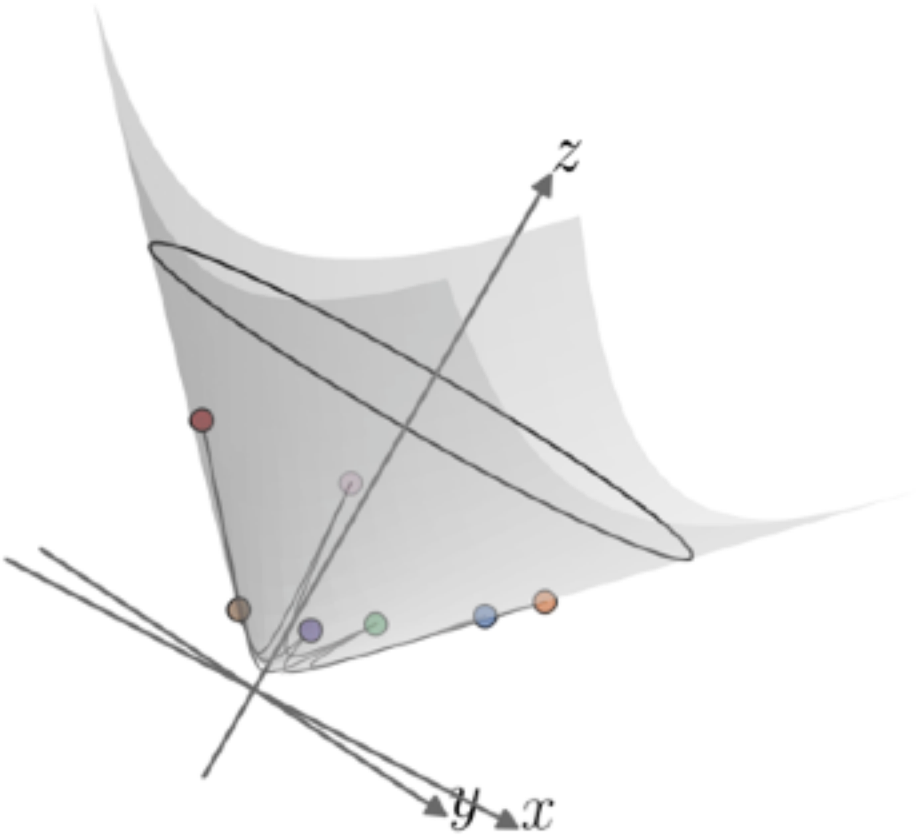
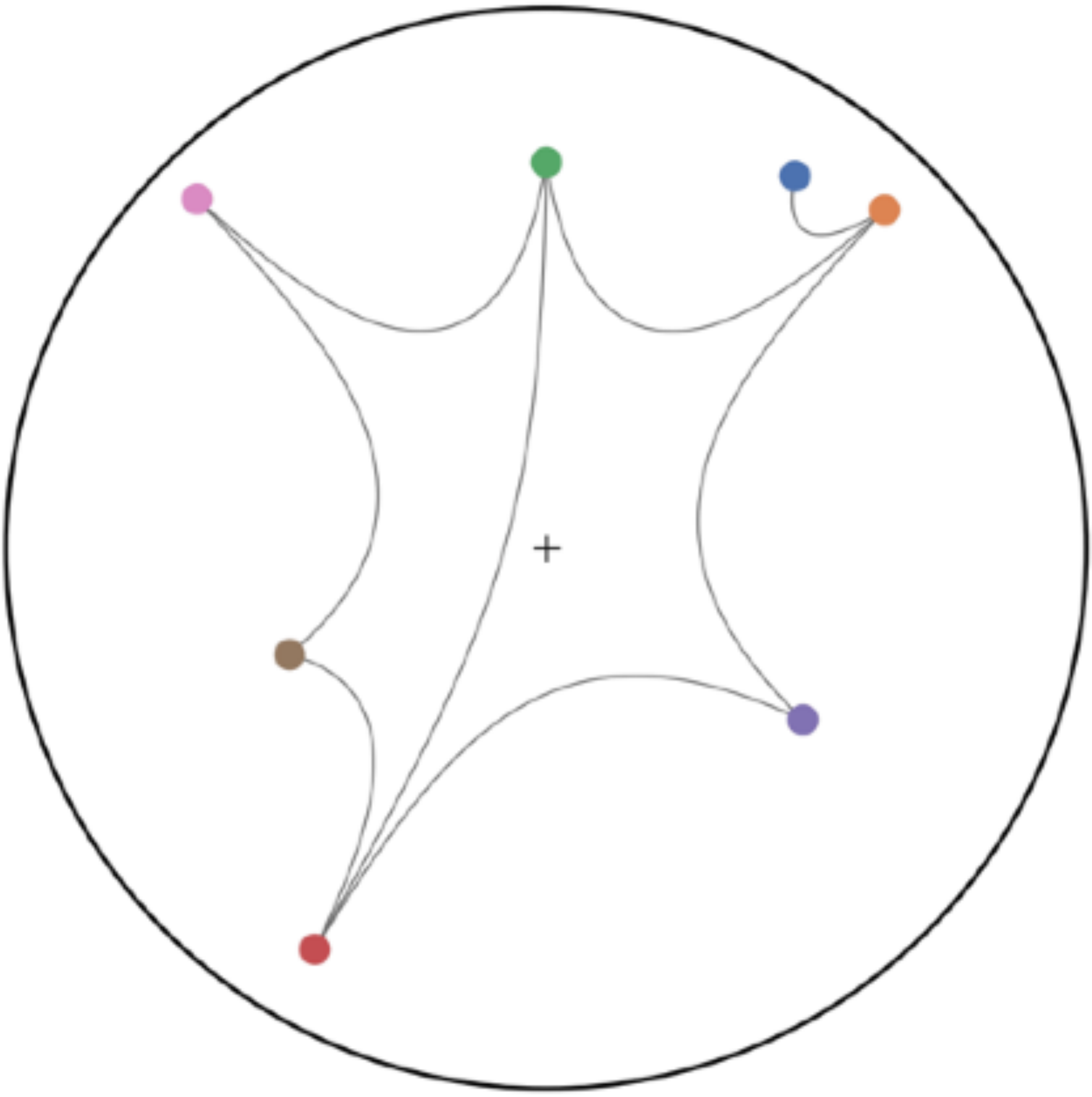
8

