The relevant appendices can be found under 'Related Documents' for each experiment at this link:
http://sky.campus.mcgill.ca/Exp/exp.a5w#

Zeeman Effect

Updated in August 2018

Warnings:

- Be careful when manipulating the Lummer-Gehrcke plate; it is a fragile and very expensive piece of glass. Manipulate it from the sides.
- If you choose to analyze the effect with the Neon lamp (optional), be extremely careful when changing the lamp. Get instruction from an instructor or technician.
- As with all optics experiment, make sure you manipulate the parts over a table, and not over the floor in your hands. If you happen to drop them, they will likely not be damaged.

Contents

1 Introduction

The object of this experiment is to investigate the interaction of atomic magnetic moments with a magnetic field (The Zeeman Interaction). This is one of the weakest interaction studied by physicists. The sophisticated optical equipment available enables the precise measurement of energy changes as small as 10^{-5} eV. This degree of sensitivity is obtained using an optical interferometer. Normal spectrometer resolving power ($\lambda/\delta\lambda$ where $\delta\lambda$ can be resolved at λ) is greatly enhanced by employing the principle of division of amplitude by multiple reflections in the interferometer. The multiple beams produced are mutually out of phase by the same amount and interfere in the image plane of the spectrometer to produce a simple interference pattern which obeys the usual criterion for constructive interference, that nλ = d(θ) where d is the path difference between any two successive beams. This pattern is very sensitive to changes in wavelength so that small wavelength (energy) changes may be determined from measurements made on the interference pattern, which may itself be recorded using a CCD camera.

2 Theory

More information on the theory of the Zeeman effect is given in Appendix A.

Quantum mechanics allows a description of the state of an atom by multiple quantum numbers. An isolated atom will have a base energy determined by its principal quantum number n, which is an integer. The angular momentum of the atom is also quantized in both magnitude and direction. The magnitude is described by the azimuthal quantum number l, which is an integer ranging between 0 and $n-1$, while the direction is given by the magnetic quantum number m_l, which is an integer ranging from $-l$ to l.

In a magnetic field $\mathbf{B}$, the magnetic moment resulting from the angular momentum in the atom will cause an interaction with the applied field that will split the energy levels according to the value of the magnetic quantum number m. The magnetic moment of the atom is given by:

$$\mu = -\left(\frac{e}{2m_e}\right)\mathbf{L} \tag{1}$$

where e is the charge of the electron (positive) and m_e its mass. Here, $L = \sqrt{l(l+1)}$. The interaction energy is given by $E_m = -\mu \cdot \mathbf{B}$, such that:

$$E_m = \left(\frac{e}{2m_e}\right) LB\cos(\vartheta) \tag{2}$$

where ϑ is the angle between $\mathbf{L}$ and $\mathbf{B}$; due to the quantization of the direction of angular momentum, we have $\cos(\vartheta) = \frac{m_l}{\sqrt{l(l+1)}}$, such that:

$$E_m = \mu_B m_l B \quad . \tag{3}$$

$\mu_B = e/2m_e$ is a quantity known as the **Bohr magneton**. You will seek to obtain its value in this experiment.

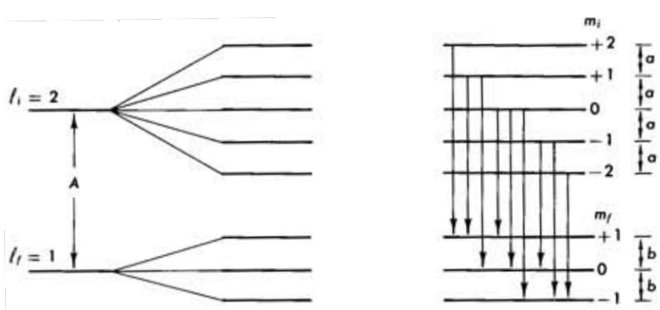

Figure 1: Illustration of the splitting in the atomic energy levels with the application of a magnetic field **B**. In Zeeman splitting, a triplet is observed as Δm_l may equal -1, 0 or 1, and the spacing between the energy levels for some l is constant. Taken from Adrian C. Melissinos and Jim Napolitano. *Experiments in Modern Physics.* Academic Press, USA, 2nd edition, 2003, p. 221.

