- There are 5 models of hyperbolic geometry in $\mathbb{R}^{D,1}$:

  H: the half-space model;

  I: the interior of the disk model;

  J: the hemisphere model;

  K: the Klein model;

  L: the hyperboloid model.

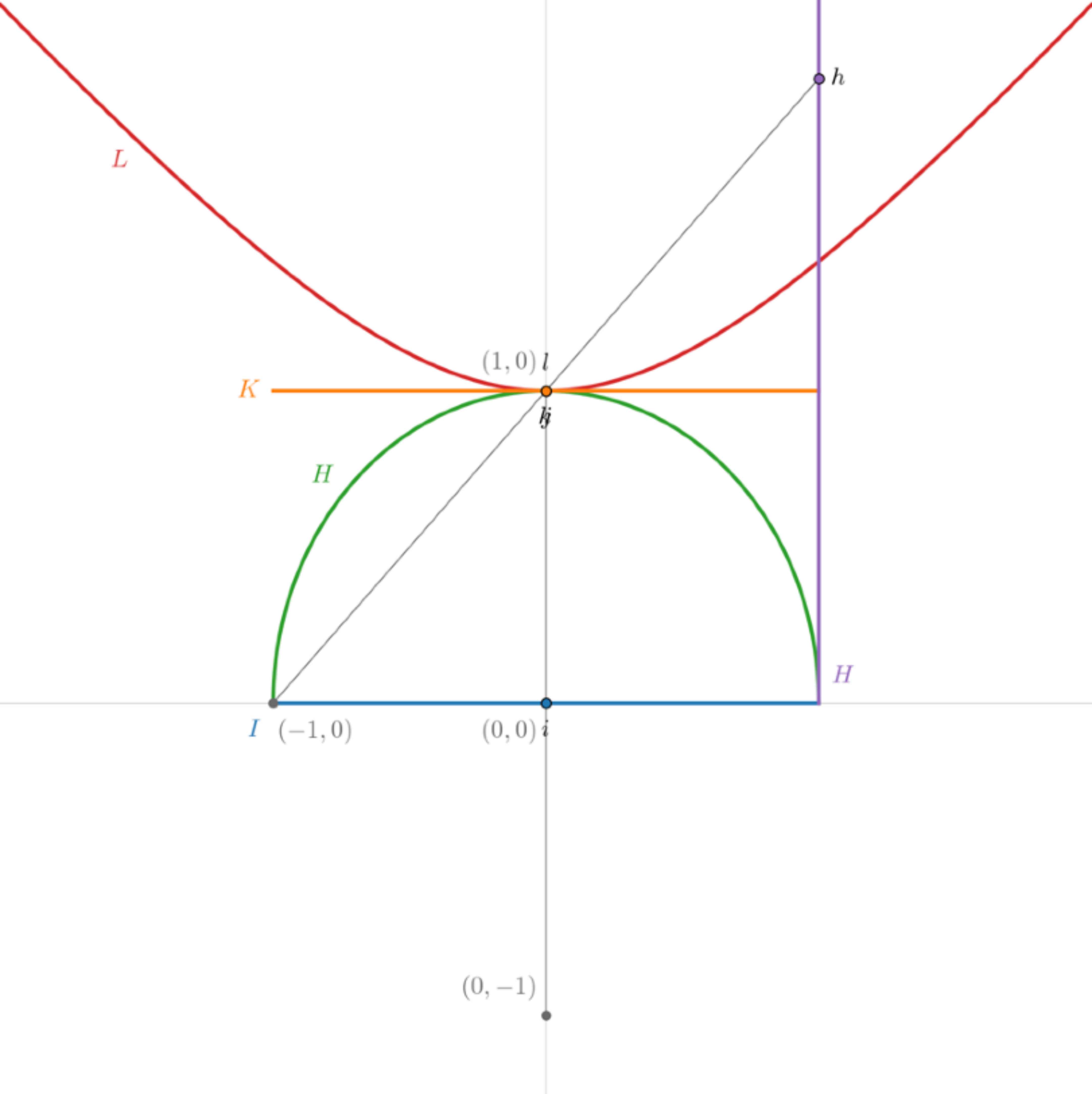
- They are isometrically equivalent.

