Inflation Inequality : Redistribution under Household Heterogeneity[1]

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Abstract

I show how non-homothetic preferences are useful to account for non linearity in consumption where households are forming their consumption bundle not only on relatives prices but also on household income/wealth level. Data shows a pro-rich inflation where lowest income household are experiencing a higher cumulative price index for their consumed bundle. Lastly, I provide an endowment model and conduct a negative supply shock in which poorest households are the most exposed and are affected twice. First by losing in wealth and secondly by experiencing subsequent price surge. I also provide an extended Aiyagari model with non-homothetic preferences. Households save not only for precautionary purpose but also for a desire to be able to acquire wealth and consume high-end product less subject to price fluctuations.

[1]I would like to thank Stéphane Auray and Pablo Winant for their surpervision and suggestions throughout this master thesis as well as Bertrand Garbinti for valuable comments.

Replication codes for data, plots and modelling part are located in this Github project folder

1 Introduction

After being at a low or null level for several years, inflation finally strikes back. In the wake of the pandemic, enormous stimulus packages and furlough schemes were warranted to prevent a global slowdown. It puts henceforth the possibility of living in a low to high inflation environment. Indeed, US annual inflation peaked recently at 8.5% in June an all-time high since 1982 and 9.1% in August for the Euro area.

This increase in the cost of living does not occur without making any casualties. Especially for foods and energy which peaked at 13.5% and 23.8%. It reduces the overall purchasing power of an agent by depreciating the value of the domestic currency held by the public. People could nevertheless hedge against it. If they adjust accordingly by keeping their purchasing power in real term constant, then we should see no impact. While it is possible, people do not hedge perfectly against inflation. Without adjusting accordingly their income and wealth in nominal terms to keep up with inflation, they lose in real terms. It is even more annoying when households consumption is mainly concentrated in commodities that are more volatile than others. These interactions demonstrates a potential for unequal inflation exposure as people are not consuming the same way and lead way to non-homothetic preferences

During the pandemic, fresh products prices rose due to high demand and disrupted supply chain. Low-income decile people were then the most exposed as they spent a high share of their revenue on necessity goods. More recently with the war in Ukraine, energy prices soared. In the wake of price rise, these commodities would represent a higher share in their expenses and erode their disposable resource for other means. Additionally, poorer households do not have access to hedging technologies or do not experience gain in wealth as their asset composition is poor in illiquid assets such as housing or retirement accounts. It would then be optimal to save for precautionary purpose regarding income risk but also for countering unexpected inflation exposure coming from a price surge. Consequently people are hedging against inflation differently and has aftereffect on asset distribution

In a high inflation environment, poorest household vulnerability toward inflation feeds concerns for inflation inequality. This heterogeneity in exposure constitutes a break away from the traditional bulk CPI and its aggregate bias. Spending patterns could then be identified with the granularity that the world can now offer to us with big data scanner level like Nielsen data sets with fast-moving consumer goods barcode level data (Jaravel [2021]). This research project will provide an overview of this concern and demonstrates to what extent it has to be highly considered to preserve purchasing power for those most vulnerable. Lastly, economic models have always been on the fact that the inflation path is known perfectly, I provide a framework in which price path is set endogenously consistent with non-homothetic preferences. Heterogenous impact of inflation in then mainly driven by different consumption bundle with a deep parameter related to an income/wealth effect changing consumption patterns. Indeed, luxury products - let us say by quality - are highly considered at higher wealth level while primary needs good like - like food and energy - are more consumed at the bottom of the distribution. Different consumption bundle would then be formed for different income level leading to different exposure to inflation.

I first find a pro-rich inflation in the Euro Area by using Household Expenditure surveys at the EU level. Poorest households in terms of revenue are then the most exposed to inflation. These findings has been extended to other degrees of heterogeneity such as age, urbanisation and activity status. Data further shows heterogenous expenditure shares along the distribution. Households are then forming their consumption bundle depending on characteristics consistent with my desire to simulate agent with non-homothetic preferences.

Secondly, I simulate an endowment model in which poorest household are hit twice by an aggregate supply shock on primary goods. First, everyone lose unequal portions of their wealth due to different endowment intensity. Poorest households are those endowed with the most of primary goods. Hence they first lose more than others. Secondly, the economy is now poorer and are, consistent with non-homothetic preferences, more interested in primary goods. As a result, supply shock effect is compounded with an increased interest in primary good and lead to a price surge. Poorest households consume the most of primary goods, their bundle become then more expensive.

Thirdly, I solve an extended Aiyagari model with non-homothetic (NH) preferences across goods. Households in the NH model are saving less on average but households at the bottom of the distribution are accumulating more assets than in the Aiyagari model. They are then not only accumulating assets for precautionary purposes to insure themselves against idiosyncratic income risk but also for a desire to acquire more revenue and be able to consume more in the next periods, especially income elastic good. Conversely, richest people are spending more because they benefit from low general equilibrium prices and have already satisfied their needs. There is then indeed more interest in consumption than saving.

Section 1 deals with literature. Section 2 provides an empirical view on consumption bundle at a more dis-aggregated level and characteristics specific inflation exposure for the Euro Area. Section 3 shows a static endowment model and a dynamic general equilibrium model with non-homothetic preferences. 4 discuss policy implications, 5 discuss the main caveats from my analysis and what have been done in the literature to quantify real inflation exposure.

Related literature.

Macroeconomics experience a departure from the traditional representative agent setup. Heterogeneity has been all around the place to the extent that heterogeneous agents models are now considered as a compelling setup for highlighting inequalities and the impact of monetary policy across households. Kaplan et al. [2018] proposes such a framework which looks like the one used by McKay et al. [2016] as they also emphasize uninsurable income shocks by the households. They further add on the difference in marginal propensity to consume (MPC) with a distinction between poor and wealthy hand to mouth (Kaplan et al. [2014]). This wealth dimension is interesting in my studies as these households are likely to not consume, save, spend the same as their wealth evolve paving then the way for non-homothetic preferences. Engel Law (1857) illustrates consumption patterns different from rich and poor. He argues that foods would represent a smaller share household expenditure as his income grows. Consumption patterns are then interacting with other dimensions like income or wealth. These patterns links my paper to the one related to structural change from PIGL preferences, Stone and Geary preferences... I decide to exploit non-homothetic preferences with Comin et al. [2021].

Heterogeneity in inflation is now gaining territory. Even if early findings shows modest evidence for inflation inequality as in Hobijn and Lagos [2005], McGranahan and Paulson [2005]. It has recently interested academics due to inflation all around the developed world. Recently, Jaravel [2021] urges the importance of studying heterogeneity in inflation across household groups and the conduct of high frequency data studies. Aggregation bias could then be overcome with large datasets on expenditure data and granular prices from different sectors. Several attempts to measure inflation inequality across the globe. van Kints and Breunig [2021] shows higher inflation experience in Australia by for low income households from 2011 to 2018. However it is mainly driven by higher price in tobacco and alcohol mainly consumed at the low income level where it represents a high share in their expenditures. Akkoç and Kizilirmak [2021] find that inflation could differs across household types and head of the household gender in which female head lead to lower inflation. In recent inflation periods shows higher burden for the poor Baez et al. [2021]. Countries like Turkey with double digit inflation are the most preoccupied, they also argue a high vulnerability of low income households relying on necessities. Zhu et al. [2022] share the same point with European recent surge in energy prices. They point also the fact that Household budget survey in Europe are most of the time not done annually so it does not permit to have a "live" exposure to measure. Cravino and Levchenko [2017] expose how high devaluation hurt lower end of the income distribution in the aftermath. This has also been replicated in Gouvêa [2022] for Brazil where the distributional impact of Lula's shock has been studied. Braun and Lein [2020] shows higher burden for the poor with household scanner data from AC Nielsen in Switzerland. Mejia and Hartley [2022] states the compounding effect of such inflation inequality that could have on the poorest households. Weissert [2022] implements PIGL preferences and use CEX CPI data to end up with a higher cost of living over time for lower end income households.

