Unveiling the hidden geometry of weighted networks

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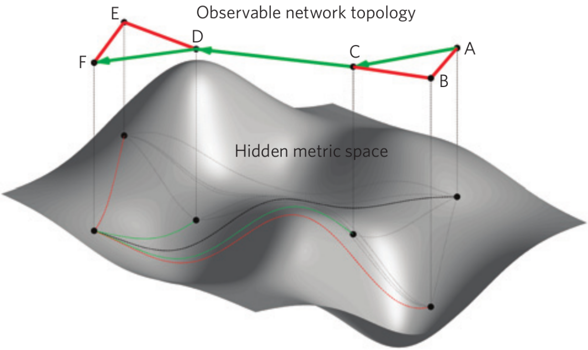
Motivation and outline

It is assumed that complex networks are embedded in a metric space in which the distances between nodes represent intrinsic similarities that determine the structure of the network.

- How can we model networks embedded in a hidden metric space ?
- What effect has a hidden metric space on the properties of the networks ?
- Can we infer a plausible hidden metric space for real networks ?



Outline

- Random networks with hidden metric space
- Random weighted networks with hidden metric space
- Illustration U.S. airports network
- Open questions

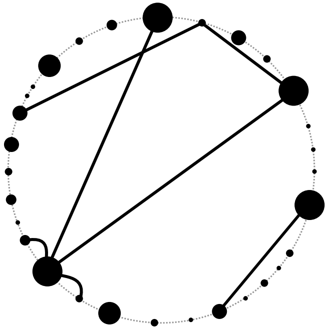
Random networks with hidden metric space

- N nodes distributed in homogeneous and isotropic D-dimensional space
- Each node is assigned a hidden variable κ

$$\rho(\kappa) \propto \kappa^{-\gamma}$$

- Nodes are connected with probability $p(\chi)$ where

$$\chi \propto \frac{d}{(\kappa\kappa')^{1/D}}$$



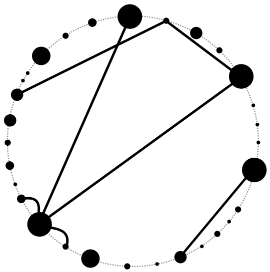
- If $p(\chi)$ is integrable over $\chi \in [0, \infty)$

$$\langle k(\kappa) \rangle \propto \kappa \qquad \Rightarrow \qquad P(k) \sim k^{-\gamma}$$

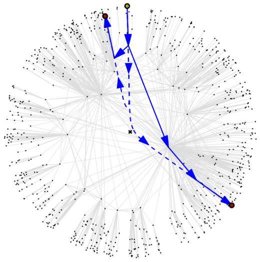
- Small-world: high degree nodes likely to be connected even at long distance.
- Triangle inequality implies strong clustering controlled by the specifics of $p(\chi)$.

Random networks with hidden metric space

Hidden variables model



Purely geometrical model

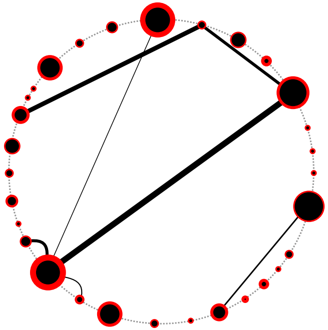


- Natural geometry of scale-free networks is hyperbolic (Krioukov *et al.* PRE 2009, PRE 2010)
- Self-similarity of real complex networks (Serrano *et al.* PRL 2008)
- Efficient navigability of the Internet without an explicit knowledge of its global structure (Boguñá *et al.* Nat. Phys. 2008, Nat. Commun. 2010)
- Identification of biochemical pathways in living organisms (Serrano *et al.* Mol. Biosyst. 2012)
- Realistic models of growing networks (Papadopoulos *et al.* Nature 2010, Zuev *et al.* Sci. Rep. 2015)

Random weighted networks with hidden metric space

- N nodes distributed in homogeneous and isotropic D-dimensional space
- Each node is assigned two hidden variables κ and σ according to $\rho(\kappa, \sigma)$
- Nodes are connected with probability $p(\chi)$ where

$$\chi \propto \frac{d}{(\kappa\kappa')^{1/D}}$$



- Links have a weight w according to $\varphi(w) = \frac{1}{\bar{w}}\, f(w/\bar{w})$ where

$$\bar{w} \propto \frac{\sigma\sigma'}{(\kappa\kappa')^{1-\alpha/D} d^{\alpha}}$$

with $0 \leq \alpha \leq D$ controlling the coupling between the weights and the metric space.

