

# Transfer learning and Curriculum learning

Distance and interactions between tasks

Defining a **geometry** of the space of learning tasks

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- Up to now:
  - we were looking at **changes of distribution** between learning and testing
  - Or to **changes of domains** in transfer learning
- Now, we turn to the **bias** introduced by controlling the **order** of the **examples** or of the **tasks**
  - This supposes that there are **interferences** between examples or tasks

Note: in **I.I.D.** and the **ERM** principle, all examples have the **same weight** and there is **no place for interference or order**

First, are all examples created **equal?**

# Outline

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1. How to measure the difficulty of a training example
2. Catastrophic forgetting
3. Curriculum building
4. A geometry on the space of tasks
5. Conclusions

How to measure the difficulty  
of examples?

# Measuring the **difficulty** of examples

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- Previously
  - A **statistical** view
    - The probability of predicting the ground truth label **for an example omitted** from the training set
  - A **learning** view
    - The **earliest training iteration** after which the model (e.g. NN) predicts the ground truth class for that example **in all subsequent iterations**

Baldock, R., Maennel, H., & Neyshabur, B. (2021). Deep learning through the lens of example difficulty. *Advances in Neural Information Processing Systems*, 34.

# Measuring the **difficulty** of examples

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- Proposition
  - The notion of “prediction depth”
  - And three distinct **difficulty types**:
    - Does this example **look mislabeled?**
    - Is classifying this example only easy if the label is given (**ambiguous example**)?
    - Is this **example ambiguous** both with and without its label?

Baldock, R., Maennel, H., & Neyshabur, B. (2021). Deep learning through the lens of example difficulty. *Advances in Neural Information Processing Systems*, 34.



Prediction depth

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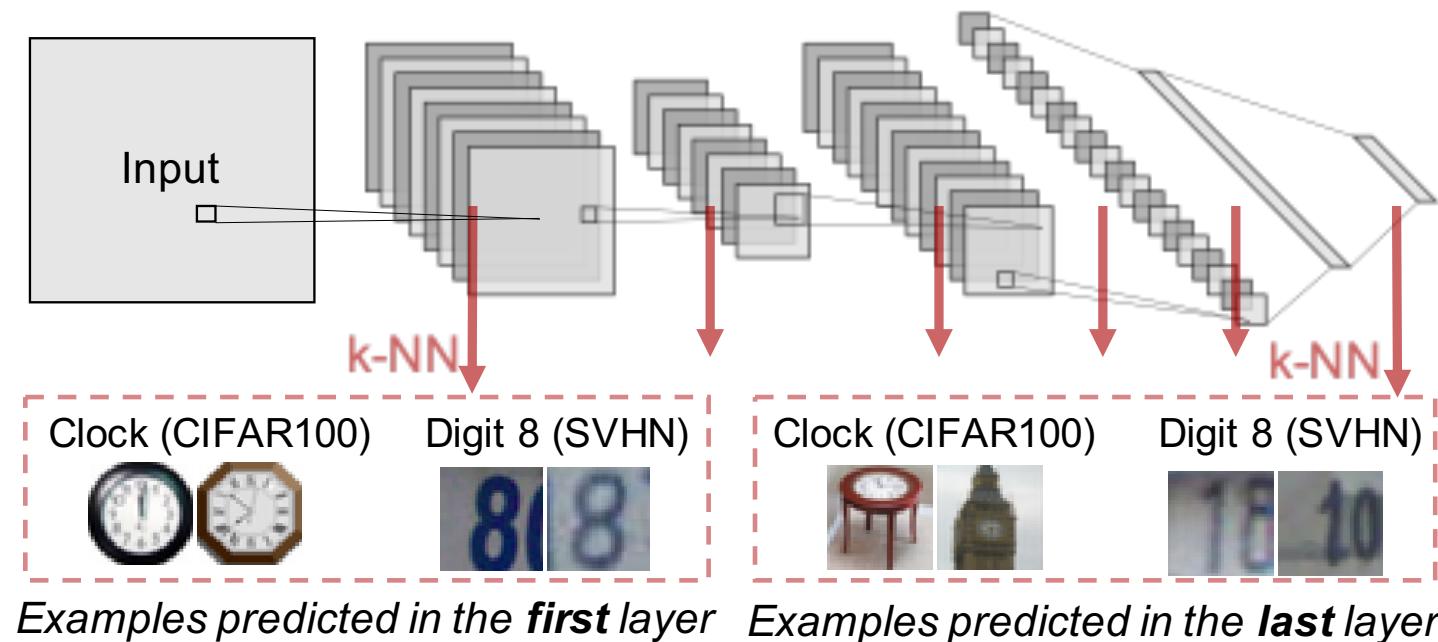
## Prediction depth

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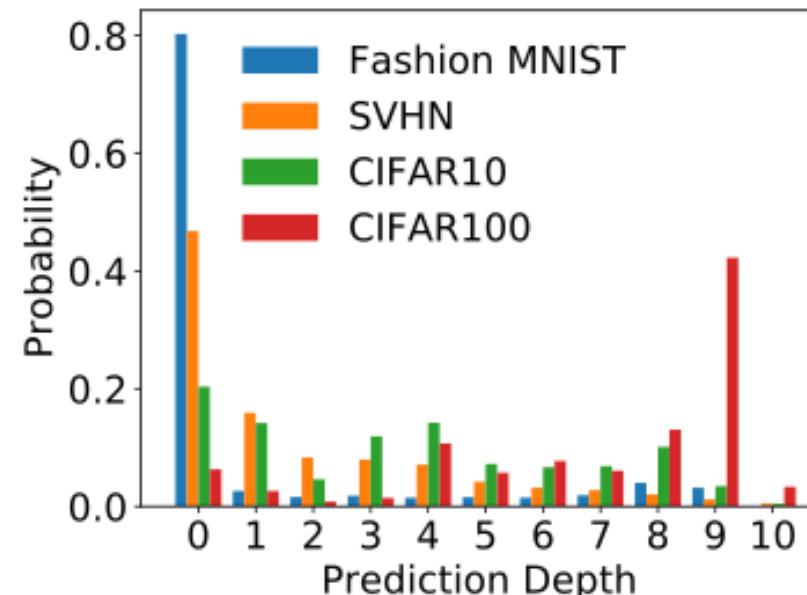
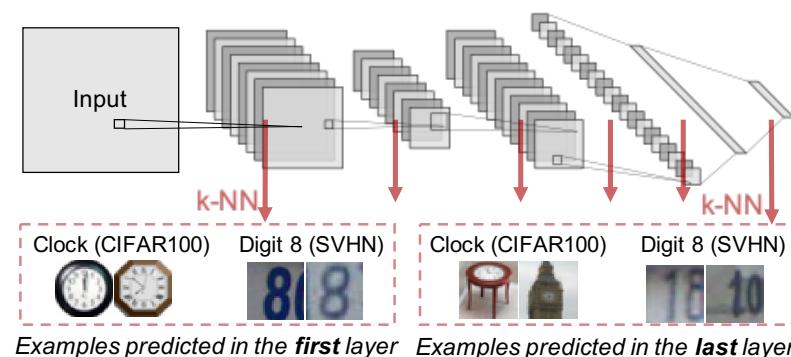
## Prediction depth

- The **number of hidden layers** after which the network's final prediction **is already determined**



# Prediction depth

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## How to measure the prediction depth?

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- k-NN classifier **probes** (with  $k = 30$ )
  - Compare the **hidden embedding** of an **input** to **those of the training set**

(what is the class of the  $k$  nearest neighbors in the embedding considered)

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- k-NN classifier **probes** (with  $k = 30$ )
  - Compare the **hidden embedding** of an **input** to **those of the training set**  
(what is the class of the  $k$  nearest neighbors in the embedding considered)
- A prediction is defined to be made at a **depth  $L = l$**  if
  - The k-NN classification **after** layer  $L = l - 1$  is **different** from the network's final classification,
  - but the classification of k-NN probes **after** **every** layer  $L \geq l$  are all **equal** to the final classification of the network

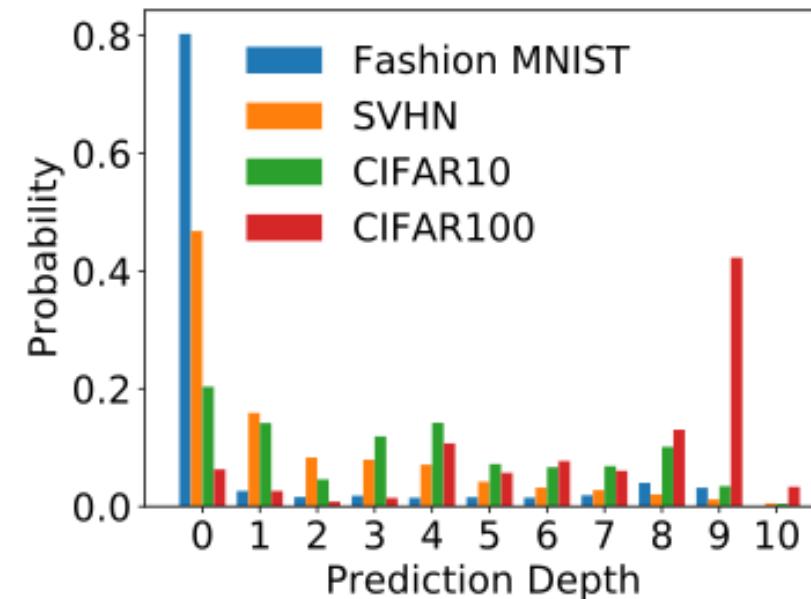
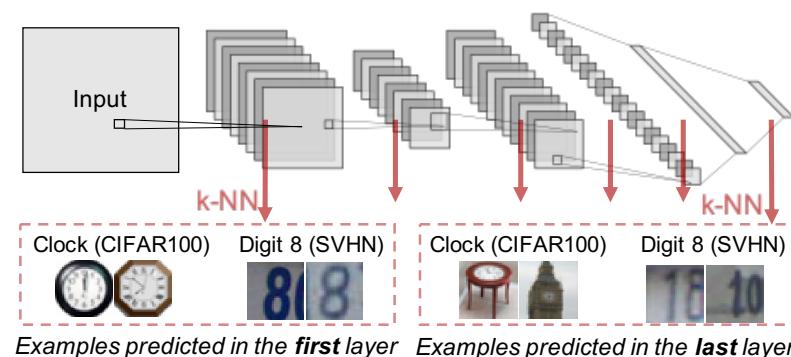
## What they claim to show

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1. The **prediction depth is larger** for examples that visually appear to be **more difficult**
  - And this is consistent between NN's architectures and random seeds
2. Predictions are on average **more accurate** for validation examples with **small prediction depths**
3. Final predictions for examples that **converge earlier** during training are typically determined in **earlier layers** (i.e. small prediction depth)
4. Both the **adversarial input margin** and **output margin** are **larger** for examples with **smaller prediction depths**

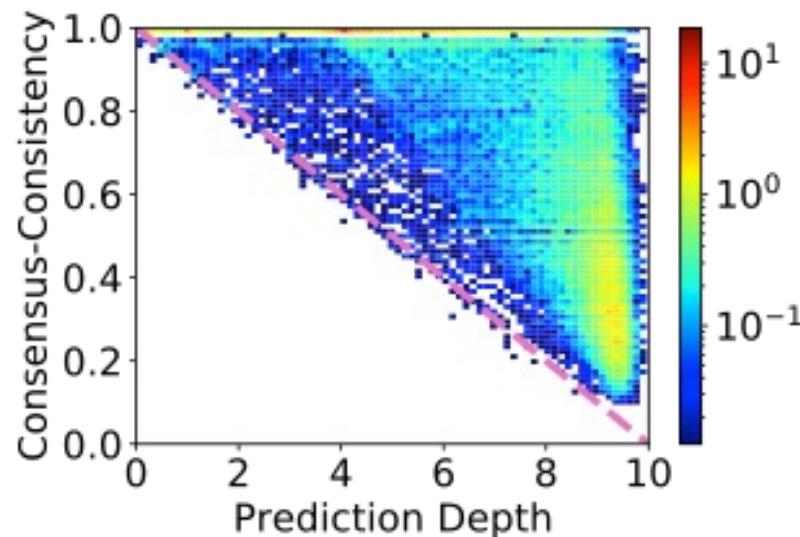
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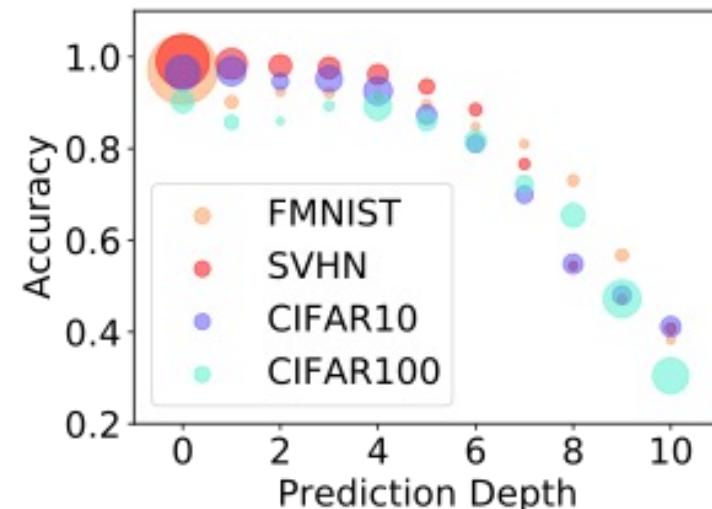
## What they claim to show

2. Predictions are on average **more accurate** for validation examples with **small prediction depths**



250 ResNet18 were trained on CIFAR100 (90:10% random train:validation splits). Comparison of the **average prediction depth** of an example to the **consensus-consistency** of the corresponding prediction.

**Consensus-consistency:** the fraction of NNs that predict the ensemble's consensus class. *The larger the fraction, the higher the consensus.*

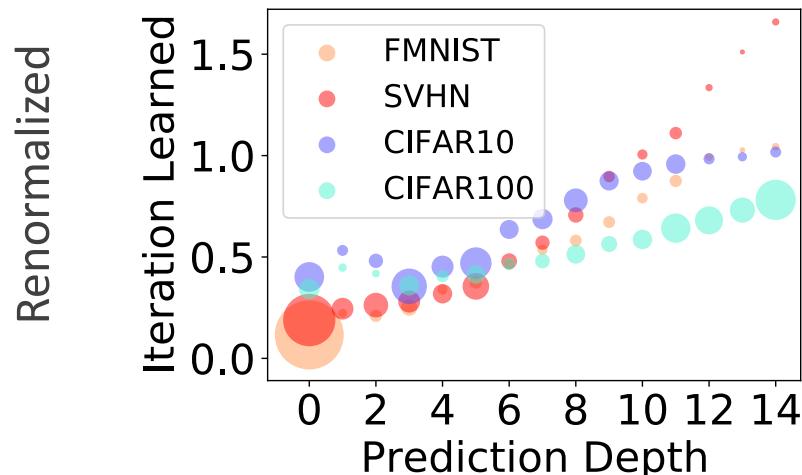


For each dataset, 250 ResNet18 were trained on CIFAR100 (90:10% random train:validation splits). For each point in the validation split, its **prediction depth** and whether the **prediction was correct** was recorded.

# What they claim to show

3. Final predictions for data points that **converge earlier** during training are typically determined in **earlier layers**

- Measure the **difficulty of learning an example** by the **speed at which the model's prediction converges** for that input during training
- **Iteration learned.** A data point is said to be learned by a classifier at training iteration  $t = \tau$  if the predicted class at iteration  $t = \tau - 1$  is different from the final prediction of the converged NN and the predictions at all iterations  $t \geq \tau$  are equal to the final prediction of the converged NN.



For each input in the validation split, the **prediction depth** and the **iteration learned** are recorded

Positive correlation between the **prediction depth** and the **iteration learned** appears for all datasets

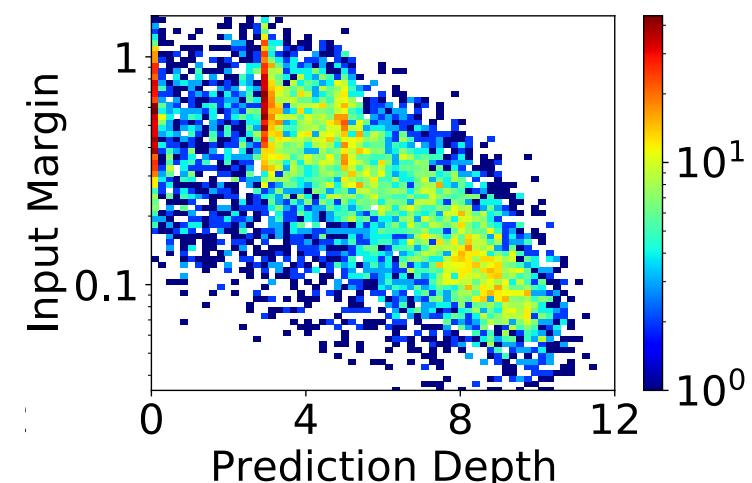
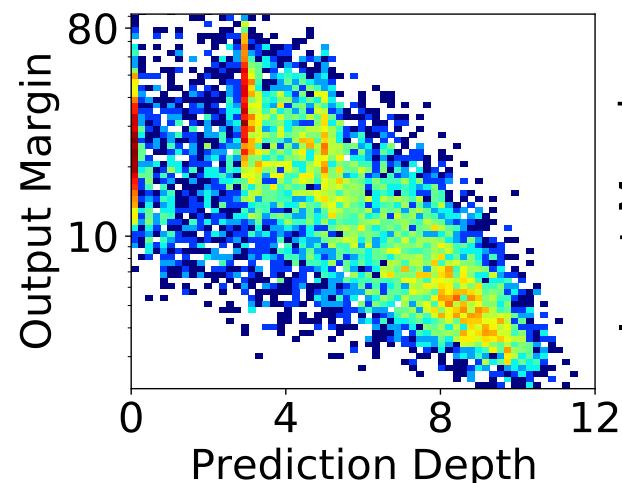
4. Both the adversarial input margin and output margin are **larger** for examples with **smaller prediction depths**

— **Output margin:** difference between the largest and second-largest **output** of the NN (logits).

*The smaller the output margin, the more **uncertain** is the learner.*

— **Input margin:** the smallest **norm** required for an adversarial perturbation **in the input** to change the NN's class prediction.

*The smaller the input margin, the **easier** it is to change the prediction of the learner.*



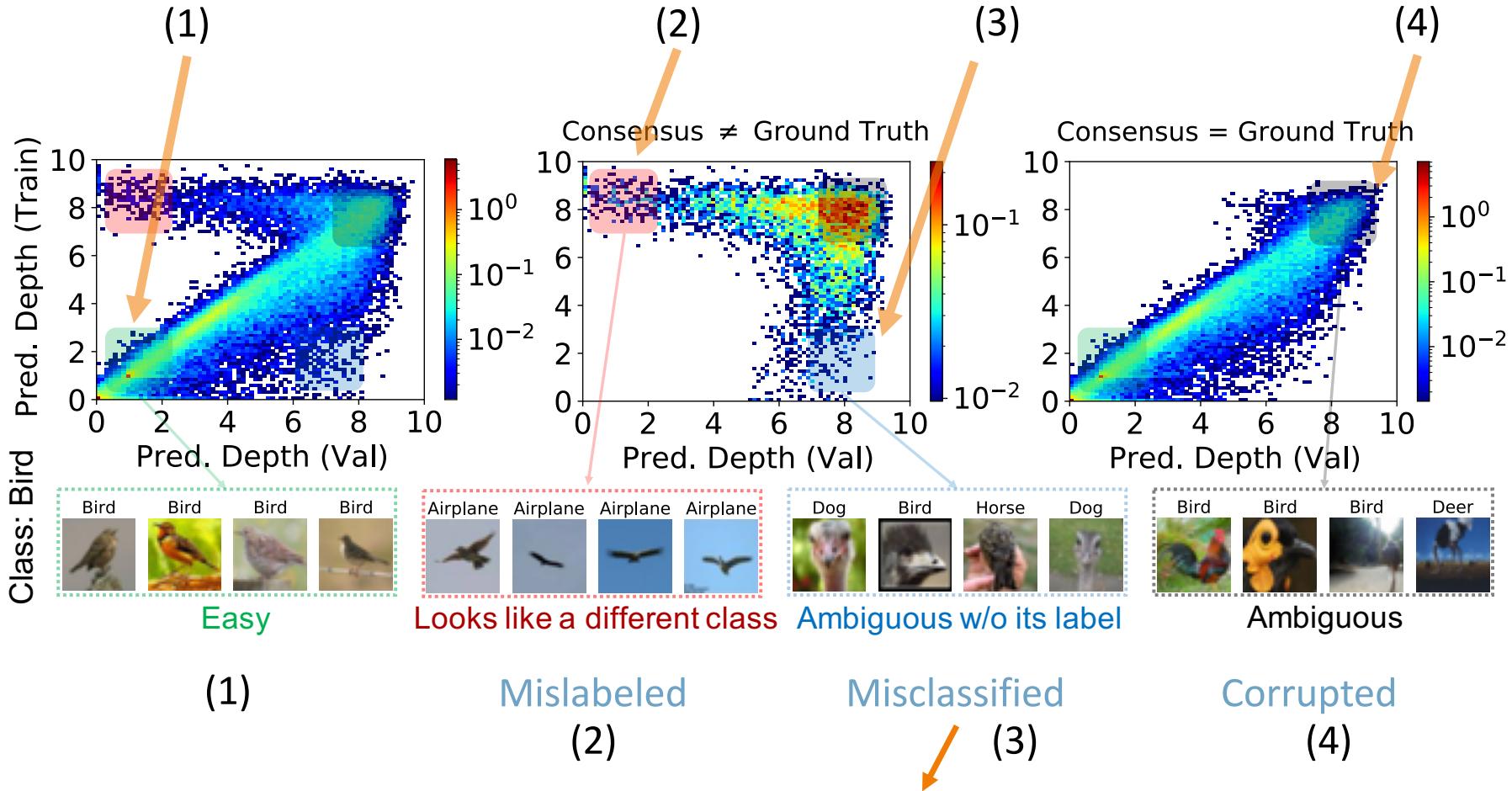
Shows that data points with **smaller prediction depths** have both **larger** input and output margins on average, and that **variances** of the input and output margins **decrease** as the prediction depth increases

- **Different forms of example difficulty**

- **Validation:** points with **low** prediction depth are “**clear**”  
**high** prediction depth are “**ambiguous**”
- **Training:** idem

1. (**Low  $PD_{val}$  and low  $PD_{train}$** ) : **Easy examples**
2. (**Low  $PD_{val}$  and high  $PD_{train}$** ) : **Look like a different class**
  - E.g. **mislabeled** examples
3. (**High  $PD_{val}$  and low  $PD_{train}$** ) : **Ambiguous unless the label is given**
  - E.g. resemble both their **own class** and **another class**  
Likely to be **misclassified**
4. (**High  $PD_{val}$  and high  $PD_{train}$** ) : **Ambiguous**
  - Examples that may be **corrupted** or of a **rare** sub-class.