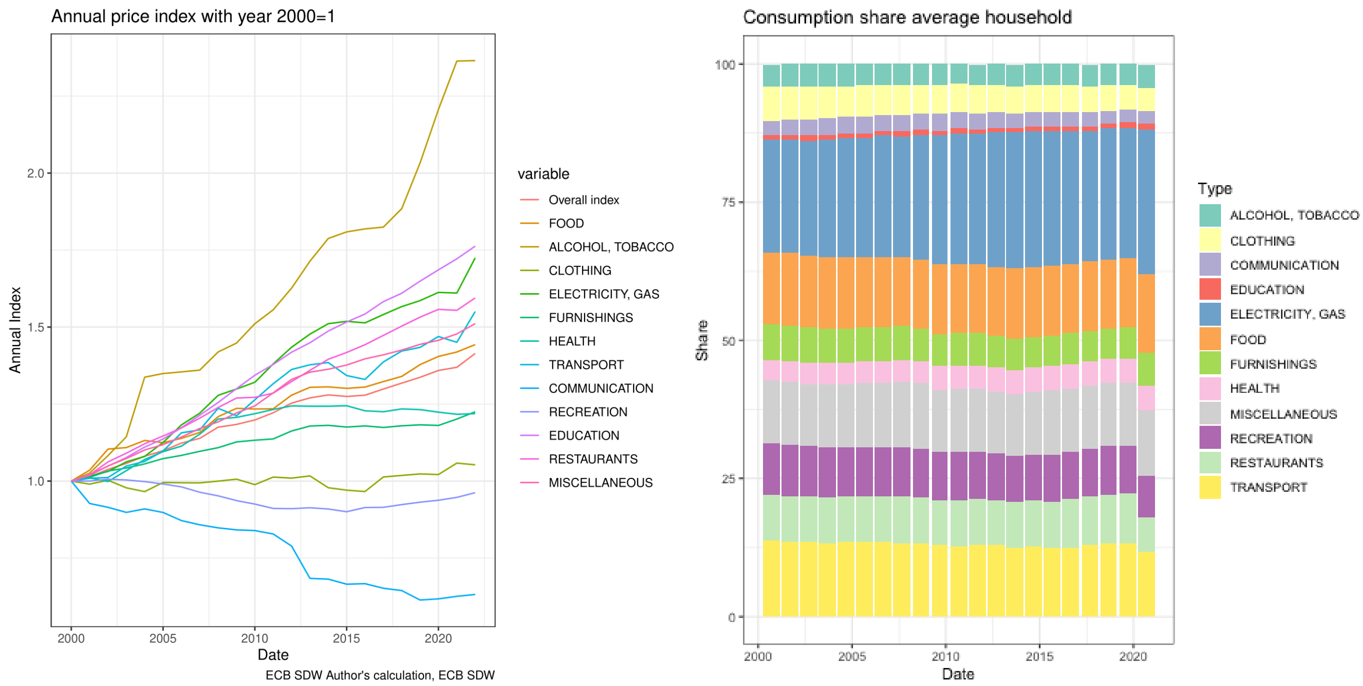
Mounting evidence, especially during major crisis, put forth concerns for inflation inequality. Argente and Lee [2021] find that low income households experienced higher inflation during the 2008 Great Recession in the United States. Jaravel and O'Connell [2020] found that during the Great Lockdown in Britain, 96% of households experienced inflation in 2020. They also emphasized evidence from sectoral data in the United States that low-income households experienced a higher annual inflation rate compared to high-income households. Hochmuth et al. [2022] use Consumer Expenditure Survey (CEX) to match CPI subindices by considering 21 consumption categories from 1995 to 2020. For them first income decile households are experiencing similar inflation as households in the last decile, execpt during grand recession. In particular, first decile faced on average a 0.37 percentage points higher annual inflation rate between 2004 and 2015. While it is similar, inflation volatility is not and is more than 2.5 times higher for the poor. They also argue by pointing out higher exposure for these households to in food, gas and utilities

Price rigidities in rich people consumption bundle seems then to play a role in inflation inequality. Nakamura and Steinsson [2008] emphasize that price rigidities vary across product categories. Clayton et al. [2018] find that more educated people seem to spend more in sectors where prices are rigid. They propose a study of monetary policy under a 2 sectors economy where one sector is price rigid and the other one is flexible. They add heterogeneous households who differ on whether they are college-educated or not. They find that college-educated people are the most exposed to a contraction in monetary policy since a decline in prices translates into a decline in relative price with respect to the flexible sector. College-educated people would then reduce their consumption by a higher margin compared to non college-educated households.

Lastly, I also link my study to redistribution from asset side inflation which constitutes one of the main caveats in my paper. Auclert [2019] introduces a decomposition of monetary policy transmission channels through which he emphasizes a Fisher credit channel with net nominal positions (NNP). Since assets are detained in nominal terms, inflation could have a impact. Cardoso et al. [2022] has argued in this favor. They use Spanish expenditure survey and link it to bank dataset from the BBVA. They find that middle-aged individuals, who have large negative NNPs due to mortgages, were roughly unaffected by inflation, while old people experienced the largest decline in real wealth, as the have large positive NNPs. Indeed, as early as Doepke and Schneider [2006] shows that main losers from inflation are rich, old households, whereas the main winners are young, middle-class households with mortgage debt. It has been replicated with Canadian data in Meh and Terajima [2011] with same results. Adam and Zhu [2016] have done this at the Euro Area level. They further argue that government are at the winning side of unexpected inflation. Especially for Greece, Italy, Portugal and Spain. They also find that countries who experience higher inflation tend to borrow more and sell fewer in nominal terms. In line with Doepke and Schneider [2006], they also find that richest households are losing more in real term due to inflation.

2 Data

Quick view on the data is necessary to have an idea of how consumption bundle are different across quintiles. Even if consumption patterns are likely to be heterogeneous across countries regarding revenue and consumption, I decide to take the Euro Area (EA) as a ensemble for patterns studies. Data are then taken at the EA level[2]. Price indexes are from the European Central Bank Statistical Data Warehouse where I can find HICP - By classification of individual consumption by purpose. I can have them on a monthly basis from 2000 to 2022. Representative household consumption shares per purpose can be obtained from Eurostat on an annual basis. Lastly these data can be obtained across quintile with Eurostat classification of individual consumption by purpose[3].



(a) Annual price index per purpose

(b) Consumption share over time

Figure 1: **Price indexes and consumption share for average household**
Notes: Panel (a) shows the annual price index per index from 2000 to 2022. Data are taken on the 1st month of the year. Indexed at 2000 = 1. Panel (b) shows consumption share per purpose for an average considered household.

Figure 1a shows 12 annual price indexes per consumption purpose from 2000 to 2022 and Figure 1b represents the consumption bundle share for an average household. At the first sight, Euro Area household consumption share seems to vary over time for several reasons.

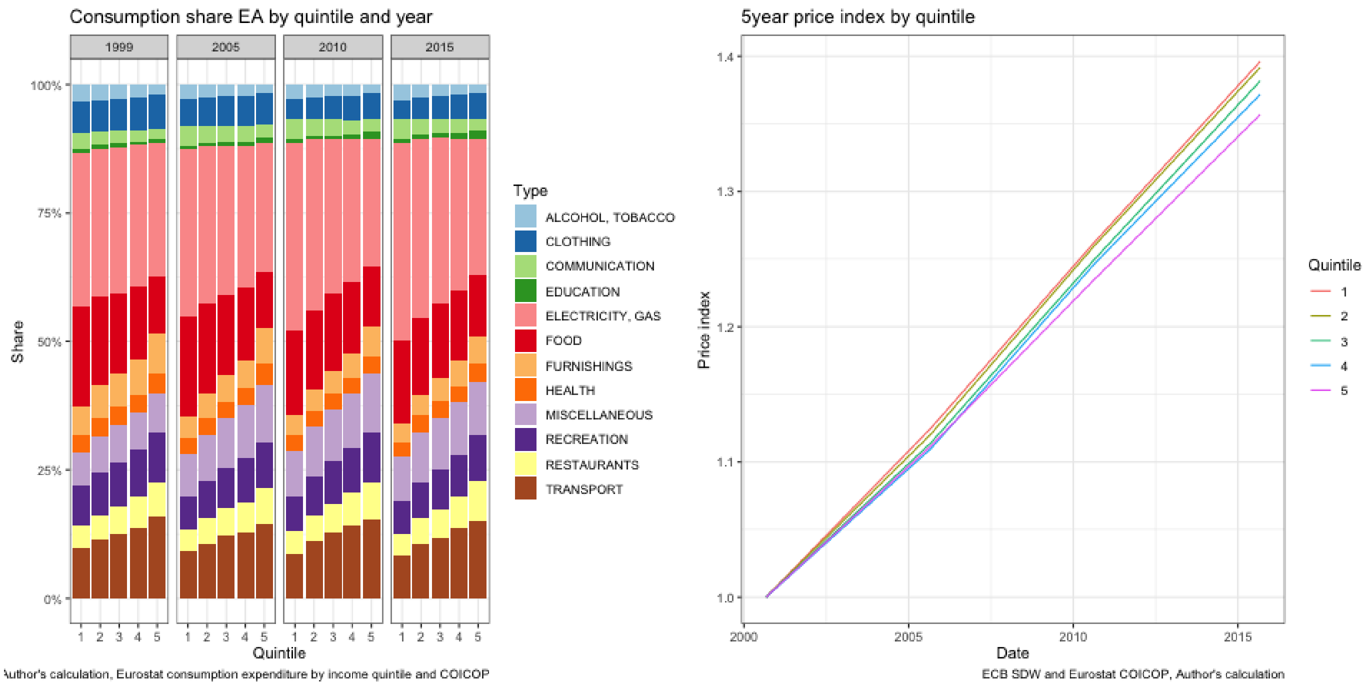
First, prices for a specific commodity might become more expensive and in the short run. If revenue do not adjust instantly, it could represent a higher share of the overall expenditure

[2] More details in appendix A.1

[3]. Data are only available on a 5 years basis such as 1999, 2005, 2010, 2015. Most of it are still estimated with low reliability.

that the household would have to dedicate to consumption. Second, the "average" person might be time varying as he could become richer or poorer over time. Hence his consumption patterns could differ from a year or another. Finally households could also get restricted from consuming a commodity or is simply short in supply. This is what happened in 2020 with the covid crisis where most of the Euro Area countries imposed a lockdown. Indeed, Figure 1b with year 2020 seems to highlight a shift from consumption patterns where less was dedicated to transport and more to housing and electricity. This sudden shift from consumption leads to an inflation exposure totally different as households concentrate their consumption in other sectors.

