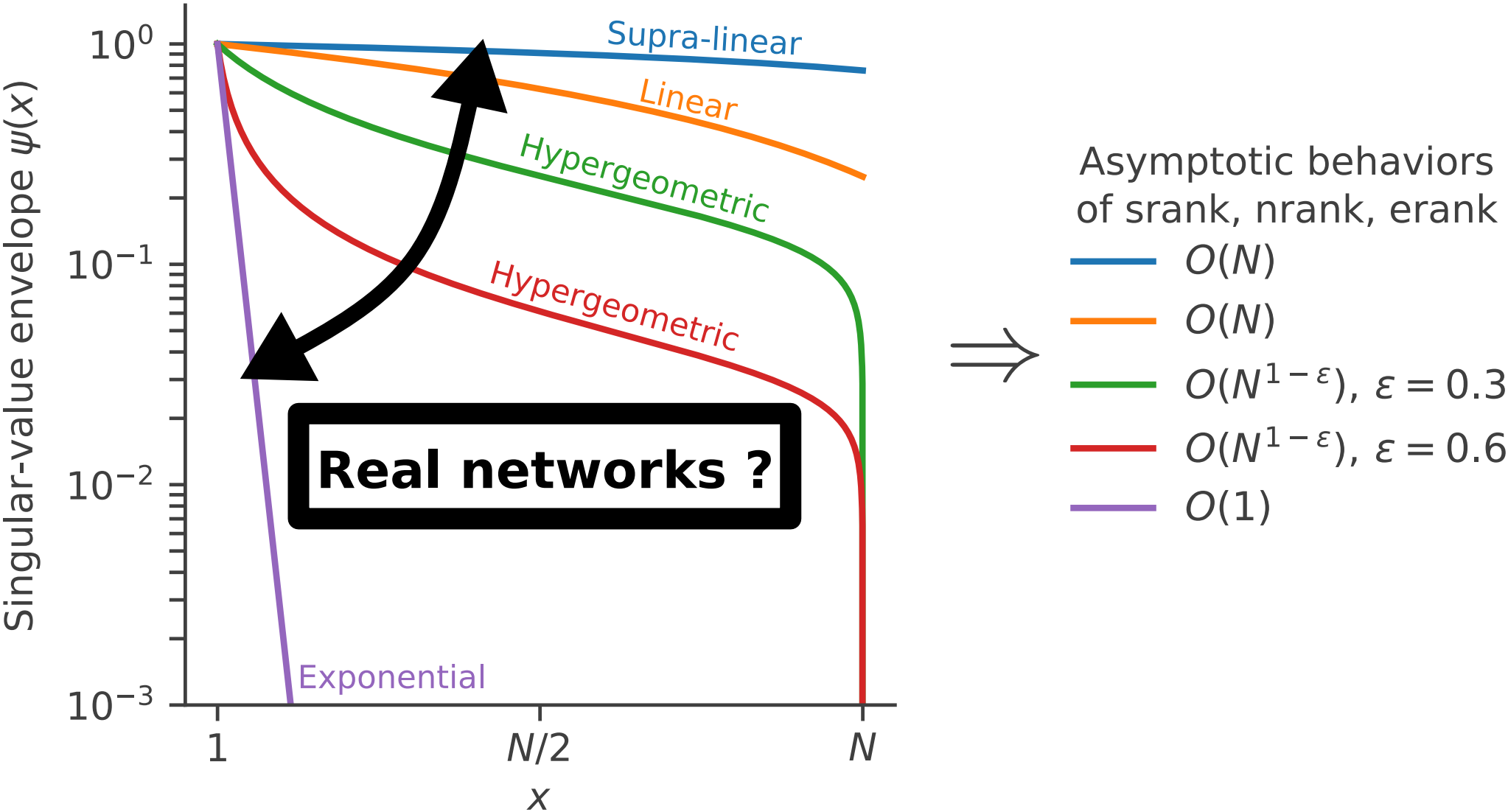


Asymptotic behaviors
