Beekeeping Economics Model: Mathematical Specification

Extracted from live_beekeeping_explorer_v2.R

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1 Core Model Equations

1.1 Forager Allocation

The share of bees allocated to foraging activities is determined by colony strength:

$$\text{Forager Share} = \frac{1}{1 + e^{\omega - \theta \cdot \text{frames per colony}}} \tag{1}$$

where:

- ω = base foraging parameter (default: 0.5)
- θ = colony strength coefficient (default: 0.3)
- frames per colony = $\frac{\text{total frames}}{\text{total colonies}}$

Total foragers are then:

$$\text{Total Foragers} = \text{Forager Share} \times \text{Total Frames} \tag{2}$$

1.2 Forage Production

Forage collection follows a simple piecewise linear production function:

Marginal forage production:

$$f'(x) = \begin{cases} A & \text{if } x < B \\ 0 & \text{if } x \geq B \end{cases} \tag{3}$$

Total forage production (integral):

$$F(x) = \begin{cases} A \cdot x & \text{if } x < B \\ A \cdot B & \text{if } x \geq B \end{cases} \tag{4}$$

where:

- $F(x)$ = total forage collected
- $f'(x)$ = marginal forage production
- x = total foragers
- A = marginal forage production capacity (varies by season)
- B = saturation point for foragers (varies by season)
- I = parameter (not currently used, reserved for future extensions)

1.3 Crop Yield (Pollination Services)

Crop yield from pollination services follows an analogous piecewise linear function:

Marginal crop yield:

$$y'(x) = \begin{cases} D & \text{if } x < E \\ 0 & \text{if } x \geq E \end{cases} \tag{5}$$

Total crop yield (integral):

$$Y(x) = \begin{cases} D \cdot x & \text{if } x < E \\ D \cdot E & \text{if } x \geq E \end{cases} \tag{6}$$

where:

- $Y(x)$ = total crop yield
- $y'(x)$ = marginal crop yield
- x = total foragers
- D = marginal crop production capacity (varies by season)
- E = saturation point for foragers (varies by season)
- G = parameter (not currently used, reserved for future extensions)

The marginal product of foragers for pollination revenue is used directly:

$$\frac{\partial Y}{\partial x} = y'(x) = \begin{cases} D & \text{if } x < E \\ 0 & \text{if } x \geq E \end{cases} \tag{7}$$

1.4 Bee Stock Dynamics

Colony and frame dynamics evolve according to:

$$\text{Frames}_{t+1} = \max(0, \text{Frames}_t + \beta \cdot \text{Colonies}_t - \delta \cdot \text{Frames}_t) \tag{8}$$

$$\text{Colonies}_{t+1} = \text{Colonies}_t \cdot (1 - \delta) \tag{9}$$

where:

- $\beta = \alpha \cdot t_{\text{dur}}$ (growth per period)
- α = weekly growth rate (frames per colony per week)
- δ = loss rate per period
- t_{dur} = period duration in weeks (default: 13)

1.5 Feed Balance

Feed consumption and requirements:

$$\text{Forage Consumed} = \text{Total Frames} \times \gamma \times t_{\text{dur}} \tag{10}$$

$$\text{Net Forage} = \text{Forage Collected} - \text{Forage Consumed} \tag{11}$$

$$\text{Feed Required} = \max(0, -\text{Net Forage}) \tag{12}$$

$$\text{Honey Harvested} = \max(0, \text{Net Forage}) \tag{13}$$

where γ = feed consumption rate (lbs per frame per week, default: 1.5)

2 Management Operations

2.1 Management Actions

At the end of each period, three management actions can be applied:

$$\begin{aligned}
\text{Culling:} \quad & \text{Colonies}_{t+1} \leftarrow \text{Colonies}_{t+1} \times (1 - r_{\text{cull}}) && (14)\\
& \text{Frames}_{t+1} \leftarrow \text{Frames}_{t+1} \times (1 - r_{\text{cull}}) && (15)\\
\text{Splitting:} \quad & \text{Colonies}_{t+1} \leftarrow \text{Colonies}_{t+1} \times (1 + r_{\text{split}}) && (16)\\
\text{Merging:} \quad & \text{Colonies}_{t+1} \leftarrow \text{Colonies}_{t+1} \times (1 - r_{\text{merge}}) && (17)
\end{aligned}$$