# What they claim to show



These examples are difficult to connect to their predicted class in the **validation** split but easy to connect to their ground truth class during **training**. These points may, for example, visually resemble both their own class and another class. They are likely to be misclassified.

## Conclusion

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Introduces a notion of **example difficulty** called the **prediction depth**

- which uses the **processing** of data **inside the network** to score the **difficulty** of an example

## Conclusion

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- **Easy examples** are learned and recognized **early** in the network

## What they claim to show

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1. **Early** layers **generalize** while **later** layers **memorize**
2. Networks converge **from** input layers **towards** output layers
3. **Easy** examples are learned **first**
4. Networks present **simpler** functions **earlier** in the training

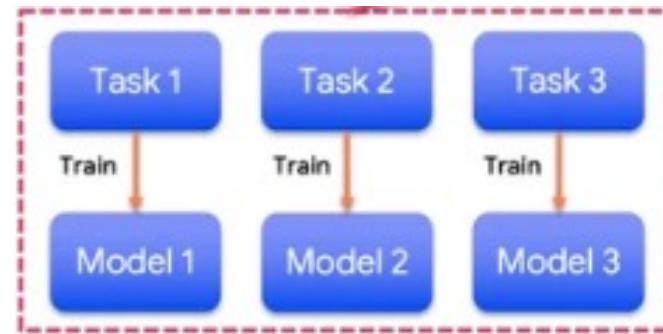
# Outline

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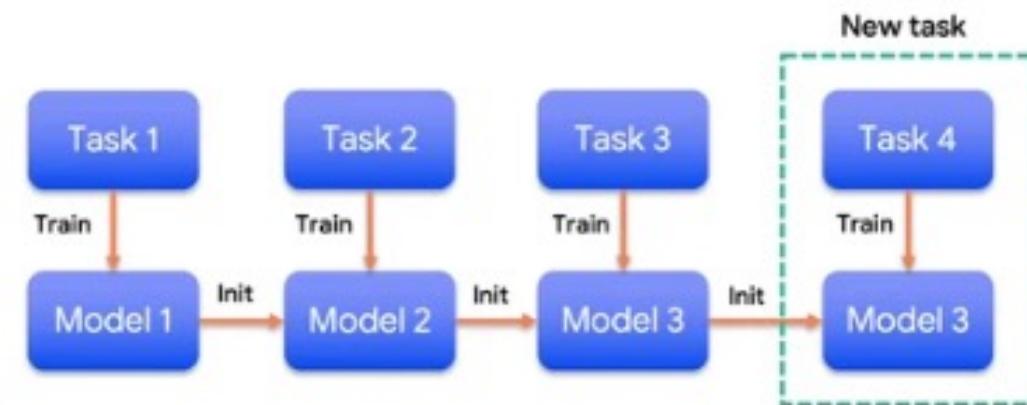
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# Continual learning of new tasks

Training new tasks  
from scratch



Continual learning



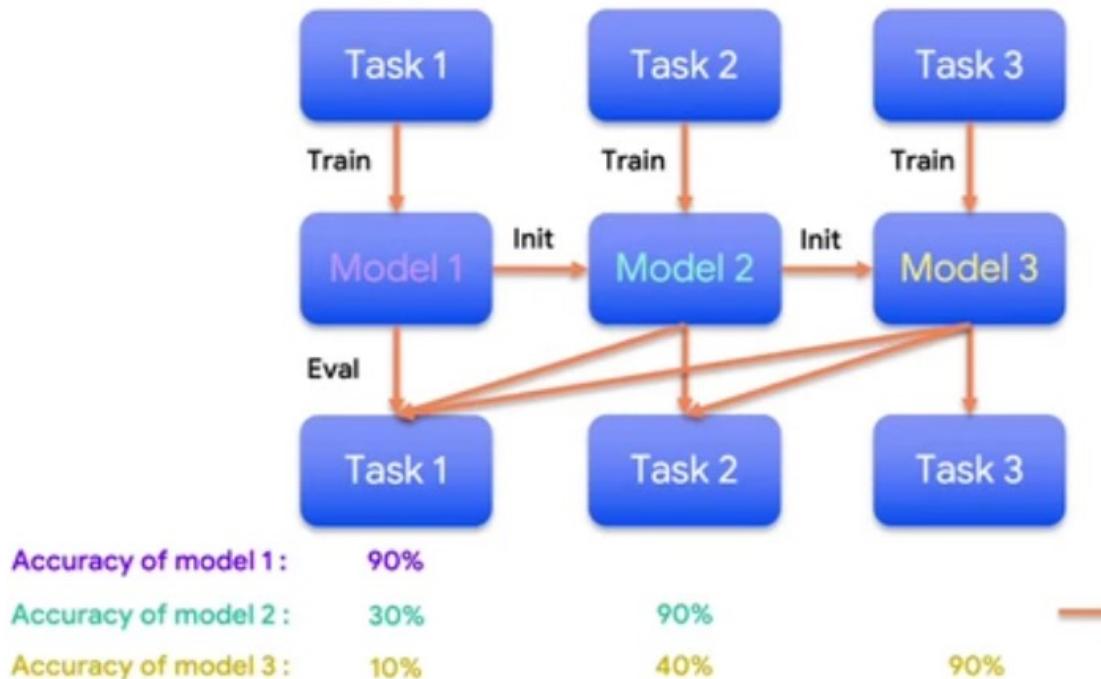
Chen, J., Nguyen, T., Gorur, D., & Chaudhry, A. (2023).

Is forgetting less a good inductive bias for forward transfer?

ICLR-2023.

# Continual learning of new tasks

Continuously updating the model on new tasks results in severely **degraded** performance on **old** tasks



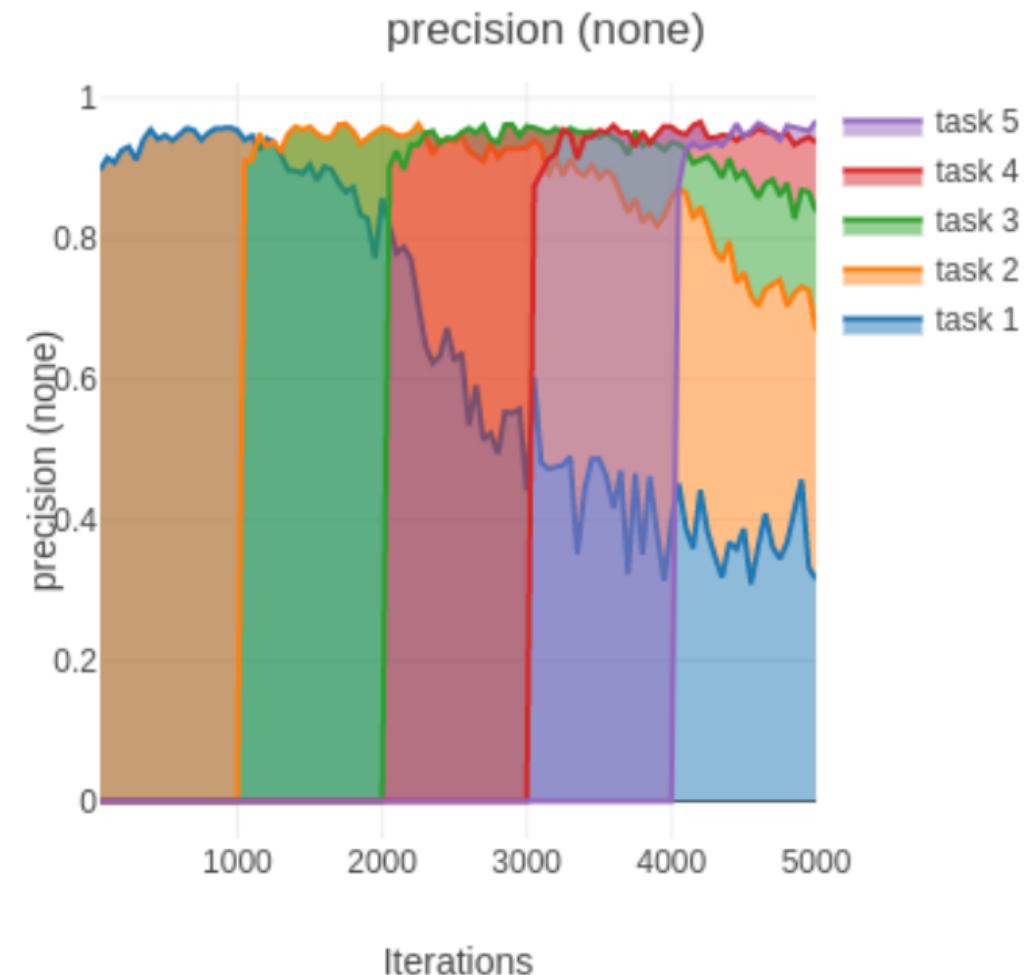
# Catastrophic forgetting

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McCaffary, D. (2021).

**Towards continual task learning in artificial neural networks: current approaches and insights from neuroscience.**

*arXiv preprint arXiv:2112.14146.*



## Catastrophic forgetting

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- ANNs have the tendency to completely and **abruptly forget** previous learned information upon learning new information
  - Therefore **ANNs** are unable to learn multiple tasks sequentially
  - Lifelong or continual learning would not be possible for **ANNs**
  - In **humans**, catastrophic forgetting does not happen
    - Learning to **drive a car** does not result in not knowing anymore how to **ride a bike**

## Catastrophic forgetting: how to avoid it

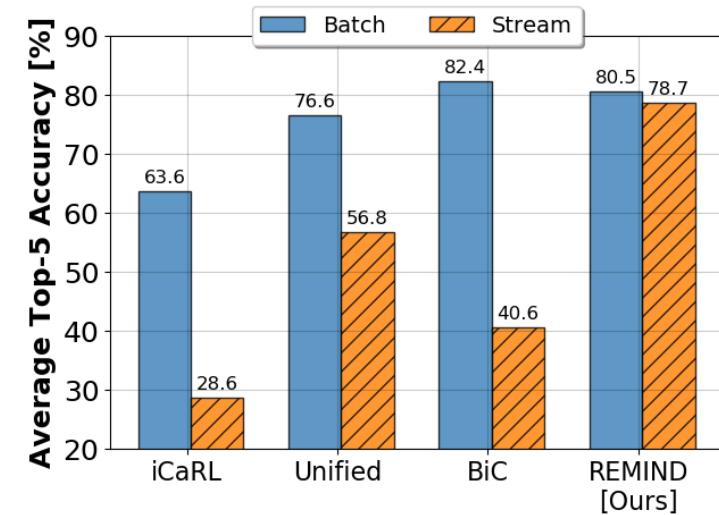
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- Training for ImageNet typically involves
  - to **break** the training dataset into **M distinct batches**,
  - where for ImageNet each batch typically has about **100,000 instances**
  - from 100 classes that are **not seen in later batches**,
  - and then the algorithm sequentially **loops** over each batch many times.
- **Not** efficient
- **Not** biologically plausible

# Catastrophic forgetting

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- In the **streaming scenario**
  - Learn with very small batches
  - Performs only a single pass through the dataset
- Large decrease in performance



Hayes, T. L., Kafle, K., Shrestha, R., Acharya, M., & Kanan, C. (2020, August). **Remind your neural network to prevent catastrophic forgetting**. In *European Conference on Computer Vision* (pp. 466-483). Springer, Cham.

**Fig. 1.** Average top-5 accuracy results for streaming and incremental batch versions of state-of-the-art models on ImageNet.

# Reasons for catastrophic forgetting

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- **Interferences** in the hidden layers
  - When training on the **B task** modifies a lot the weights learnt for **task A**
    - No guarantee that the representation of deeper layers learned for **task A** will be sufficient to losslessly encode novel information, for **task B**, that is not representative of the original training data (i.e. task A)
- The major issue is **balancing**
  - the **stability** of existing representations
  - with the **plasticity** required to efficiently learn new ones

# Catastrophic forgetting

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- Questions
  - What happens to the **internal representations** of neural networks as they undergo catastrophic forgetting?
  - Does the degree to which a network forgets depend on the ***semantic similarity*** between the successive tasks?

- What do we expect?

# Catastrophic forgetting and hidden representations

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- What role **hidden layers** play in forgetting?
  - Do **all** parameters (and layers) **forget equally**?
  - Or are **specific components** of the network particularly **responsible** for the drop in accuracy in sequential training?

# Catastrophic forgetting and hidden representations

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- What role **hidden layers** play in forgetting?
  - Do **all** parameters (and layers) **forget equally**?
  - Or are **specific components** of the network particularly **responsible** for the drop in accuracy in sequential training?

How to approach these questions?

# Catastrophic forgetting and hidden representations

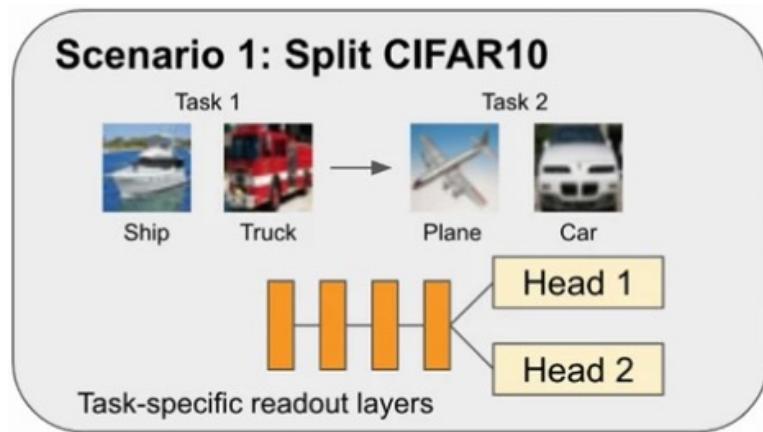
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Study the effect of **individual layers** on forgetting

- Two tasks: task 1 and task 2
  1. First learn task 1
  2. Then **freeze** some layers: starting from the lowest ones
  3. Learn task 2 only **training** the **remaining** layers  
and measure the **performance**

# Catastrophic forgetting and hidden representations

- What role **hidden layers** play in forgetting?

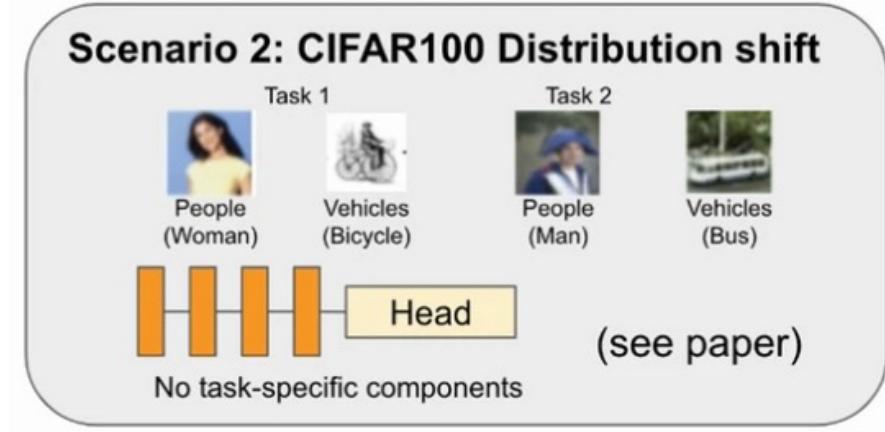
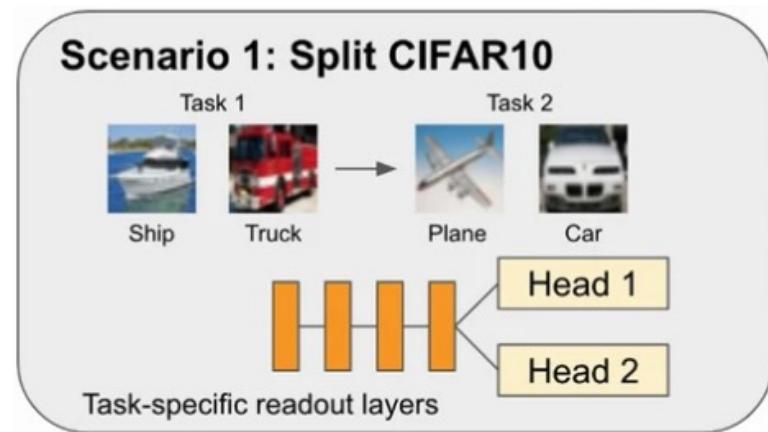


On CIFAR-10:

**task 1** (5 classes) then **task 2** ( $5 \neq$  classes)

# Catastrophic forgetting and hidden representations

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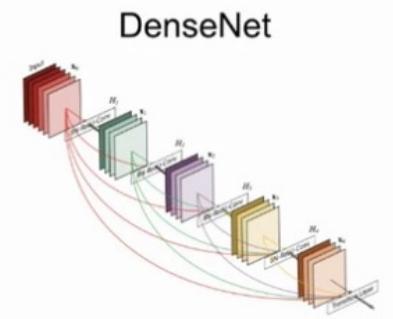
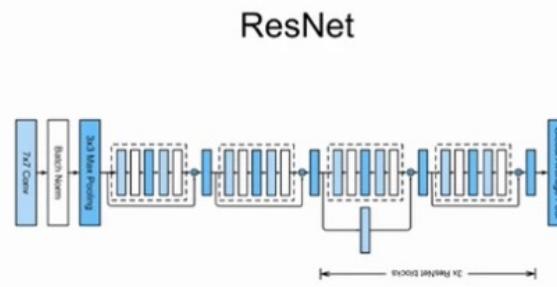
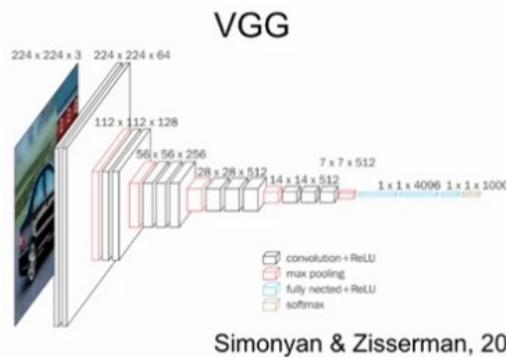
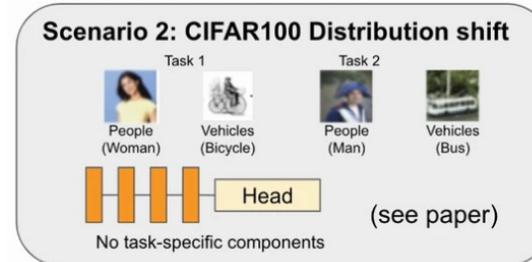
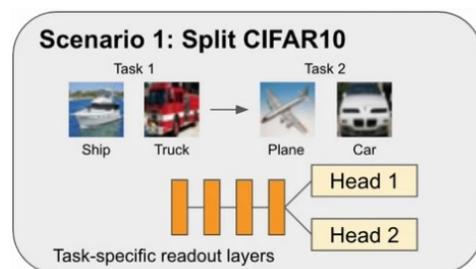
On CIFAR-100:

**task 1** (examples of 5 subsets of 5 superclasses) then **task 2** (examples of  $5 \neq$  subsets of same 5 superclasses).

Each task is to distinguish between the CIFAR-100 superclasses, but input data for each task is a different subset of the constituent classes of the superclass.

# Catastrophic forgetting and hidden representations

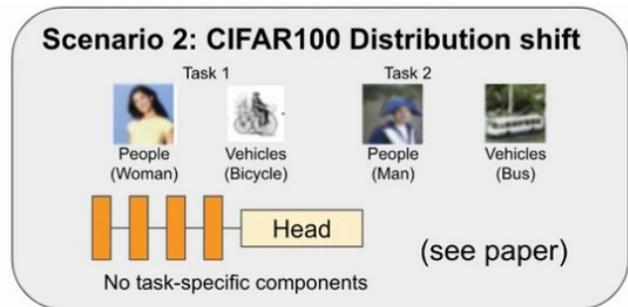
- What role hidden layers play in forgetting?
    - Tested on 3 different Deep Neural Networks



RAMASESH, Vinay V., DYER, Ethan, et RAGHU, Maithra (2021). **Anatomy of catastrophic forgetting: Hidden representations and task semantics.** *ICLR-2021*.