of srank, nrank, erank

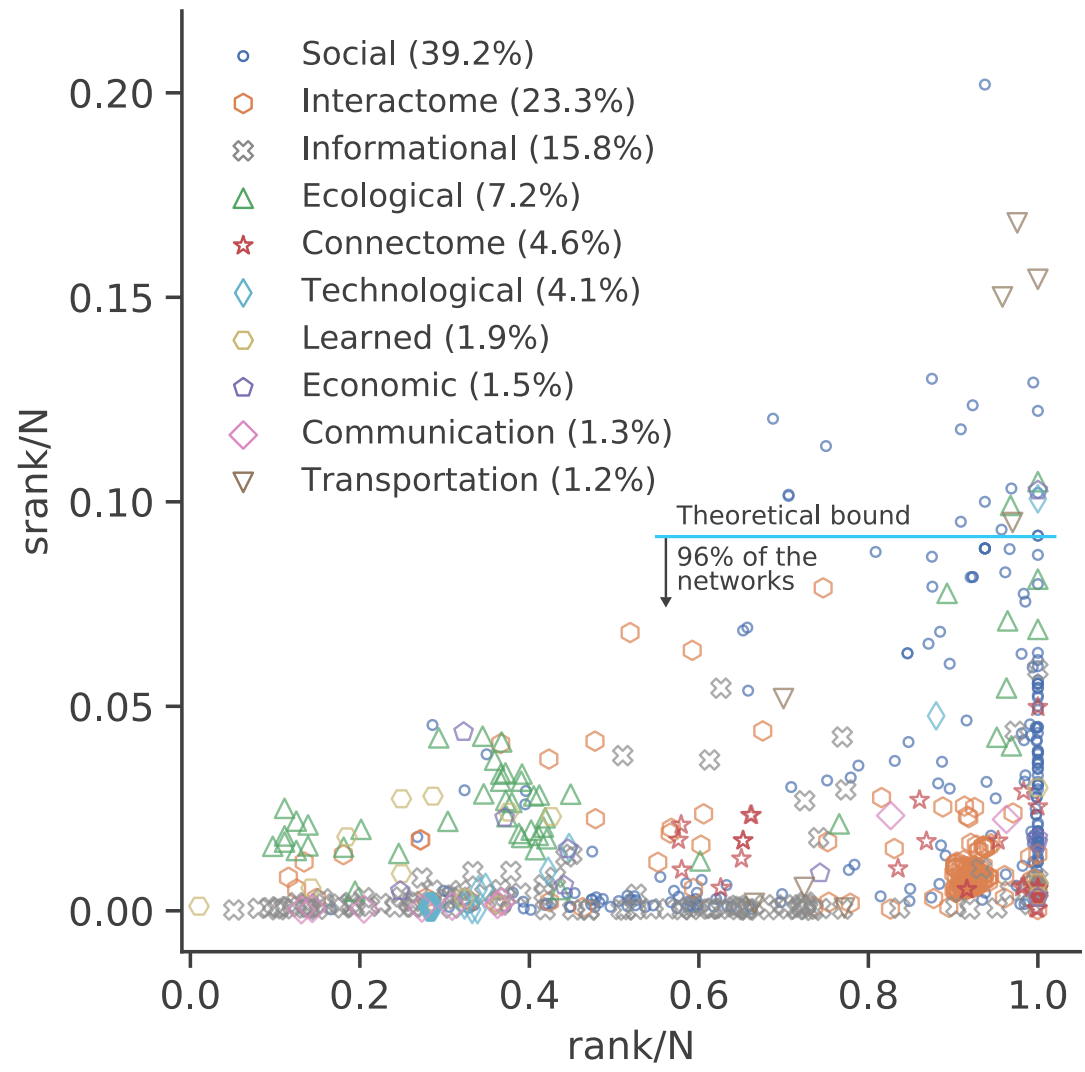
