Introduction Transfer Learning (2)

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(Recap) UDA : Definition

Definition 1 (Unsupervised Domain Adaptation (UDA))

Let's consider a feature space $\mathcal{X}$ and a label space $\mathcal{Y}$. We define $P_S(X, Y)$ and $P_T(X, Y)$ the respective source and target joint distributions over $\mathcal{X} \times \mathcal{Y}$.

We call **Unsupervised Domain Adaption** the learning setting composed of :

- A source **labeled** sample $\mathcal{S} = \{(x_1, y_1), ..., (x_m, y_m)\} \in \mathcal{X} \times \mathcal{Y}$ where $x_i \sim P_S(X)$ and $y_i \sim P_S(Y|X = x_i)$
- A target **unlabeled** sample $\mathcal{T}_{\mathcal{X}} = \{x'_1, ..., x'_m\} \in \mathcal{X}$ where $x'_j \sim P_T(X)$

Feature-based approach

Let Φ be a space of feature transformation : $\phi \in \Phi, \phi : \mathcal{X} \rightarrow \mathcal{X}$.
Feature-based approaches assume that there exist $\phi \in \Phi$ such that :

$$P_T(\phi(X), Y) = P_S(\phi(X), Y) \ \ \text{(symetric)} \tag{1}$$

or

$$P_T(\phi(X), Y) = P_S(X, Y) \ \ \text{(assymetric)} \tag{2}$$

UDA Feature-based

A UDA feature-based approach consist in solving the following optimization problems :

$$\widehat{\phi^*} = \underset{\phi \in \Phi}{\text{argmin}}\, D(\phi(\mathcal{S}_\mathcal{X}), \phi(\mathcal{T}_\mathcal{X}))$$

$$\widehat{h^*} = \underset{h \in \mathcal{H}}{\text{argmin}} \sum_{(x_i, y_i) \in \mathcal{S}} \ell(h(\widehat{\phi^*}(x_i)), y_i)$$

With D a discrepancy measure between distributions.

Feature-based Examples

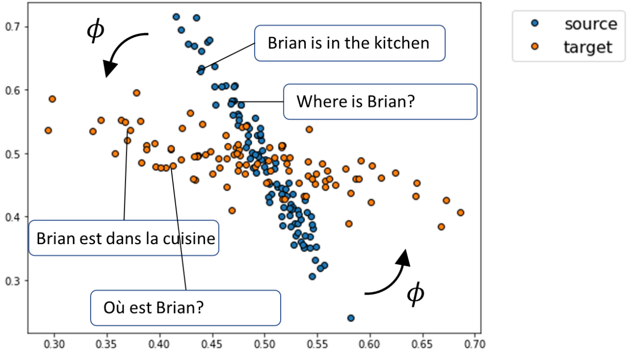


Figure – **Rotation** In this example, the space of feature transformation is $\Phi = \{x \to Mx^T; M \in \mathbb{R}^{p \times p}, M^T M = \mathrm{Id}_p\}$ and is used to match a dataset of english sentences to a dataset of french sentences.

Feature-based Examples

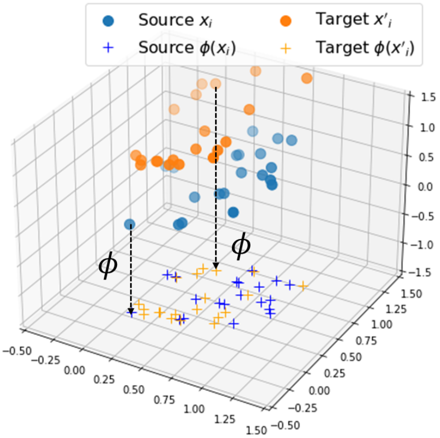


Figure – **Projection** In this example, the space of feature transformation is $\Phi = \{x \to Px^T; P \in \text{Proj}(\mathbb{R}^{p \times p})\}$

