

# Simple random geometric graph

1. Sprinkle  $N$  nodes uniformly on the **hyperbolic** disk of radius  $R$ .
2. Connect any nodes separated by a distance less than  $r = R$ .

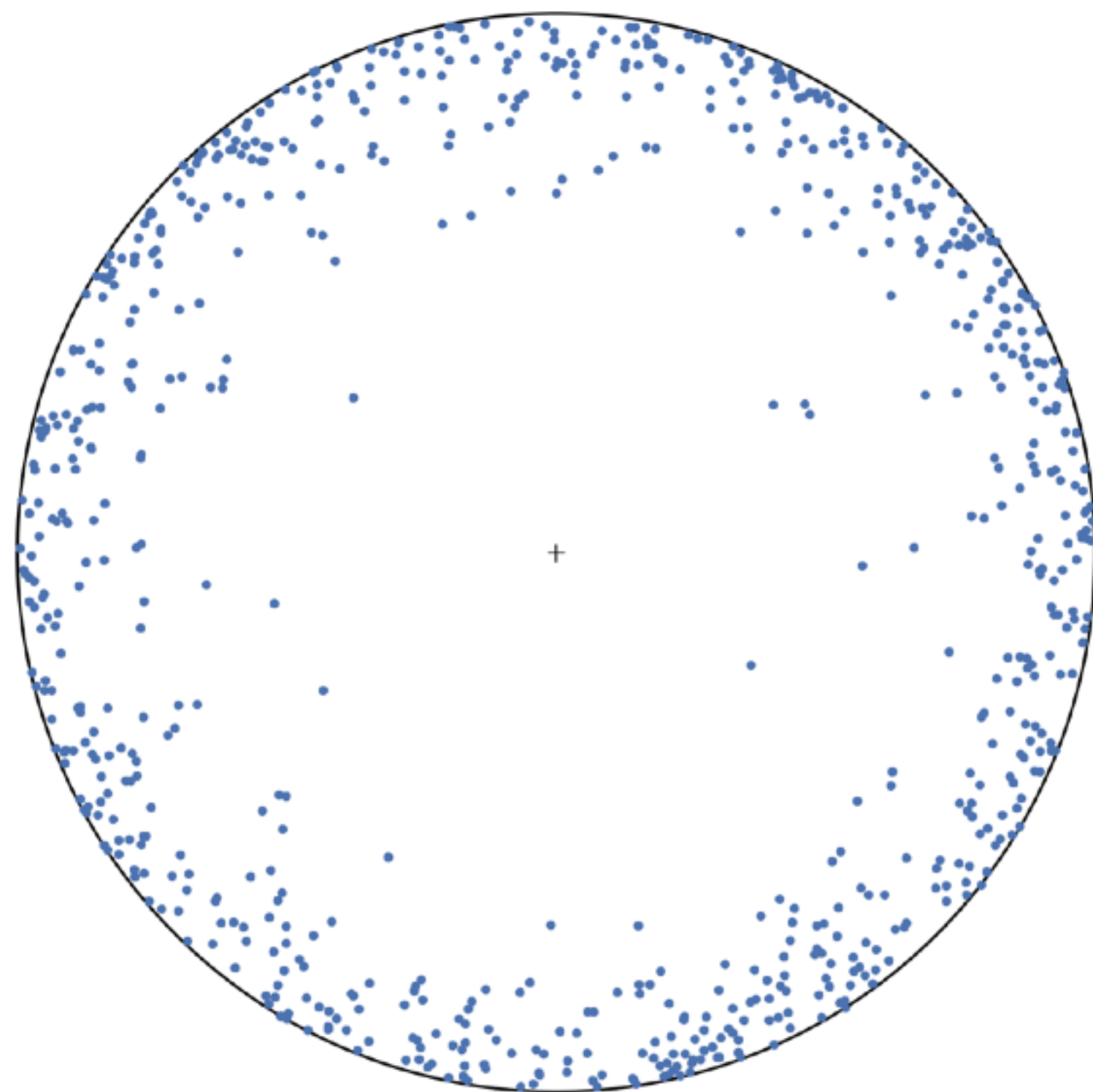
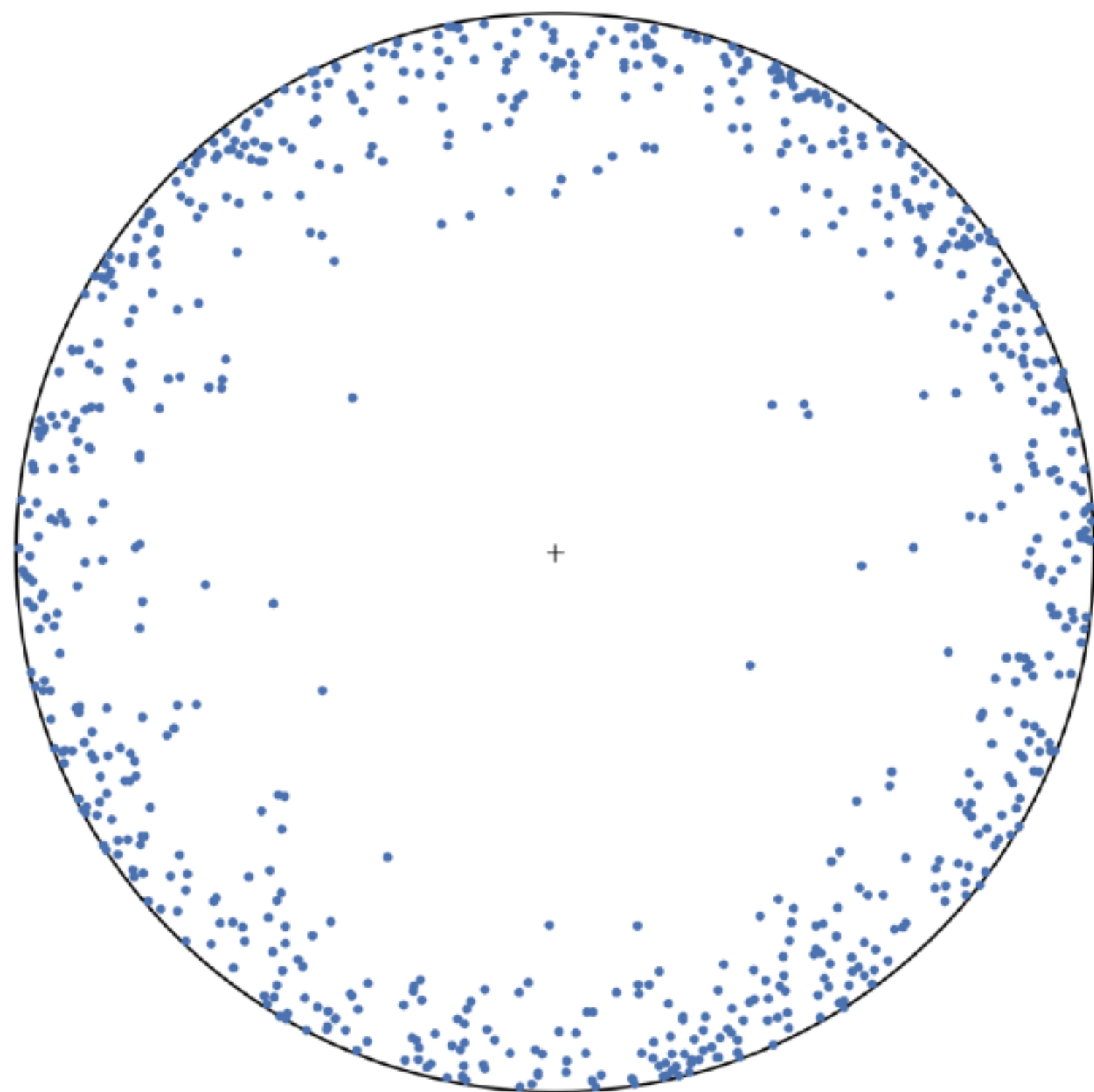
✓ high clustering

