

# Solving Blind Inverse Problems: Adaptive Diffusion Models for Motion-Corrected Sparse-View 4DCT

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# **Introduction**

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# Introduction to Lung CT

- **Lung CT Reconstruction:** The reconstruction of lung CT images can be formulated as the following optimization problem:

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathcal{R}\mathbf{x} - \mathbf{b}\|_W^2 + \mathcal{R}(\mathbf{x})$$

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- **Introduction to 4DCT:**

- Captures 3D images across respiratory phases, enabling dynamic lung motion visualization.
- Essential for applications like radiotherapy planning [1].

# Introduction to Dynamic Lung CT

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- **4DCT Acquisition Modes:**

- Helical mode: The scanner rotates continuously as the patient table moves through the gantry.
- Cine mode: The scanner acquires multiple images at the same table position over a period.
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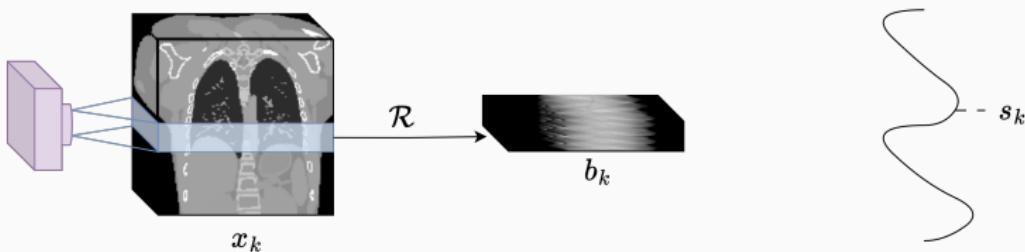
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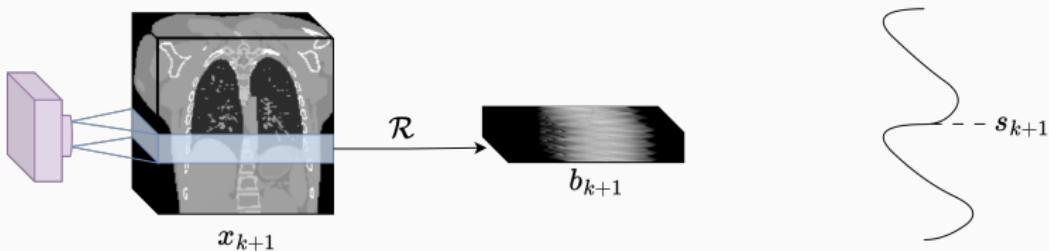


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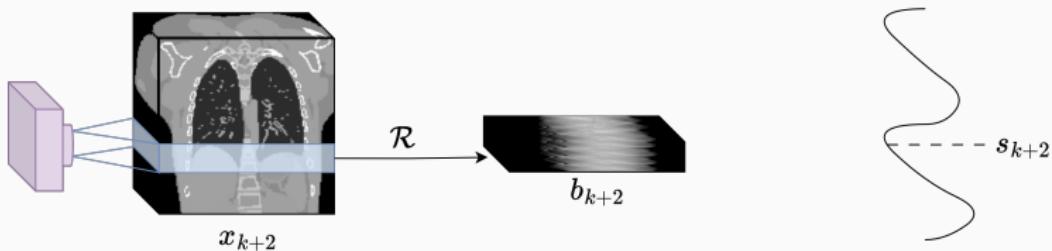


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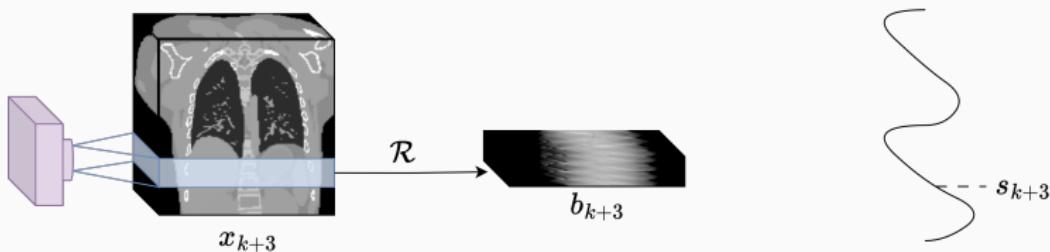


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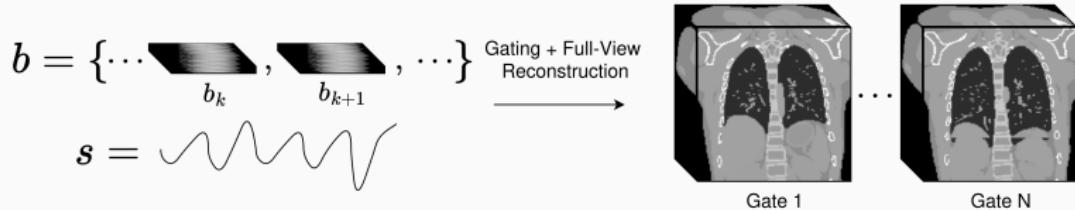
$$b = \{\cdots, \overbrace{\quad}^{b_k}, \overbrace{\quad}^{b_{k+1}}, \cdots\}$$

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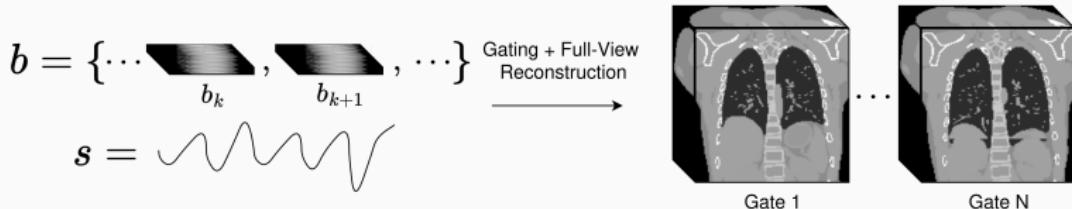
# Challenges in 4DCT

$$b = \{\dots, b_k, b_{k+1}, \dots\} \xrightarrow{\text{Gating + Full-View Reconstruction}} s = \text{wavy line}$$

# Challenges in 4DCT



# Challenges in 4DCT



- **Challenges:**

- Motion artifacts due to irregular breathing patterns.
- Surrogate signals may not always be stored.
- Increased radiation dose from multiple phase acquisitions; necessitates dose reduction strategies.

