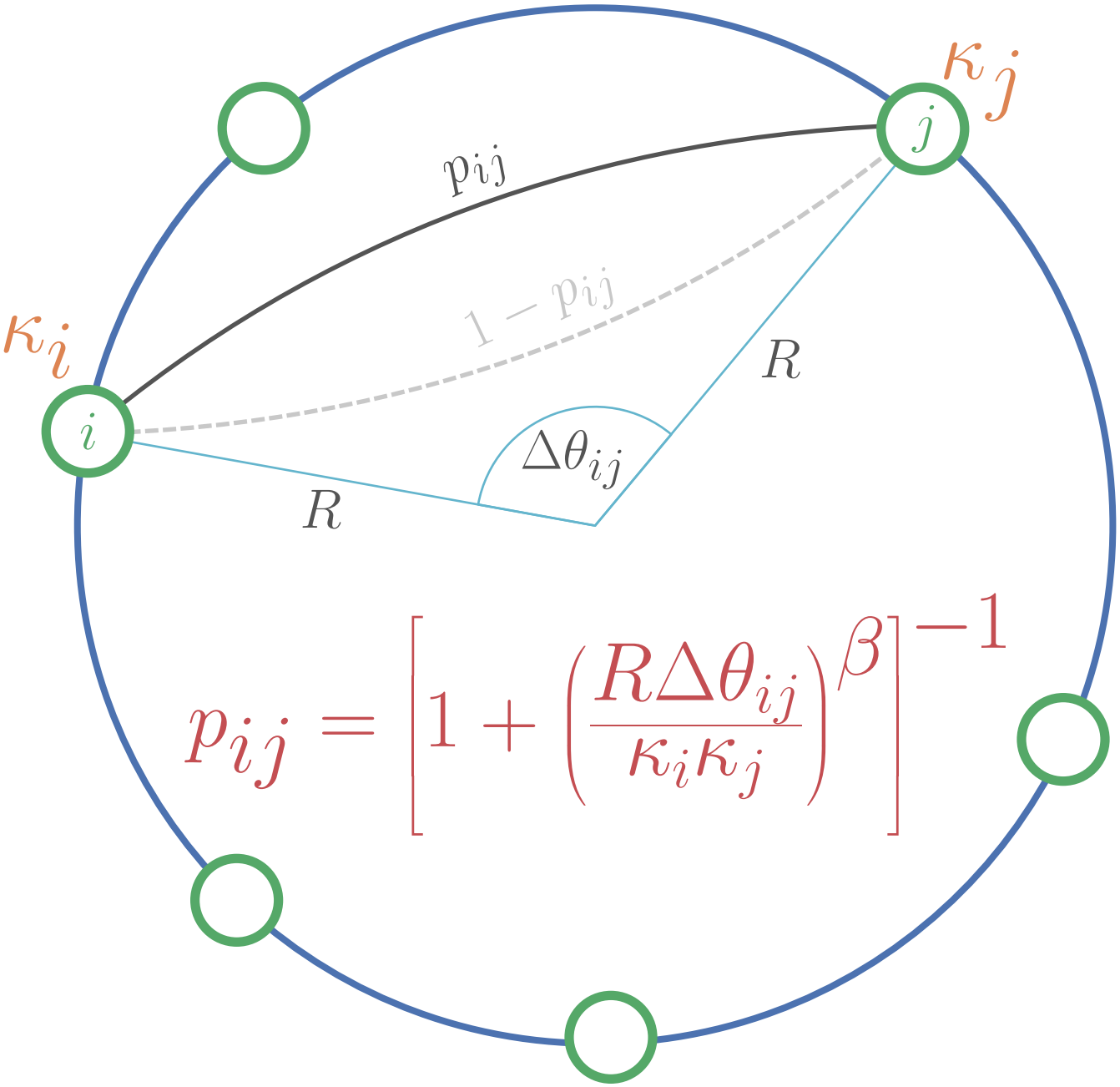


1

0



c.