✓ power-law degree distribution with exponent  $-3$

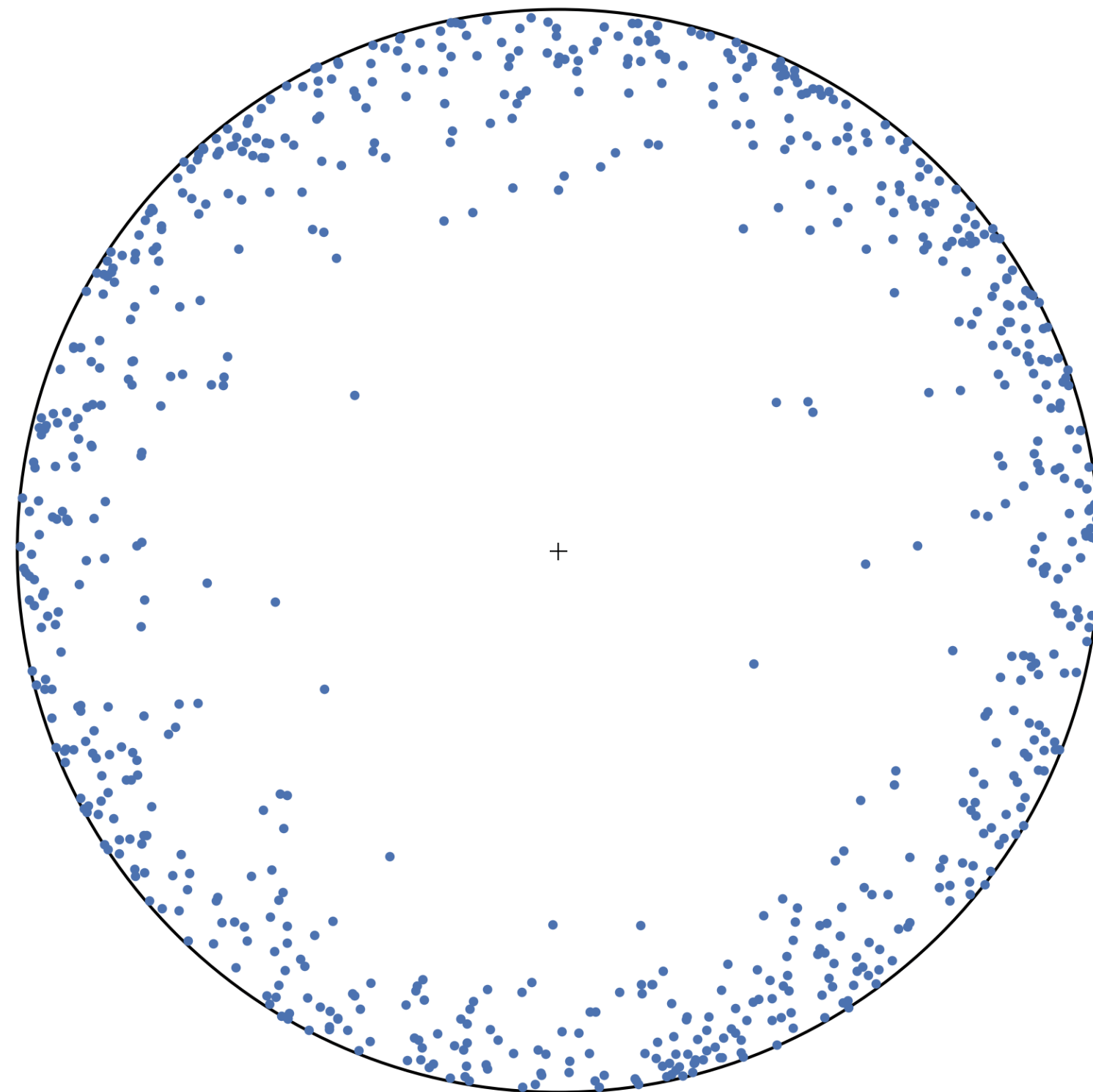