Random weighted networks with hidden metric space

- The hidden variable κ corresponds to the expected degree

$$\langle k(\kappa,\sigma)\rangle \propto \kappa \; ; \qquad P(k) = \int \frac{\mathrm{e}^{-\kappa}\kappa^{k}\rho(\kappa)}{k!} d\kappa$$

- The hidden variable σ corresponds to the expected strength

$$\langle s(\kappa,\sigma)\rangle \propto \sigma$$

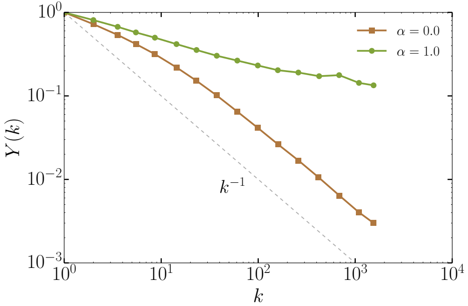
- The joint density $\rho(\kappa,\sigma)$ controls the correlation between the strength and the degree

$$\langle s(k)\rangle = \iint \frac{\sigma \mathrm{e}^{-\kappa}\kappa^{k-1}\rho(\kappa,\sigma)}{(k-1)!P(k)} d\sigma d\kappa$$

Random weighted networks with hidden metric space

- The free parameter α indirectly controls the *disparity* and the weight distribution

$$Y_i = \sum_j \left(\frac{w_{ij}}{s_i} \right)^2 \quad ; \quad \frac{1}{k_i} \leq Y_i \leq 1$$



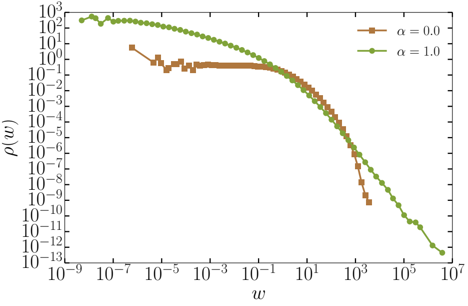


Illustration: U.S. airports network

- Hidden variables

$$\rho(\kappa) = \frac{(\gamma-1)\kappa_0^{\gamma-1}\kappa^{-\gamma}}{1-(\kappa_0/\kappa_c)^{\gamma-1}}$$

$$\rho(\sigma|\kappa) = \frac{\lambda^{a\kappa^\eta/\lambda}}{\Gamma(a\kappa^\eta/\lambda)}\sigma^{a\kappa^\eta/\lambda-1}\mathrm{e}^{-\sigma/\lambda}$$

- Probability of connection

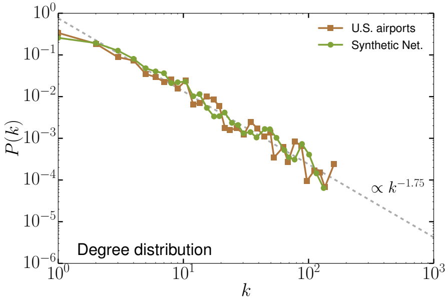
$$p(\chi) = \frac{1}{\chi^\beta + 1}$$

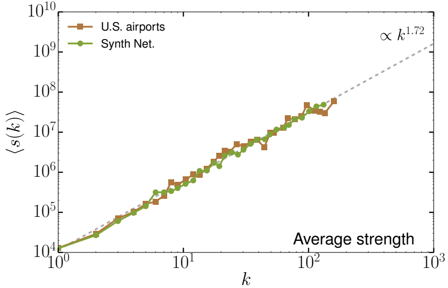
- Probability of weights

$$\varphi(w) = \frac{1}{\bar{w}}\mathrm{e}^{-w/\bar{w}}$$

- Strength distribution

$$\rho(s) \sim s^{(\gamma+\eta-1)/\eta}$$





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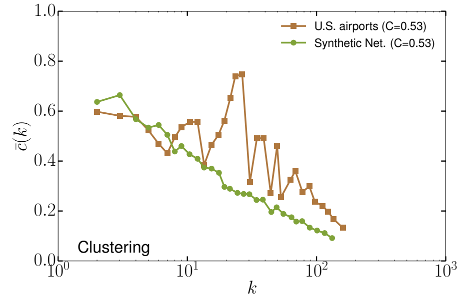