2.2 Management Constraints

The model enforces mutual exclusion between splitting and merging:

$$r_{\text{split},s} > 0 \Rightarrow r_{\text{merge},s} = 0 \quad \forall s \in \{\text{seasons}\} \tag{18}$$

This constraint is implemented via penalty function in optimization:

$$\text{Penalty} = \sum_s 10^6 \times r_{\text{split},s} \times r_{\text{merge},s} \times \mathbf{1}_{r_{\text{split},s} > 0.1, r_{\text{merge},s} > 0.1} \tag{19}$$

3 Economic Model

3.1 Revenue Streams

$$\begin{aligned}
\text{Revenue}_{\text{honey}} &= \text{Honey Harvested} \times P_h && (20)\\
\text{Revenue}_{\text{crop}} &= \frac{\partial Y}{\partial x} \times P_c \times \text{Total Foragers} && (21)
\end{aligned}$$

where:

- P_h = honey price per unit (seasonal, default: \$3.0)
- P_c = crop price per unit (seasonal, default: \$0.5)

3.2 Cost Structure

$$\begin{aligned}
\text{Cost}_{\text{maintenance}} &= \text{Colonies} \times \frac{\text{Cost}}{4} && (22)\\
\text{Cost}_{\text{feed}} &= \text{Feed Required} \times P_{\text{feed}} && (23)\\
\text{Cost}_{\text{management}} &= \sum_{a \in \{\text{cull, split, merge}\}} r_a \times \text{Units}_a \times P_a && (24)
\end{aligned}$$

where:

- Cost = annual maintenance cost per colony (default: \$50)
- P_{feed} = feed price per unit (default: \$0.02)
- $P_{\text{cull}}, P_{\text{split}}, P_{\text{merge}}$ = management operation costs

3.3 Profit Function

Period profit:

$$\pi_t = \text{Revenue}_{\text{honey},t} + \text{Revenue}_{\text{crop},t} - \text{Cost}_{\text{maintenance},t} - \text{Cost}_{\text{feed},t} - \text{Cost}_{\text{management},t} \tag{25}$$

Present value of total profits:

$$\Pi = \sum_{t=1}^{T} \frac{\pi_t}{(1+r_q)^{t-1}} \tag{26}$$

where $r_q = (1+r_a)^{1/4} - 1$ is the quarterly discount rate derived from annual rate r_a.

4 Parameter Values

4.1 Parameter Count Summary

The beekeeping economics model contains a total of **59 parameters** across several categories:

4.1.1 Biological Parameters (25 total)

Core Biological Parameters (9):

- α (growth rate)
- δ (loss rate)
- ω (base foraging parameter)
- θ (colony strength coefficient)
- γ (feed consumption rate)
- B (forage saturation parameter)
- E (crop saturation parameter)
- I (forage collection threshold)
- G (crop yield threshold)

Season-Specific Biological Parameters (16):

- Growth factors (4 seasons)
- Loss factors (4 seasons)
- Forage capacities A (4 seasons)
- Crop capacities D (4 seasons)
- Cost (annual maintenance cost)
- P_{feed} (feed price)
- P_{split} (splitting cost)
- P_{merge} (merging cost)
- P_{cull} (culling cost)
- r_a (annual discount rate)
- Colonies_0 (initial colonies)
- Frames_0 (initial frames)
- Years (simulation length)

4.1.2 Fixed Parameters (5 total)

- t_{dur} (period duration) - fixed at 13 weeks
- Plus the seasonal capacities A and D for each season (derived from seasonal table)

4.1.3 Additional Economic Seasonal Parameters (8 total)

From the seasonal variations table, seasonal multipliers for:

- Honey prices P_h (4 seasons)
- Crop prices P_c (4 seasons)

4.1.4 Management Parameters (12 total)

Decision variables for 12 management rates (3 actions × 4 seasons): culling, splitting, merging rates.

