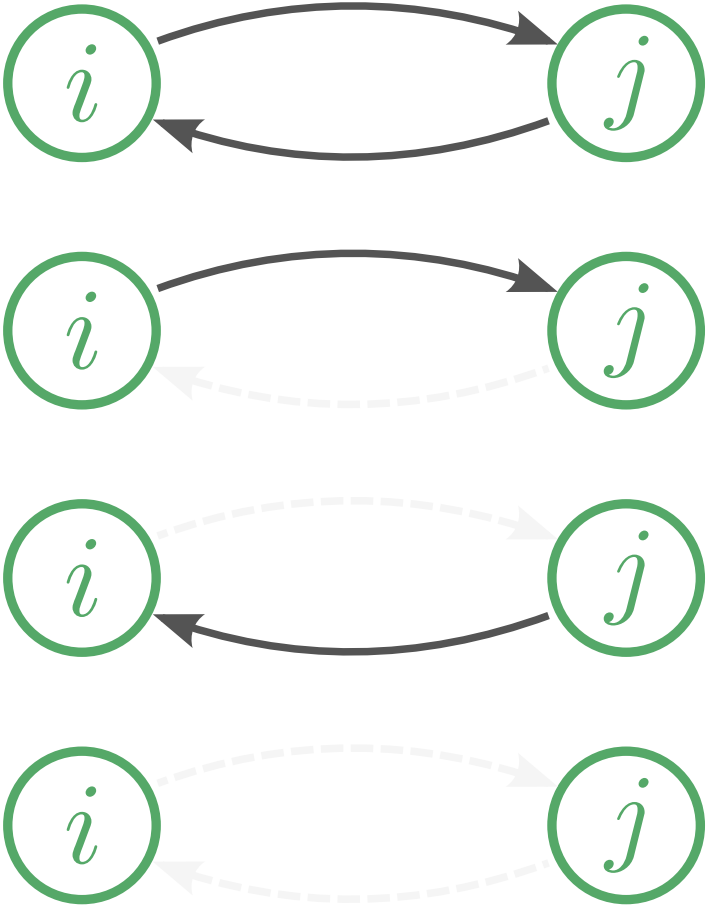
14

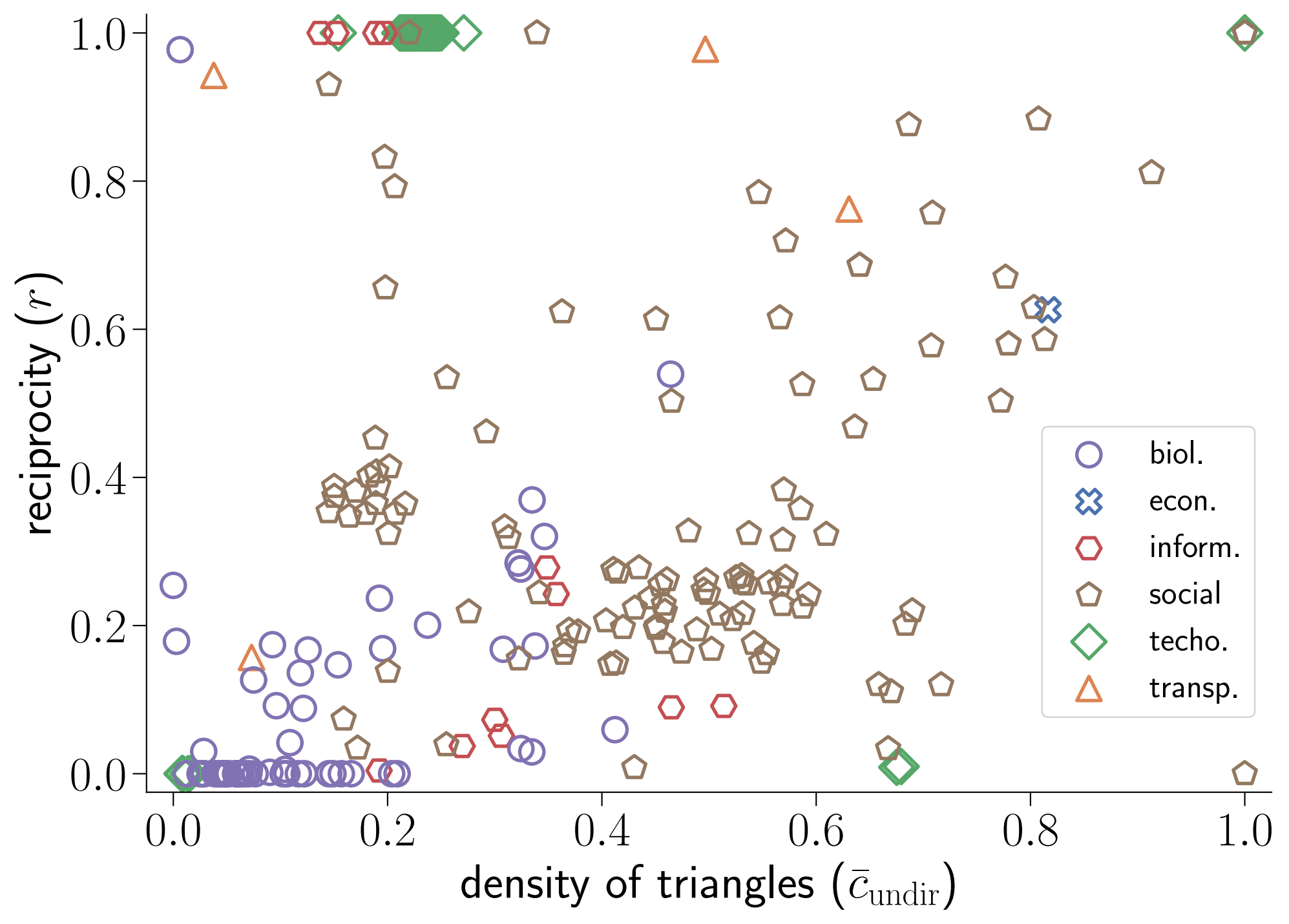
Fortuitous reciprocity



Reciprocity in the directed $\mathbb{S}^1$ model

A reciprocal connection between node i and node j occurs with probability $p_{ij}p_{ji}$.

292 network datasets downloaded from Netzschleuder (networks.skewed.de).


1.0
0.8
0.6
0.4
0.2
0.0
reciprocity (r)
0.0
0.2
0.4
0.6
0.8
1.0
density of triangles ($\bar{c}_{\text{undir}}$)
biol.
econ.
inform.
social
techo.
transp.

A realistic model will need to go beyond fortuitous reciprocity.

κ^{in} : in-degree

κ^{out} : out-degree

β : density of triangles

$$r = \mathbb{E}\left[\frac{L^{\leftrightarrow}}{L}\right] = \mathbb{E}\left[\frac{k^{\leftrightarrow}}{k^{\text{out}}}\right] \approx \frac{\mathbb{E}\left[k^{\leftrightarrow}\right]}{\mathbb{E}\left[k^{\text{out}}\right]}$$

$$\simeq \iiint \frac{\kappa_i^{\text{out}} \kappa_j^{\text{in}}}{\langle \kappa \rangle^2} \frac{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}}\right)^{\beta-1}}{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}}\right)^{\beta}}$$

$$\times \rho(\kappa_i^{\text{in}}, \kappa_i^{\text{out}}) \rho(\kappa_j^{\text{in}}, \kappa_j^{\text{out}}) d\kappa_i^{\text{in}} \kappa_i^{\text{out}} d\kappa_j^{\text{in}} \kappa_j^{\text{out}}$$

