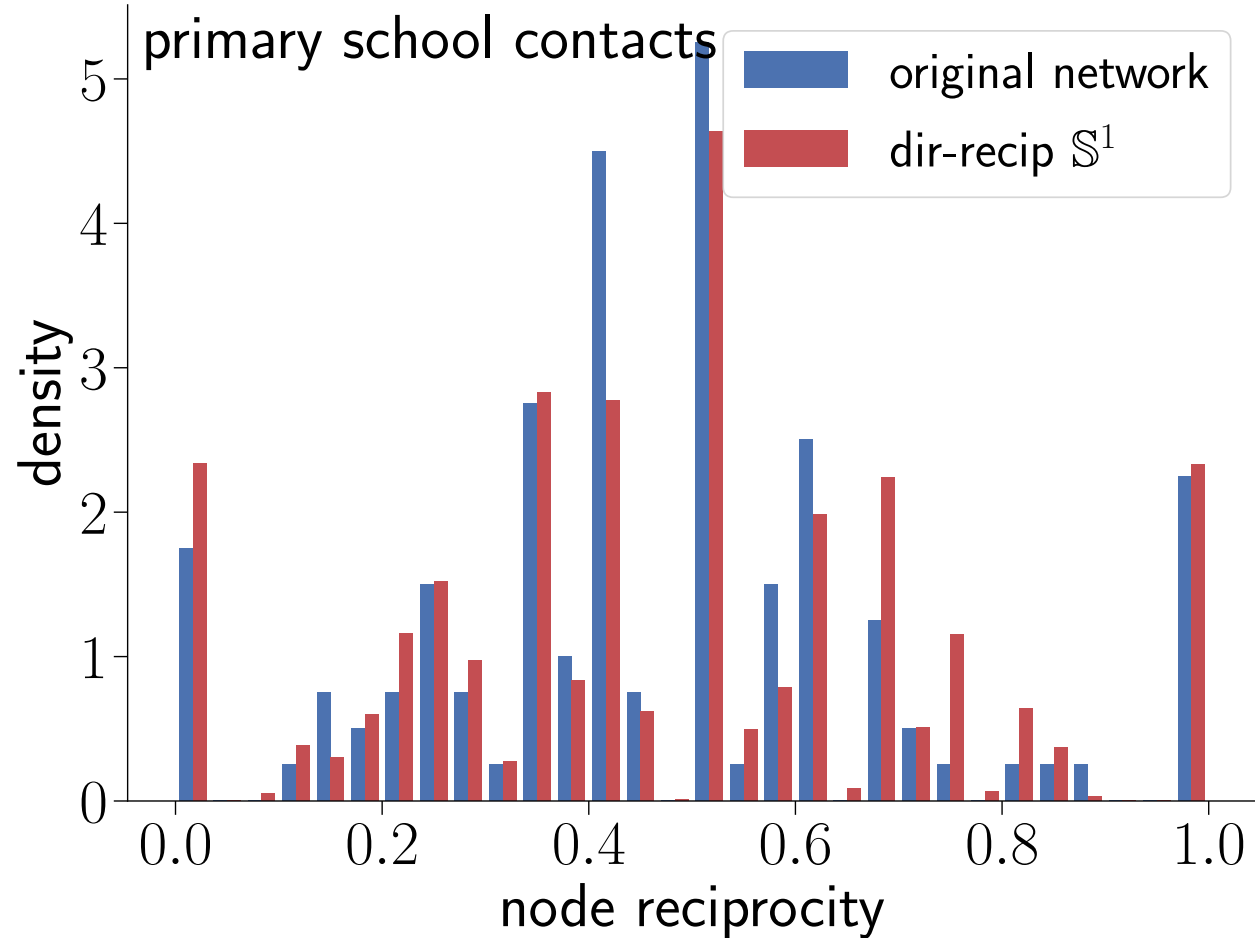
