

The rank of adjacent matrices

$$\mathbf{W} = \sum_{i=1}^r \sigma_i \mathbf{u}_i \mathbf{v}_i^\top = \sigma_1 \begin{bmatrix} | \\ \mathbf{u}_1 \\ | \\ \vdots \end{bmatrix} \left[\frac{\mathbf{v}_1^\top}{\mathbf{v}_2^\top} \frac{\mathbf{v}_2^\top}{\mathbf{v}_3^\top} \dots \right] + \sigma_2 \begin{bmatrix} | \\ \mathbf{u}_2 \\ | \\ \vdots \end{bmatrix} \left[\frac{\mathbf{v}_1^\top}{\mathbf{v}_2^\top} \frac{\mathbf{v}_2^\top}{\mathbf{v}_3^\top} \dots \right] + \dots$$

Singular value decomposition (SVD)



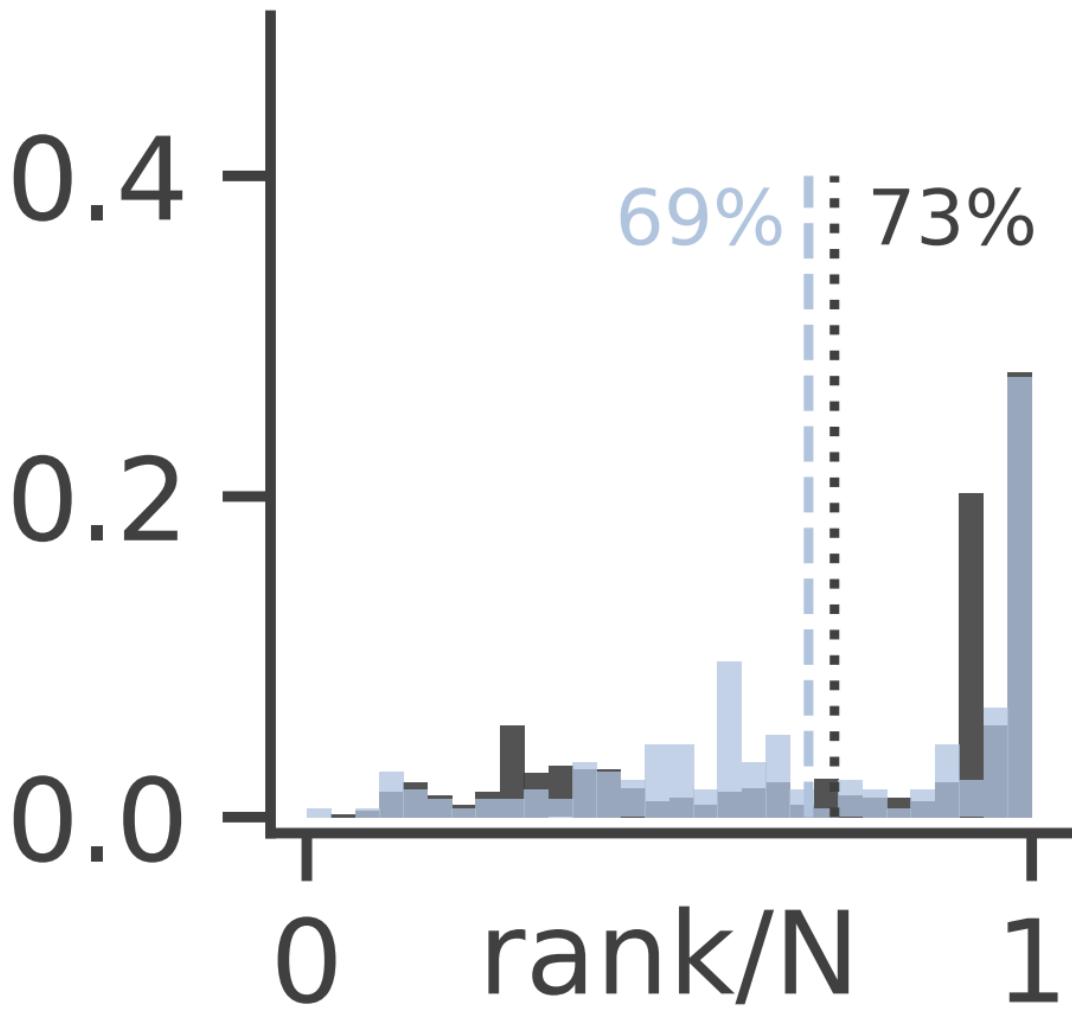
W
Real matrix

=
 U
Orthogonal matrix

σ_1
 \dots
 σ_r
0
 \dots
0
 V^T
Diagonal matrix
Orthogonal matrix

Rank r of a matrix

- ▷ number of linearly independent rows/columns
- ▷ dimension of the vector space generated by its rows/columns
- ▷ number of nonzero singular values