- They have their own metric, geodesics, isometries, and so on.

- Each model supplies its own natural intuitions.

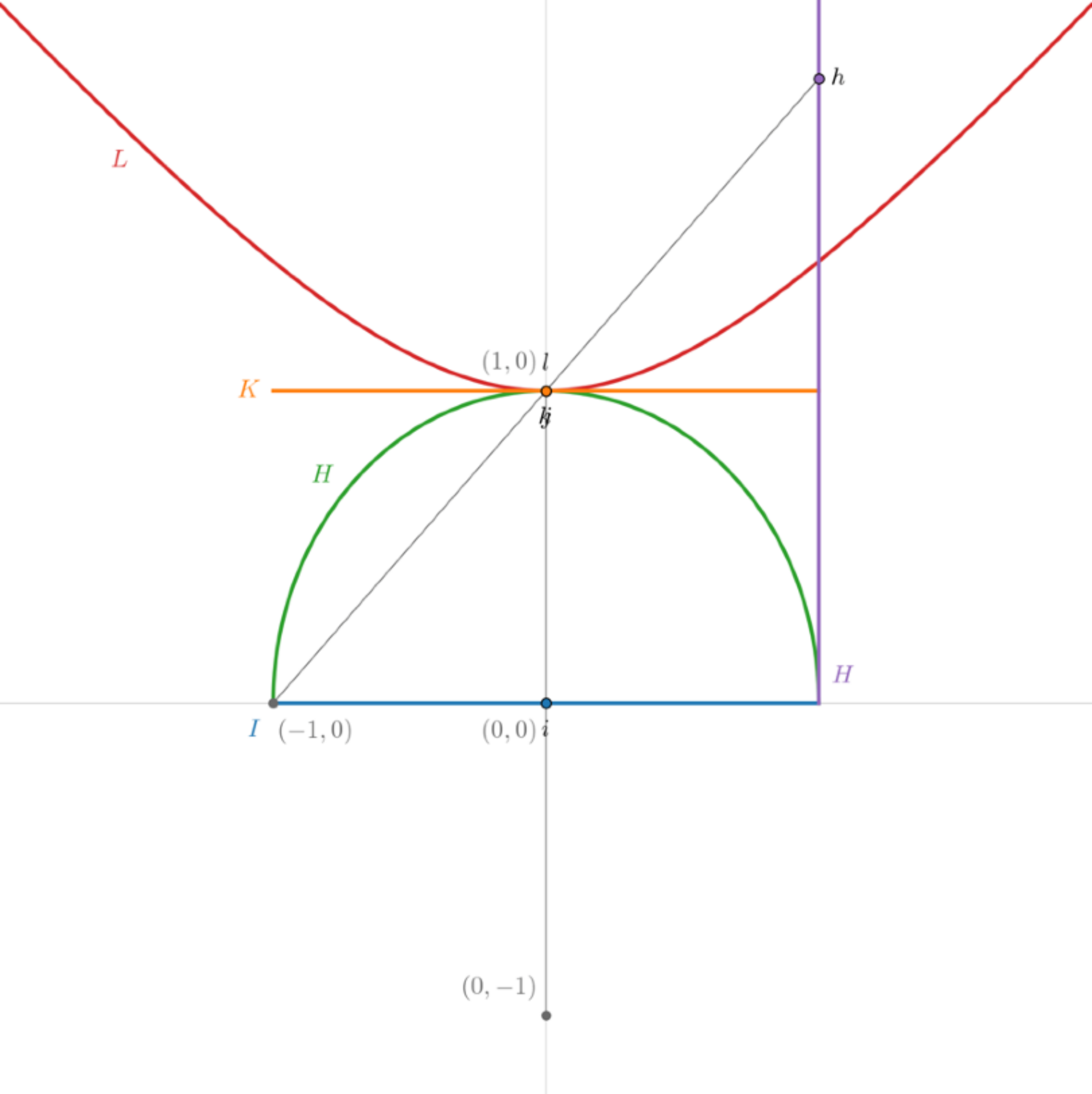
7

Hyperbolic geometry