# Catastrophic forgetting and hidden representations

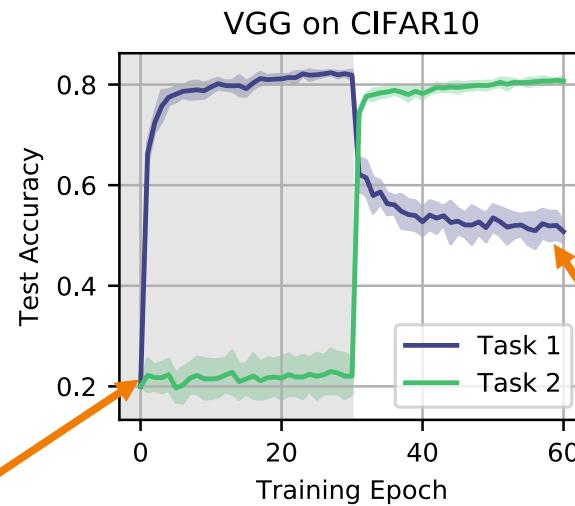
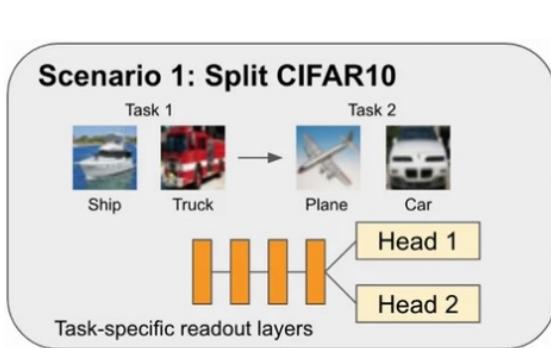
- Manifestation of catastrophic forgetting?



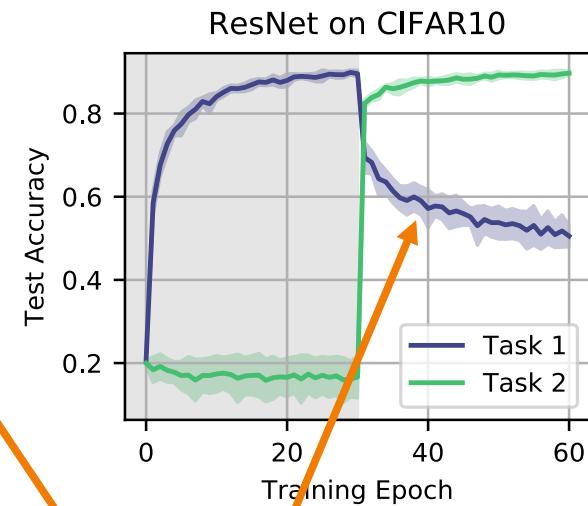
What do we expect?

# Catastrophic forgetting and hidden representations

- Manifestation of catastrophic forgetting?



Starts at ~20% recognition rate: normal



Significant drop of performance on task 1 when learning task 2

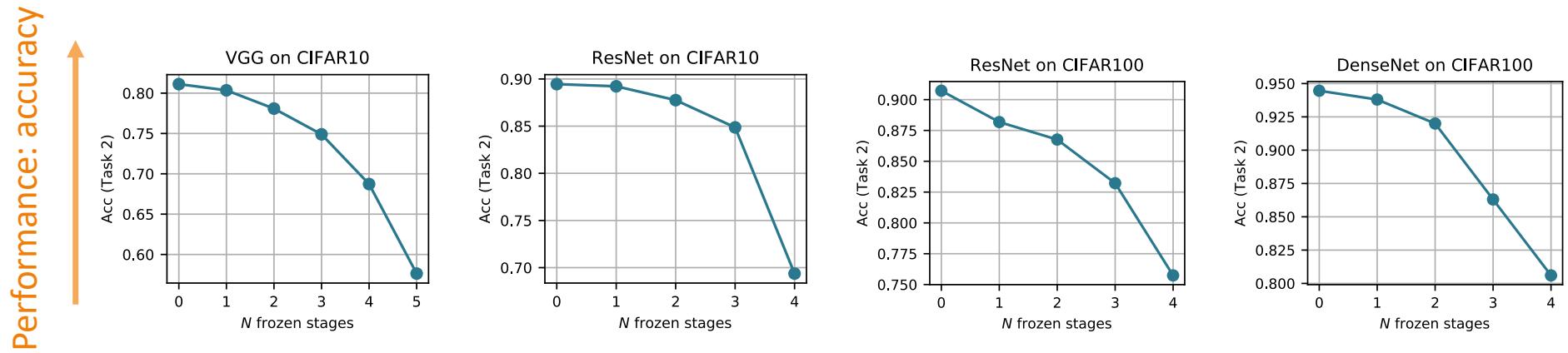
# Catastrophic forgetting and hidden representations

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# Catastrophic forgetting and hidden representations

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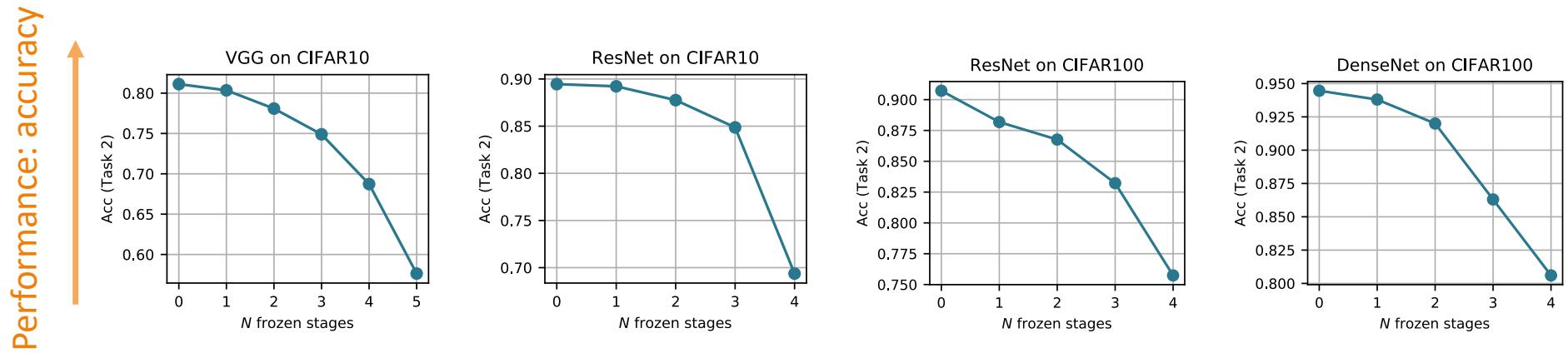


*Stages* = hidden layers starting from the earliest ones

Which conclusions?

# Catastrophic forgetting and hidden representations

- What role **hidden layers** play in forgetting?



*Stages* = hidden layers starting from the earliest ones

- Freezing the **earliest hidden layers** after learning **task 1** has **little impact** on the performance of **task 2**
- **Higher layers** are disproportionately **responsible** for catastrophic forgetting

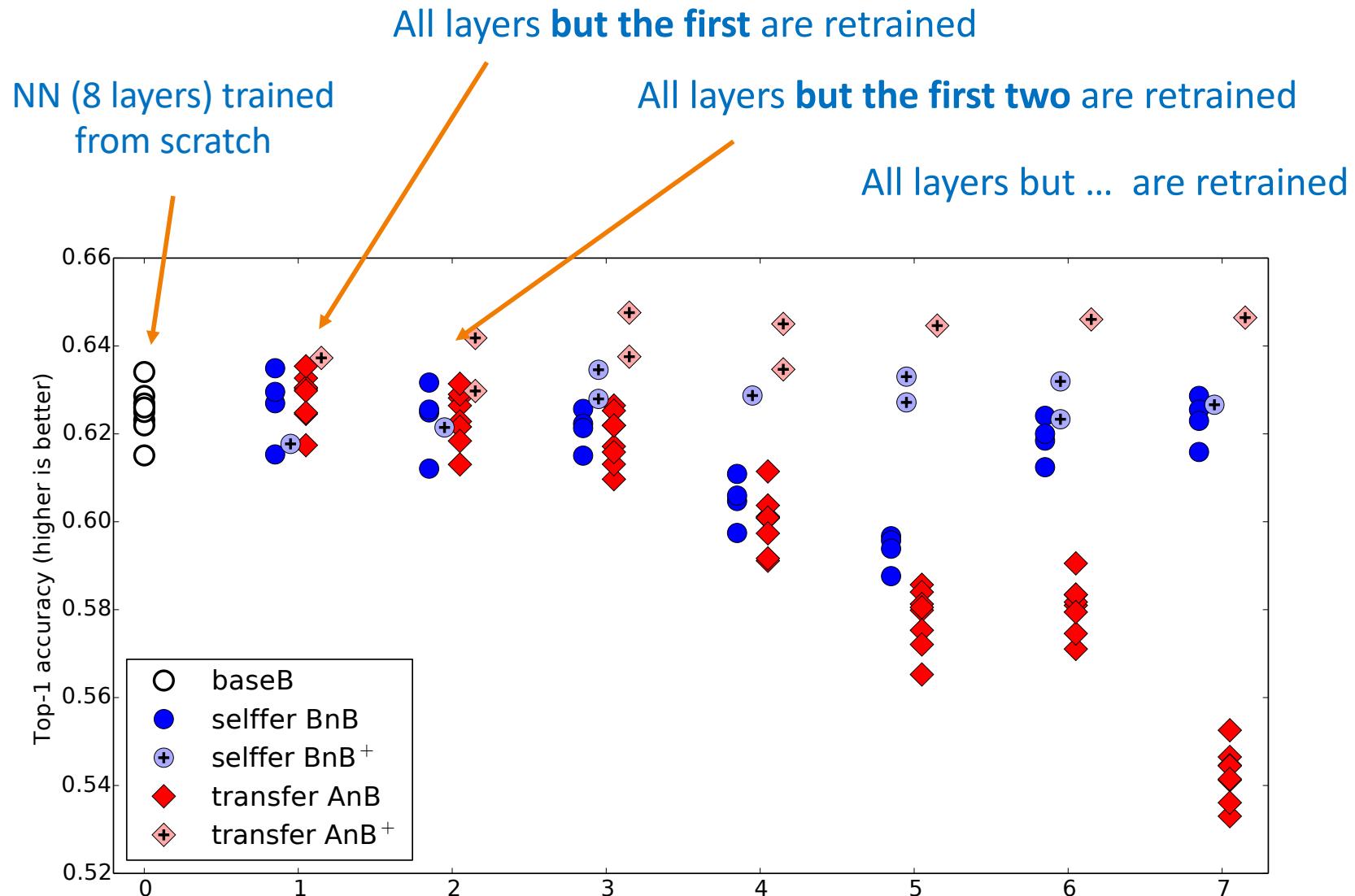
# Catastrophic forgetting and hidden representations

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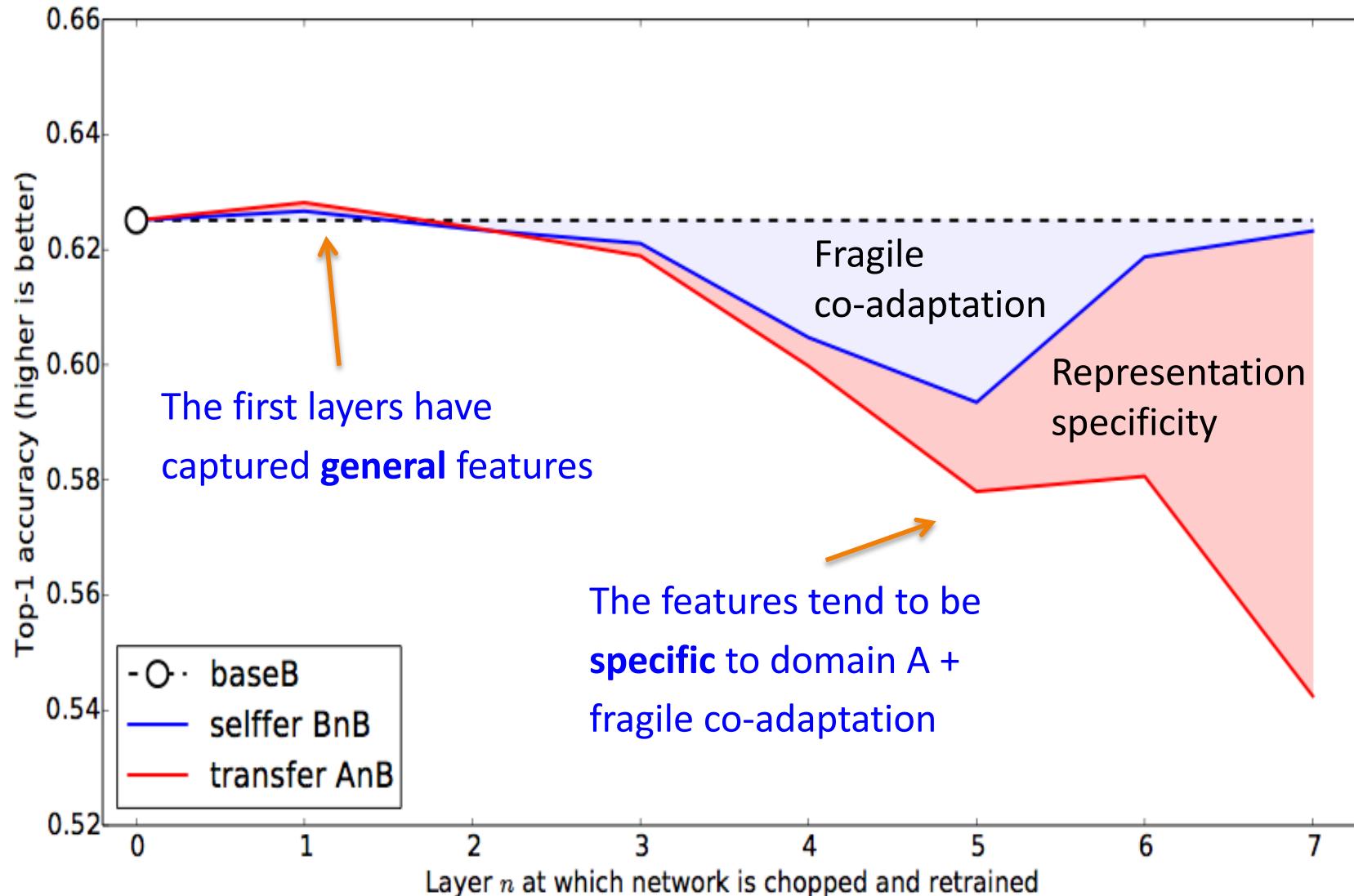
- What role **hidden layers** play in forgetting?
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Why is it so?

# Results: what to think of them?



# Interpretation



# Catastrophic forgetting and hidden representations

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- What role **hidden layers** play in forgetting?
  - Measure *how similar* is each hidden layer **before** and **after** learning **task 2**
    - Use Centered Kernel Alignment (CKA)

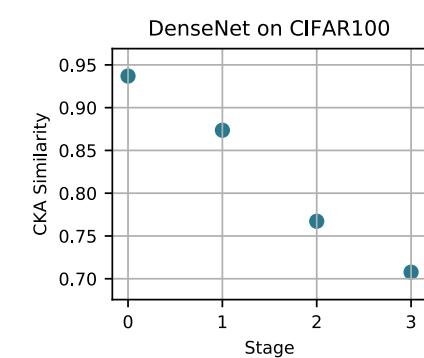
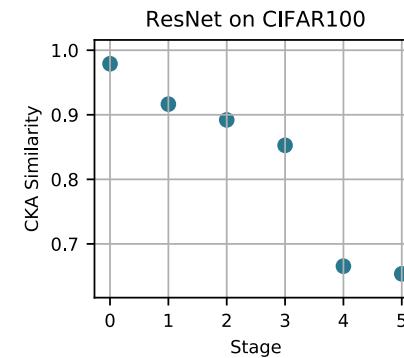
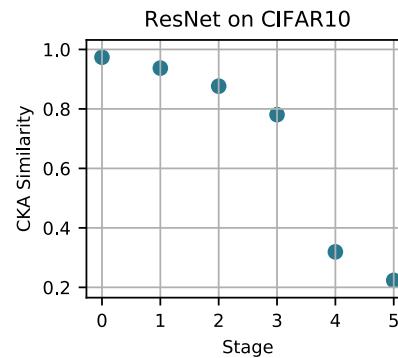
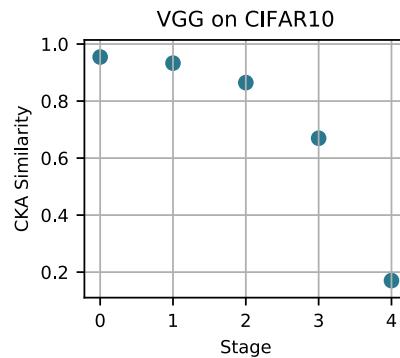
Specifically, letting  $X \in \mathbb{R}^{n \times p}$  and  $Y \in \mathbb{R}^{n \times p}$  be (centered) layer activation matrices of (the same)  $n$  datapoints and  $p$  neurons, CKA computes

$$\text{CKA}(X, Y) = \frac{\text{HSIC}(XX^T, YY^T)}{\sqrt{\text{HSIC}(XX^T, XX^T)}\sqrt{\text{HSIC}(YY^T, YY^T)}} \quad (1)$$

for HSIC Hilbert-Schmidt Independence Criterion (Gretton et al., 2005). We use linear-kernel CKA.

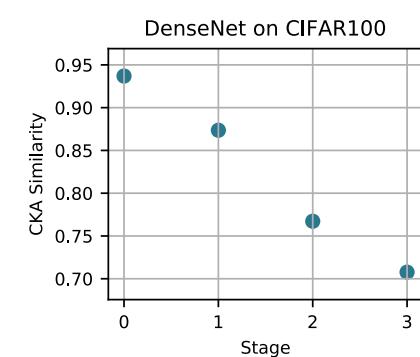
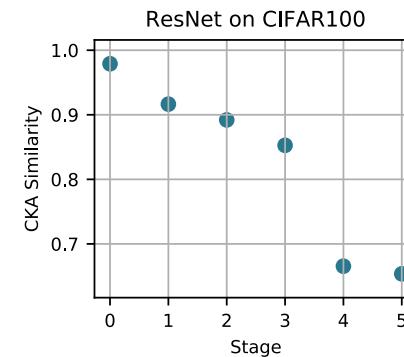
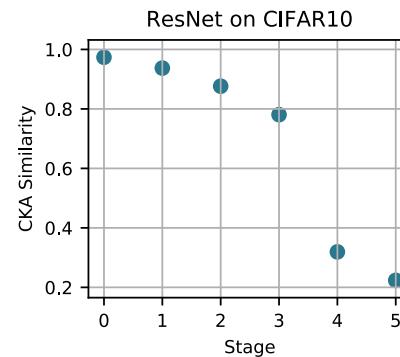
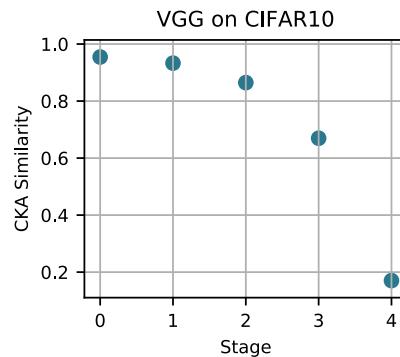
# Catastrophic forgetting and hidden representations

- What role **hidden layers** play in forgetting?
  - Measure **how similar** is each hidden layer **before** and **after** learning **task 2**
    - Use Centered Kernel Alignment (CKA)



# Catastrophic forgetting and hidden representations

- What role hidden layers play in forgetting?
  - Measure how similar is each hidden layer **before** and **after** learning **task 2**
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Across architectures and tasks, we observe that the **lower layers have high representation similarity**, suggesting that lower layer Task 1 features are reused in Task 2.

For all tasks and all NNs

# Catastrophic forgetting and hidden representations

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- What role **hidden layers** play in forgetting?
  - Measure how similar is each **subspace** (PCA of activations) of the hidden layers **before** and **after** learning **task 2**

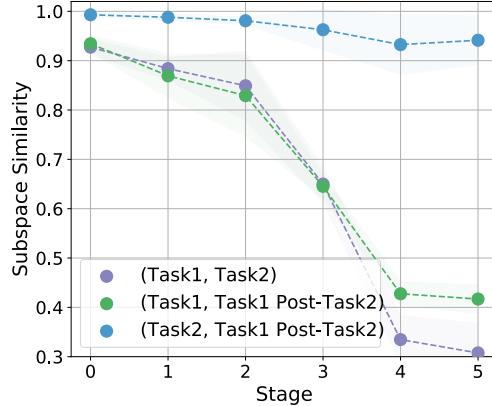
Letting  $X \in \mathbb{R}^{n \times p}$  be the (centered) layer activation matrix of  $n$  examples by  $p$  neurons, we compute the PCA decomposition of  $X$ , i.e. the eigenvectors  $(v_1, v_2, \dots)$  and eigenvalues  $(\lambda_1, \lambda_2, \dots)$  of  $X^\top X$ . Letting  $V_k$  be the matrix formed from the top  $k$  principal directions,  $v_1, \dots, v_k$  as columns, and  $U_k$  the corresponding matrix for a different activation matrix  $Y$ , we compute

$$\text{SubspaceSim}_k(X, Y) = \frac{1}{k} \|V_k^\top U_k\|_F^2$$

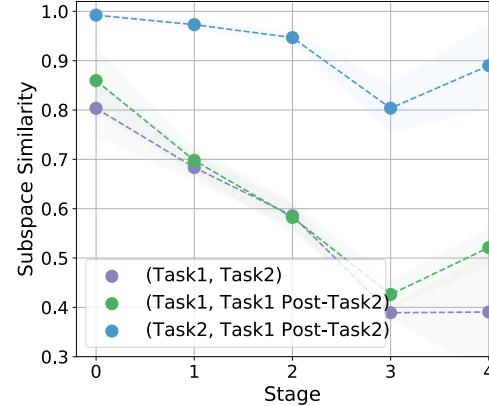
This measures the **overlap** in the subspaces spanned by  $(v_1, \dots, v_k)$  and  $(u_1, \dots, u_k)$ . Concretely, if  $X$  and  $Y$  correspond to layer activation matrices for two different tasks,  $\text{SubspaceSim}_k(X, Y)$  measures **how similarly the top  $k$  representations** for those tasks are stored in the network.