Several mechanisms are then at play and I decide to spend more time on the second one where wealth effect could shift consumption bundle leading to heterogeneous consumption bundle composition. I also point out how heterogeneous sectoral inflation dynamics could redistribute welfare in real terms over time. To do that, I obtain household consumption expenditure across quintiles of revenue from the Eurostat. My first guess is that they gather household expenditure surveys from each country of the Euro Area at the micro levels to end up with these aggregate measures. Indeed, it coincides with European Expenditure Survey[4] which is done every 5 years. Due to data restriction, I can only work a 5 year basis where at year 1999, 2005, 2010 and 2015 I know how households differ by their consumption bundle across quintiles of revenues. I end up with 4 data points per quintile at the aggregate level.



(a) Annual price index per purpose

(b) Price index across quintiles

Figure 2: **Heterogeneous consumption bundle and experienced inflation**
Notes: Panel (a) shows the annual price index per index from 2000 to 2022. Data are taken on the 1st month of the year. Indexed at 2000 = 1. Panel (b) shows consumption share per purpose for an average considered household. Due to price index data restriction I compute (b) by setting price level at year 1999 to the one at year 2000

Figure 2a shows that 1st quintile households are spending more into foods and housing,

[4]For example France Budget des Familles is a household budget survey where micro data are gathered such that household job occupancy, martial status, highest diploma achieved, age and revenue etc. It permits to measure how heterogeneous consumption bundle are depending on household characteristics

electricity gas compared to household at the 5th quintile. Indeed, when primary needs are satisfied, there should be a point at which the richer a household is, the more he would spend more into sectors related to leisure like recreation spending which is steadily growing by quintile. I do not say consumption will drop for specific sectors as households get richer, rather consumption in any commodities should grow with income/ wealth level but at a decreasing pace for necessities and increasing pace for superior ones. An additional interesting feature is how "ELECTRICITY, GAS" which corresponds to "Housing, water, electricity, gas and other fuels", represents a higher share of expenditure for all households but gap is now wider across revenue. Which consists of almost 10 percentage point difference between 1st and 5th quintile.

Finally, different consumption bundle are leading to different inflation exposure at the micro level. I illustrate at the quintile level with a pro-rich inflation for the Euro Area from 2000 to 2015. Figure 2b illustrates the fact that households at the first quintile are constantly having a higher cumulative inflation. On a 5 year basis, household quintile specific price index is the highest for lowest revenue households. In 2015, I find a unit-wise consumption bundle that is around 3% more expensive for households at the first quintile relative to households at the fifth. I attribute this difference to an energy and food dependence for these households. It translates later into a higher inflation exposure as prices of these goods have been on a increasing pace in Figure 1a. These data points are computed by taking household income specific share per purpose multiplied by the price index at each period which consists of this formula

$$\mathcal{P}_q = \sum_{j}^{\mathcal{J}} \frac{P_t^j}{P_0^j} \omega_{q,t}^j. \tag{1}$$

Where $\mathcal{J}$ is the total number of consumption per purpose, $\mathcal{P}_q$ is quintile q specific price index, P_t^j is variety j price index at date t and $\omega_{q,t}^j$ corresponds to quintile q consumption share in variety j ($\sum_j^{\mathcal{J}} \omega_{q,t}^j = 1$). To make these statements holds, households at the first quintile in 2000 should also ranked be at the first quintile in 2015. Assuming that, it permits to evaluate inflation exposure at different quintile over time.

With available data, I illustrate how household consumption bundle price indexes could differ across quintile. It is then unlikely that the average/ representative household inflation exposure is exactly the same as the one at the first quintile. These findings are useful to account for targeted public policies such as fiscal policies that produce efficiency at minimal cost. Policy implications is discussed in section 4. Other dimensions of heterogeneity are discussed and quantified in appendix A.1.2, namely activity status, degree of urbanisation, age etc. By taking taking as reference time 2000 and look for how prices and consumption bundle evolved over time. I end up with higher cumulative price level for people in rural areas, professionally active, retired and manual workers.

3 Models with non-homothetic preferences

I provide an endowment economy and a competitive decentralized economy with non-homothetic preferences across 3 types of good: necessary, normal and superior goods. In the endowment model, I perform a supply shock in which a share of the total production is destroyed and see how prices at the general equilibrium fluctuates consistent with non-homothetic preferences. By assuming sticky prices, I look at how these supply shocks redistribute welfare across household via heterogeneous exposure to inflation. In the decentralized competitive model, I perform a productivity shock.

3.1 Endowment model

3.1.1 Household.

Household receives random endowment level for goods $\in \mathcal{I}$. He solves a constrained optimization problem given non-homothetic preferences.

$$\max_{\{C_i\}_{i=1}^{\mathcal{I}}} U(C_1, ..., C_I) \quad st \quad E(\mathbf{C}, \mathbf{P}) \equiv \sum_{i=1}^{I} p_i W_i \geq \sum_{i=1}^{I} p_i C_i. \tag{2}$$

$(C_1, ..., C_{\mathcal{I}})$ correspond to a consumption bundle, E is its expenditure level that coincide with its endowment level at given prices p_i. Finally C_i correspond to the amount of good i consumed by household h.

To account for non-homothetic preferences, an income effect in the allocation of resources across consumption goods is included. I adopt the implicit utility function with $g(U) = U^{1-\sigma}$ and income specific with sector specific non-homothetic parameter $\boldsymbol{\epsilon}$ proposed in Sato [1977] and Comin et al. [2021]. It is better understood as a non linear problem where a bundle is formed following non-homothetic paramaters and a specific utility level match this constraint:

$$\sum_{i}^{I} \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{g(U)^{\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 1. \tag{3}$$

In this model, I give different endowment level to each household and look for their inflation exposure over time following different shocks given that they optimally choose their consumption bundle with available resources. This inflation exposure is mainly derived due to heterogeneous consumption bundle and income rigidity. If rigidity is removed, then households benefit from how much their endowments are asked in the market. I will adopt for 3 types of goods with different income elasticity. I impose a $\epsilon_i \leq 1$ elasticity for inferior good, $\epsilon_n = 1$ for normal good and $\epsilon_l \geq 1$ for high quality luxury type good. By doing that, households benefit from an income effect where they consume more income elastic good as their revenue side grows. It is then intuitively translated as a higher consumption in luxury good relative to inferior one as their endowment level is high.

Optimal demands are obtained for a given price level and endowment level for the household. Let C_i^* denotes these demands:

$$\forall i \in \mathcal{I} \quad C_i = \left(\frac{p_i}{E(\mathbf{C}, \mathbf{P})}\right)^{-\sigma} \gamma_i g(U)^{\epsilon_i(1-\sigma)}. \tag{4}$$

Since household decision to consume good i is influenced by its utility level, we can solve for $U = F(\mathbf{C})$ by replacing C_i^* into the non-homothetic constraint (3), this equation should hold at the optimum

$$\sum_{i}^{I} \gamma_i^{\frac{1}{\sigma}} \left[\frac{\left(\frac{p_i}{E(\mathbf{C},\mathbf{P})} \right)^{-\sigma} \gamma_i}{U^{(1-\sigma)\sigma\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 1 \tag{5}$$

I can then infer about all the quantities asked for good $i \in \mathcal{I}$. Figure 3 illustrates non linear demand for luxury good and primary good. I see that poorest individual are likely to consume Primary good more than richest person since it is cheaper and yield a higher return in their utility level. At the opposite, when wealth grows for the household, he demand for of luxury good than primary good. In this model, wealthy households are more interested into income elastic goods as their wealth level grows. This is reflected by the fact that luxury good demand relatively supersedes primary good demand when household is wealthy in endowments. Figure 1 does have a flaw by considering $p = [2.0, 3.0, 5.0]$ as the price level that clear the market. Simulation 3.1.3 shows how endowment model optimal demands are influenced by initial conditions.