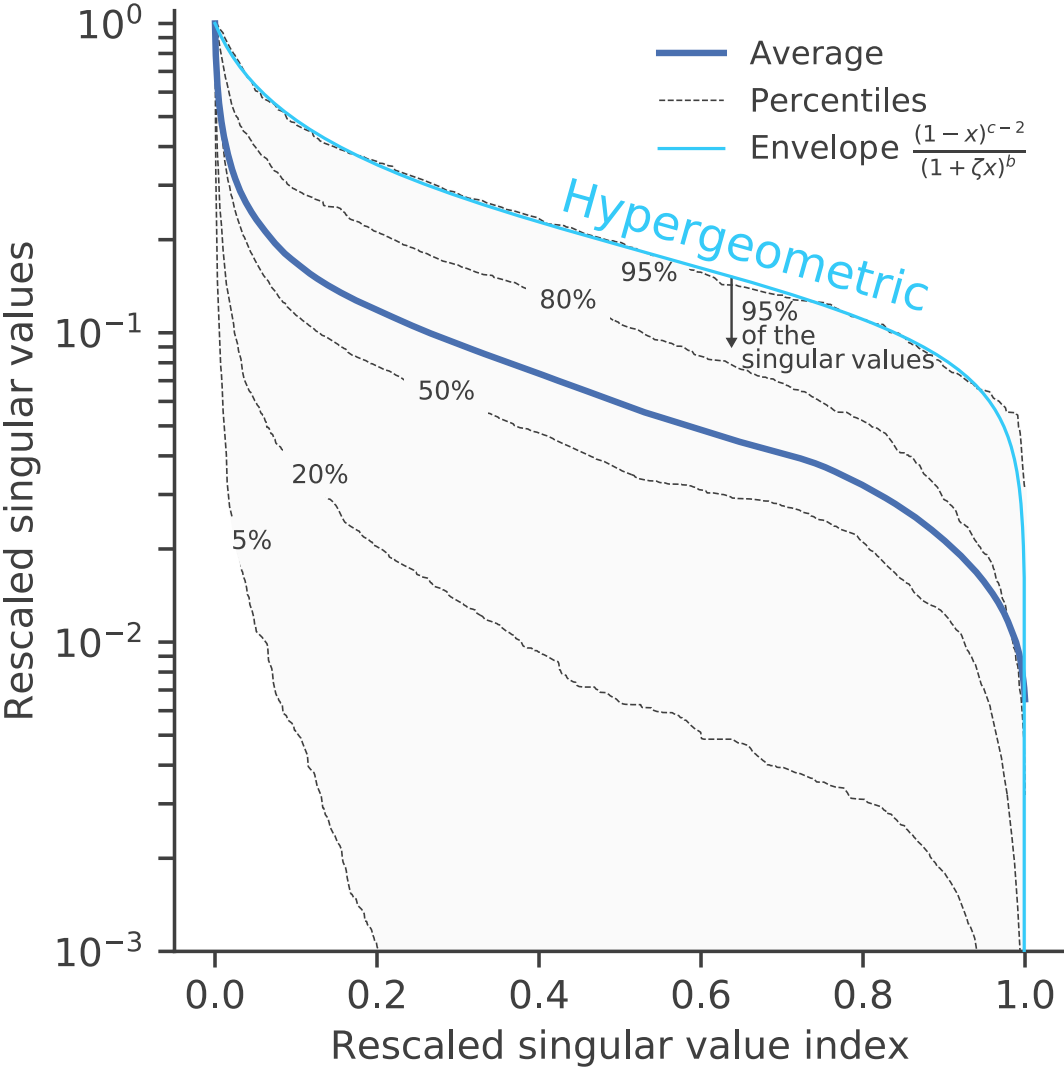
\Rightarrow

