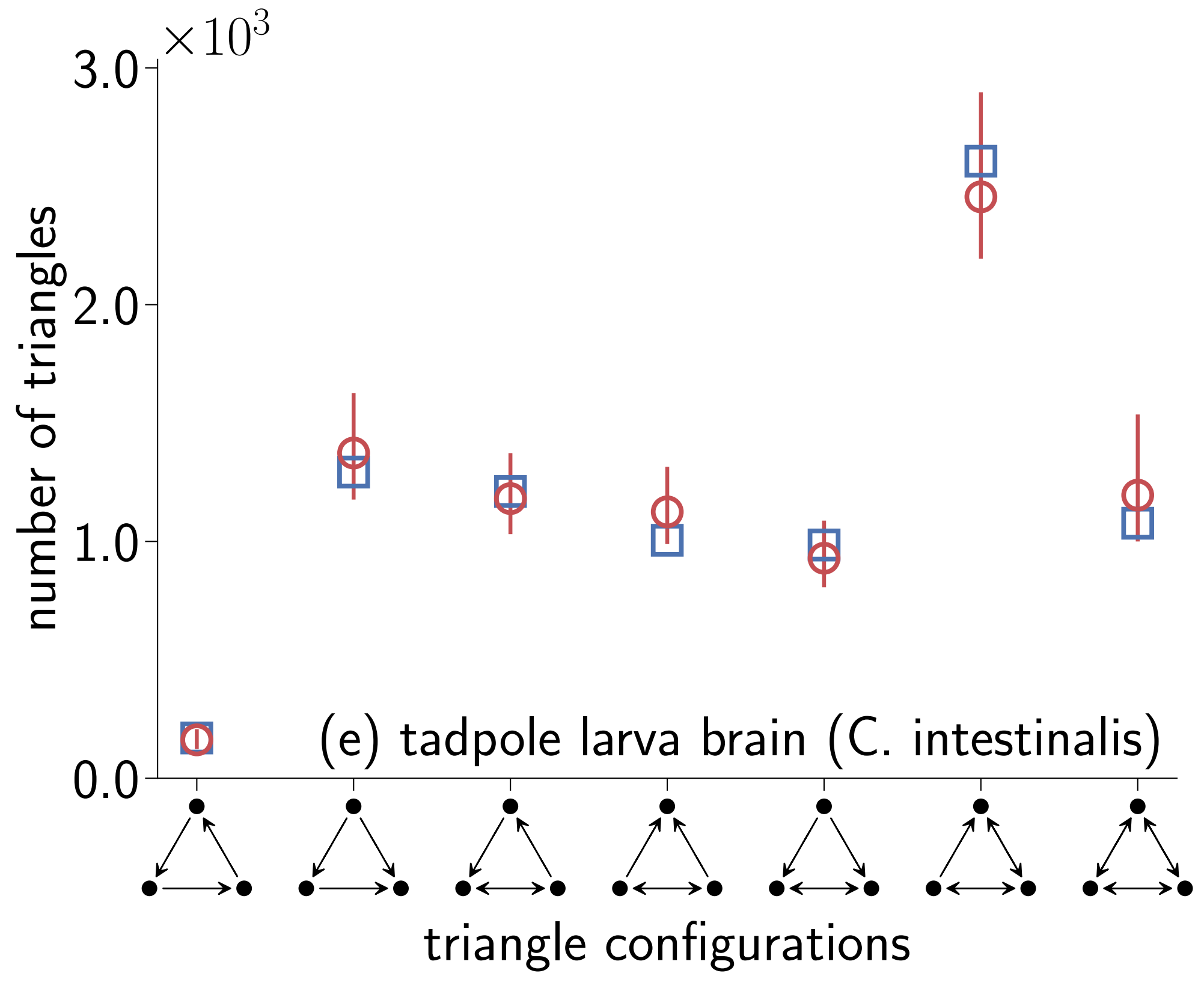
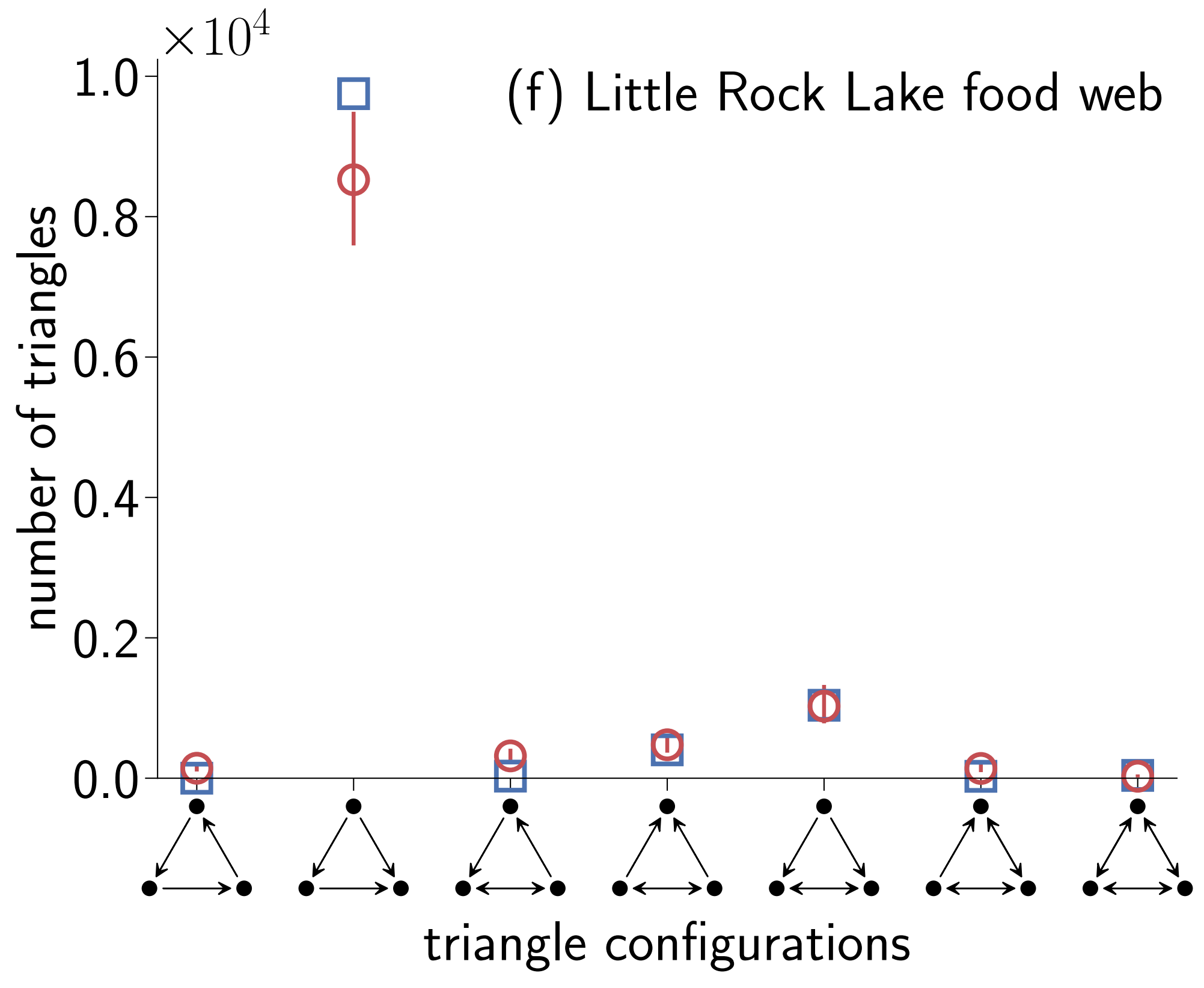
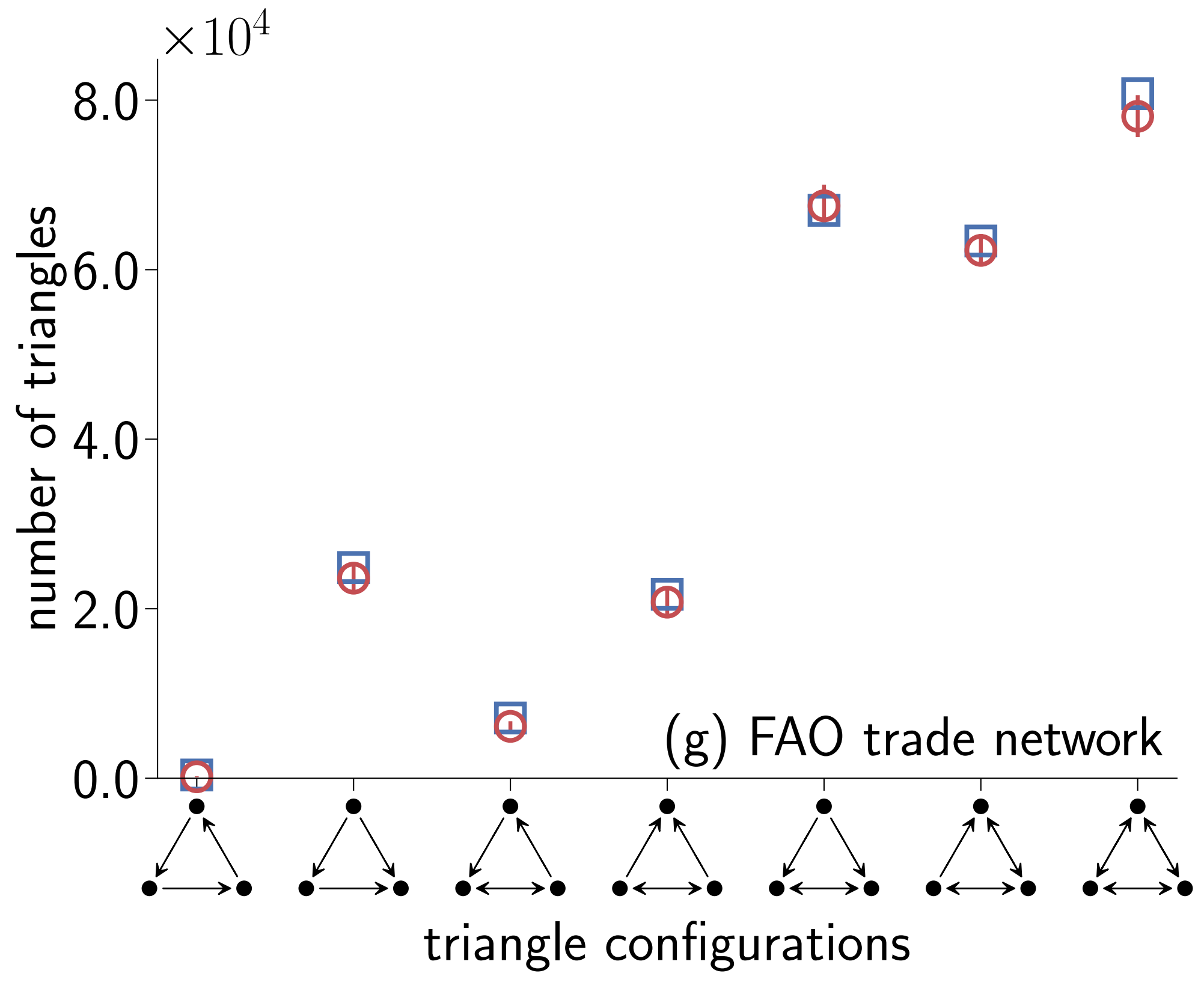
