

Hyperbolic geometry

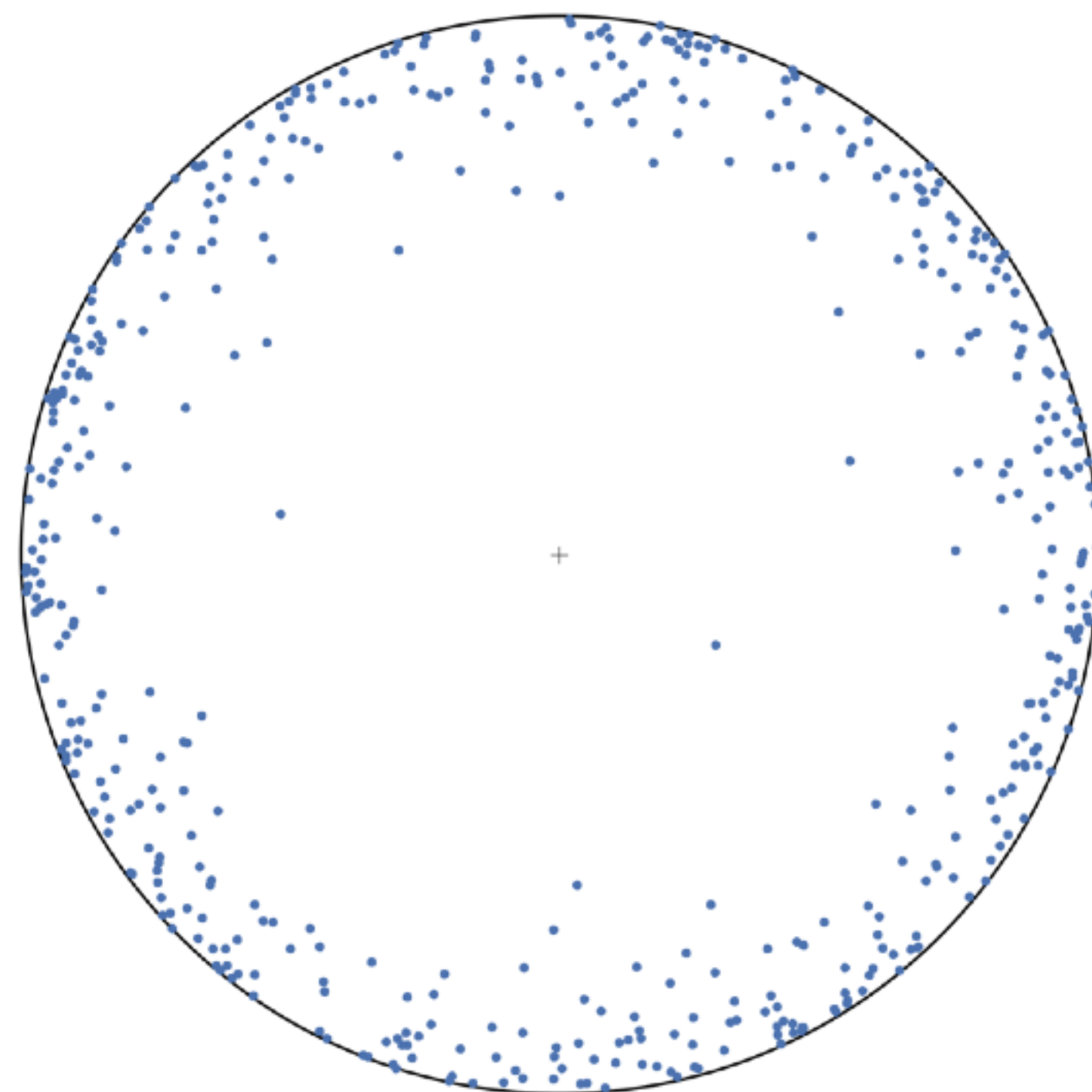
