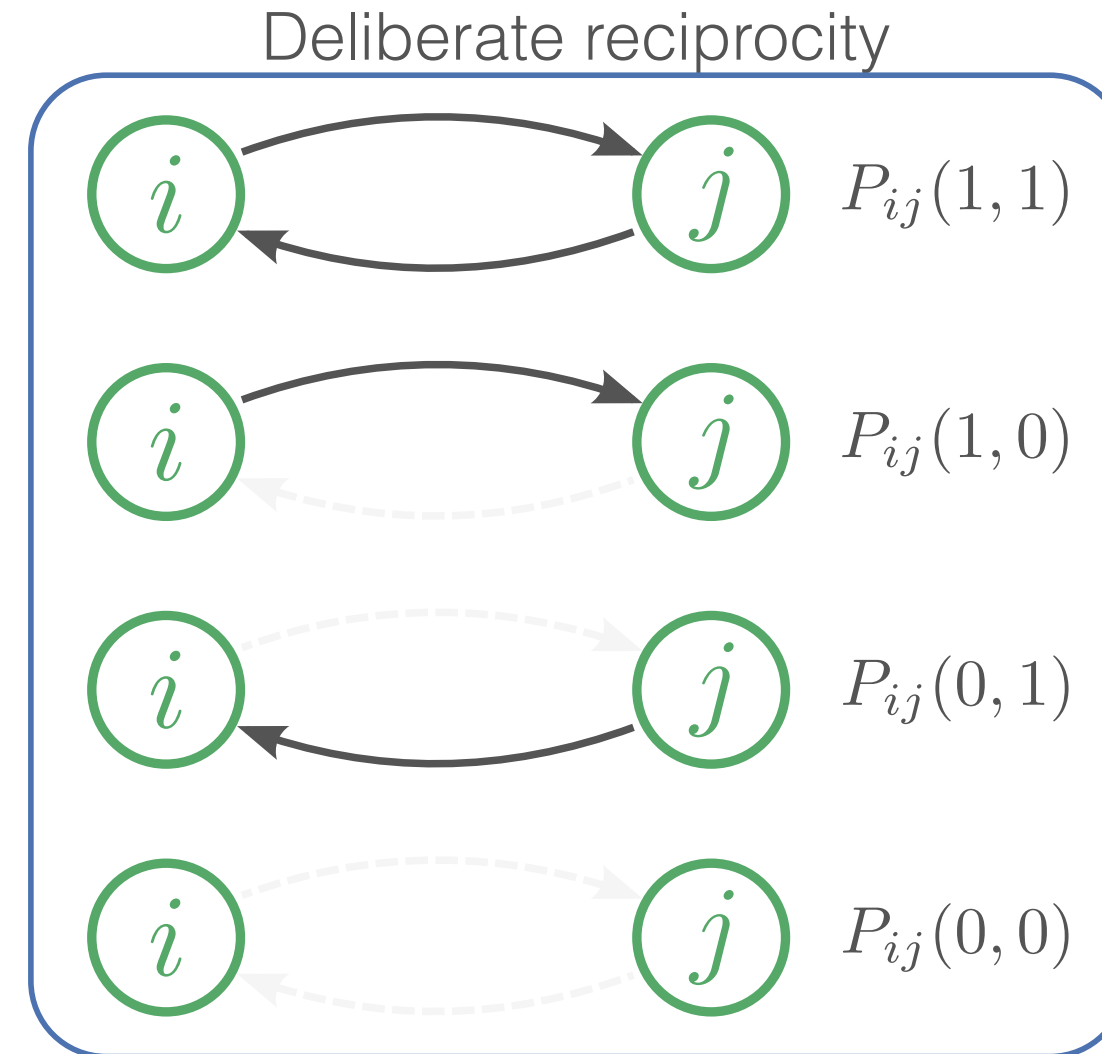
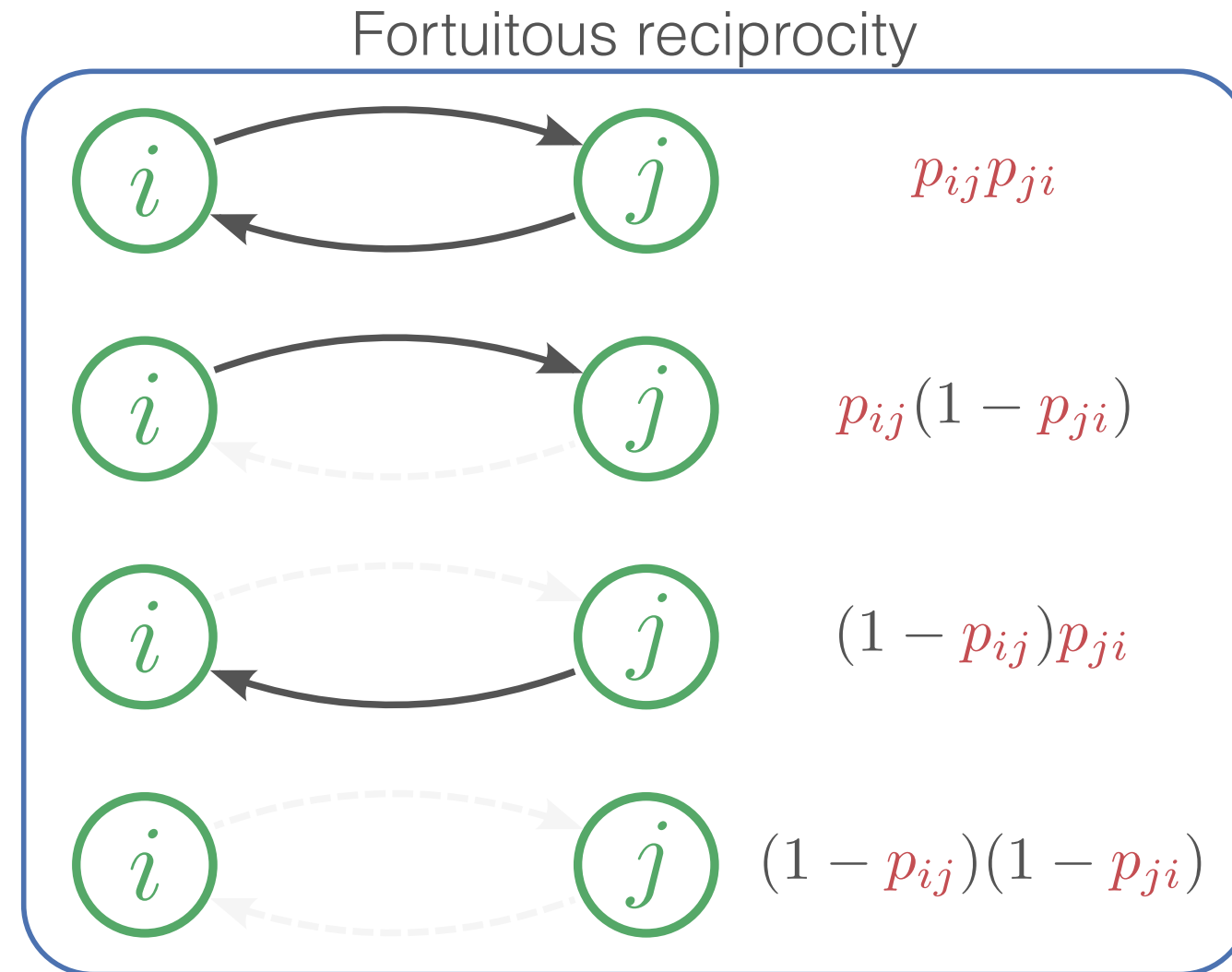


Deliberate reciprocity in random directed networks

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j .

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Fitting the directed \mathbb{S}^1 model to real networks

Inputs from a real network :

1. joint degree distribution $P(k^{\text{in}}, k^{\text{out}})$
2. reciprocity r
3. density of triangles

Assuming uniform angular positions for nodes,

1. infer $(\kappa^{\text{in}}, \kappa^{\text{out}})$ to replicate $P(k^{\text{in}}, k^{\text{out}})$ on average (analytical)
2. set ν to reproduce r (analytical)
3. adjust β to recreate the density of triangles (semi-analytical)

Generate a sample of random directed networks :

1. assign angular positions randomly
2. draw directed links using the probabilities defined by the framework for deliberate reciprocity