- What role **hidden layers** play in forgetting?
  - Measure how similar is each subspace (PCA of activations) of the hidden layers **before** and **after** learning task 2

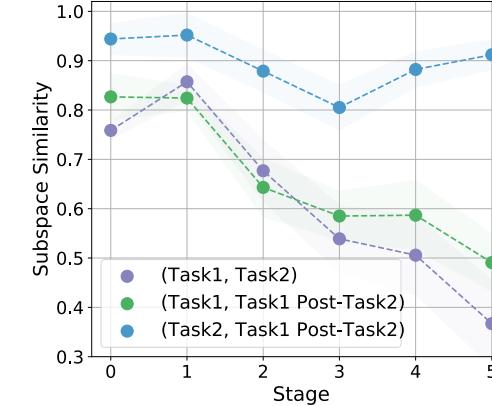
CIFAR10 Resnet: Subspace Similarity Between Task 1, Task 1 post-Task 2 training and Task 2



CIFAR10 VGG: Subspace Similarity Between Task 1, Task 1 post-Task 2 training and Task 2



CIFAR100 Resnet: Subspace Similarity Between Task 1, Task 1 post-Task 2 training and Task 2



- (Task 1, task 2) : low similarity for higher hidden layers
- (Task 1, task 2 and again on task 1) : much has been lost
- (Task 2, task 1 after training on task 2): higher hidden layers are more similar to task 2 than to task 1! The Task 1 subspace in higher layers is erased through Task 2 training

All comparisons show **high similarity in lower layers**, indicative of **feature reuse**

# Catastrophic forgetting and hidden representations

---

- During sequential training,
  - effective **feature reuse happens in the lower layers**,
  - but in **the higher layers**, after Task 2 training, **Task 1 representations are mapped into the same subspace as Task 2**.

Specifically, **Task 2 training causes subspace erasure of Task 1 in the higher layers.**

# Catastrophic forgetting and hidden representations

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Specifically, **Task 2 training causes subspace erasure of Task 1 in the higher layers.**

→ Do popularly used mitigation methods act to **stabilize higher layers?**

# Mitigation strategies and hidden representations

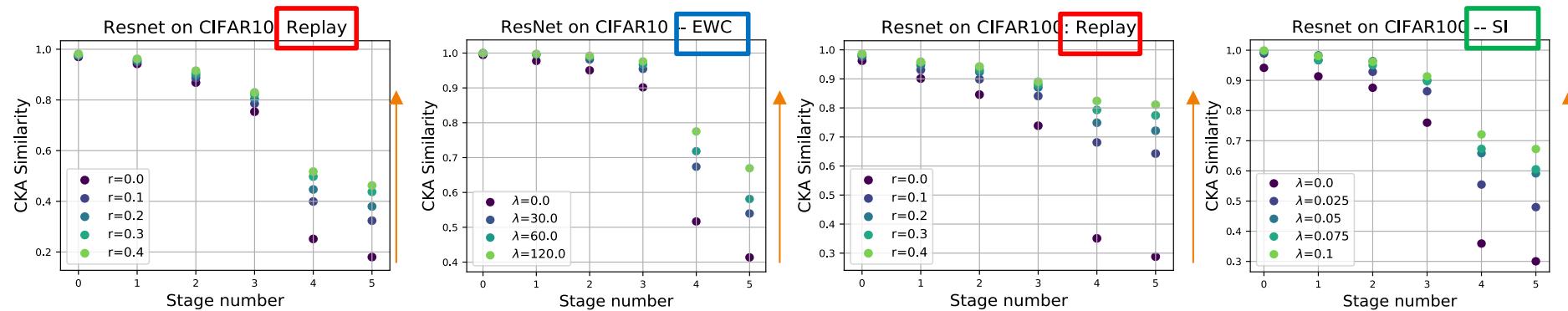
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## Types of mitigation **strategies**

- **Regularization-based approaches**
- **Replay-based approaches**
- What are their **impact**? Are they successful?
- **How do they act** on the hidden layers?
  - Do they **stabilize higher** hidden layers?
    - If yes, by keeping **non interfering** orthogonal **representations**?
    - Or by promoting **high features reuse**?

# Mitigation strategies and hidden representations

- CKA analysis
  - Measures how similar a pair of hidden layer representations are



- Compute CKA between layer representations of Task 1 **before** and **after** Task 2 training
- With varying amounts and types of mitigation.

→ Mitigation methods **stabilize the higher layer representations**

# Mitigation strategies - (1) Regularization approaches

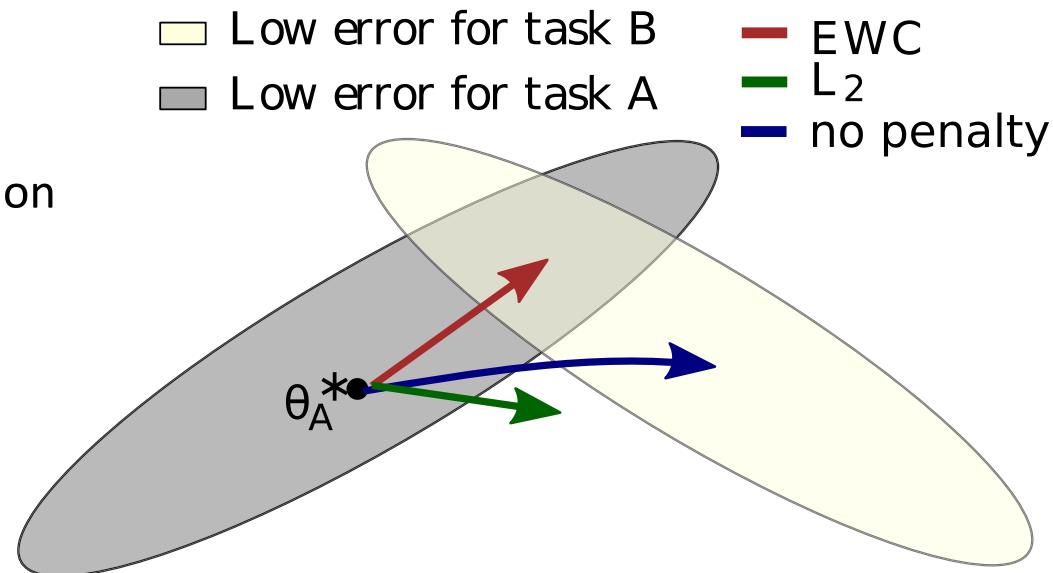
## EWC

Elastic Weight consolidation

Kirkpatrick, J., Pascanu, R., Rabinowitz, N., Veness, J., Desjardins, G., Rusu, A. A., ... & Hadsell, R. (2017).

Overcoming catastrophic forgetting in neural networks.

*Proceedings of the national academy of sciences*, 114(13), 3521-3526.



While learning **task B**, **EWC** protects the performance in **task A** by **constraining the parameters to stay in a region of low error**

for **task A** centered around  $\theta_A^*$

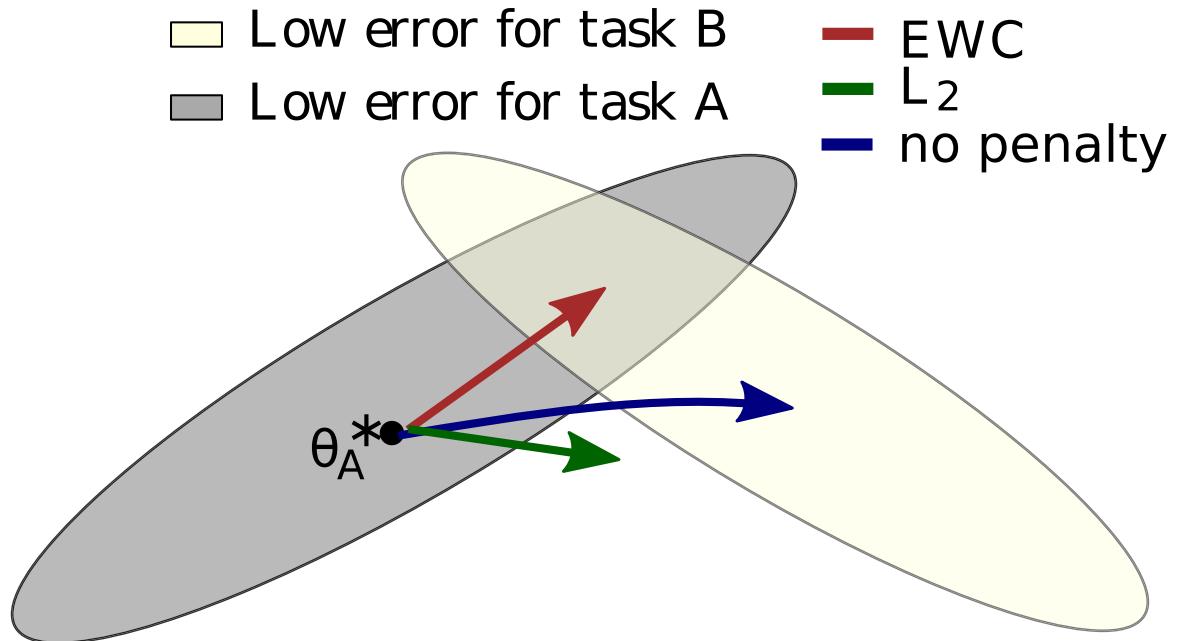
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...

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**Fig. 1.** EWC ensures task A is remembered while training on task B. Training trajectories are illustrated in a schematic parameter space, with parameter regions leading to good performance on task A (gray) and on task B (cream color). After learning the first task, the parameters are at  $\theta_A^*$ . If we take gradient steps according to task B alone (blue arrow), we will minimize the loss of task B but destroy what we have learned for task A. On the other hand, if we constrain each weight with the same coefficient (green arrow), the restriction imposed is too severe and we can remember task A only at the expense of not learning task B. EWC, conversely, finds a solution for task B without incurring a significant loss on task A (red arrow) by explicitly computing how important weights are for task A.

## Mitigation strategies - (1) Regularization approaches

- “Elastic Weight consolidation” (EWC)
  - EWC works by slowing learning of the network **weights** which are most relevant for solving **previously encountered tasks**

$$\mathcal{L}(\theta) = \mathcal{L}_B(\theta) + \sum_i \frac{\lambda}{2} F_i (\theta_i - \theta_{A,i}^*)^2$$

Hyperparameter indicating the relative importance of previously encountered tasks compared to new ones

Parameters of the network

Loss for task B

Fisher information matrix

Parameters important for solving task A

The diagram illustrates the EWC loss function  $\mathcal{L}(\theta)$ . It consists of two main parts: the base loss  $\mathcal{L}_B(\theta)$  and a regularization term. The regularization term is a sum over parameters  $i$ , where each term is proportional to the Fisher information  $F_i$  times the squared difference between the current parameter  $\theta_i$  and its value  $\theta_{A,i}^*$  from a previous task. Orange arrows point from the labels to their corresponding terms in the equation. For example, an arrow points from 'Parameters of the network' to  $\mathcal{L}(\theta)$ , and another from 'Fisher information matrix' to the term involving  $F_i$ .

## Mitigation strategies - (1) **Regularization** approaches

---

- Elastic Weight consolidation (**EWC**)
  - the **Fisher information matrix** is used to give an estimation of the **importance of weights** for solving tasks
    - The **importance weighting** is proportional to the diagonal of the Fisher information metric over the old parameters for the previous task

$$\mathcal{L}(\theta) = \mathcal{L}_B(\theta) + \sum_i \frac{\lambda}{2} F_i (\theta_i - \theta_{A,i}^*)^2$$



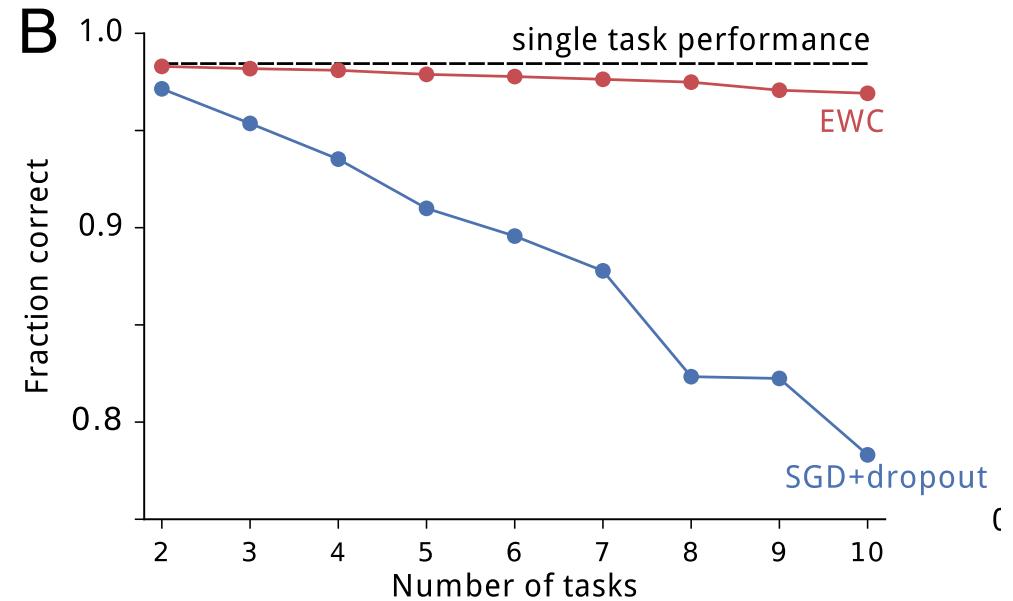
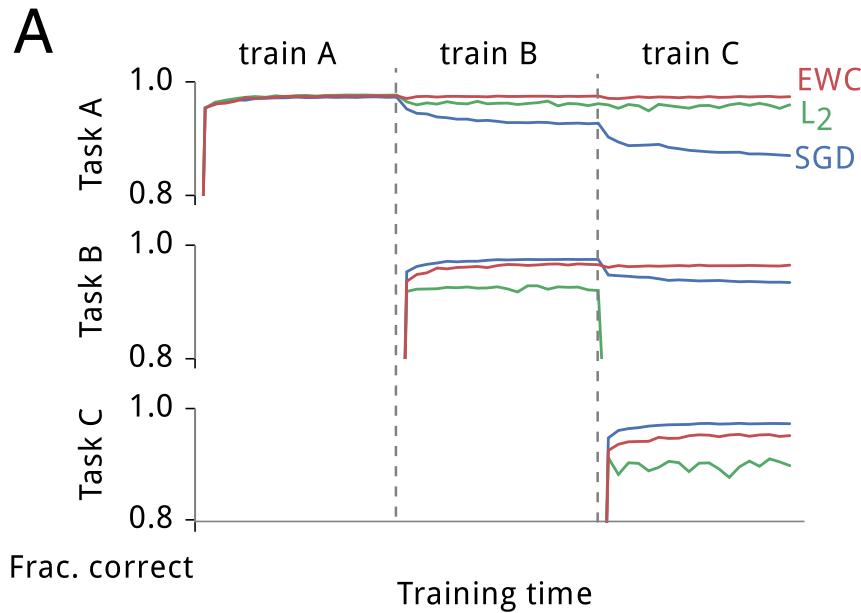
Parameters important for solving task A

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# Mitigation strategies - (1) Regularization approaches



- **(A) Training curves** for three random permutations A, B, and C, using **EWC** (red),  **$L_2$  regularization** (green), and **plain SGD** (blue).  
Note that only EWC is capable of maintaining a high performance on old tasks, while retaining the ability to learn new tasks.
- **(B) Average performance across all tasks**, using **EWC** (red) or **SGD** with dropout regularization (blue). The dashed line shows the performance on a single task only.

---

## EWC is **efficient**

- **How does it do it?**
  - High features **reuse**?
  - Promoting **non interfering** higher subspaces?

## Mitigation strategies – (2) Replay-based approaches

---

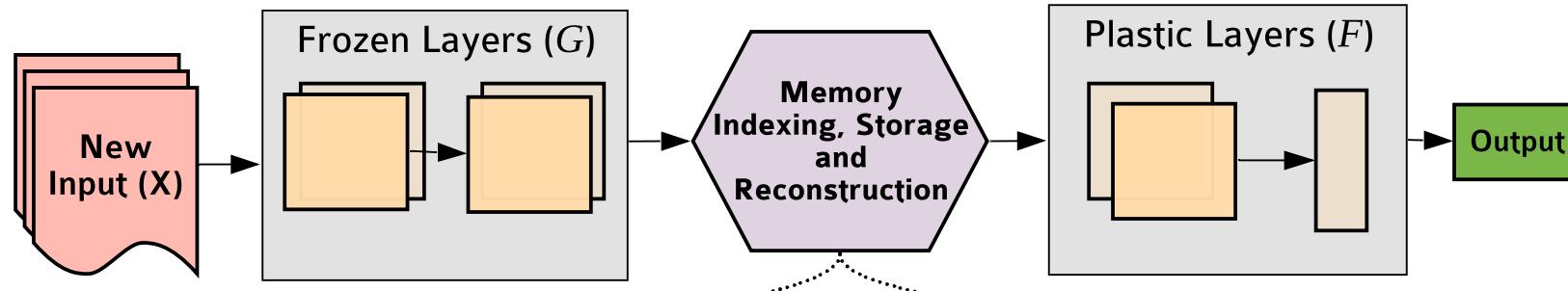
- Deep generative replay
  - A generative model is used to **generate representative data from previous tasks**
  - From which a sample is **selected** and **interspersed** with the dataset of the new task
  - Example: REMIND (Replay using Memory Indexing)
    - Replays a compressed representation of previously encountered training data
    - Using hidden layers (e.g. a feature map)



Hayes, T. L., Kafle, K., Shrestha, R., Acharya, M., & Kanan, C. (2020, August). **Remind your neural network to prevent catastrophic forgetting.** In *European Conference on Computer Vision* (pp. 466-483). Springer, Cham.

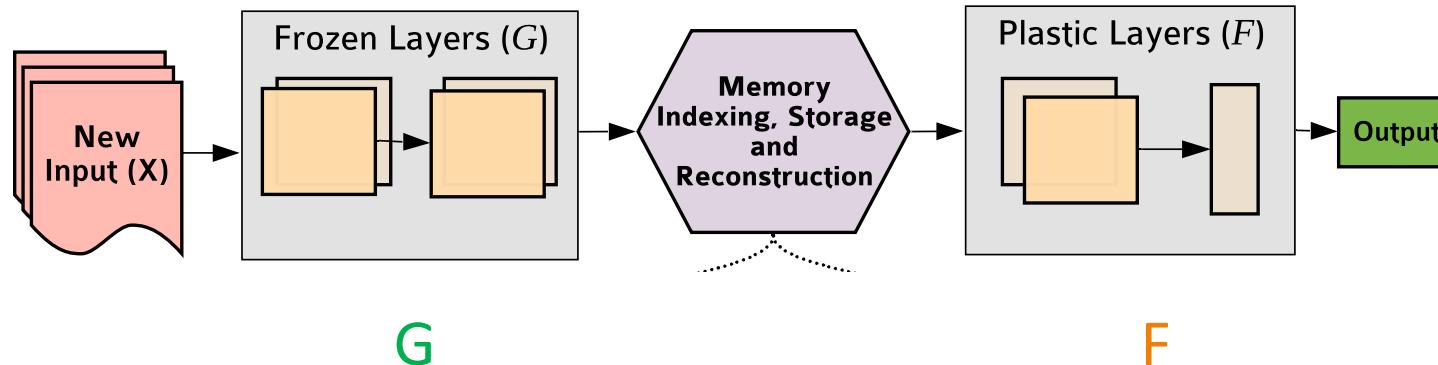
## (2) REMIND

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...