There are then an income and price channel through which household demand goods. Equation for relative demand (6) in log term identify the channels through which these demands are formed:

$$\log\left(\frac{C_i}{C_j}\right) = \underbrace{\sigma \log\left(\frac{p_j}{p_i}\right)}_{\text{Relative price Effect}} + \underbrace{(\epsilon_i - \epsilon_j)(1-\sigma)\log g(U)}_{\text{Non-homothetic preferences Income effect}} \underbrace{+\log\left(\frac{\gamma_i}{\gamma_j}\right)}_{\text{Weights}}. \tag{6}$$

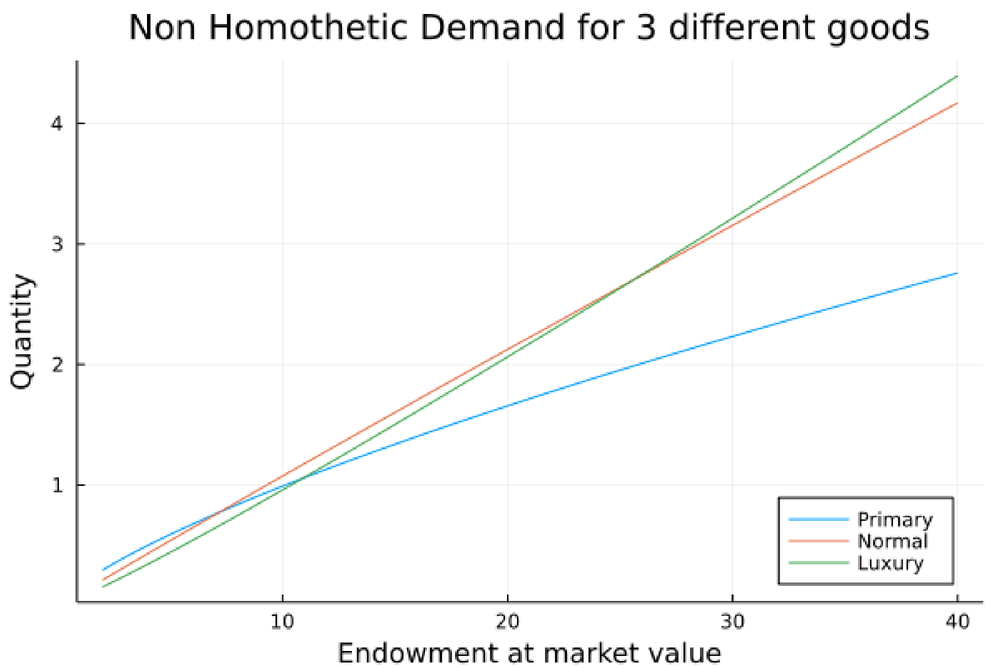


Figure 3: **Simulated non-homothetic Demand for 3 different goods**
Notes: To illustrate non linear demand function, I set up 27 000 different endowment levels and fix a market value (price vector) of [2.0, 3.0, 5.0] with respect to each good. I rank the simulated wealth level at market value so that I can look for optimal non-homothetic demand from the poorest household to the richest in terms of endowment. Following this, I compute the non-homothetic demand functions according to equation (4) for primary, normal and luxury good. This non linear curvature for our demand are coming from ϵ_i in the non-homothetic utility constraint (5).

3.1.2 Market clearing, resource constraints at given prices

Our endowment model is a closed economy with the sum of good i supply equals to the sum of all households non-homothetic demand of good i. Let $h \in \mathcal{H}$ indexes household h so that C_i^h is the demand of good i by household h. Market clearing condition

$$\forall i \in \mathcal{I} \quad \sum_{h=1} W_i^h = \sum_{h=1} C_i^h$$

is satisfied when the market excesses at the aggregate level are offset with a price vector $\mathbf{P}$ that allow non-homothetic demand to fit overall good i supply level.

3.1.3 Results

Calibration and distribution.

Calibration needed for this endowment model are non-homothetic parameters $\boldsymbol{\epsilon}$, CES consumption intensity γ_i and the elasticity of substitution between sectors σ. Without sufficient data at the micro level for the Euro Area, around 4 to 5 data points per quintile from 2000 to 2020, I decide to not estimate expenditure elasticities for setting the non-homothetic parameters. I decide to rely on what the literature can offer me.

I set $\sigma = 0.50$, as in Comin et al. [2021], so that goods are complementary across sectors. Demand intensity per sector are equally set at $\gamma_i = \frac{1}{3} \quad \forall i$, so that goods are equally weighted in non-homothetic utility. Finally, I declare non-homothetic parameters for primary, normal, luxury goods at $\boldsymbol{\epsilon} = [0.80, 1.00, 1.25]$. Expenditure elasticity for normal good is set at 1.00 for comparison purpose across sectors and enables a constant return to scale demand. Elasticity for primary good is set below 1.00 so that its demand decay with richer households. Superior/luxury good has an elasticity higher than 1.00 consistent with what is reflected in service sector in Giarda and Romero [2022][5] and Comin et al. [2021][6].

I finally declare sectoral endowments wit primary goods following $\mathcal{N}(2,1)$, normal goods following $\mathcal{N}(2,1)$ and superior goods following a $\mathit{Lognormal}(0.02, 0.5)$, all truncated at 0.

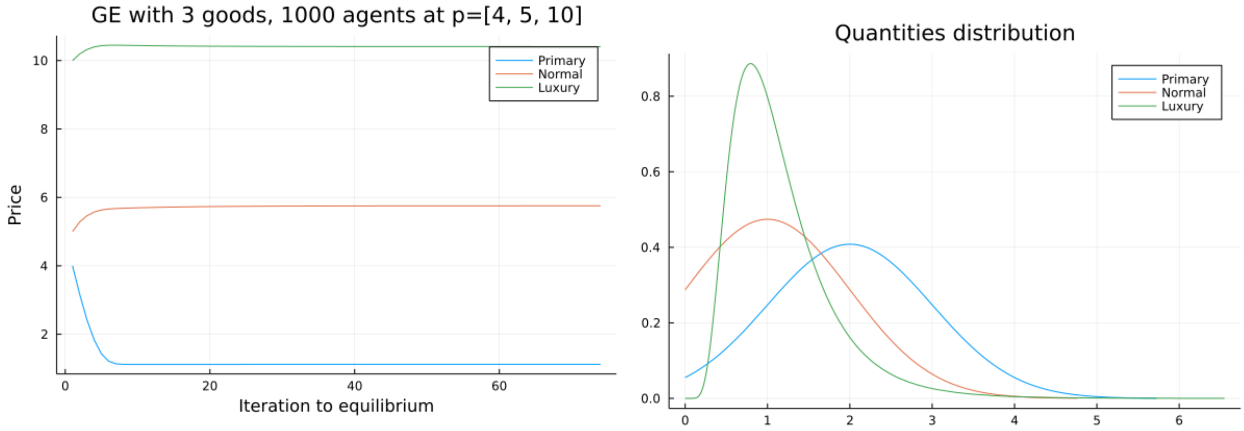
Simulation.

I perform an algorithm that find prices consistent with non-homothetic preferences and overall distributed endowment level. However, I have to point out that initial price level guess has an impact on the general equilibrium prices. Since our first price vector guess infers on the household wealth level, price guess on good i that is high constitutes a gain for those who is endowed with an important amount of good i. Indeed, households could sell the good with overpriced guess to smooth his consumption toward income elastic goods. At the end, prices clear the market but is not independent from first price guess.

In this setup demand for good i relatively to good j is not only shaped by the price of it but also by the wealth of the household. Indeed, richer households will tend to buy more superior good as their endowment level is high. When I perform a first price guess in figure 4, demand for primary good is well below the endowment, it is updated to the point when both quantities offset each other.

[5] $\epsilon_s = 1.16$ obtained from Spanish Consumption Expenditure survey for service sector

[6] $[\epsilon_a, \epsilon_m, \epsilon_s] = [0.37, 0.83, 1.20]$ for agricultural, manufacture and service sector estimated from US Consumption Expenditure survey and CPI prices



(a) General equilibrium price iteration with 3 goods and 1000 households

(b) Endowments distribution

Figure 4: **General equilibrium distribution and how market clears**
Notes: This figure plots our endowment model economy when households are assumed to detain goods following distribution laws at figure 4b. Figure 4a finds the general equilibrium prices consistent with households preferences. I consider a first price guess at $p = [4.0, 5.0, 10.0]$ and update price level to the point where it clears the market consistent with non-homothetic demand. Price level stabilizes when $p = [1.12, 5.75, 10.40]$

Negative supply shock on primary goods

I decide to perform a negative supply shock on primary goods. I would like to draw a scenario in which supply in primary goods are cut by $X\%$ and see what happens on price. Let us take $X = 20\%$ so a fifth of primary goods quantity is then destroyed (bad luck in growing crops, bad international trade supply). I then look at the overall general equilibrium impact on prices with non-homothetic preferences. Figure 5 displays a supply shock on primary goods and a price reset at equilibrium. Let also say that the shock is done within a short lapse of time so that households can't react immediately. The price displayed by our simulation is then the one at the end after market interaction. What I want is to see what happened when people are surprised by a supply shock given a level of endowment and non-homothetic preferences. Without any surprise, post shock price for primary goods jumped due to scarcity and high demand for it before the shock. This shock has also an impact on other goods general equilibrium prices when market clears.