Feature-based Examples

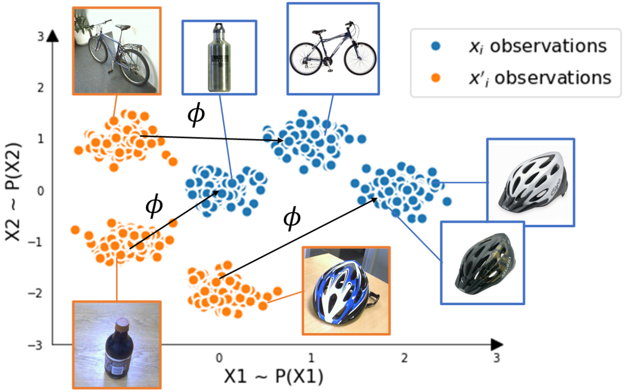
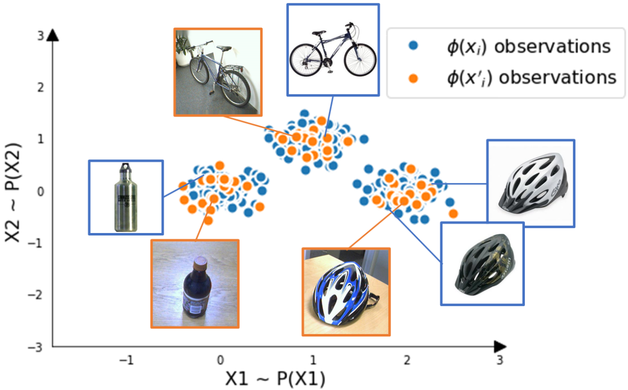


Figure – **Continuous Transformation** In this example, the space of feature transformation is $\Phi = \mathcal{C}_0(\mathcal{X})$ and is used to match a dataset of pictures from Amazon to a dataset of webcam pictures.

Feature-based Examples



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Feature-based approach : Subspace Alignment

Subspace Alignment (SA)

The purpose of Subspace Alignment [Fernando et al., 2013] is to find a linear transformation of the source PCA eigenvectors which match as close as possible the target PCA eigenvectors :

$$\widehat{\phi_T^*} = x \to xW_T^d$$
$$\widehat{\phi_S^*} = x \to xW_S^d M^*$$
$$M^* = \underset{M \in \mathbb{R}^{d\times d}}{\operatorname{argmin}} ||W_S^d M - W_T^d||_2^2$$

Where W_T^d and W_S^d are respectively the matrixes of the d first eigenvectors of the target and source PCA.

Subspace Alignment Example

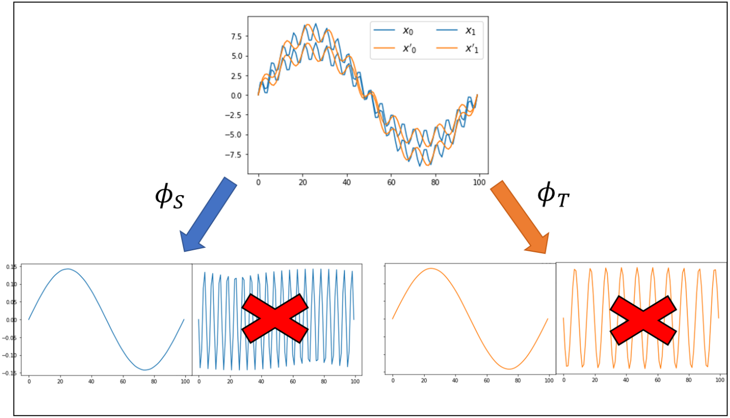

Figure – **Subspace Alignment** In this example, the source dataset is composed of signals : $t \to 5X_1 \sin(2\pi t) + X_2 \sin(40\pi t)$ and the target dataset of signals : $t \to 5X_1 \sin(2\pi t) + X_2 \sin(20\pi t)$ $(X_1, X_2 \sim \mathcal{U}([0, 2]))$

Correlation Alignment (CORAL)

The purpose of Correlation Alignment [Sun et al., 2016] is to find a linear transformation of the source data which covariance matrix match as cloas as possible the target data covariance matrix :

$$\widehat{\phi_S^*} = x \to xA^*$$

$$A^* = \underset{A \in \mathbb{R}^{p \times p}}{\operatorname{argmin}} ||A^T C_S A - C_T||_2^2$$

Where C_T and C_S are respectively the covariance matrixes of the target and source dataset.