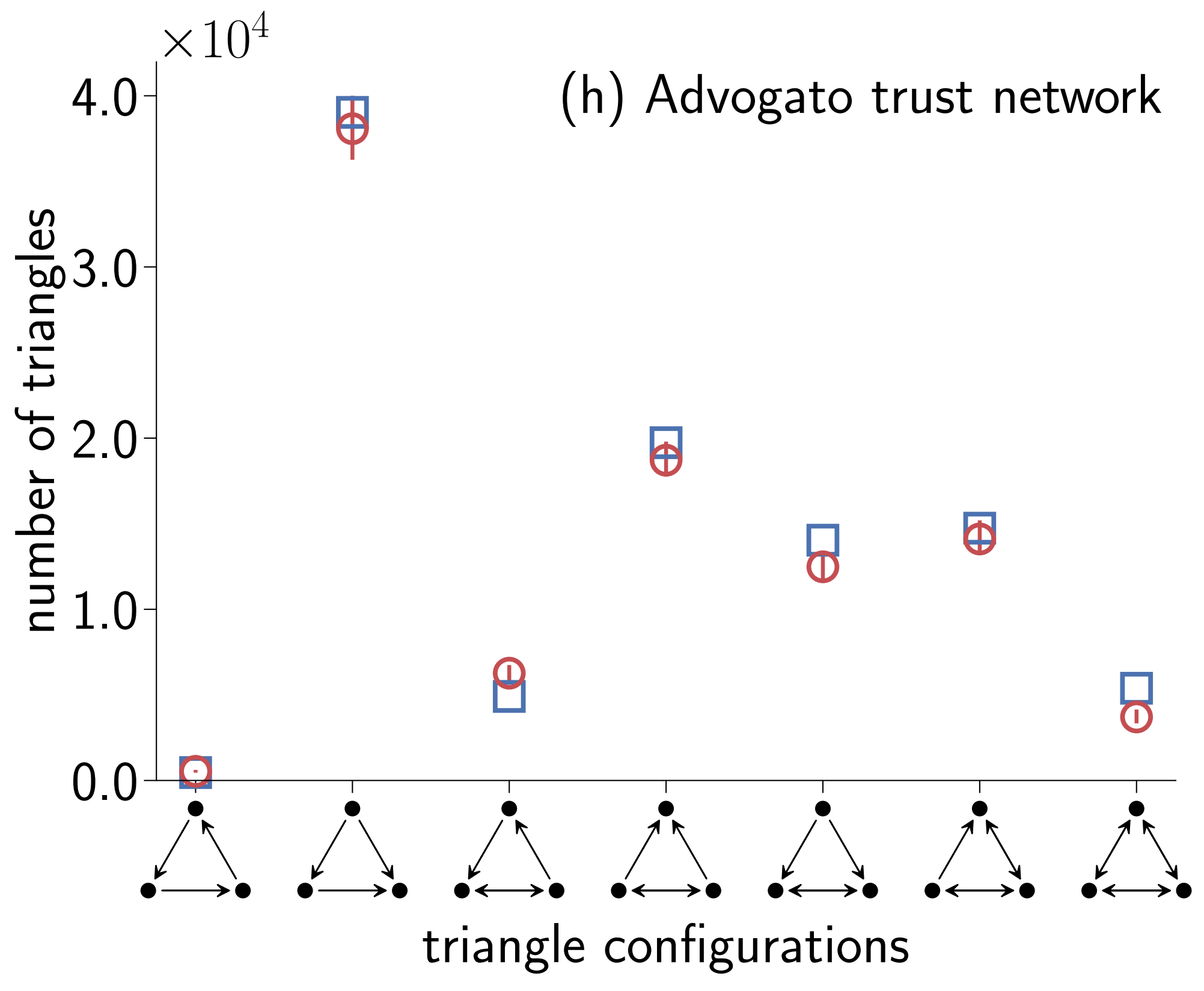
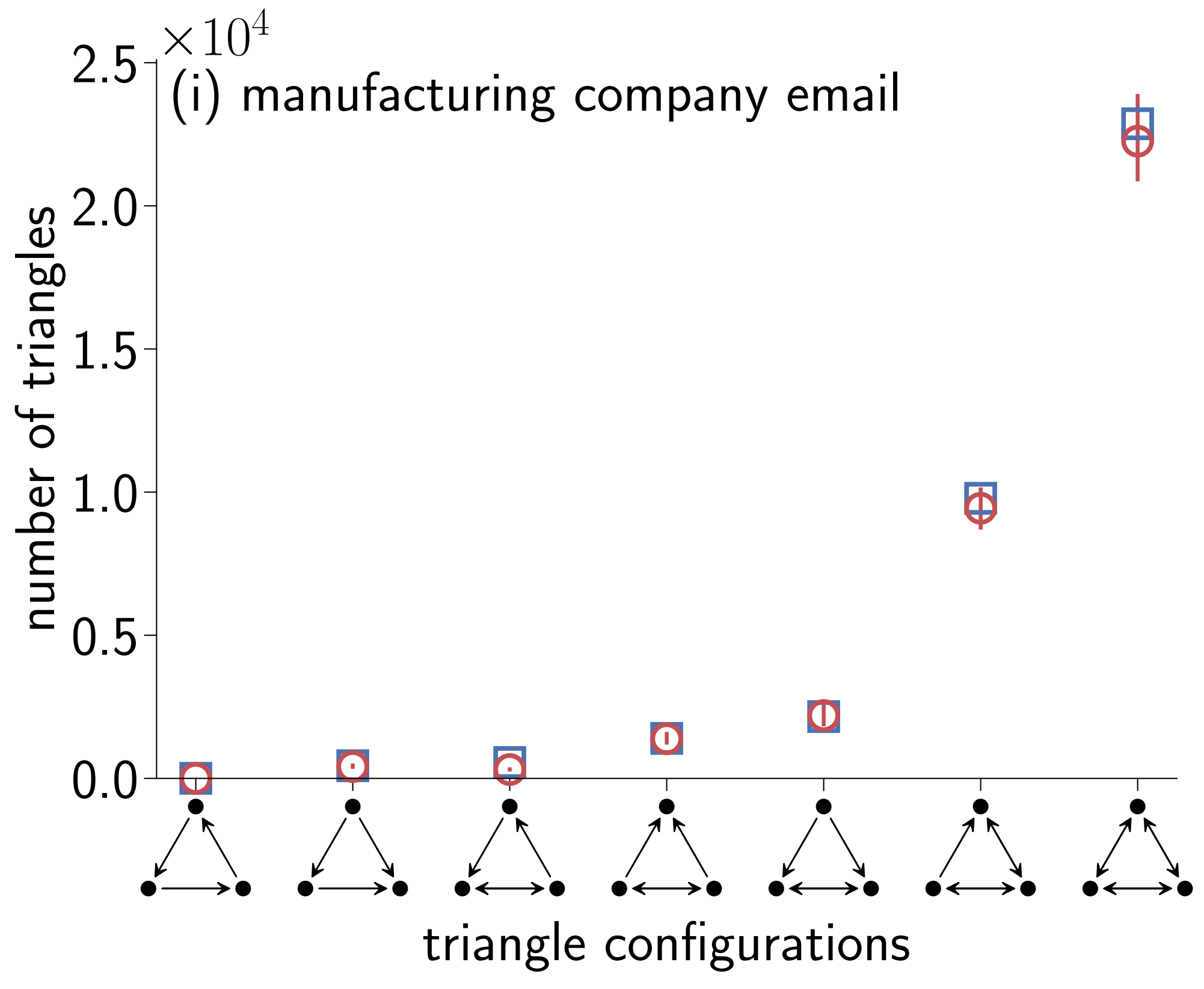
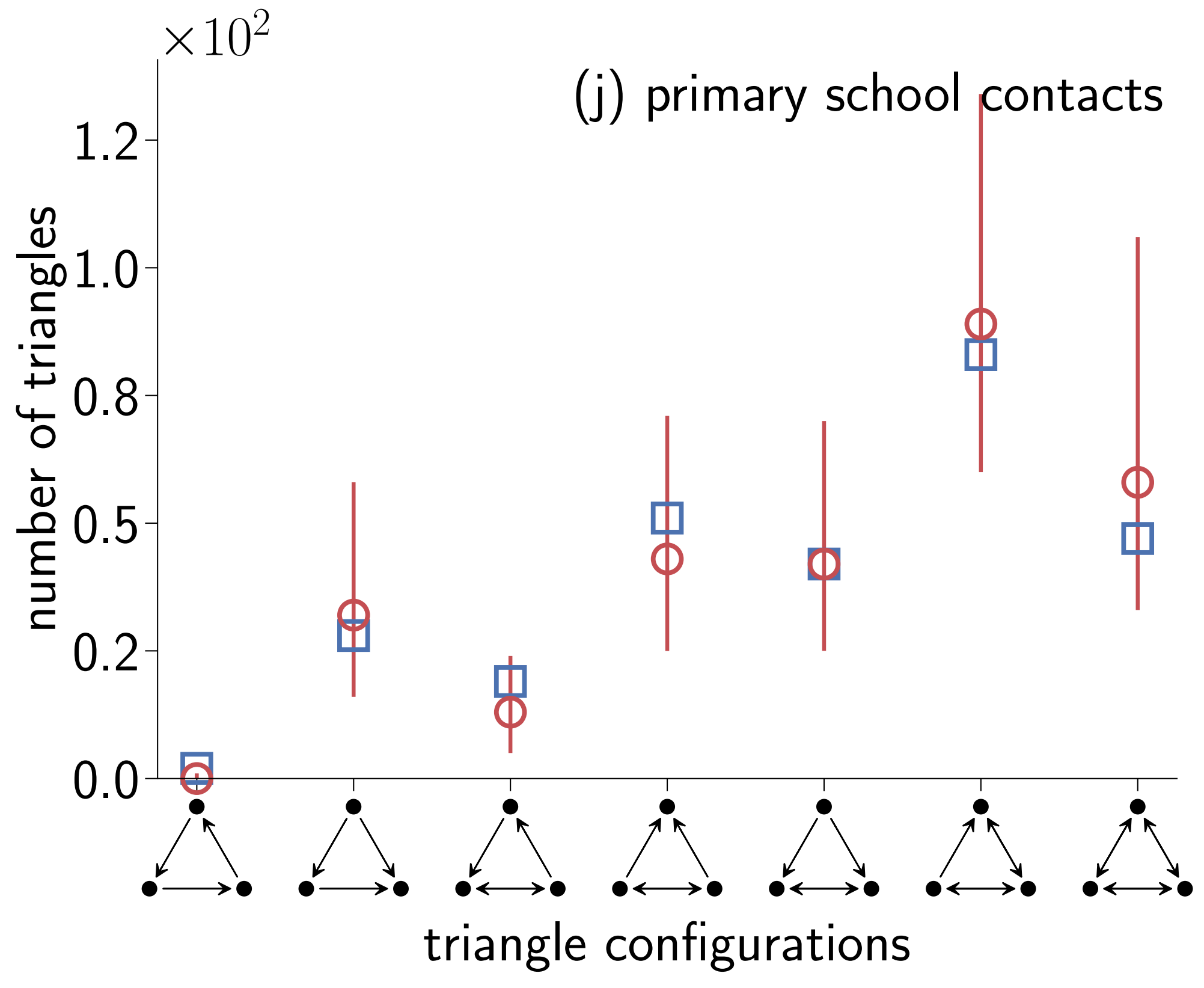