In a way I simulate a recession in which the overall wealth is cut by a fifth but is disproportionately cut at the household level. Some households detain relatively more primary goods and are thus the most inclined to be impacted during this recession.

Mechanism at play.

As seen in figure 3, primary goods are essential to low endowment households. A negative supply shock resulting in a price increase for primary good might switch their consumption habit to avoid inflation. But how could they do that if they can't afford other goods? In a case where there is a recession, they are even less inclined to buy luxury good due to lower wealth. Primary goods would then still represent a big share in their expenditures. There is thus a mass of people in our 1000 agents that do switch their consumption behavior so that they would rather prefer consume more of something else instead of the primary goods due to relative price effect. However, another mass would not do it because the benefit from switching its consumption to another one is small even if primary good become more costly.

These trade-offs are captured by this model with non-homothetic preferences. These results are in line with Orchard [2022] who presents cyclical demand shifts in which contractionary shocks are leading household to cut back on luxuries but continue to buy necessities. Which disproportionally affect poorer household since the relative price of these commodities go up and households at the bottom of the income distribution devote a higher share of their revenue into necessities. Poorer households are then hit twice, first by the recession and second indirectly by rising price on what they consume the most.

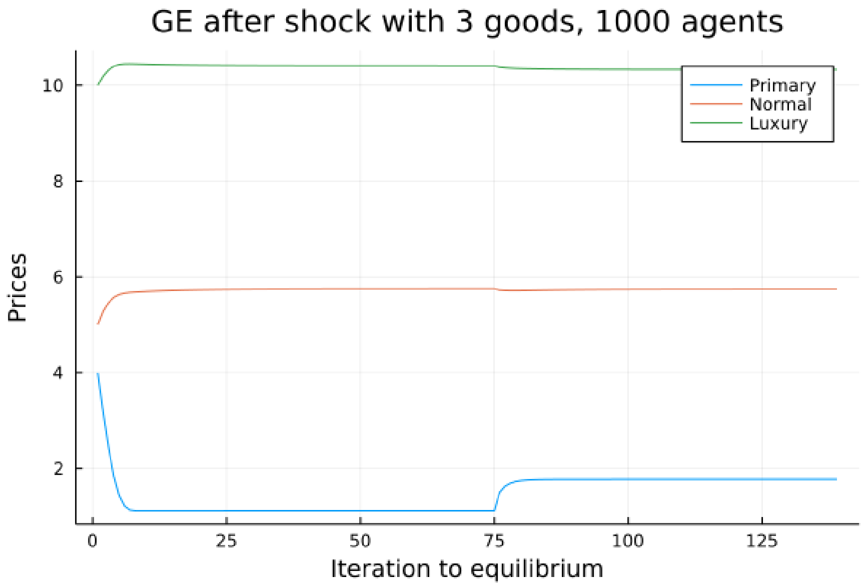


Figure 5: **Post negative primary supply shock market reaction and iteration to general equilibrium price**
Notes: This figure plots market reactions after a supply shock on primary goods. I perform after the steady state prices consistent with non-homothetic preferences a supply shock that destroys 1/5 of the endowment in primary goods. Equilibrium with new endowment level is achieved with a price vector equal to: $p = [1.78, 5.75, 10.33]$ after the shock.

Table 1: Primary good supply shock impact on prices

Good	Before shock	Post shock	Change
Primary	1.12	1.77	+ 58%
Normal	5.75	5.75	- 0.08%
Luxury	10.40	10.33	- 0.74%

Notes: Simulate a overall 20% quantity cut in primary good. Prices are reached endogenously, consistent with non-homothetic preferences.

For those people, the marginal gain from consuming primary good post shock is higher than consuming an extra good $j \in \mathcal{I} \setminus Primary$. As a result, a drop in demand occur for those who stick their demand into primary goods. To satisfy their budget constraint, those people would then cut their demand for Normal and Luxury goods as illustrated in Figure 5 where prices dropped due to lower demand.

Overall, primary good price goes up by 58%, normal good price drops by 0.08% and luxury good drops by 0.74% for a 20% cut in primary goods quantity. Since supply shock is experienced instantaneously, it simulates a drop in overall utility perceived by those who consume relatively more primary goods in share due to a costlier after shock consumption bundle. Indeed, they are not fully compensated by a higher revenue coming from their endowment as prices are sticky in the short term. Figure 6 display on average 3.5 percent points instantaneous welfare loss difference between first quintile and fifth quintile household following a supply shock in primary goods.

Following an overall supply shock on primary goods, households are impacted disproportionately. First income quintile household lose the most with on average 6% welfare loss. It is mainly due to the fact that these households are holding/consuming a large relative primary good compared to the richest in the distribution. A supply shock in primary goods translates into less wealth dedicated to utility for those who hold a lot of them. Consistent with non-homothetic preferences, this supply shock translates into a 58% price increase in primary goods which further dig into household purchasing power.

However, these conclusions are not consistent if I impose flexible prices. In the latter case, those who benefit are the ones who detain a large quantity of primary goods in their vault because their goods is currently on high demand. Their endowment level is then reevaluated at a higher market value and offer the possibility to redistribute welfare toward those who detain scarce goods, explained in appendix A.3.

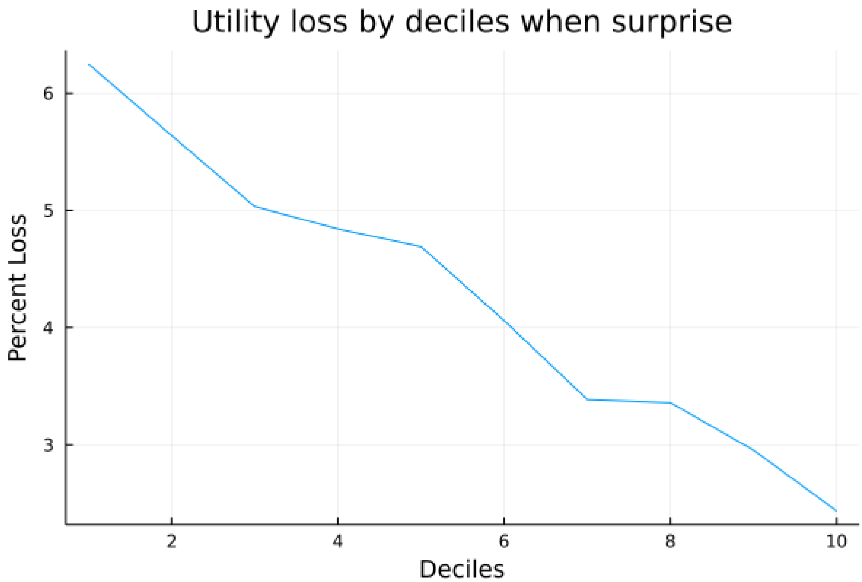


Figure 6: **Post shock welfare loss**

Notes: This figure plots welfare loss across deciles after a supply shock. I group people by deciles to obtain a mean loss across deciles.

3.2 Decentralized Competitive Equilibrium

First note: whenever I put a bold character, it represents a collection of term across households and sector or it is the value at the aggregate level for economy $\mathcal{E}_d$. The following problem will be in discrete time, I further provide the same problem in continuous time in appendix A.5.

3.2.1 Households.

I present a decentralized competitive equilibrium with a distribution of labor productivity and wealth at the household level. Households can then form their consumption/saving decisions with capital and labor revenue as counterparts. I further add non-homothetic preferences to account for wealth effect in consumption bundle composition.

