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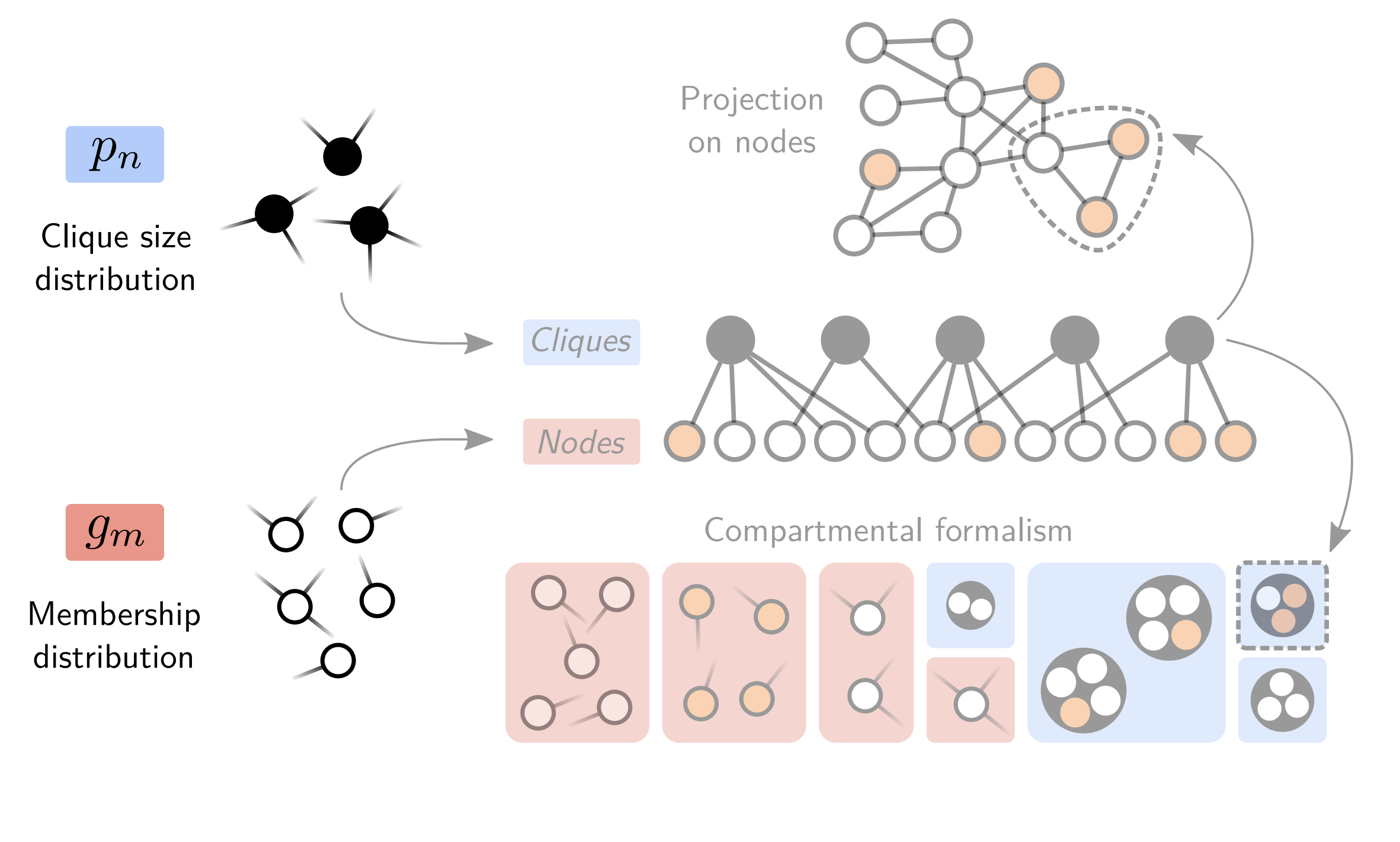
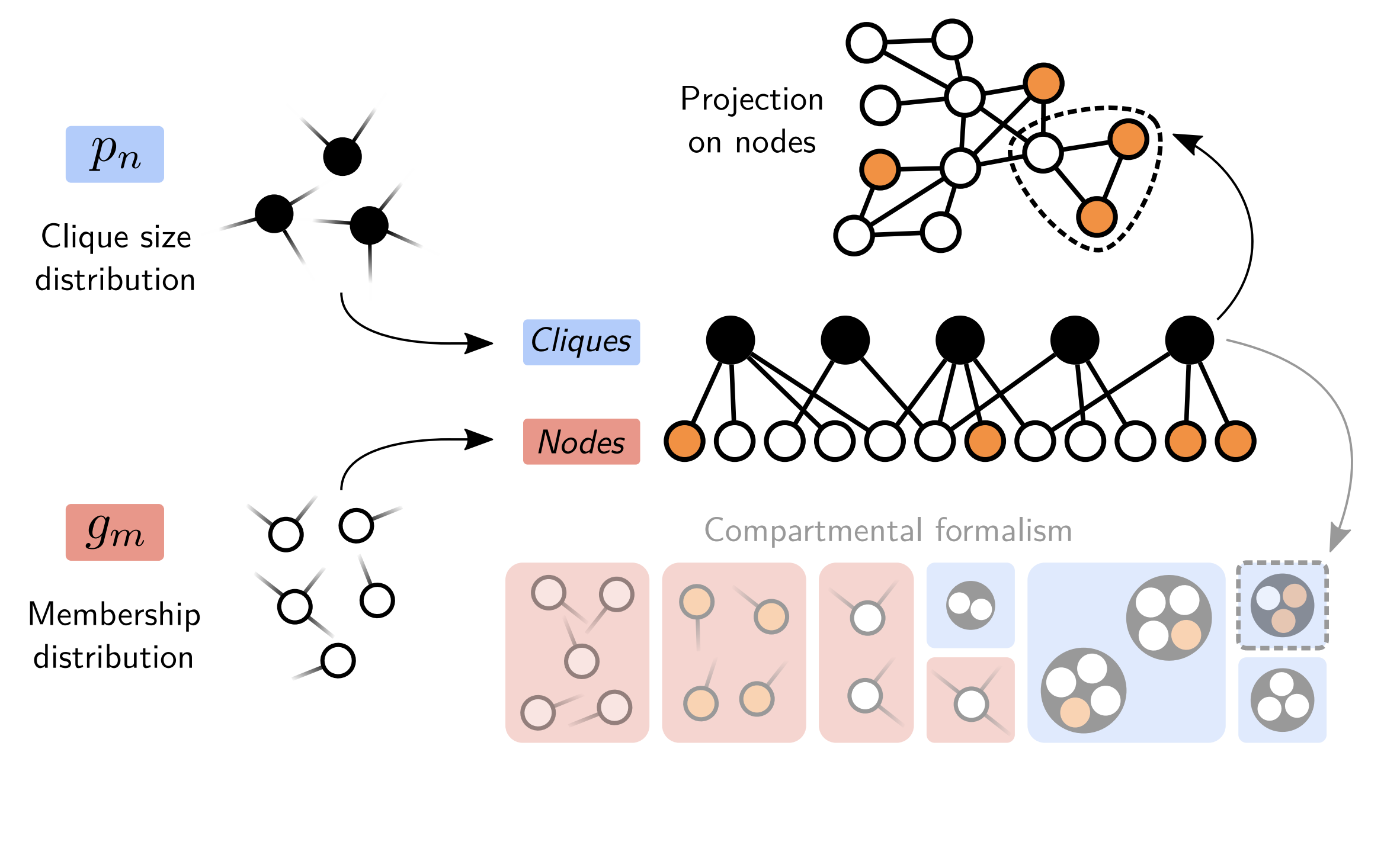