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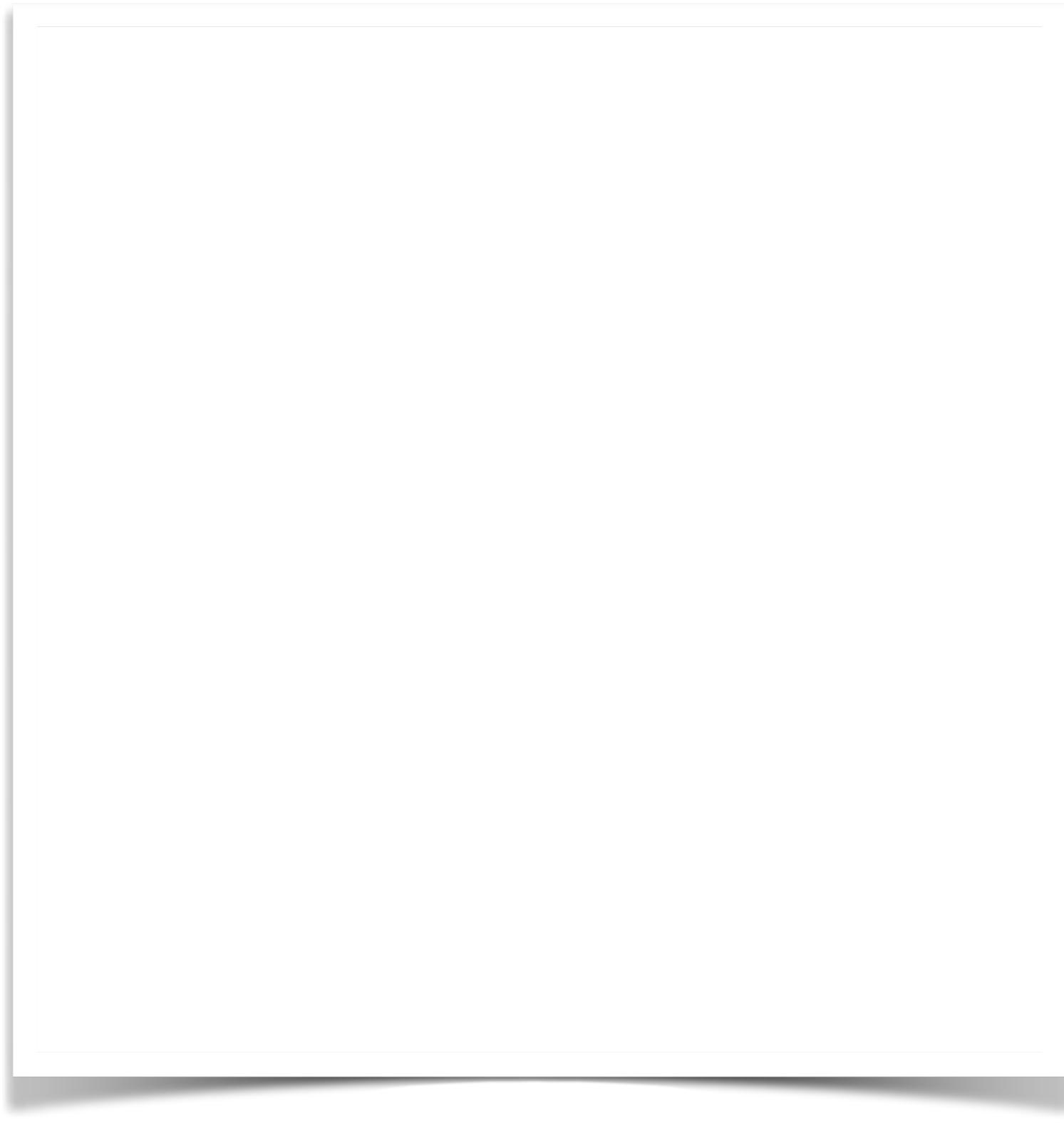
Unweighted networks