# Joint Reconstruction & Motion Estimation (JRM)

- General JRM Framework:

$$\min_{\boldsymbol{x}, \varphi} \frac{1}{2} \|\mathcal{A}_{\varphi} \boldsymbol{x} - \boldsymbol{b}\|_W^2 + \boldsymbol{R}(\boldsymbol{x})$$

where  $\varphi$  are motion parameters.

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- 4DCT JRM Framework [2]:

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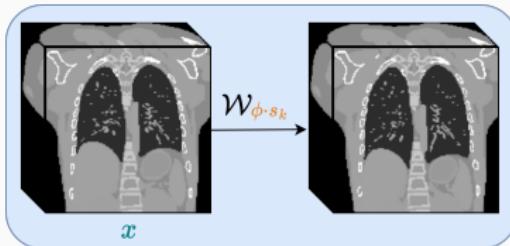
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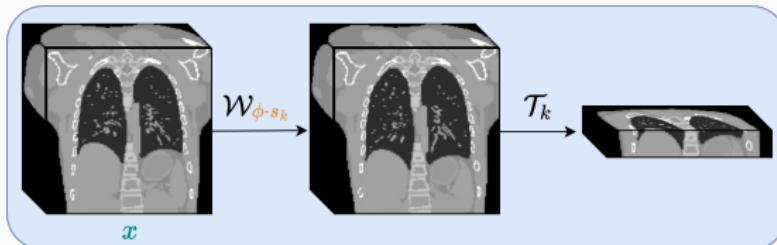
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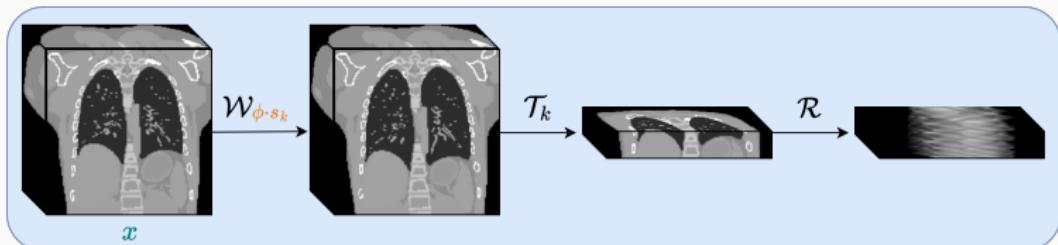
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## Methods

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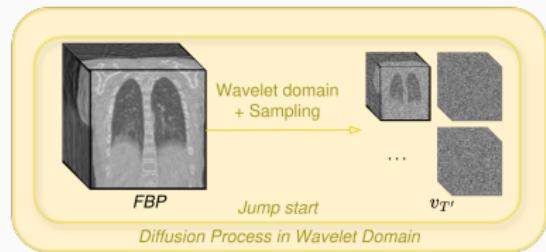
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- **Challenges:**
  - Sparse-view data increases the ill-posedness of the problem.
  - Hand-crafted regularizers (e.g., Total Variation) tend to oversmooth and erase subtle image features.

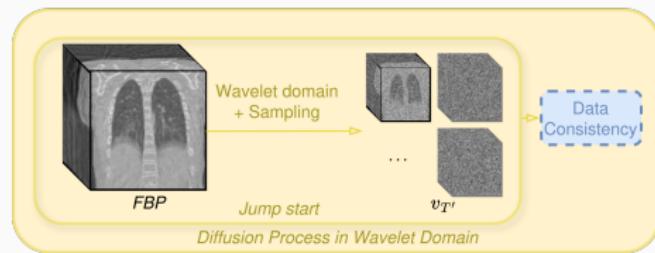
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- **Solution Overview:**
  - JRM via Adaptive Diffusion Models (ADMs).
  - Motion-free  $\mathbf{x}$  is estimated via a Deep Posterior Sampling (DPS) approach [3].
  - Motion parameters  $\phi, \mathbf{s}$  are estimated through a classical optimization pipeline.
  - Wavelet Diffusion Models (WDMs) [4] are used to enhance computational efficiency.

