

Mesoscopic level: The k-core/onion decomposition

Onion decomposition: k-core decomposition with additional information about the positions of nodes within every k-shell (layers). Information about layers is obtained from the k-core decomposition with minimal additional computational cost.

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Stub matching scheme
One type of nodes per layer

 ► Three types of stubs (red, green, black)

> Rules:
     1. Allowed links: red-red, red-green, red-black
     2. Nodes in layer \ell and shell k must
         (a) have exactly k links to nodes in layers \ell' > \ell (if layer
             \ell is the first layer of the k-shell).
         (b) have at least k+1 links to nodes in layers \ell' > \ell-1
             and at most k links to nodes in layers \ell' > \ell (if it is
             not in the first layer of the k-shell).
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Information about layers is obtained from the k-core decomposition with minimal additional computational cost.

Stub matching scheme

- > One type of nodes per layer
- ▷ Three types of stubs (red, green, black)
- > Rules:
 - 1. Allowed links: red-red, red-green, red-black
 - 2. Nodes in layer ℓ and shell k must
 - (a) have exactly k links to nodes in layers $\ell' \geq \ell$ (if layer ℓ is the first layer of the k-shell).
 - (b) have at least k+1 links to nodes in layers $\ell' \geq \ell-1$ and at most k links to nodes in layers $\ell' \geq \ell$ (if it is not in the first layer of the k-shell).

Challenges

- ▶ Many analytical approaches (PGF, ODE) can be adapted to account for stub types, but the graphs considered are infinite in size.
- Sampling naively is easy, sampling right is tricky (ex.: uniform sampling via edge swaps).