Atomic transitions with $|\Delta m_l| > 1$ are highly improbable and are said not to be allowed. Depending on the initial and the final state of the atom, we might thus expect to obtain a triplet of peaks when a magnetic field strong enough to resolve the individual spectral lines is applied to the atoms, depending on if Δm_l equals -1, 0 or 1. This is illustrated in figure (1); on a frequency scale, the peaks are equally spaced by $\mu_B B/h$ and the triplet is centered where the transition would occur if no magnetic field was to be applied.

Anomalous Zeeman Effect

Supplementary splitting may occur when the net spin angular momentum S (quantized by the spin quantum number s; here, $S = \sqrt{s(s+1)}$) of the atoms is non-zero, due to coupling of the angular momenta L and S. Without going into the full derivation, which is detailed in Appendix A, the energy shift is given by:

$$E_m = \frac{e}{2m_e} g\mathbf{J} \cdot \mathbf{B} \tag{4}$$

where we have used the Lande g factor, given by

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad .$$

3 Measurement Using the Lummer-Gehrcke Plate Interferometer

More information on the theory of the LG plate is given in Appendix B. It is highly recommended to consult the Appendix to gain sufficient physical insight on the instrument.

In the first part of the experiment, you will use a Lummer-Gehrcke interferometer (**LG**) in combination with a constant deviation prism; it is schematized in figure (2). The instrument is a glass plate, which surfaces are accurately plane and parallel to each other. Monochromatic parallel light is introduced into the plate at almost grazing incidence, and undergoes successive internal reflections at the surfaces of the plate. At each reflection, some of the light emerges from the plate. Under appropriate conditions, the light at each reflection is in phase with the light which emerged at the previous reflection. All these emergent rays of parallel light reinforce, and may be brought to a focus by a lens. The light is usually introduced by means of a prism cemented to the plate, to avoid loss of intensity by reflection at the surface of the plate.

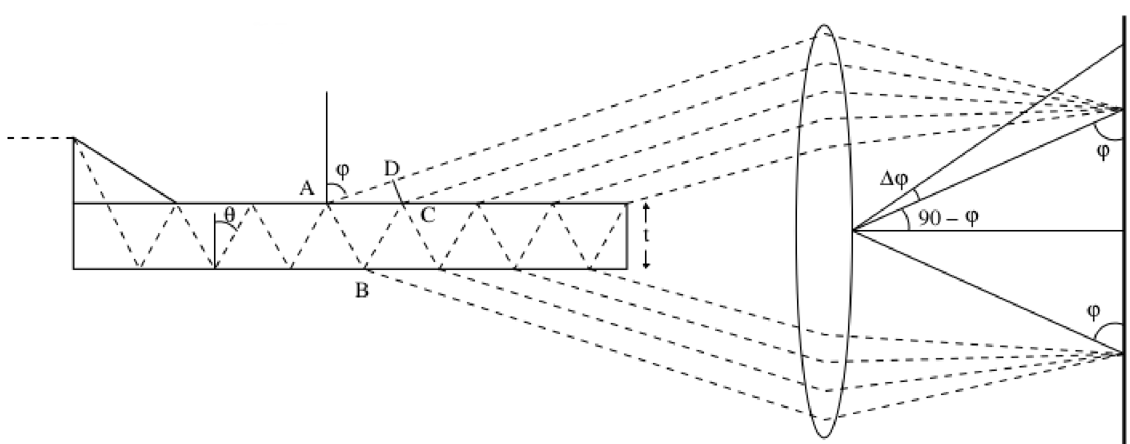

Figure 2: Illustration of the LG plate and its functioning mechanism.

3.1 Theory

It can be derived that constructive interference occurs in an LG plate of thickness t and of refraction coefficient ρ if

$$\rho t \cos(\theta) = t\sqrt{\rho^2 - \sin^2(\varphi)} = n\lambda/2 \tag{5}$$

where λ is the wavelength of the incoming monochromatic light and n is the order of the spectrum - in our case a very large value - and θ and φ are angles as defined on figure (2). By successive differentiation and substitutions on this equation, we may estimate the distance $\Delta\lambda_m$ (not to be confused with a wavelength variation!) between two successive orders of a same line:

$$\Delta\lambda_m = \frac{n\lambda^2}{n^2\lambda - 4t^2\rho\frac{d\rho}{d\lambda}} \tag{6}$$

When splitting from a same line is present, we might then relate the distance $\Delta\lambda_s$ between a line resulting from Zeeman splitting to the line obtained without a magnetic field to $\Delta\lambda_m$:

$$\frac{\Delta\lambda_s}{\Delta\lambda_m} = \frac{s}{m} \tag{7}$$

as is evidenced by figure (3).