Hyperbolic geometry



hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

For further info, see Flavors of geometry (Cambridge University Press, 1997) or Foundations of Hyperbolic Manifolds (Springer, 2019)

- Space of constant negative curvature (as opposed to flat or Euclidean space, or spherical space)
- Model for the $D=2$ hyperbolic space : positive sheet of the hyperboloid defined by

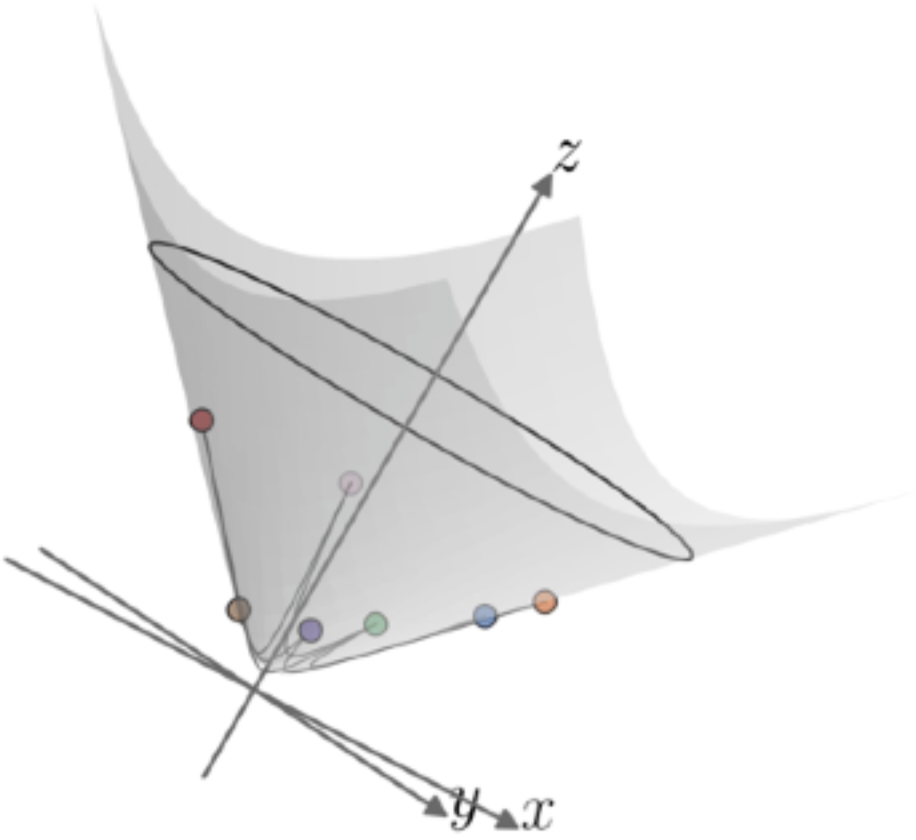
$$x^2+y^2-z^2=-1$$

- Distance between points (x_1,y_1,z_1) and (x_2,y_2,z_2) is

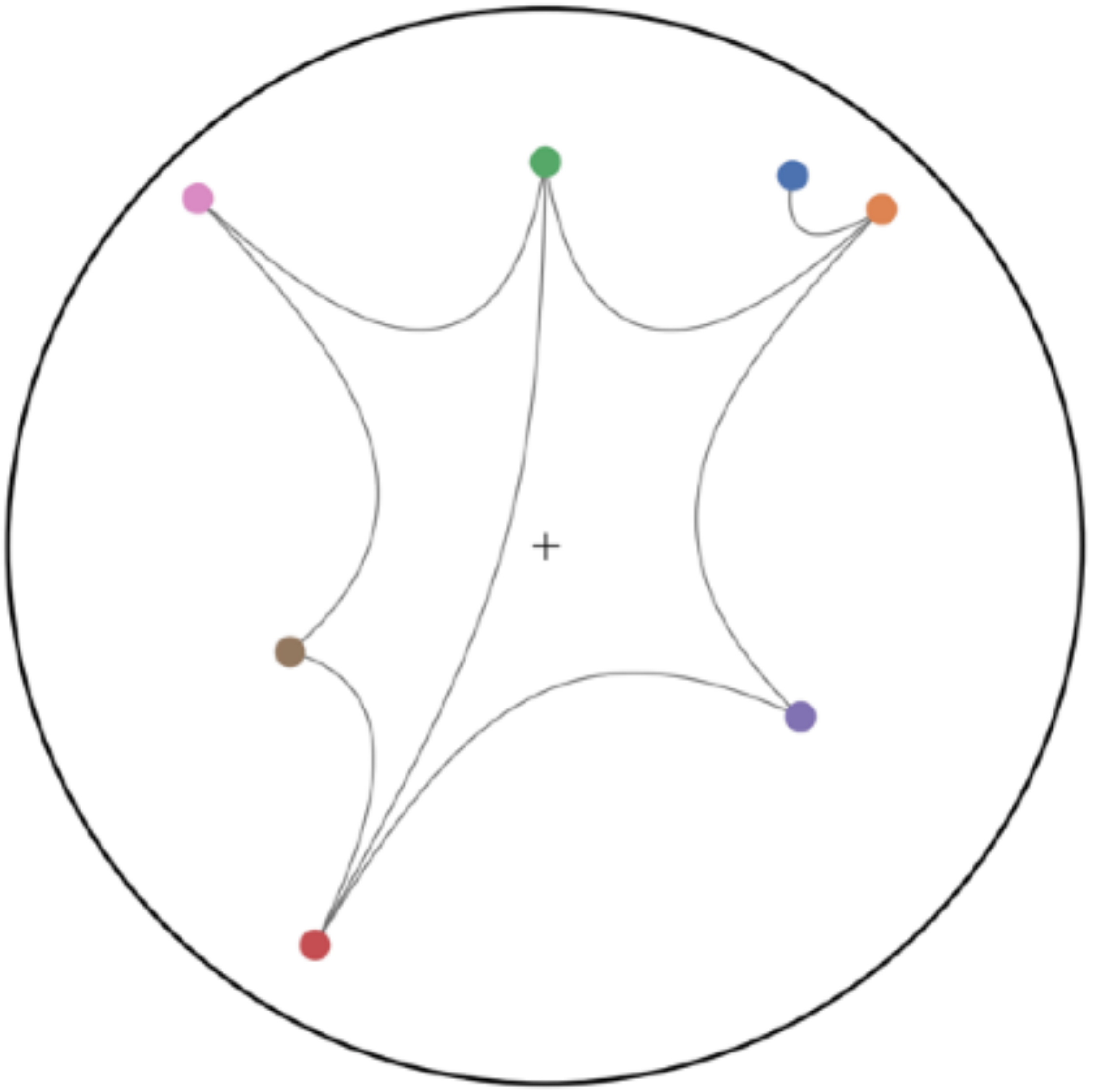
$$d(1,2)=\operatorname{arccosh}\big(z_1z_2-x_1x_2-y_1y_2\big)$$