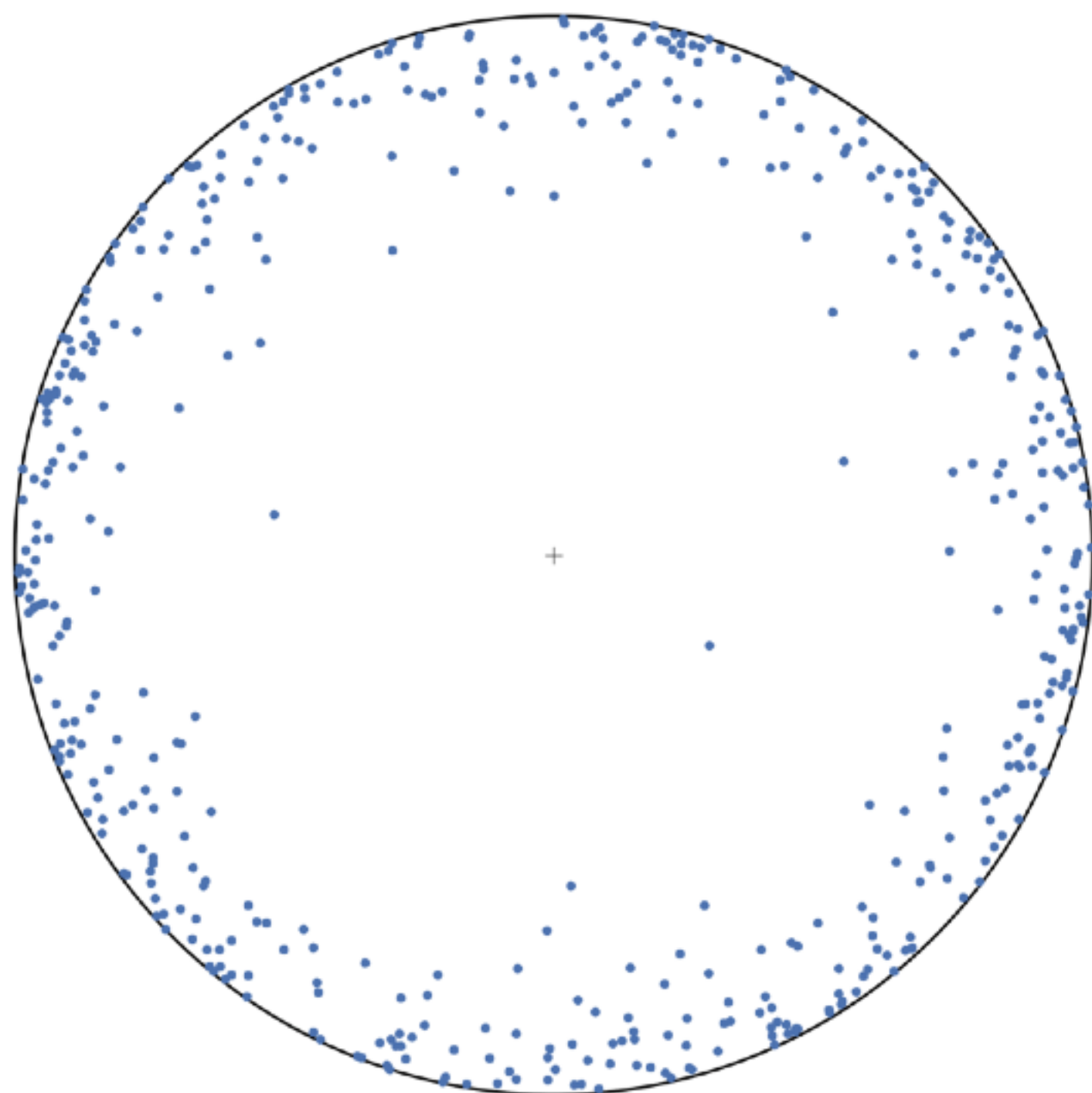
# Simple random geometric graph

