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Maximally random graph ensembles

Phys. Rev. E 70, 066117 (2004)

The probability, $P(\mathbb{A})$, for a $N \times N$ adjacency matrix $\mathbb{A} = \{a_{ij}\} =\in [0,1]^{\binom{N}{2}}$ that maximizes the entropy subjected to the L constaints $(l = 1, 2, \ldots, L)$

$$S(\{\mathbb{A}\}) = -\sum_{\mathbb{A}} P(\mathbb{A}) \ln P(\mathbb{A}) \qquad \bar{F}_l = \sum_{\mathbb{A}} F_l(\mathbb{A}) P(\mathbb{A})$$

is (α_l being the l-th Lagrange multiplier)

$$P(\mathbb{A}) \propto \exp\left(-\sum_{l=1}^{L} \alpha_l F_l(\mathbb{A})\right) .$$

Phys. Rev. E 68, 026112 (2003)

Phys. Rev. E 86, 026120 (2012)

Example 1: fixing the expected number of edges

$$\bar{F}_1 = \sum_{i=1}^{N} \sum_{j=i+1}^{N} a_{ij} = M$$

yields the Bernouilli random graph ensemble

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p^{a_{ij}} (1-p)^{1-a_{ij}} \quad \text{with} \quad p = \frac{1}{e^{\alpha_1}+1}\,.$$

Example 2: fixing the expected degree sequence

$$\bar{F}_l = \sum_{j=1}^{N} a_{lj} = k_l$$

for $l = 1, \ldots, N$ yields the soft configuration model

$$P(\mathbb{A}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N} p_{ij}^{a_{ij}} (1 - p_{ij})^{1-a_{ij}} \quad \text{with} \quad p_{ij} = \frac{1}{e^{-(\alpha_i + \alpha_j)} + 1} \, .$$

Redefining $\kappa_l = \sqrt{\langle \kappa \rangle N} e^{\alpha_l}$ for $l = 1, \ldots, N$ yields the Chung-Lu model

$$p_{ij} = \frac{1}{1 + \frac{\langle \kappa \rangle N}{\kappa_i \kappa_j}} \simeq \frac{\kappa_i \kappa_j}{\langle \kappa \rangle N} \, .$$

Studying the conditions for which p_{ij} can be factorised informs us on the degree-degree correlations observed in real networks.

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EPL 81, 28005 (2007)
Phys. Rev. E 68, 026112 (2003)
Phys. Rev. E 86, 026120 (2012)

Modeling clustering

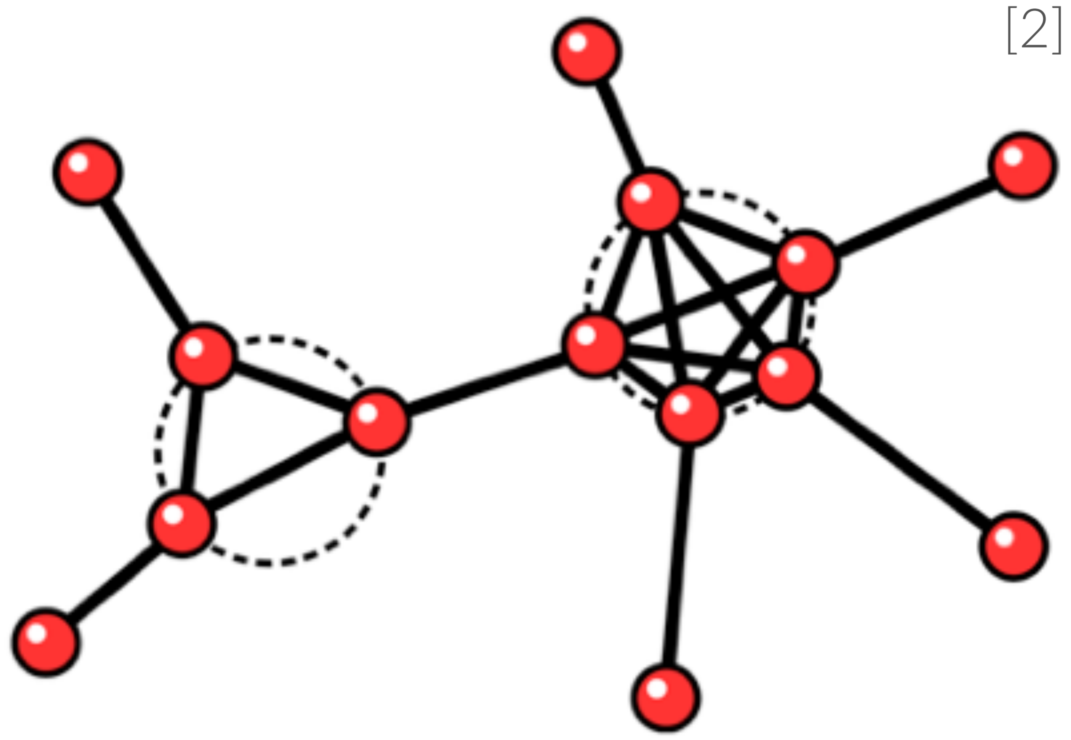
Tricky because clustering consists in three-node interactions while our mathematical tools rely on pairwise interactions either explicitly or implicitly.

