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Example 4: fixing the expected degree sequence and the expected total energy

$$\bar{F}_l = \sum_{j=1}^{N} a_{lj} = k_l \quad (l = 1, \ldots, N)$$

$$\bar{F}_{N+1} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \varepsilon_{ij} a_{ij} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} f(x_{ij}) a_{ij} = E$$

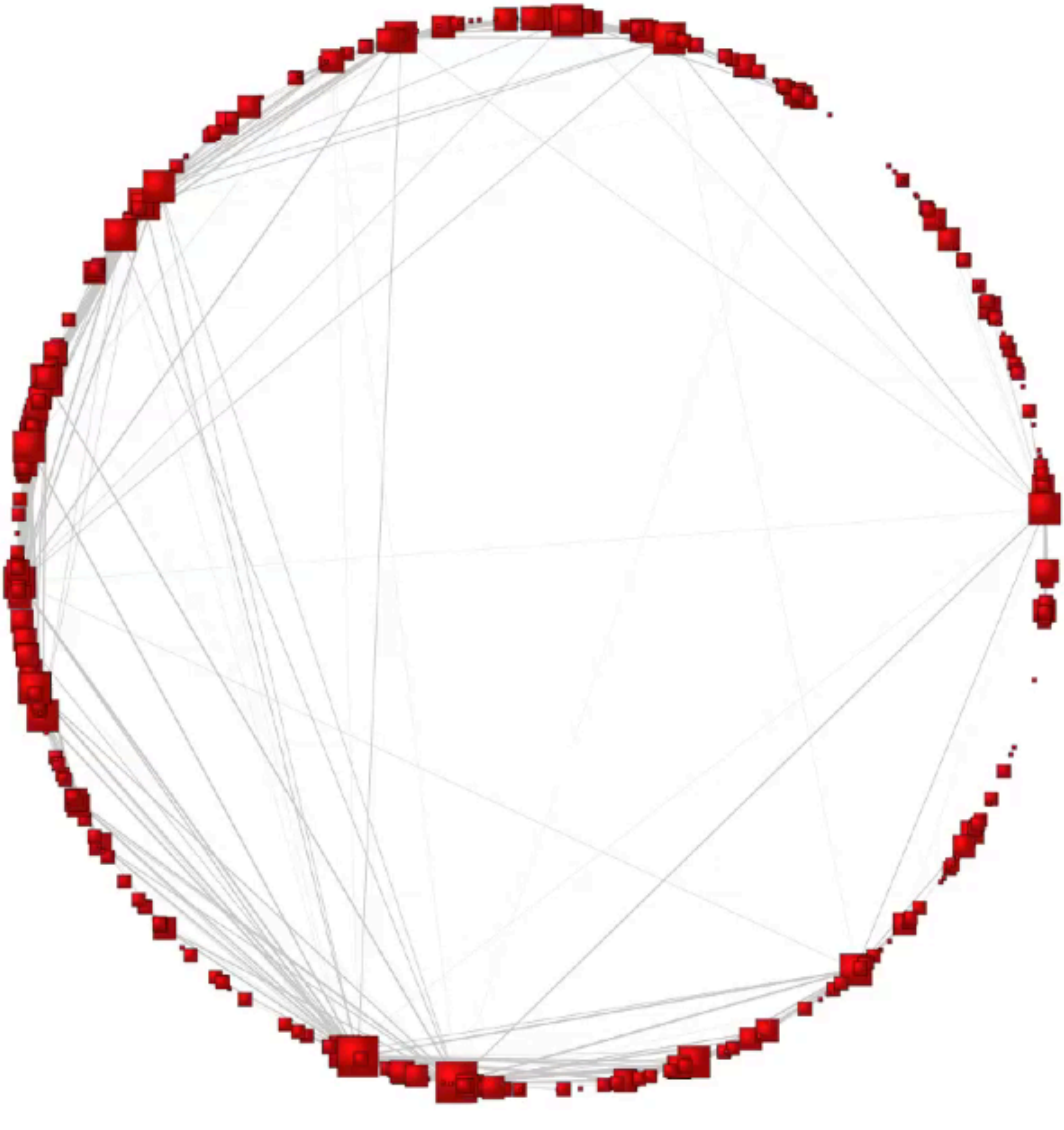
yields the heterogeneous random geometric graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1 - p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{\beta\varepsilon_{ij} - \alpha_i - \alpha_j} + 1} \, .$$

The graphs will be sparse, highly clustered, small-world and devoid of non-structural degre-degre correlation iif $f(x_{ij}) = \ln x_{ij}$ and $\beta \in [D, D+2]$[a]. Redefining $\alpha_l = -(\beta/D) \ln(\sqrt{\mu}\kappa_l)$ yields

$$p_{ij} = \frac{1}{e^{\beta(\varepsilon_{ij} - \mu)} + 1} \quad \text{with} \quad \varepsilon_{ij} = \ln\left(\frac{x_{ij}}{(\kappa_i \kappa_j)^{\frac{1}{D}}}\right) \, .$$

[a] No upper bound if expected degree sequence is scale-free.



