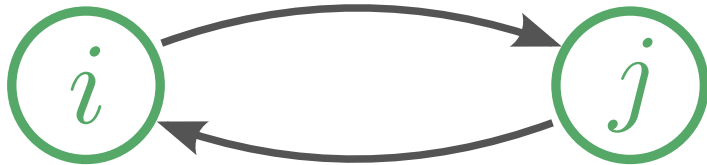


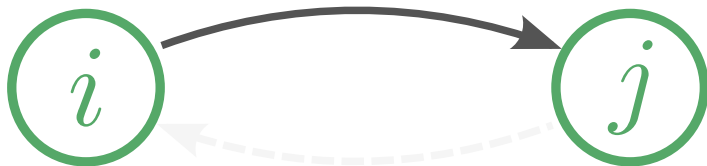
1

0

Fortuitous reciprocity



$$p_{ij}p_{ji}$$



$$p_{ij}(1 - p_{ji})$$



$$(1 - p_{ij})p_{ji}$$

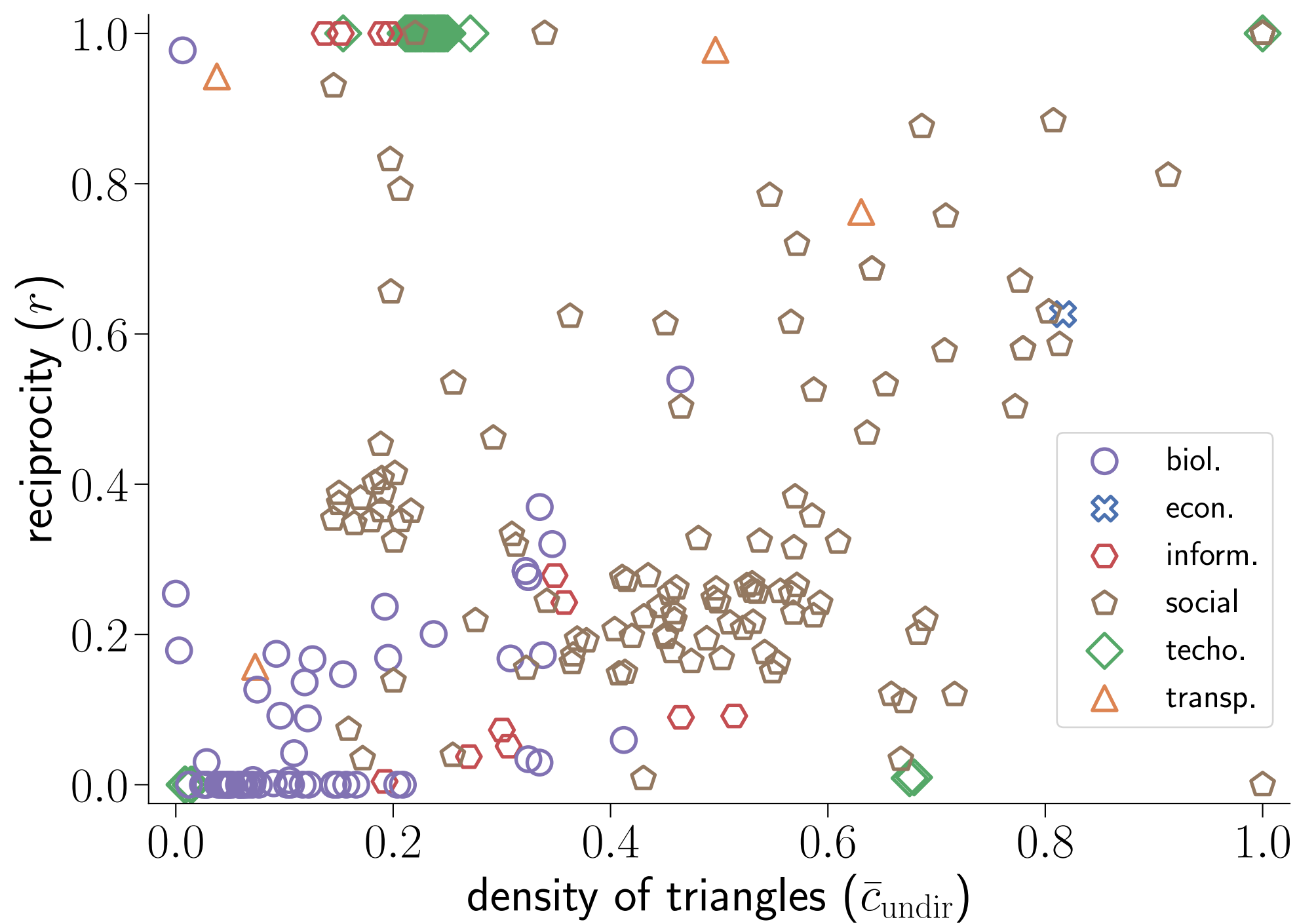


$$(1 - p_{ij})(1 - p_{ji})$$

Reciprocity in the directed S^1 model

A reciprocal connection between $\text{node } i$ and $\text{node } j$ occurs with probability $p_{ij}p_{ji}$.

292 network datasets downloaded from Netzhelder (networks.skewed.de).



A realistic model will need to go beyond reciprocity.