Correlation Alignment Example

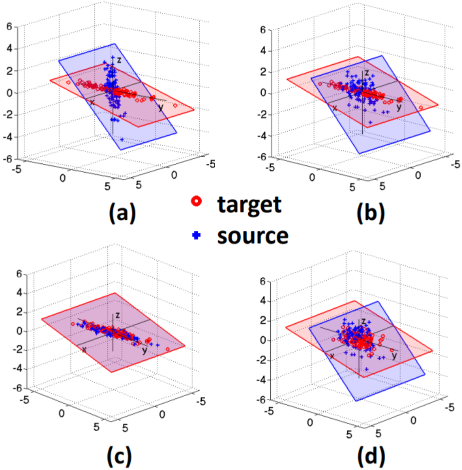

Figure – **Correlation Alignment** (source [Sun et al., 2016]) (a) : initial situation, (b) : source data are "whitened", (c) : source data are "re-colored" with the target covariance.

Feature-based approach : Optimal Transport

Optimal Transport for Domain Adaptation (OTDA)

The purpose of OTDA [Courty et al., 2016] is to find the optimal transportation from the source data to the target data :

$$\widehat{\phi_S^*} = x \to \sum_{x' \in \mathcal{T}} \gamma^*(x, x') x'$$

$$\gamma^* = \underset{\gamma \in \Gamma}{\text{argmin}} \sum_{x \in \mathcal{S}} \sum_{x' \in \mathcal{T}} \gamma(x, x') ||x - x'||_2^2$$

Where, for any $\gamma \in \Gamma$, $\gamma : \mathcal{S} \times \mathcal{T} \to [0, 1]$ and :

$$\sum_{x \in \mathcal{S}} \gamma(x, x') = 1 \ \forall x' \in \mathcal{T}$$

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OTDA Example

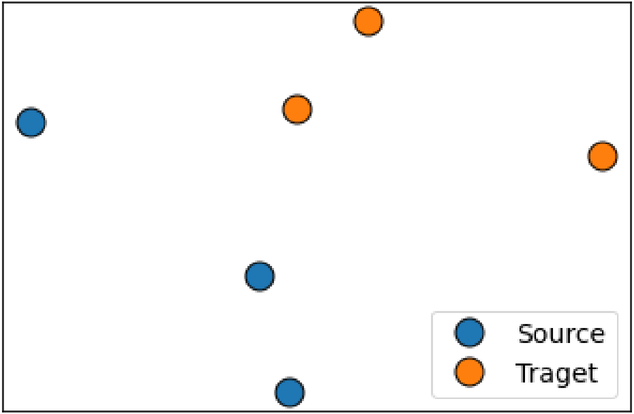

Figure – **OTDA** : In this example, we are looking at the optimal pairing between three source and target data points. The optimal pairing minimizes the sum of distances between paired data points.

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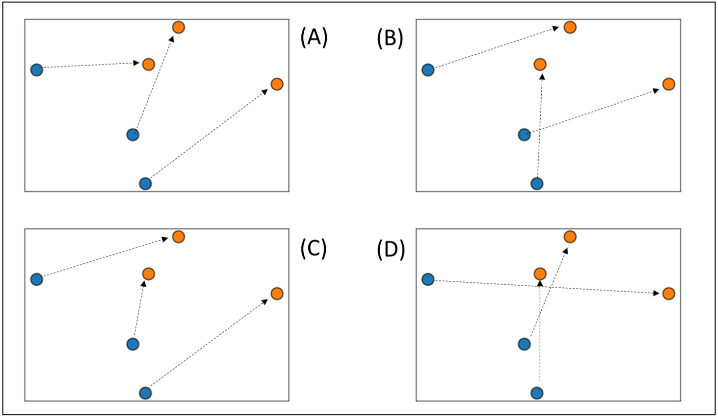

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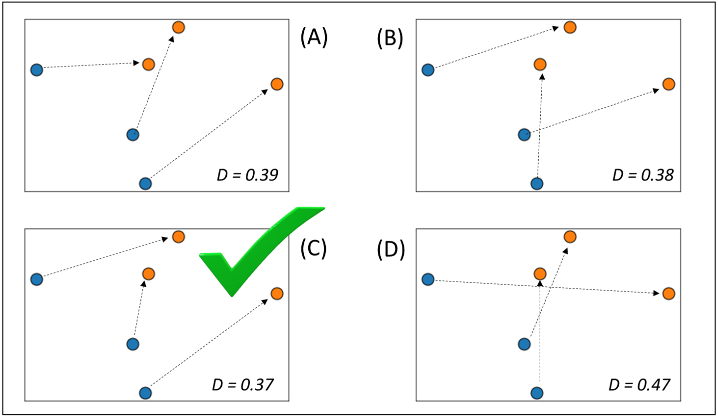


Figure – **OTDA** : The optimal pairing is the pairing (C). Finding the optimal pairing for large datasets is often intractable, we are then looking at approximated solution.

