Geodesic Optimization for Predictive Shift Adaptation on EEG data

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Code available at:

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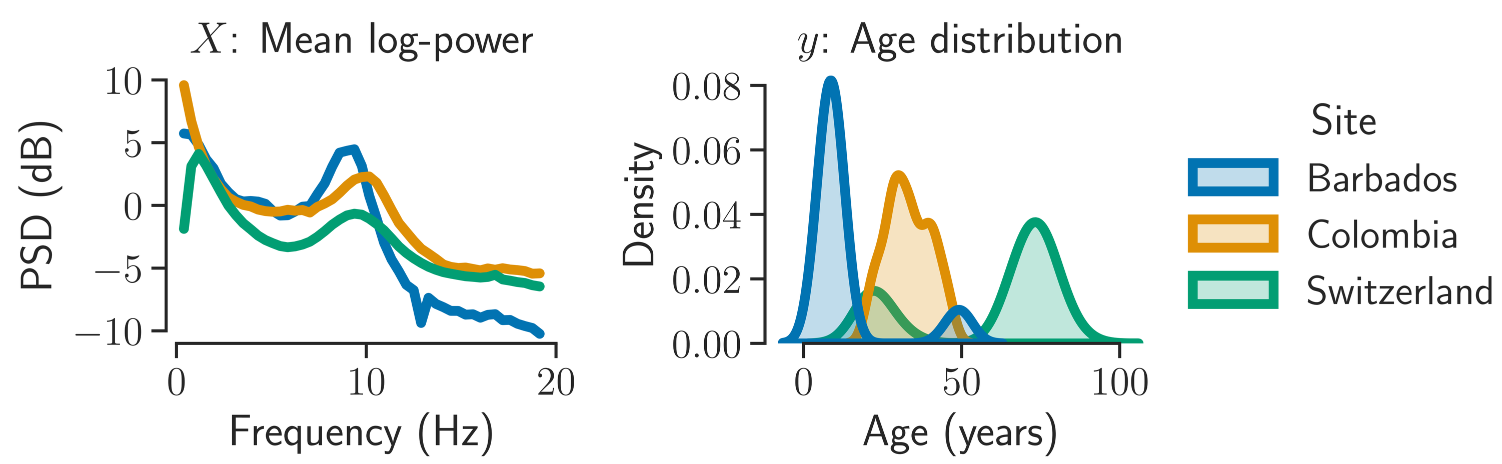
Multi-source domain adaptation

- **Distribution shift** between multiple source and target domains
- Domains in **neuroscience data**: subjects, recording sessions, experimental conditions, hospitals, acquisition protocols, ...



- In neuroscience: extensive use of **covariance matrices** that belong to the **Riemannian manifold**:

$$\mathbb{S}_d^{++} = \{\mathbf{\Sigma} \in \mathbb{R}^{d \times d} \mid \mathbf{\Sigma}^\top = \mathbf{\Sigma}, \text{ and } \mathbf{x}^\top \mathbf{\Sigma} \mathbf{x} > 0, \text{ for all } \mathbf{x} \neq \mathbf{0}\}$$

Question: How can we handle joint shifts in both Riemannian features $\mathbf{\Sigma}$ and the label y ?

Machine learning on $\mathbb{S}_d^{++}$

Classical geometric tools:

- **Riemannian mean** of $\{\mathbf{\Sigma}_i\}_{i=1}^N$:

$$\overline{\mathbf{\Sigma}} \triangleq \underset{\mathbf{\Sigma} \in \mathbb{S}_d^{++}}{\arg\min} \sum_{i=1}^{N} \left\| \log \left(\mathbf{\Sigma}^{-1/2} \mathbf{\Sigma}_i \mathbf{\Sigma}^{-1/2} \right) \right\|_F^2$$

- **Logarithmic mapping** projects $\mathbf{\Sigma}'$ to the tangent (Euclidean) space at $\mathbf{\Sigma}$:

$$\log_{\mathbf{\Sigma}}(\mathbf{\Sigma}') \triangleq \mathbf{\Sigma}^{1/2} \log \left(\mathbf{\Sigma}^{-1/2} \mathbf{\Sigma}' \mathbf{\Sigma}^{-1/2} \right) \mathbf{\Sigma}^{1/2} \in \mathbb{S}_d$$