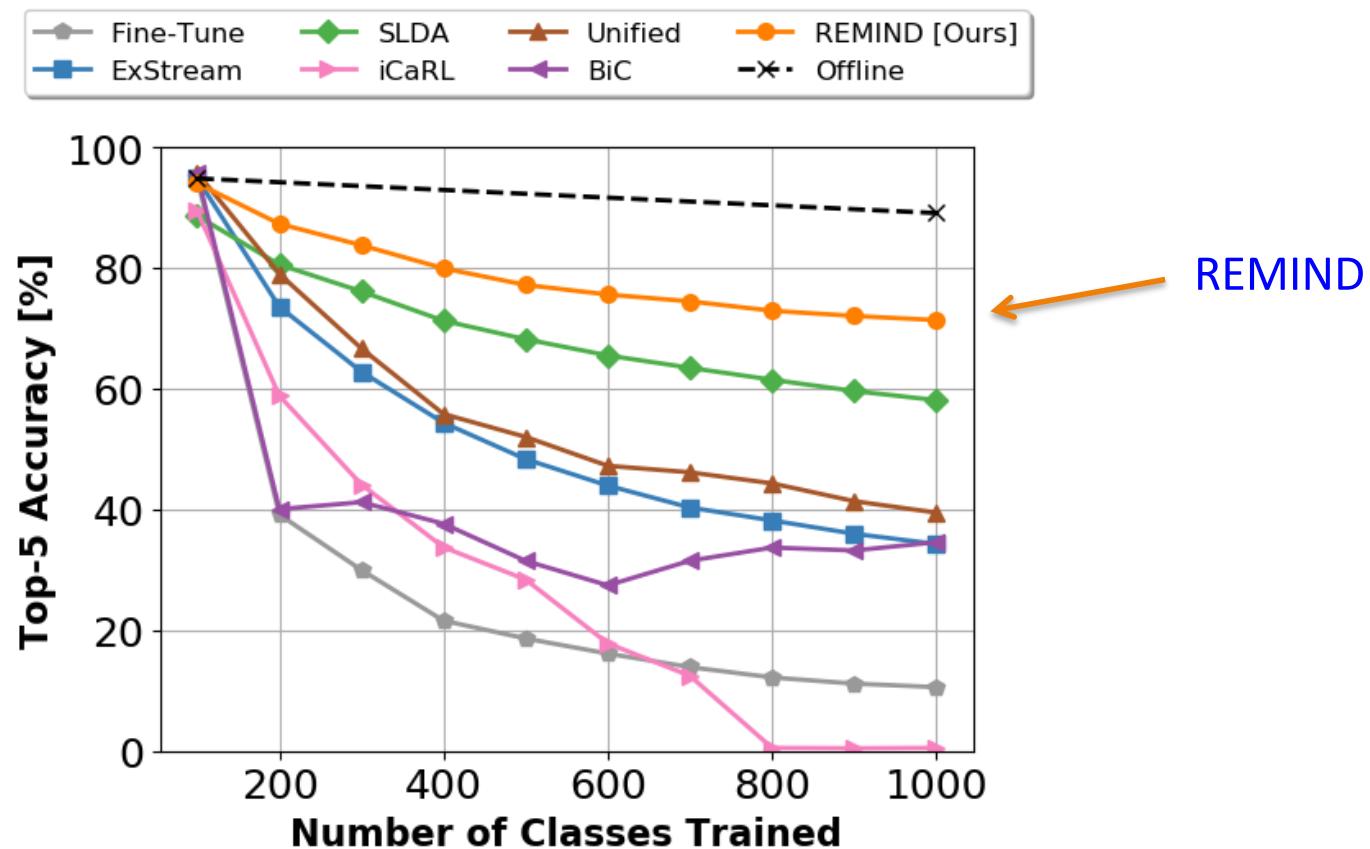
(2)



- First, **train the complete network**:  $G + F$  layers on the training set
- Froze ( $G$ ) and **store** sort of **prototype features** of the training examples
- Later, during training of **new tasks**, **use** the stored prototype features to **generate training instances** before ( $F$ ) related to the previous tasks together with new training examples and train only ( $F$ )

## (2) REMIND

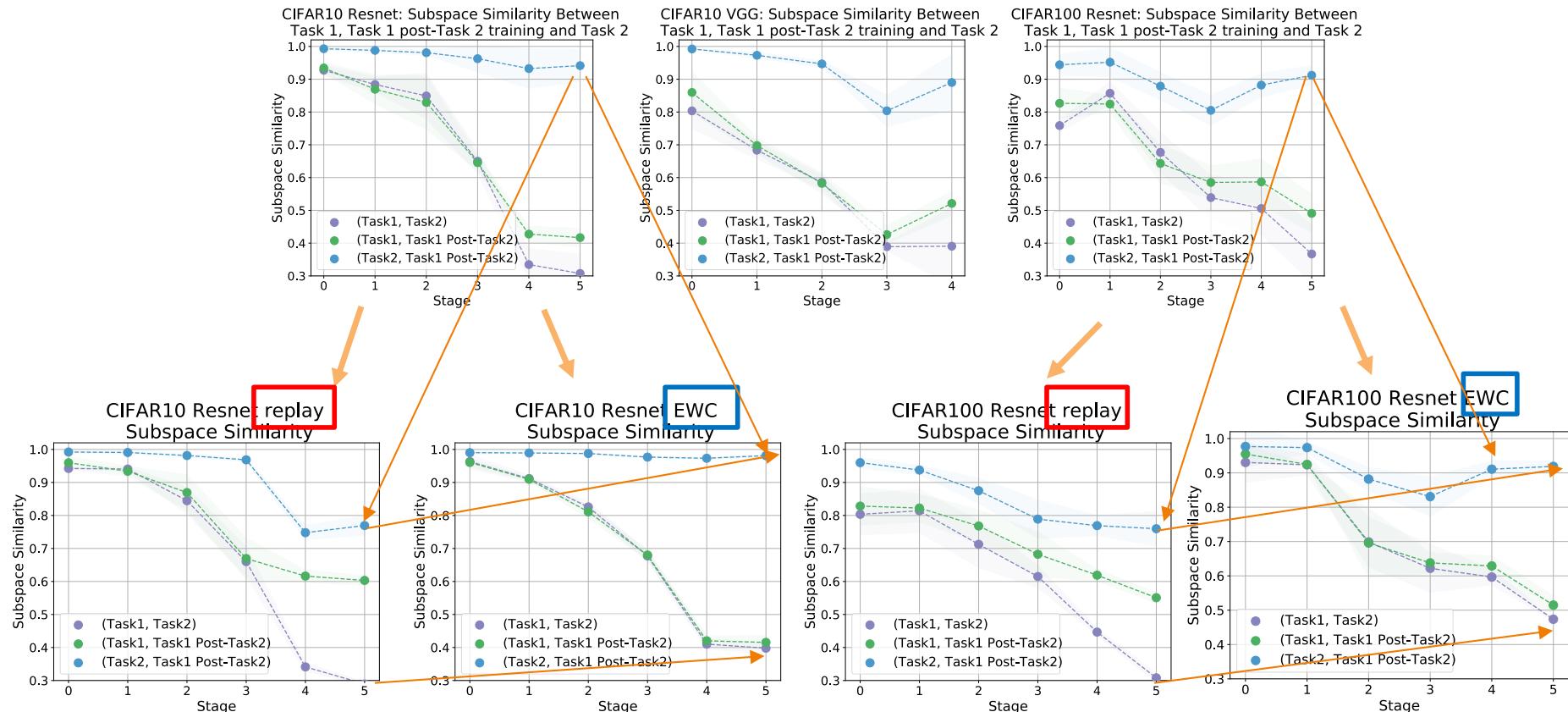
- Performances when learning additional classes of ImageNet



Now the **analysis**

# Mitigation strategies and hidden representations

- But what about **subspace similarity**?

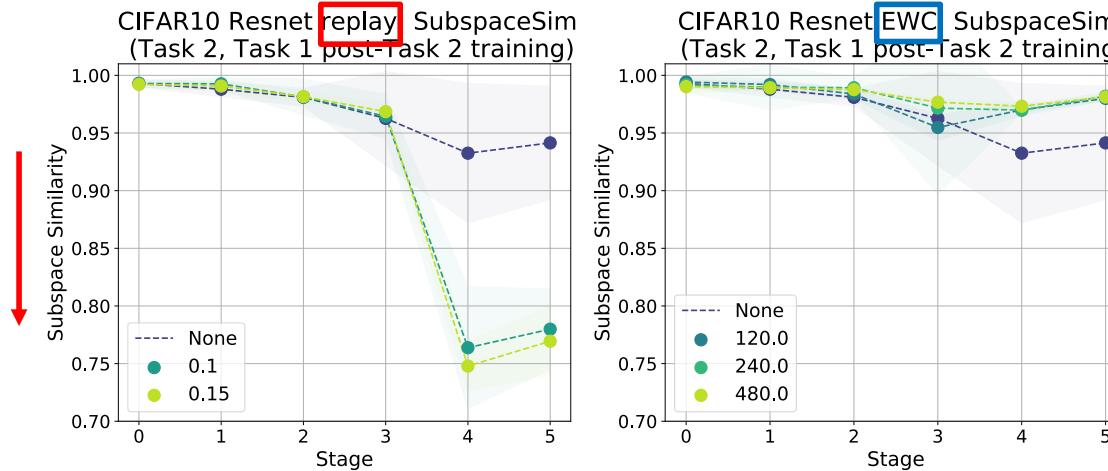


- (Task 2, Task 1 post-Task 2 training) similarity is lower in **replay** compared to **EWC regularization-based methods**
- As is (Task1, task 2)

# Mitigation strategies and hidden representations

- But what about **subspace similarity**?

- With varying **degree** of mitigation



- Again (Task 2, Task 1 post-Task 2 training) is **much lower in replay** compared to no mitigation
- When **EWC** maintains **similar** subspaces for (Task2, Task 1 post Task 2 training)

## Conclusion

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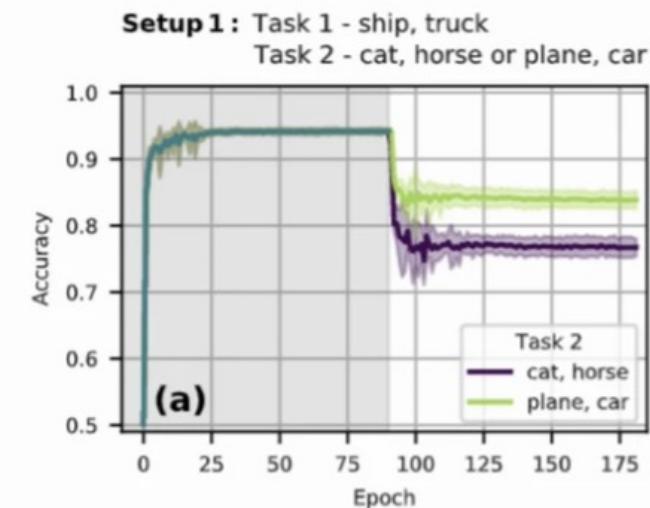
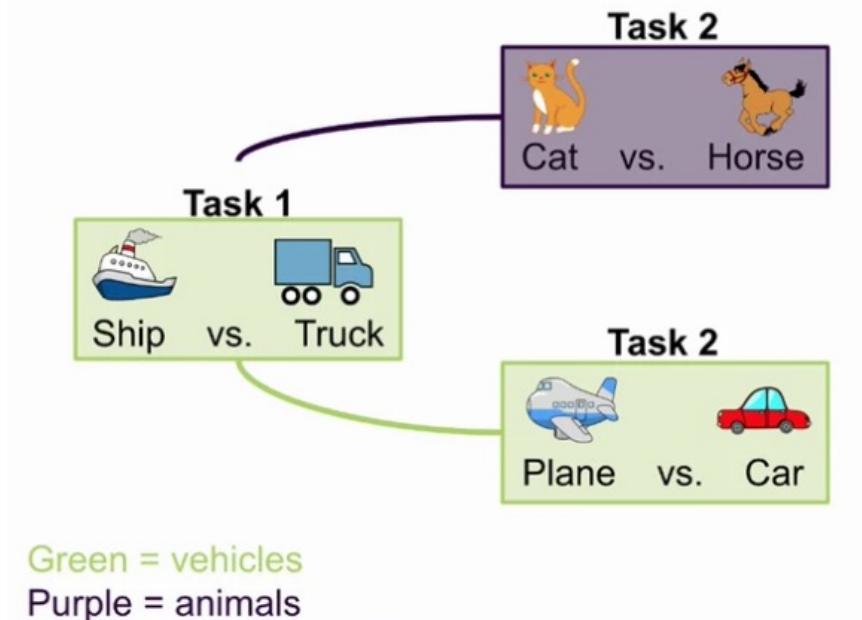
- REPLAY adds **non interfering** additional features in higher layers
- EWC promotes the **reuse** of features in higher layers

# Catastrophic forgetting

---

- Questions
  - What happens to the **internal representations** of neural networks as they undergo catastrophic forgetting?
- — Does the degree to which a network forgets depend on the ***semantic similarity*** between the successive tasks?

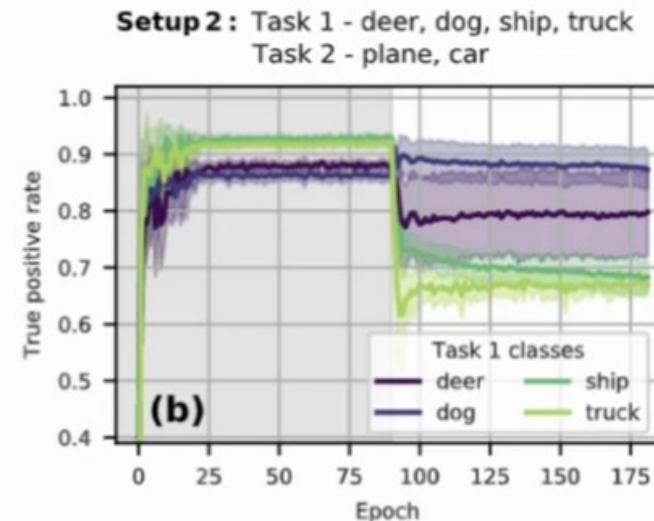
# Catastrophic forgetting and semantic similarity between tasks



In this scenario: More similar tasks cause less forgetting

# Catastrophic forgetting and semantic similarity between tasks

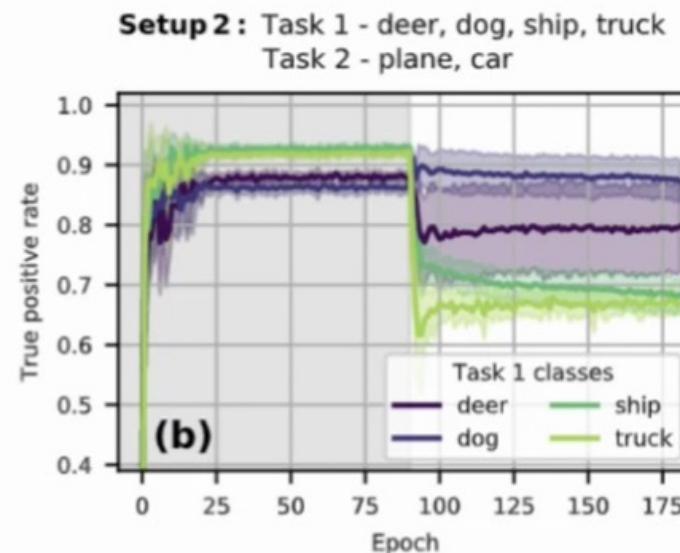
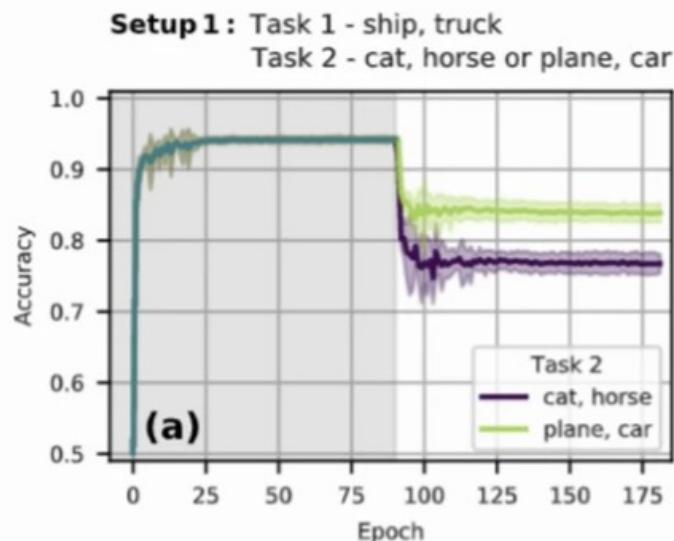
- But ...



In this scenario: similar categories are forgotten more

# Catastrophic forgetting and semantic similarity between tasks

- A contradiction?

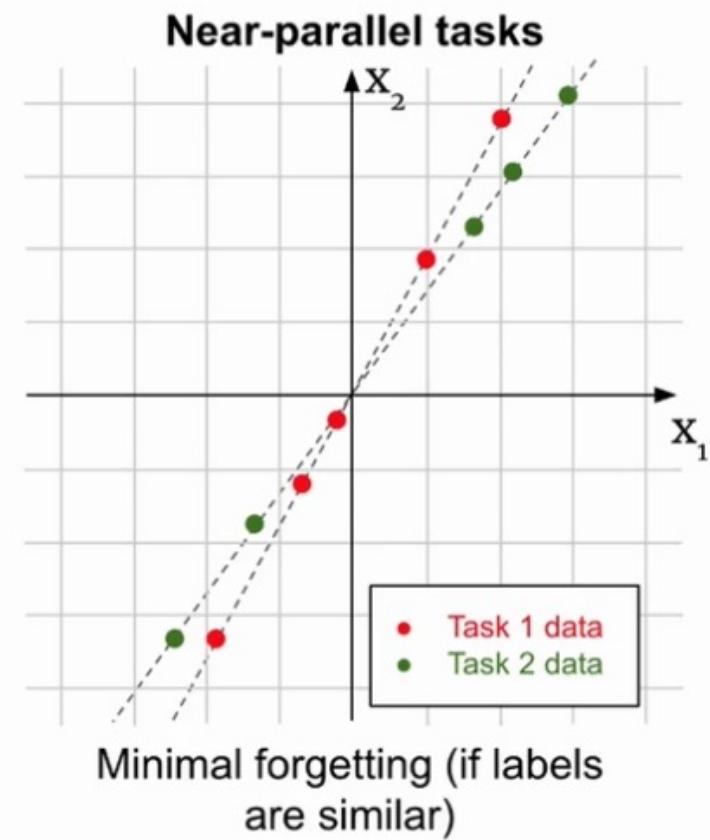
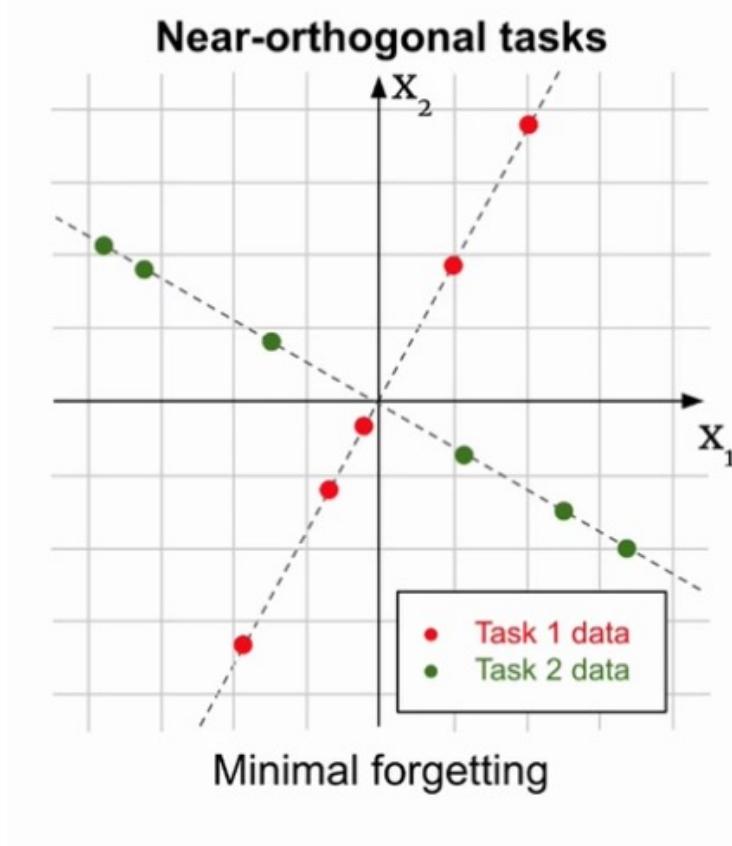


Similar tasks cause less forgetting

Similar categories are forgotten more

# Catastrophic forgetting and semantic similarity between tasks

- Alignment of subspaces



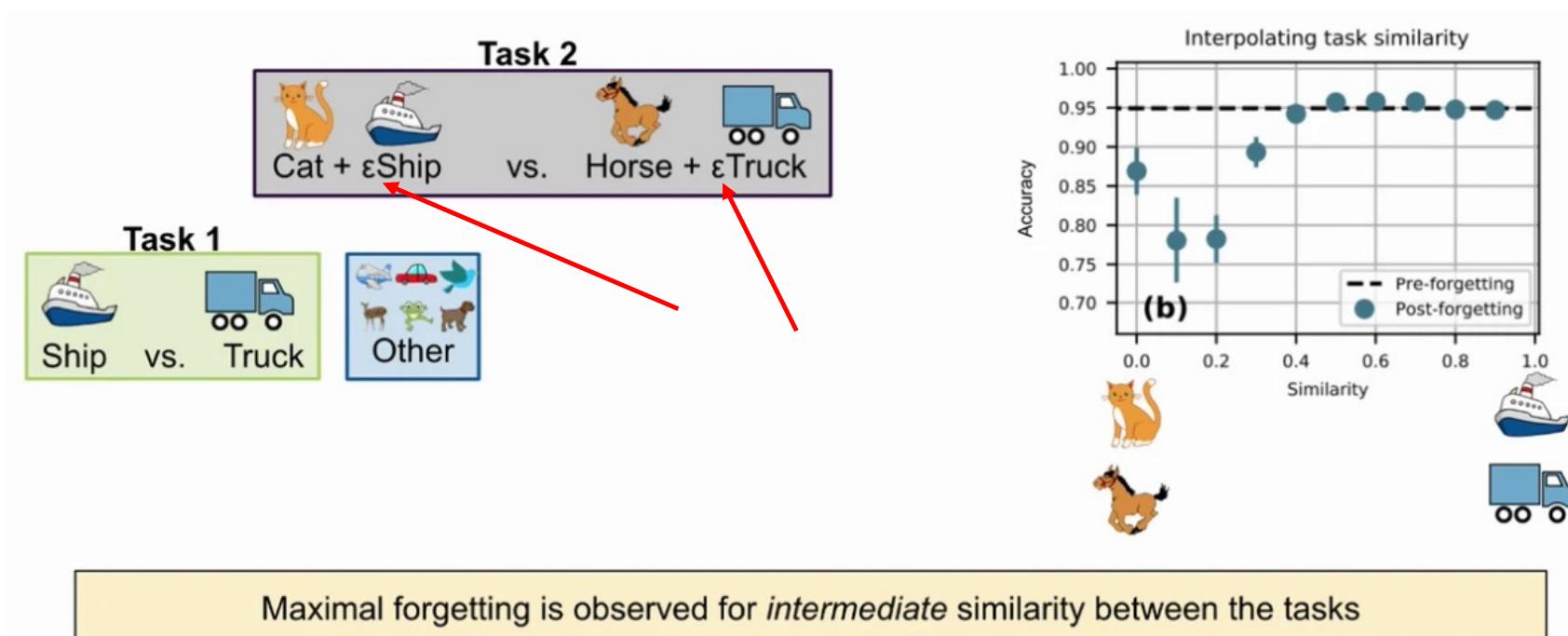
Near **orthogonal** model representations

Near **equal** model representations

Little forgetting

# Catastrophic forgetting and semantic similarity between tasks

- ...