κ^{in} : in-degree

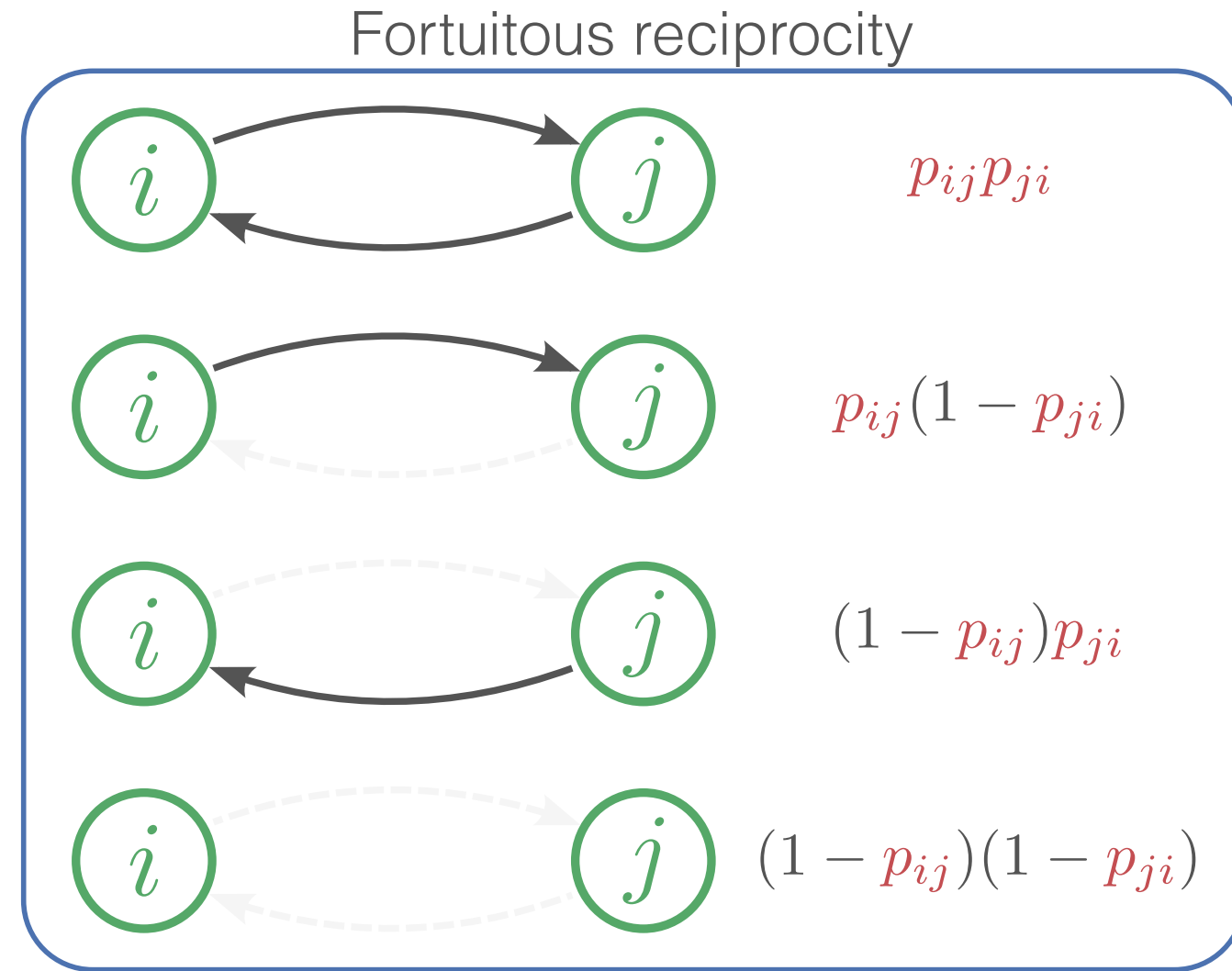
κ^{out} : out-degree

β : density of triangles

$$\begin{aligned}
r &= \mathbb{E} \left[\frac{L^{\leftrightarrow}}{L} \right] = \mathbb{E} \left[\frac{k^{\leftrightarrow}}{k^{\text{out}}} \right] \approx \frac{\mathbb{E} [k^{\leftrightarrow}]}{\mathbb{E} [k^{\text{out}}]} \\
&\simeq \iiint \frac{\kappa_i^{\text{out}} \kappa_j^{\text{in}}}{\langle \kappa \rangle^2} \frac{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}} \right)^{\beta-1}}{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}} \right)^{\beta}} \\
&\quad \times \rho(\kappa_i^{\text{in}}, \kappa_i^{\text{out}}) \rho(\kappa_j^{\text{in}}, \kappa_j^{\text{out}}) d\kappa_i^{\text{in}} \kappa_i^{\text{out}} d\kappa_j^{\text{in}} \kappa_j^{\text{out}}
\end{aligned}$$

Reciprocity in the directed \mathbb{S}^1 model

A reciprocal connection between node i and node j occurs with probability $p_{ij}p_{ji}$.