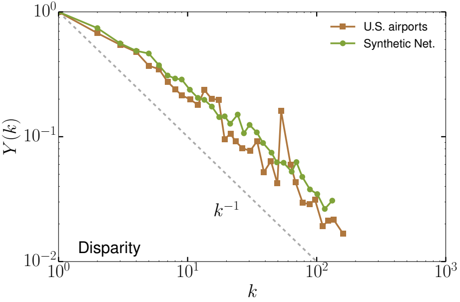

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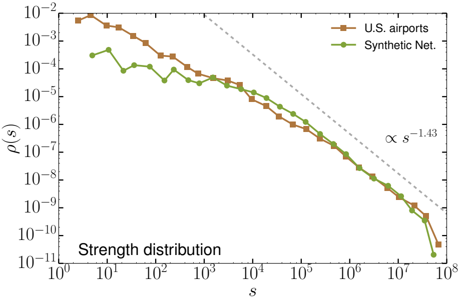
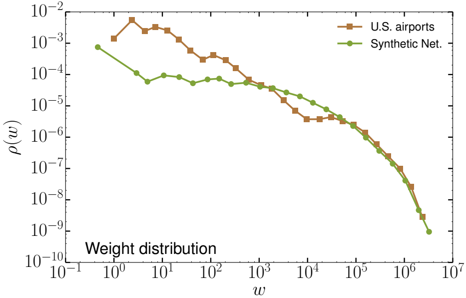
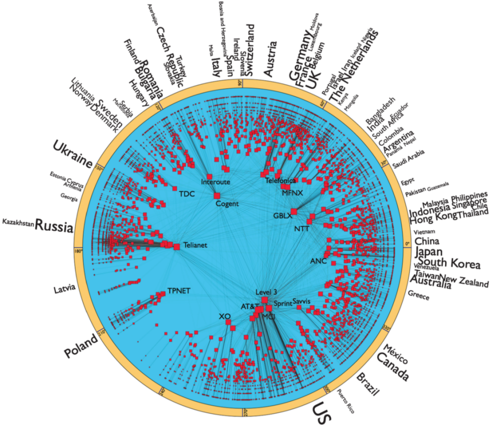
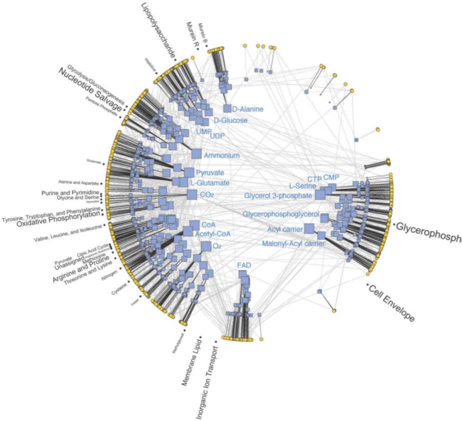




Open questions

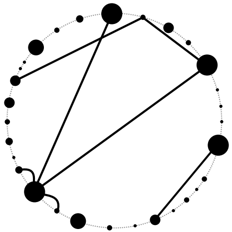
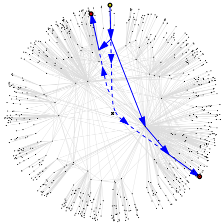
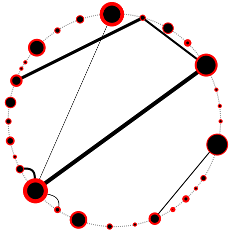
- Embedding of real complex networks





Open questions

- Natural geometry of scale-free weighted networks

	Hidden variables model	Purely geometrical model
Unweighted networks		
Weighted networks		?

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