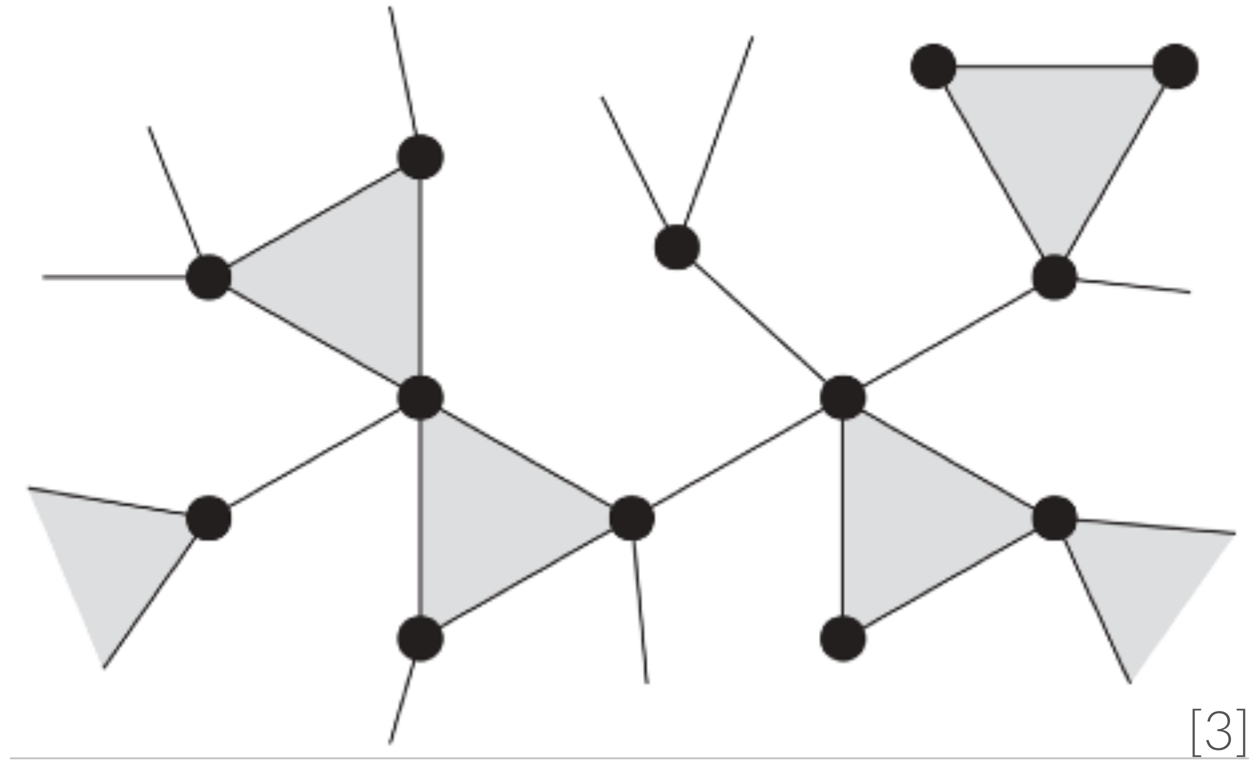
Straightforward inclusion of triangles to the maximally random graph ensemble formalism yields unwanted behavior (ex.: triangle agglutination in the Strauss model [6]).

