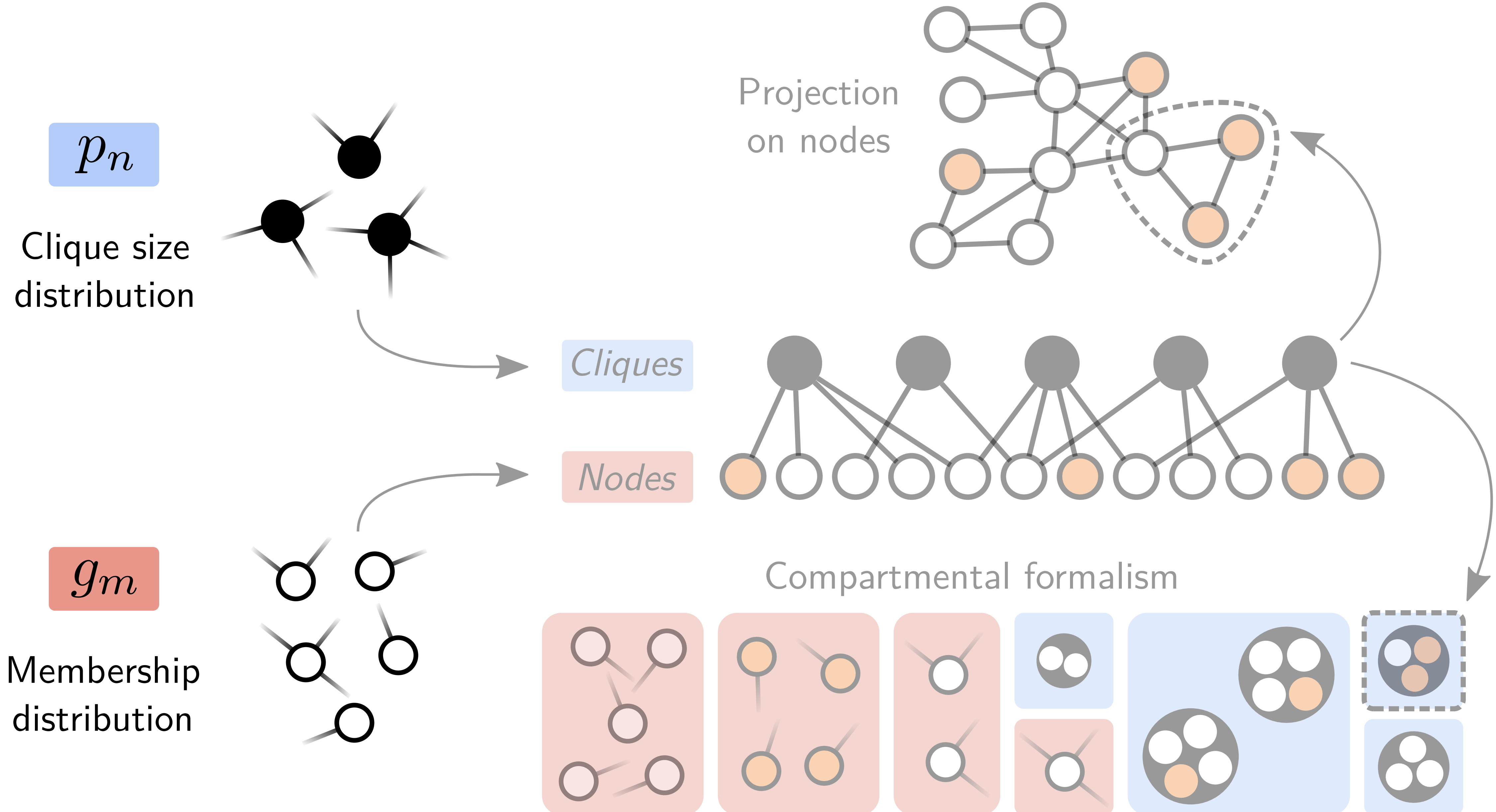