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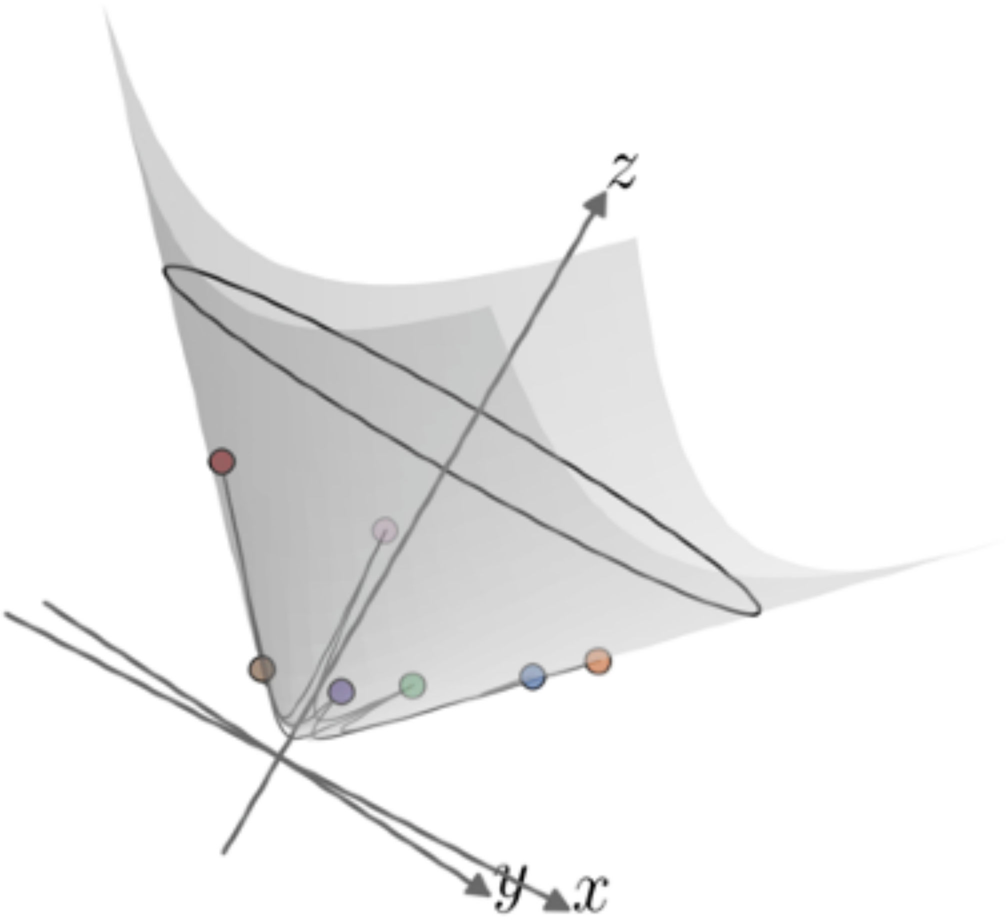
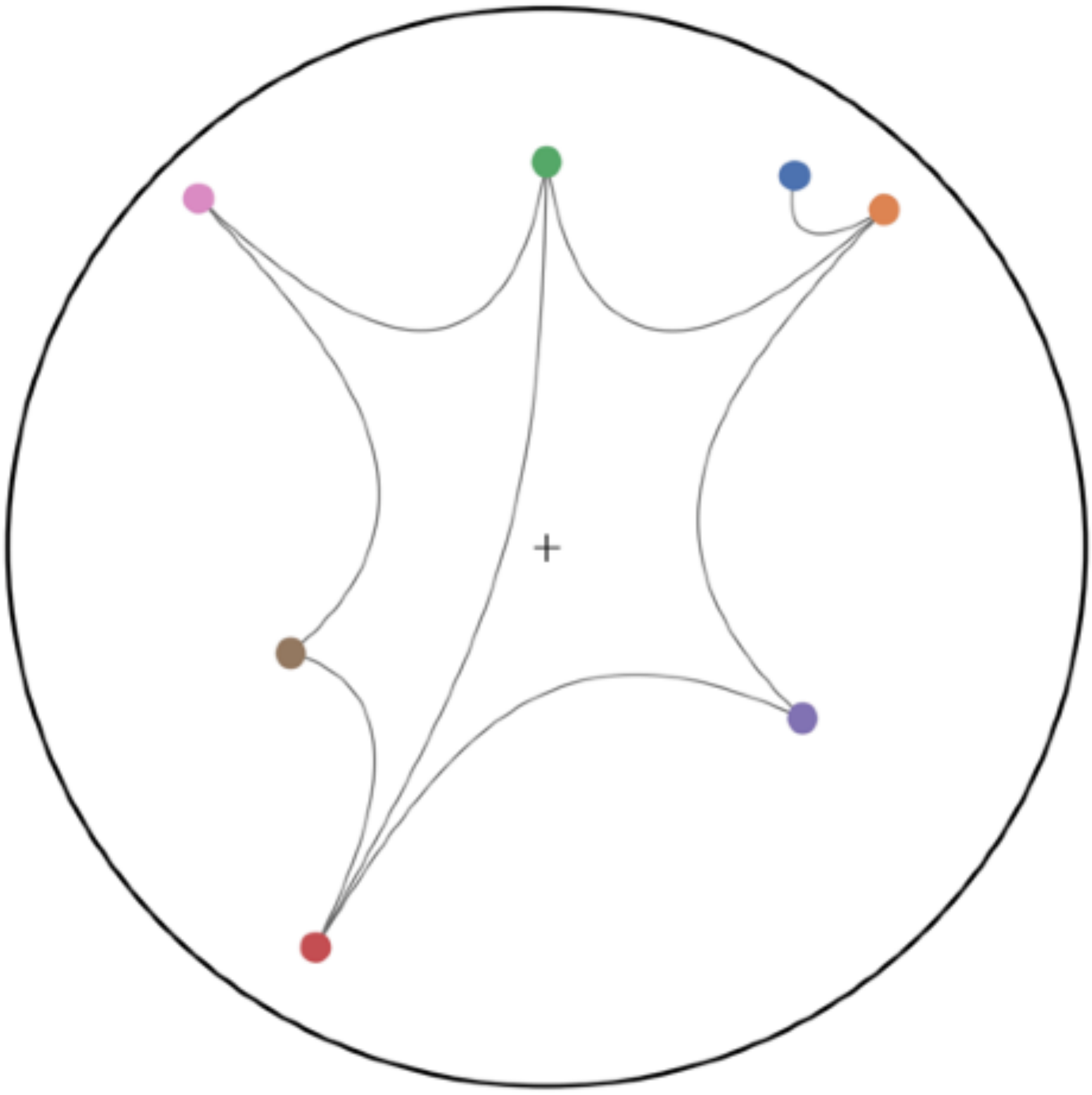
Phys. Rev. E 80, 035101 (2009)