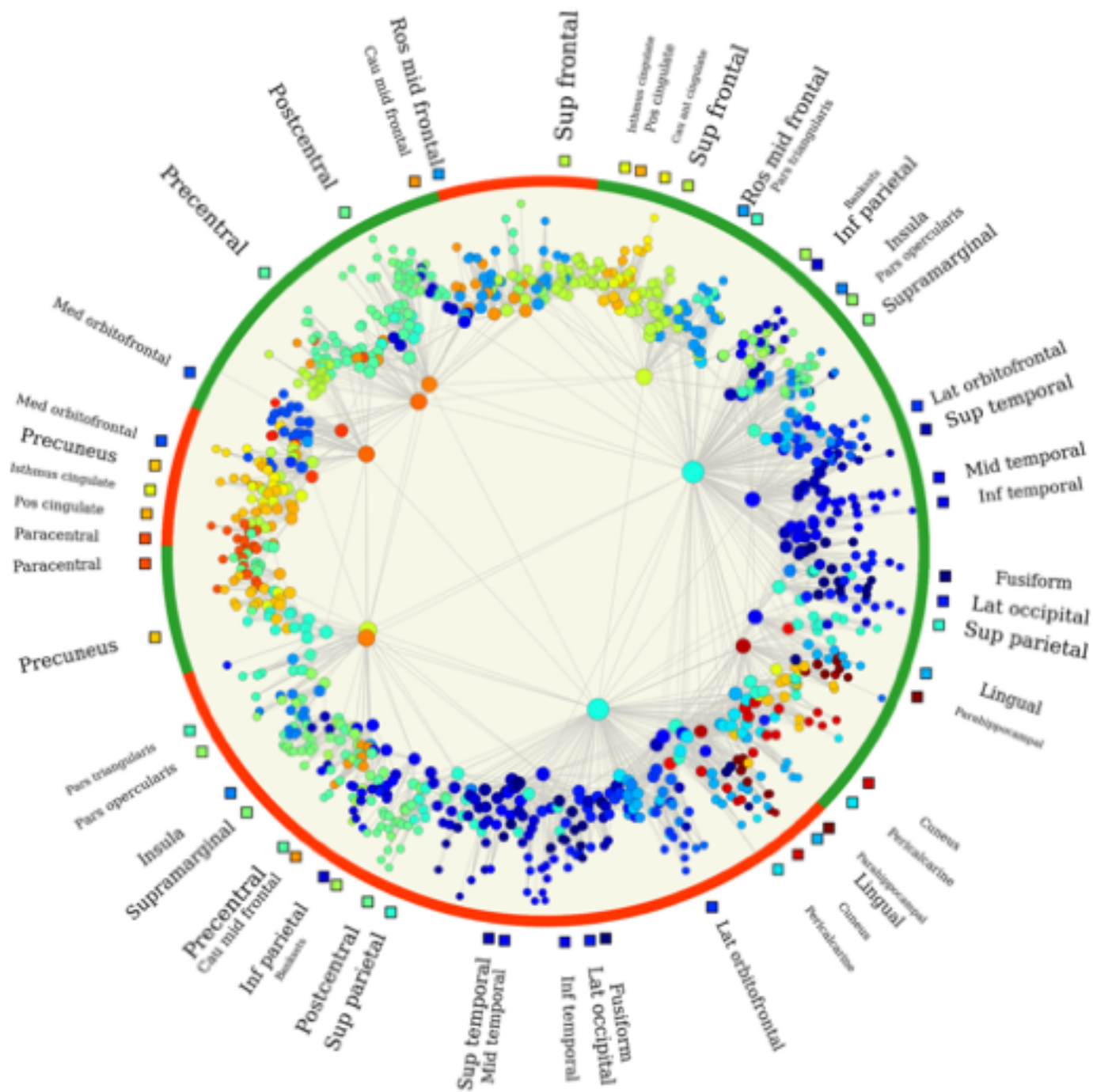