clustering coefficient

0.8

0.6

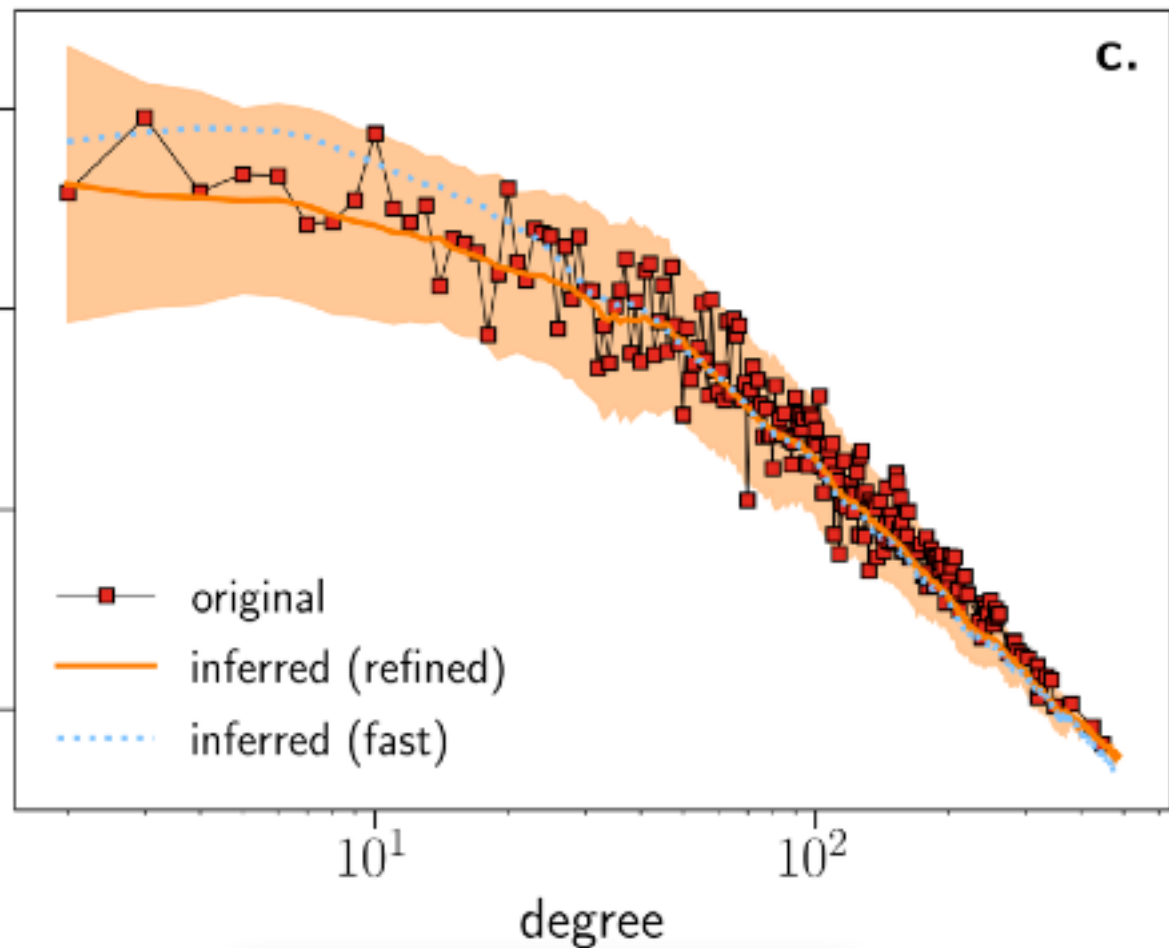
0.4

0.2

 10^1 10^2

degree

- original
- inferred (refined)
- inferred (fast)



A geometric approach to clustering: the S^1/\mathbb{H}^2 model

[15]

The S^1 model

1. Sprinkle N nodes uniformly on a circle of radius R .
2. Assign an expected degree κ to each node according to some pdf $\rho(\kappa)$.
3. Draw a link between node i and node j with probability p_{ij} .