1. Sprinkle  $N$  nodes uniformly on the **hyperbolic** disk of radius  $R$ .
2. Connect any nodes separated by a distance less than  $r = R$ .

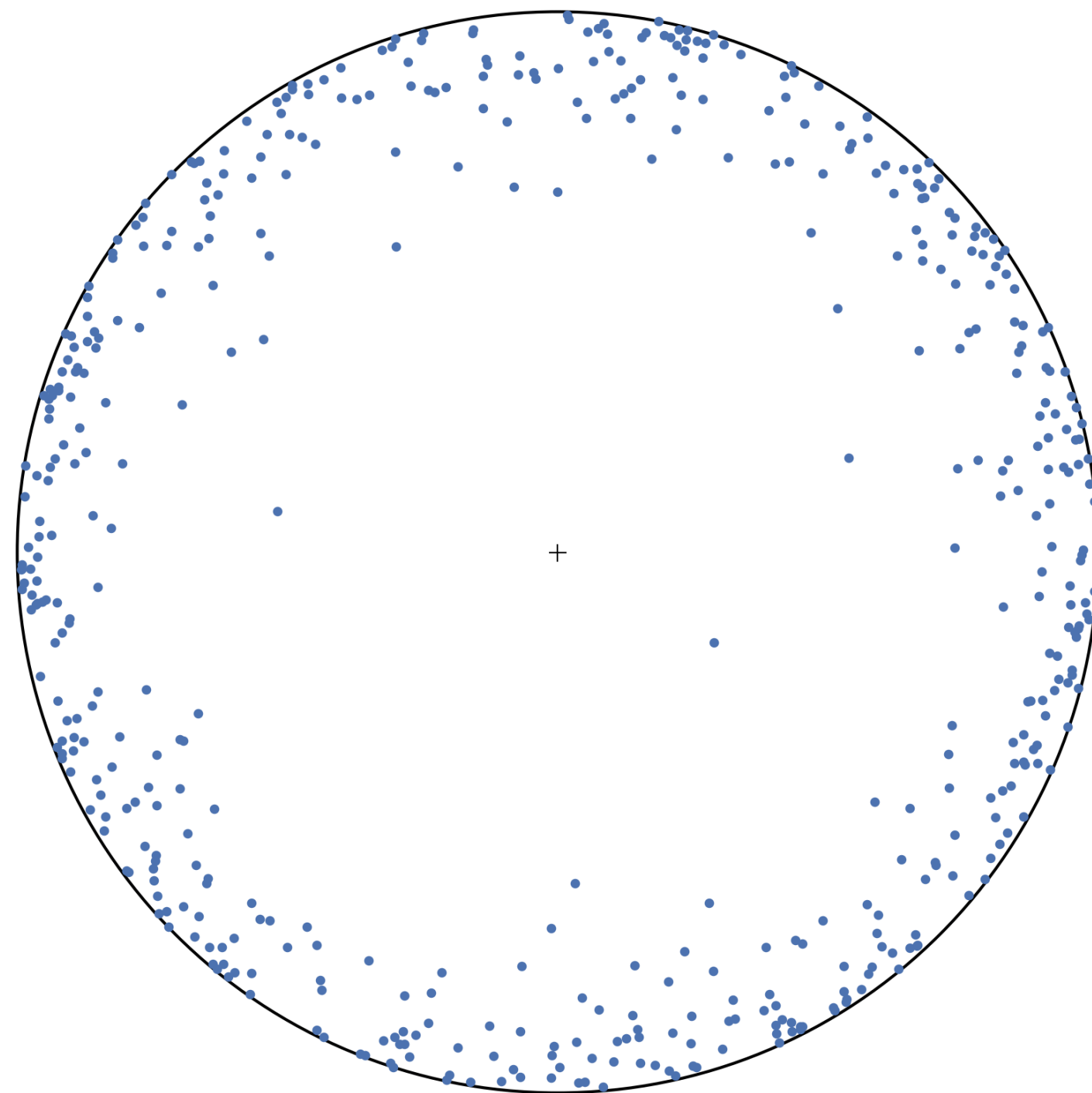
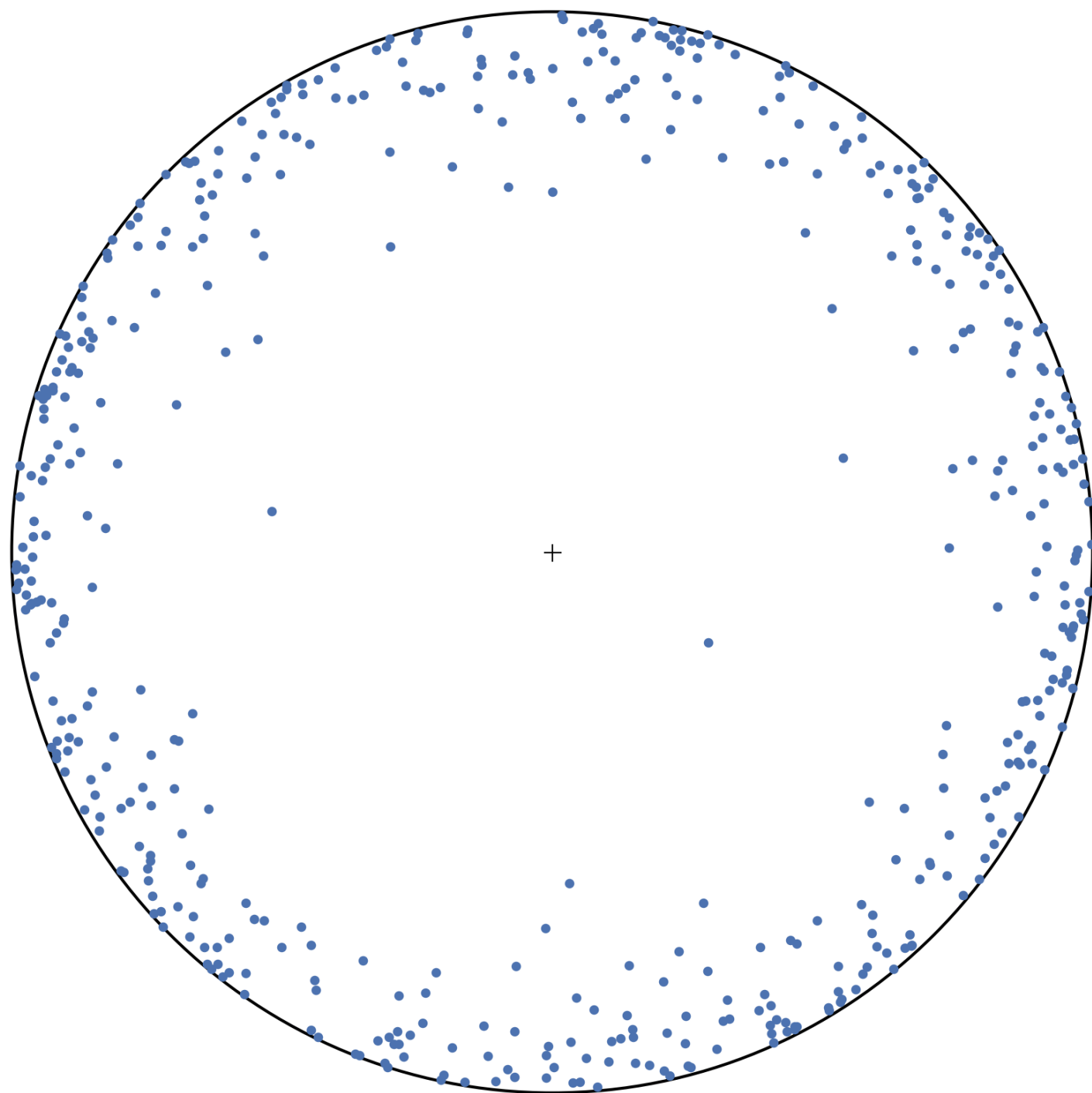
✓ high clustering

