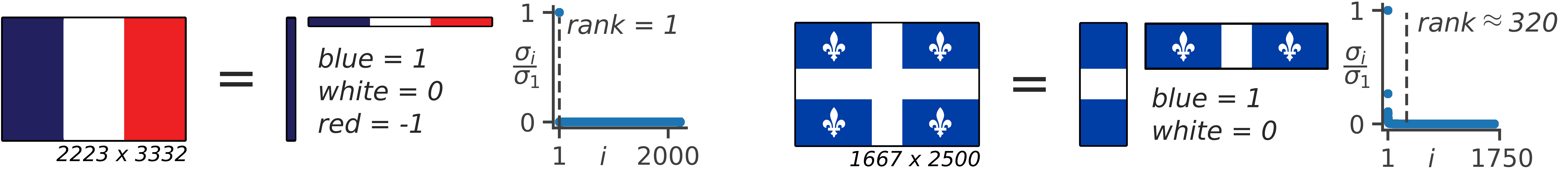
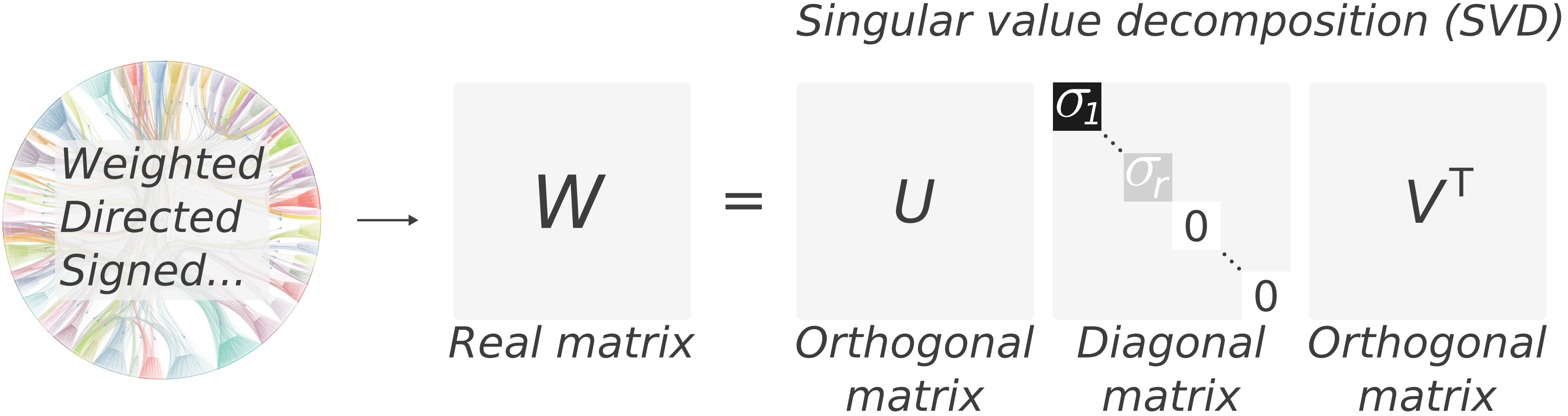


2223 x 3332
=
blue = 1
white = 0
red = -1
1
rank = 1
σi / σ1
0
1
i
2000
1667 x 2500
=
blue = 1
white = 0
1
rank ≈ 320
σi / σ1
0
1
i
1750