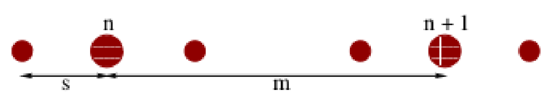

Figure 3: Illustration of the splitting and the different orders in an LG spectrum.

We finally need to relate those distances to the shift in energy resulting from the magnetic field. The energy of a photon is $E = hc/\lambda$; differentiating with respect to λ_s, we may approximate that

$$\Delta E = -hc\Delta\lambda_s/\lambda^2 \tag{8}$$

We then identify ΔE as the shift in energy due to the magnetic field E_m; using equations (3), (6) and (7), we get:

$$\mu_B B = \frac{hc}{\lambda^2}\frac{s}{m}\left(\frac{n\lambda^2}{n^2\lambda - 4t^2\rho\frac{d\rho}{d\lambda}}\right) \tag{9}$$

which, finally, as $\sin(\beta) \sim 1$ as the screen is far from the plate, and by using equation (5), totally relates the Bohr magneton to parameters measurable with our setup:

$$\mu_B B = \frac{s}{m}\frac{hc\sqrt{\rho^2 - 1}}{2t(\rho^2 - 1 - \lambda\rho\frac{d\rho}{d\lambda})} \ . \tag{10}$$

3.2 Setup and Procedure

The setup is shown in figure 4. The spectral tube (1), in our case a cadmium lamp, is placed at the center of the electromagnet (2), which provides a magnetic field of near constant magnitude and direction in between its coils when a current is driven through it. The light from the tube goes through a lens (3) that focuses the beam on the slit at the entrance of the collimator (4). Optionally, it may pass through a polarizing filter (5) - this is required only for some observations. It then enters the L-G plate (6), then it is deviated by the constant deviation prism (7), which can be rotated with the milled ring (8). As the amount of refraction on the prism depends on the wavelength of the incident light, the wavelengths are separated when leaving the prism. The resulting spectrum is either observed through the telescope or sent to the camera at (9), whose telemetric photo objective (zoom) must be set to maximum and focus set to infinity. The computer (10) analyzes the resulting pictures. The intensity of the magnetic field is adjusted by the power supply (11), whose output voltage is monitored by a voltmeter (12).

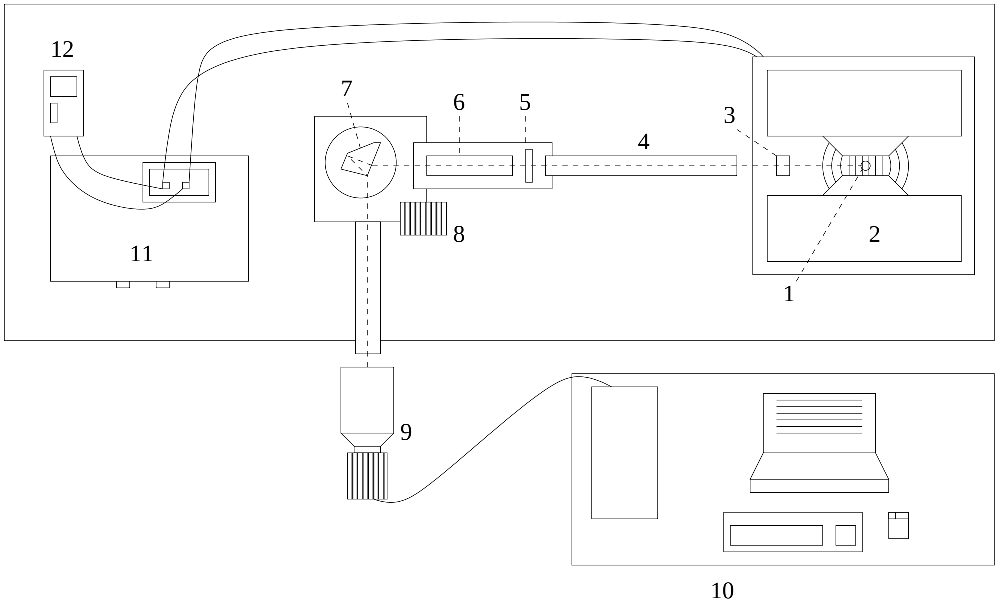

Figure 4: Experimental setup for the Zeeman experiment.