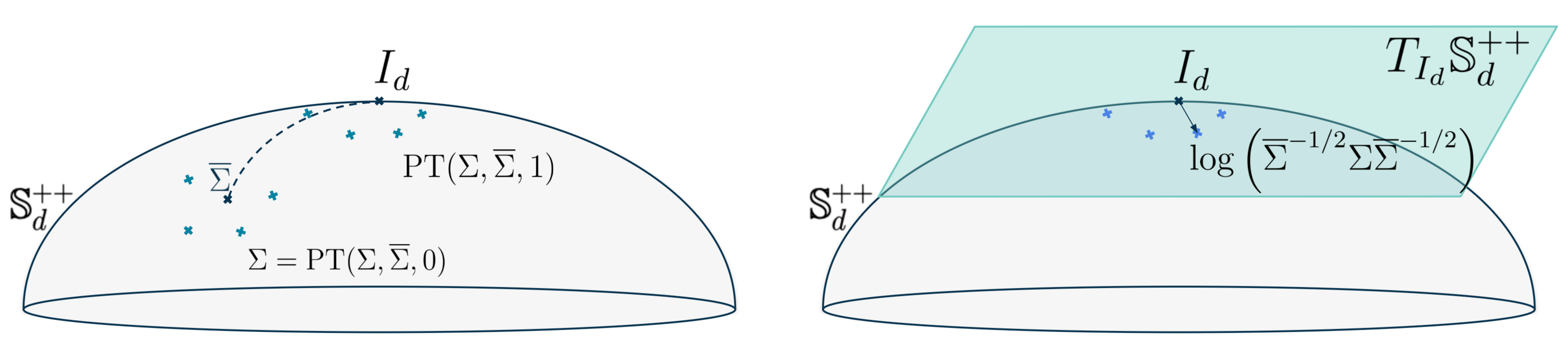
- **Parallel transport** moves $\mathbf{\Sigma}'$ along the geodesic from $\mathbf{\Sigma}$ to $\mathbf{I}_d$:

$$\mathrm{PT}\left(\mathbf{\Sigma}', \mathbf{\Sigma}, \alpha\right) \triangleq \mathbf{\Sigma}^{-\alpha/2} \mathbf{\Sigma}' \mathbf{\Sigma}^{-\alpha/2}$$

Learning from the dataset $\{(\mathbf{\Sigma}_i, y_i)\}_{i=1}^N$ by transforming it into Euclidean vectors:

$$\phi\left(\mathbf{\Sigma}_i, \overline{\mathbf{\Sigma}}\right) \triangleq \mathrm{uvec}\left(\log_{\mathbf{I}_d}\left(\mathrm{PT}\left(\mathbf{\Sigma}_i, \overline{\mathbf{\Sigma}}, 1\right)\right)\right) = \mathrm{uvec}\left(\log\left(\overline{\mathbf{\Sigma}}^{-1/2} \mathbf{\Sigma}_i \overline{\mathbf{\Sigma}}^{-1/2}\right)\right) \in \mathbb{R}^{d(d+1)/2}$$

where uvec vectorizes the upper triangular part [1]