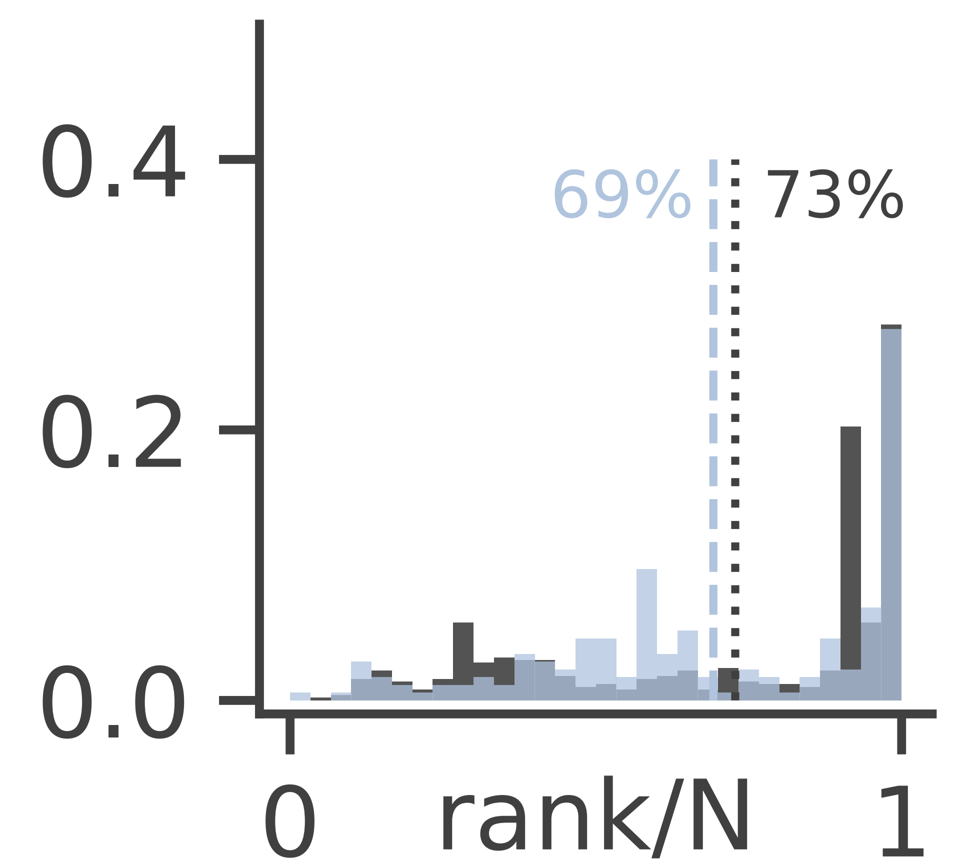
The rank of adjacency matrices

$$\mathbf{W} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^\intercal = \sigma_1 \begin{bmatrix} | \\ \mathbf{u}_1 \\ | \end{bmatrix} \begin{bmatrix} \text{---} & \mathbf{v}_1^\intercal & \text{---} \end{bmatrix} + \sigma_2 \begin{bmatrix} | \\ \mathbf{u}_2 \\ | \end{bmatrix} \begin{bmatrix} \text{---} & \mathbf{v}_2^\intercal & \text{---} \end{bmatrix} + \sigma_3 \begin{bmatrix} | \\ \mathbf{u}_3 \\ | \end{bmatrix} \begin{bmatrix} \text{---} & \mathbf{v}_3^\intercal & \text{---} \end{bmatrix} + \dots$$


Singular value decomposition (SVD)
Weighted
Directed
Signed...
W
=
U
σ1
σr
0
0
V^T
Real matrix
Orthogonal matrix
Diagonal matrix
Orthogonal matrix

Rank r of a matrix

- number of linearly independent rows/columns
- dimension of the vector space generated by its rows/columns
- number of nonzero singular values



0.4
0.2
0.0
69%
73%
0
rank/N
1

Fraction
of networks

Unweighted networks
Weighted networks

Results for 679 empirical networks (502 unweighted networks and 177 weighted networks) downloaded from Netzschleuder.

Full-rank (or almost full rank) adjacency matrices are very common!