$\times 10^3$
3.0
2.0
1.0
0.0
number of triangles
(e) tadpole larva brain (C. intestinalis)
triangle configurations

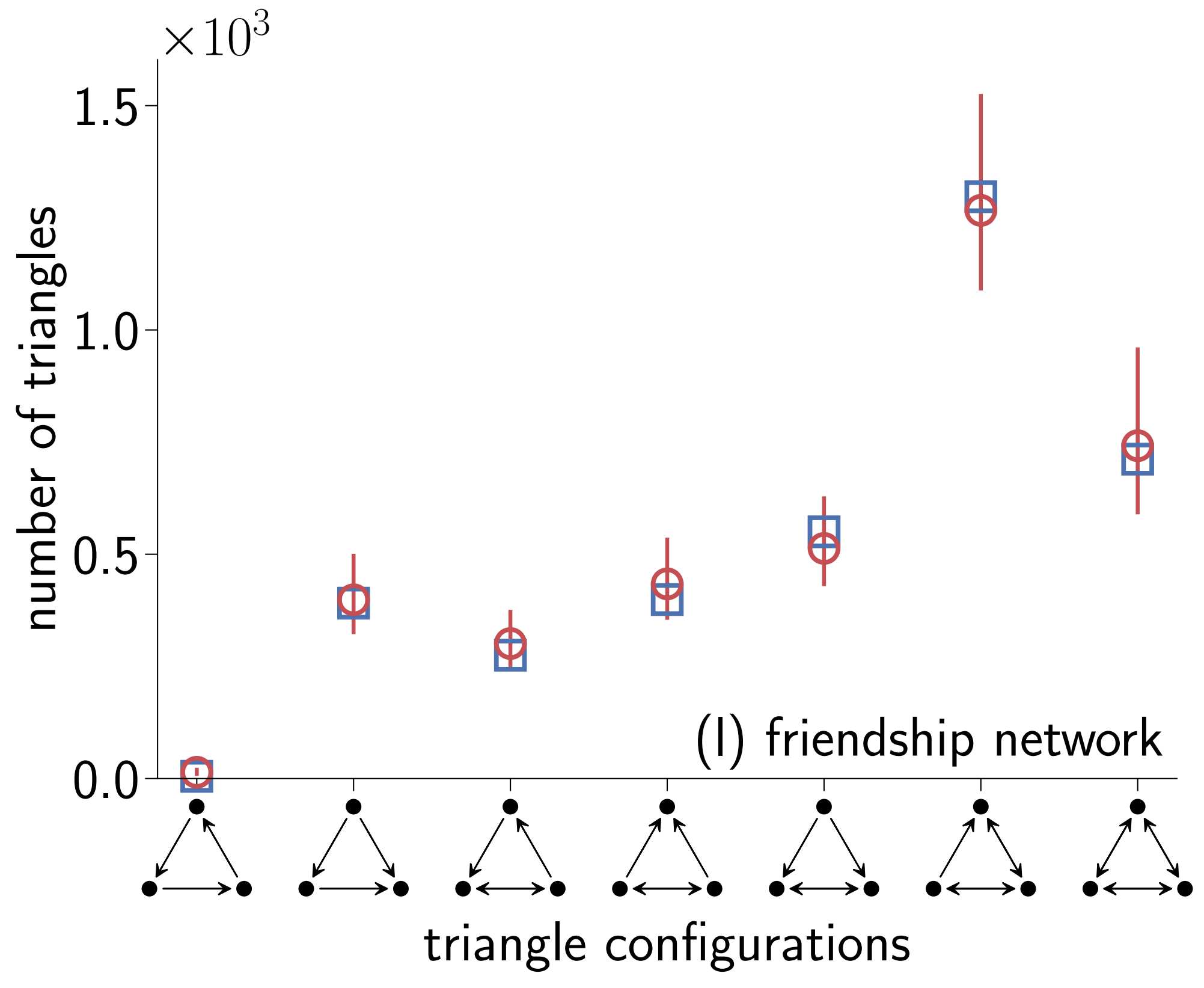

(f) Little Rock Lake food web
$\times 10^4$
1.0
0.8
0.6
0.4
0.2
0.0
number of triangles
triangle configurations