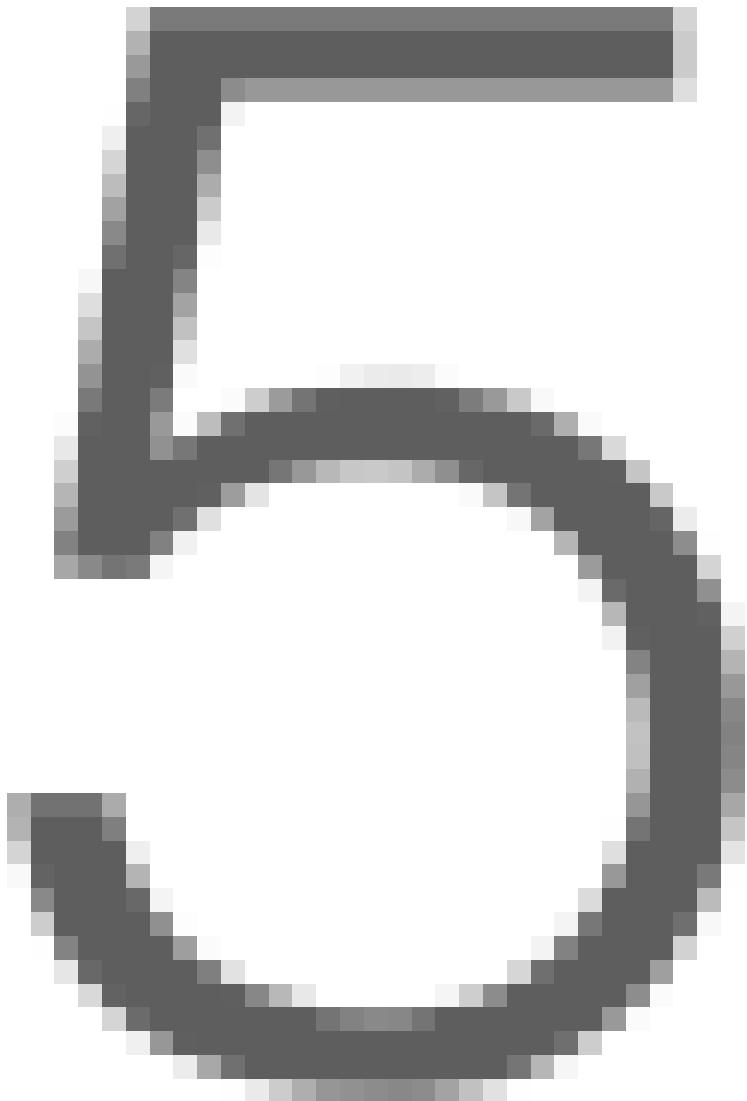
Weighted networks

Results for 679 empirical networks (502 unweighted networks and 177 weighted networks) downloaded from Netzschluder.

Full-rank (or almost full rank) adjacency

matrices are very common!