Let us first denote a collection of consumption $\mathbf{C}_t$ across sectors at time t such that $(C_{1t}, ...C_{It}) = \mathbf{C}_t$. Each household in economy $\mathcal{E}_d$ is then maximizing their utility under constraint with non-homothetic preferences:

$$\begin{array}{lc}\max_{\{\mathbf{C}_t, B_{it+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(\mathcal{C}_t) & \\ \text{s.t.} & w_t L_{it} + (1+r_t)B_{it} - B_{it+1} - E_t(\mathbf{C_t}, \mathbf{P_t}) \geq 0 \\ & E_t(\mathbf{C_t}, \mathbf{P_t}) - \sum_{i=1}^{I} P_{it} C_{it} \geq 0 \\ & B_{it} \geq 0 \end{array} \tag{7}$$

Where $E_t(\mathbf{C_t}, \mathbf{P_t})$ correspond to the expenditure level with $\mathbf{P_t} = (P_{1t}, ...P_{It})$, a collection of prices across sectors. Their demands are non linear, they consume more elastic goods as their income grows so we still have the implicit intratemporal utility of the following form with $\mathcal{C}_t = F(\mathbf{C_t})$, a value obtained from a composite consumption function with non-homothetic preferences:

$$\sum_{i}^{I} \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_{it}}{g(\mathcal{C}_t)^{\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 1. \tag{8}$$

The problem is consequently two folds. I have first the intratemporal problem with non-homothetic demand solved by finding the value $\mathcal{C}_t$ from the consumption bundle $\mathbf{C_t}$ chosen by the households via maximising $\mathcal{C}_t$ under budget constraint. Secondly, this intratemporal value is used in the intertemporal optimization problem where household are forming their consumption/saving problem. I have thus $u(\mathcal{C}_t) = \frac{\mathcal{C}_t^{1-\theta}}{1-\theta}$, where θ is the inverse of intertemporal elasticity of substitution.

I can then a standard recursive form utility function with nested non-homothetic preferences. $\mathcal{C}_t$ is consequently solving the static optimization problem with non-homothetic demand whereas u correspond to the instantaneous intertemporal utility at time t from non-homothetic utility value $\mathcal{C}_t$ obtained at each time t. Using the derivation displayed in the appendix A.2.3 equation (22), I see that we can solve for the value of $\mathcal{C}_t$ by having the level of expenditure, the non-homothetic demands across goods and the price vector . I therefore obtain $\mathcal{C}_t$ as function of $\mathbf{E_t}$, $\mathbf{C_t}$ and $\mathbf{P_t}$ so that $\mathcal{C}_t = \mathcal{U}_t(\mathbf{E_t}, \mathbf{C_t}, \mathbf{P_t})$. The intertemporal problem take thus this form become

$$\max_{\{B_{it+1}\}} \sum_{t=0}^{\infty} \beta^t u(\mathcal{U}_t(w_t L_{it} + (1+r_t)B_{it} - B_{it+1}, \mathbf{C_t}, \mathbf{P_t})) \tag{9}$$

I consider a normal distribution for household productivity so that $L_t \sim \mathcal{N}(\mu, \sigma)$ and stands for household labor efficiency. Households are then providing their labor productivity in exchange of w_t. Households are randomly assigned to each sector so that the distribution of productivity is identical across sectors. There is no arbitrage across sectors so there is no premium over working in a specific sector. Consequently, there is unique wage w_t at which labor market across sectors clears.

For now let us also suppose that there is a distribution of wealth that provides capital to the production sector via households borrowing/lending system. For a given date and a given sector of activity, households aggregate capital level correspond to $\sum_{h=1} B_{it} = B_{it}$. Capital remuneration for households r is then obtained through aggregate market conditions regarding supply and demand of capital at a given time and how labor and capital are substitutables.

From above (9) equation, the value function takes the following form:

$$V_t(B_{it}) = \max_{B_{it+1}} \quad u(\mathcal{C}_t) + \beta V_{t+1}(B_{it+1}). \tag{10}$$

From standard discrete optimization, I can provide necessary and sufficient conditions for $\{B_{it+1}\}_{t=0}^{\infty}$ so that it coincides with the solution. I obtain the Euler equation by exploiting the first order conditions and envelope theorem. Together with the transversality conditions we obtain:

$$\begin{aligned} u'(\mathcal{C}_t)\frac{\partial \mathcal{U}_t}{\partial \mathbf{E_t}} &= \beta(1+r_t)u'(\mathcal{C}_{t+1})\frac{\partial \mathcal{U}_{t+1}}{\partial \mathbf{E_{t+1}}} \\ \lim_{T\to\infty} \beta^T(1+r_t)B_{it}u'(\mathcal{C}_t)\frac{\partial \mathcal{U}_t}{\partial \mathbf{E_t}} &= 0 \end{aligned} \tag{11}$$

Replacing the terms with $u'(C_t) = \mathcal{C}_t^{-\theta}$ and $\frac{\partial \mathcal{U}_t}{\partial \mathbf{E_t}} = \frac{\partial \log \mathcal{U}_t}{\partial \log \mathbf{E_t}}\frac{\mathcal{U}_t}{\mathbf{E_t}} = \varepsilon_t^{G/E}\frac{\mathcal{U}_t}{\mathbf{E_t}}$, where $\varepsilon_t^{G/E}$ correspond to the elasticity of expenditure level ($\mathbf{E_t}$) on intra temporal consumption bundle utility level ($\mathcal{U}_t$), I get

$$\begin{aligned} \frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} &= \left[\beta(1+r_t)\frac{\frac{\partial \mathcal{U}_{t+1}}{\partial \mathbf{E_{t+1}}}}{\frac{\partial \mathcal{U}_t}{\partial \mathbf{E_t}}}\right]^{\frac{1}{\theta}} \\ \frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} &= \left[\beta(1+r_t)\frac{\varepsilon_{t+1}^{G/E}}{\varepsilon_t^{G/E}}\frac{\mathcal{U}_{t+1}}{\mathcal{U}_t}\frac{E_t}{E_{t+1}}\right]^{\frac{1}{\theta}} \\ \lim_{t\to\infty} \beta^t(1+r_t)B_{it}\mathcal{C}_t^{-\theta}\varepsilon_t^{G/E}\frac{\mathcal{U}_t}{E_t} &= 0 \end{aligned} \tag{12}$$

3.2.2 Firms.

Multiple firms produce good $i \in \mathcal{I}$ in sector i by hiring capital and labor. They act competitively within their own sector. Returns to scale are then constants and firms are price takers. We can thus take a representative firm within each sector and present its following Cobb Douglas form:

$$\forall i \in \mathcal{I}, \quad Y_{it} = A_i K_{it}^{\alpha} L_{it}^{1-\alpha} \tag{13}$$

where A_i is the sector specific productivity, α captures capital intensity, L_{it}, K_{it} are the sum of labor and capital provided by households $h \in \mathcal{H}$ in specific sector i. Indeed, $\sum_{h=1} L_{it} = L_{it}$ and $\sum_{h=1} B_{it} = K_{it}$. Firms are then solving static profit maximization problem, where Π_t stands for profits at each date t.

$$\forall i \in \mathcal{I}, \quad \max_{K_{it}, L_{it} \geq 0} \Pi_t = P_{it}Y_{it} - R_t K_{it} - w_t L_{it} \tag{14}$$

where P_i is the price of sector i good and $R = r + \delta$ captures the cost of capital as inputs with capital depreciation. First order conditions are then obtained for all sectors such that $\{r_t, w_t\}$ are obtained at the aggregate level. I can thus define the labor market wage, with r at the equilibrium

$$r_t = A\alpha\left(\frac{\mathbf{L_t}}{\mathbf{K_t}}\right)^{1-\alpha} - \delta,$$

$$w_t(r) = A(1-\alpha)\left(\frac{A\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

where L_t, K_t results from the aggregate labor and capital provided by households

$$\sum_{h=1}\sum_{i=1}^{I} L_{it} = \mathbf{L_t},$$

$$\sum_{i=1}^{I} B_{it} = \mathbf{B_t} \equiv \sum_{i=1}^{I} K_{it} = \mathbf{K_t}.$$

3.2.3 Market clearing

Let $\mathbf{C_t} = (C_{1t}, ...C_{It})$ be a collection of aggregate consumption level from all sectors at the household level, forming a aggregate consumption bundle $\mathbf{C}$ at each date t. Let us also denote $\mathbf{Y_t} = (Y_{1t}, ..., Y_{It})$. Consider also $\mathbf{P_t} = \{P_{it}\}_{i=1}^{I}$, a collection of price for all sectors $i \in I$ at a given date t. Decentralized competitive market equilibrium in our economy $\mathcal{E}_d$ consist of:

- $\{\mathbf{C_t}, \mathbf{B_t}, \mathbf{L_t}, \mathbf{K_t}, \mathbf{P_t}, R_t, w_t\}_{t=0}^{\infty}$ such that $r_t = R_t - \delta$ at each date t
- $\{\mathbf{C_t}, \mathbf{B_t}\}_{t=0}^{\infty}$ solves households intertemporal optimization problems (7) given B_0 and $\{\mathbf{P_t}, r_t, w_t\}_{t=0}^{\infty}$
- $\{\mathbf{K_t}, \mathbf{L_t}\}_{t=0}^{\infty}$ solves firms profit maximization problems (14) given $\{\mathbf{P_t}, R_t, w_t\}_{t=0}^{\infty}$
- Markets must clear with aggregate non-homothetic demands equalizing aggregate production level for all our sectors with $\{\mathbf{P_t}, R_t, r_t, w_t\}_{t=0}^{\infty}$, so:
  - $\mathbf{L_t} \sim \mathcal{N}(\mu, \sigma)$ which is iid.
  - $\mathbf{K_t} = \mathbf{B_t}$ Saving provided are used by firms to produce goods.
  - $\mathbf{Y_t} = \mathbf{C_t}$, Non-homothetic demands equal Production.