★ fixes the expected degree of nodes (κ) \rightarrow soft configuration model (CM)

★ triangle inequality of the underlying metric space \rightarrow triangles from pairwise interactions

★ level of clustering tuned with parameter β

Other properties and generalizations

- ▷ Amenable to many **analytical calculations**
- ▷ Geometric interpretation in terms of **hyperbolic geometry** (the \mathbb{H}^2 model) [1,2]
- ▷ Parsimonious explanation of **self-similarity** [3,4]
- ▷ Generalizable to **weighted** [5], **bipartite** [6,7,8], **multiplex** [9,10] and **growing** [11] networks
- ▷ Generalizable to networks with **community structure** [12,13,14]
- ▷ **Mapping of real complex networks** unto hyperbolic space [15,16]
- ▷ Identification of biochemical pathways in E. Coli [8]
- ▷ Efficient Internet routing protocols [17]
- ▷ Multiscale organization of the human connectome [18]
- ▷ Geometrical interpretation of preferential attachment [11]
- ▷ ...

[1] Phys. Rev. E 80, 035101 (2009)

[2] Phys. Rev. E 82, 036106 (2010)

[3] Phys. Rev. Lett. 100, 078701 (2008)

[4] Nat. Rev. Phys. 3, 114 (2021)

[5] Nat. Commun. 8, 14103 (2017)

[6] Phys. Rev. E 84, 026114 (2011)

[7] Phys. Rev. E 95, 032309 (2017)

[8] Mol. Biosyst. 8, 843 (2012)

[9] Nat. Phys. 12, 1076 (2016)

[10] Phys. Rev. Lett. 118, 218301 (2017)

[11] Nature 489, 537 (2012)

[12] Sci.Rep. 5, 9421 (2015)

[13] J. Stat. Phys. 173, 775 (2018)

[14] New J. Phys. 20, 052002 (2018)