- Polar coordinates

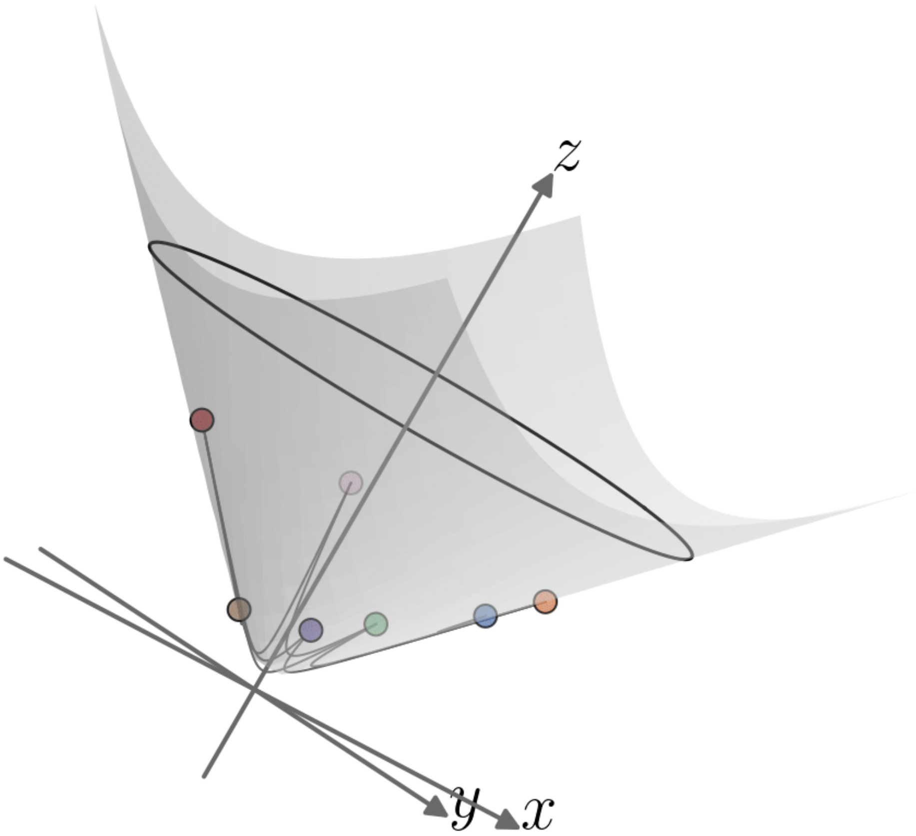
$$\begin{aligned}x&=\sinh(r)\cos(\theta)\\y&=\sinh(r)\sin(\theta)\\z&=\cosh(r)\end{aligned}$$