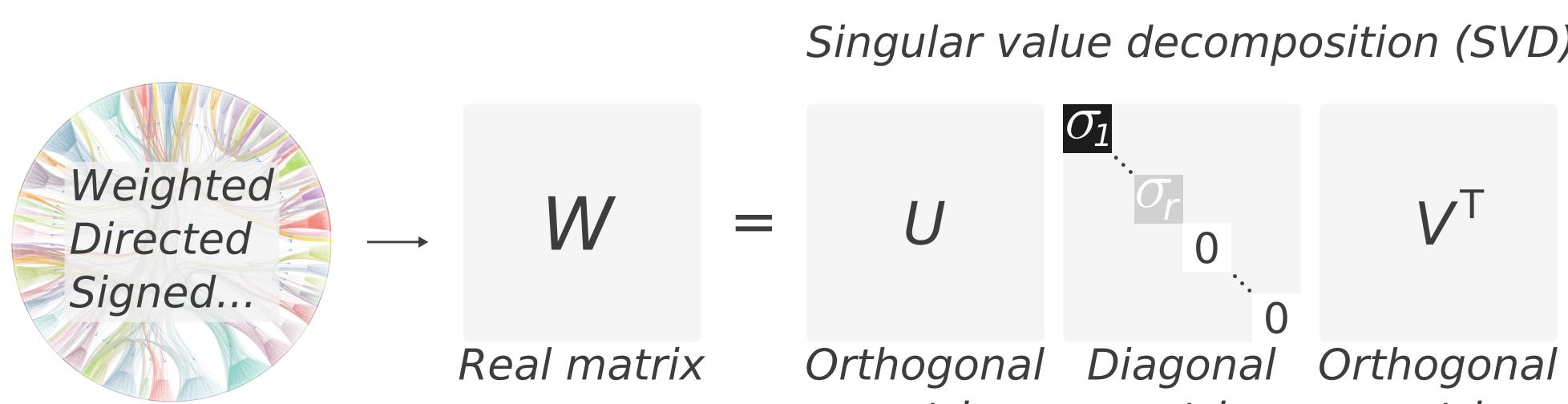




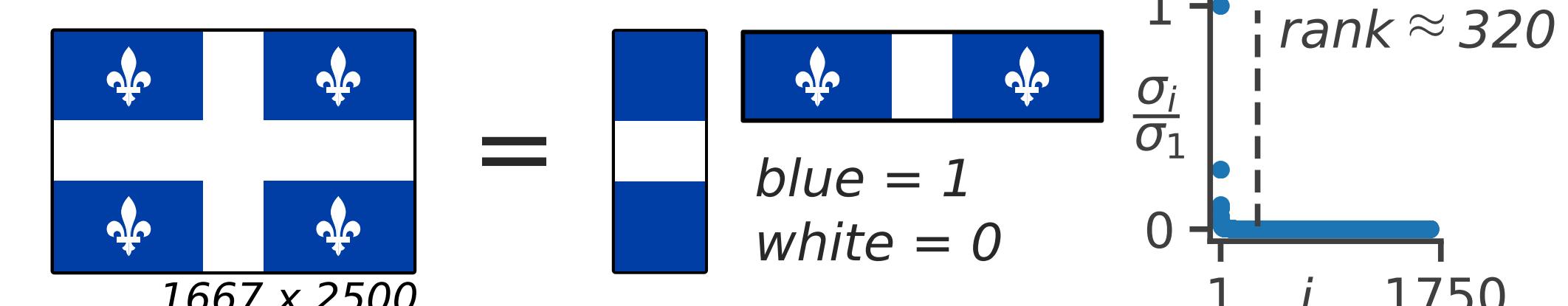
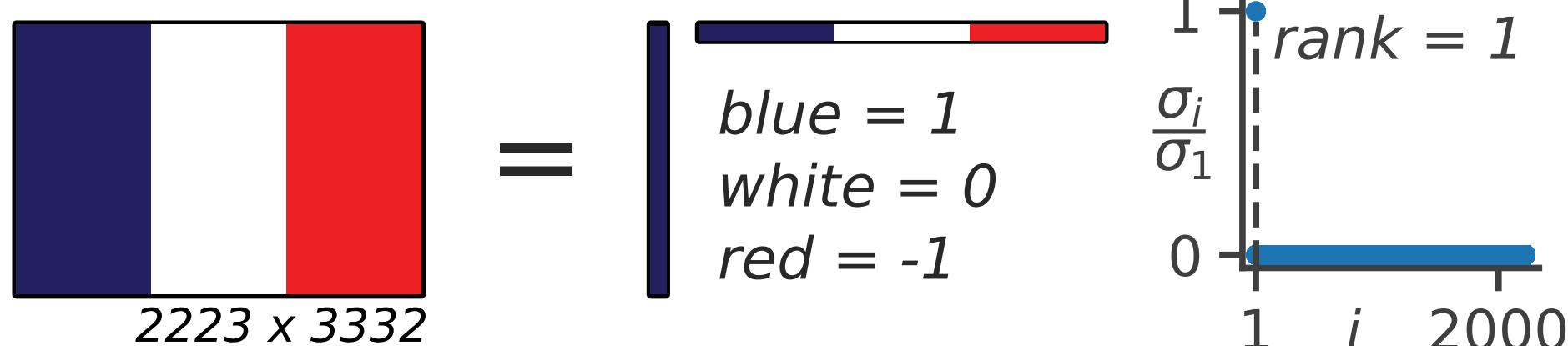
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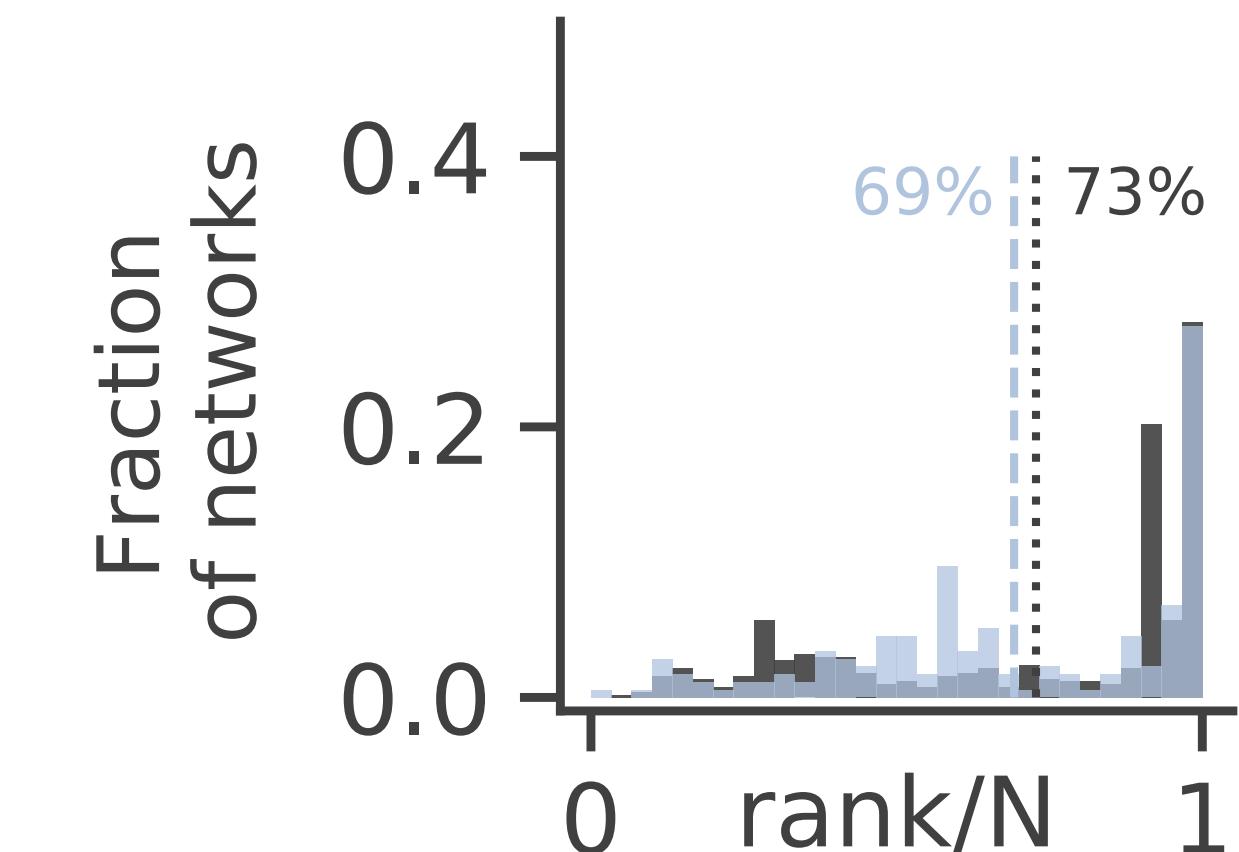


$$W = \sum_{i=1}^r \sigma_i u_i v_i^\top = \sigma_1 \begin{bmatrix} | \\ u_1 \\ | \end{bmatrix} \left[\quad v_1^\top \quad \right] + \sigma_2 \begin{bmatrix} | \\ u_2 \\ | \end{bmatrix} \left[\quad v_2^\top \quad \right] + \sigma_3 \begin{bmatrix} | \\ u_3 \\ | \end{bmatrix} \left[\quad v_3^\top \quad \right] + \dots$$



Full-rank (or almost full rank) adjacency matrices are very common!

■ Unweighted networks
■ Weighted networks



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The effective ranks of adjacency matrices

$$\mathbf{W} = \sum_{i=1}^{\textcolor{red}{r}} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top \simeq \sum_{i=1}^{\textcolor{red}{n}} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top$$