✓ power-law degree distribution with exponent  $-3$

Phys. Rev. E 82, 036106 (2010)



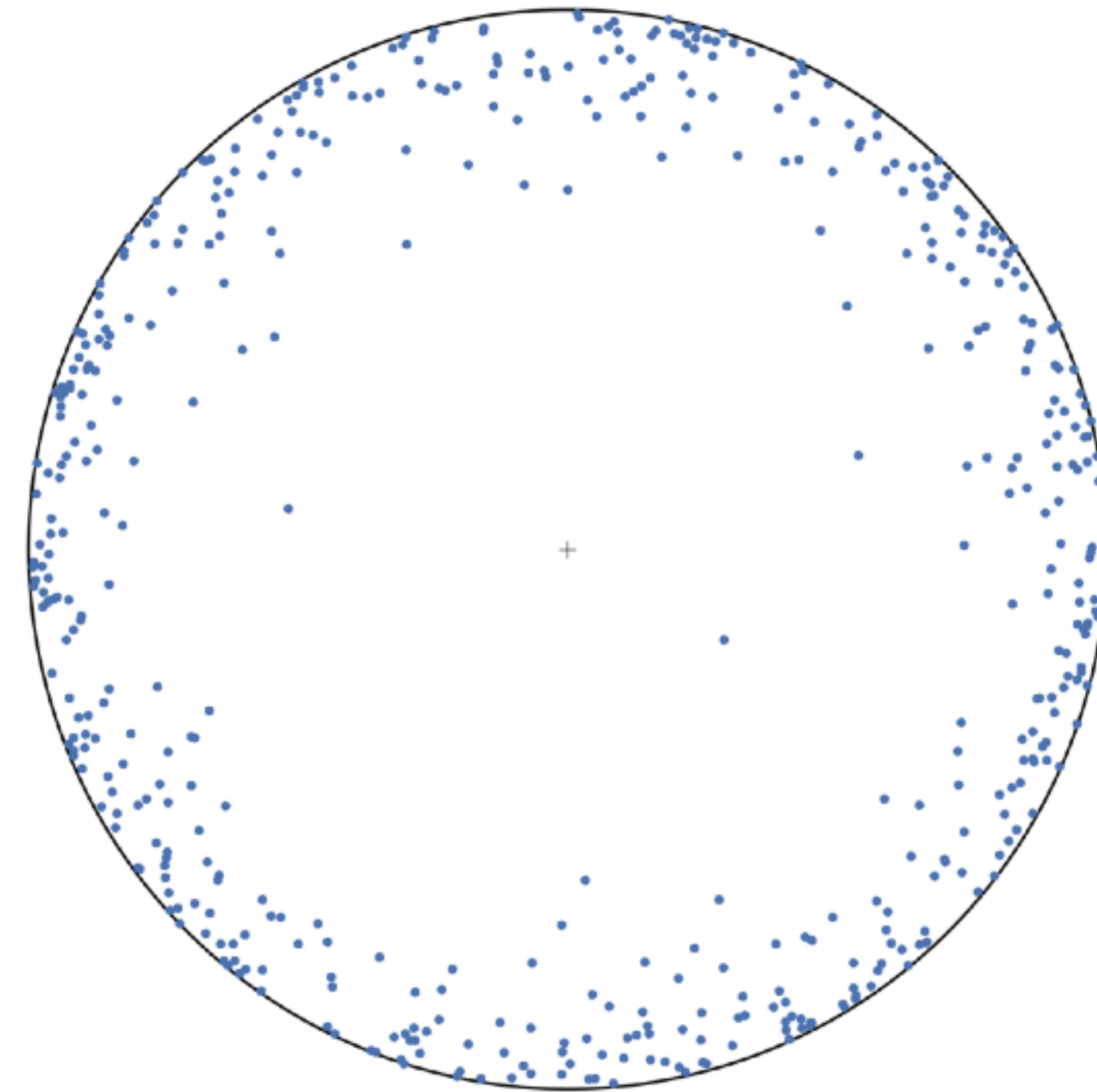
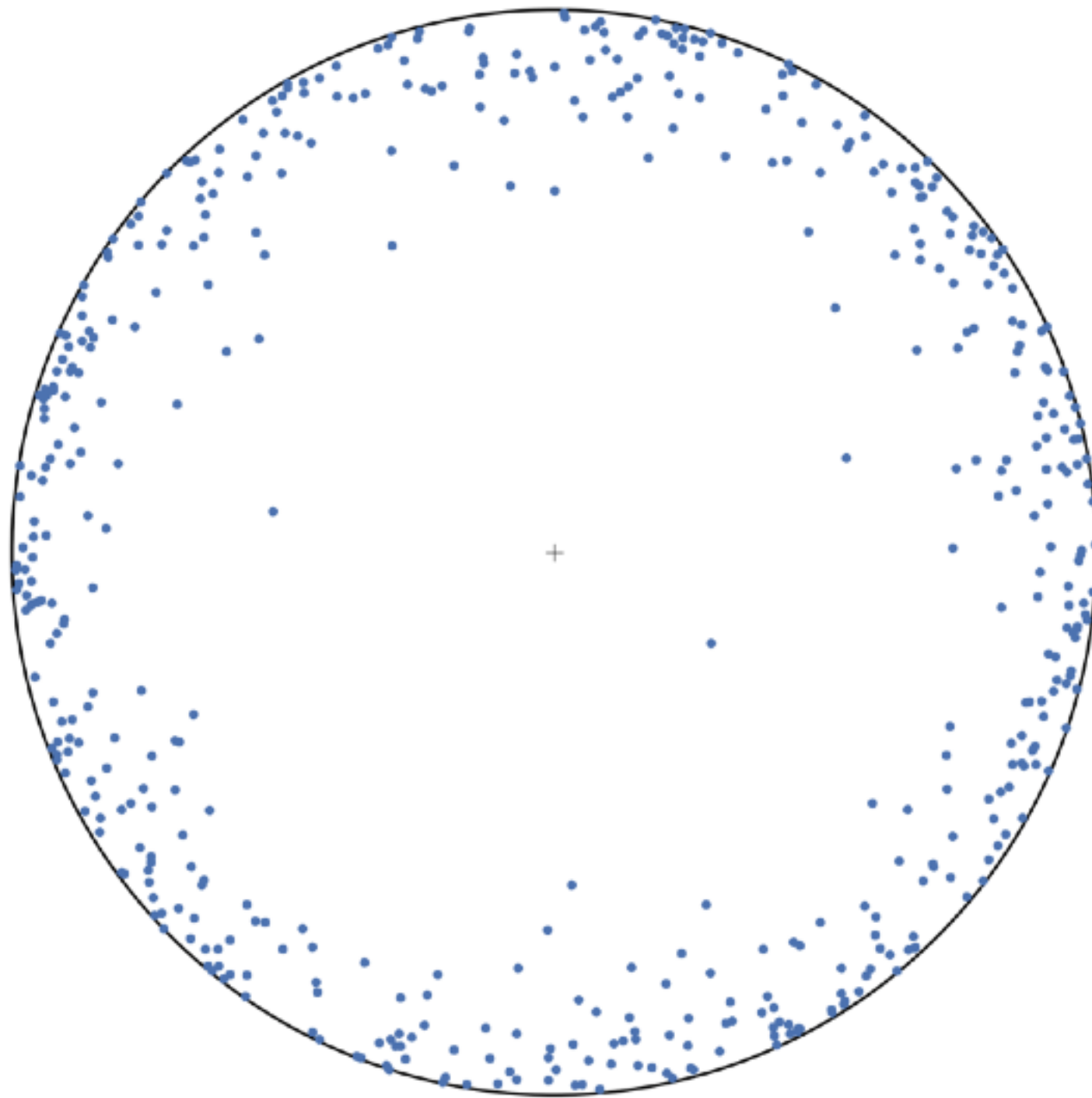