4.1.5 Total Parameter Count

- **Biological parameters:** 25 parameters (9 core + 16 season-specific)
- **Economic parameters:** 6 parameters
- **Initial conditions:** 2 parameters
- **Simulation parameters:** 1 parameter
- **Fixed parameters:** 5 parameters
- **Economic seasonal variations:** 8 parameters
- **Management decisions:** 12 parameters
- **Grand total:** 59 parameters

4.2 Default Parameter Values

Parameter	Description	Default	Min	Max
Biological Parameters				
α	Growth rate (frames/colony/week)	0.1	0	1.0
δ	Loss rate per season	0.05	0.01	0.2
ω	Base foraging parameter	0.5	0.1	1.0
θ	Colony strength coefficient	0.3	0.1	0.5
γ	Feed consumption (lbs/frame/week)	0.5	0	5.0
Economic Parameters				
Cost	Annual maintenance cost per colony ($)	50	0	400
P_{feed}	Feed price per unit ($)	0.02	0	10
P_{split}	Splitting cost per operation ($)	25	0	100
P_{merge}	Merging cost per operation ($)	10	0	100
P_{cull}	Culling cost per operation ($)	10	0	100
r_a	Annual discount rate (%)	3	0	15
Initial Conditions				
Colonies_0	Initial number of colonies	1,000	100	5,000
Frames_0	Initial number of frames	6,000	500	30,000
t_{dur}	Period duration (weeks)	13	–	–

Simulation Parameters				
Years	Number of years to simulate	5	1	30

4.3 Seasonal Parameter Variations

Parameter	Spring	Summer	Fall	Winter	Range
Growth/Loss Factors					
Growth Factor	1.0	1.0	0.3	0.3	[0, 2.0]
Loss Factor	1.0	1.0	1.2	2.0	[0, 5.0]
Production Capacities					
A (Forage)	100	300	0	0	[0, 500]
D (Crop)	15	15	0	0	[0, 2000]
Market Prices					
P_h (Honey, \$)	3.0	3.0	3.0	3.0	[1.0, 10.0]
P_c (Crop, \$)	0.5	0.5	0.5	0.5	[0.1, 2.0]
Management Rates (%)					
Culling	0	0	0	5	[0, 100]
Splitting	30	20	0	0	[0, 100]
Merging	0	0	5	10	[0, 100]

5 Optimization Problem

The beekeeping management optimization problem is:

$$\max_{r_{a,s}} \quad \Pi = \sum_{t=1}^{T} \frac{\pi_t(r_{a,s})}{(1+r_q)^{t-1}} \tag{27}$$

$$\text{subject to:} \quad 0 \leq r_{a,s} \leq 1 \quad \forall a \in \{\text{cull, split, merge}\}, s \in \{\text{seasons}\} \tag{28}$$

$$r_{\text{split},s} \cdot r_{\text{merge},s} = 0 \quad \forall s \in \{\text{seasons}\} \tag{29}$$

where $r_{a,s}$ represents the rate of management action a in season s.

6 Model Function Visualizations

This section presents graphical representations of the three core production functions under different parameter scenarios to illustrate the model's behavioral characteristics.

6.1 Forager Share Function

Figure 1 shows how the forager allocation responds to colony strength (measured by frames per colony) under minimum, default, and maximum parameter values. The logistic shape ensures smooth transitions while preventing unrealistic allocations.

6.2 Forage Collection Functions by Season

Figure 2 illustrates forage collection across the four seasons, showing how seasonal capacity (A) and diminishing returns (B) interact to determine total forage collected as a function of forager effort.