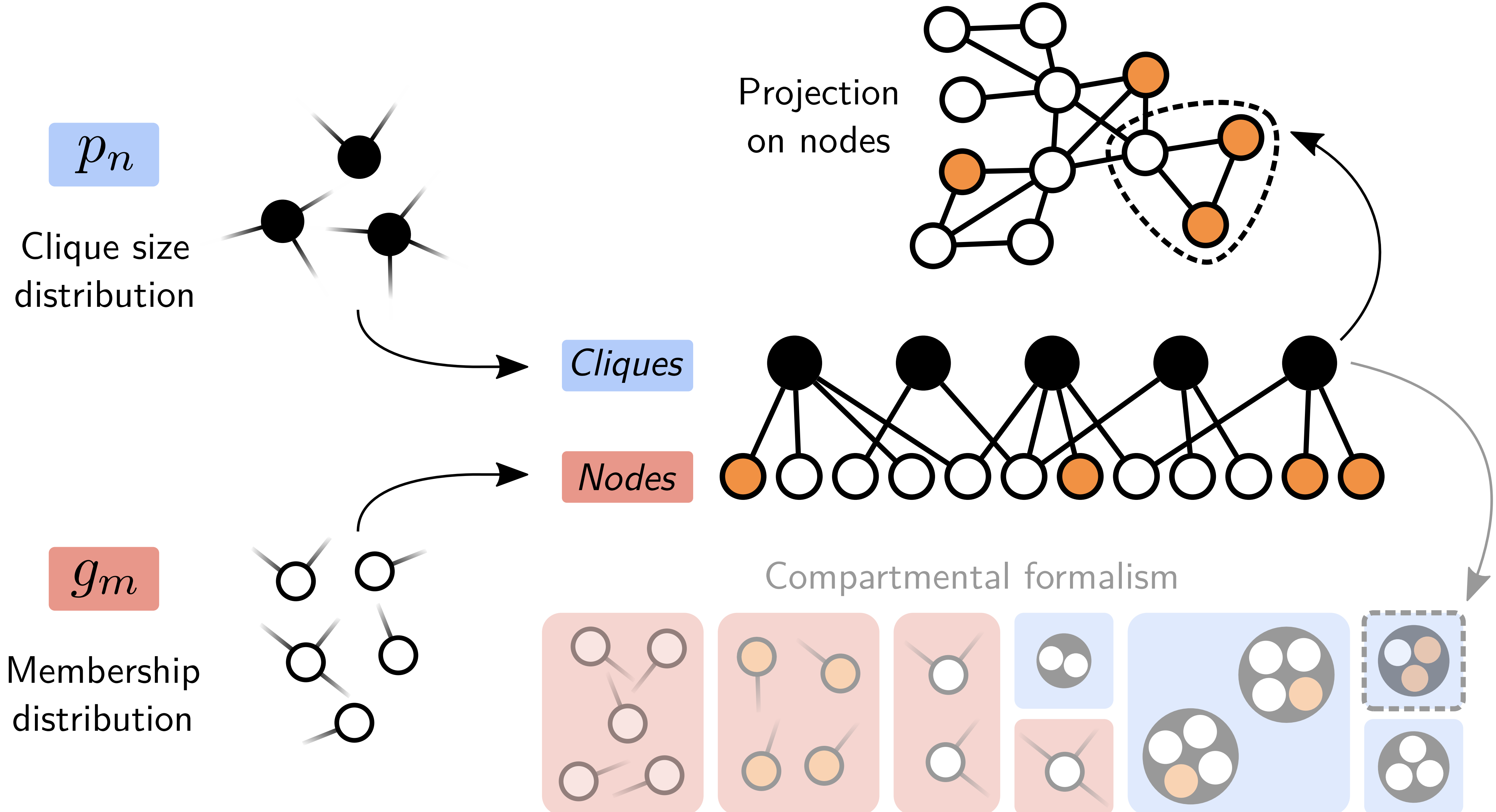
5