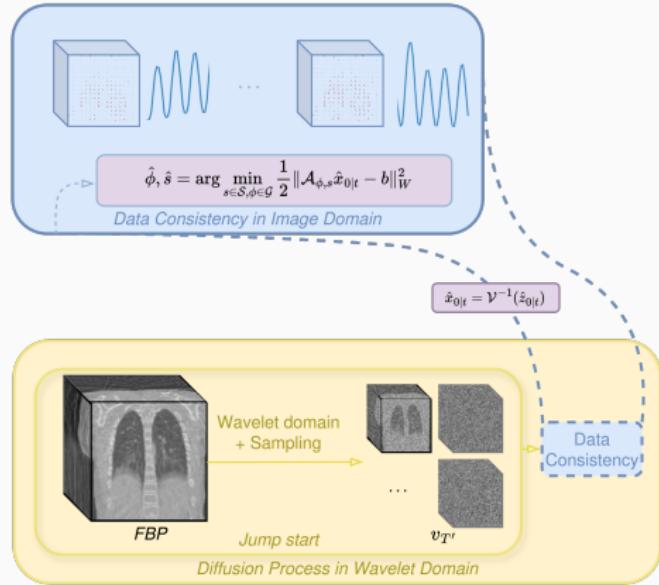
# Overview of JRM-ADM



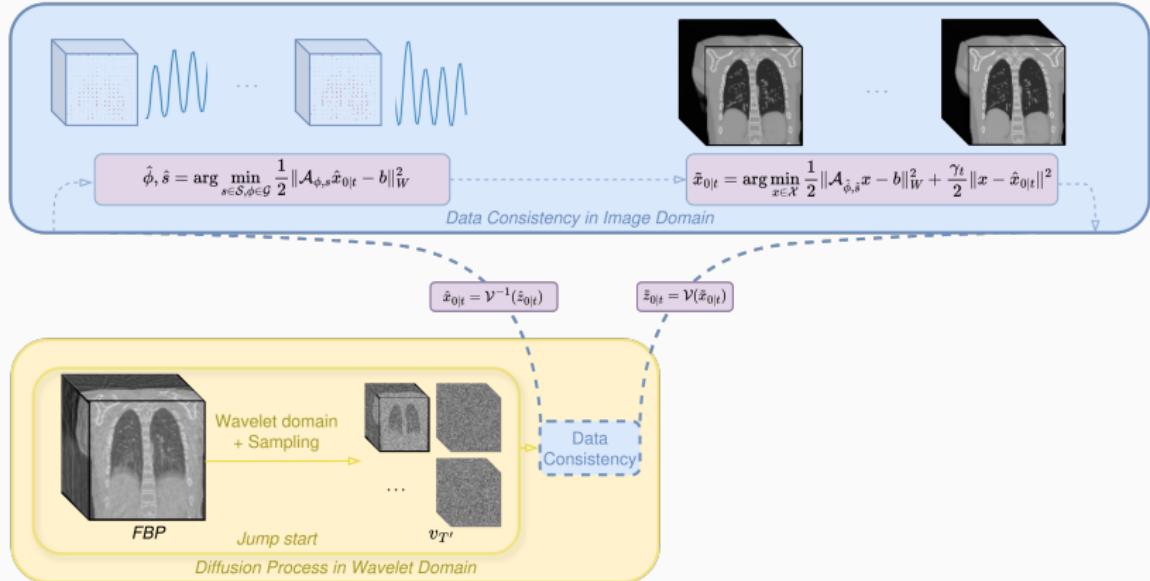
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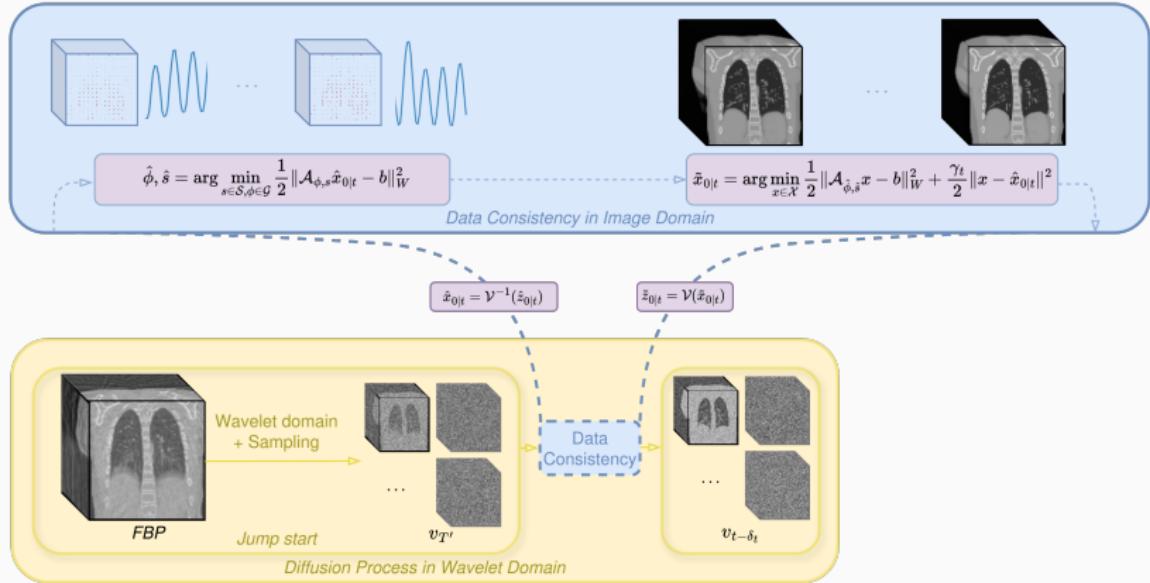
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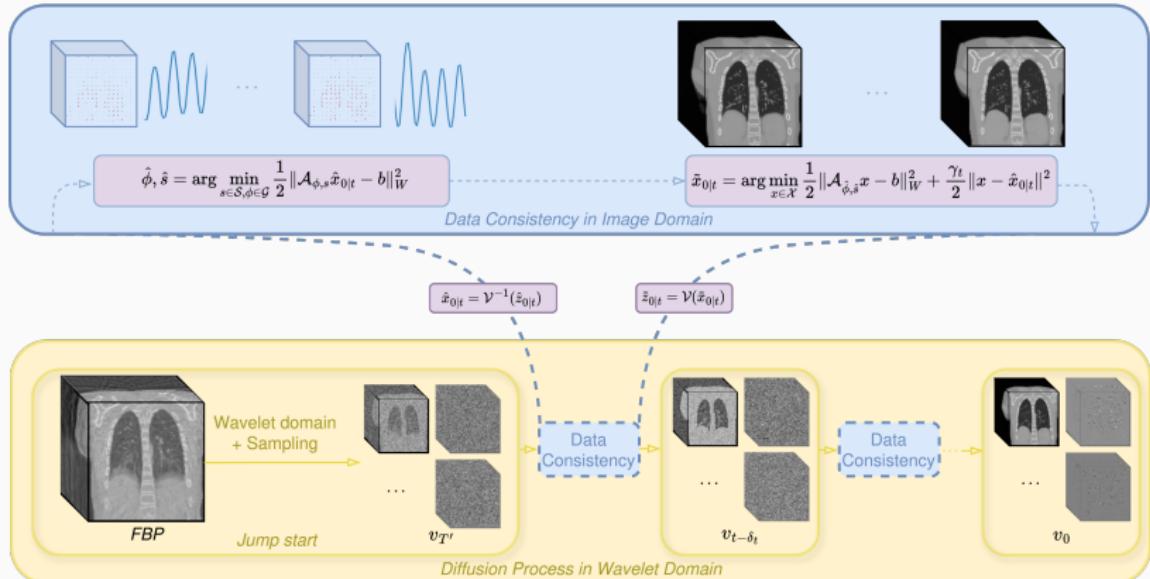
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# Experiments