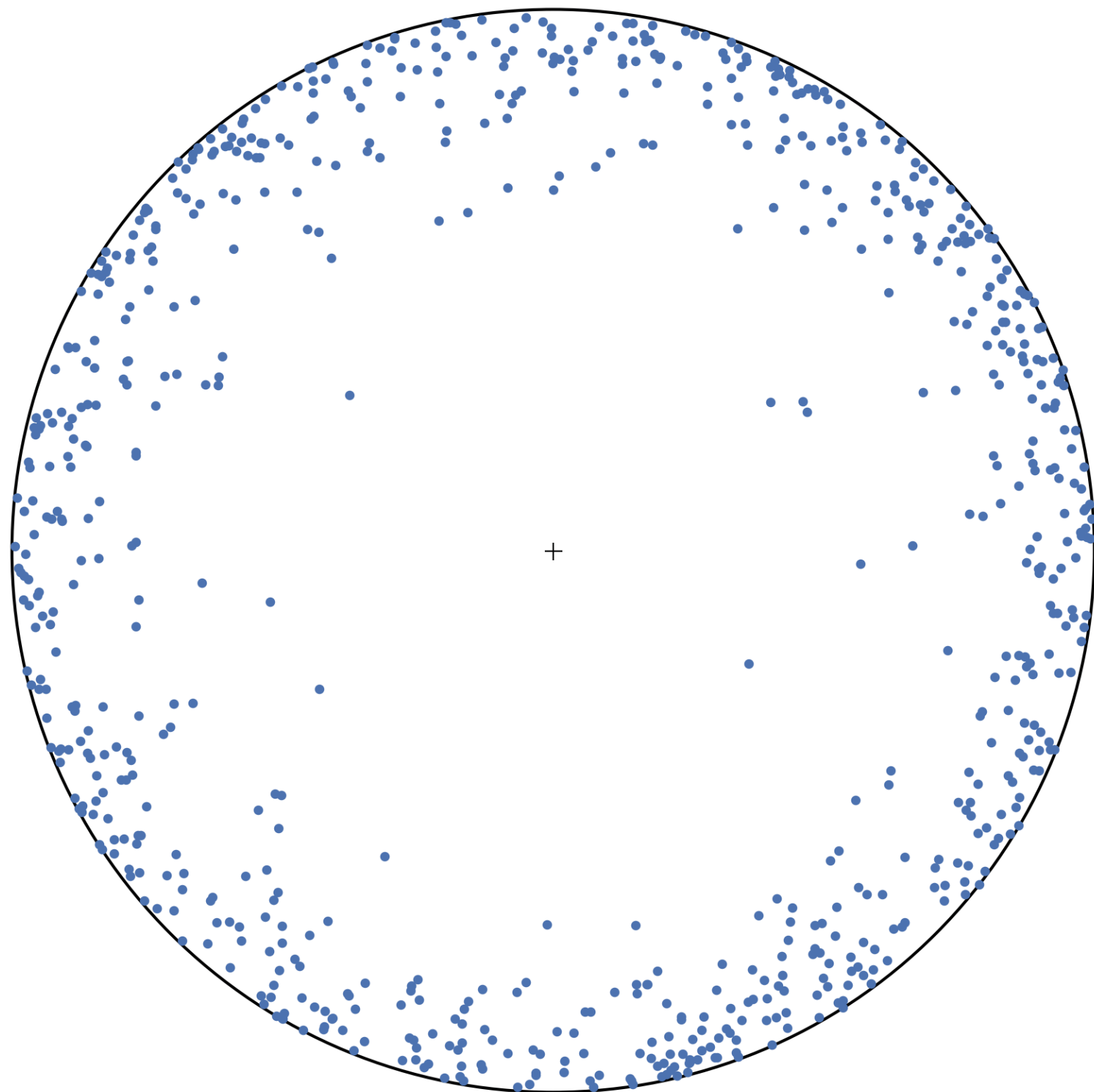
Phys. Rev. E 82, 036106 (2010)