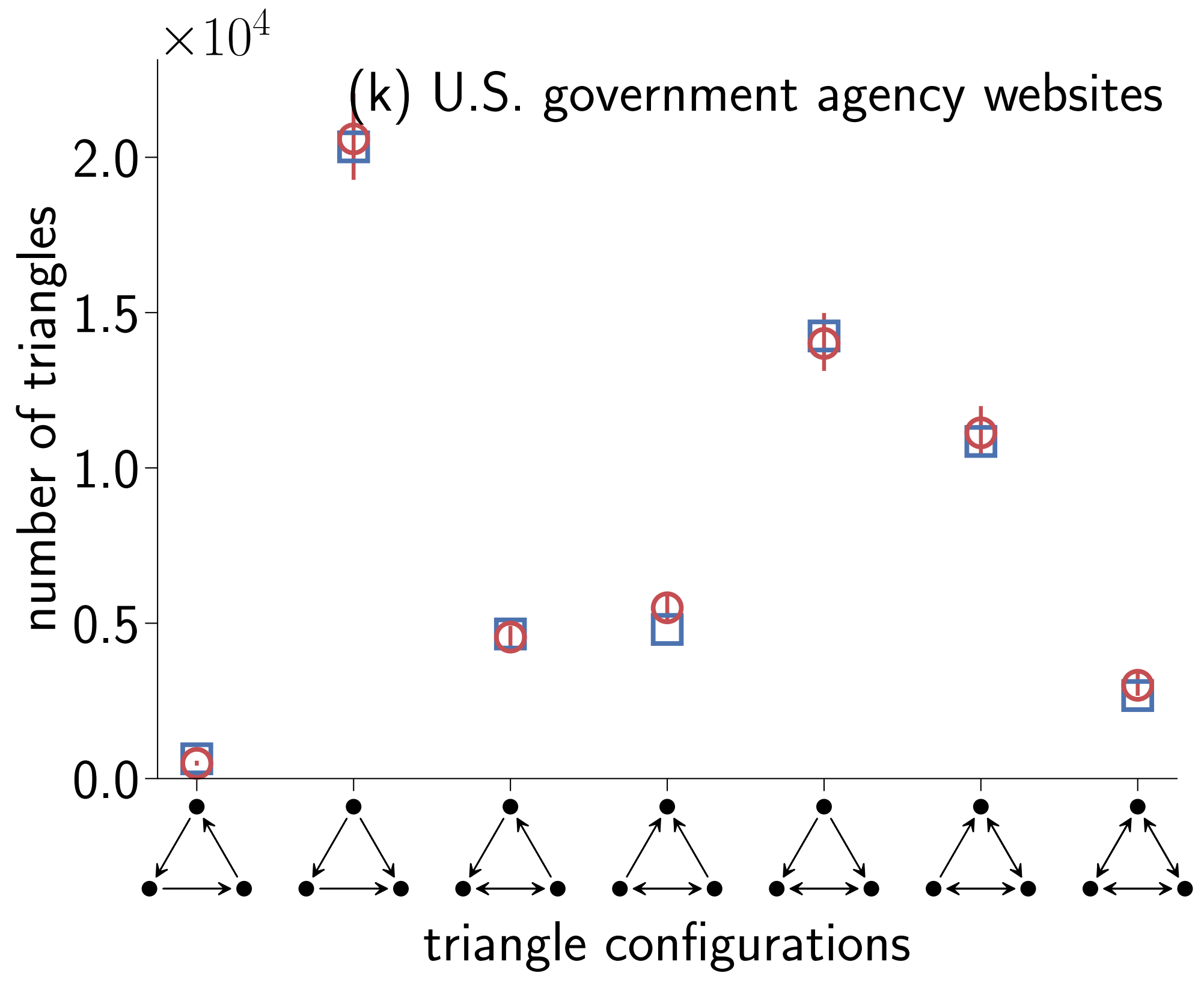

×10^4
8.0
6.0
4.0
2.0
0.0
number of triangles
(g) FAO trade network
triangle configurations

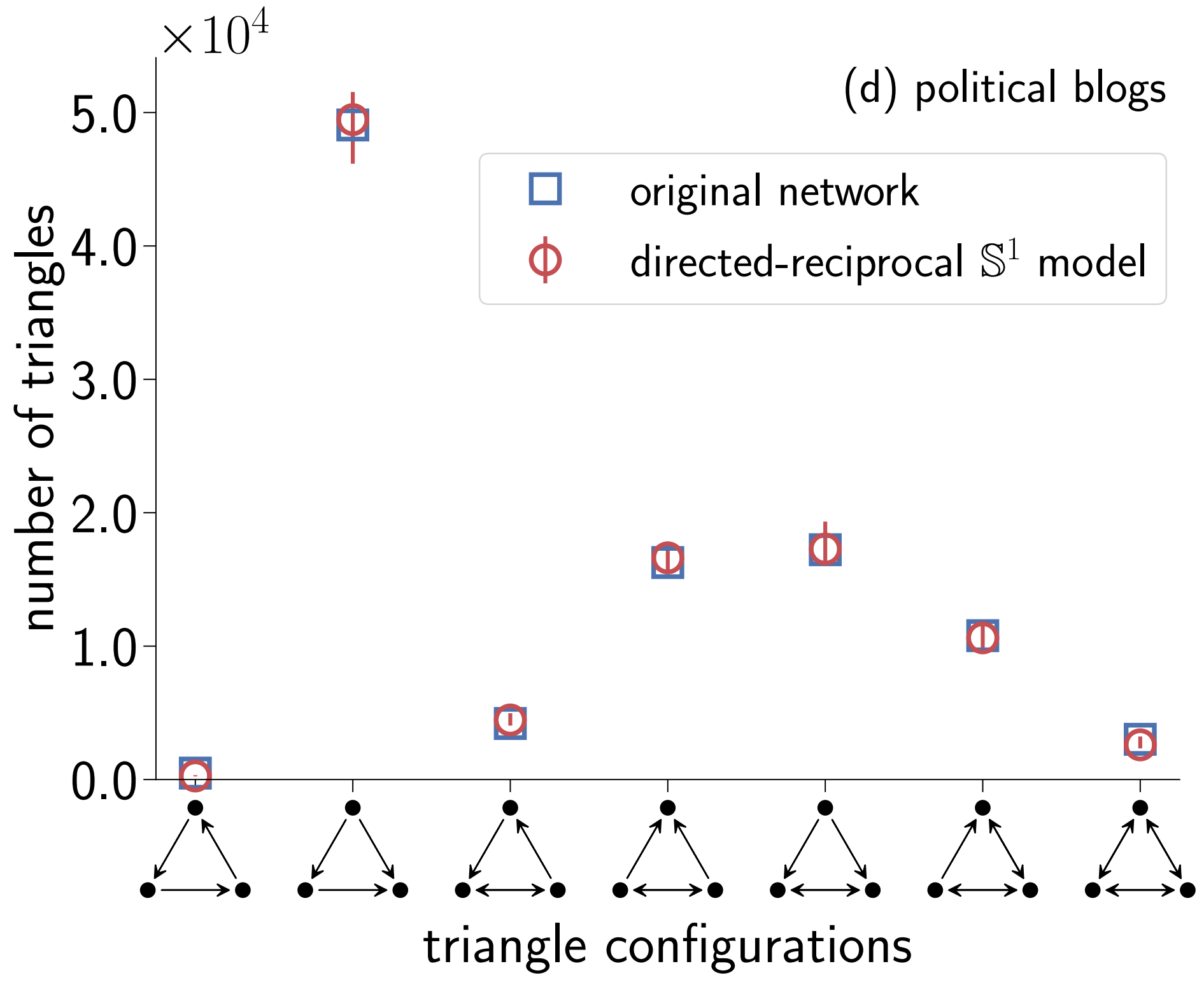

(h) Advogato trust network
$\times 10^4$
4.0
3.0
2.0
1.0
0.0
number of triangles
triangle configurations