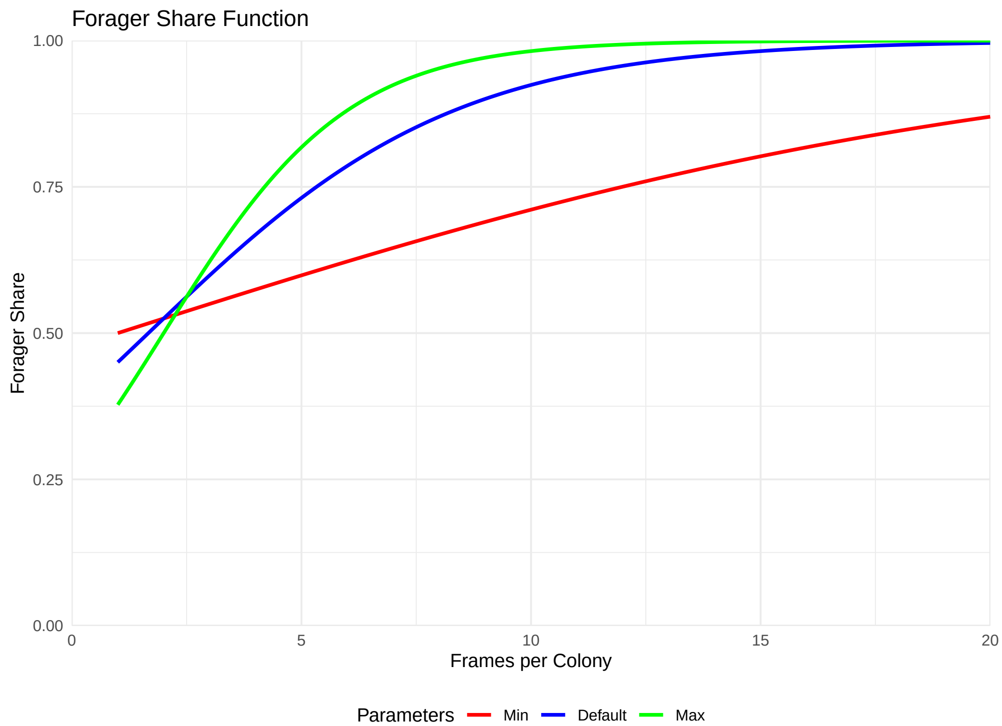

Figure 1: Forager share as a function of frames per colony under different parameter scenarios. The Min scenario uses $\omega = 0.1, \theta = 0.1$; Default uses $\omega = 0.5, \theta = 0.3$; Max uses $\omega = 1.0, \theta = 0.5$.

6.3 Crop Yield Functions by Season

Figure 3 shows pollination-derived crop yields across seasons, demonstrating the economic returns from deploying foragers to agricultural contracts rather than natural forage collection.

6.4 Function Characteristics

Key observations from the function plots:

1. **Forager Share:** Exhibits classic logistic behavior with steeper transitions under higher θ values, ensuring realistic forager allocation constraints.
2. **Seasonal Variation:** Both forage and crop functions show strong seasonal patterns, with Spring and Summer being productive periods while Fall and Winter show minimal activity.
3. **Diminishing Returns:** All production functions display diminishing marginal returns, preventing unrealistic linear scaling of outputs with inputs.
4. **Parameter Sensitivity:** The functions show substantial sensitivity to capacity parameters (A, D) but more moderate sensitivity to curvature parameters (B, E).
5. **Threshold Effects:** The logistic formulations create natural threshold effects where small increases in forager allocation can yield large increases in output near inflection points.