Outline

1. Feature-based Transfer Learning
2. Parameter-based Transfer Learning
3. Conclusion

Regularization Transfer LR

The purpose of Regularization Transfer LR [Chelba et al., 2007] is to find the target linear regression coefficients that fit target data while being "close" to the source coefficients :

$$\widehat{\beta^*_S} = \underset{\beta \in \mathbb{R}^p}{\operatorname{argmin}} ||X_S \beta^T - Y_S||_2^2$$

$$\widehat{\beta^*_T} = \underset{\beta \in \mathbb{R}^p}{\operatorname{argmin}} ||X_T \beta^T - Y_T||_2^2 + \lambda ||\beta - \beta_S||_2^2$$

With X_S, Y_S and X_T, Y_T the source and target input data and labels.

Exercise : Transfer Learning with Linear Regression

Let's consider $X_T \in \mathbb{R}^{n\times p}$, $Y_T = \mathbb{R}^{n\times 1}$ and $\beta_S \in \mathbb{R}^p$. Find a close-form solution to the problem :

$$\widehat{\beta_T^*} = \underset{\beta\in\mathbb{R}^p}{\operatorname{argmin}} ||X_T\beta^T - Y_T||_2^2 + \lambda||\beta - \beta_S||_2^2$$

Hint : Write $||.||_2^2$ as a scalar product $\langle ., . \rangle$ and find the vector $D(\beta)$ such that :

$$G(\beta + h) - G(\beta) = \langle D(\beta), h\rangle + o(h)$$

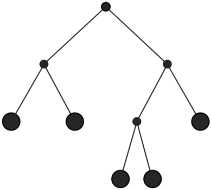
With $G(\beta) = ||X_T\beta^T - Y_T||_2^2 + \lambda||\beta - \beta_S||_2^2$ and $h \in \mathbb{R}^p$

The solution β_T^* verifies $D(\beta_T^*) = 0$

(Reminder) Decision Trees and Random Forests

[Segev et al., 2017]

Structure Expansion Reduction (SER)



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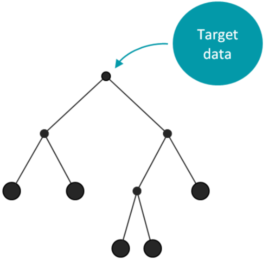

Supervised Transfer Learning on Decision Tree

[Segev et al., 2017]

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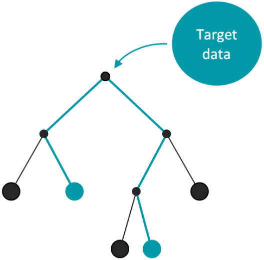

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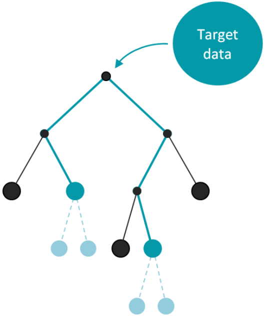

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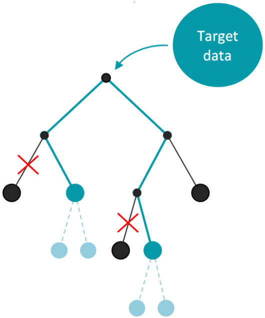

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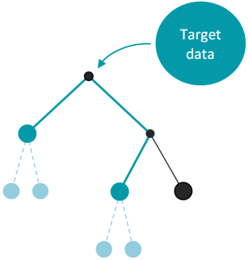