Phys. Rev. E 82, 036106 (2010)



hyperboloid in $\mathbb{R}^{2,1}$

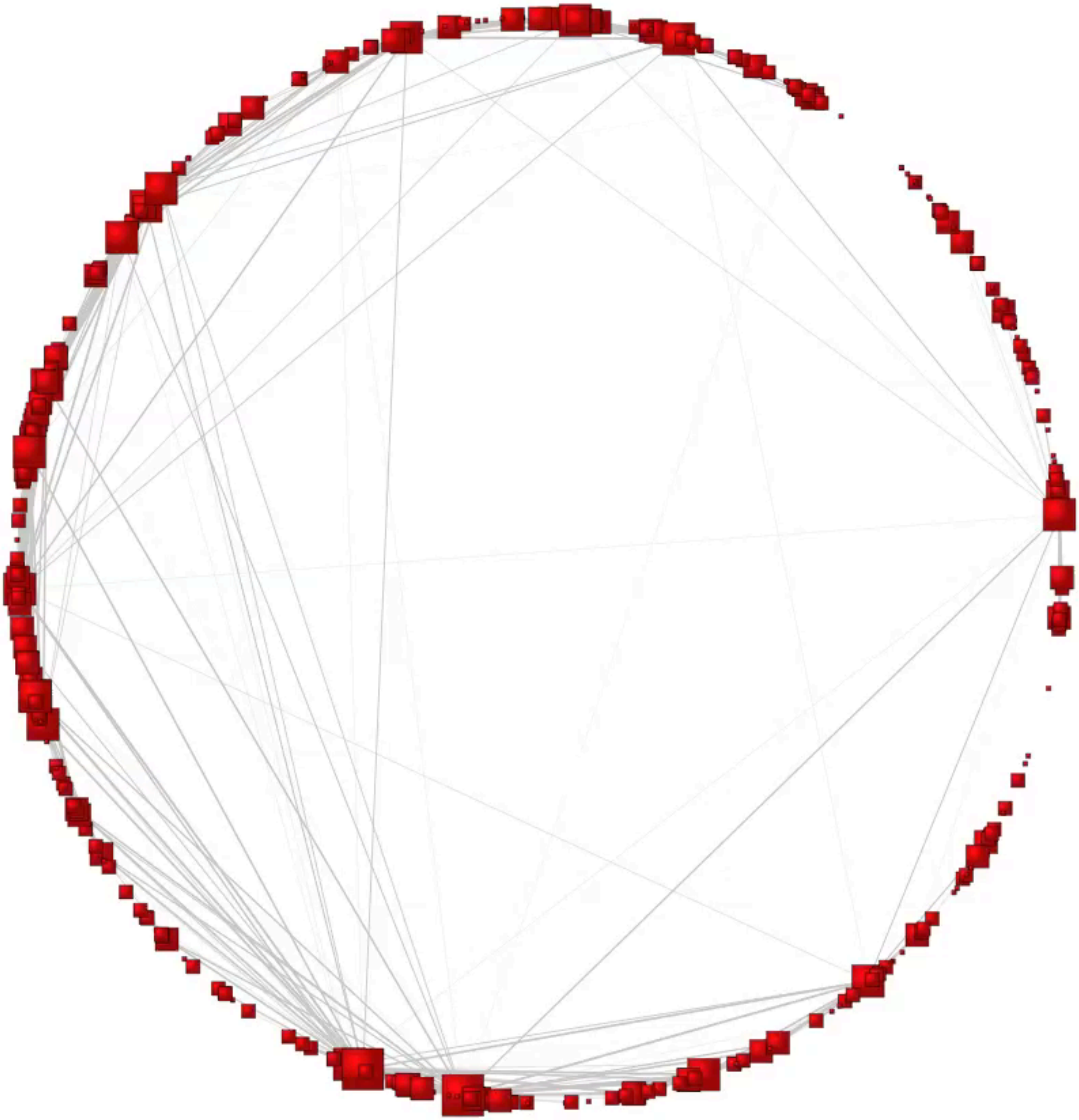


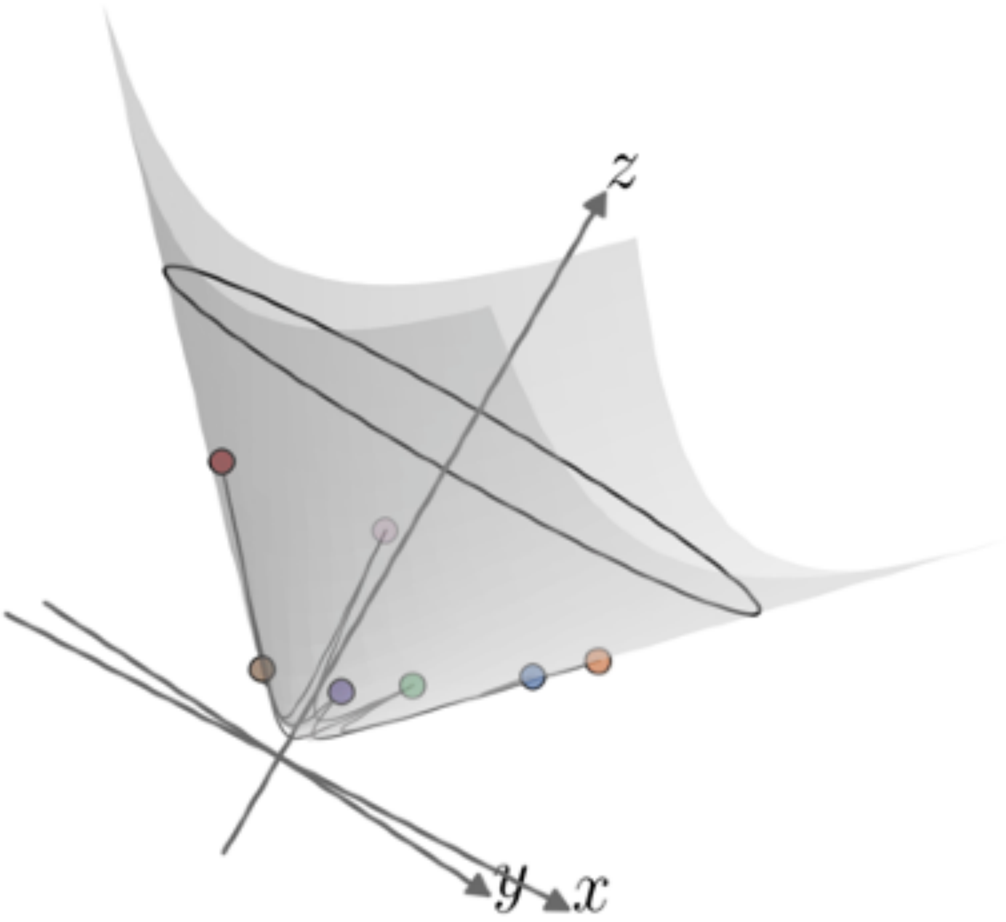
hyperbolic disk (r, θ)