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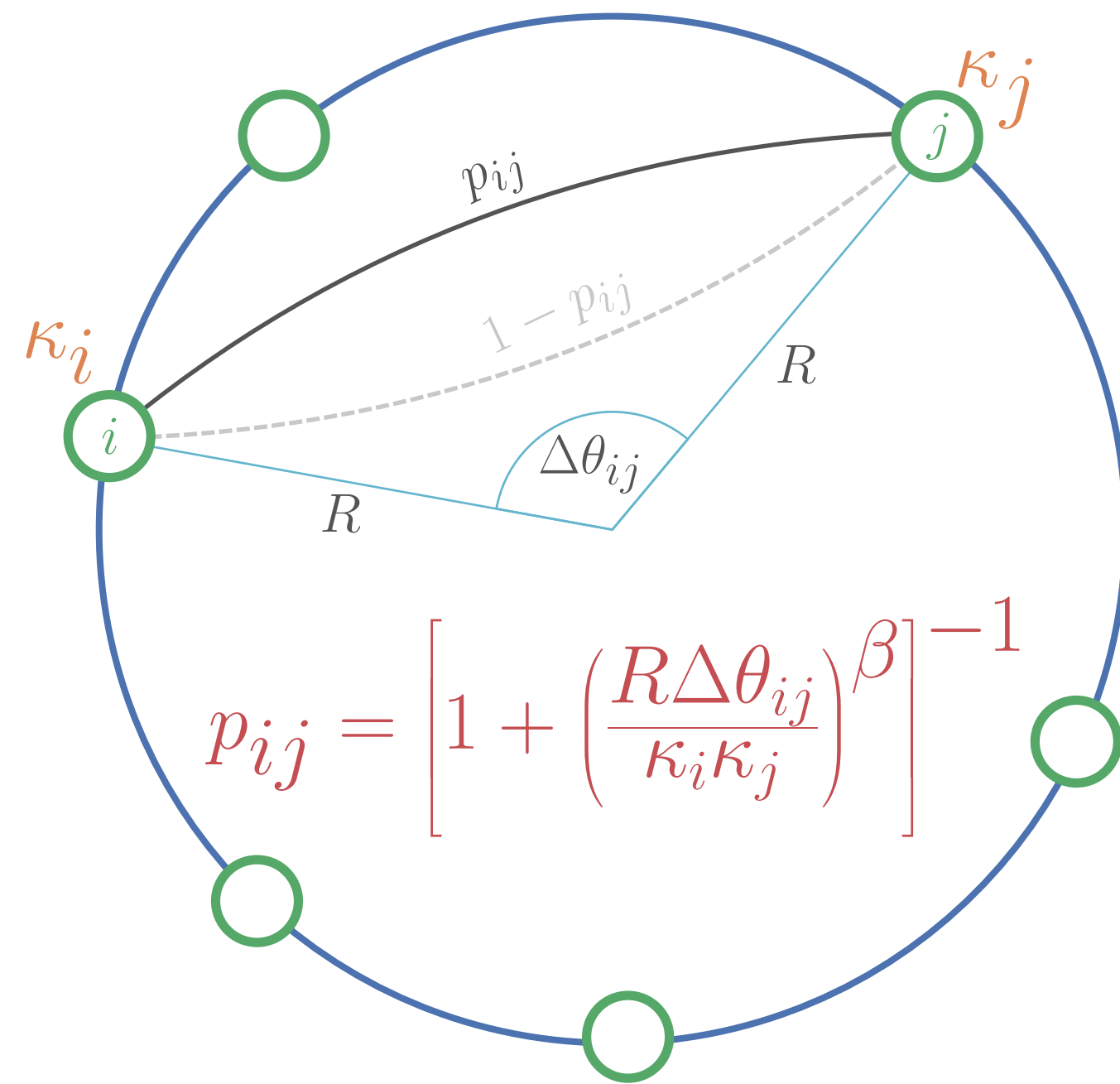
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- ✓ high clustering
- ✓ power-law degree distribution with exponent  $-3$

# A geometric approach to clustering : the $\mathbb{S}^1/\mathbb{H}^2$ model



## The $\mathbb{S}^1$ model

1. Sprinkle  $N$  nodes uniformly on a circle of radius  $R$ .
2. Assign an expected degree  $\kappa$  to each node according to some pdf  $\rho(\kappa)$ .
3. Draw a link between node  $i$  and node  $j$  with probability  $p_{ij}$ .

- ★ fixes the expected degree of nodes ( $\kappa$ ) → soft configuration model (CM)
- ★ triangle inequality of the underlying metric space → triangles from pairwise interactions
- ★ level of clustering tuned with parameter  $\beta$

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