Supervised Transfer Learning on Decision Tree

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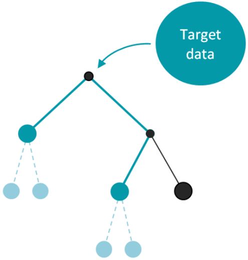
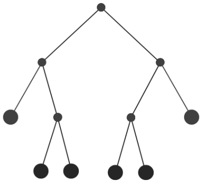
Structure Expansion Reduction (SER) Structure Transfer (STRUT)





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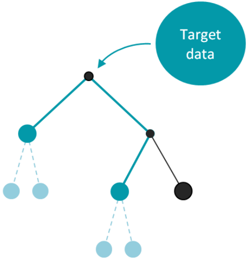

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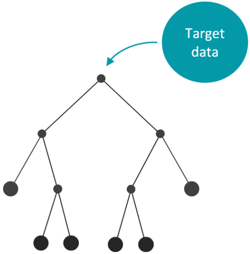

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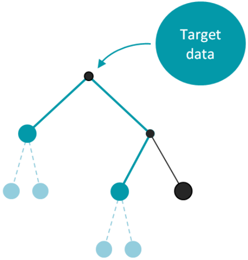

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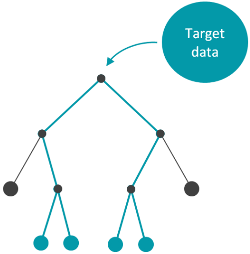

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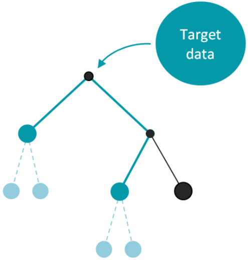

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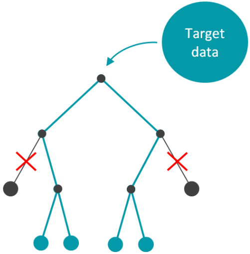

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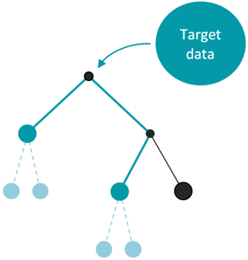

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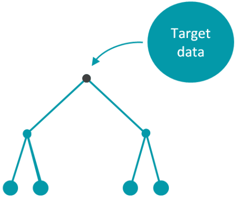

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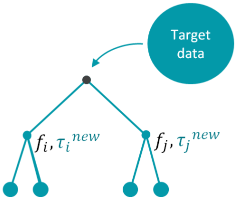

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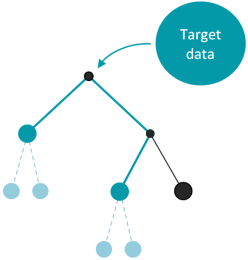
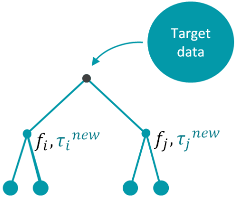


Partition refinement/simplification

Structure Transfer (STRUT)

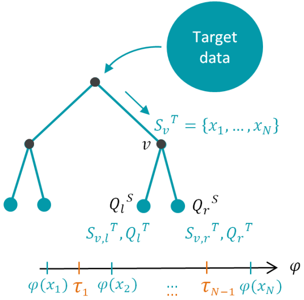


Local Drifts / Squeezes and Stretches

STRUT divergence gain optimization

Try to keep split left/right target distributions as close as possible to source ones by updating the threshold.

STRUT node update



Q_l^S, Q_r^S : class proportions of source data in children w.r.t. the **original** split.

$S_{v,l}^T$, $S_{v,r}^T$: subsets of S_v^T that fall in the children nodes of v.

$Q_l^T(\tau)$, $Q_r^T(\tau)$: class proportions of target data in children w.r.t. the **new** split.

Goal : Maximize DG while being in a local maximum of Information Gain (IG) (here IG = Gini gain).