primary school contacts





1

8

clustering coefficient

0.8

0.6

0.4

0.2

—■—

original

—

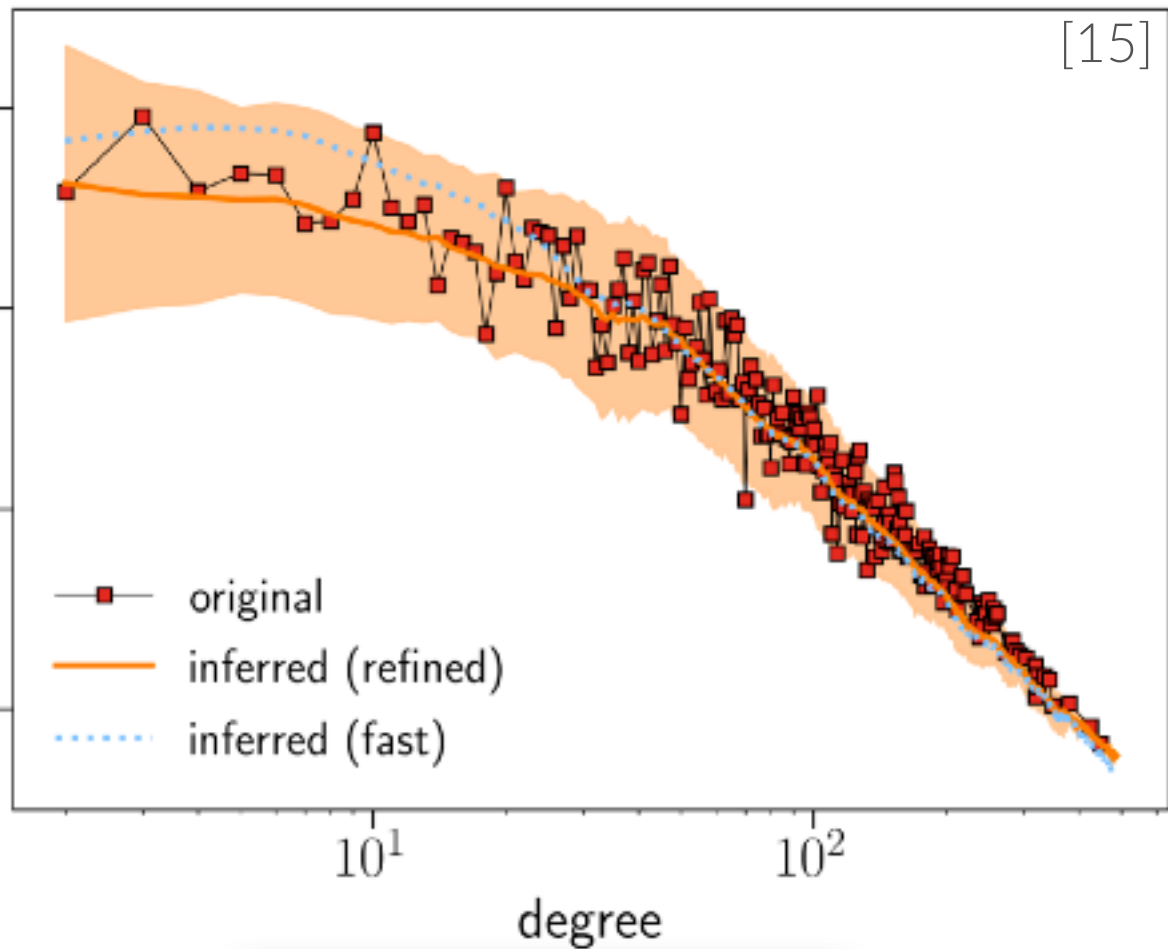
inferred (refined)

⋯

inferred (fast)

 10^1 10^2

degree



[1] Phys. Rev. E 80, 035101 (2009)

[2] Phys. Rev. E 82, 036106 (2010)

[3] Phys. Rev. Lett. 100, 078701 (2008)

[5] Nat. Commun. 8, 14103 (2017)

[6] Phys. Rev. E 84, 026114 (2011)

[7] Phys. Rev. E 95, 032309 (2017)

[8] Mol. Biosyst. 8, 843 (2012)

[9] Nat. Phys. 12, 1076 (2016)

[10] Phys. Rev. Lett. 118, 218301 (2017)

[11] Nature 489, 537 (2012)

[12] Sci.Rep.5, 9421 (2015)

[13] J. Stat. Phys. 173, 775 (2018)

[14] New J. Phys. 20, 052002 (2018)

[15] New J. Phys. 21, 123033 (2019)

[16] Nat. Commun. 8, 1615 (2017)

[17] Nat. Commun. 1, 62 (2010)

[18] PNAS 117, 20244 (2020)

A powerful and versatile framework

- ▷ Amenable to many **analytical calculations** [1,2]
- ▷ Generalizable to **weighted** [5], **bipartite** [6,7,8], **multiplex** [9,10], **directed** [4] and **growing** [11] networks
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- ▷ Parsimonious explanation of **self-similarity** [3]
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 - ...
- ▷ ...