Maximally random geometric graph ensembles

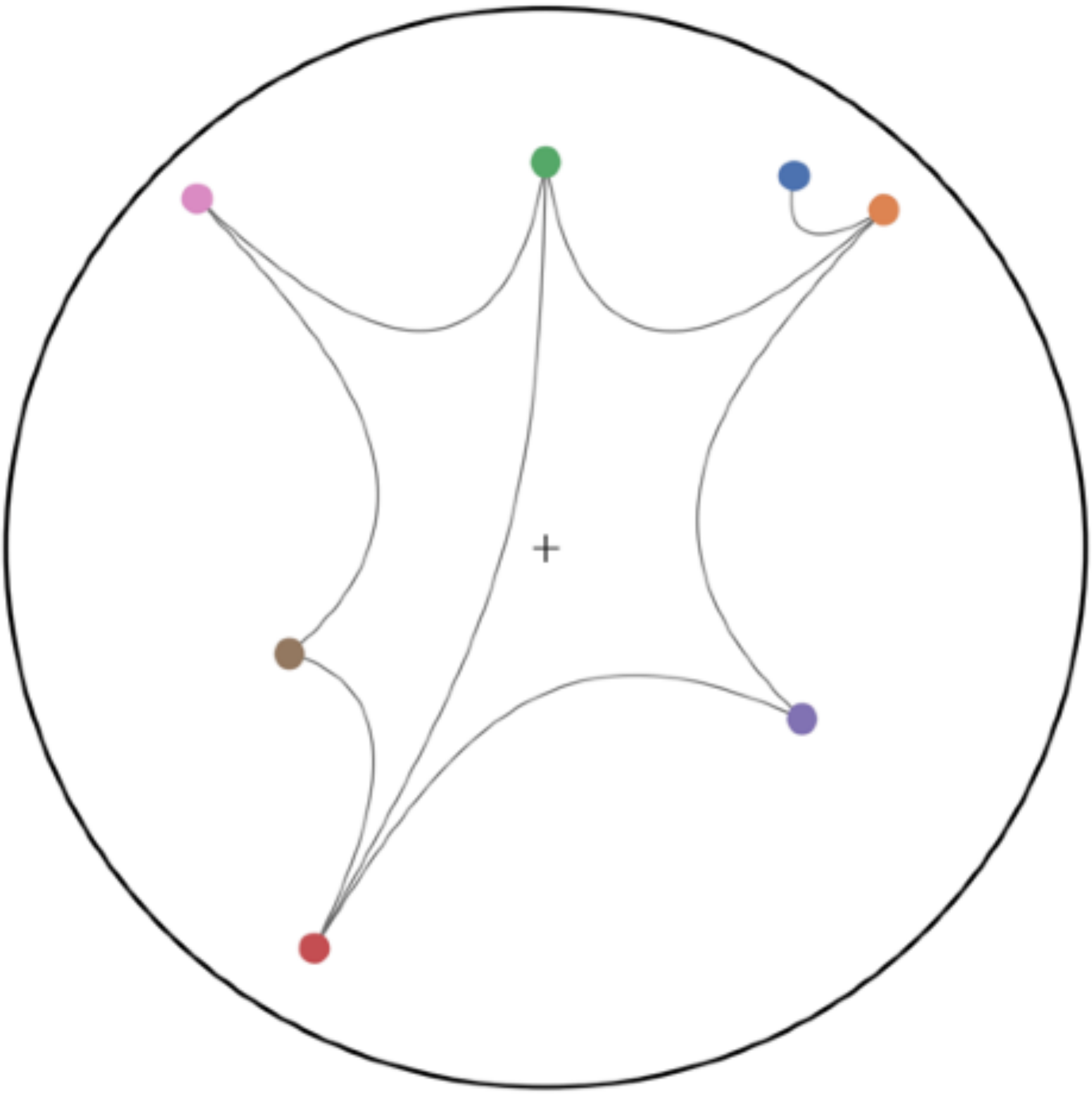
Courtesy of M. Boguñá

When the geometry is a D-dimensional sphere, $\mathbb{S}^D$ the model can be mapped to a purely geometric model in hyperbolic space $\mathbb{H}^{D+1}$.