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# Quantitative Results

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## Experiments on XCAT

### Phantoms:

- Generated 3D thorax CT volumes ( $128^3$  voxels,  $2.6 \text{ mm}^3$  voxel size) using the XCAT phantoms [5].
- Trained WDMs on these volumes to learn anatomical priors.
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## Experiments on Pseudo-Real Data:

- Using LIDC-IDRI [6] 3D thorax CT volumes ( $256^3$  voxels,  $1.25 \text{ mm}^3$  voxel size)
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- **Evaluation:** Assessed reconstruction quality using metrics such as SSIM and PSNR.

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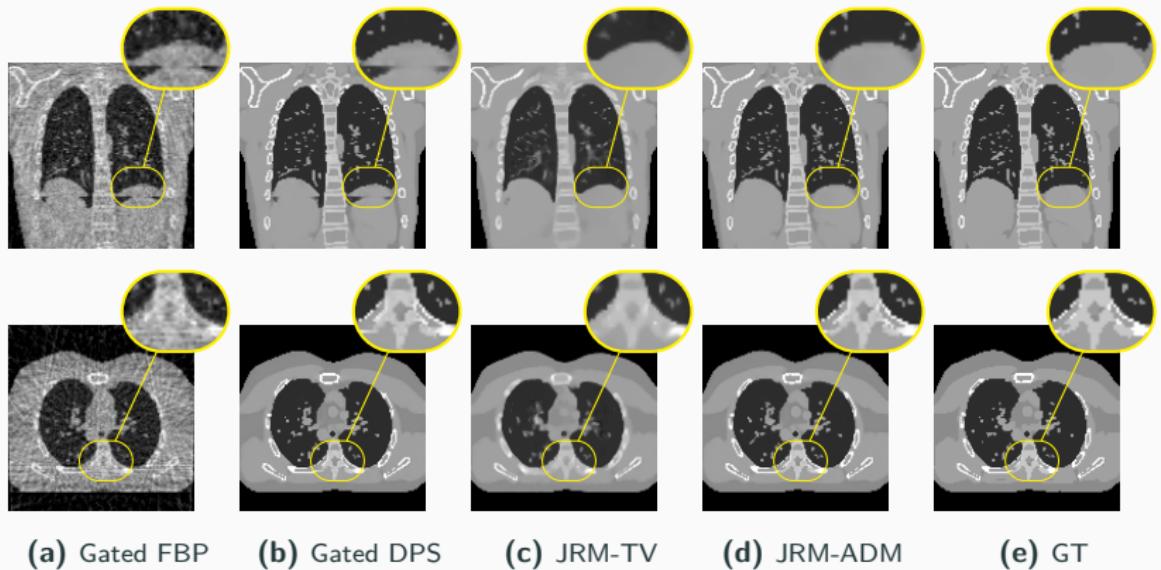
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# Quantitative Evaluation of Reconstruction Methods

Metric	Gated FBP	Gated DPS	JRM-TV	JRM-ADM
PSNR ↑	20.59	24.09	25.04	<b>27.05</b>
SSIM ↑	0.37	0.90	0.89	<b>0.94</b>

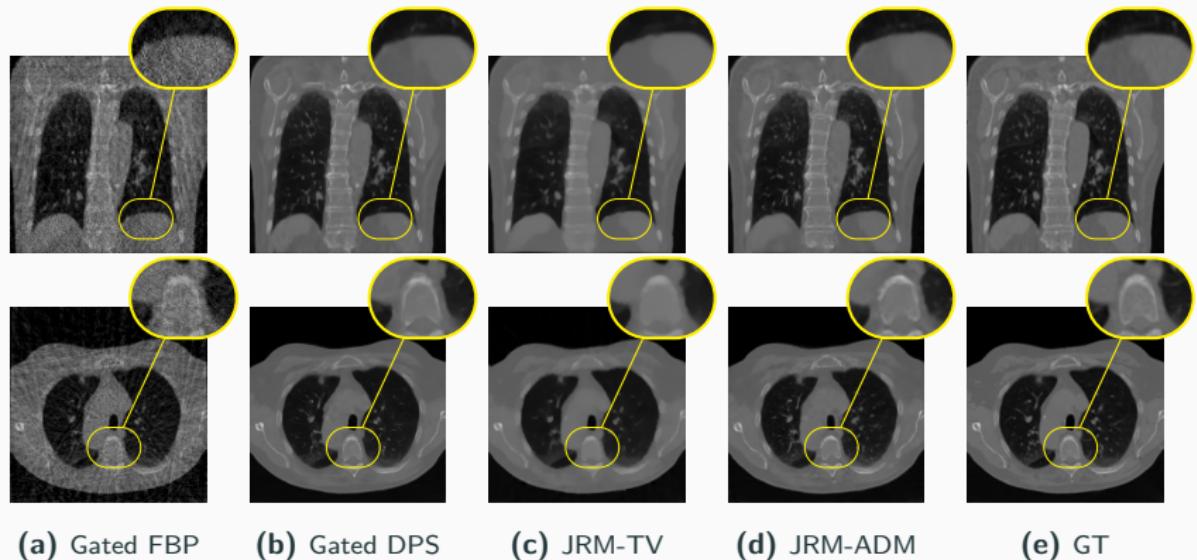
**Table 1:** Quantitative evaluation (PSNR, SSIM) of four different reconstruction methods on the end-inhale phase for five XCAT phantoms.