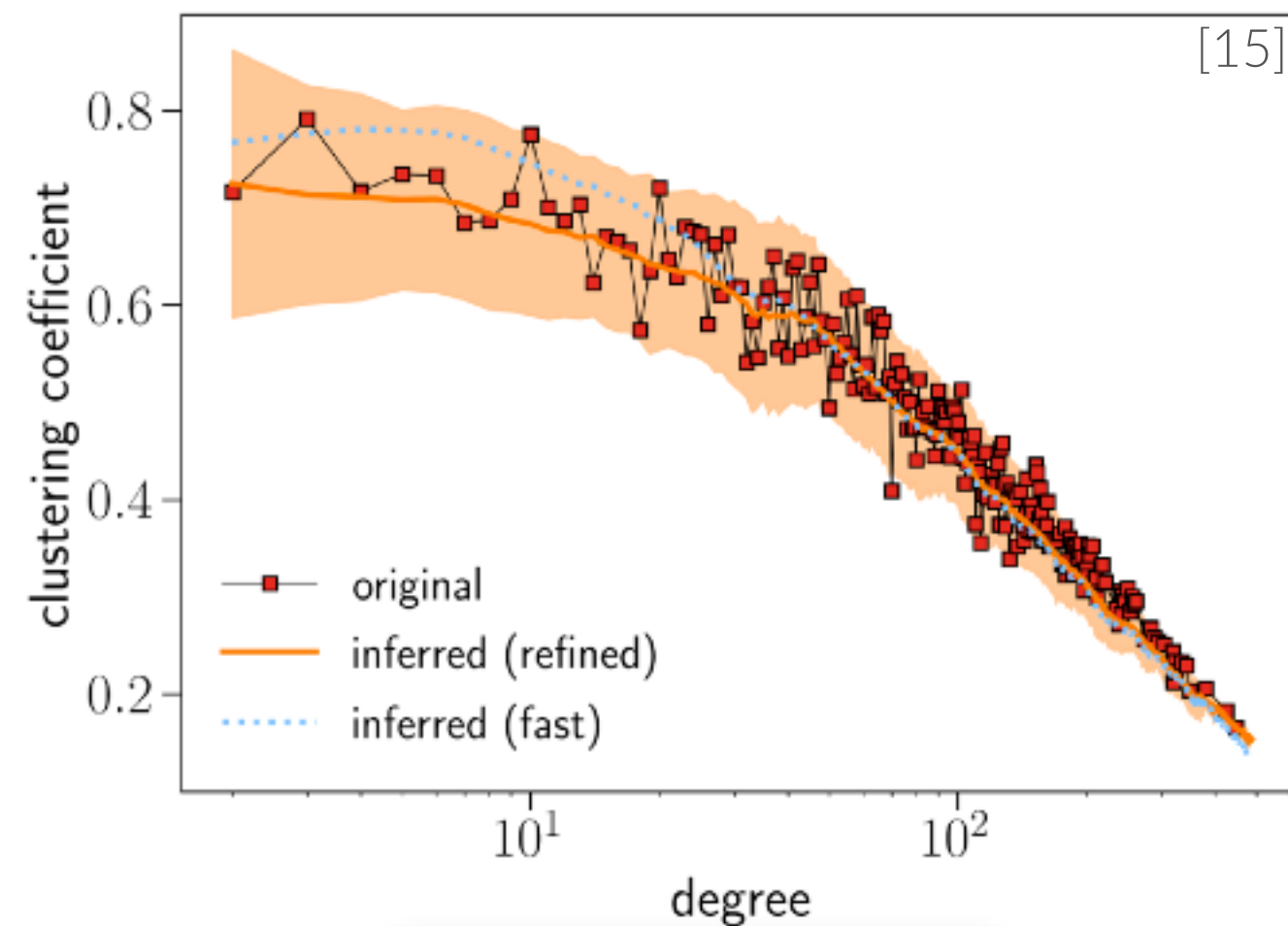
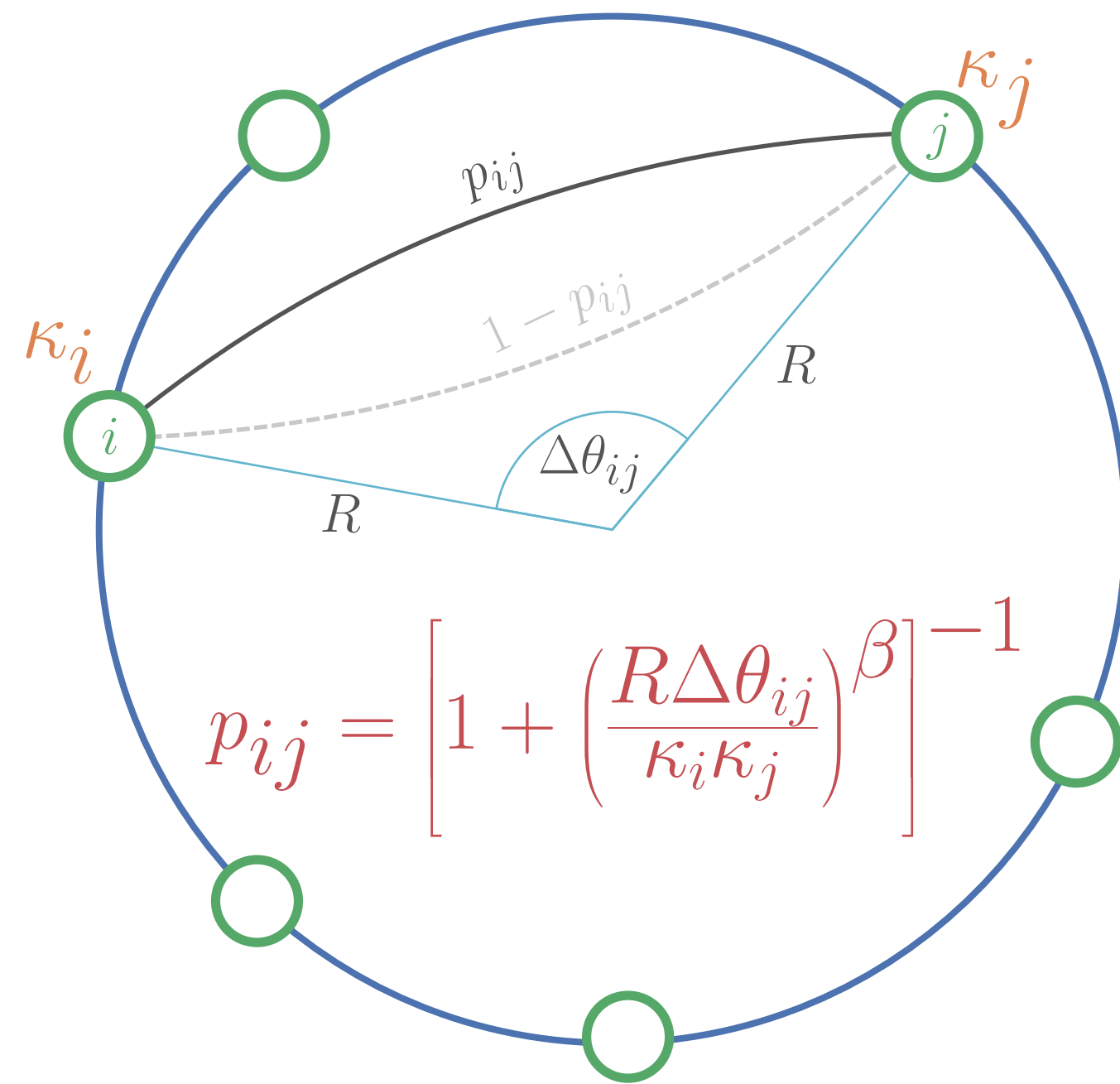
[15] New J. Phys. 21, 123033 (2019)