3.2.1 Calibration

The very first thing you will have to do in the experiment is to calibrate the electromagnet (use excess voltage with the electromagnetic power supply, then control the current to vary the magnetic field; do not exceed labelled maximum values or ask the tech what are the acceptable levels to run it at); to do so, place a Hall probe in between the coils of the electromagnet and measure the magnetic field for various power supply current values. The voltmeter measures the voltage across a low value shunt resistor placed in series with the coil of the magnet and the measured voltage is thus linearly related to the current in the magnet. The relationship between the voltage and the magnetic field should be linear for low field magnitudes and deviate from linearity at high field magnitudes.

Using the universal stand, set the hall probe in the middle of the field generator. Before you set it up, it is very important that you check that the stand and the clamps are made of a non-ferromagnetic material, such as brass or aluminum. Using a ferromagnetic stand in conjunction with the electromagnet will send parts flying at high velocity and may hurt/kill you. Reusable painters tape (usually green; do not use duct, scotch, masking tape, etc. since these are harder to remove) may be used also.

You will want to position the hall probe in the dead middle of the magnet. Beware, the plugs on the probe may be misleading: the usb end is NOT for data transfer, it acts merely as a power source; plug it into an outlet with a converter, not into a computer. You want to connect the two banana plugs of the probe into a voltmeter, not a voltage source! Place roughly the probe in the middle of the magnet with the stand, then turn on the magnet. The location of the hall probe affects the voltage reading. Figure 5 shows the relevant axes. The center position can be found as follow: the voltage reading of the probe is minimized when moving the probe in the x direction, but it is maximized when moving the probe in the y or z direction.

The whole point of this calibration is to relate the voltage measured on the plugs under the table with the strength of the field. Therefore, you need to make a voltage measurement on both the probe and the bottom plugs. You may want to use two voltmeters to speed up the process. You can then convert the probe voltage to field strength with the table seen in the relevant appendix. Make sure you use the correct model to find the conversion ratio (the model should be listed on your equipment list; when in doubt, ask the technician. It is likely to be HR66).

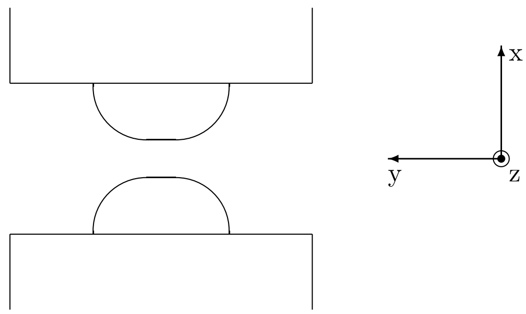

Figure 5: Magnets, how do they work? Picture of the top of the magnet.

3.2.2 LG setup

- Set the cadmium lamp in the middle of the magnet. There is no systematic way to set it properly in the middle of the magnet; use your judgement.
- Put the spectrometer telescope in the place of the camera. Focus it at infinity and adjust the alignment until you can observe the cadmium spectrum through it. Locate the red line corresponding to $\lambda = 643.8$nm.
- Move the spectrometer across the field of illumination to obtain a maximal intensity. Align the source, the collimator axis, the focusing lens and the prism angle carefully to further optimize the intensity while keeping a good level of resolution. A cylindrical lens and wooden stand is available to focus the source to a line.
- Place the Lummer-Gehrcke (LG) interferometer on the spectrometer table **with caution**. This piece requires no adjustment prior to use. The prism is broken due to incompetent students that dropped it on the ground, but the plate is still usable. Place it anyway in the support: the cadmium lamp is bright enough that the plate works mostly well even without the prism. Please manipulate the plate with its unpolished sides.
- Observe the spectrum once again. Decrease the slit width until only a broken line or series of “dots” is seen - this is your interference pattern resulting from the LG plate. It should look similar to that in figure (8).
- Observe that each position of constructive interference (dot) is split into 3 when a magnetic field is applied - this is the normal Zeeman effect.
- Include the polarizing filter so that only polarized light enters the spectrometer and observe that the three “dots” correspond to light linearly polarized in orthogonal directions.
- Replace the telescope with the digital camera (remove the post to screw it into the camera; do not screw the camera in by spinning it on the post while it is in the holder) and try to put it on the optical axis. You must focus at infinity and determine a suitable exposure time - the steps to do so will be detailed below. Acquire images at different magnetic field values. The data analysis steps will be detailed later.
- You may choose another line to observe the anomalous Zeeman effect.