# Qualitative Evaluation of Reconstruction Methods



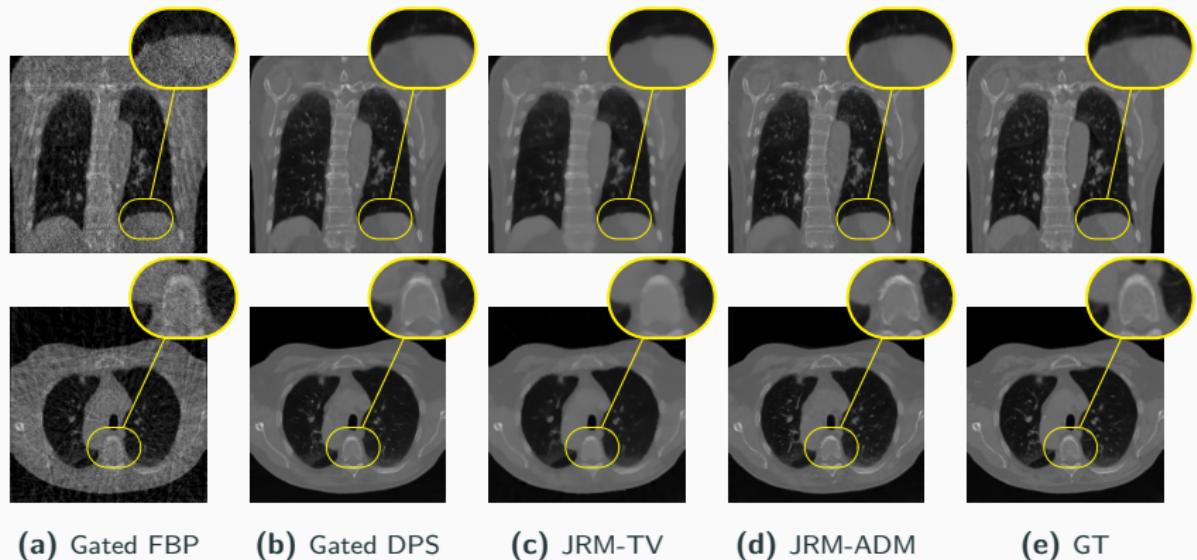
**Figure 1:** GT and end-inhale phase reconstructions on XCAT phantoms.

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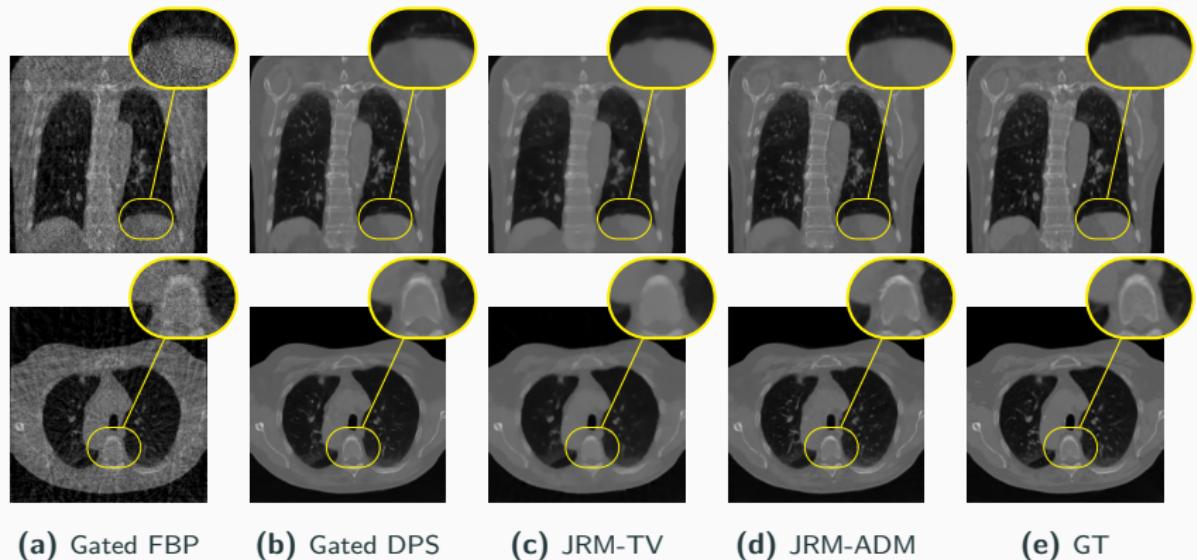
**Figure 2:** GT and reconstructions on pseudo-real data.