p_n
Clique size distribution
g_m
Membership distribution
Cliques
Nodes
Projection on nodes
Compartmental formalism



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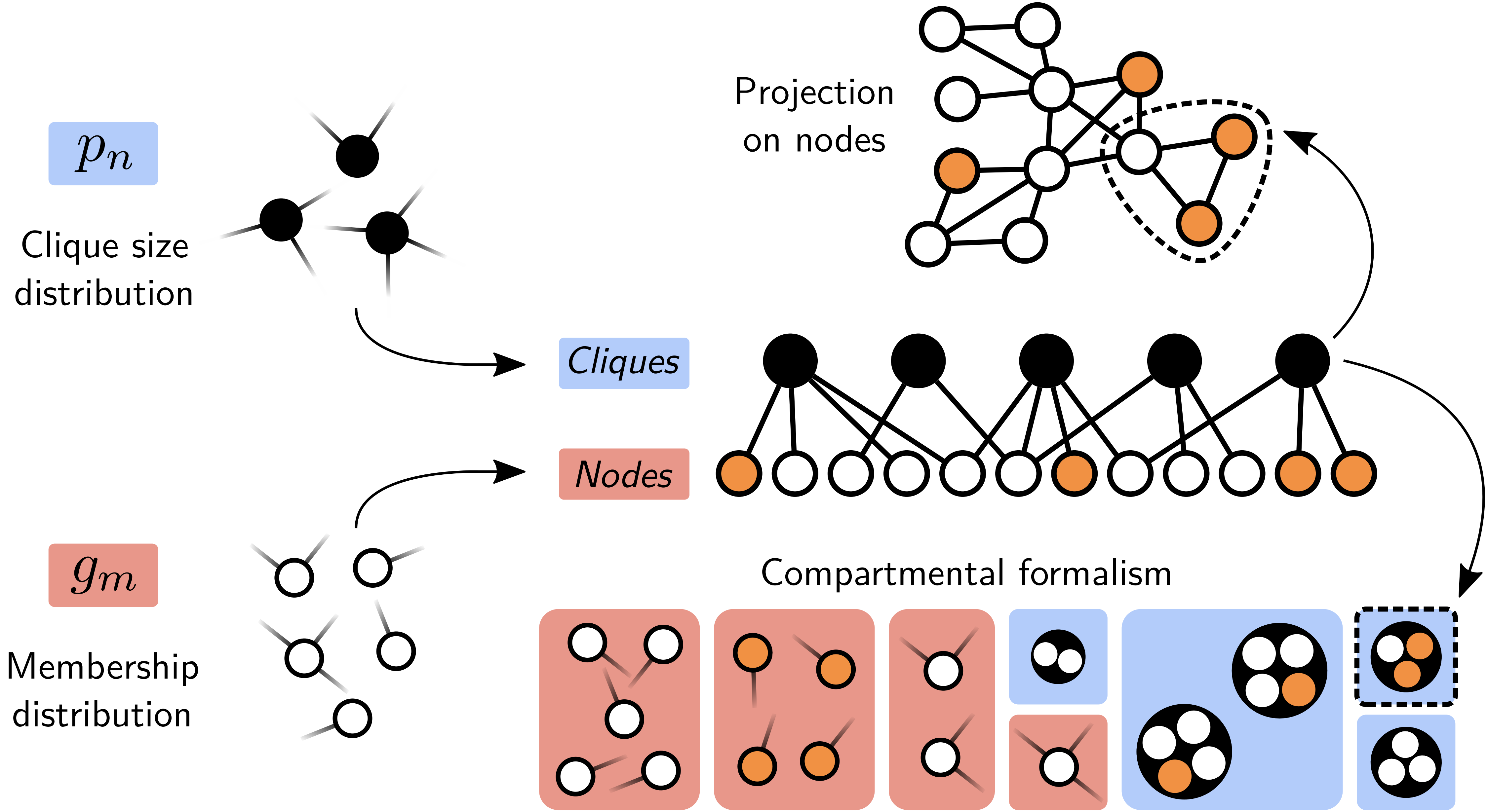

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M.E.J. Newman, *Properties of highly clustered networks*, Phys. Rev. E 68, 026121 (2003)

Dynamics at the groups level

$$\frac{\mathrm{d}G_{n,i}}{\mathrm{d}t} = \mu(i+1)G_{n,i+1} - \mu i G_{n,i} + (n-i+1)\Theta_{n,i-1,\beta}G_{n,i-1} - (n-i)\Theta_{n,i,\beta}G_{n,i} + (n-i+1)\rho G_{n,i-1} - (n-i)\rho G_{n,i}$$

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$$\frac{\mathrm{d}S_m}{\mathrm{d}t} = \mu(g_m - S_m) - mrS_m$$

Mean-field quantities

$$\rho(t) = r(t)\frac{\sum_m m(m-1)S_m}{\sum_m mS_m} \quad ; \quad r(t) = \frac{\sum_{n,i}(n-i)\Theta_{n,i,\beta}G_{n,i}}{\sum_{n,i}(n-i)G_{n,i}}$$

Global prevalence

$$I(t) = \sum_m [g_m - S_m(t)]$$

The model and its mathematical description

$r(t)$: mean infection rate from a random group to which a random node belongs

$\rho(t)$: mean infection rate from all *other* groups to which a random node in a random group belongs

S_m : fraction of nodes with membership m and that are susceptible

$G_{n,i}$: fraction of group of size n and that contain i infectious nodes

g_m : fraction of nodes with membership m
p_n : fraction of groups of size n

μ : recovery rate
β : *weight* associated with transmission
$\Theta_{n,i,\beta}$: transmission rate in a group of size n, with i infected nodes, and weight β