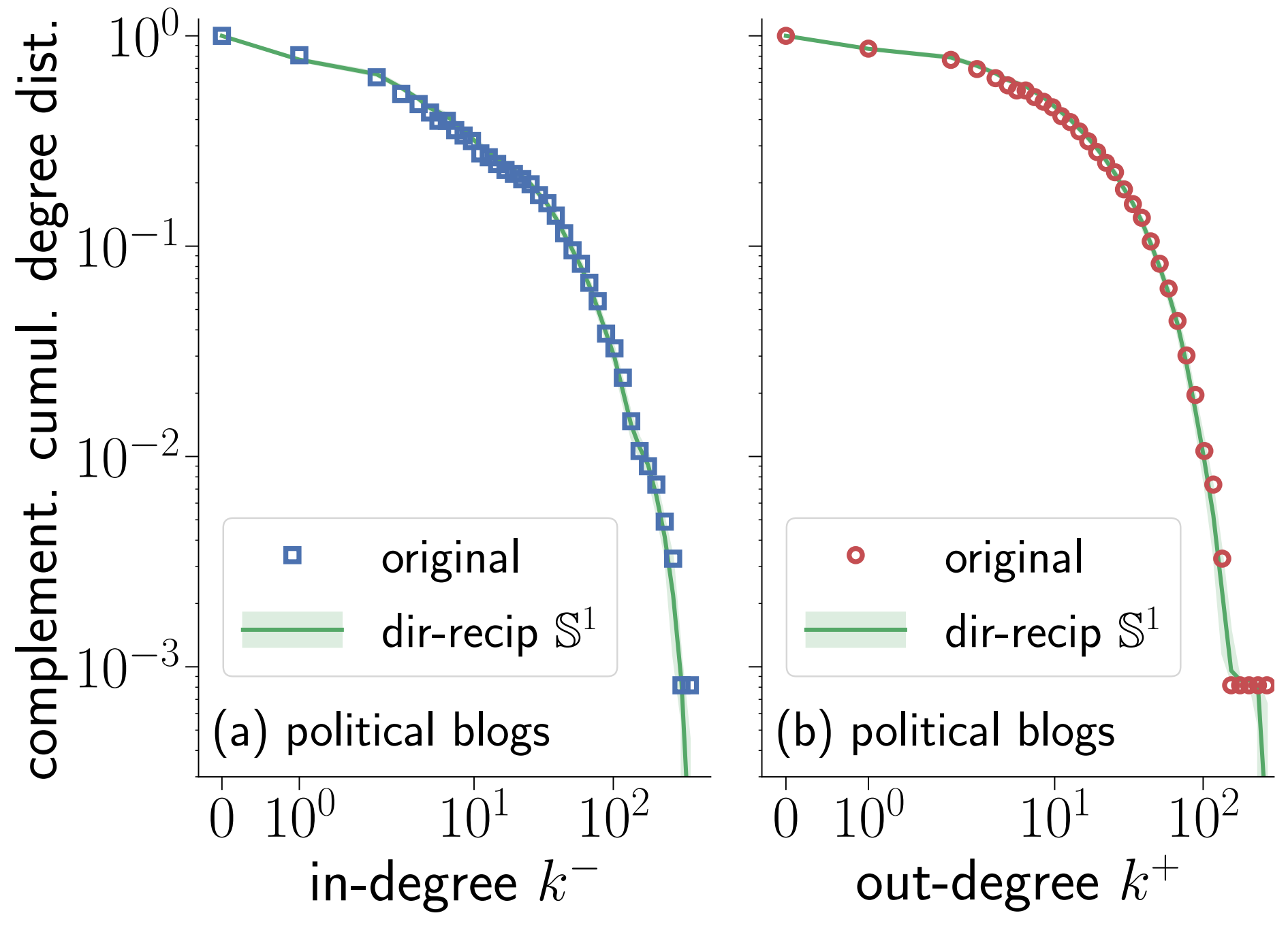

(i) manufacturing company email
$\times 10^4$
2.5
2.0
1.5
1.0
0.5
0.0
number of triangles
triangle configurations


(j) primary school contacts
$\times 10^2$
1.2
1.0
0.8
0.5
0.2
0.0
number of triangles
triangle configurations


$\times 10^3$
1.5
1.0
0.5
0.0
number of triangles
(I) friendship network
triangle configurations


(k) U.S. government agency websites
$\times 10^4$
2.0
1.5
1.0
0.5
0.0
number of triangles
triangle configurations


(d) political blogs
original network
directed-reciprocal $\mathbb{S}^1$ model
number of triangles
$\times 10^4$
5.0
4.0
3.0
2.0
1.0
0.0
triangle configurations


complement. cumul. degree dist.
10^0
10^{-1}
10^{-2}
10^{-3}
original
dir-recip $\mathbb{S}^1$
(a) political blogs
0
10^0
10^1
10^2
in-degree k^-
original
dir-recip $\mathbb{S}^1$
(b) political blogs
0
10^0
10^1
10^2
out-degree k^+

Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks

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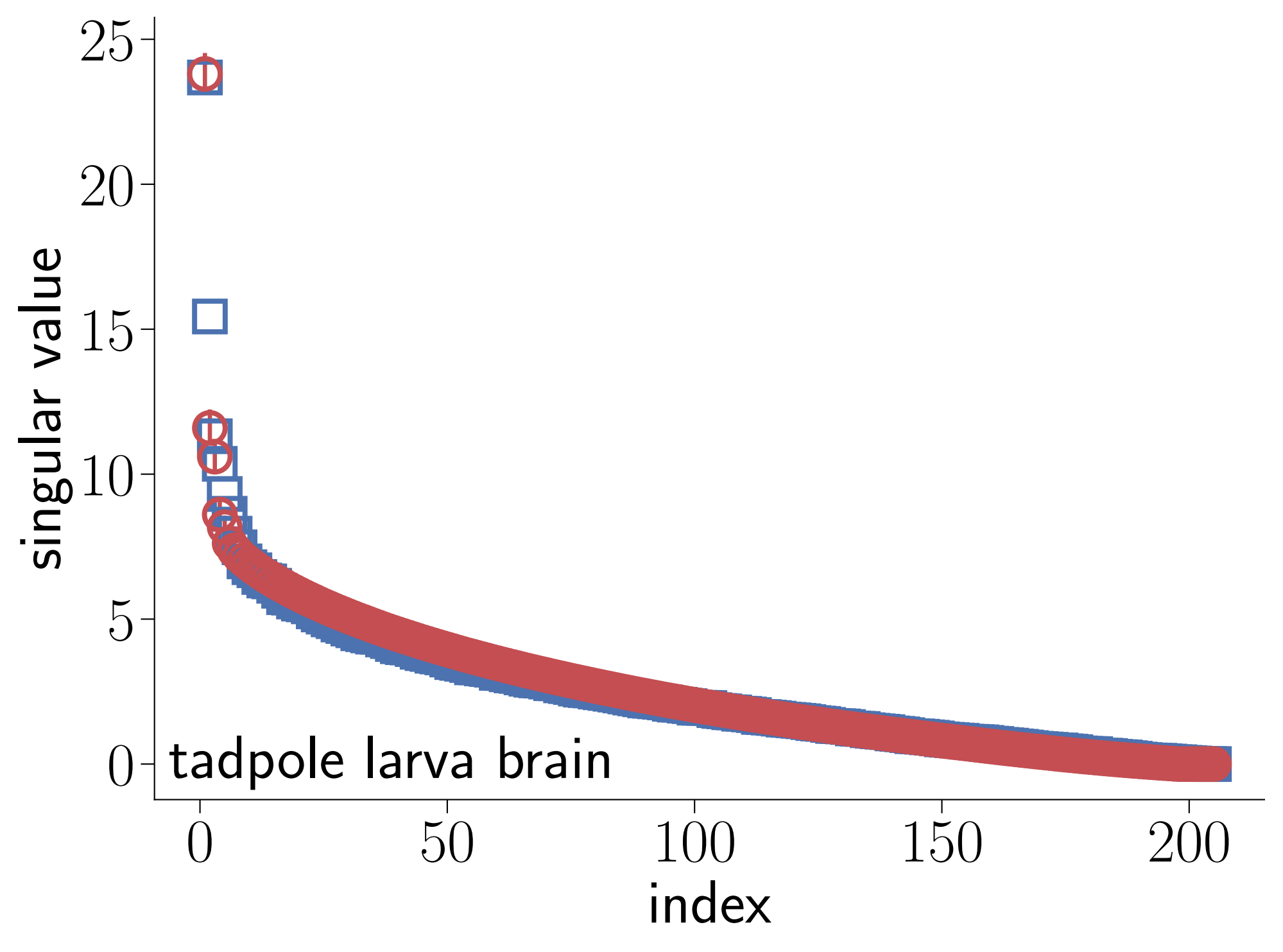
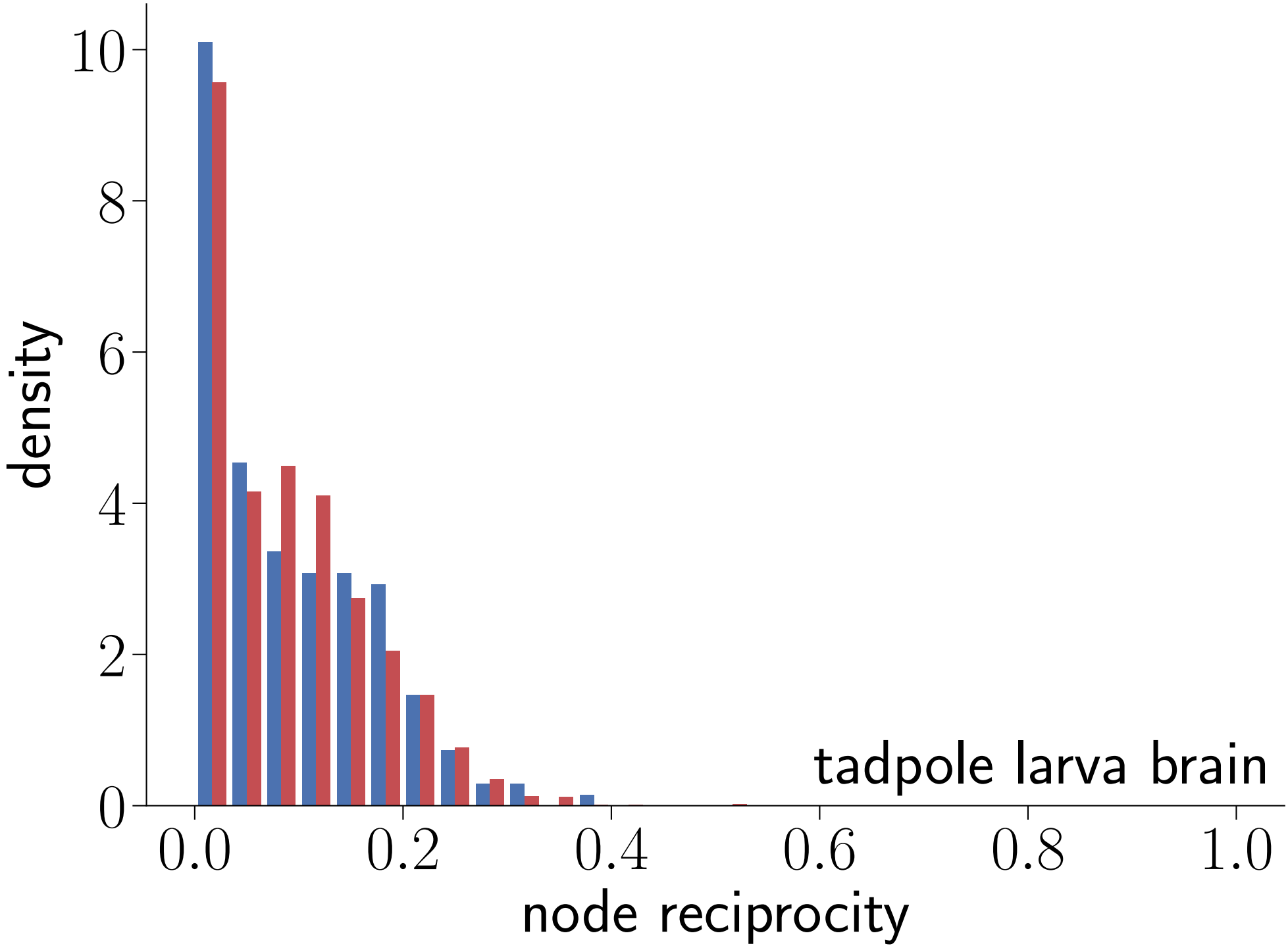