5

The rank of adjacency matrices

Rank r of a matrix

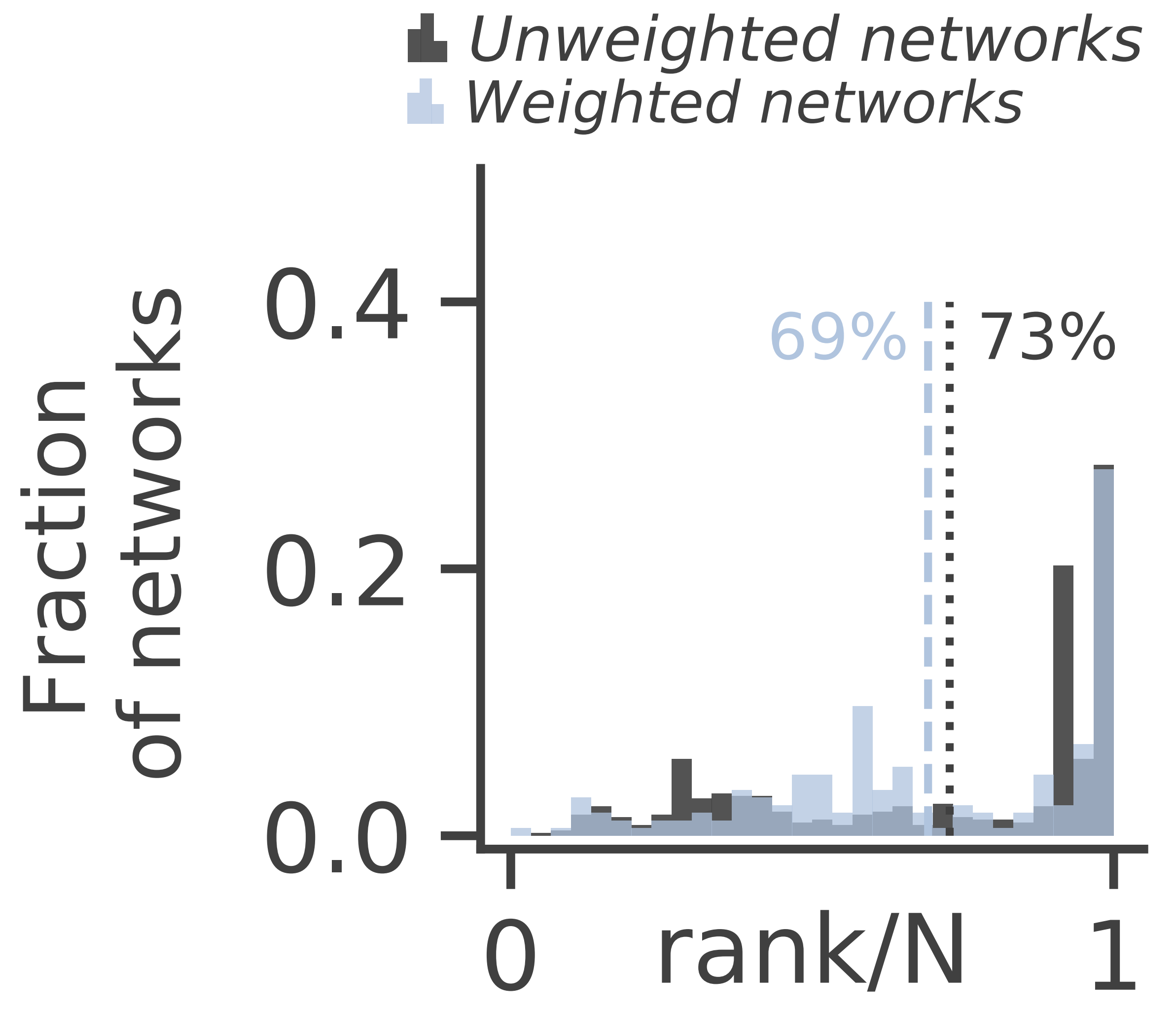
- ▷ number of linearly independent rows/columns
- ▷ dimension of the vector space generated by its rows/columns
- ▷ number of nonzero singular values

Singular value decomposition (SVD)