DG optimization

$$\tau_m = \underset{\tau \in \mathrm{T}_v}{\operatorname{argmax}} \left(DG\left(Q_l^S, Q_r^S, Q_l^T(\tau), Q_r^T(\tau) \right) \right)$$

$$\text{s.t. } IG(\tau_{m-1}) < IG(\tau_m) \text{ and } IG(\tau_m) > IG(\tau_{m+1})$$

Divergence Gain :

$$DG(\tau) = 1 - \frac{|S_{v,l}^T(\tau)|}{|S_v^T|} J(Q_l^S, Q_l^T(\tau)) - \frac{|S_{v,r}^T(\tau)|}{|S_v^T|} J(Q_r^S, Q_r^T(\tau))$$

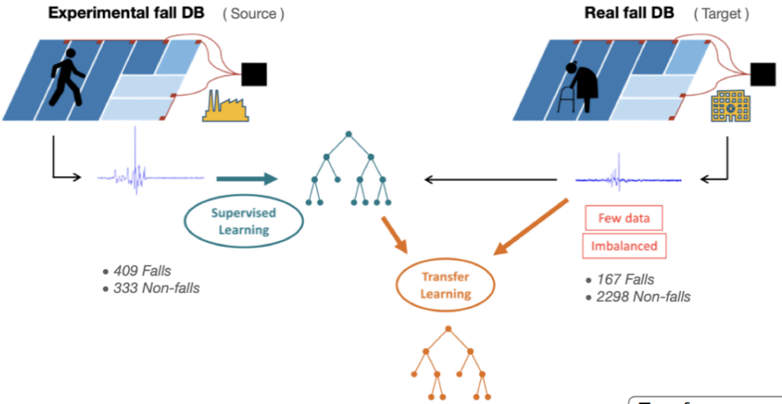
Jensen-Shannon divergence :

$$J(P, Q) = \frac{1}{2}\left(KL(P||M) + KL(Q||M)\right), \text{ with } M = \frac{1}{2}(P + Q)$$

Kullback-Leibler divergence :

$$KL(P||Q) = \sum_{k=1}^{K} P_k \ln\left(\frac{P_k}{Q_k}\right)$$

Transfer learning with Class Imbalance (Fall detection)



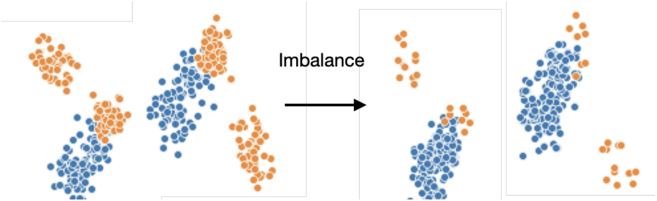
Source domain : $\mathcal{D}_S = \{\mathcal{X}_S, P^S(X_S)\}$
Target domain : $\mathcal{D}_T = \{\mathcal{X}_T, P^T(X_T)\}$
Source task : $\mathcal{T}_S = \{\mathcal{Y}_S, P^S(Y_S/X_S)\}$
Target task : $\mathcal{T}_T = \{\mathcal{Y}_T, P^T(Y_T/X_T)\}$

$\mathcal{X}_S = \mathcal{X}_T, \mathcal{Y}_S = \mathcal{Y}_T$
$P^S(X_S) \neq P^T(X_T)$
$P^S(Y_S/X_S) \neq P^T(Y_T/X_T)$

Transfer context :
Homogeneous
Inductive
Model-based

Transfer Learning with Class Imbalance

Class	Src	Targ	Ser	Strut
Maj class	10.9	6.3	13.8	4.8
Min class	10.7	**5.0**	**7.0**	**2.2**



SER and *STRUT* are not suited for target class imbalance.

Target shift

Conditional distribution for a given label y stays the same but the overall proportion of each label changes between source and target. $\forall y \in \mathcal{Y}$,

- $P_T(Y = y) \neq P_S(Y = y)$
- $P_T(X|Y = y) = P_S(X|Y = y)$

Comparably to **covariate shift**, the **density ratio** can be expressed depending on Y only ($w(X, Y) = w(Y)$).
We can deduce $P_T(Y|X)$ from $P_S(Y|X)$ and $w(Y)$.

$$
\begin{aligned}
P_T(X, Y) \quad &= P_T(X|Y)P_T(Y) = w(Y)P_S(Y)P_S(X|Y) = w(Y)P_S(X|Y) \\
P_T(Y|X) \quad &= \frac{w(Y)P_S(X, Y)}{P_T(X)} = \frac{w(Y)P_S(X, Y)}{\sum_{k=1}^{K} w(y_k)P_S(X, Y = y_k)} = \frac{w(Y)P_S(Y|X)P_S(X)}{\sum_{k=1}^{K} w(y_k)P_S(Y = y_k|X)P_S(X)} \\
&= \frac{w(Y)P_S(Y|X)}{\sum_{k=1}^{K} w(y_k)P_S(Y = y_k|X)}
\end{aligned}
$$

Transfer Learning with Class Imbalance

- What is the "simplest" target class imbalance situation ?
  - How to quantify the risk of pruning relevant leaves ?