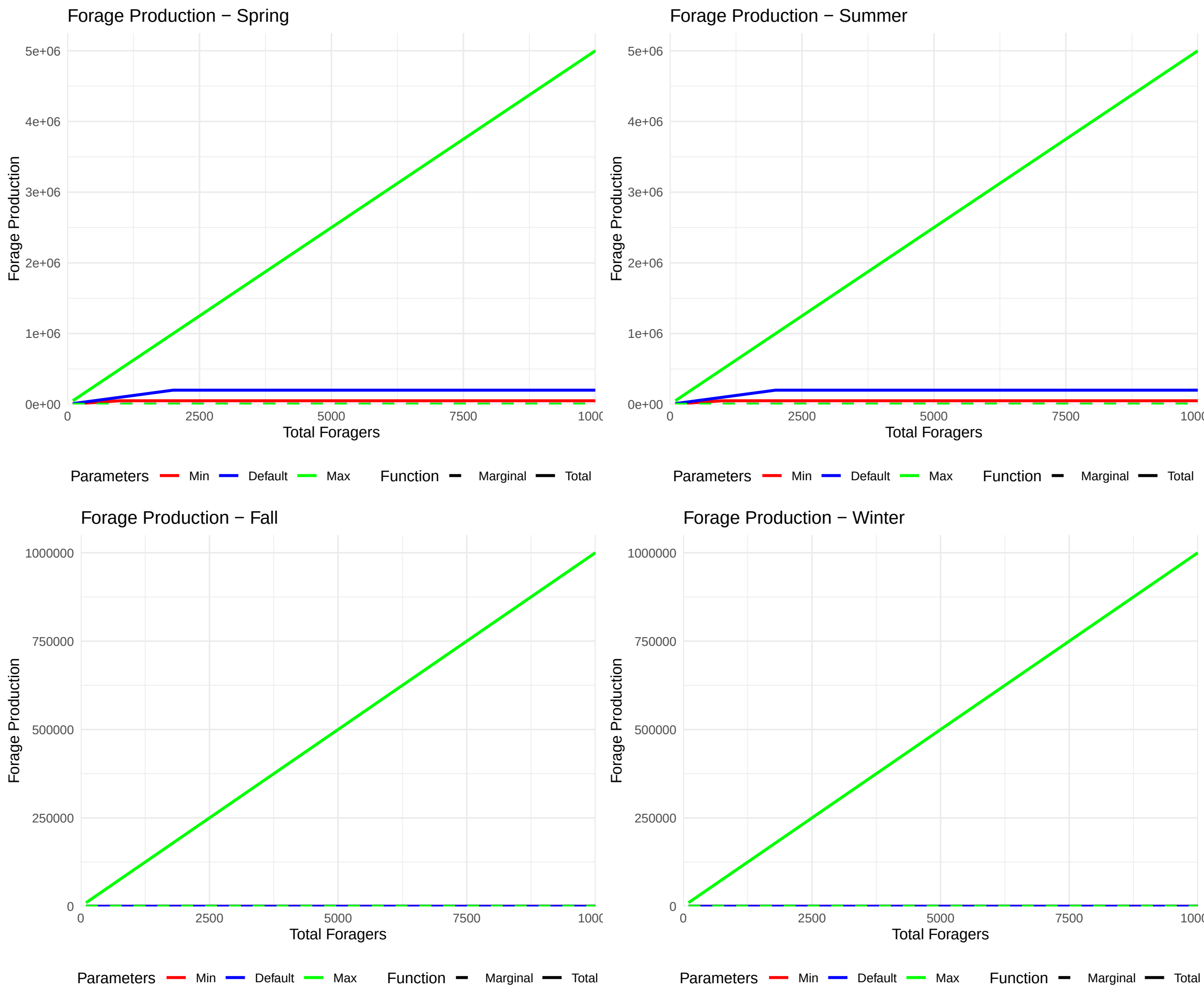

Figure 2: Forage collection functions by season under different parameter scenarios. Spring and Summer show active forage periods, while Fall and Winter have minimal capacity. Min scenarios use $A = 0$ (inactive seasons) or $A = 100/300$ (active seasons) with $B = 0.0001$; Default uses base values; Max uses $A = 500$ with $B = 0.01$.