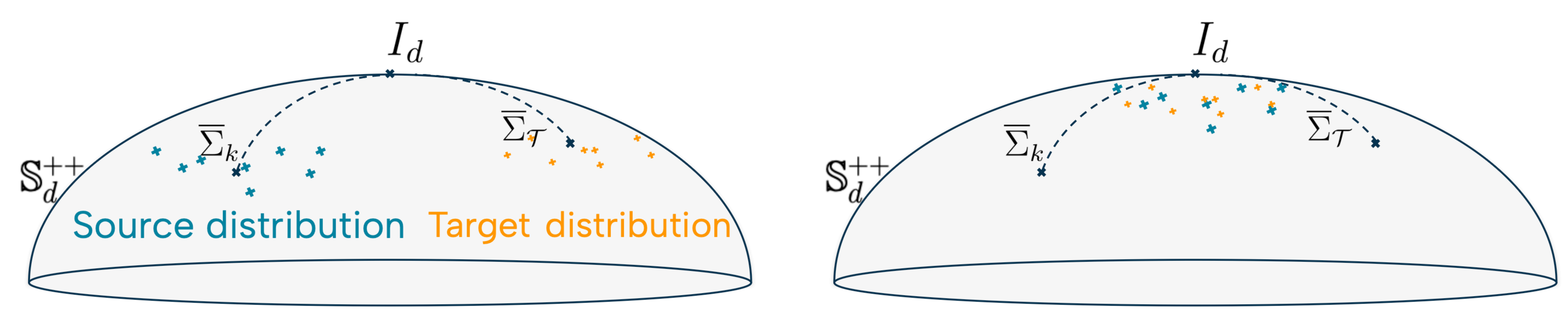
Multi-source domain adaptation on $\mathbb{S}_d^{++}$

K source domains: $\{(\mathbf{\Sigma}_{k,i}, y_{k,i})\}_{i=1}^{N_k}$ for all $k \in [\![1, K]\!]$ and $N_{\mathcal{S}} = \sum_{k=1}^K N_k$
One target domain: $\{(\mathbf{\Sigma}_{\mathcal{T},i}, y_{\mathcal{T},i})\}_{i=1}^{N_\mathcal{T}}$

Assumption: $\overline{\mathbf{\Sigma}}_\mathcal{T}$ is known.

- **Recenter** per domain $\mathcal{D} \in \{1, \dots, K, \mathcal{T}\}$ [2, 3]:

$$\mathbf{Z}_\mathcal{D} \triangleq \left[\phi\left(\mathbf{\Sigma}_{\mathcal{D},1}, \overline{\mathbf{\Sigma}}_\mathcal{D}\right), \dots, \phi\left(\mathbf{\Sigma}_{\mathcal{D},N_\mathcal{D}}, \overline{\mathbf{\Sigma}}_\mathcal{D}\right)\right]^\top \in \mathbb{R}^{N_\mathcal{D} \times d(d+1)/2}$$



Problem: If $\mathbb{P}(y)$ varies across domains, then "Recenter" performs poorly.

Geodesic Optimization for Predictive Shift Adaptation: `GOPSA`

Assumption: $\overline{\mathbf{\Sigma}}_\mathcal{T}$ and $\bar{y}_\mathcal{T}$ are known.

- **Learn the parallel transport** $\alpha \in [0, 1]$ **per domain:**

$$\phi\left(\mathbf{\Sigma}, \overline{\mathbf{\Sigma}}, \alpha\right) \triangleq \mathrm{uvec}\left(\log_{\mathbf{I}_d}\left(\mathrm{PT}\left(\mathbf{\Sigma}, \overline{\mathbf{\Sigma}}, \alpha\right)\right)\right) = \mathrm{uvec}\left(\log\left(\overline{\mathbf{\Sigma}}^{-\alpha/2} \mathbf{\Sigma} \overline{\mathbf{\Sigma}}^{-\alpha/2}\right)\right)$$

- Learnable **source data matrix** w.r.t. $\gamma \in \mathbb{R}^K$:

$$\mathbf{Z}_\mathcal{S}(\gamma) \triangleq \left[\phi\left(\mathbf{\Sigma}_{1,1}, \overline{\mathbf{\Sigma}}_1, \sigma(\gamma_1)\right), \dots, \phi\left(\mathbf{\Sigma}_{K,N_K}, \overline{\mathbf{\Sigma}}_K, \sigma(\gamma_K)\right)\right]^\top$$

where $\sigma(\gamma) \triangleq (1 + \exp(-\gamma))^{-1}$ is the sigmoid function.