## General conclusions

---

- Higher layers are disproportionately **responsible** for catastrophic forgetting
- Different methods for **mitigating** forgetting exist
  - all **stabilize higher layer** representations,
    - But some methods encourage **greater feature reuse** in higher layers, (e.g. EWC)
    - Others **store task** representations **as orthogonal subspaces**, preventing interference (e.g. REPLAY)
- **Semantic similarity** between subsequent tasks consistently **controls** the degree of forgetting
  - forgetting is **most severe** for tasks with **intermediate similarity**

# Can forgetting be useful in transfer learning?

---

- we want our models to capture generalizable concepts rather than purely statistical regularities
- Forgetting = any process that results in a decrease in training accuracy.

Zhou, H., Vani, A., Larochelle, H., & Courville, A. (2022). **Fortuitous forgetting in connectionist networks.**  
*ICLR-2022.*

# Can forgetting be useful in transfer learning?

---

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# Can forgetting be useful in transfer learning?

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Forgetting = any process that results in a **decrease** in training accuracy.

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*ICLR-2022.*

# Can **forgetting** be **useful** in transfer learning?

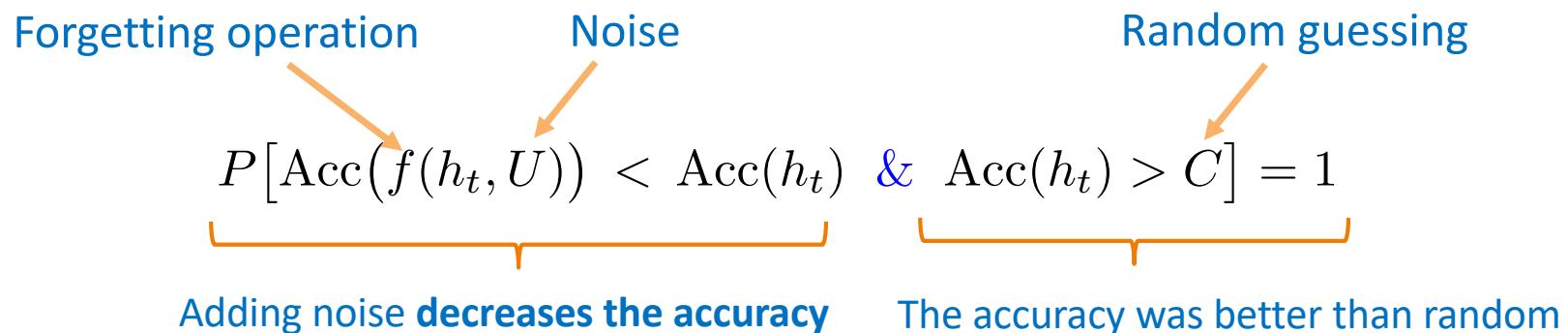
---

- **Forgetting** is an inescapable component of **human memory**.
- It is **often thought** to be an **undesirable** characteristic of the human mind
- **However**, substantial evidence in psychology and neuroscience have shown that **forgetting** and **learning** have a **symbiotic** relationship

Could that be the same for **machine learning**?

- “Forgetting”  $h_t \in \mathcal{H}$  trained on  $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{1 \leq i \leq n}$

$$\text{Acc}(h) = \frac{1}{n} \sum_{i=1}^n \mathbb{I}\{h(\mathbf{x}_i) = y_i\}$$



$$I(f(N_t, U), \mathcal{D}) > 0$$

Adding noise equates to a **partial removal of information** (still “aligned”)

# Can forgetting be useful in transfer learning?

- The **forget-and-relearn** hypothesis
  - Given an appropriate forgetting operation, iterative **re-training AFTER forgetting** will amplify **unforgotten features** that are **consistently useful** under different learning conditions induced by the forgetting step.
  - A forgetting operation that favors the preservation of **desirable features** can thus be used to steer the model towards those desirable characteristics.

Many existing algorithms which have successfully demonstrated improved generalization **have a forgetting step** that disproportionately **affects undesirable information** for the given task.

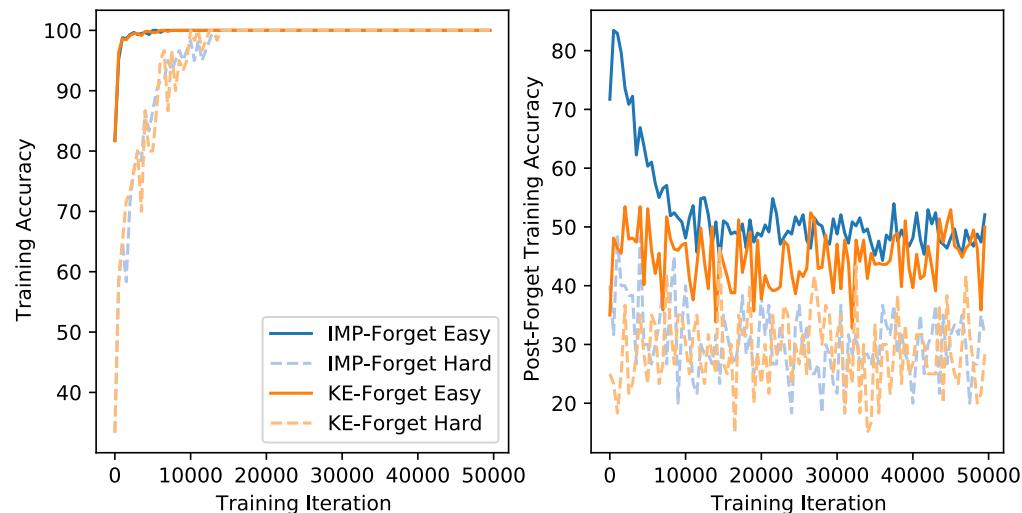
# Can forgetting be useful in transfer learning?

- Easy vs. Hard examples
  - Use the **output margin** between the **largest** and **second-largest** logits (outputs) for each example

KE-style perturbation: the **reinitialization** of a random subset of the weights

IMP-style perturbation: the **zeroing** of **small magnitude weights** and rewinding of the remaining weights

Hard examples are **more adversely affected** than easy ones by weight perturbations



Normal training

Training with weight perturbations at each iteration

# Targeted forgetting

---

- Later-Layer Forgetting (LLF)
  - Reinitialization of later layers at each learning iteration

Method	Flower	CUB	Aircraft	MIT	Dog
Smth (N1)	$51.02 \pm 0.09$	$58.92 \pm 0.24$	$57.16 \pm 0.91$	$56.04 \pm 0.39$	$63.64 \pm 0.16$
Smth long (N3)	$59.51 \pm 0.17$	$66.03 \pm 0.13$	$62.55 \pm 0.25$	$59.53 \pm 0.60$	$65.39 \pm 0.55$
Smth + KE (N3)	$57.95 \pm 0.65$	$63.49 \pm 0.39$	$60.56 \pm 0.36$	$58.78 \pm 0.54$	$64.23 \pm 0.05$
Smth + LLF (N3) (Ours)	<b><math>63.52 \pm 0.13</math></b>	<b><math>70.76 \pm 0.24</math></b>	<b><math>68.88 \pm 0.11</math></b>	<b><math>63.28 \pm 0.69</math></b>	<b><math>67.54 \pm 0.12</math></b>

- Importance of having **variable conditions for refining first layers**
- Keeps and **amplifies the useful features of the first layers**

## Lesson

---

- Forgetting is **useful**
  - If it promotes the **amplification of useful features in the first layers**

# Outline

---

1. How to measure the difficulty of a training example
2. Catastrophic forgetting
3. Curriculum building
4. A geometry on the space of tasks
5. Conclusions

- Out of Distribution learning (OOD)

How the past can be used to solve the present task

An **intelligent** agent must have the capacity to **accumulate** and  
**maintain** task proficiency across experiences

An **intelligent** agent must have the capacity to **accumulate** and  
**maintain** task proficiency across experiences

The sequence of tasks may **not be explicitly labeled**,  
tasks may **switch unpredictably**,  
and any individual task may **not recur for long time** intervals

- Curriculum learning
  - Teacher – student
  - Interferences between examples and/or learning tasks
  - Mostly interested in the performance on the final task

- Curriculum learning
  - Teacher – student
  - Interferences between examples and/or learning tasks
  - Mostly interested in the performance on the final task
- ≠ Continual learning
  - The learner is expected to be good on each task
  - and to retain its performance on all tasks

## How to use the past

---

- This question is **already present in I.I.D. learning** (classical inductive learning)

Training examples are useful only insofar as **there exists a bias**

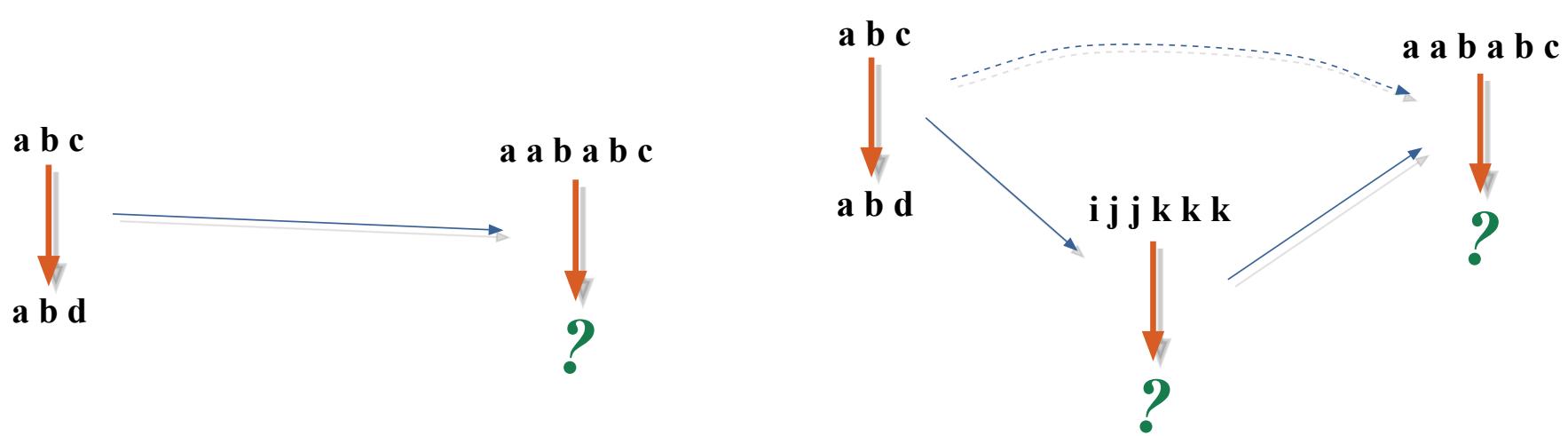
Otherwise, only **rote learning** is possible

## Sequencing effects

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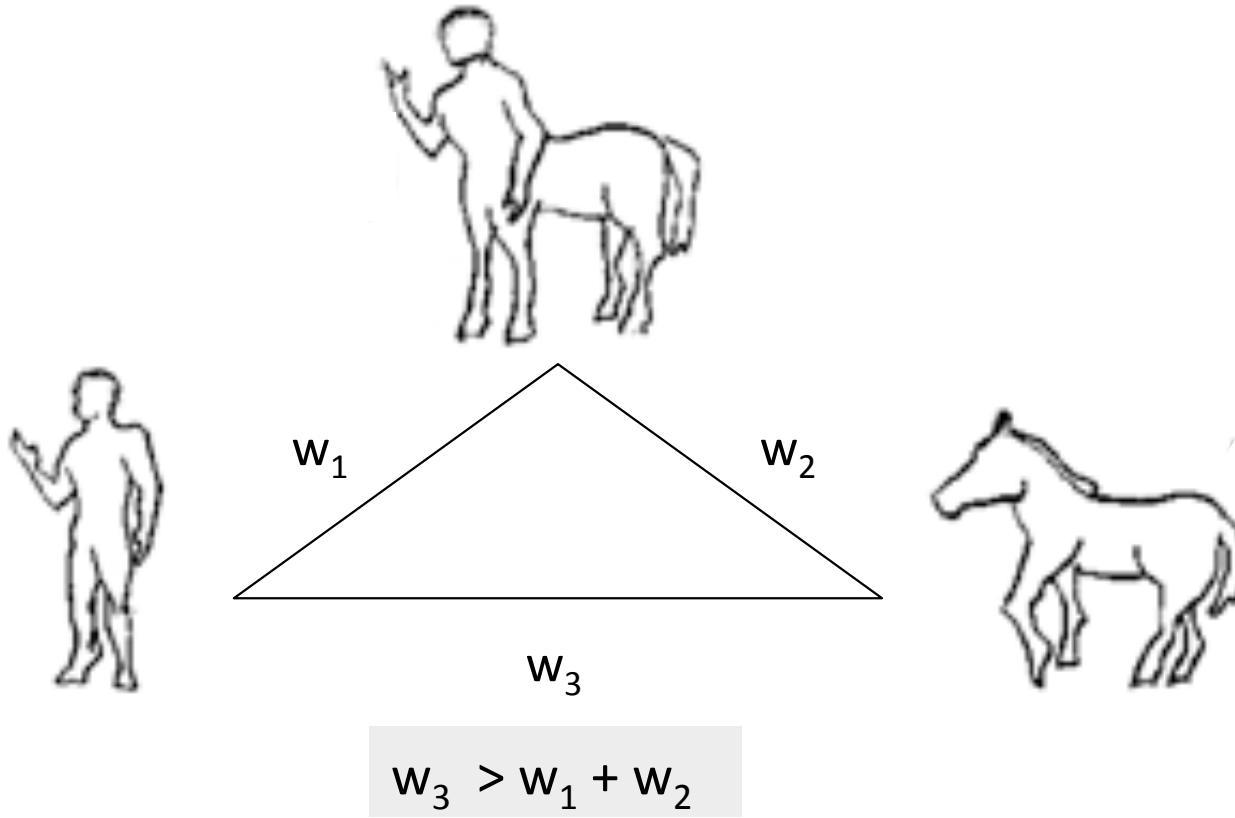
An example of **ANTI-curriculum**

# Transfer and path dependence



## Need for non-symmetrical similarity

---



Adapted from: D.W. Jacobs, D. Weinshall, and Y. Gdalyahu. Classification with non-metric distances: Image retrieval and class representation. PAMI 2000.

The **order** of the examples (learning tasks) is in itself a **bias**

# Curriculum building

# Curriculum learning

---

- Curriculum learning is a biologically-inspired learning paradigm. It is motivated by the observation that humans and animals **learn better** when they are exposed to **examples that are organized in a meaningful way**.
  - For example, the **visual system** of human infants develops gradually, focusing first on simple geometric patterns or objects with prominent contours before processing more complex patterns [Anderson et al., 2024].
  - **Works in ANNs** have also shown that the presence of an ordered set of **progressively more difficult tasks** can help a model to learn better and faster [Elman, 1993; Krueger and Dayan, 2009; Bengio et al., 2009].
    - In particular, the experiments of Bengio et al. [2009] suggest that a curriculum, defined as **an ordered set of experiences**, makes the loss function of the model **converge towards better local minima** and **reduces the number of optimization steps** needed to reach a given performance.

## Curriculum learning

---

- Curriculum strategies can be considered a **special case of transfer learning**, in which knowledge from early experiences guides the learning process for more challenging tasks.

## Incremental learning

---

- Usually when there is **no teacher** to organize the order of the training examples or tasks
- It is a particular form of continual learning

- 
- “... Unlike (statistical) machine learning, in human learning supervision is often accompanied by a **curriculum**. Thus **the order of presented examples is rarely random** when a human teacher teaches another human.
  - Likewise, the task may be divided by **the teacher** into smaller sub-tasks, a process sometimes called shaping (Krueger & Dayan, 2009) and typically studied in the context of reinforcement learning (e.g. Graves et al., 2017).
  - Although it remained for the most part in the fringes of machine learning research, **curriculum learning** has been identified as **a key challenge** for machine learning throughout.”

[Daphna Weinshall et al. (2018) « **Curriculum Learning by Transfer Learning: Theory and Experiments with Deep Networks** ». ICML-2018.]

# Curriculum learning

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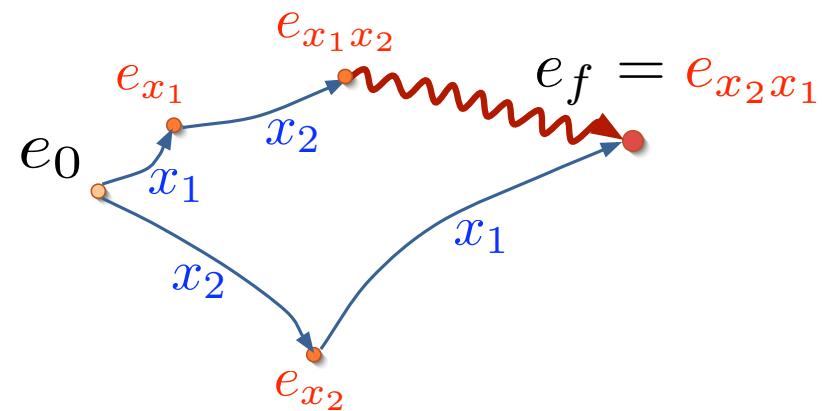
- “Humans need about two decades to be trained as fully functional adults of our society.
- That **training is highly organized**, based on **an education system** and a **curriculum** which introduces different concepts at different times, **exploiting previously learned concepts to ease the learning of new abstractions**.
- By **choosing which examples to present and in which order to present them** to the learning system, one can guide training and remarkably increase the speed at which learning can occur.”

[Joshua Bengio (2018) « **Learning deep architectures for AI** ». Now Publishers Inc, 2009.]

# Order effects

---

- How to **predict** them?
- How to **quantify** them?
- How to **formalize** them?
- How to **control** them?



## Sequencing effects

---

- How to **eliminate** them?
- How to organize them and **guide learning**?
- How to **build a curriculum** for machines?

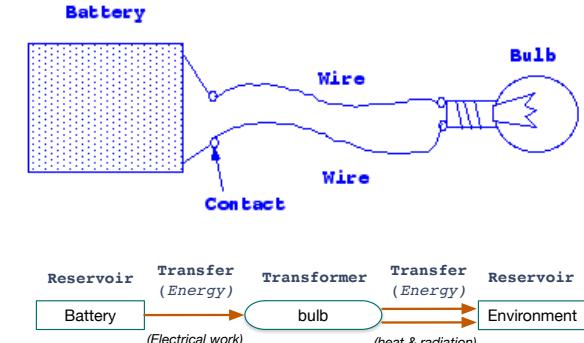
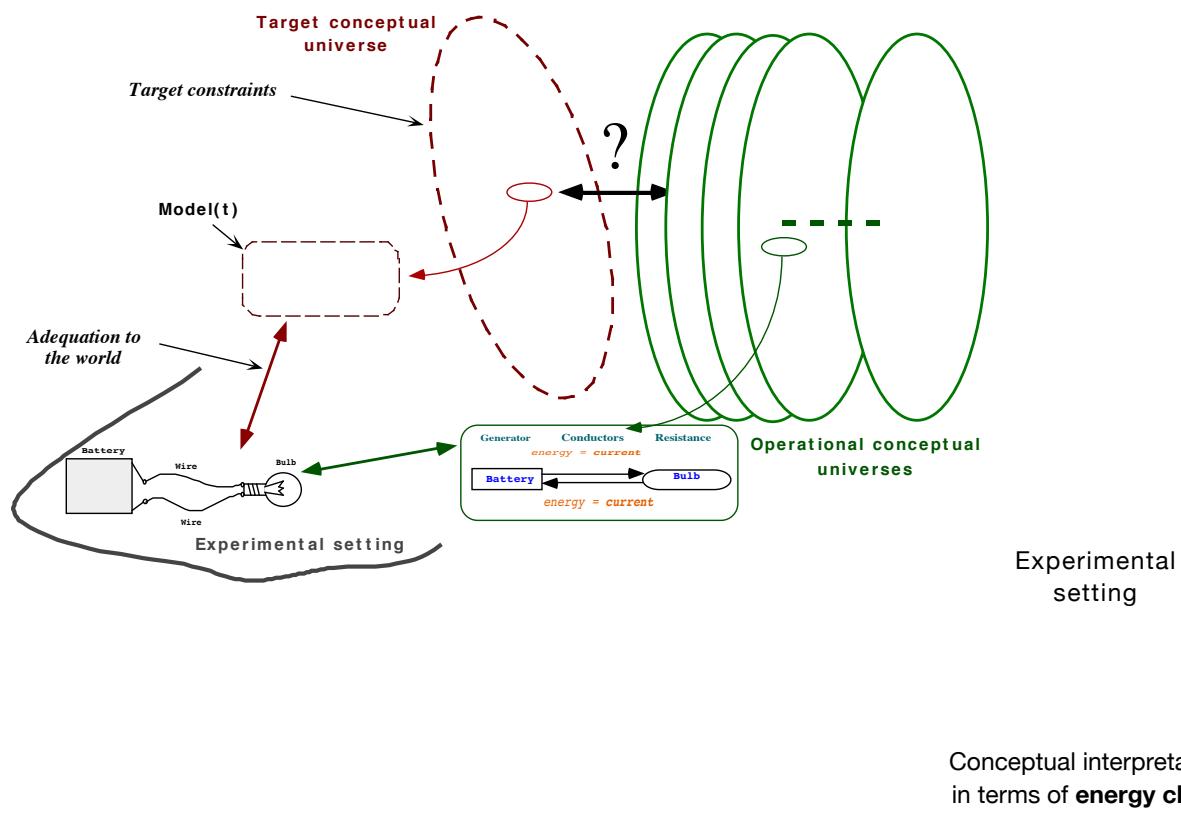
NO!