4 Measurement Using the Fabry-Pérot Interferometer (FP)

More information on the theory of the FP interferometer is given in Appendix A.

The second part uses an alternative interferometry method - the Fabry-Pérot interferometer, which is illustrated in figure (6). The instrument is made of parallel glass plates which are partially transmitting ($\sim 70\%$). A collimating lens L1 in the cavity renders the incident rays parallel; they then undergo multiple reflections on plates A and B and are made to interfere by being converged at F by the lens L2, producing concentric constructive interference circles.

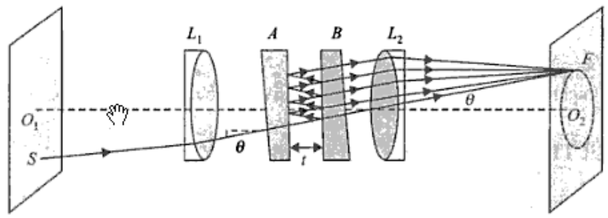

Figure 6: Visualization of the Fabry-Pérot interferometer. Taken from Devaraj Singh, *Fundamentals of Optics*, p.154.

4.1 Theory

It can be shown (see Appendix A for a derivation) that the difference in wave number between two satellite lines in the FP spectrum is given by:

$$\Delta \bar{v} = \frac{\delta}{2t\Delta} \tag{11}$$

where t is the distance between the interferometer's mirrors[1]. Δ and δ are parameters that depend on the radius of the various observed rings as such:

$$\delta = \frac{1}{4}\sum_{n=1}^{4}(r^2_{n+1,\alpha} - r^2_{n+1,\beta}) \tag{12}$$

$$\Delta = \frac{1}{4}\sum_{n=1}^{2}(r^2_{2n,\alpha} - r^2_{2n-1,\alpha} + r^2_{2n,\beta} - r^2_{2n-1,\beta})\Delta = \frac{1}{4}\sum_{n=1}^{2}(r^2_{2n,\alpha} - r^2_{2n-1,\alpha} + r^2_{2n,\beta} - r^2_{2n-1,\beta}) \tag{13}$$

where n is the order of the ring, r its radius and α and β represent the two components of the splitting (for a doublet, α would consistently represent the leftmost peak, and β the rightmost peak, say). Note that this equation indicates that you need to see at least four order of rings to obtain the values for Δ and δ.

As the difference in wave number between the lines of the doublet and the central line is $\Delta\bar{v}/2$, we can obtain the following expression for the Bohr magneton:

$$E_m = \mu_B B = \hbar c \Delta \bar{v}/2 \tag{14}$$

where equation (3) was used with $m_l = 1$.

4.2 Setup and Procedure

- First, remove the LG plate, then unscrew the slit at the entrance of the collimator (4). Place a red colour filter on one of the holds for the telescope; you can tape it in place.
- Install the Fabry-Pérot on a mount that goes on the optical rail. Adjust the FP cavity at the right height and bring it close to the exit of the deviation prism. You want to use the remainder of the light and keep only the proper wavelength. It is important that the light goes throught the interferometer *after* going through the red color filer.
- Install the camera close to the end of the optical rail. Fine-tune the alignment of the FP cavity and the camera to observe a diffraction pattern, taking the form of multiple rings, with a maximum intensity. You should align the center of the rings near a corner of the pictures to see a maximal number of rings and to have a maximal pixel resolution.
- Acquire images at different magnetic field values. The camera should be setup as for the first part of the experiment. The data analysis steps will be detailed later.

[1] As of August 2012, this value is 4mm. See Appendix D for the data sheet on the FP etalon used in this experiment.