Most models therefore assume

- ▷ an underlying tree-like structure
- ▷ that the networks are dense

[2]





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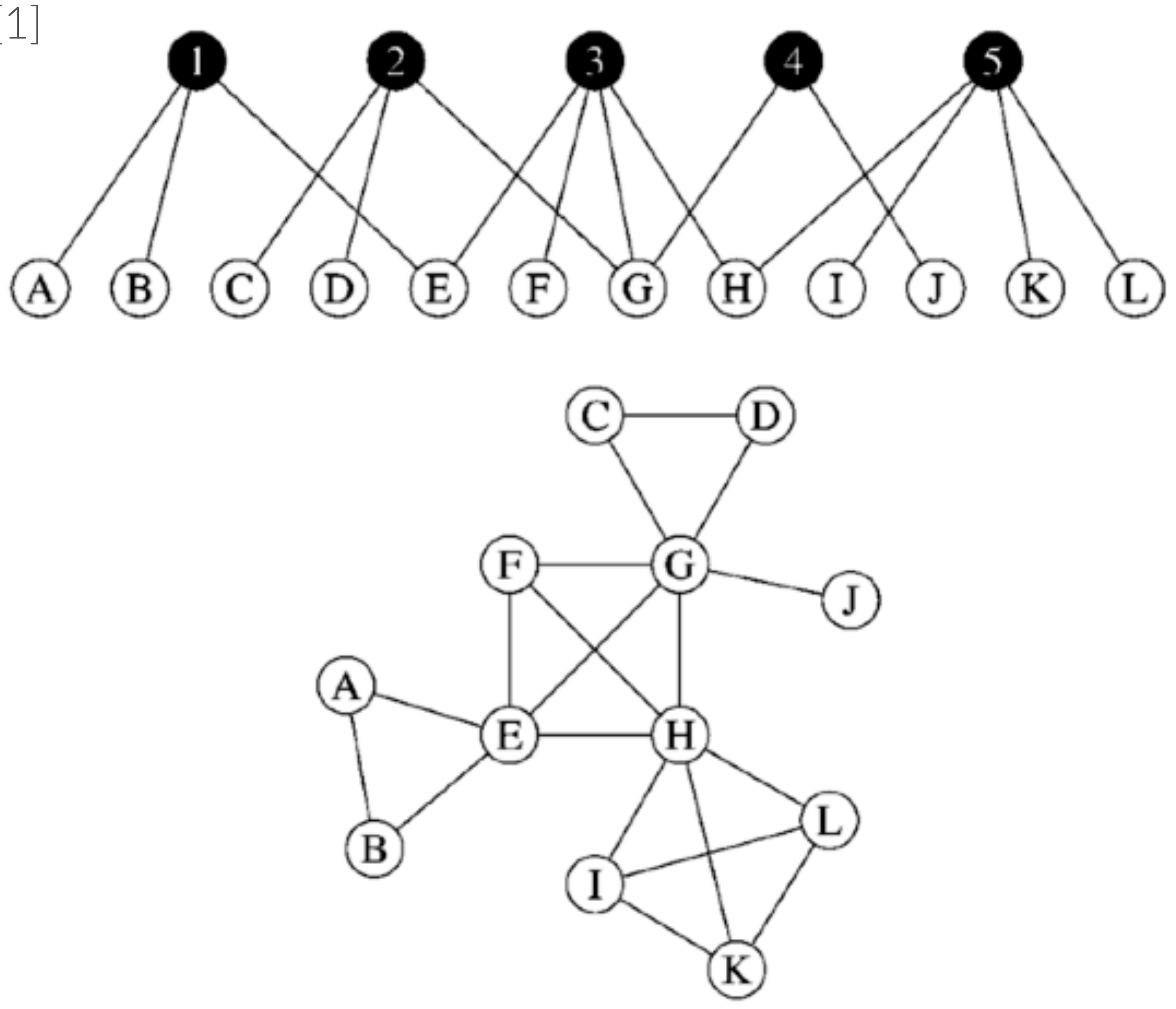
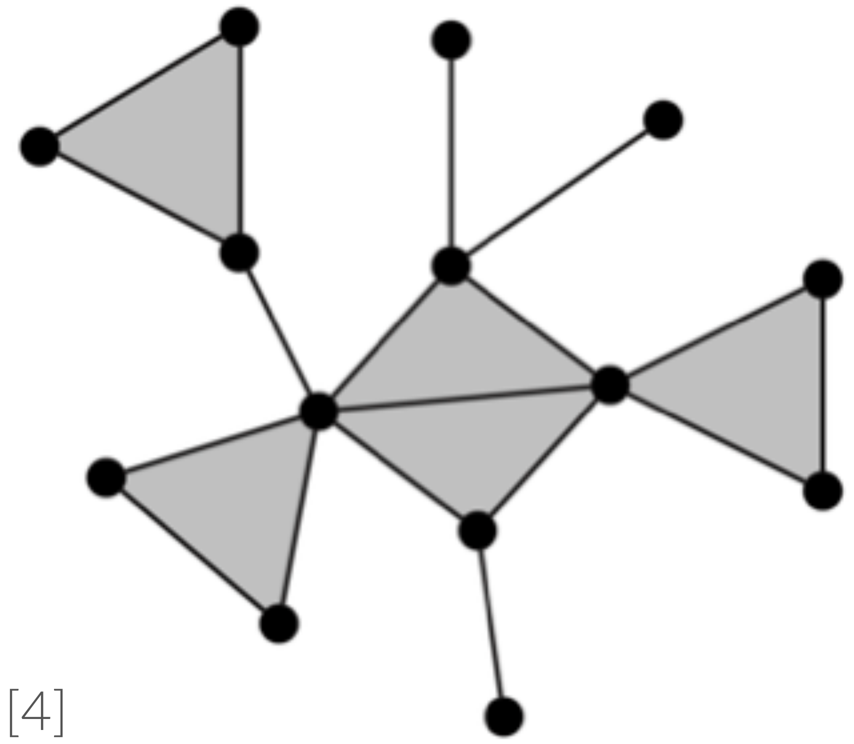
[1]