[16] Nat. Commun. 8, 1615 (2017)

[17] Nat. Commun. 1, 62 (2010)

[18] PNAS 117, 20244 (2020)

A geometric approach to clustering : the $\mathbb{S}^1/\mathbb{H}^2$ model



The \mathbb{S}^1 model

1. Sprinkle N nodes uniformly on a circle of radius R .
2. Assign an expected degree κ to each node according to some pdf $\rho(\kappa)$.
3. Draw a link between node i and node j with probability p_{ij} .

- ★ fixes the expected degree of nodes (κ) → soft configuration model (CM)
- ★ triangle inequality of the underlying metric space → triangles from pairwise interactions
- ★ level of clustering tuned with parameter β

Other properties and generalizations

- ▷ Amenable to many analytical calculations
- ▷ Geometric interpretation in terms of hyperbolic geometry (the \mathbb{H}^2 model) [1,2]
- ▷ Parsimonious explanation of self-similarity [3,4]
- ▷ Generalizable to weighted [5], bipartite [6,7,8], multiplex [9,10] and growing [11] networks
- ▷ Generalizable to networks with community structure [12,13,14]
- ▷ Mapping of real complex networks unto hyperbolic space [15,16]
- ▷ Identification of biochemical pathways in E. Coli [8]
- ▷ Efficient Internet routing protocols [17]
- ▷ Multiscale organization of the human connectome [18]
- ▷ Geometrical interpretation of preferential attachment [11]
- ▷ ...

[1] Phys. Rev. E 80, 035101 (2009)
[2] Phys. Rev. E 82, 036106 (2010)
[3] Phys. Rev. Lett. 100, 078701 (2008)
[4] Nat. Rev. Phys. 3, 114 (2021)
[5] Nat. Commun. 8, 14103 (2017)
[6] Phys. Rev. E 84, 026114 (2011)

[7] Phys. Rev. E 95, 032309 (2017)
[8] Mol. Biosyst. 8, 843 (2012)
[9] Nat. Phys. 12, 1076 (2016)
[10] Phys. Rev. Lett. 118, 218301 (2017)
[11] Nature 489, 537 (2012)
[12] Sci. Rep. 5, 9421 (2015)

[13] J. Stat. Phys. 173, 775 (2018)
[14] New J. Phys. 20, 052002 (2018)
[15] New J. Phys. 21, 123033 (2019)
[16] Nat. Commun. 8, 1615 (2017)
[17] Nat. Commun. 1, 62 (2010)
[18] PNAS 117, 20244 (2020)

Outline

1. Why models and the challenge of clustering
2. A geometric approach to clustering
3. Euclid and hyperbolic geometry
4. A hyperbolic solution to clustering
- 5. Rethinking interactions: the case of directed graphs**
6. Rethinking interactions: the case of modular structure