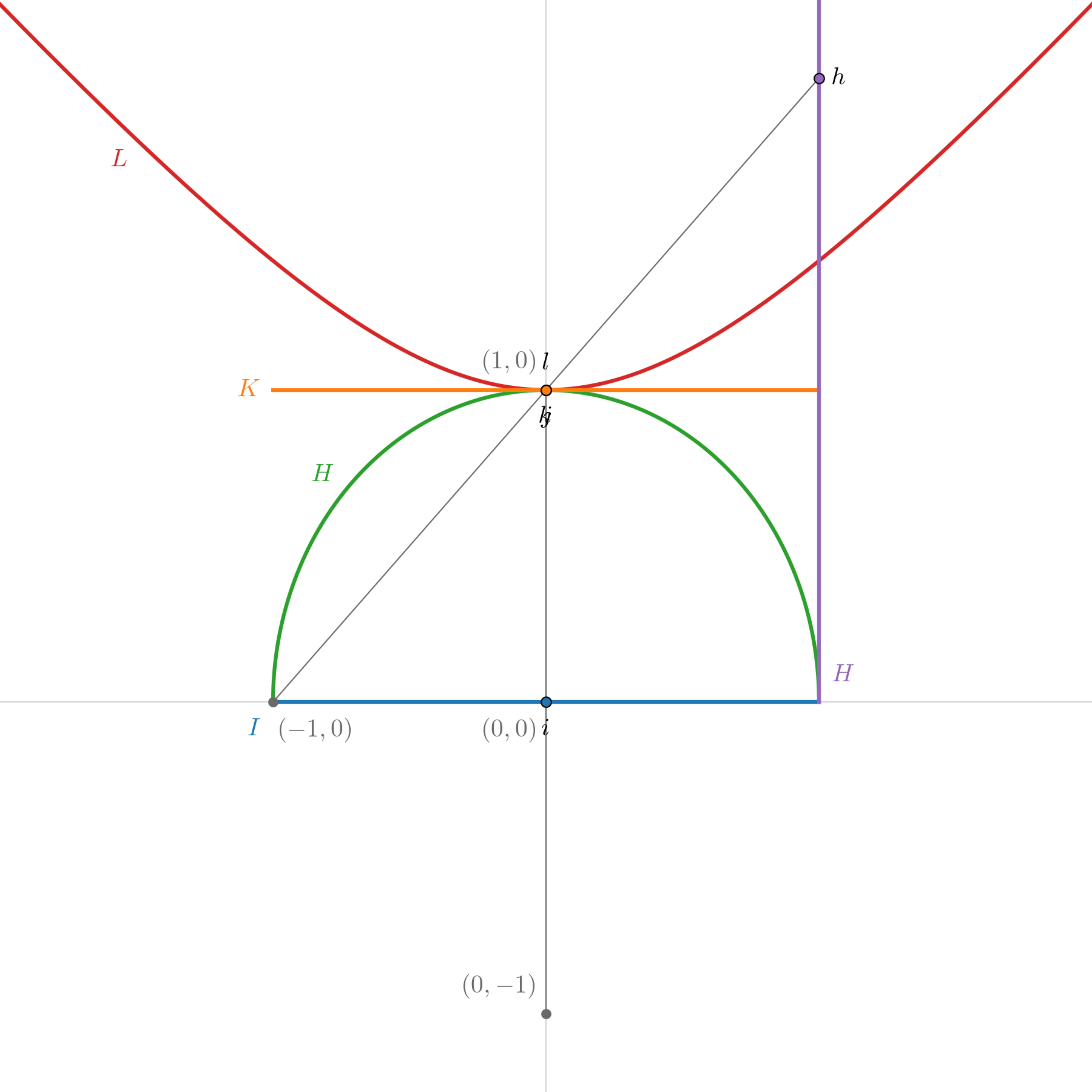


L
h
(1, 0) l
K
H
H
I
(−1, 0)
(0, 0) i
(0, −1)

The five analytic models and their connecting isometries in $D = 1$.