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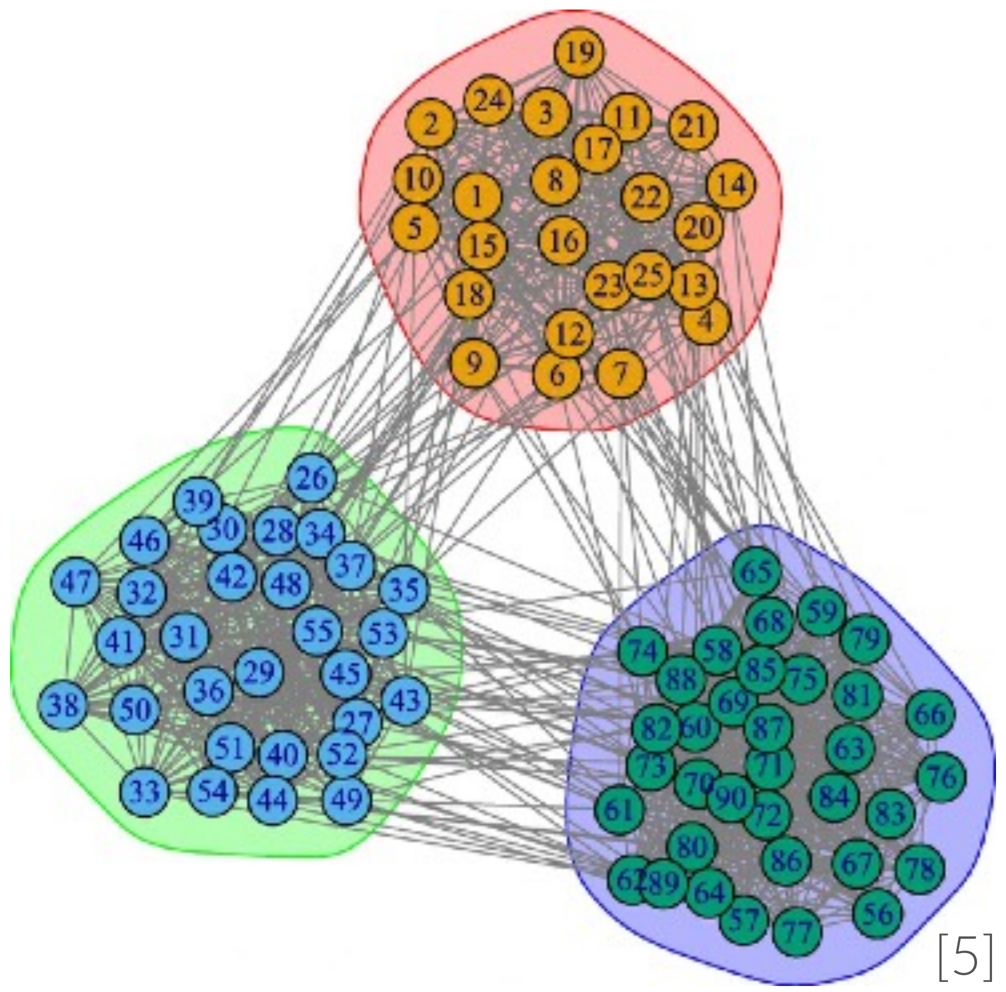

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[1] Phys. Rev. E 68, 026121 (2003)
[2] Phys. Rev. E 80, 036107 (2009)
[3] Phys. Rev. Lett. 103, 058701 (2009)
[4] Phys. Rev. E 82, 066118 (2010)
[5] Appl. Netw. Sci. 4, 122 (2019)
[6] Phys. Rev. E 72, 026136 (2005)