- $O(N)$
- $O(N)$
- $O(N^{1-\varepsilon}), \varepsilon = 0.3$
- $O(N^{1-\varepsilon}), \varepsilon = 0.6$
- $O(1)$



A workable definition of "low" effective rank





The singular values of approx. 96% of considered networks are bounded from above by an hypergeometric envelope \Rightarrow **sublinear effective ranks!**

Model definition low effective rank: $\sim 10\%$ of the number of nodes N

Approx. 96% of the 679 networks qualify for having a low effective rank!

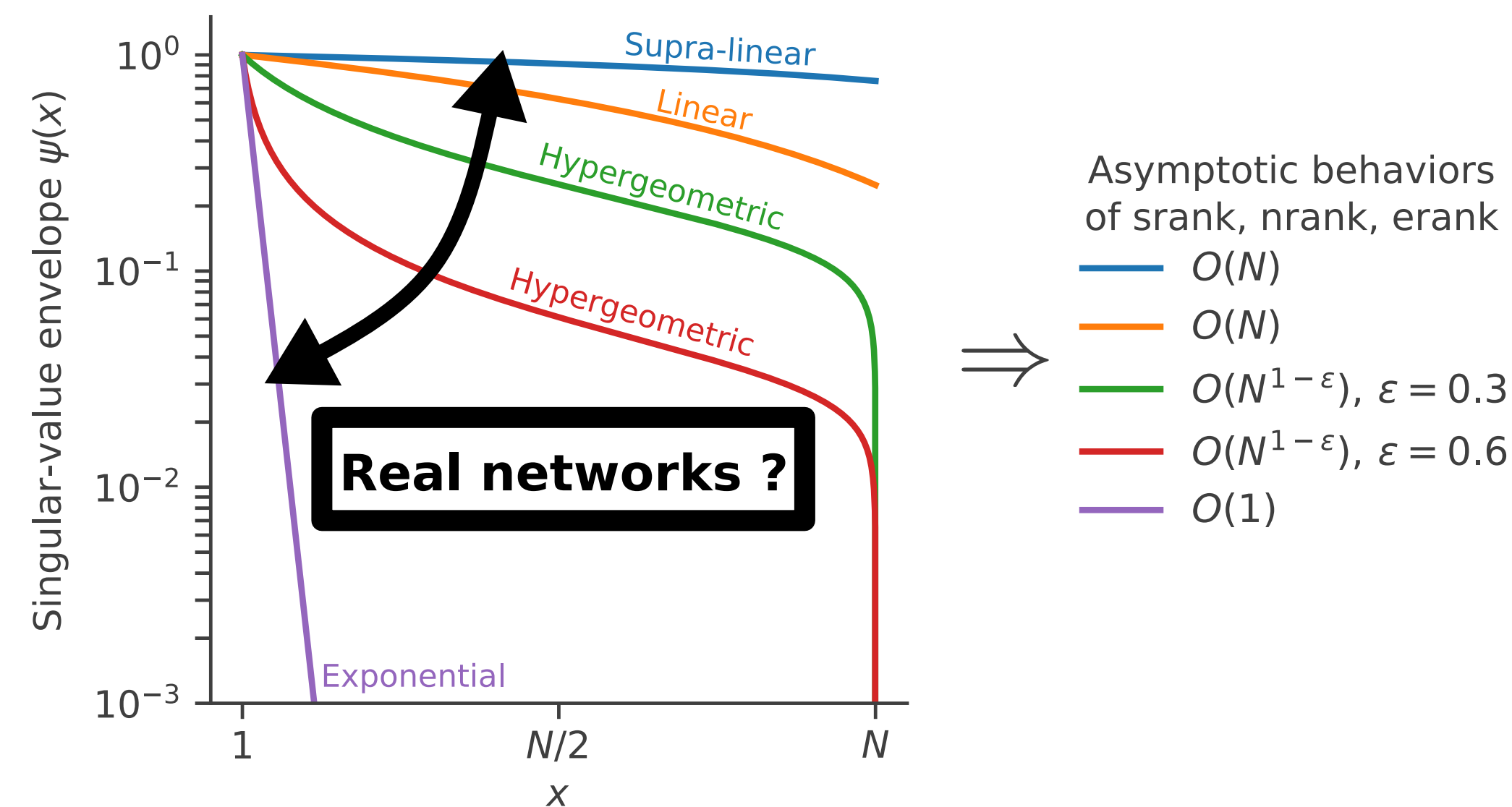
Hint: the rapid decrease of the dominant singular values of the adjacency matrix implies a low effective rank

- ▷ low effective rank? \Rightarrow effective rank scales at most sublinearly as the number of nodes, N , goes to infinity ($N^{1-\varepsilon}$ with $\varepsilon \in (0, 1]$)