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Homogeneous class imbalance :

$$p^T(x|y) = p^S(x|y) \tag{3}$$

$$p^T(y = k|x) = \lambda_k \frac{p^S(y = k|x)}{\sum_{j=1}^{K} \lambda_j p^S(y = j|x)}, \text{with} \quad \lambda_k = \frac{P^T(y = k)}{P^S(y = k)} \tag{4}$$

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Pruning risk :

Goal : *Estimate the risk of loosing a source leaf l of minority class k_m with n_m target data.*
Minority class leaves that should stay unchanged :

$$\forall k \neq k_m, p^T(y=k_m|x \in l) > p^T(y=k|x \in l)$$

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$$\boxed{\overset{(4)}{\Longrightarrow} \quad \lambda_{k_m} p^S(y=k_m|x \in l) > \lambda_k p^S(y=k|x \in l)}$$

Risk of leaf l being unreached by n_m minority class target data :

$$PRR_{n_m}(l) \overset{def}{=} p^T(\text{No data among } n_m \text{ of class } k_m \text{ class reaching } l) = p^T(x \notin l|y=k_m)^{n_m}$$

Transfer Learning with Class Imbalance

- What is the "simplest" target class imbalance situation ?
  - How to quantify the risk of pruning relevant leaves ?

Homogeneous class imbalance :

$$p^T(x|y) = p^S(x|y) \tag{3}$$

$$p^T(y=k|x) = \lambda_k \frac{p^S(y=k|x)}{\sum_{j=1}^{K} \lambda_j p^S(y=j|x)}, \text{with} \quad \lambda_k = \frac{P^T(y=k)}{P^S(y=k)} \tag{4}$$

Pruning risk :

Goal : *Estimate the risk of loosing a source leaf l of minority class k_m with n_m target data.*
Minority class leaves that should stay unchanged :

$$\forall k \neq k_m, p^T(y=k_m|x \in l) > p^T(y=k|x \in l)$$

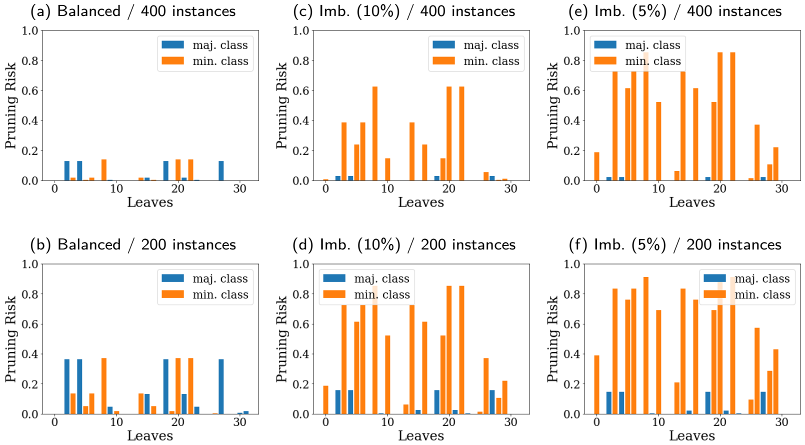
$$\boxed{\overset{(4)}{\Longrightarrow} \quad \lambda_{k_m} p^S(y=k_m|x \in l) > \lambda_k p^S(y=k|x \in l)}$$

Risk of leaf l being unreached by n_m minority class target data :

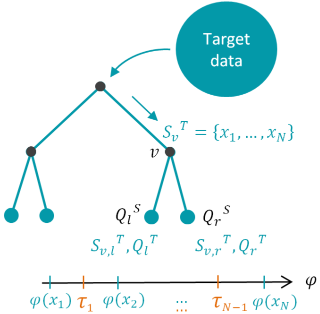
$$PRR_{n_m}(l) \overset{def}{=} p^T(\text{No data among } n_m \text{ of class } k_m \text{ class reaching } l) = p^T(x \notin l|y=k_m)^{n_m}$$

$$\boxed{\overset{(3)}{\Longrightarrow} \quad PRR_{n_m}(l) = p^S(x \notin l|y=k_m)^{n_m}}$$

The pruning risk increases rapidly with class imbalance and data rarity.