p_n
Clique size distribution
g_m
Membership distribution
Cliques
Nodes
Projection on nodes
Compartmental formalism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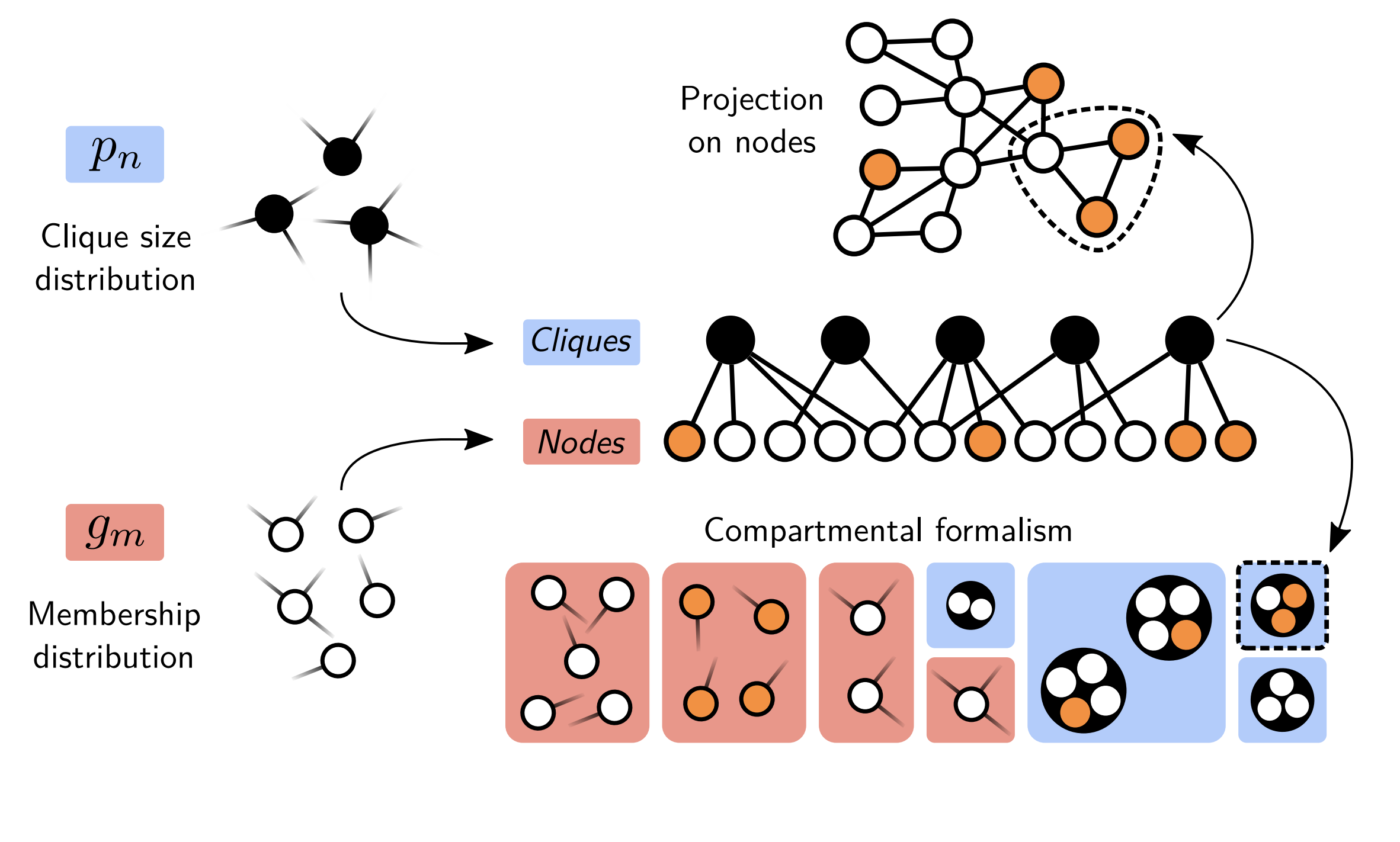
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M.E.J. Newman, *Properties of highly clustered networks*, Phys. Rev. E 68, 026121 (2003)