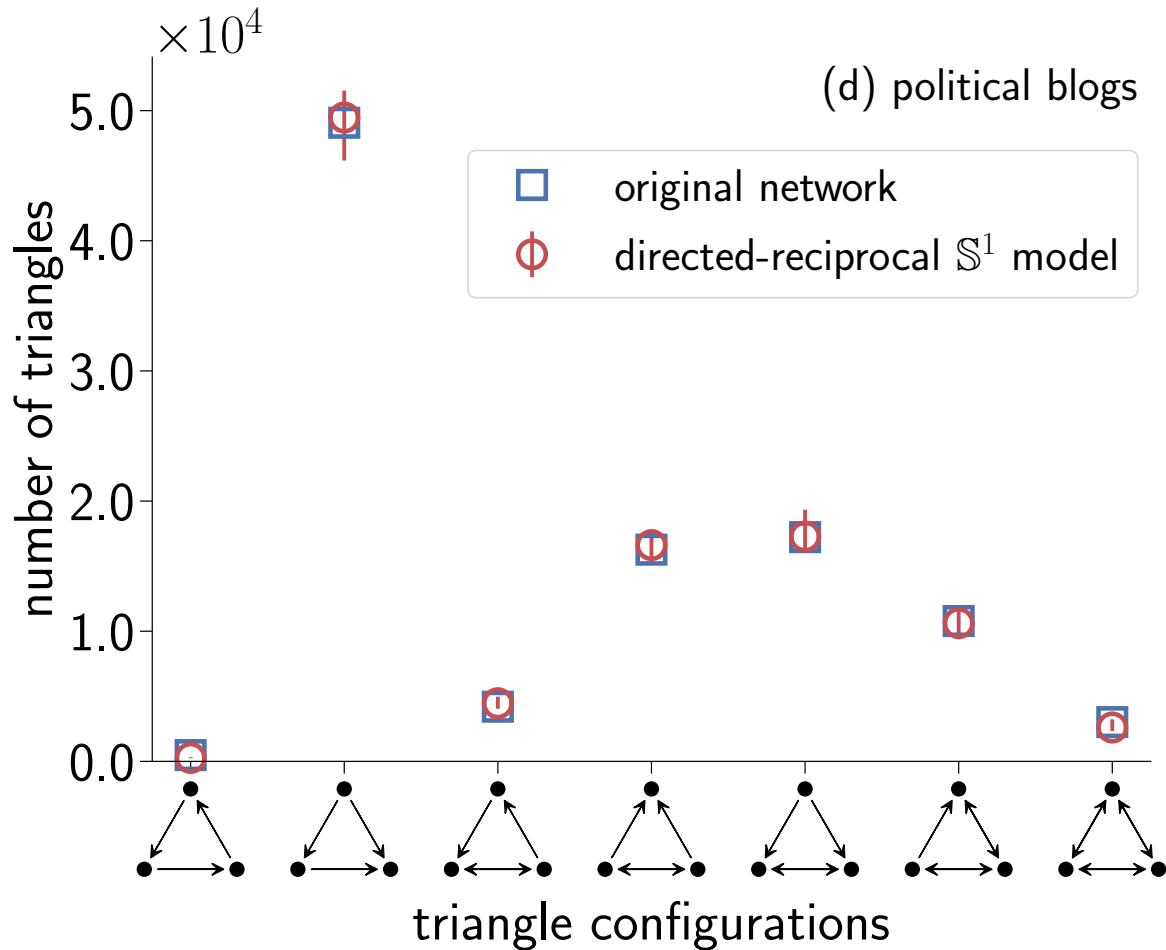
[4] Nat. Phys. 20, 150 (2024)