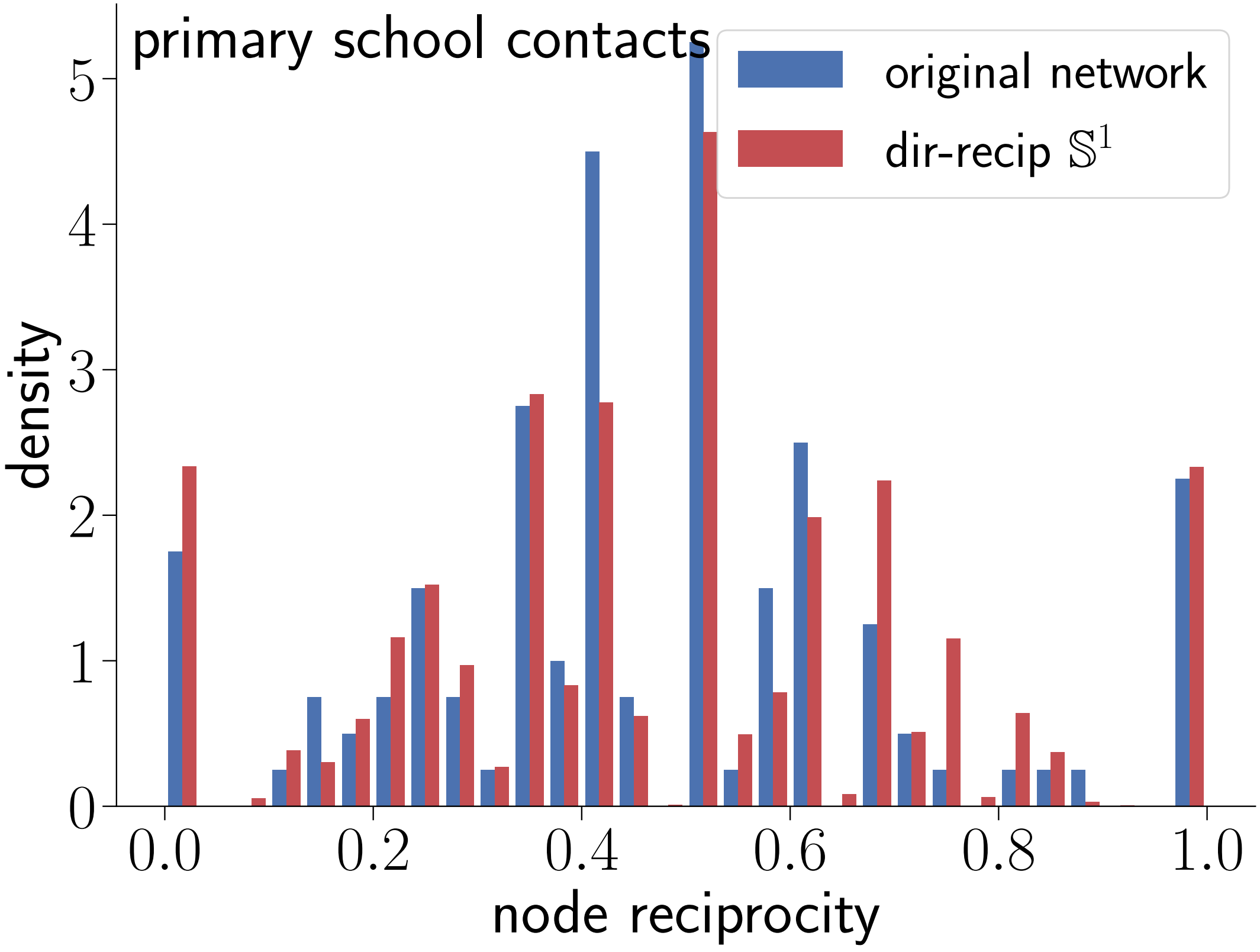
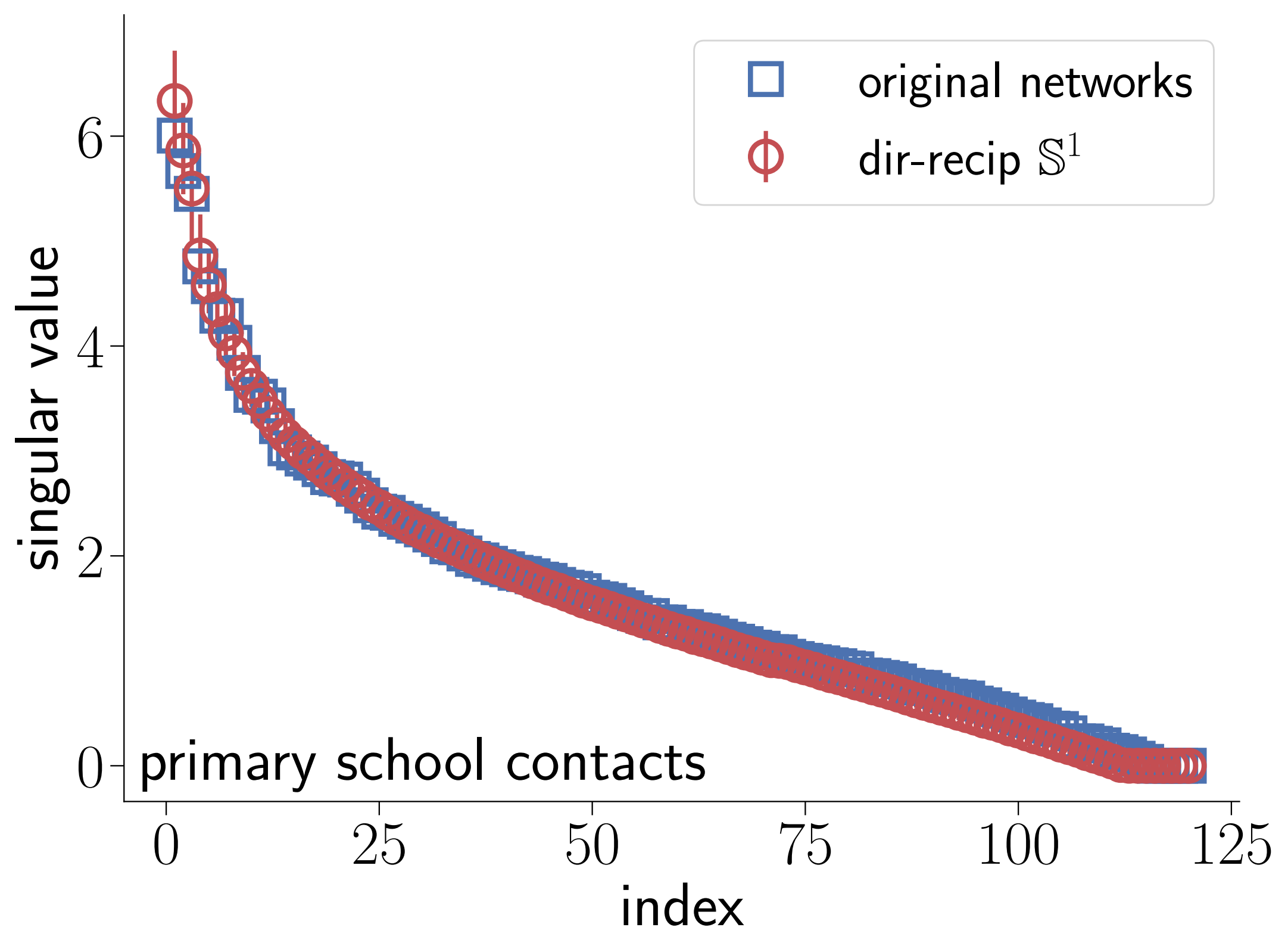
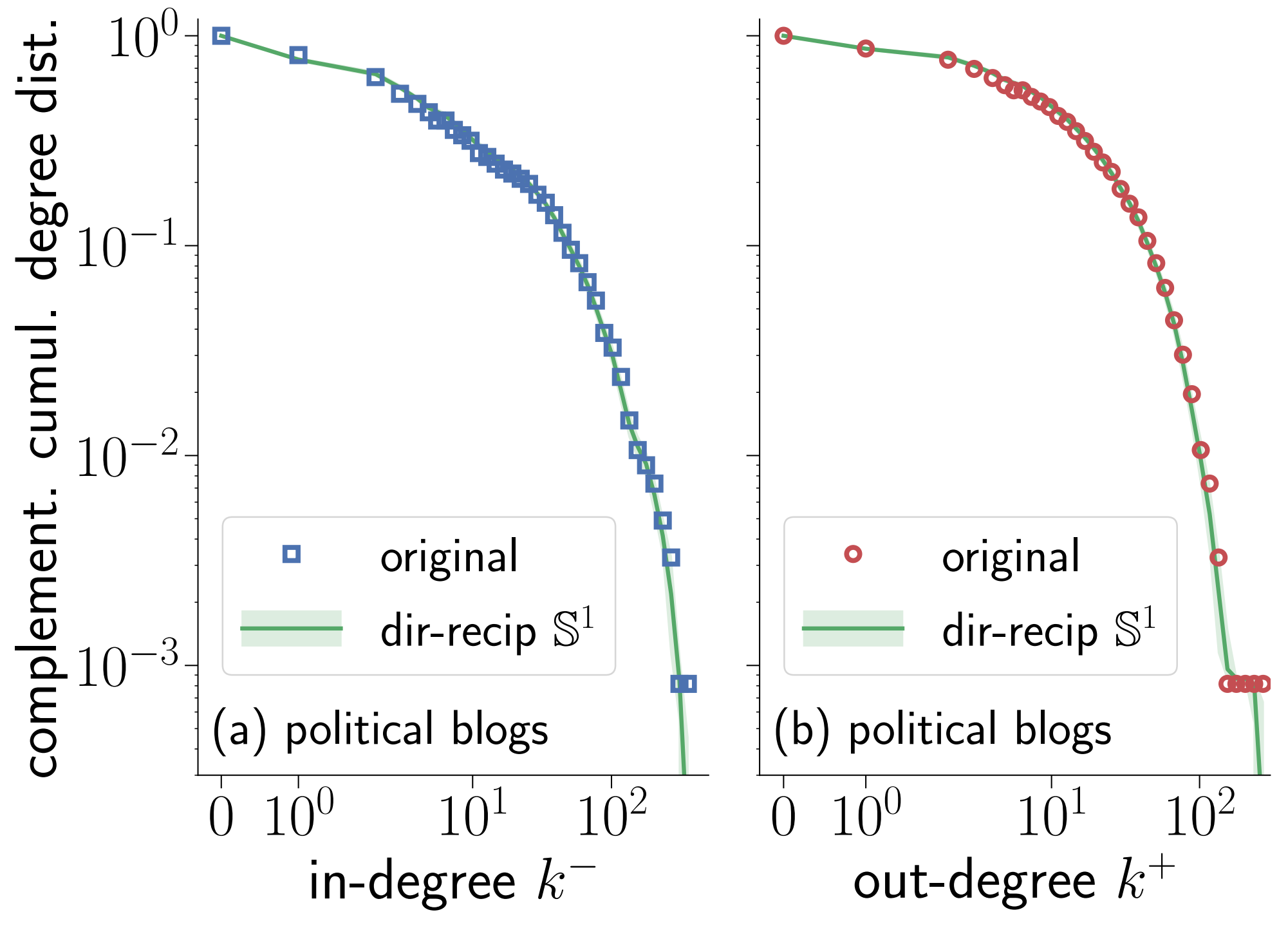
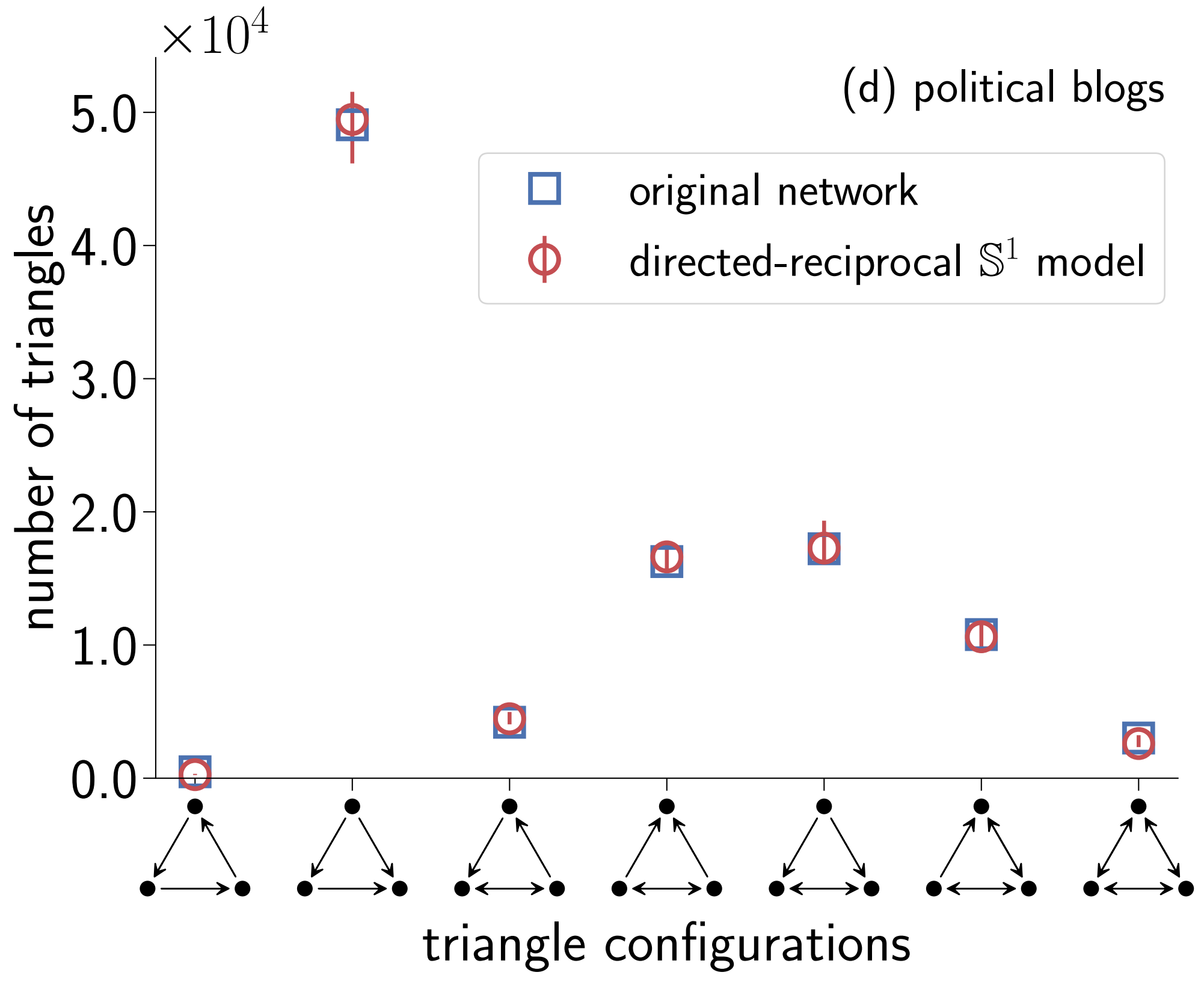