μ : recovery rate

β : *weight* associated to transmission

$\Theta_{n,i,\beta}$: transmission rate in a group of size n, with i infected nodes, and weight β

Dynamics at the groups level

$$\begin{aligned}\frac{\mathrm{d}G_{n,i}}{\mathrm{d}t} &= \mu(i+1)G_{n,i+1} - \mu i G_{n,i} \\ &+ (n-i+1)\Theta_{n,i-1,\beta}G_{n,i-1} - (n-i)\Theta_{n,i,\beta}G_{n,i} \\ &+ (n-i+1)\rho G_{n,i-1} - (n-i)\rho G_{n,i}\end{aligned}$$

Dynamics at the nodes level

$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \mu(g_m - S_m) - m r S_m$$

Mean-field quantities

$$\rho(t) = r(t)\frac{\sum_m m(m-1)S_m}{\sum_m mS_m}\ ; \qquad r(t) = \frac{\sum_{n,i}(n-i)\Theta_{n,i,\beta}G_{n,i}}{\sum_{n,i}(n-i)G_{n,i}}$$

Global prevalence

$$I(t) = \sum_m [g_m - S_m(t)]$$

The model and its mathematical description

$r(t)$: mean infection rate from a random group to which a random node belongs

$\rho(t)$: mean infection rate from all *other* groups to which a random node in a random group belongs

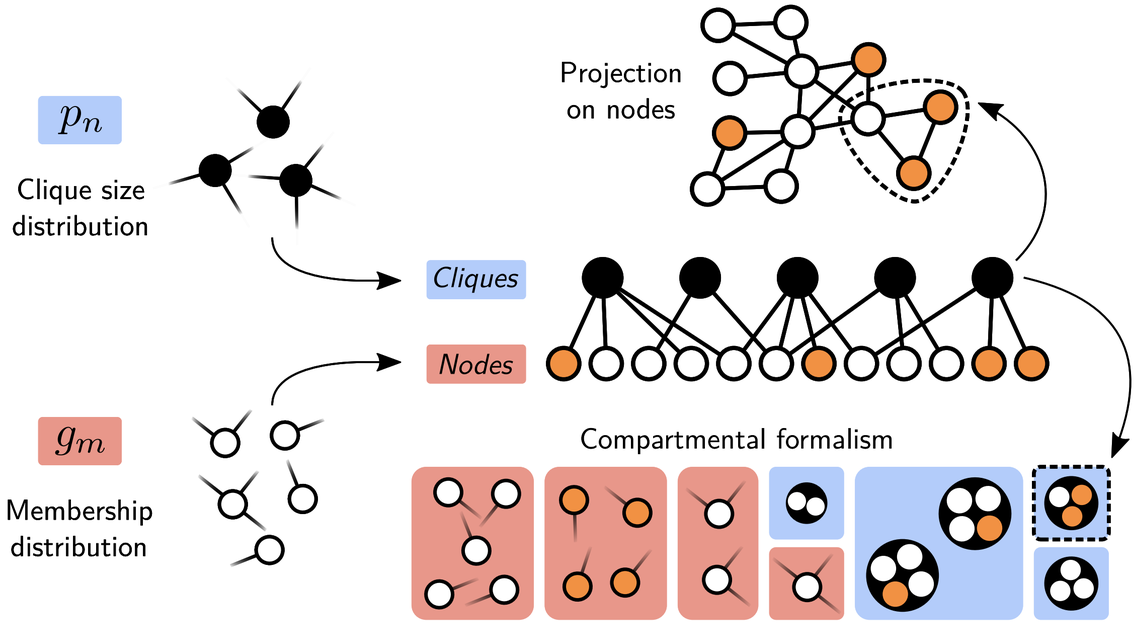
S_m : fraction of nodes with membership m and that are susceptible

$G_{n,i}$: fraction of group of size n and that contain i infectious nodes

g_m : fraction of nodes with membership m

p_n : fraction of groups of size n

The model and its mathematical description



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The model and its mathematical description

Stationary state

$$
\begin{aligned}
S_m^* &= \frac{g_m}{1+mr} \\
\mu(i+1)G_{n,i,\beta}^* &= \left[\mu i + (n-i)(\Theta_{n,i,\beta}+\rho)\right] G_{n,i,\beta}^* \\
&\quad - (n-i+1)(\Theta_{n,i-1,\beta}+\rho) G_{n,i-1,\beta}^*
\end{aligned}
$$

Epidemic threshold

$$
\left.\frac{\mathrm{d}F}{\mathrm{d}\rho}\right|_{\rho\to 0} > 1
$$

where

$$
F(\rho) \equiv r(\rho)\frac{\sum_m m(m-1)S_m(\rho)}{\sum_m mS_m(\rho)}
$$