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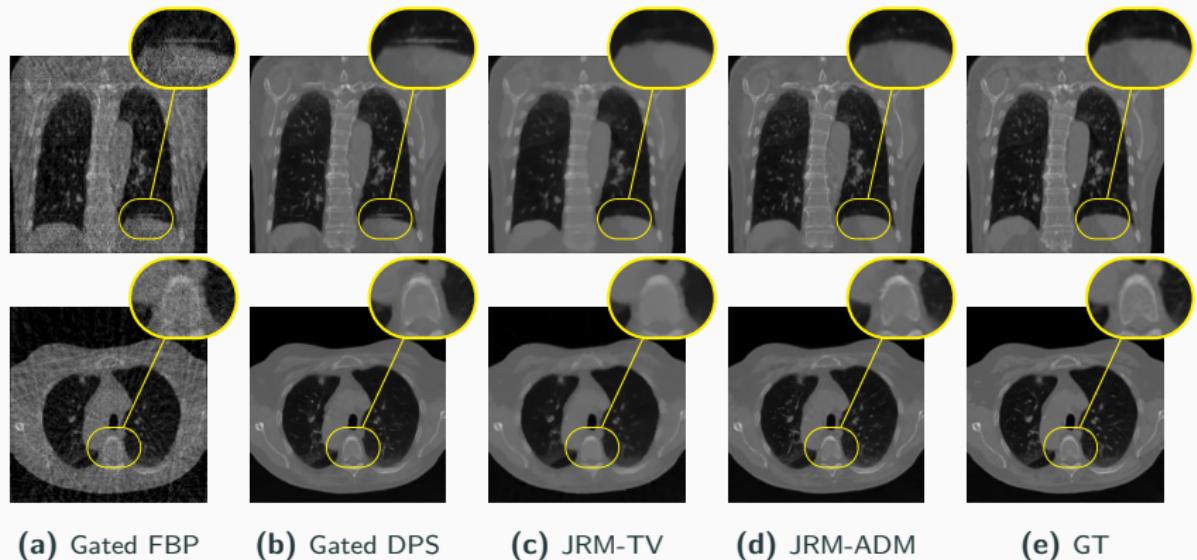
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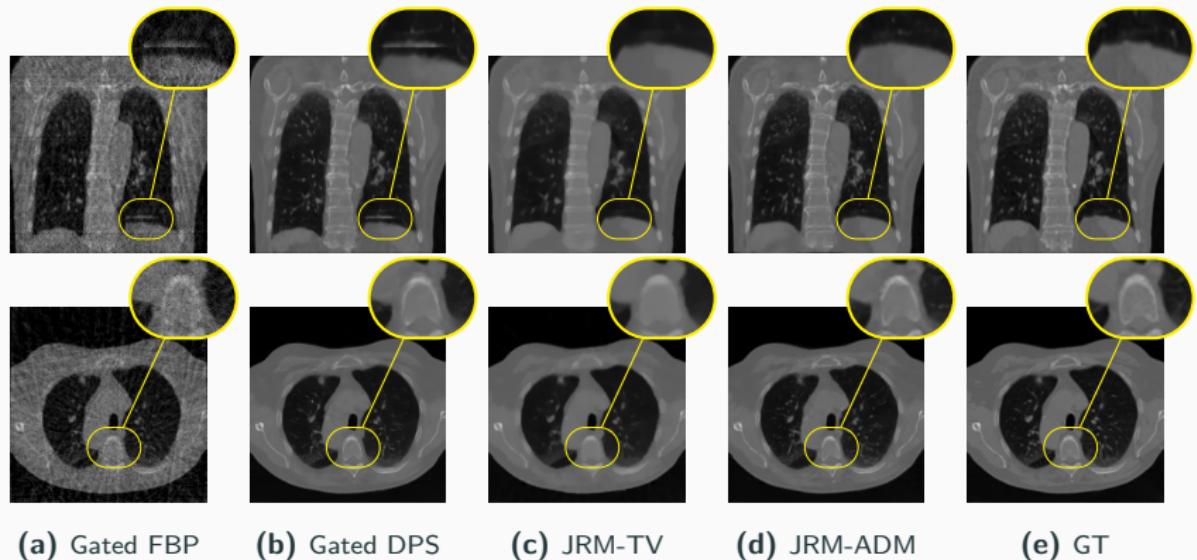
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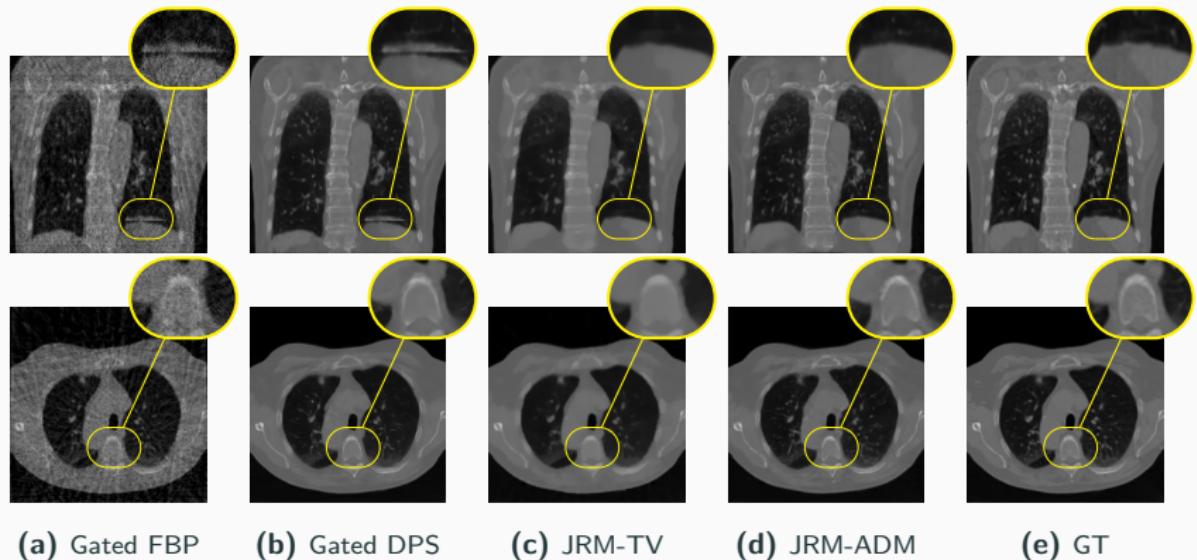
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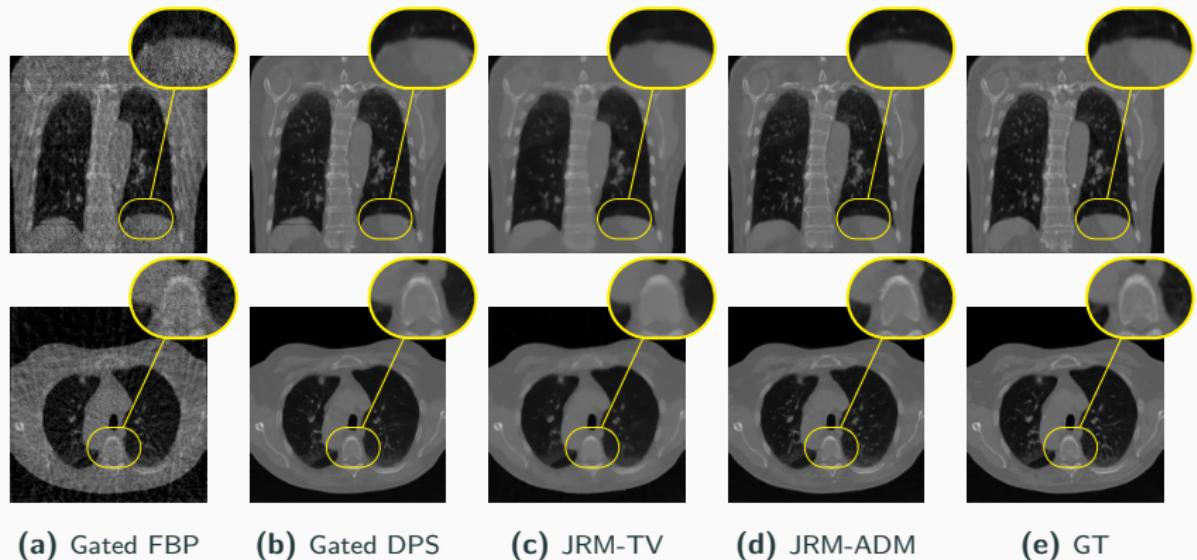
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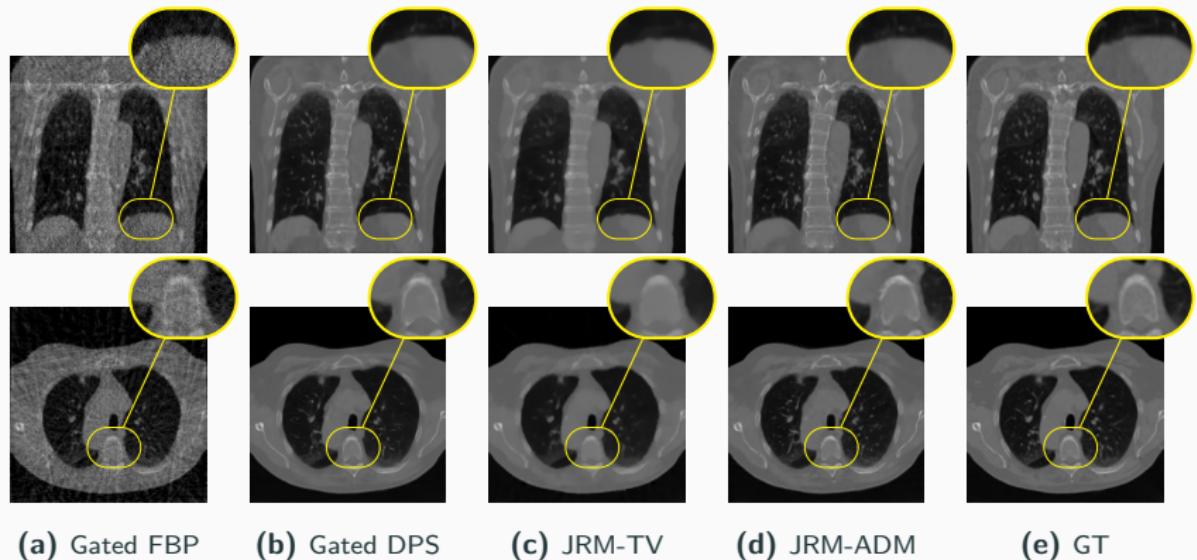