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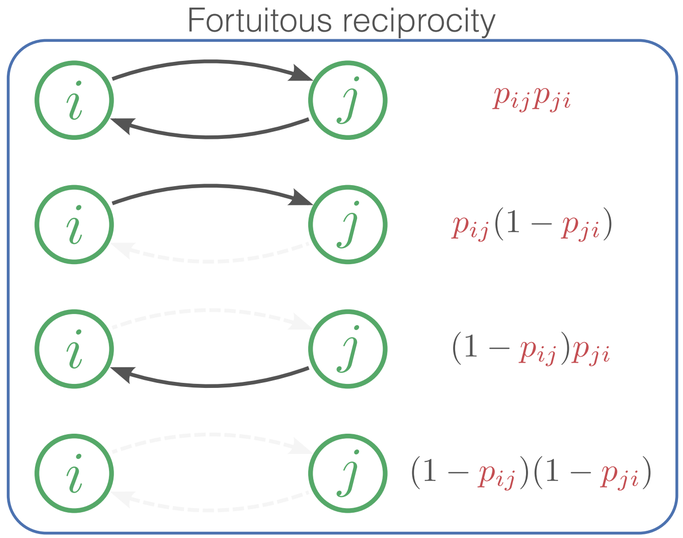

$$r = \mathbb{E}\left[\frac{L^{\leftrightarrow}}{L}\right] = \mathbb{E}\left[\frac{k^{\leftrightarrow}}{k^{\text{out}}}\right] \approx \frac{\mathbb{E}\left[k^{\leftrightarrow}\right]}{\mathbb{E}\left[k^{\text{out}}\right]}$$

$$\simeq \iiiint \frac{\kappa_i^{\text{out}}\kappa_j^{\text{in}}}{\langle\kappa\rangle^2} \frac{1-\left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}}\frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}}\right)^{\beta-1}}{1-\left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}}\frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}}\right)^{\beta}}$$

$$\times\, \rho(\kappa_i^{\text{in}}, \kappa_i^{\text{out}})\rho(\kappa_j^{\text{in}}, \kappa_j^{\text{out}}) d\kappa_i^{\text{in}}\kappa_i^{\text{out}} d\kappa_j^{\text{in}}\kappa_j^{\text{out}}$$

κ^{in} : in-degree

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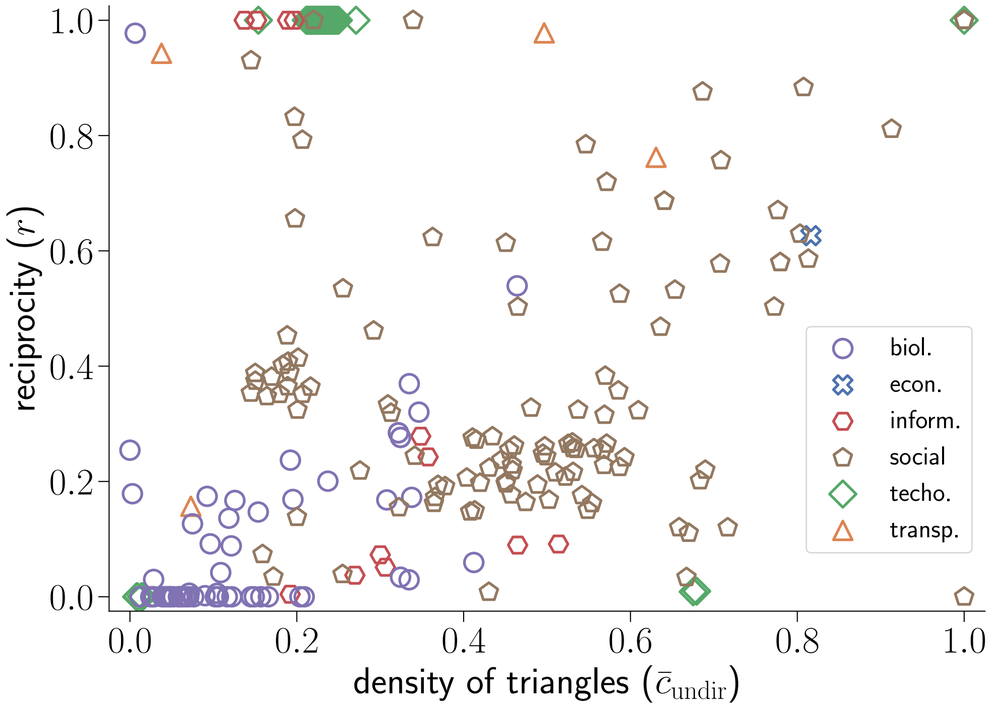
β : density of triangles



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A realistic model will need to go beyond fortuitous reciprocity.

Deliberate reciprocity in random directed networks

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j.