A geometric approach to clustering: Hyperbolic geometry

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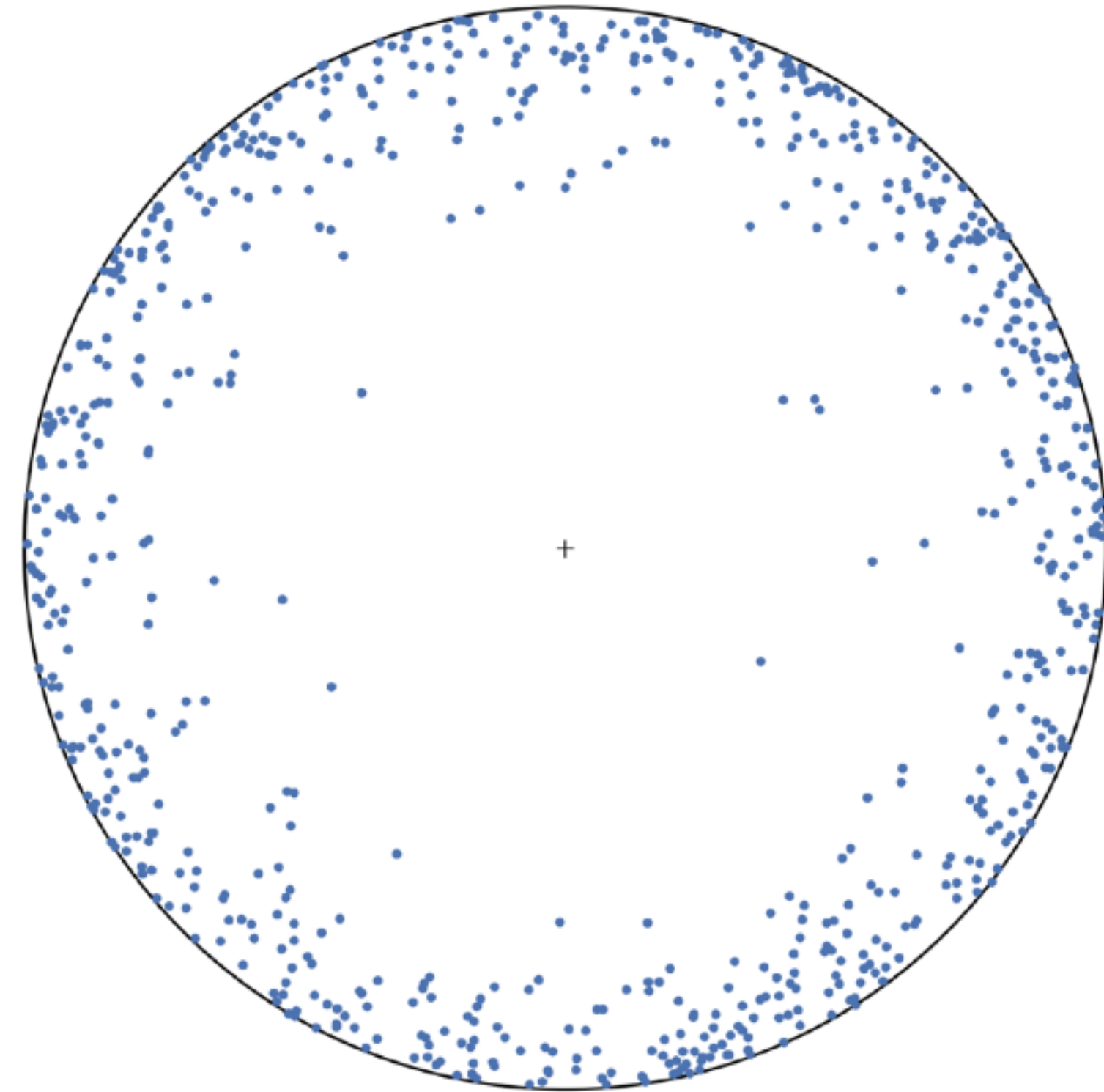
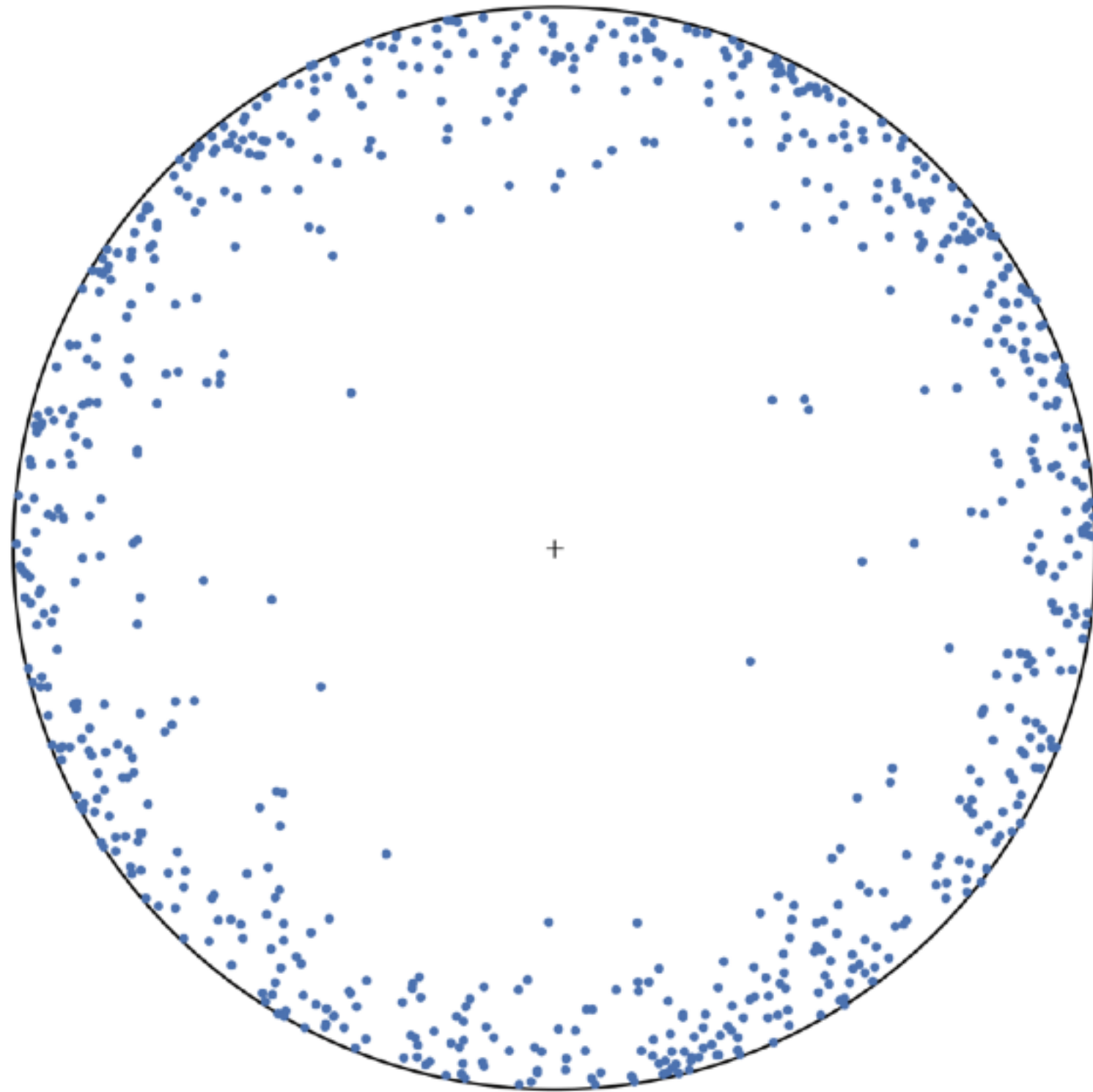