L
h
(1, 0) l
K
H
H
I
(−1, 0)
(0, 0) i
(0, −1)



h
L
$(1,0)$ l
K
kj
H
H
I
$(-1,0)$
$(0,0)$ i
$(0,-1)$

Hyperbolic geometry

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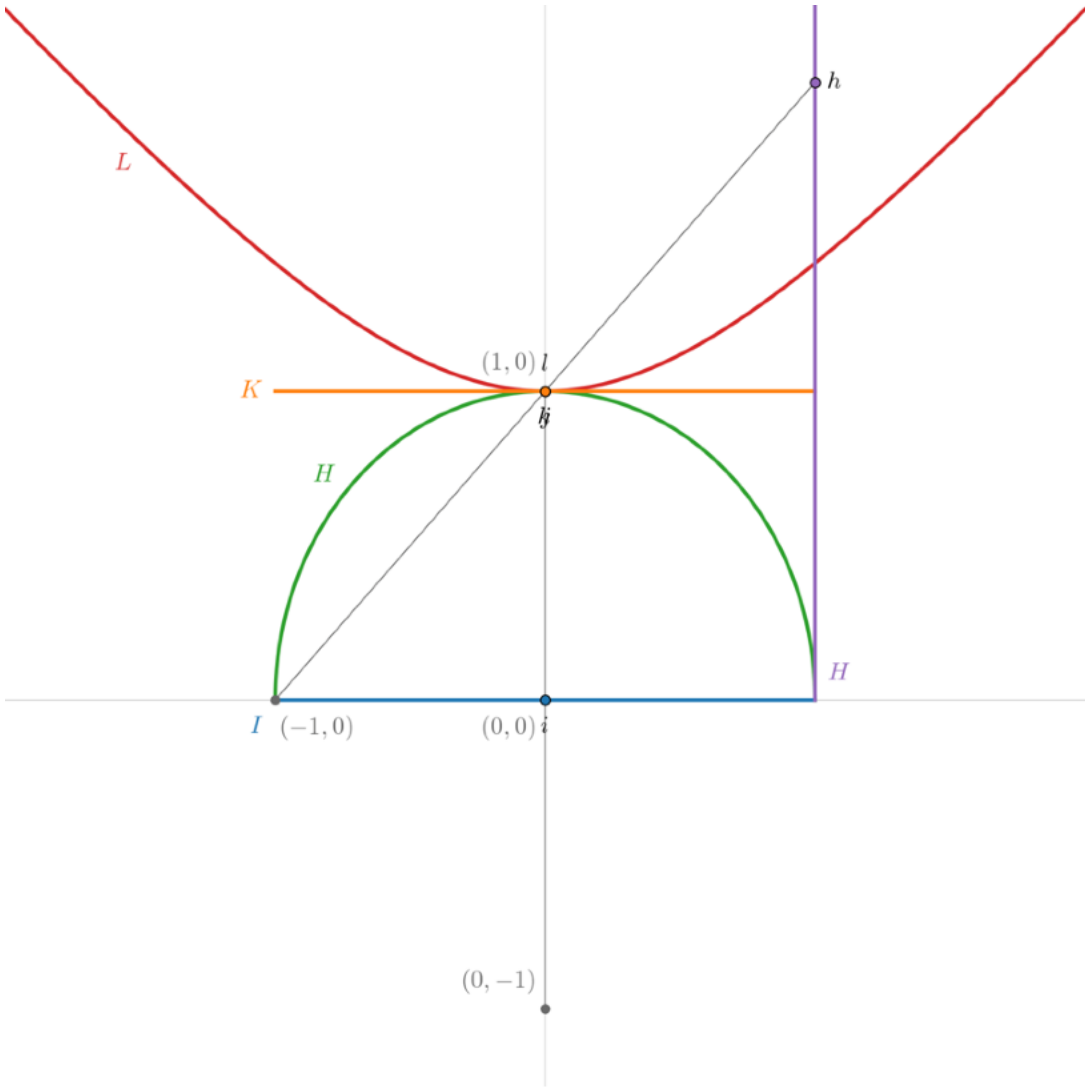