3.2.4 Numerically solve problems

I adapt Benjamin Moll discrete heterogeneous agent model codes in Julia written by Tom Sweeney such that non-homothetic preferences for goods are considered. As in the endowment model 3.1, I take 3 goods: Primary, Normal and Luxury with elasticities below, equal and above 1. I calibrate the model the same way as in section 3.1.3. My problem has now an intertemporal dimension in which Euler equation is considered to account for household saving decisions. I provide solutions with Endogenous Grid Point (EGP) Method Iteration and Euler Equation Iteration (EE).

The main challenge from this problem is the fact that intratemporal utility $\mathcal{C}_t$ is a composite utility in which we have 3 different goods. To obtain this value, household cash in hand is the way to go with salary and capital revenue. Following this I obtain a first guess for the intratemporal utility which has to be updated since saving decisions are taking into account in the intertemporal problem. Lastly non-homothetic demands derived from the intratemporal problem have to be equal to the total production by all sectors so that market clears. At the end I have a general equilibrium consistent with non-homothetic preferences. All of these ingredients help to solve the Bellman Equation for the intertemporal utility problem as described in section 3.2.

Steps with codes here:

I adapt the algorithm such that non-homothetic preferences are included in the solving procedure. I use the following steps:

- Compute income distribution following a $\mathbf{L_t} \sim \mathcal{N}(1, 0.2)$, set an asset grid, define the number of households L=10000 and time horizon T=500.
- Define tolerance values used for the iterating over households saving problem, firm production with non-homothetic preferences coming from the households.
- Define market size in quantity for primary (35%), normal (35%) and luxury (30%) sector so that total production in quantity proportional to market size. Capital and Labor will then be used according to this size factor.

- Define guesses for the 3 sectors as well as intratemporal utility consistent with non-homothetic preferences

-
  - **Endogenous Grid Point method (used in general and partial equilibrium sections (3.2.5, 3.2.5):** I use intertemporal utility function invertibility to compute current a guess for the intratemporal value from the Euler equation
  - **Euler Equation Iterations (used in partial equilibrium section 3.2.5):** Compute cash in hand for the household and compile a saving decision consistent with the Euler Equation.

- Iterate until convergence and prices consistency with production level and demands.

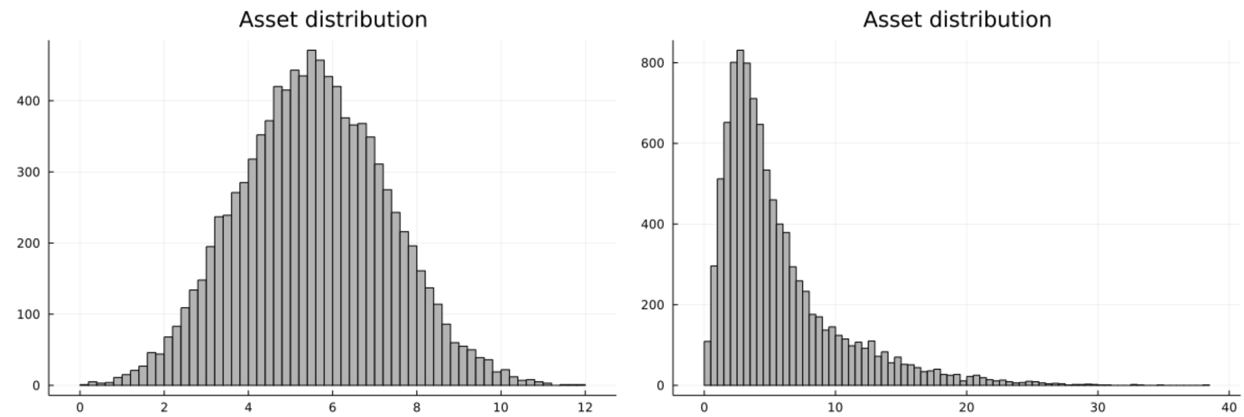
3.2.5 Results

General equilibrium. (GE)

I first compare my non-homothetic (NH) preferences model with an Aiyagari model on a general equilibrium setup. solved via endogenous grid point, I find an overall saving slightly higher on average in the Aiyagari model. I relate it to 2 main factors:

- There are 3 prices in the NH model different from unity and good have their income elasticity. It permits to form consumption bundle via relative prices and income channels. Cheaper goods might then be used extensively by everyone to acquire additional welfare while low income household consume more due to their preferences for less income elastic good (primary goods). It is the opposite for wealthier households.

- People save in the Aiyagari model for precautionary saving. In the NH model, precautionary saving also exist but act differently on household along the distribution. In this model, poor save more because it yields a higher intertemporal value as saving is subsequently used to acquire additional revenue in the next period so that the household is rewarded from consumption more income elastic good. Whereas rich consume more due to a threshold effect.



(a) NH Preferences model with 3 sectors

(b) Aiyagari model

Figure 7: **Comparison between NH with 3 sectors and Aiyagari Model**
Notes: These figures shows saving/asset distribution for non-homothetic (NH) preferences intertemporal consumption saving problem and an Aiyagari model. NH model has 3 different goods in which households consume whereas in the Aiyagari model there is a unique consumption good. 10000 households are considered.

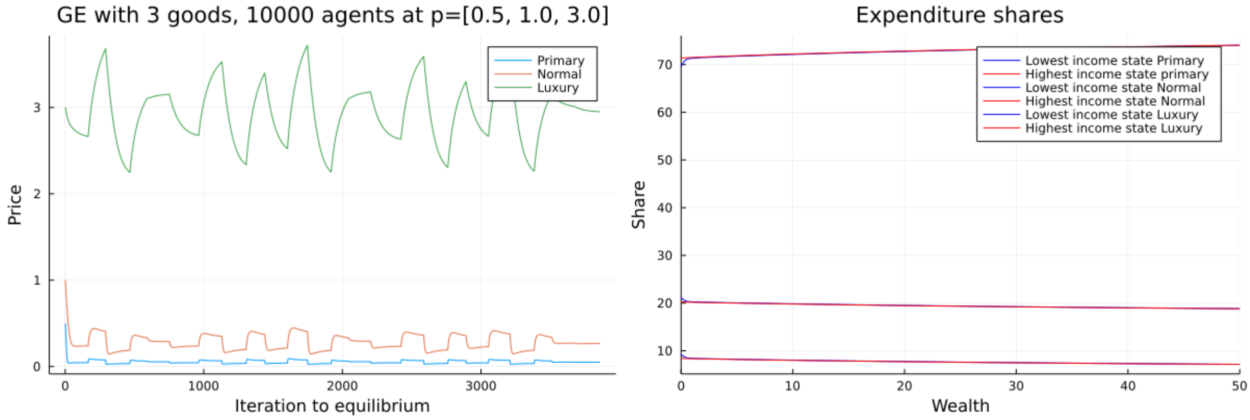
Figure 7 shows how these models differ in asset distribution. NH model displays a normal distribution with few people with low capital level while Aiyagari model display a Lognormal right skewed distribution. Table 2 shows households save on average 5.652 unit of capital in the Aiyagari model while this amount is at around 5.528 for the NH model. I also notice a tendency for richest households to spend more in the NH model. I attribute this to first a threshold effect in wealth. Any additional savings would then be dominated by a preference to consume goods (with a higher preferences for luxury ones as they are richer). Secondly, prices are set consistently with non-homothetic preferences, hence mass of people along the distribution play a role in shaping price level. Indeed, a low equilibrium price encourage richest to consume more by relative price effect. For those reasons, savings go down in the NH model for the richest while people at the bottom of the distribution save more. Median household in the NH model acquire 5.507 units of capital whereas in the Aiyagari model only 4.205 units. There is a even higher difference for household at the 10th percentile.

Table 2: non-homothetic model vs Aiyagari model

Components/Methods	NH EGP with market clearing price	Aiyagari model
Price	[0.05, 0.267, 2.950]	1
Computation time in seconds	30400	80
Interest rate	4.31%	4.15%
Average asset level	5.528	5.652
Asset 10th Percentile	3.281	1.571
Asset 50th Percentile	5.507	4.205
Asset 99th Percentile	9.647	22.132

Notes: Both methods use Endogenous Grid Point methods (EGP). There is income risk in both setup. NH models has 3 different goods. **NH model run overnight so I don't have a precise estimate of computation time (8.44 hours but has likely stopped)**. Since it runs forever, I could not try again for better estimates.