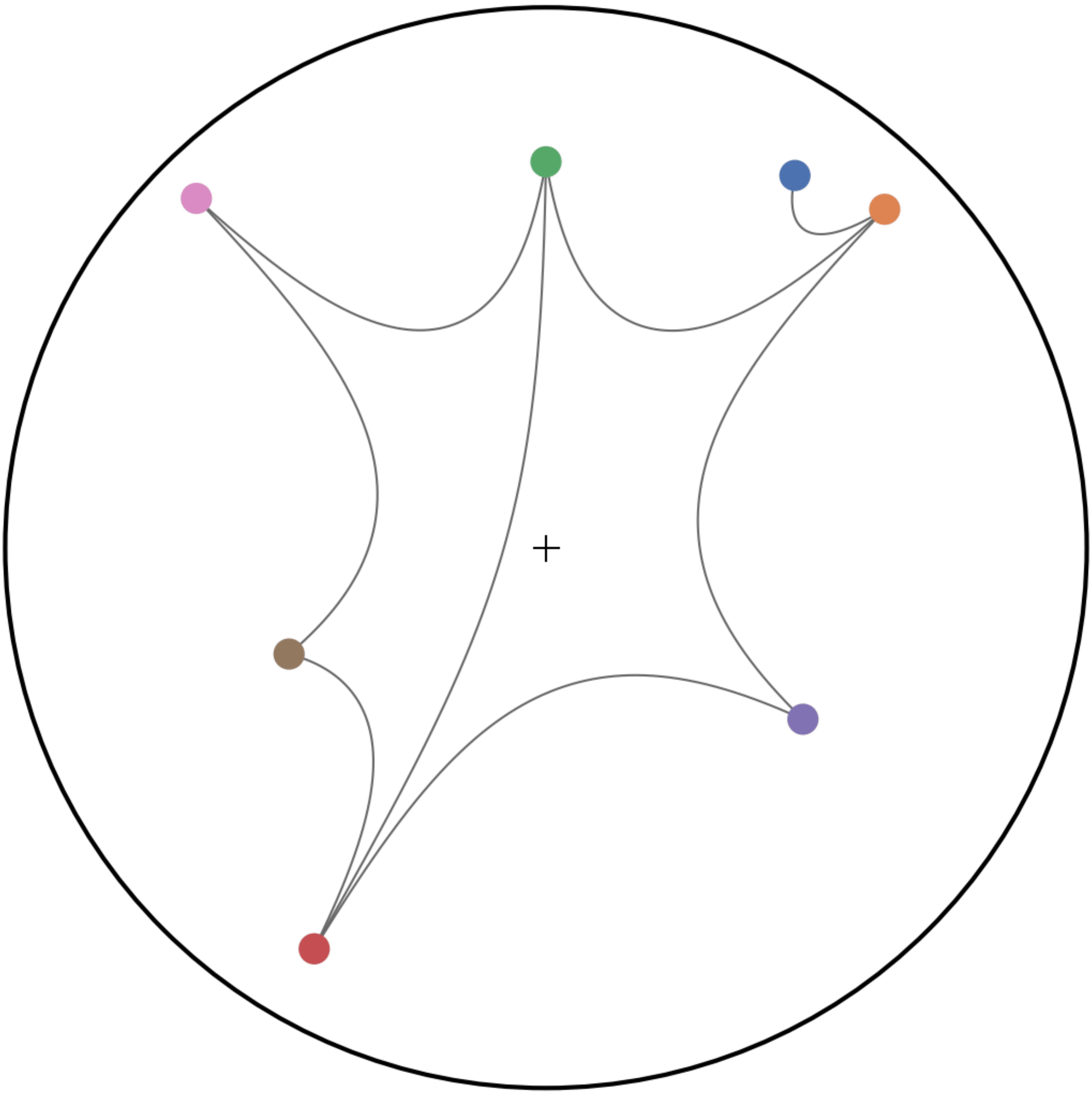
hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)



hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

Hyperbolic geometry

- ▷ Space of constant negative curvature (as opposed to flat or Euclidean space, or spherical space)

- ▷ Model for the $D = 2$ hyperbolic space : positive sheet of the hyperboloid defined by