STRUT node update



Q_l^S, Q_r^S : class proportions of source data in children w.r.t. the **original** split.

$S_{v,l}^T$, $S_{v,r}^T$: subsets of S_v^T that fall in the children nodes of v.

$Q_l^T(\tau)$, $Q_r^T(\tau)$: class proportions of target data in children w.r.t. the **new** split.

DG optimization

$$\tau_m = \operatorname*{argmax}_{\tau \in \mathrm{T}_v} \left(DG \left(Q_l^S, Q_r^S, Q_l^T(\tau), Q_r^T(\tau) \right) \right)$$

s.t. $IG(\tau_{m-1}) < IG(\tau_m)$ and $IG(\tau_m) > IG(\tau_{m+1})$

STRUT(λ)

Use of equation (2) :

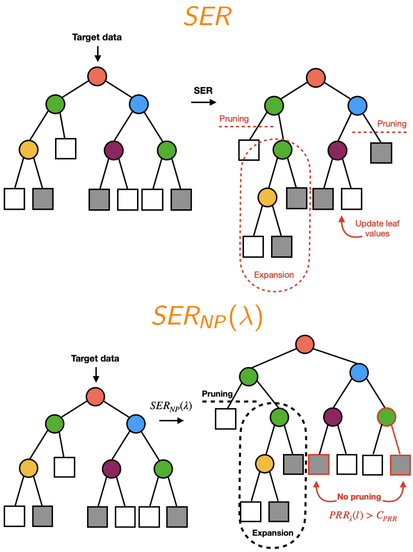
$$p^T(y = k|x) = \lambda_k \frac{p^S(y = k|x)}{\sum_{j=1}^{K} \lambda_j p^S(y = j|x)}$$

to change the source class proportions in DG :

$$Q_{l,k}^{S'} = \lambda_k \frac{Q_{l,k}^S}{\sum_k \lambda_j Q_{l,j}^S} \qquad Q_{r,k}^{S'} = \lambda_k \frac{Q_{r,k}^S}{\sum_j \lambda_j Q_{r,j}^S}$$

STRUT(λ) can be seen as a generalization (original *STRUT* $\Leftrightarrow \lambda \simeq 1$)

SER generalization for Class Imbalance [Minvielle et al., 2019]



$PRR_{n_m}(l) = p^S(x \notin l|k_m)^{n_m}$: (1)

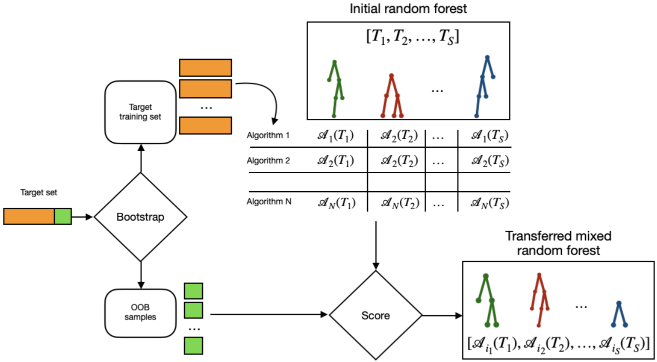
$\lambda_{k_m} p^S(k_m|x \in l) > \lambda_k p^S(k|x \in l)$: (2)

Adaptation for class imbalance :

- Expansion step unchanged
- Flag each leaf l following Eq. (2) and with $PRR_{n_m}(l) > C_{PRR}$
- Avoid any pruning involving leaf l

Same pruning limitation strategy on STRUT : $STRUT_{NP}(\lambda)$

Selective Transferred Random Forest (STRF)



Returns : A mixed random forest using several transfer algorithms and a selection ratio for each algorithm.

Conclusion

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