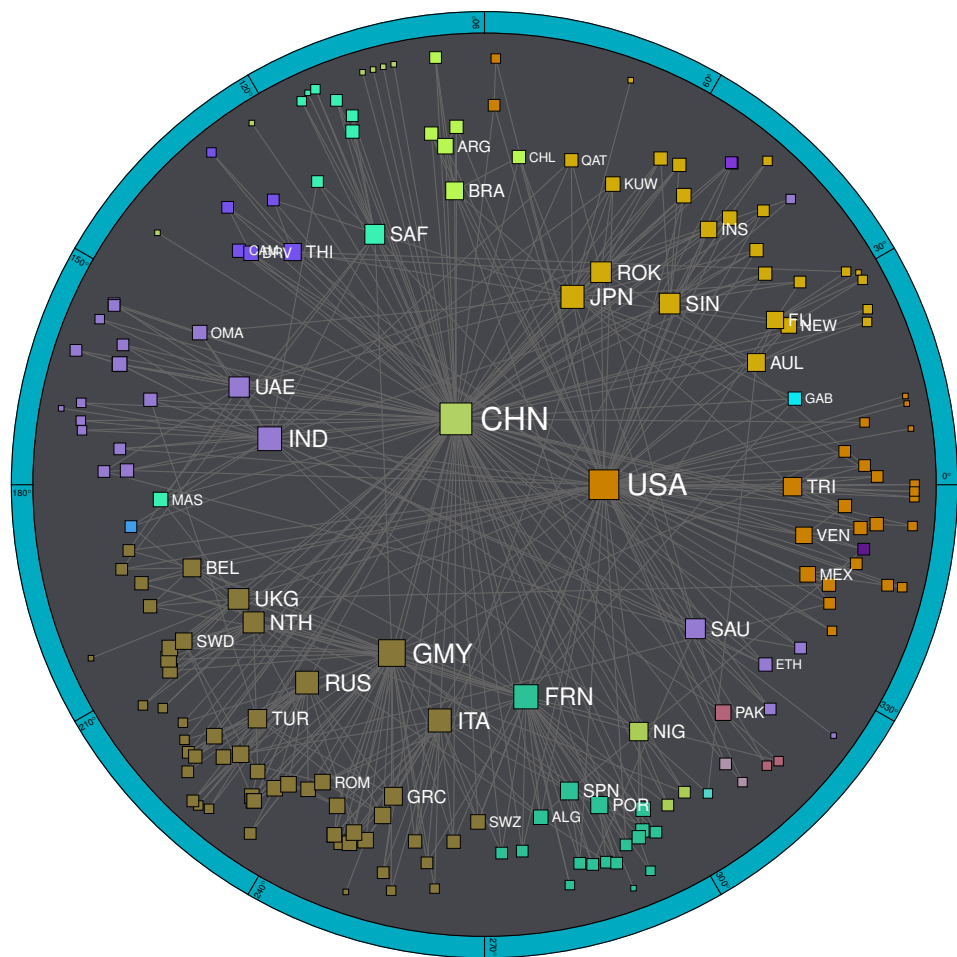
[19] Nat. Phys. 14, 583 (2018)