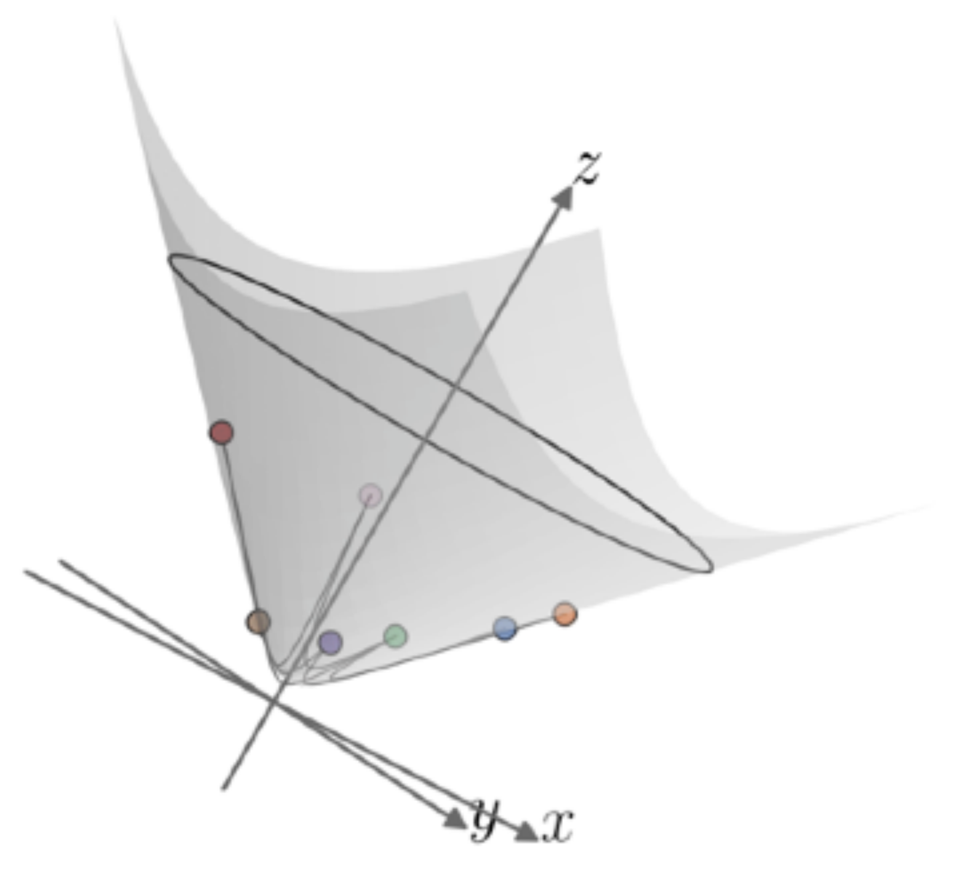
$$x^2 + y^2 - z^2 = -1$$

- ▷ Distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

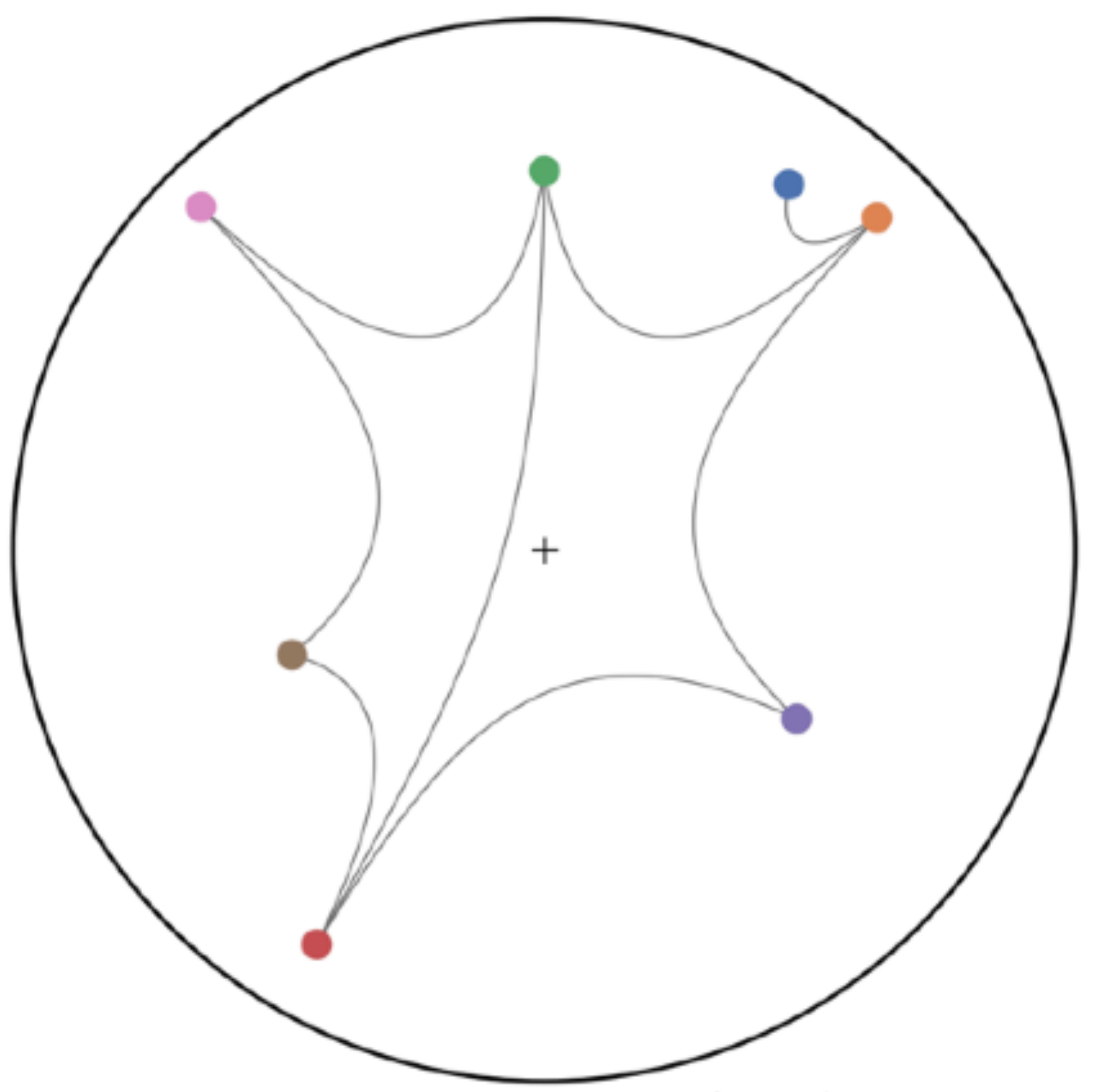
$$d(1,2) = \operatorname{arccosh}\big(z_1 z_2 - x_1 x_2 - y_1 y_2\big)$$