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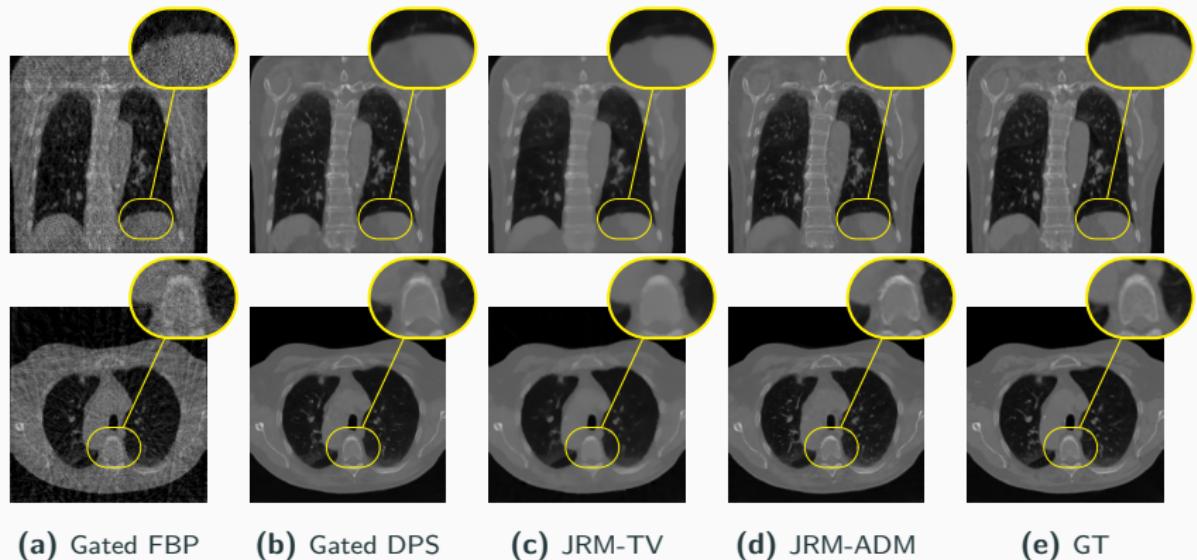
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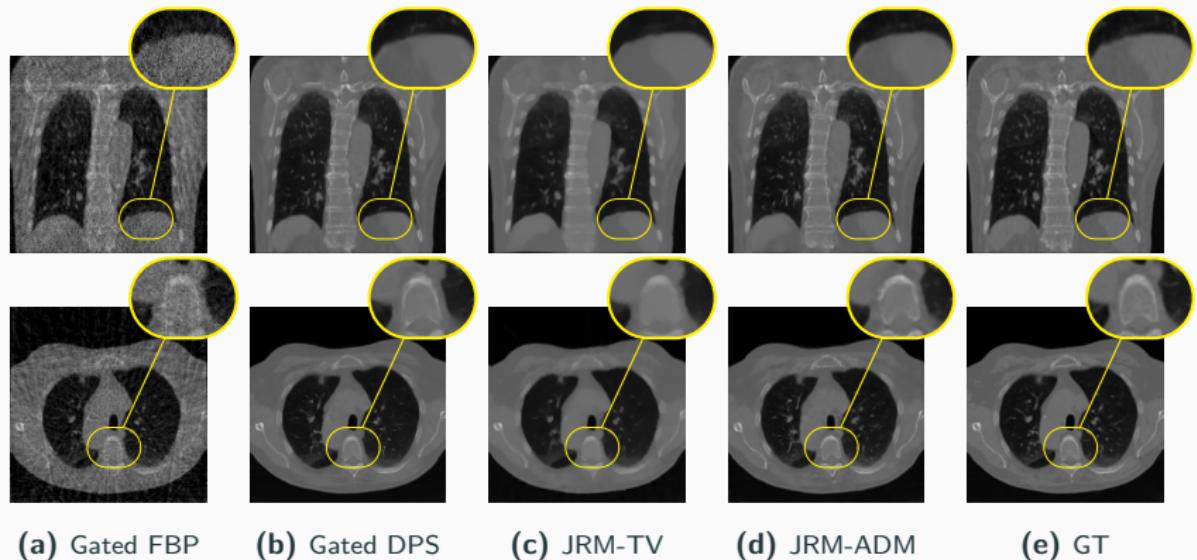
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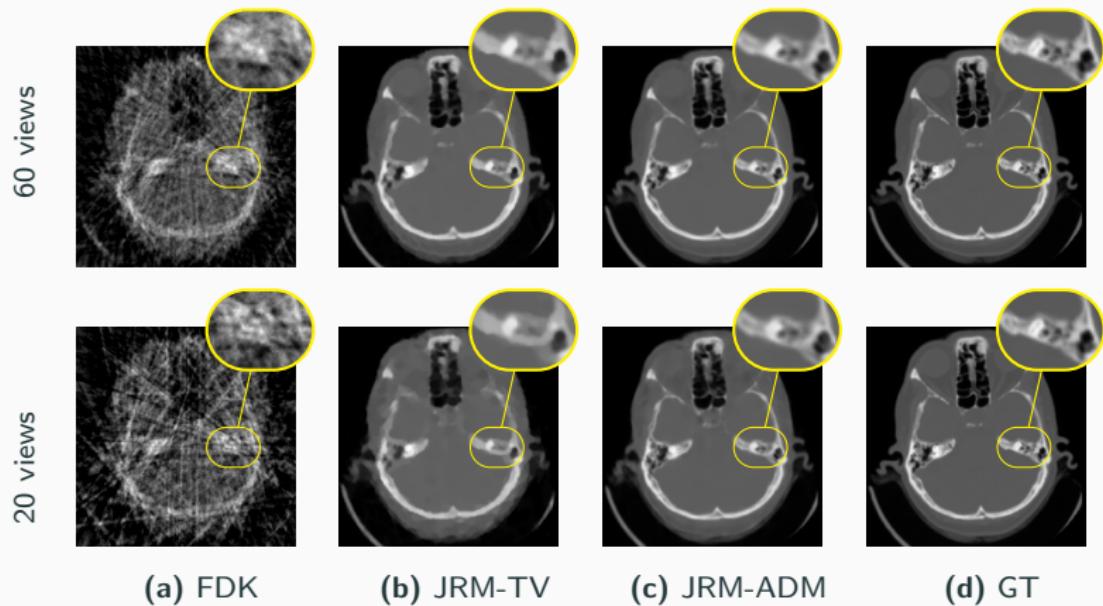
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# Extension to Motion-Corrected Sparse-View Head CBCT [9]: Quantitative Results