YES!

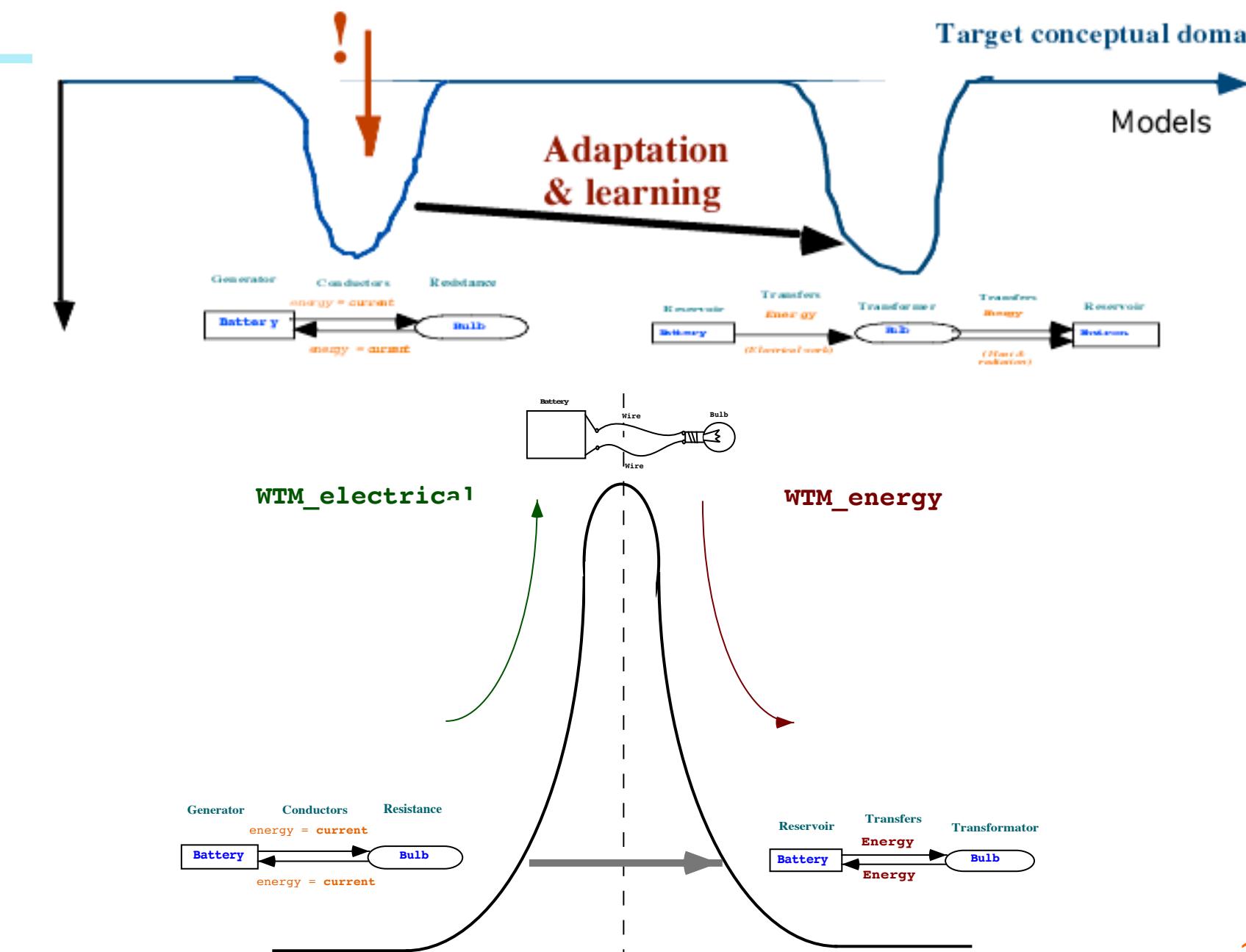


# Cognitive tunnel effect

[A. Cornuéjols, A. Tiberghien, G. Collet. *Tunnel Effects in Cognition: A new Mechanism for Scientific Discovery and Education.* Arxiv-1707.04903- Tue, 18 Jul 2017 00:00:00 GMT]



## Target conceptual domain



What did we learn so far?  
on curriculum learning

## Teacher - student

---

- E.g. Distillation
  - Change gradually the **targets**
  - Change gradually the **source data points**
  - Change gradually the **learning task** (real curriculum learning)
    - With a teacher aware of the student

# Continual Learning

---

- the ability to **learn continually from a stream of data**
  - **building on** what was learned previously
  - and being able to **remember those learnt tasks.**
- What humans are capable of, and what is also **the end goal for an artificially intelligent machine.**

# Outline

---

1. How to measure the difficulty of a training example
2. Catastrophic forgetting
3. Curriculum building
4. A geometry on the space of tasks
5. Conclusions

## A geometry on the space of tasks

- 
- We expect that transfer is **easy** when source and target tasks are “**close**”
  - And it may be **difficult** to transfer across tasks that are “**far away**”

But **how to measure** “closeness”  
and “*far away*” for learning tasks?

Define a **geometry** over the space of tasks

- 
- How to measure the difficulty of a new example?
  - Curriculum learning
  - How to best help learning?

# Outline

---

1. A reminder about O.O.D.: fundamental questions
2. Curriculum building: a geometry on the space of tasks
3. How to measure the difficulty of a training example
4. Defining a distance between tasks
5. Conclusions

## Curriculum building

And the **geometry** of the space of **learning tasks**

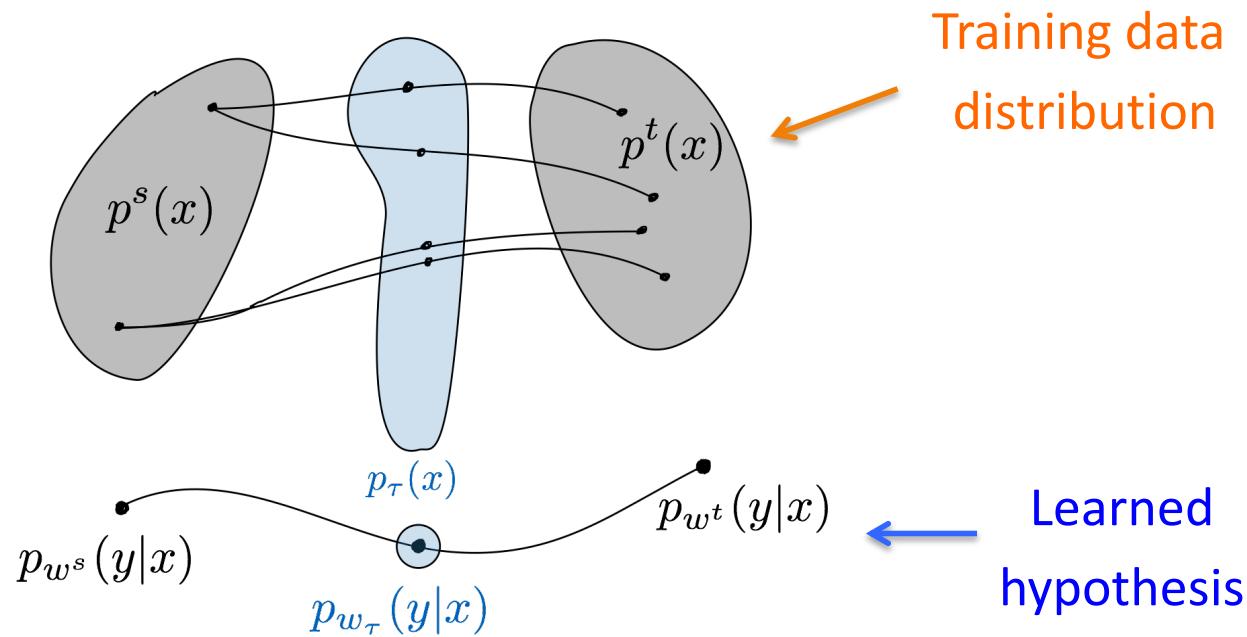
# Geometry of the space of tasks

---

- Desiderata
  - 1. Should incorporate the hypothesis space, and not only the “distance” between the inputs (as is usually done)
    - For instance, it is often observed that *transferring larger models is easier.* The geometry should reflect this.
  - 2. The distance between tasks is not symmetrical

Gao, Y., & Chaudhari, P. (2021, July). An information-geometric distance on the space of tasks. In *International Conference on Machine Learning* (pp. 3553-3563). PMLR.

# Idea

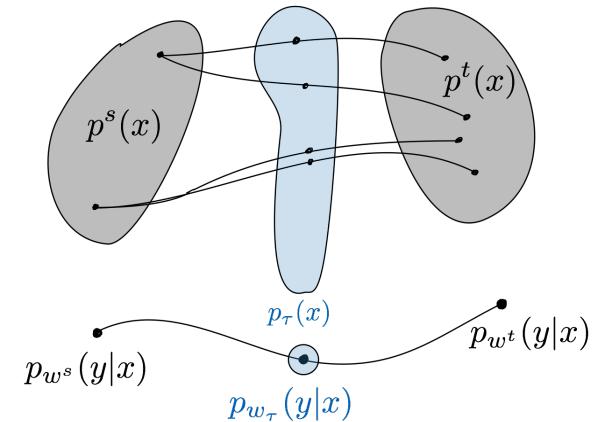


Modify **conjointly** the training data distribution and the **learned hypothesis**

Compute iteratively the intermediate training sets such that

- at each step  $\tau$  the new task is close to
- what can be learned by the current learner  
(characterized by its **current hypothesis**)

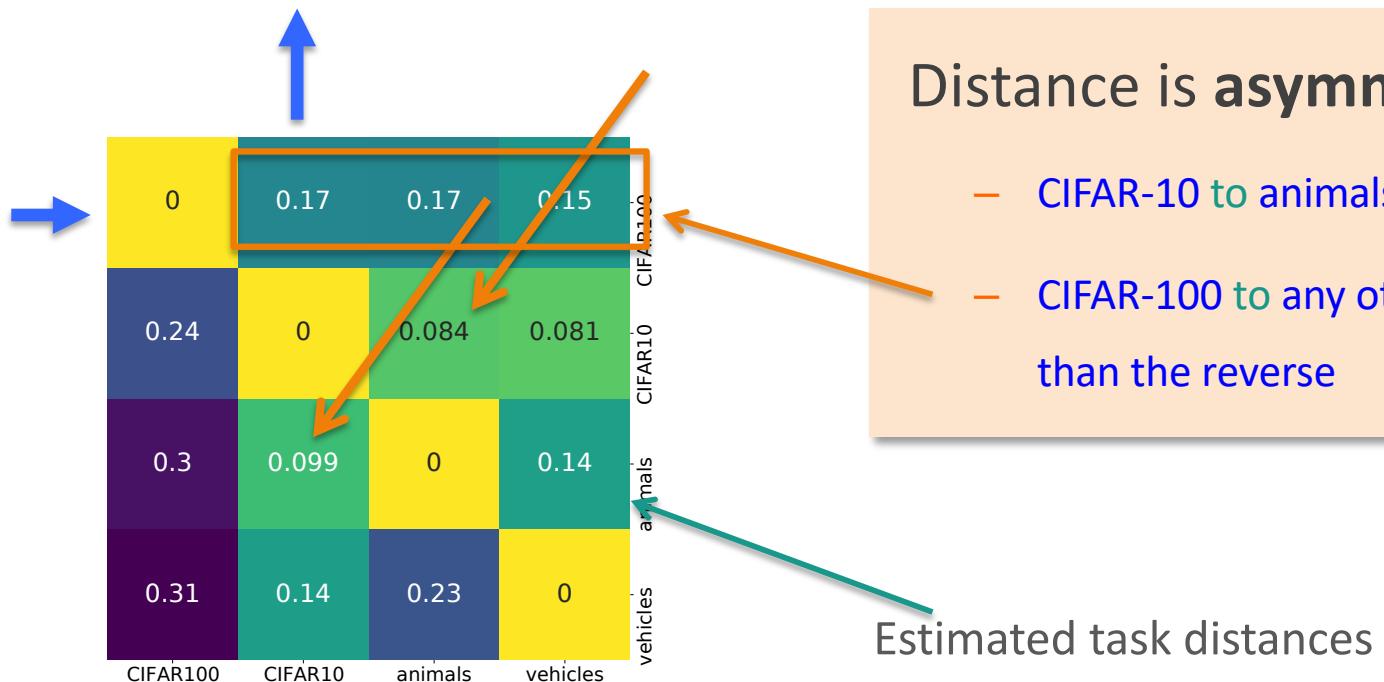
# Contributions



- They formalize a “**coupled transfer distance**” between learning tasks
  - as the **length of the shortest trajectory** on a Riemannian manifold that **the weights** of a classifier travel on when they are adapted **from the source task to the target task**.
  - At each instant during this transfer, weights are fitted on an interpolating task that evolves along the **optimal transportation (OT)** trajectory between source and target tasks.
- Given an optimal transport between tasks, the Fisher-Rao distance between the initial and final weights, which the coupled transfer distance computes, corresponds to finding a **weight trajectory** that **keeps the generalization gap small** on the interpolated tasks

# Experimental results

- Using an **8-layer convolutional NN** (ReLU, dropout, batch-normalization) with a final fully connected layer



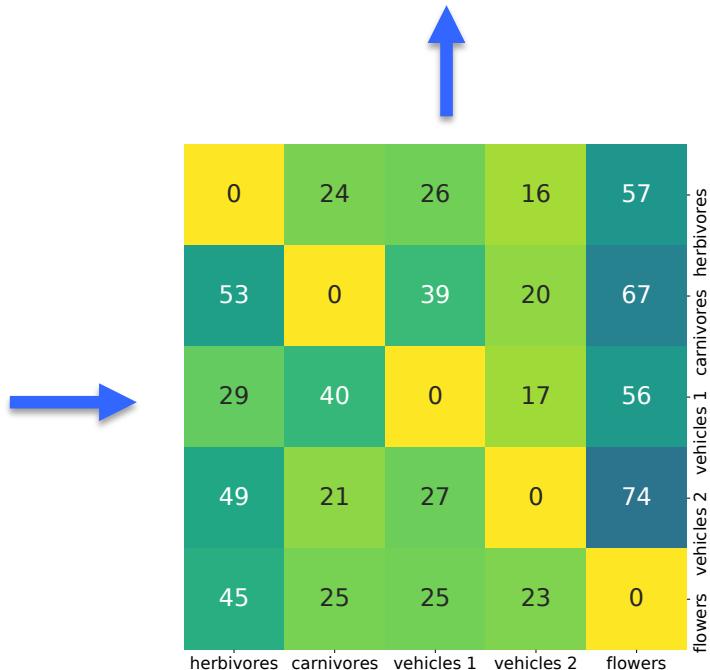
Distance is **asymmetrical**

- CIFAR-10 to animals < animals to CIFAR-10
- CIFAR-100 to any other is much easier than the reverse

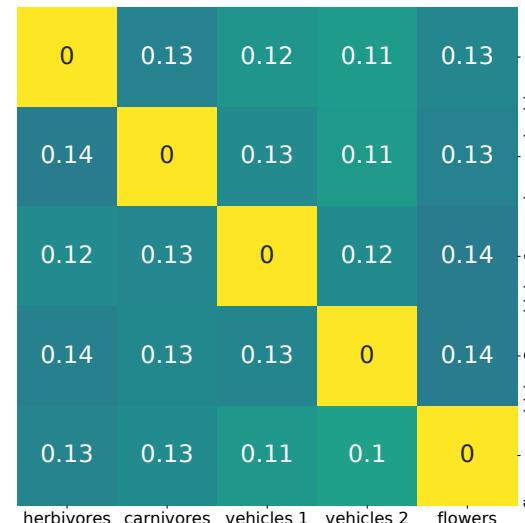
Estimated task distances

# Experimental results

- Using an **8-layer convolutional NN**



- And a **wide residual network (WRN-16-4)**: larger capacity



Distance is much **reduced**  
using a **larger capacity** model

# Conclusions

---

- Interesting work
  - New definition of **distance** between tasks
    - Asymmetrical
    - Depends on the **capacity** of the learning system
  - New way to build a **curriculum**

# Conclusions

---

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    - Depends on the **capacity** of the learning system
  - New way to build a **curriculum**
- Limits
  - Still a **crude** way to build intermediate tasks
  - **Same** input-output source and target domains!!!
    - Because the distance is based on the Kullback-Leibler divergence
  - **Same hypothesis space** in both source and target domains!!!

# Conclusions

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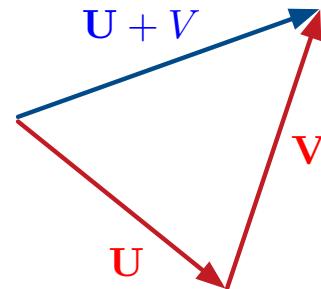
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Not **general** enough for  
transfer learning

# Parallel Transport and Covariant Derivative

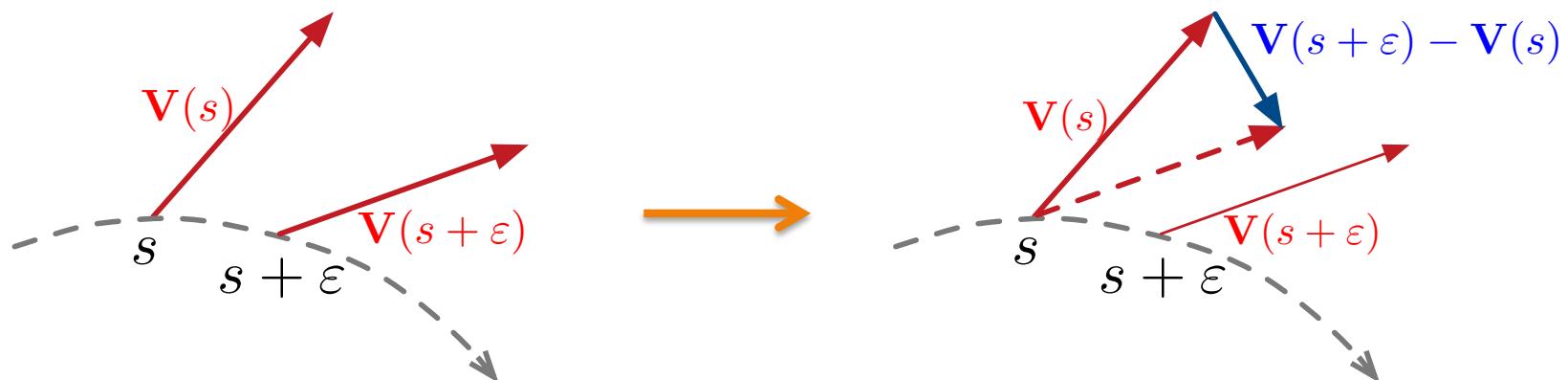
# Euclidian geometry

- **Addition of vectors**



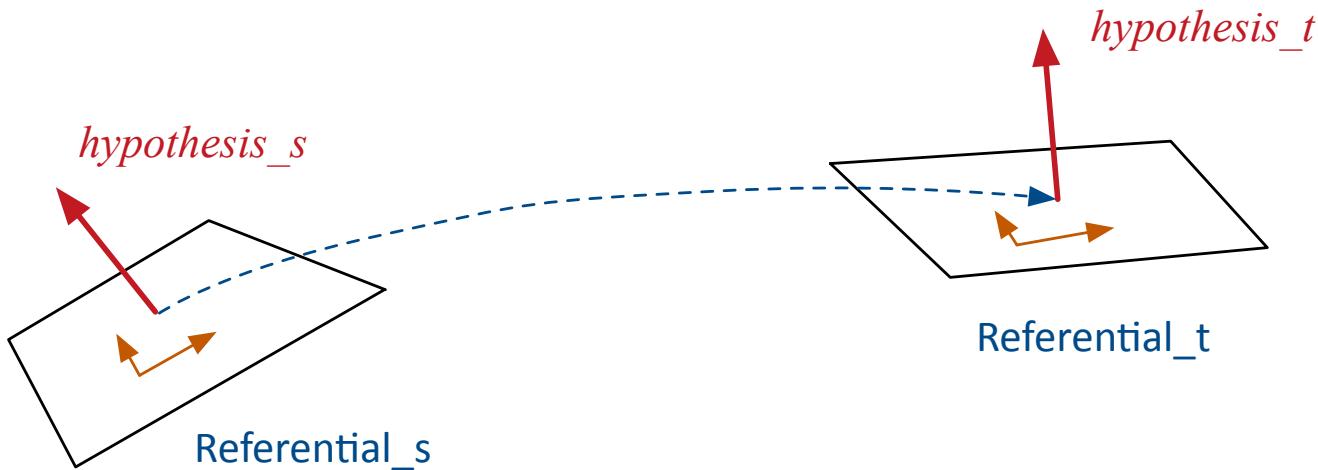
- **Substraction of vectors and derivative**

$$\frac{d\mathbf{V}}{ds} = \lim_{\varepsilon \rightarrow 0} \frac{\mathbf{V}(s + \varepsilon) - \mathbf{V}(s)}{\varepsilon}$$