- ▷ Polar coordinates

$$\begin{aligned} x &= \sinh(r)\cos(\theta) \\ y &= \sinh(r)\sin(\theta) \\ z &= \cosh(r) \end{aligned}$$



hyperboloid in $\mathbb{R}^{2,1}$



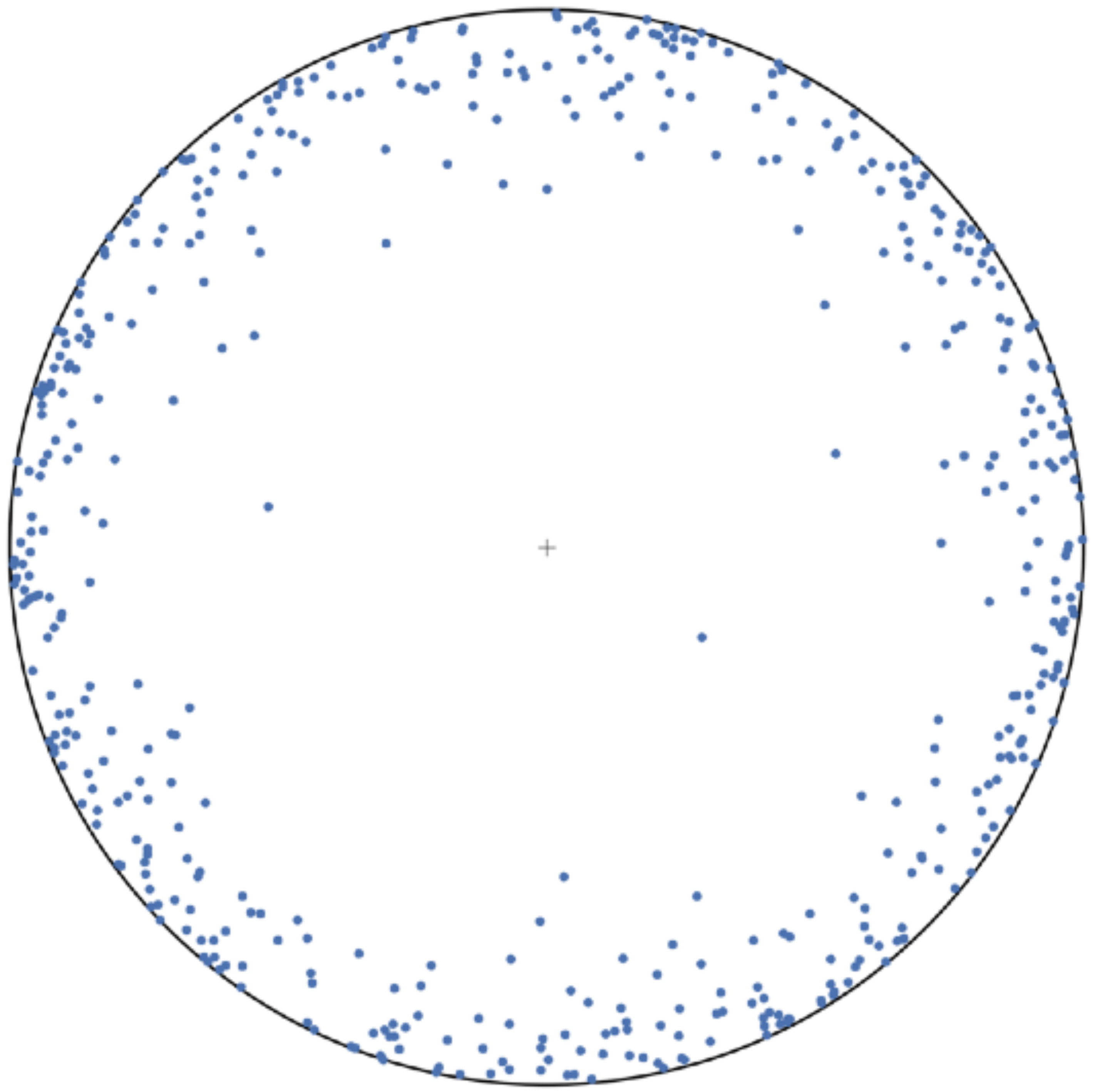
hyperbolic disk (r, θ)