Train-time:

$$\gamma_\mathcal{S}^\star \triangleq \underset{\gamma \in \mathbb{R}^K}{\arg\min} \|\mathbf{y}_\mathcal{S} - \mathbf{Z}_\mathcal{S}(\gamma) \boldsymbol{\beta}_\mathcal{S}^\star(\gamma)\|_2^2$$

subject to $\quad \boldsymbol{\beta}_\mathcal{S}^\star(\gamma) \triangleq \mathbf{Z}_\mathcal{S}(\gamma)^\top (\lambda \mathbf{I}_N + \mathbf{Z}_\mathcal{S}(\gamma)\mathbf{Z}_\mathcal{S}(\gamma)^\top)^{-1} \mathbf{y}_\mathcal{S}$

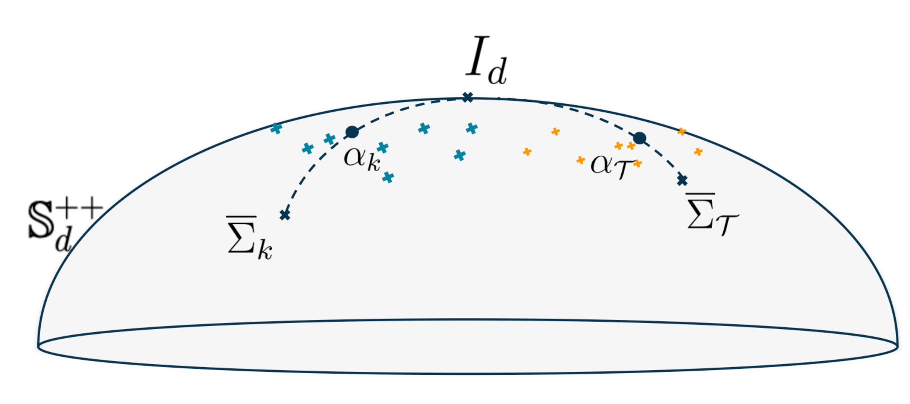
- Learnable **target data matrix** w.r.t. $\gamma \in \mathbb{R}$:

$$\mathbf{Z}_\mathcal{T}(\gamma) \triangleq \left[\phi\left(\mathbf{\Sigma}_{\mathcal{T},1}, \overline{\mathbf{\Sigma}}_\mathcal{T}, \sigma(\gamma)\right), \dots, \phi\left(\mathbf{\Sigma}_{\mathcal{T},N_\mathcal{T}}, \overline{\mathbf{\Sigma}}_\mathcal{T}, \sigma(\gamma)\right)\right]^\top$$

Test-time:

(Optimization) $\quad \gamma_\mathcal{T}^\star \triangleq \underset{\gamma \in \mathbb{R}}{\arg\min} \left(\bar{y}_\mathcal{T} - \frac{1}{N_\mathcal{T}} \mathbf{1}_{N_\mathcal{T}}^\top \mathbf{Z}_\mathcal{T}(\gamma) \boldsymbol{\beta}_\mathcal{S}^\star(\gamma_\mathcal{S}^\star)\right)^2$

(Prediction) $\quad \hat{\mathbf{y}}_\mathcal{T} \triangleq \mathbf{Z}_\mathcal{T}(\gamma_\mathcal{T}^\star) \boldsymbol{\beta}_\mathcal{S}^\star(\gamma_\mathcal{S}^\star)$



