## Contact network epidemiology

Contents lists available at ScienceDirect

Infectious Disease Modelling

Infectious Disease Modelling

journal homepage: www.keaipublishing.com/idm

A primer on the use of probability generating functions in infectious disease modeling

Joel C. Miller

PHYSICAL REVIEW E 66, 016128 (2002)

Spread of epidemic disease on networks

M. E. J. Newman

Probability generating functions (PGFs) formalism

- probability u that an outbreak eventually dies out

- the fraction of the population infected in an epidemic wave (and the probability of such wave) is

$$R(\infty) = \sum_{k>0}^{\infty} p_k (1 - u^k) = 1 - G_0(u)$$

 $-H_0(x)$ : PGF of the distribution of the size of outbreaks that will eventually die out

$$H_1(x) = \square = \P + \P + \P + \Pi + \Pi + \dots = x \sum_{k\geq 0}^{\infty} \frac{(k+1)p_{k+1}}{\langle k \rangle} [H_1(x)]^k = xG_1(H_1(x))$$

- the distribution of the size of outbreaks that will eventually die out can be extracted from

$$H_0(x) = xG_0(H_1(x))$$

## Message #1: the friendship paradox

- on average, your friends have more friends than you do
  - $\rightarrow$  a random individual has k friends with probability  $p_k$
  - $\rightarrow$  however, their friends have k friends with probability  $\propto kp_k$
- by spreading on a contact network, the disease naturally oversamples individuals more likely to cause a larger number of secondary infections
- ignoring this effect leads back to the mass-action assumption

## JOURNAL OF THE ROYAL SOCIETY INTERFACE



Research articles

Beyond R<sub>0</sub>: heterogeneity in secondary infections and probabilistic epidemic forecasting

Laurent Hébert-Dufresne, Benjamin M. Althouse, Samuel V. Scarpino and Antoine Allard ☑