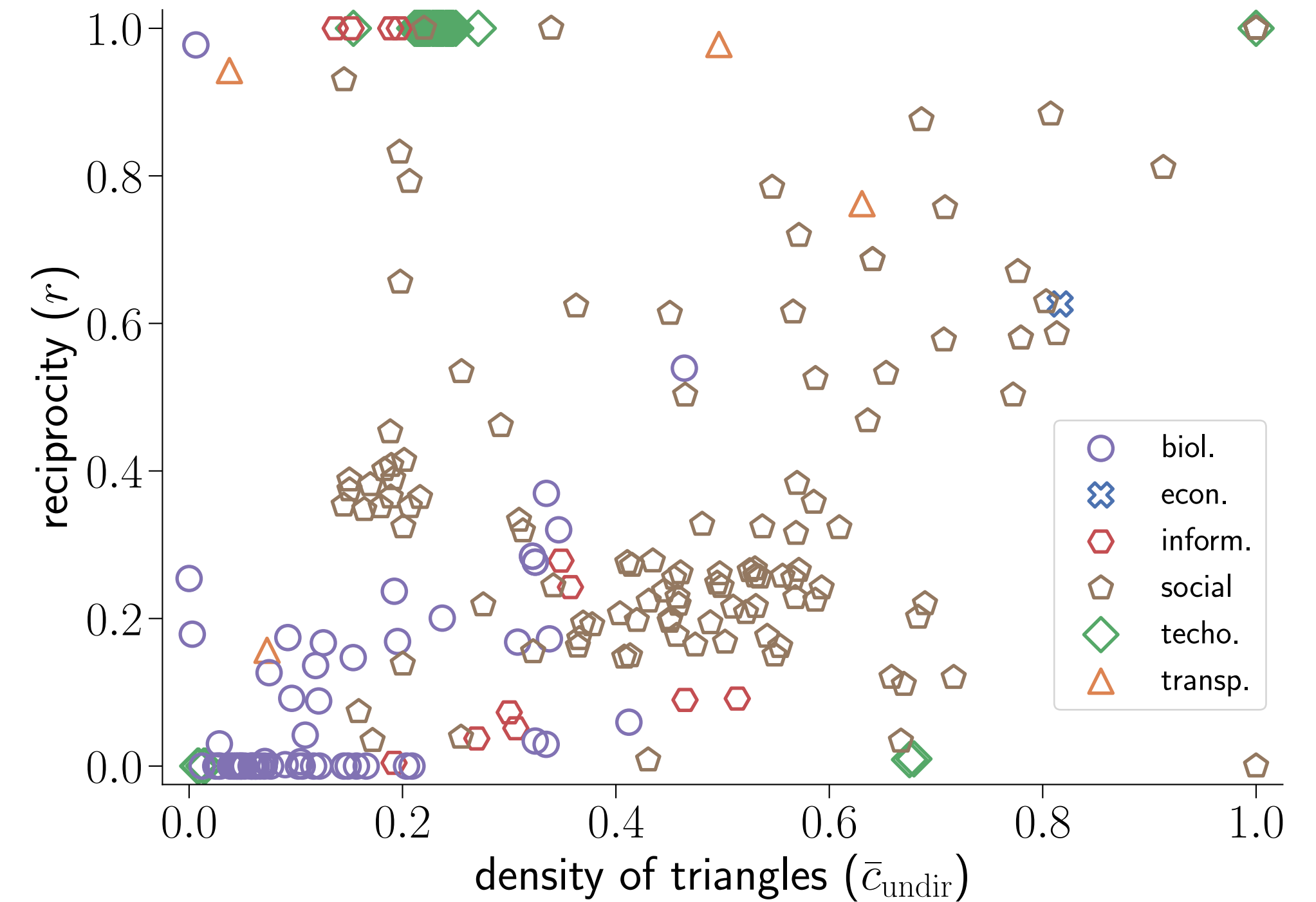


$$\begin{aligned}
 r &= \mathbb{E} \left[\frac{L^{\leftrightarrow}}{L} \right] = \mathbb{E} \left[\frac{k^{\leftrightarrow}}{k^{\text{out}}} \right] \approx \frac{\mathbb{E} [k^{\leftrightarrow}]}{\mathbb{E} [k^{\text{out}}]} \\
 &\simeq \iiint \frac{\kappa_i^{\text{out}} \kappa_j^{\text{in}}}{\langle \kappa \rangle^2} \frac{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}} \right)^{\beta-1}}{1 - \left(\frac{\kappa_i^{\text{out}}}{\kappa_i^{\text{in}}} \frac{\kappa_j^{\text{in}}}{\kappa_j^{\text{out}}} \right)^{\beta}} \\
 &\quad \times \rho(\kappa_i^{\text{in}}, \kappa_i^{\text{out}}) \rho(\kappa_j^{\text{in}}, \kappa_j^{\text{out}}) d\kappa_i^{\text{in}} \kappa_i^{\text{out}} d\kappa_j^{\text{in}} \kappa_j^{\text{out}}
 \end{aligned}$$

κ^{in} : in-degree

κ^{out} : out-degree

β : density of triangles



292 network datasets downloaded from Netzschleuder (networks.skewed.de).

A realistic model will need to go beyond fortuitous reciprocity.

Deliberate reciprocity in random directed networks

A random network model defines the probability p_{ij} for a directed link to exist from node i to node j .