Hyperbolic geometry

- Space of constant negative curvature (as opposed to flat or Euclidean space, or spherical space)
- Model for the $D = 2$ hyperbolic space : positive sheet of the hyperboloid defined by

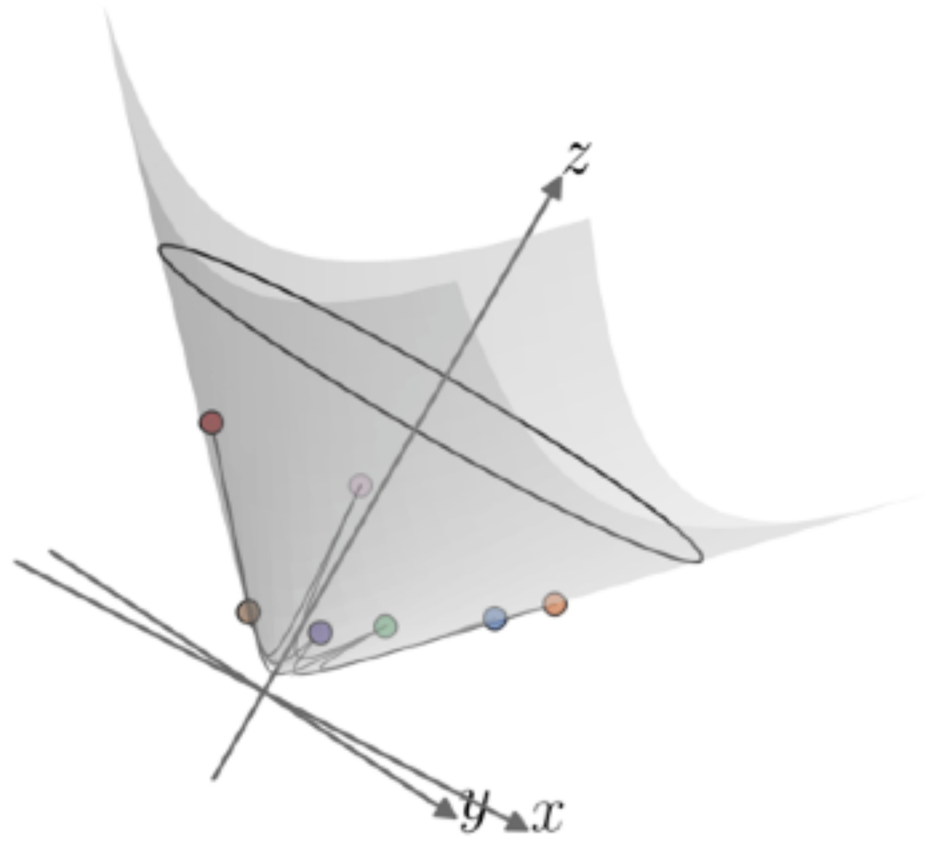
$$x^2 + y^2 - z^2 = -1$$

- Distance between points (x_1, y_1, z_1) and (x_2, y_2, z_2) is

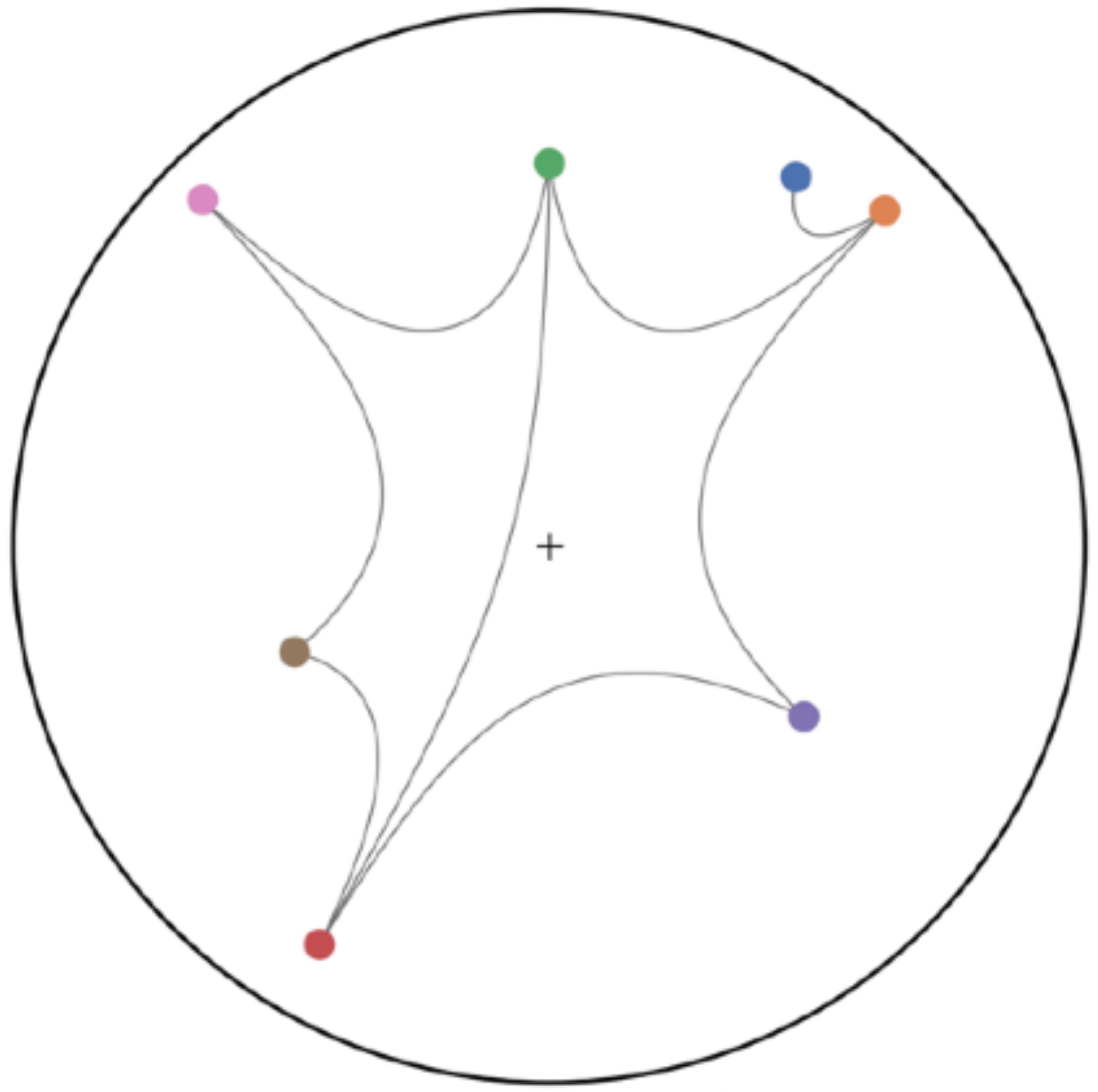
$$d(1,2) = \operatorname{arccosh}\big(z_1 z_2 - x_1 x_2 - y_1 y_2\big)$$

- Polar coordinates

$$x = \sinh(r)\cos(\theta)$$
$$y = \sinh(r)\sin(\theta)$$
$$z = \cosh(r)$$



hyperboloid in $\mathbb{R}^{2,1}$



hyperbolic disk (r, θ)

For further info, see Flavors of geometry (Cambridge University Press, 1997)
or Foundations of Hyperbolic Manifolds (Springer, 2019)