# A geometric approach to clustering: Hyperbolic geometry

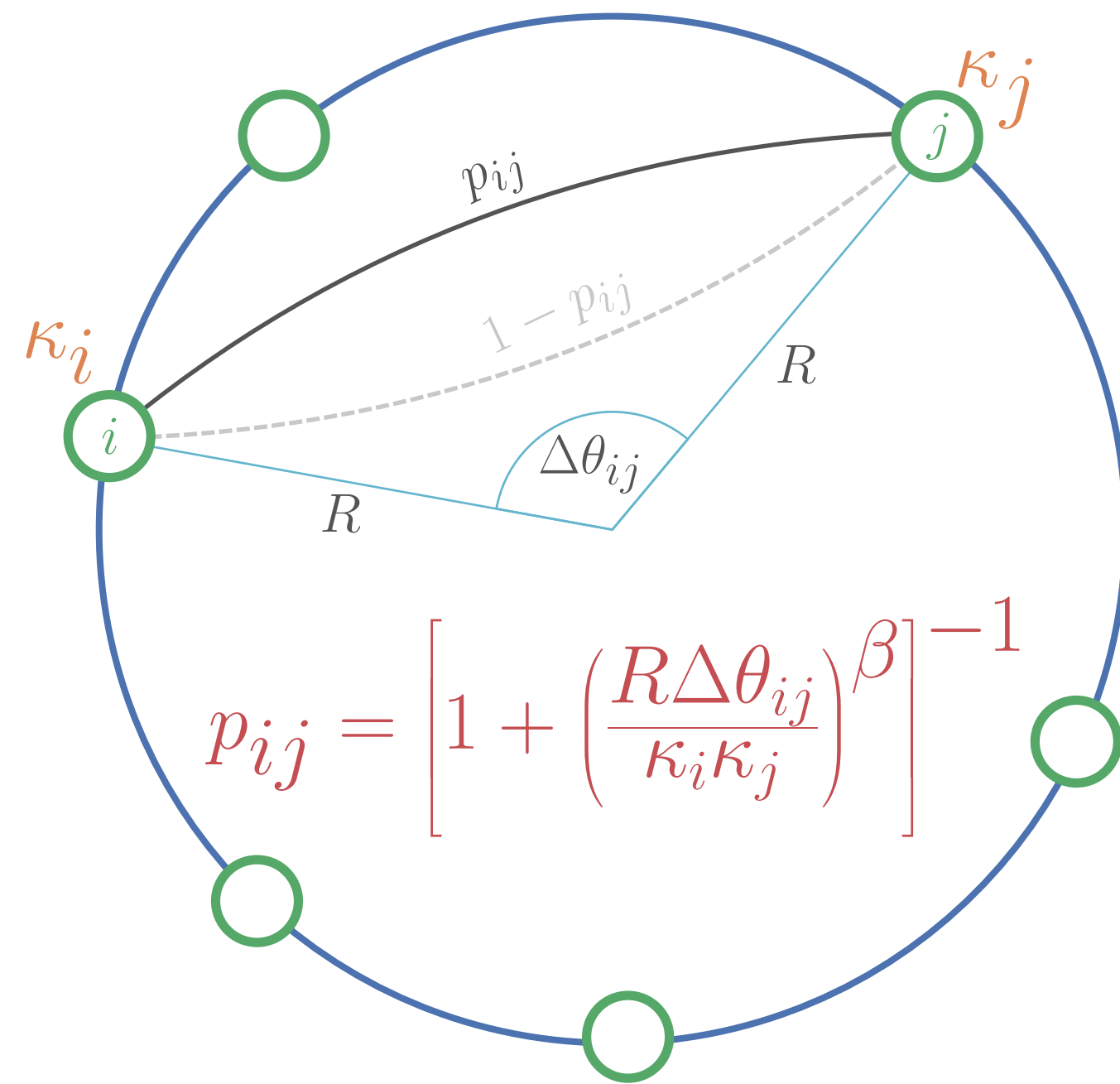
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- ✓ high clustering
- ✓ power-law degree distribution with exponent  $-3$

# A geometric approach to clustering: the $\mathbb{S}^1/\mathbb{H}^2$ model



## The $\mathbb{S}^1$ model

1. Sprinkle  $N$  nodes uniformly on a circle of radius  $R$ .
2. Assign an expected degree  $\kappa$  to each node according to some pdf  $\rho(\kappa)$ .
3. Draw a link between node  $i$  and node  $j$  with probability  $p_{ij}$ .

- ★ fixes the expected degree of nodes ( $\kappa$ ) → soft configuration model (CM)
- ★ triangle inequality of the underlying metric space → triangles from pairwise interactions
- ★ level of clustering tuned with parameter  $\beta$

[1] Phys. Rev. E 80, 035101 (2009)  
[2] Phys. Rev. E 82, 036106 (2010)  
[3] Phys. Rev. Lett. 100, 078701 (2008)  
[4] Nat. Rev. Phys. 3, 114 (2021)  
[5] Nat. Commun. 8, 14103 (2017)  
[6] Phys. Rev. E 84, 026114 (2011)

[7] Phys. Rev. E 95, 032309 (2017)  
[8] Mol. Biosyst. 8, 843 (2012)  
[9] Nat. Phys. 12, 1076 (2016)  
[10] Phys. Rev. Lett. 118, 218301 (2017)  
[11] Nature 489, 537 (2012)  
[12] Sci. Rep. 5, 9421 (2015)

[13] J. Stat. Phys. 173, 775 (2018)  
[14] New J. Phys. 20, 052002 (2018)  
[15] New J. Phys. 21, 123033 (2019)  
[16] Nat. Commun. 8, 1615 (2017)  
[17] Nat. Commun. 1, 62 (2010)  
[18] PNAS 117, 20244 (2020)