# Non Euclidian geometry

- Subtraction of vectors and derivative



We can no longer directly compare vectors (or tensors)

Necessity of the covariant derivative

# Parallel transport

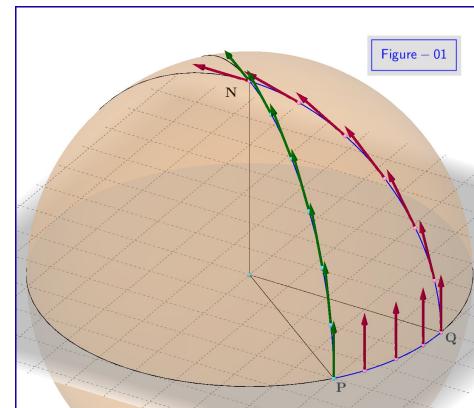
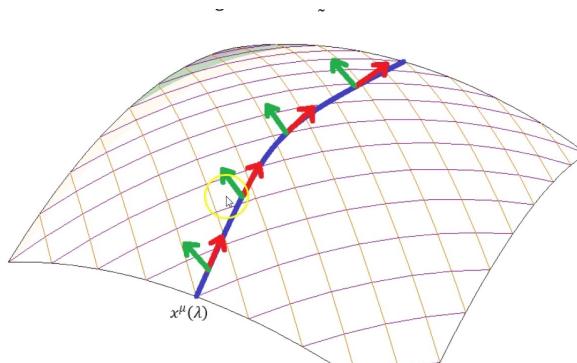
- Transport a vector (or a tensor) parallel to itself along a curve

Covariant derivative = 0

$$(\partial_k V^i)^{\text{covariant}} = \partial_k V^i + \Gamma_{jk}^i V^j$$

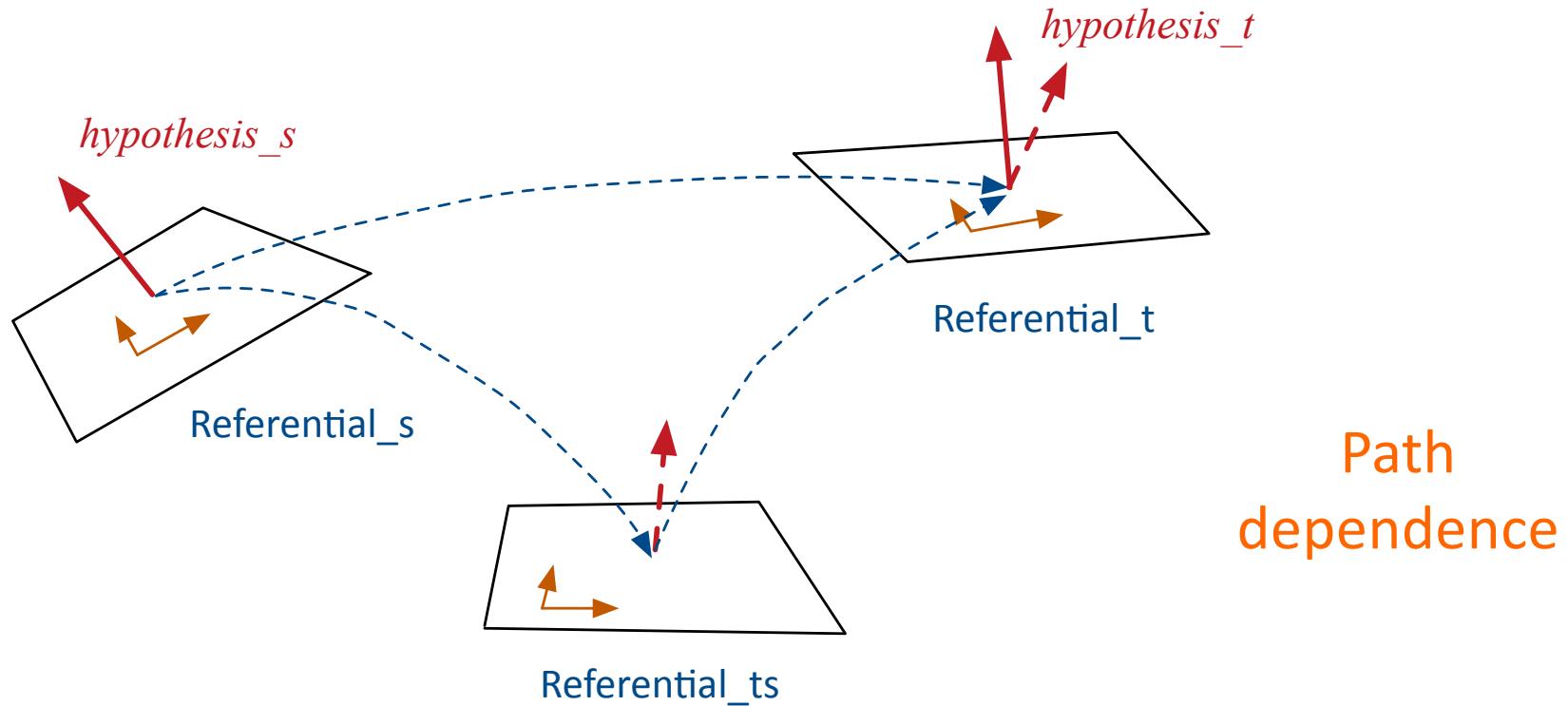
Kronecker symbol

$$V^i(x^k)^{\text{parallel transported}} = V^i(x^k) + \Gamma_{jk}^i V^j \Delta x^k$$



Path  
dependent!

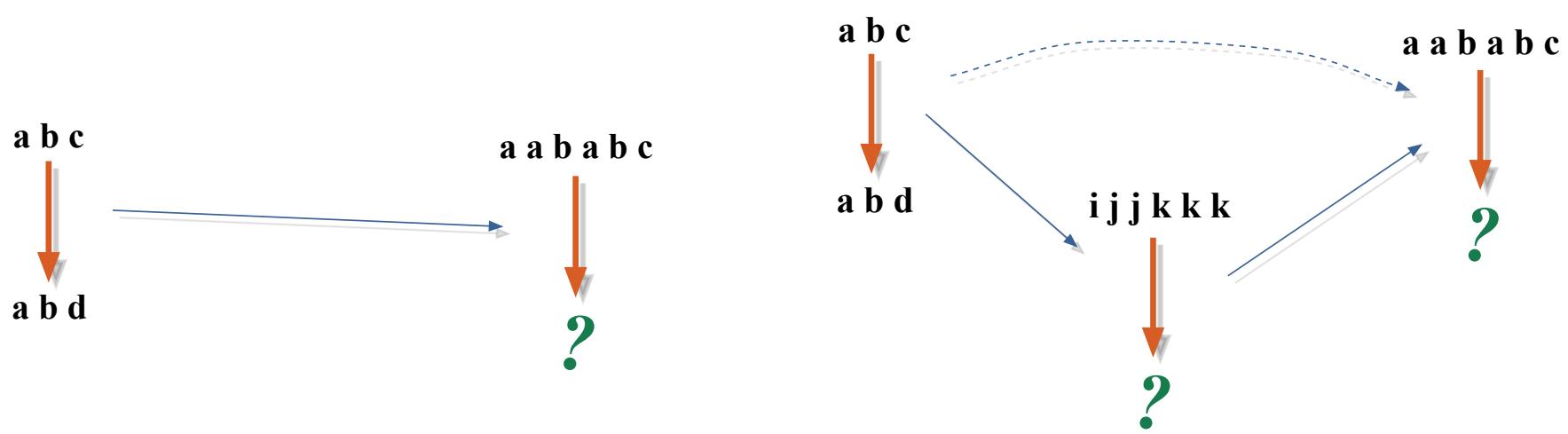
# Transfer and path dependence



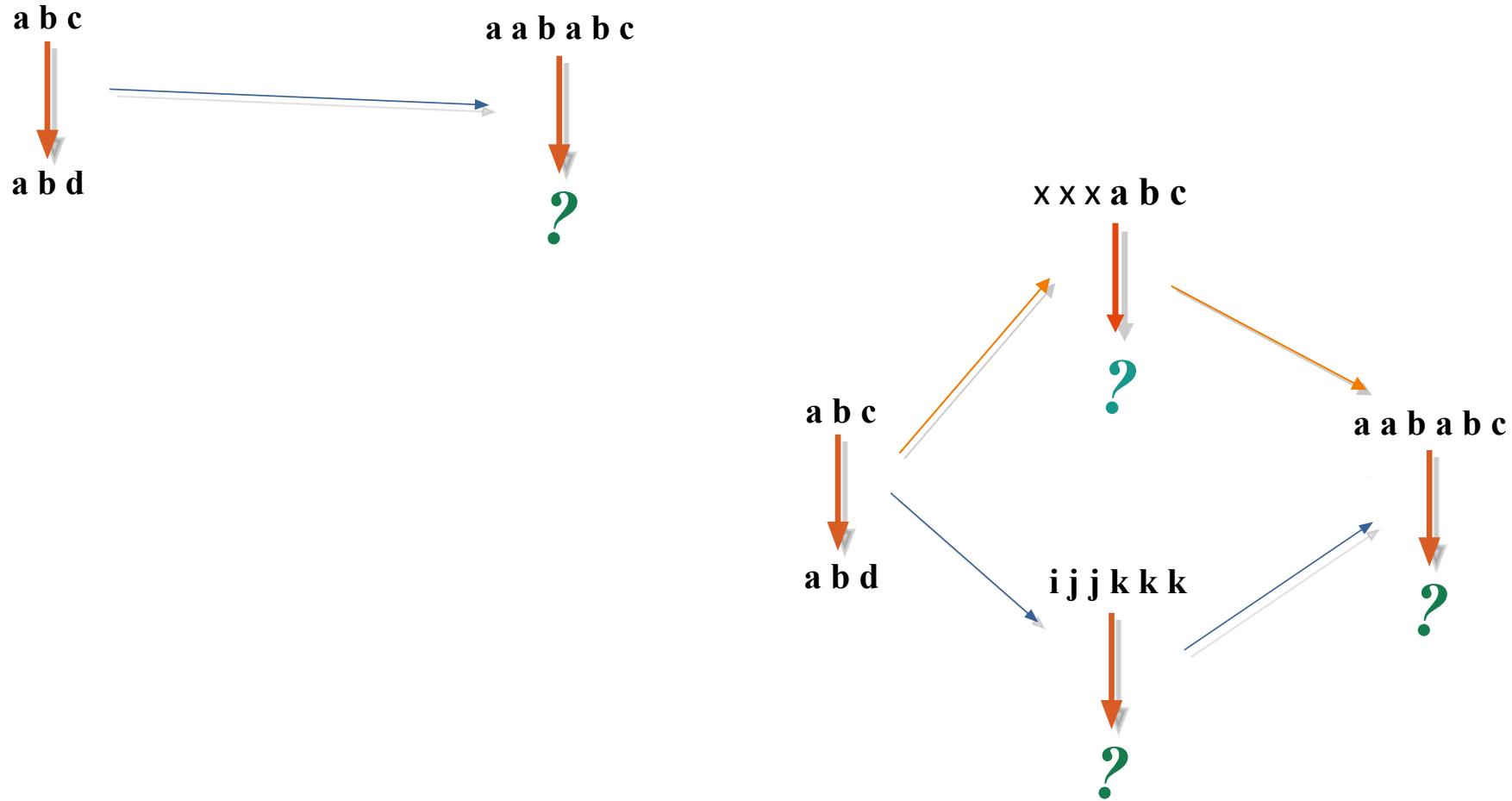
Transfer ? **Parallel transport** of hypothesis from source to target

But, how to compute it between different source and target domains?

# Transfer and path dependence



# Transfer and path dependence



# Outline

---

1. How to measure the difficulty of a training example
2. Catastrophic forgetting
3. Curriculum building
4. A geometry on the space of tasks
5. Conclusions

- 
- **Transfer** learning
    - ability to **use** what has been learned **from a previous task on a new task.**

The difference with continual learning is that transfer learning is not concerned about keeping the ability to solve previous tasks.

- **Curriculum** learning
  - a training process that proposes a **sequence of more and more difficult tasks** to a learning algorithm in order to make it able to **learn, at last**, a generally **harder task**.

The sequence of tasks is designed in order to be able to **learn the last one**.

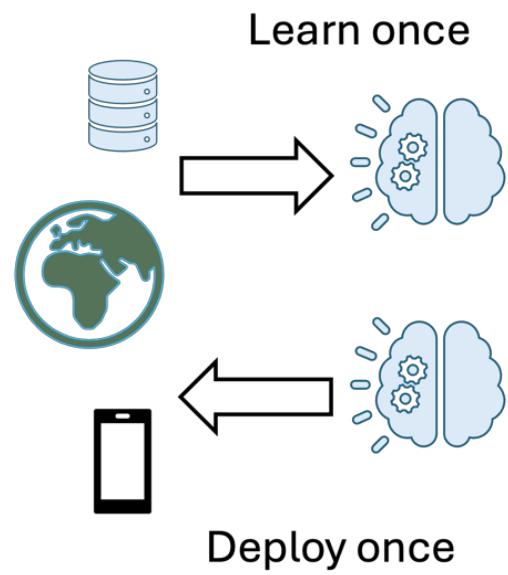
# Continual Learning

---

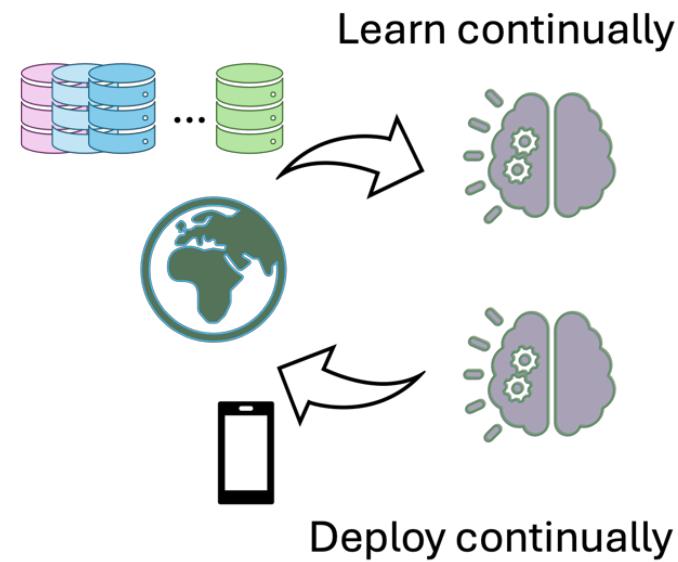
- the ability to **learn continually from a stream of data**
  - **building on** what was learned previously
  - and being able to **remember those learnt tasks.**
- What humans are capable of, and what is also **the end goal for an artificially intelligent machine.**

# LLMs and Continual learning

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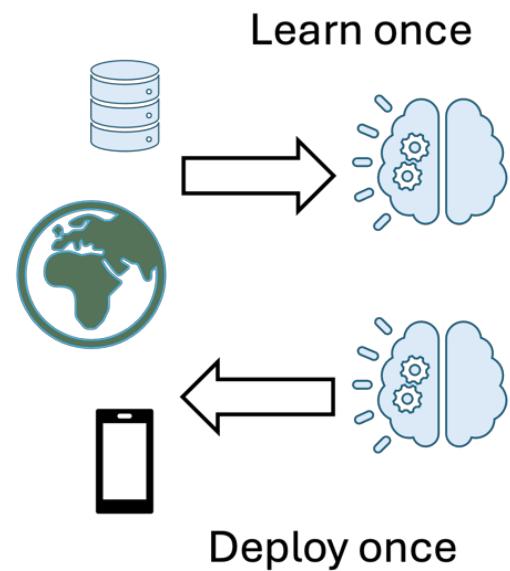


Static Machine Learning

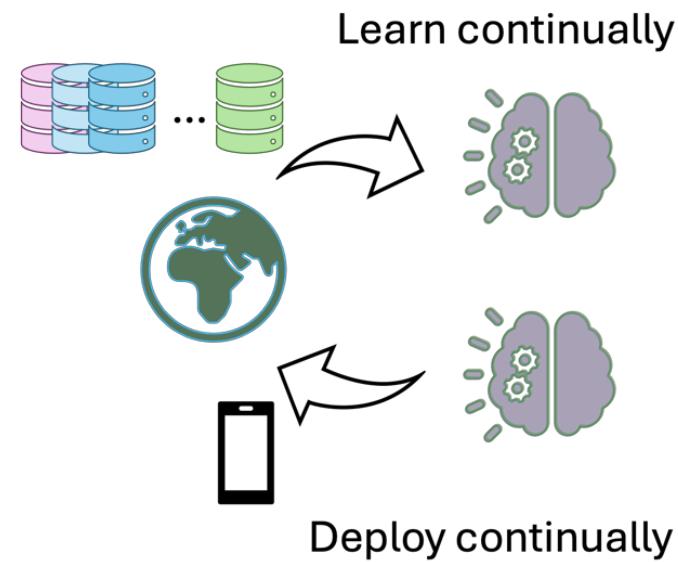


Continual Machine Learning

# LLMs and Continual learning: not solved yet



Static Machine Learning



Continual Machine Learning

## Issues

---

- In numerous cases, transfer learning works well
- But in other cases, it does not
  - A pretrained model on ImageNet leads to poor performance on MRI images [Merkow, et al. 2017]
- And we still cannot predict how transfer will fare from one learning task to another and the reasons for success or failure

# Transfer learning and curriculum learning

---

- An active and constructive viewpoint:
  - Training a system for a target task through **successive intermediate learning tasks**
  - Necessitates
    - To **identify** relevant intermediate subtasks
    - To **order** them

Curriculum learning

# Conclusions (1)

Transfer learning → mostly heuristical approaches so far

## 1. Parallel transport is a natural way for looking at transfer learning

- The covariant derivative is then a measure of difference
  - How to compute it?
    - Pioneering works in computer vision
  - What about when the source and target domains are different?
    - TransBoost: a proposal

## 2. Transfer learning is path dependent in general

- The study of these path dependencies is important ...
  - Curriculum learning
  - Longlife learning
- ... and a wide open research question

## Conclusions (2)

---

- The **theoretical guarantees** for transfer learning:
  - Do not necessarily depend on the **performance of the source hypothesis  $h_s$**   
But depend on the **bias** that  $h_s$  determines
  - **Involve** the **capacity** of the space of **transformations**  
(and the path followed between source and target)

Still to be explored



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What did we learn so far?

- Domain adaptation

- $X_S = X_T$  &  $Y_S = Y_T$  &  $P_{Y|X} = C^{te}$

- Translation

- Reweighting the training examples

- So as to put more weight on the training examples

- “close” to the target distribution

- But few test examples → Estimate target distribution?

- How to estimate the amount of reweighting?

- Domain adaptation / Transfer learning

- $(X_S = \text{or } \neq X_T) \ \& \ (Y_S = \text{or } \neq Y_T) \ \& \ (P_{Y|X} = \text{or } \neq C^{\text{te}})$
- Assumption: **something is shared** between the *source* and the *target*
  - A representation
    - E.g. First layers of NNs
  - The decision function
    - E.g. TransBoost (there are theoretical guarantees! Same as boosting)
  - Invariant representation between environments
    - E.g. Invariant Risk Minimization (IRM)
  - Causality

- **Analogy making**

Maximum relevance  
of the source

- $(X_S = \text{or } \neq X_T) \ \& \ (Y_S = \text{or } \neq Y_T) \ \& \ (P_{Y|X} \neq C^{\text{te}})$
- Identify both:
  - Good latent spaces (local representations) of the source and of the (incomplete) target
  - And an economical translation between these representations

E.g. using MDLp: Minimum Description Length principle

- In
  - **Domain adaptation**
    - $X_S = X_T$  &  $Y_S = Y_T$  &  $P_{Y|X} = C^{te}$
  - **Transfer learning**
    - $X_S \neq X_T$  and/or  $Y_S \neq Y_T$  &  $P_{Y|X} \neq C^{te}$
    - The goal is to be good on the target, not to stay good on the source
  - **Continual learning** (incremental learning)
    - $X_S = X_T$  &  $Y_S = Y_T$  &  $P_{Y|X} \neq C^{te}$
    - The goal is to be good on the target AND to stay good on the source
  - **On line learning**
    - $X_S = X_T$  &  $Y_S = Y_T$  &  $P_{Y|X} \neq C^{te}$
    - The goal is to be good at each time step

The source is part of the bias

# Questions

- In which cases is the **source useful**?
  - What are the conditions
    - On the **source**?
    - On the **relationships** between the source and the target?
      - Kullback-Leibler ; MDLp ; boosting of weak translators ; ...
- Can we obtain **theoretical guarantees**?
  - As in PAC learning
- **What should be transmitted** from the source to the target?
  - A **representation**?
    - Common to source and target (e.g. as in deep NNs)
    - Changed (e.g. as in analogy making)
  - A **decision function**? (e.g. as in TransBoost)
  - ...
- **How** should the transmission be done?
  - Reweighting
  - translation
  - ...

$h_s$  does **not** have to be good  
on the source

$h_s$  must act as a **good bias**

How to measure this before hand?  
(Same problem already in I.I.D.  
learning!)