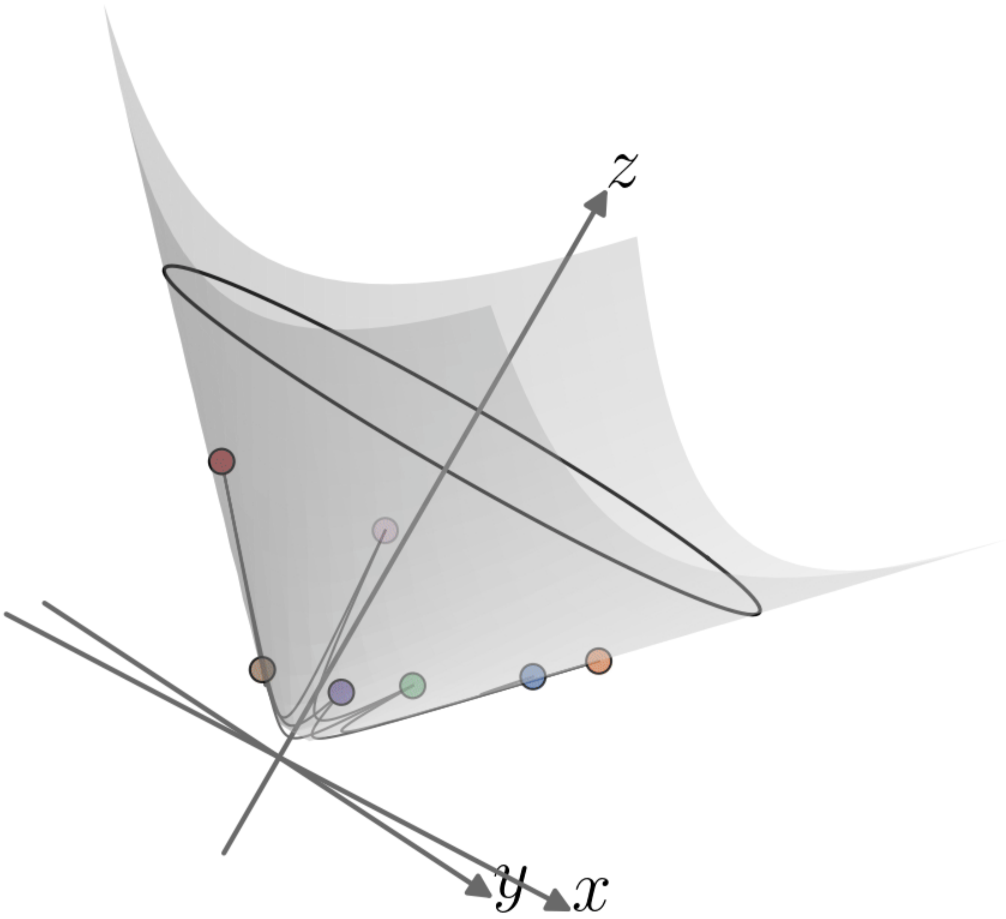




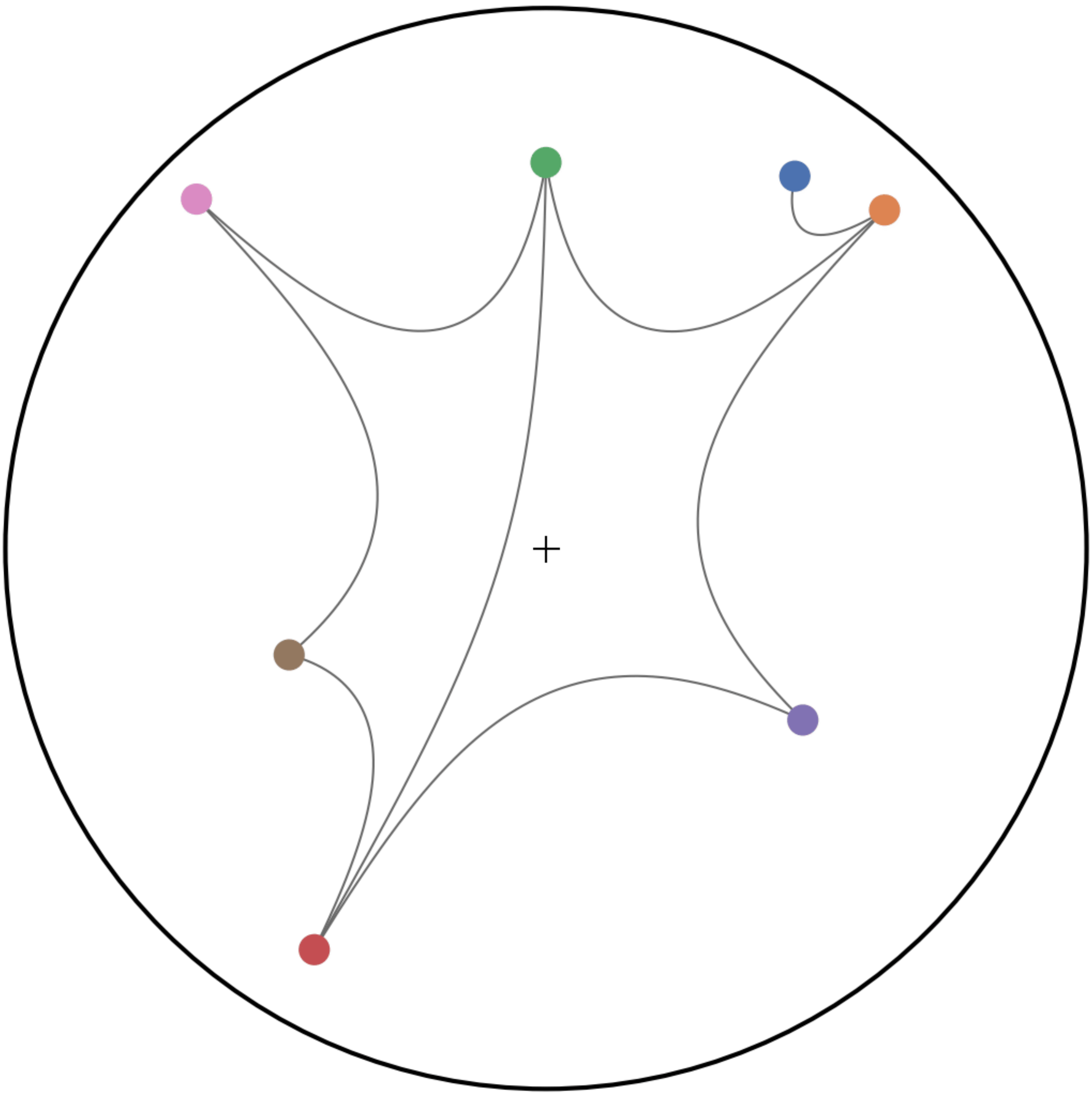
hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)



hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

Maximally random geometric graph ensembles

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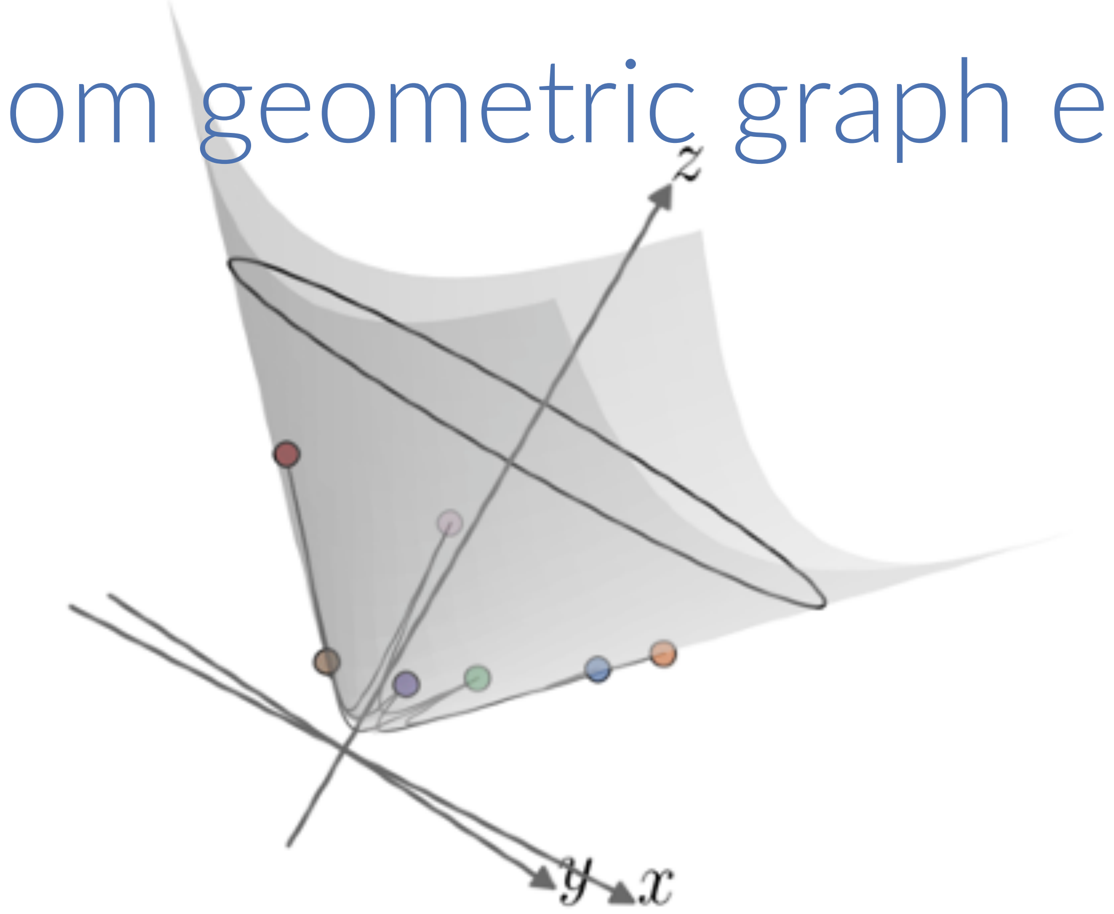
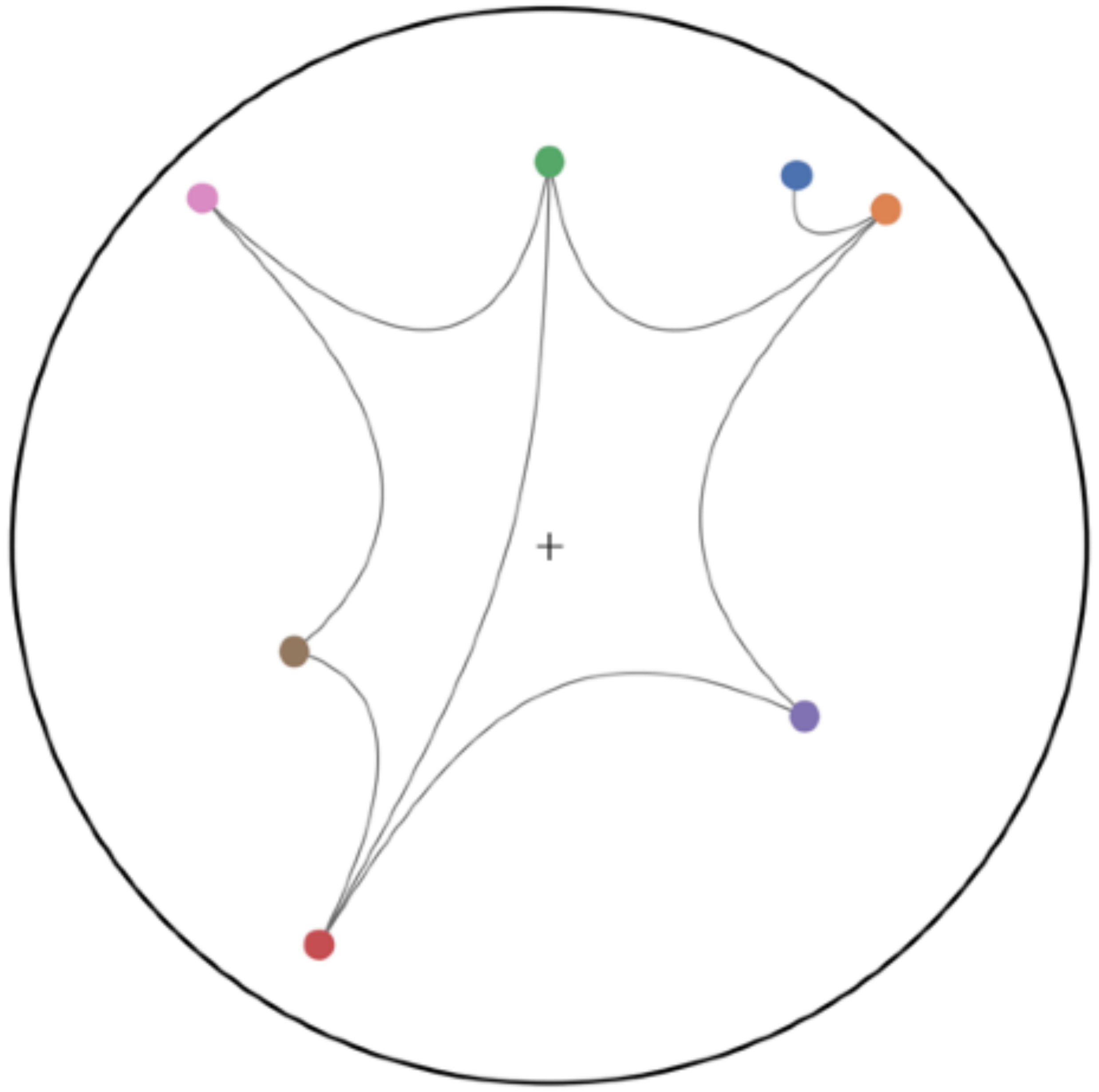
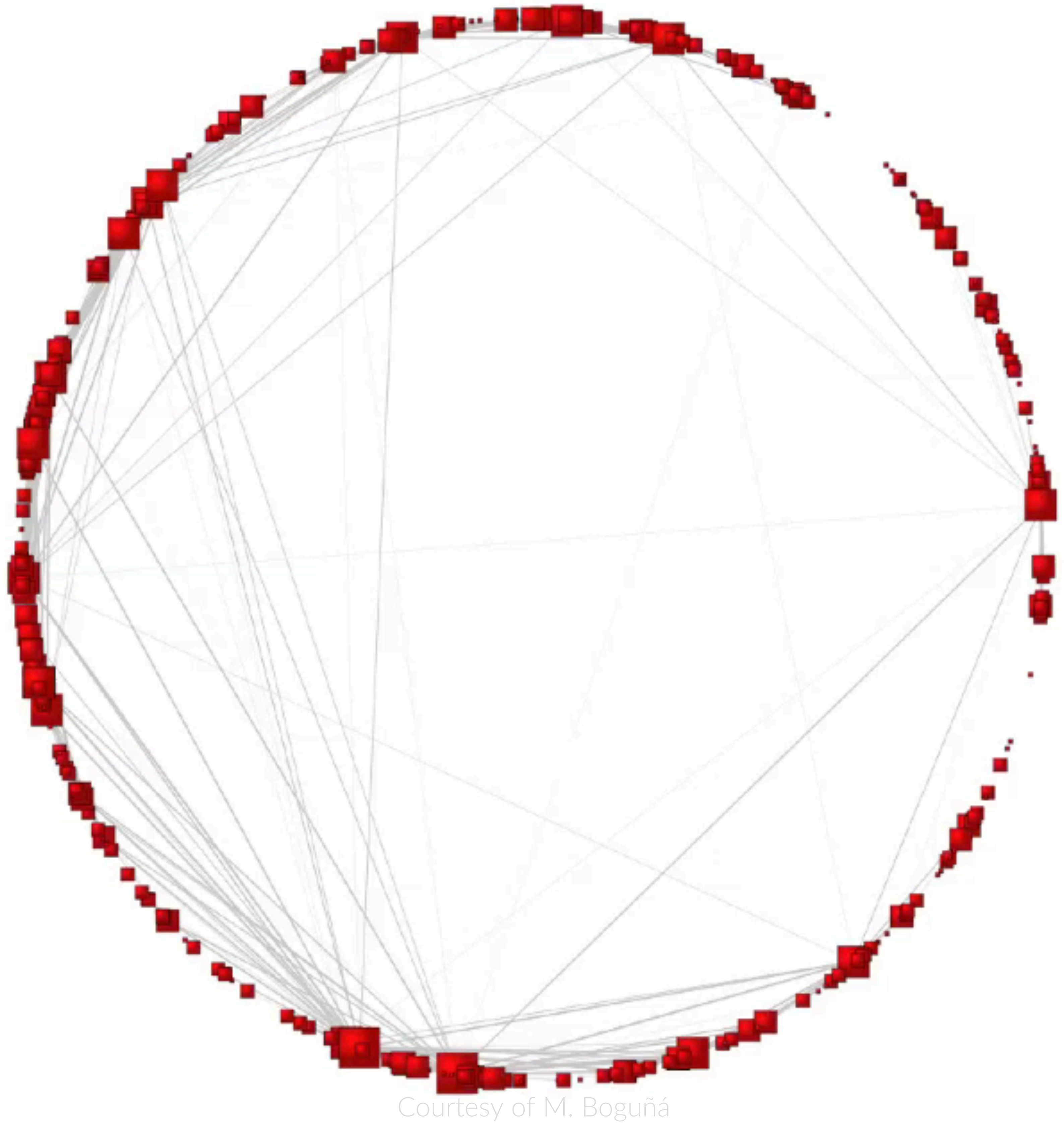


hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)



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A powerful and versatile framework

- ▷ Amenable to many analytical calculations [1,2]
- ▷ Generalizable to weighted [5], bipartite [6,7,8], multiplex [9,10], directed [4] and growing [11] networks
- ▷ Geometrical interpretation of preferential attachment [11]
- ▷ Parsimonious explanation of self-similarity [3]
- ▷ Generalizable to networks with community structure [12,13,14]
- ▷ Mapping of real complex networks unto hyperbolic space [15,16]
  - Reproduction of addtionnal properties than the ones used to fit the parameters [4,15].
  - Identification of biochemical pathways in E. Coli [8]
  - Efficient Internet routing protocols [17]
  - Organization of the human connectome [18,20]
  - Self-similar architecture [19]
  - Evolution of hierarchy in international trade [21]
  - ...
- ▷ ...

[1] Phys. Rev. E 80, 035101 (2009)
[2] Phys. Rev. E 82, 036106 (2010)
[3] Phys. Rev. Lett. 100, 078701 (2008)
[4] Nat. Phys. 20, 150 (2024)
[5] Nat. Commun. 8, 14103 (2017)
[6] Phys. Rev. E 84, 026114 (2011)
[7] Phys. Rev. E 95, 032309 (2017)
[8] Mol. Biosyst. 8, 843 (2012)
[9] Nat. Phys. 12, 1076 (2016)
[10] Phys. Rev. Lett. 118, 218301 (2017)
[11] Nature 489, 537 (2012)
[12] Sci. Rep. 5, 9421 (2015)
[13] J. Stat. Phys. 173, 775 (2018)
[14] New J. Phys. 20, 052002 (2018)
[15] New J. Phys. 21, 123033 (2019)
[16] Nat. Commun. 8, 1615 (2017)
[17] Nat. Commun. 1, 62 (2010)
[18] PNAS 117, 20244 (2020)
[19] Nat. Phys. 14, 583 (2018)
[20] PLOS Comput. Biol. 16, e1007584 (2020)
[21] Sci. Rep. 6, 33441 (2016)