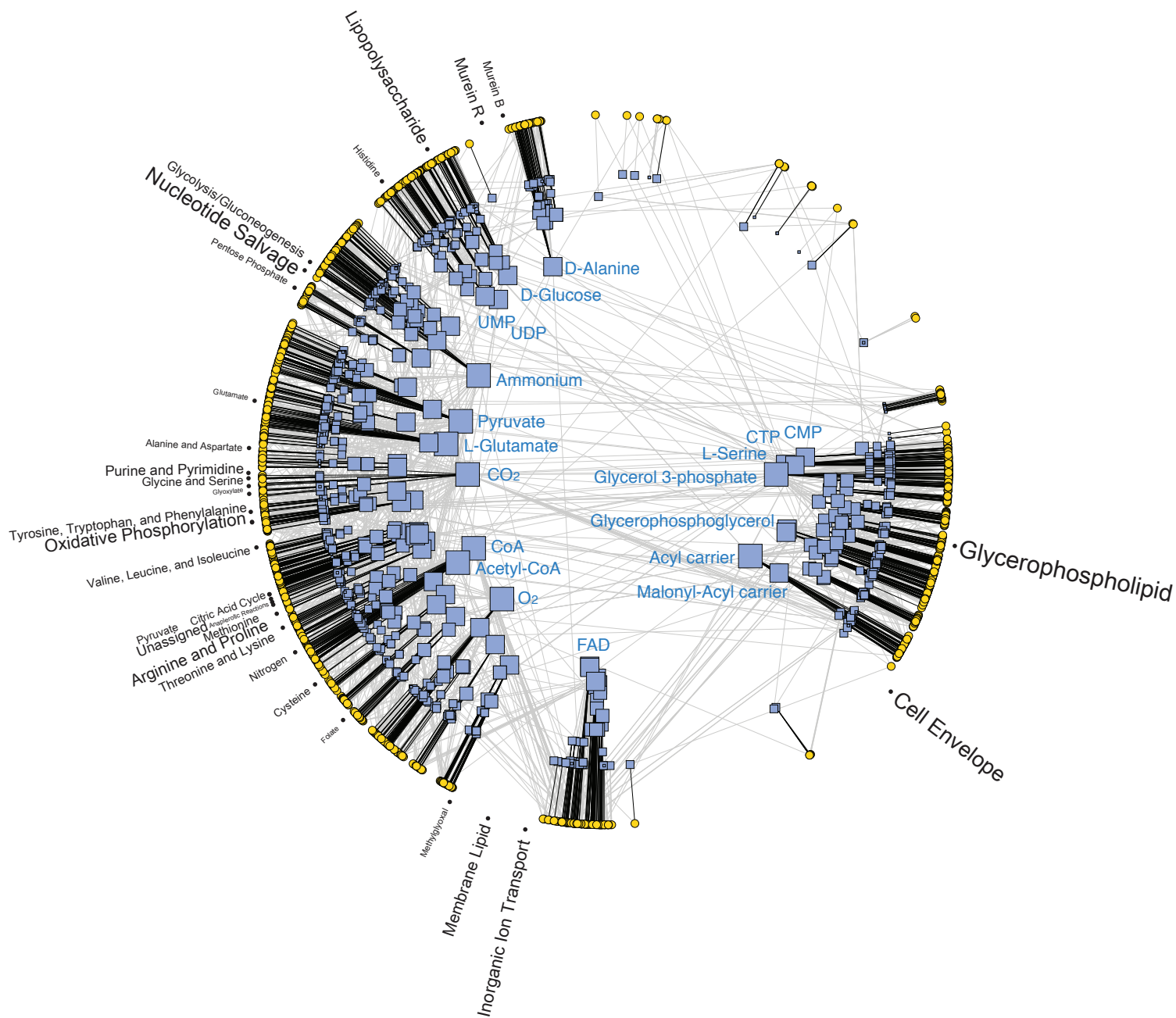
[20] PLOS Comput. Biol. 16, e1007584 (2020)

[21] Sci. Rep. 6, 33441 (2016)

(d) political blogs







Review Article | Published: 29 January 2021

Network geometry

[Marián Boguñá](#), [Ivan Bonamassa](#), [Manlio De Domenico](#) , [Shlomo Havlin](#),
[Dmitri Krioukov](#) & [M. Ángeles Serrano](#)

[Nature Reviews Physics](#) **3**, 114–135 (2021)

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Challenges

Heterogeneous random geometric graph models are prime candidate to model real networked complex systems.

But they rely heavily on our capacity to find high quality embeddings of the original datasets.

▷ Difficult optimization problem

- rugged landscape
- numerous symmetries (rotation, reflection, graph automorphisms)
- gradient not always well defined

▷ Out-of-the-box solutions do not work well

- Hamiltonian Monte-Carlo
- gradient descent

▷ Current state-of-the-art embedding methods

- rely on heuristics
- do not provide uncertainties (loglikelihood maximization)