$W = U \,\Sigma\, V^{\top}$

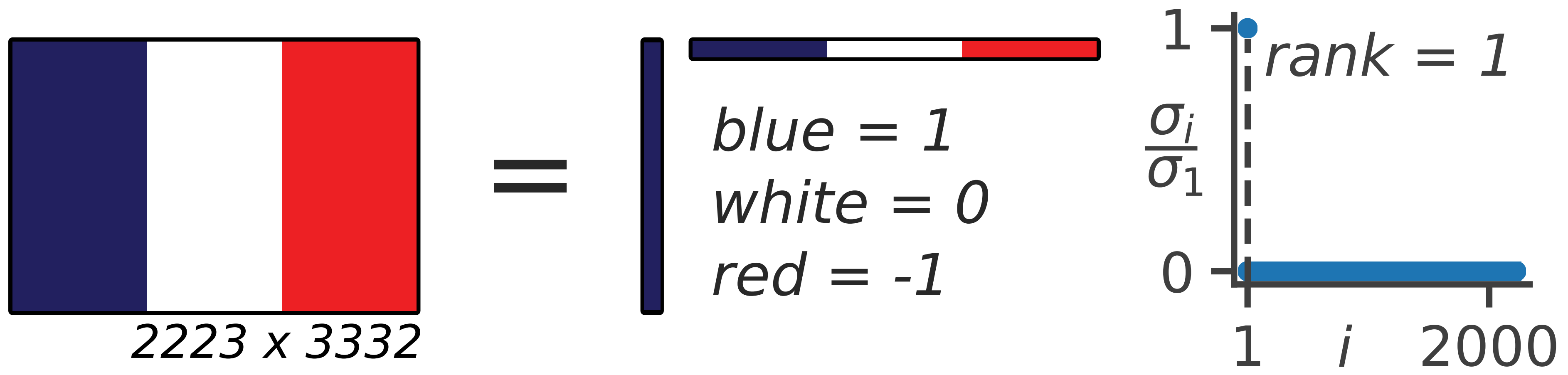
W	=	U	$\mathrm{diag}(\sigma_1, \dots, \sigma_r, 0, \dots, 0)$	$V^{\top}$
Real matrix		*Orthogonal matrix*	*Diagonal matrix*	*Orthogonal matrix*

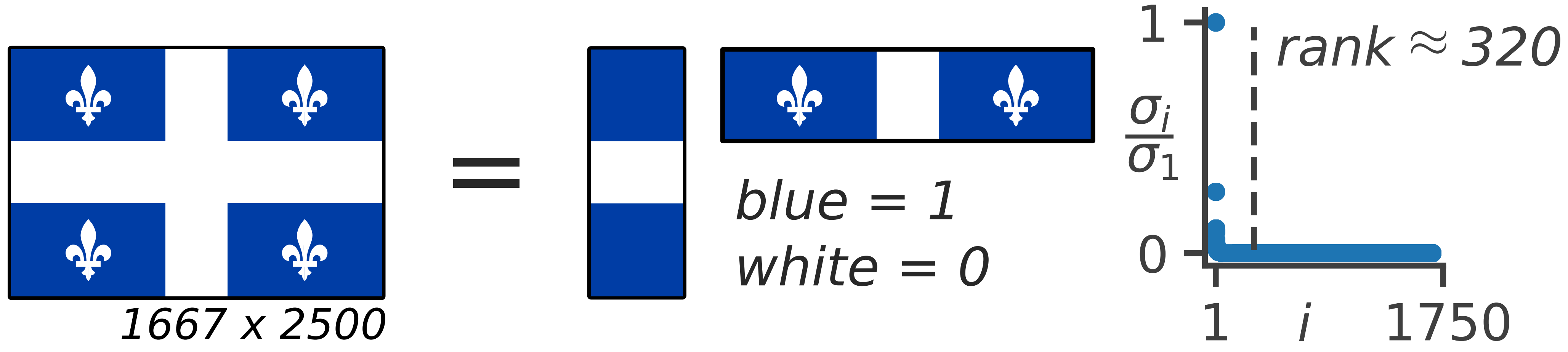
Full-rank (or almost full rank) adjacency matrices are very common!



Results for 679 empirical networks (502 unweighted networks and 177 weighted networks) downloaded from Netzschleuder.

$$\mathbf{W} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^{\top} = \sigma_1 \begin{bmatrix} | \\ \mathbf{u}_1 \\ | \end{bmatrix} \begin{bmatrix} — & \mathbf{v}_1^{\top} & — \end{bmatrix} + \sigma_2 \begin{bmatrix} | \\ \mathbf{u}_2 \\ | \end{bmatrix} \begin{bmatrix} — & \mathbf{v}_2^{\top} & — \end{bmatrix} + \sigma_3 \begin{bmatrix} | \\ \mathbf{u}_3 \\ | \end{bmatrix} \begin{bmatrix} — & \mathbf{v}_3^{\top} & — \end{bmatrix} + \dots$$





The effective ranks of adjacency matrices

$$\mathbf{W} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top \simeq \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^\top$$