25
20
15
10
5
0
singular value
tadpole larva brain
0
50
100
150
200
index

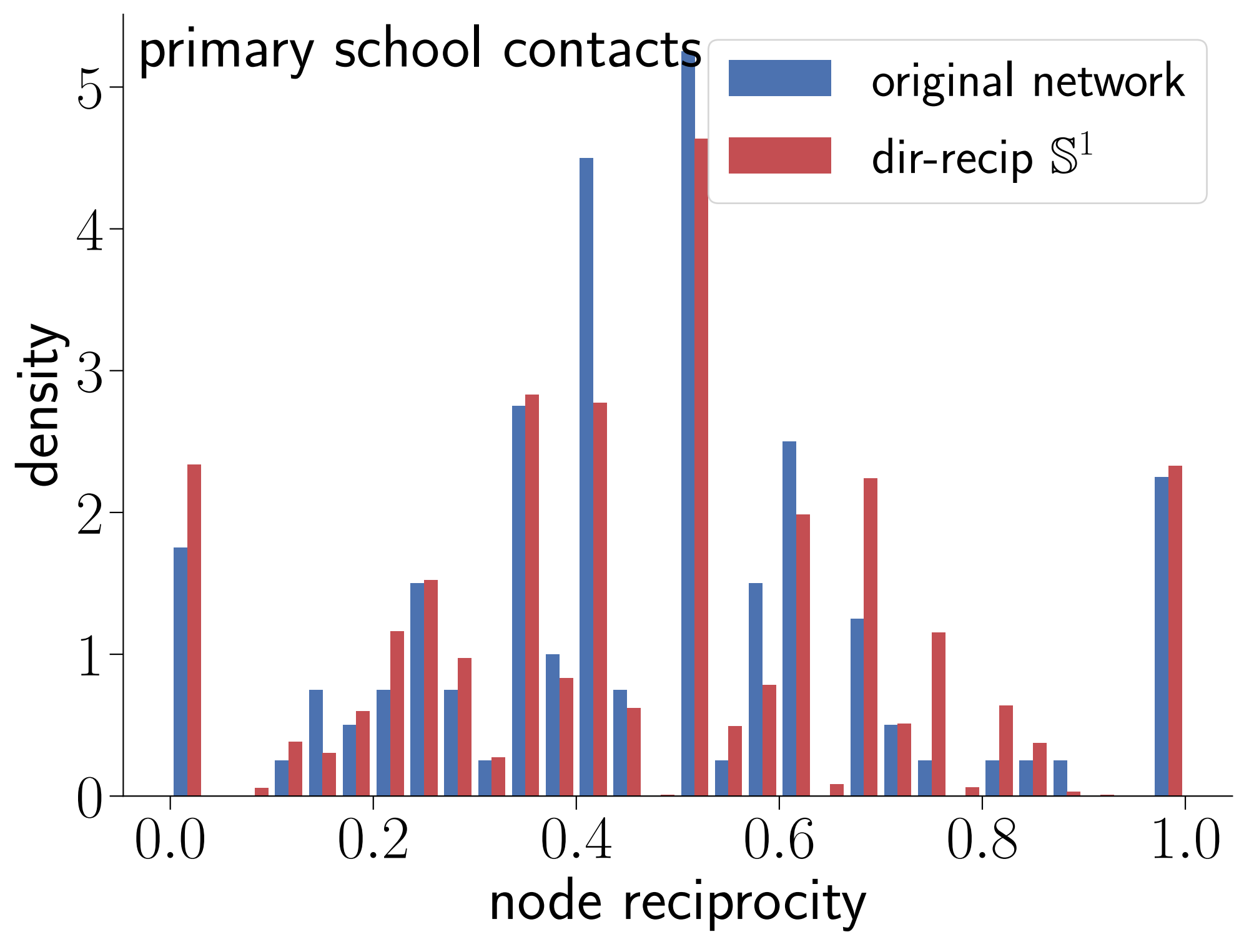

density
10
8
6
4
2
0
0.0
0.2
0.4
0.6
0.8
1.0
node reciprocity
tadpole larva brain


primary school contacts
original network
dir-recip $\mathbb{S}^1$
density
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0
node reciprocity

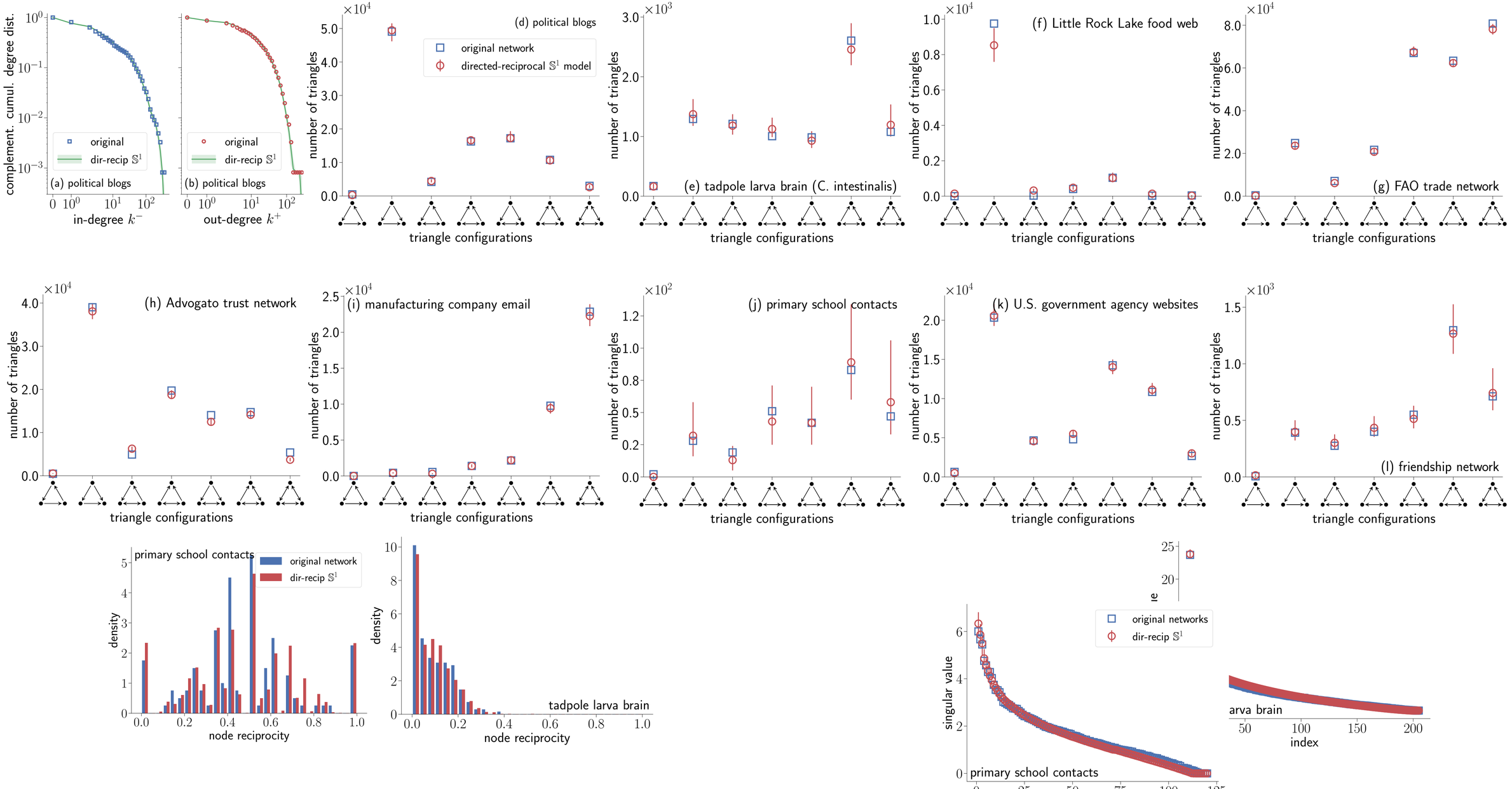

original networks
dir-recip $\mathbb{S}^1$
primary school contacts
singular value
index
0
2
4
6
0
25
50
75
100
125


complement. cumul. degree dist.
10^0
10^{-1}
10^{-2}
10^{-3}
original
dir-recip $\mathbb{S}^1$
(a) political blogs
0
10^0
10^1
10^2
in-degree k^-
original
dir-recip $\mathbb{S}^1$
(b) political blogs
0
10^0
10^1
10^2
out-degree k^+


(d) political blogs
original network
directed-reciprocal $\mathbb{S}^1$ model
$\times 10^4$
5.0
4.0
3.0
2.0
1.0
0.0
number of triangles
triangle configurations


primary school contacts
original network
dir-recip $\mathbb{S}^1$
density
node reciprocity
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0

Fitting the directed-reciprocal $\mathbb{S}^1$ model to real networks



Main takeaways

- Rethinking the link between connections and distances allows the extension of the network geometry framework to directed networks.
- Assuming an underlying geometry is a simple modeling approach that can reproduce several complex topological properties observed in real networks.

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Geometric description of clustering in directed networks

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Hyperbolic Embedding of Directed Networks

Jérémi Lesage, M. Ángeles Serrano, Marián Boguñá and Antoine Allard

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