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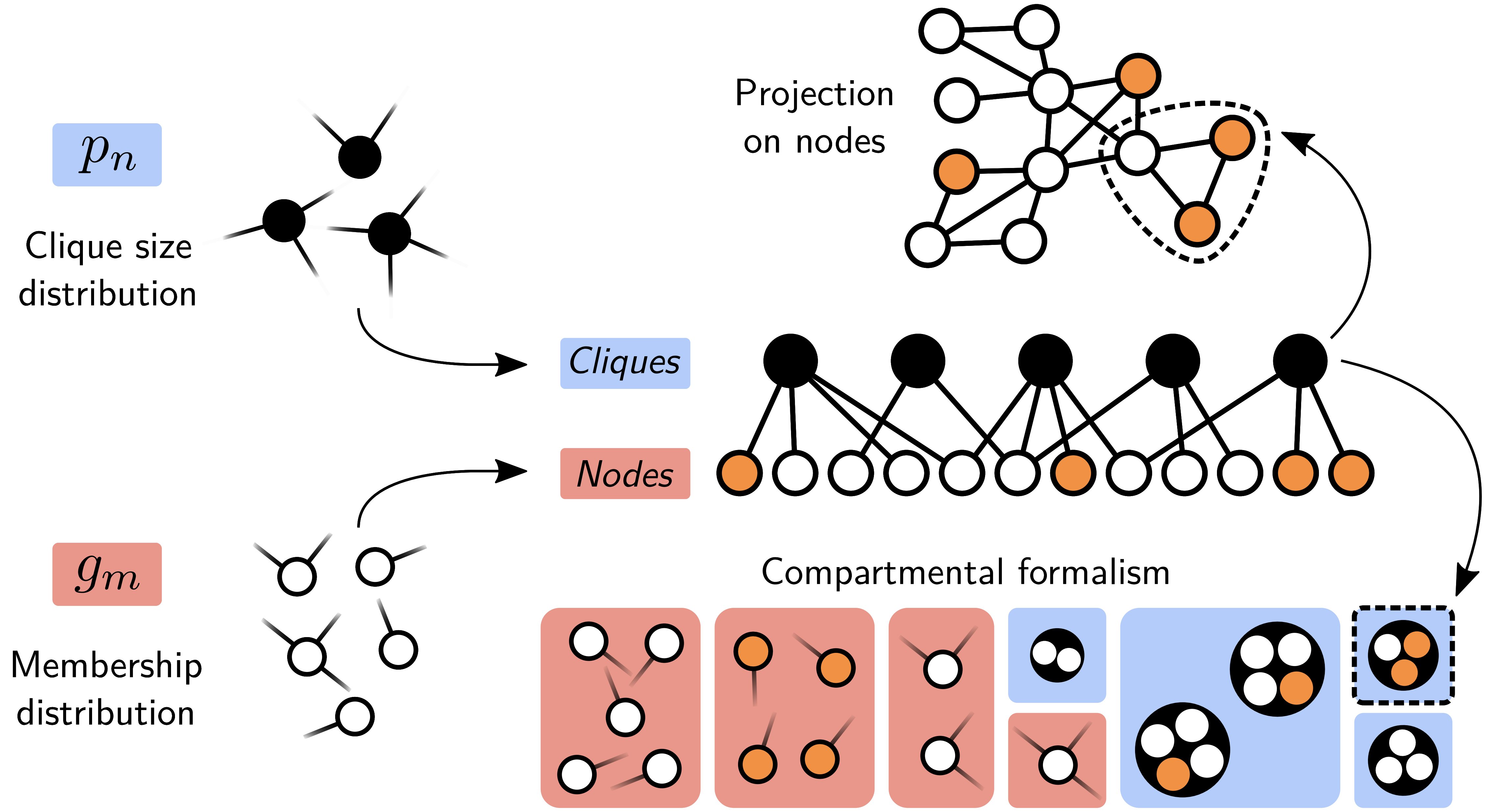

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The model and its mathematical description

Stationary state

$$\begin{aligned} S_m^* &= \frac{g_m}{1+mr} \\ \mu(i+1)G^*_{n,i,\beta} &= \left[\mu i + (n-i)(\Theta_{n,i,\beta}+\rho)\right]G^*_{n,i,\beta} \\ &\quad - (n-i+1)(\Theta_{n,i-1,\beta}+\rho)G^*_{n,i-1,\beta} \end{aligned}$$

Epidemic threshold

$$\left.\frac{\mathrm{d}F}{\mathrm{d}\rho}\right|_{\rho\to 0} > 1$$

where

$$F(\rho) \equiv r(\rho)\frac{\sum_m m(m-1)S_m(\rho)}{\sum_m mS_m(\rho)}$$