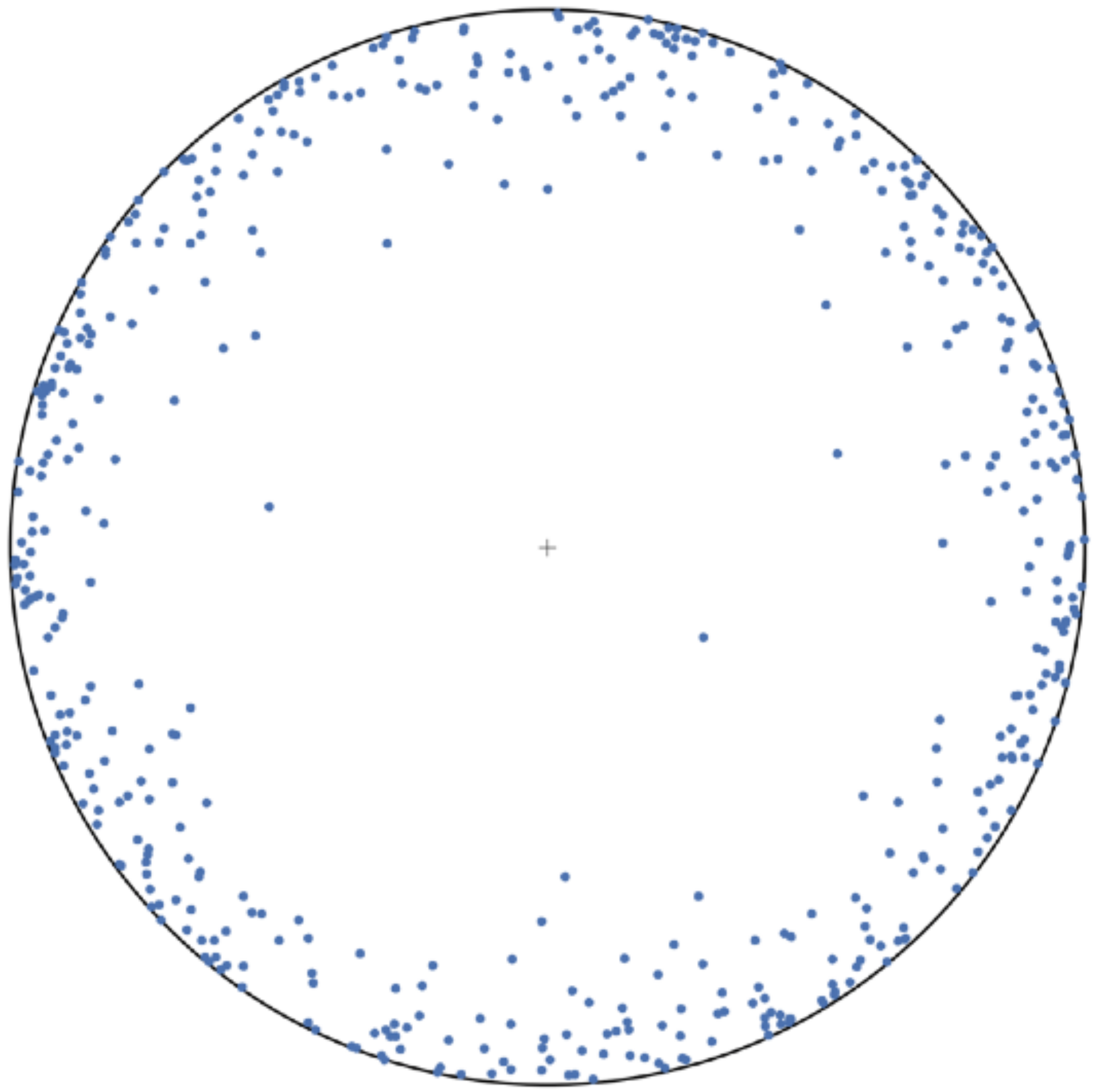
For further info, see Flavors of geometry (Cambridge University Press, 1997)
or Foundations of Hyperbolic Manifolds (Springer, 2019)

Hyperbolic geometry

Simple random geometric graph

1. Sprinkle N nodes uniformly on the hyperbolic disk of radius R.
2. Connect any nodes separated by a distance less than $r = R$.





✓ high clustering

✓ power-law degree distribution with exponent -3

Phys. Rev. E 82, 036106 (2010)