You may notice that your field of view is rather small. This is because the collimator evens out the light across the FoV. If you want larget pictures (more rings), you may want to simply remove the collimator. Doing so will greatly widen your FoV, but will add a bright point (the lamp itself). You will want to adjust the prism and the camera such that you can see most of the rings without seeing the lamp itself. This leads to some intensity inconsistensies in the ring distribution (some darker shades of red all over the place), but allows many more rings to be photographed.

Since you will be comparing radii of rings, you obviously always want to see the center in any picture you take, or your pictures will be much harder to analyse.

5 Camera Settings, Software and Data Analysis

5.1 Camera Settings

The camera is shown in figure (7) so that identification of the components in the text is easy.

Figure 7: Identified visualization of the digital camera.

In order to use the manual focus option on the PowerShotSX130 Digital Camera, select “M” on the dial near the on/off button (1), then press MF (2) until the manual focus adjustment appears on the display. The focus can be adjusted with the rotation of the disk (2). Press (3) to go from MF to shutter speed to aperture.

Try different values for the exposure time and aperture. You should start with the lowest aperture (i.e. most light) and high shutter speeds (most light as well). You may also want to reduce the sensitivity (ISO) of the camera to clear out image noises, but this requires increasing aperture.

The camera’s USB connection is not working; remove the SD card and use the USB SD card reader to upload the images to the computer. If the files are saved in the JPEG format, open them in MS Paint and save them in turn as 24-bit bitmaps (.bmp) - this is necessary for importation in the Eyespy software. However, ImageJ can use JPEG files.

5.2 Eyespy

Once you have the pictures in .bmp format, the EyeSpy software allows you to analyze the intensity spectrum of each pixel along a given line. To do so, open the EyeSpy software, then go to **File** — **Open**, then open the picture file. The graphical user interface is shown in figure (8).

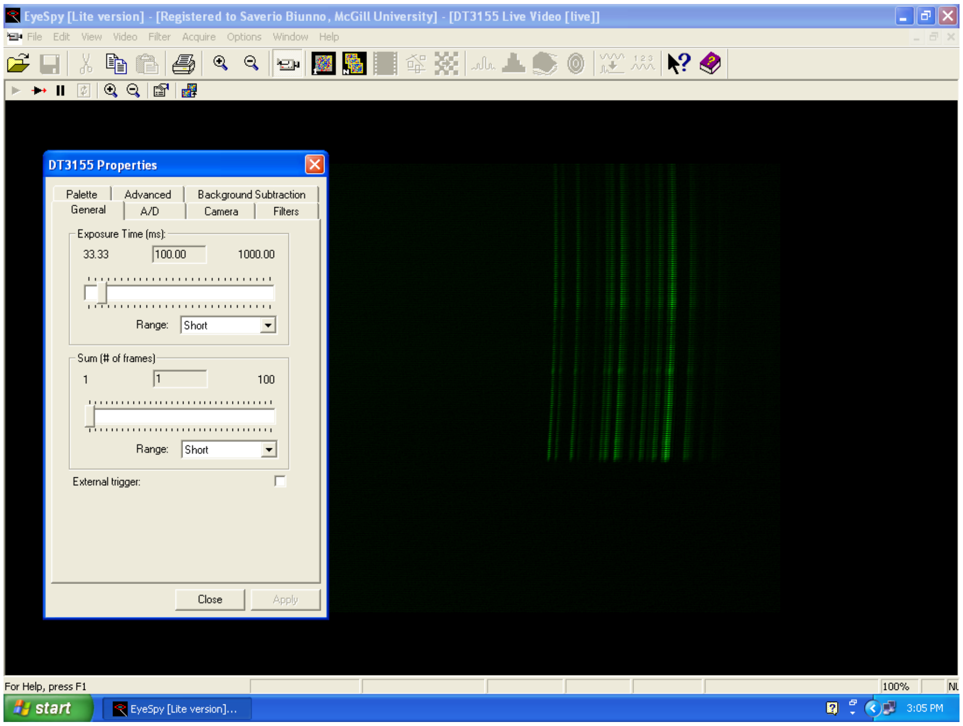

Figure 8: EyeSpy's GUI, along with a spectrum obtained with the neon lamp and the LG interferometer.