Experimental results

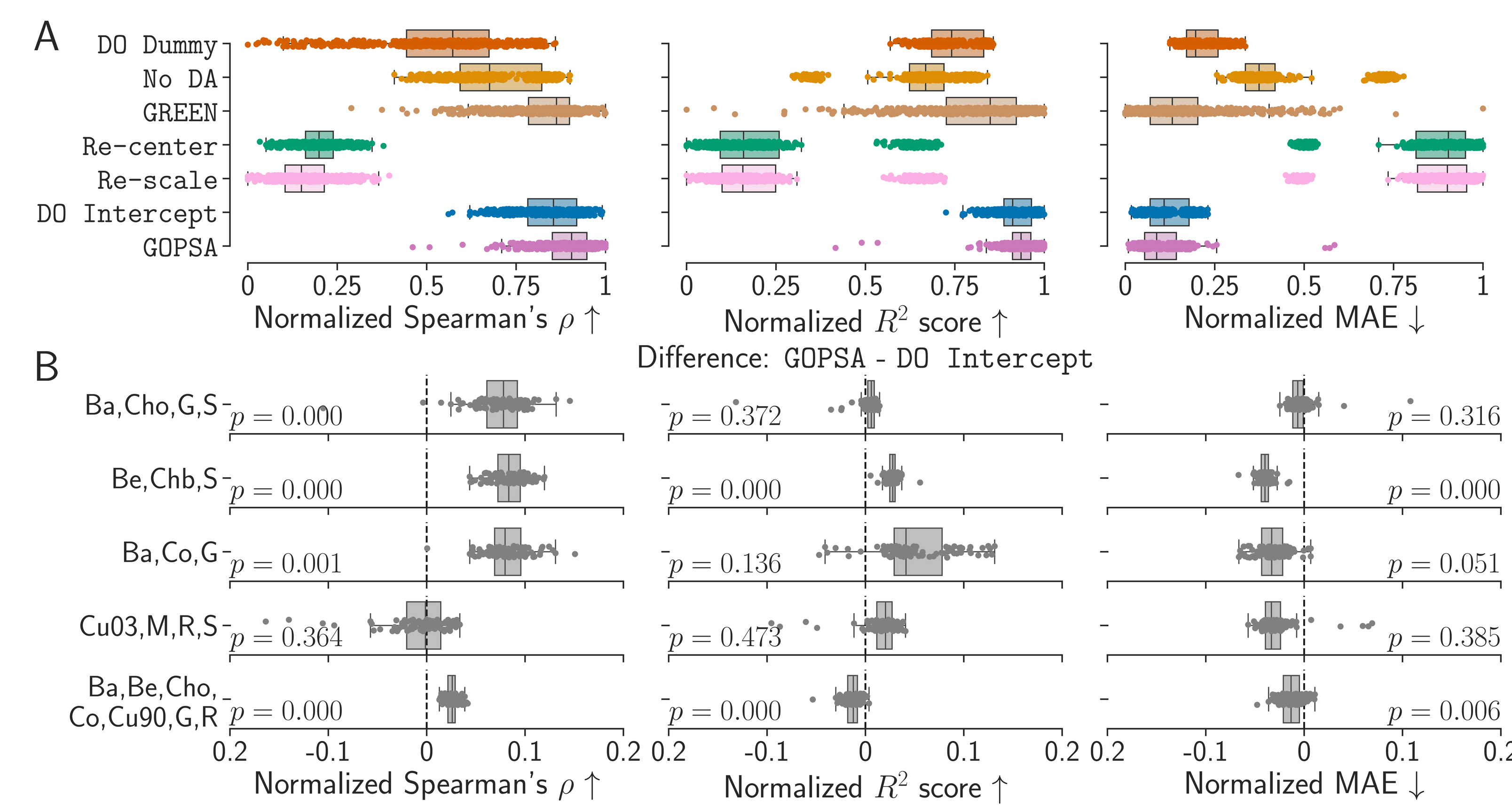
Benchmark on the HarMNqEEG dataset [4]:

- Brain age prediction task evaluated with 3 metrics: Spearman's ρ, R^2, and MAE
- Cross power spectrum densities from 1500 participants with 49 frequency bins and 19 EEG sensors
- 5 source-site combinations from 11 sites: Barbados (Ba), Bern (Be), CHBMP Cuba (Chb), Columbia (Co), Chongqing (Cho), Cuba2003 (Cu03), Cuba90 (Cu90), Germany (G), Malaysia (M), Russia (R), and Switzerland (S)

Baselines:

- `DO Dummy`: mean y per domain
- `No DA`: Ridge regression without domain adaptation
- `GREEN` [5]: SPDNet deep learning model
- `Re-center/Re-scale` [2, 3]: Ridge regression with a re-centering per domain
- `DO Intercept`: Ridge regression with domain-specific bias

Results:



- Top performers: `GOPSA` and `DO Intercept`, best average performance with low variance
- Paired t-test results: `GOPSA` outperformed `DO Intercept` in 4 site combinations for Spearman's ρ, and 3 for MAE.

References

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Neurips 2024, Vancouver, Canada

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