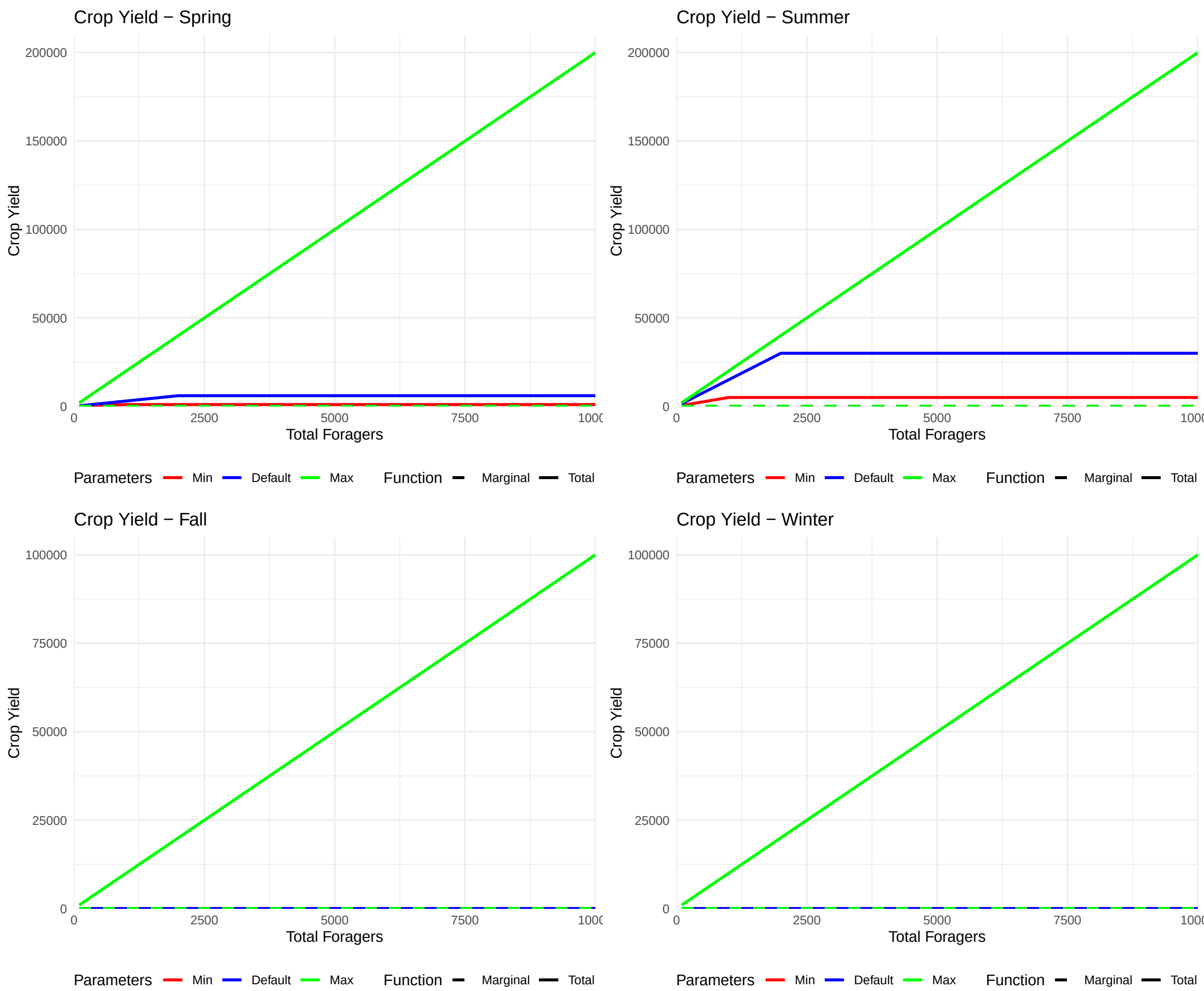

Figure 3: Crop yield functions by season under different parameter scenarios. The functions show similar seasonal patterns to forage collection but with different scaling parameters (D and E) reflecting economic rather than biological constraints.