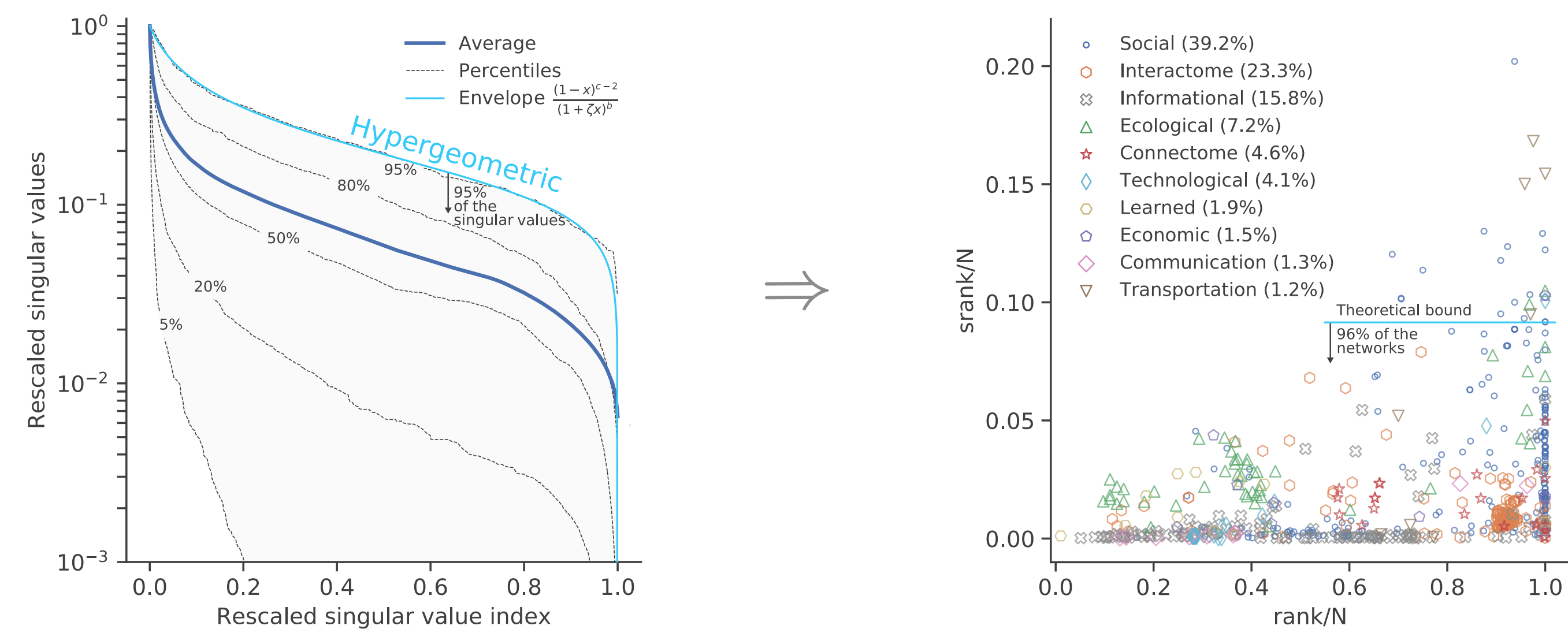
A workable definition of “low” effective rank

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Workable definition of low effective rank: $\sim 10\%$ of the number of nodes N

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The impact of effective low rank on the dynamics

Many dynamics on networks have the form

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{g}(\mathbf{x}, \mathbf{W}\mathbf{x}) = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

with $\mathbf{x} \in \mathbb{R}^N$.

Examples:

- ▷ SIS (mean-field) : $\dot{x}_i = -d_i x_i + \gamma(1 - x_i) y_i$
- ▷ Wilson-Cowan: $\dot{x}_i = -d_i x_i + (1 - ax_i) \frac{1}{1 + e^{-b(\gamma y_i - c)}}$
- ▷ Recurrent Neural Networks (RNN): $\dot{x}_i = -d_i x_i + \tanh(\gamma y_i + c_i)$
- ▷ Kuramoto-Sakaguchi: $\dot{z}_j = i\omega_j z_j + \gamma e^{-i\alpha} y_j - \gamma e^{i\alpha} z_j^2 \bar{y}_j$ with $z_j = e^{i\theta_j}$
- ▷ Population dynamics: $\dot{x}_i = -dx_i + \gamma x_i y_i$ (Lotka-Volterra)
 $\dot{x}_i = -dx_i - sx_i^2 + \gamma \frac{x_i y_i}{\alpha + y_i}$
 $\dot{x}_i = a - dx_i + bx_i^2 - cx_i^3 + \gamma x_i y_i$

for $i, j \in \{1, \dots, N\}$ and $y_i = \sum_{j=1}^N W_{ij} x_j$.