Views	Method	PSNR ( $\uparrow$ )	SSIM ( $\uparrow$ )	MAE $\tau$ ( $\downarrow$ )	MAE $\vartheta$ ( $\downarrow$ )
60	FDK	17.00	0.23	-	-
	JRM-TV	28.91	0.90	0.34	0.11
	JRM-ADM	<b>29.67</b>	<b>0.92</b>	<b>0.27</b>	<b>0.09</b>
40	FDK	16.92	0.19	-	-
	JRM-TV	27.19	0.87	0.43	0.12
	JRM-ADM	<b>28.66</b>	<b>0.91</b>	<b>0.29</b>	<b>0.11</b>
20	FDK	14.96	0.14	-	-
	JRM-TV	23.38	0.75	0.86	0.17
	JRM-ADM	<b>27.18</b>	<b>0.87</b>	<b>0.30</b>	<b>0.12</b>

**Table 2:** Quantitative results of methods in comparison on the CQ500 [8] dataset for the motion corrected reconstruction tasks under sparse-view settings of 60, 40, and 20 views.

# Extension to Motion-Corrected Sparse-View Head CBCT [9]: Qualitative Results



**Figure 3:** GT and reconstructions (axial plane) for motion-affected sparse-view CBCT: results are shown for 40- and 20-view acquisition settings.

## **Conclusion - Discussion**

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- **High-fidelity 4DCT:** JRM-ADM yields artifact-free, high-resolution reconstructions under irregular breathing, outperforming classical reconstruction methods.

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- **Real World Challenges:** Working on real sinograms would be the best path toward clinical translation.

# Acknowledgments

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This work was supported by the French **National Institute of Health and Medical Research (Inserm)**, by the **University of Western Brittany (UBO)**, by **France Life Imaging** under grant No ANR-11-INBS-0006 and by **CPER 2021–2027 IMAGIIS (INNOV-XS)**.



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