Click on the Line Profile icon (the squiggly line with numerous maxima in the top toolbar). A red line will appear, along which the intensity spectrum is taken. You can rotate the line, resize it and move it. If you right click on the line and select "properties", you can change the width of the red line over which the integral is made, say to about 10, to get a smoother profile of the intensity spectrum. A graph showing the intensity as a function of the position along the line should appear, and if your line is chosen properly, you should see the splitting clearly. If the red filter was inadequate in filtering out blue and green light, you can go use the Filter/RGB option and type in 100% for both green and blue to remove them.

While you are in the graph screen, if you go in **File** — **Export**, you will be able to export the graph as a picture, but also you can choose its format to be .txt or .xls to have the relative intensity numerical value of each pixel in a spreadsheet. This allows you to plot the values on say, Matlab, and fit Gaussians to the peaks using your favorite analysis program.

5.3 ImageJ

Alternatively, you may use ImageJ. This software is a free, comprehensive image processing open source Java program. Many of its functions can be used to improve the acquired images. However, its main use here will be to improve the precision in the readings to find to pixel distance between two dots.

Because the light intensity distribution of the dots is Gaussian, i.e. there is no sharp edge, we can use interpolation to have a good estimate of sub-pixel values. In other words, we will use ImageJ to have a better resolution on our images. ImageJ uses bi-linear interpolation, which means that every new pixel value is determined by a weighted average of the nearest 4 known-value pixels. Although this algorithm is far from perfect, it gives a good approximation of the position of the peaks and reduces the uncertainty. Launch the ImageJ applet from `http://rsb.info.nih.gov/ij/`. Import your image by clicking on **File** — **Open** from the ImageJ window (fig 15). Click on **Image** — **Scale...** to select the multiplication factor and make sure that the box **Interpolate** is checked.

Now you have your new image in a new window. By clicking on the box containing a line and a red triangle in the top toolbar, you can draw a line on the image, and then you can get the line profile intensity plot corresponding to the red line with **Analyse** — **Plot Profile**. Finally, you can have the list of intensity value at each pixel by clicking on **List** under the profile plot image.

5.4 Data Analysis

LG Interferometer

For this part, from the line profiles that you have obtained in the software, you should get the position of all of the peaks. By using their position, you should then get the shift in wavelength by using the s/m ratio defined in the discussion of the theory of the apparatus. Verify the linear nature of the Zeeman effect as the field is changed; a linear fit will return a value for $s/(mB)$. By looking at equation (10), a measurement of the Bohr magneton is possible if the index of refraction of the LG plate ρ as well as $d\rho/d\lambda$ are known. In our case, we have a borosilicate crown glass (BK7); Appendix C indicates that the equation for the index of refraction is

$$\rho^2 - 1 = \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} \tag{15}$$

where λ is in μm. All constants B_i and C_i are tabulated in table (1). You may obtain both variables from this distribution, and complete the calculation of the value of the Bohr magneton.

B_1	B_2	B_3	C_1	C_2	C_3
1.03961	$2.31792 \cdot 10^{-1}$	1.01047	$6.00070 \cdot 10^{-3}$	$2.00179 \cdot 10^{-2}$	$1.03561 \cdot 10^{2}$

Table 1: Constants associated to equation (15)

FP Interferometer

Using the line profiles obtained for this part of the experiment, you should obtain the radii of all of the rings in the pictures in pixel units. You will have to find a way to determine the center accurately. Alternatively, as we may deduce from equations (12) and (13), the order of the ring should be proportional to the square of its radius if no field is applied (as $r_\alpha = r_\beta$) - you may add a constant parameter to the radii of the rings and perform a least-squares fit to obtain the value of this offset. You may then calculate the parameters Δ and δ in equations (12) and (13) and then obtain $\Delta\bar{v}$ from equation (11). As per equation (14), $\Delta\bar{v}$ should be proportional to the applied magnetic field - the value of the Bohr magneton may then be extracted by performing a linear fit on the distribution.

6 Goals

In short, you must:

- Calibrate the electromagnet used to provide the magnetic field;
- Obtain the value of the Bohr magneton by using the Lummer-Gehrcke interferometer;
- Obtain the value of the Bohr magneton by using the Fabry-Pérot interferometer;
- (Optional) Be creative! For example, you may choose to observe and study the anomalous Zeeman splitting in one of the other two lines of the Cadmium lamp (green and turquoise).

Good luck!

Minor Changes by Liam Halloran, Aug. 2018
Updated by Gregory Bell, May 2017
Updated by Guillaume Huot, May 2016