Note that these goods have different income elasticity and availability. In our simulation, luxury good is scarce and income elastic while other are equally abundant but with an income/wealth elasticity below 1 for the primary good. As such, prices are higher for luxury goods and the lowest for primary goods. Figure 8a displays prices path until market clearing convergence. Primary good is the the least preferred good even if it is as abundant as normal good in quantity. These different prices feed into household consumption choice by relative prices and income/wealth elasticity. Indeed, figure 8b shows how consumption bundle differs by asset level. Household with the highest capital level are those who would spend a higher share in luxury good relatively to low capital level. I also see that expenditure share is income risk dependant. Households with the lowest asset level who experience a lower income state would spend most of their revenue into normal and primary goods whereas people with at least a bit of capital won't change much the course of their expenditure even if a low income state realizes. It is emphasized by the extremely small blue spike in figure 8b when asset is around 0 and a drop in the share of luxury good in the expenditure.



(a) Price path for NH model with 3 sectors (b) Consumption differs by income and wealth

Figure 8: **Prices and consumption pattern**

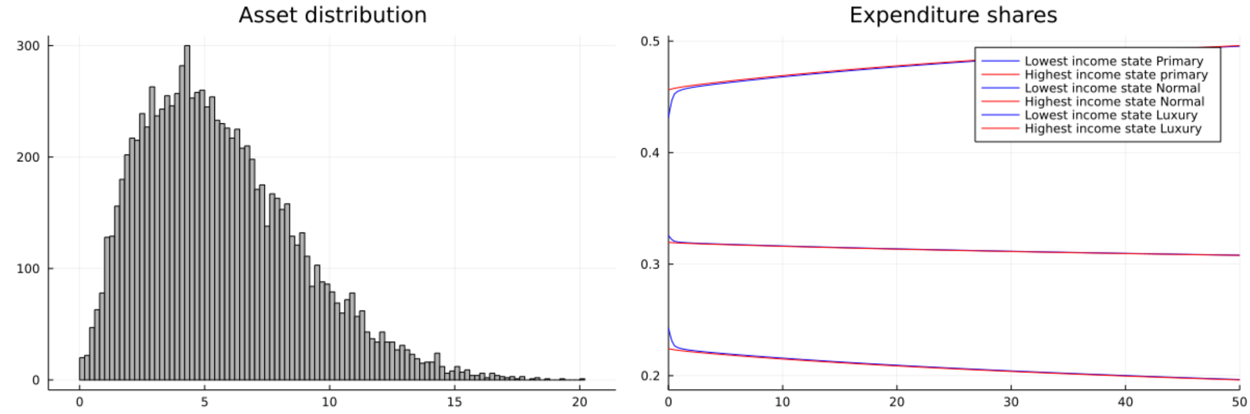
Notes: Figure 8a shows price path after first guess in the NH model solved with endogenous grid point. First guess is p=[0.5, 1.5, 3.0]. Market clearing price p=[0.05, 0.267, 2.950]. Figure 8b shows Expenditure shares for different asset grid points with non-homothetic preferences.

I observe good features from the NH model to account for a preference in income/wealth elastic good when households are rich. Consumption bundle are then constructed at the household level with income/wealth specific shares. It permits to account for household level heterogeneity and further rings a bell with section 2 where households are exposed differently to inflation via their consumption bundle. Aiyagari [1994] emphasized the presence of precautionary saving. My feeling is I further add a feature reasons for households to save by including a preference for high-end good when household is rich. There is then an added value at the intertemporal level from acquiring additional saving not only as buffer stocks as in Aiyagari but also to reach higher welfare by consuming more income elastic good through savings.

Partial equilibrium. (PE)

I compute now a partial equilibrium in which I choose a price level on the 3 goods as the one that clears the market. According to table 3, asset level is on average lower compared to an Aiyagari model used in section 3.2.5. In this section I do not let prices in the NH models to be a leading factor in households consumption saving decisions which is why saving is just slightly lower in the NH models with Euler Equation and Endogenous Grid point methods. As a matter of comparison, adding prices set consistently with non-homothetic preferences account on average for 2% less assets (5.528 (GE) /5.638 (PE) - 1.0). Additionally poorer households save more due to an intertemporal preferences for luxury good. It is more reflected in the model with non-homothetic preferences with consistent price level. Median households is indeed saving more 5.507 units in general equilibrium versus 5.165 units in partial equilibrium.

Lastly richer households are saving more than in general equilibrium because prices consistent with non-homothetic preferences don't play a role. They are then interested in consuming everything but can't benefit from low prices in primary and normal goods as in general equilibrium case. For people at the 99th percentile, prices dynamics in general equilibrium account for 33% of the drop in assets.



(a) Price path for NH model with 3 sectors (b) Consumption differs by income and wealth

Figure 9: **Prices and consumption pattern**

Notes: Figure 9a shows partial equilibrium asset distribution with endogenous grid point. I set p=[0.5, 1.5, 3.0] as price vectors that clear the market. Figure 9b shows Expenditure shares for partial equilibrium with non-homothetic preferences.

Table 3: Partial equilibrium comparison.

Components/Methods	Endogeneous Grid point	Euler Equation	Aiyagari
Price	[0.5, 1.5, 3.0]	[0.5, 1.5, 3.0]	1
Computation time in seconds	181	139	80
Interest rate	4.18%	4.18%	4.15%
Average asset level	5.638	5.625	5.652
Asset 10th Percentile	1.963	1.919	1.571
Asset 50th Percentile	5.165	5.172	4.205
Asset 99th Percentile	14.336	14.510	22.132

Notes:[0.5, 1.5, 3.0] is imposed in both methods. EGP and EE methods have non-homothetic preferences. Both methods reach similar distribution.

Productivity shock.

I perform a 5% increase in overall production productivity. Before that productivity was equal to 1.00, now I increase it to 1.05. Prices for goods are at [0.5, 1.5, 3.0][7]. Table 4 serves for comparative static comparing before and after productivity shock.

With productivity shock, households earn additional earning and can save and consume more from wage and interest rate linked to this productivity shock. Consequently, it leads to an overall increase in savings as people are now richer and display an intertemporal preference for luxury good as poorer households are also saving more in NH model. A 5% increase in productivity led to 8.8% increase in savings. Shock effect on savings is likely to have compounded due to a intertemporal desire to acquire luxury goods. Figure 10 shows widely similar distribution but higher on average asset unit per capita after productivity shock. I provide additional graphs for consumption, savings decisions before and after shock in the appendix A.4.

Post shock, households are then reacting differently along the distribution. Poorest tend to save more to acquire more income elastic goods whereas richest are consuming because

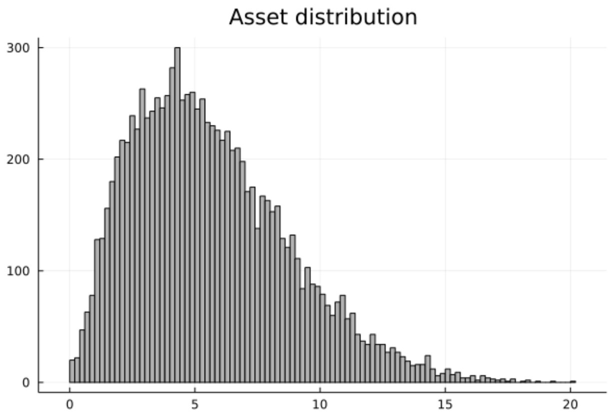
[7]I decide to impose prices here because it could run forever as shown in 3.2.5.

basic needs have been satisfied even after controlling for precautionary savings. In such an environment, richest consumption bundle is less intensive in primary. As such, they are less impacted by inflation if there is a price surge in primary goods. Meanwhile consumption bundle for the poorest are primary good intensive.

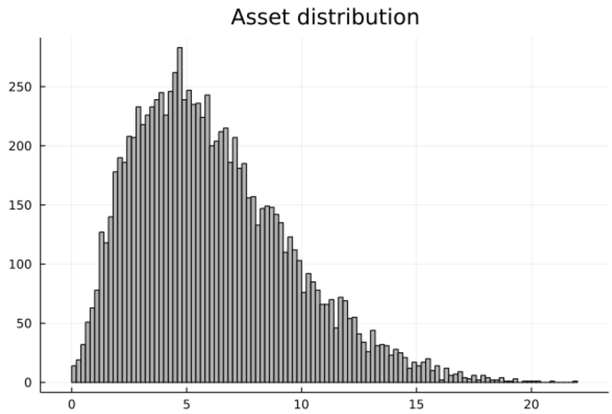
Table 4: NH models before and after productivity shock.

Components/Methods	NH EGP before	NH EGP post shock
Price	[0.5, 1.5, 3.0]	[0.5, 1.5, 3.0]
Computation time in seconds	181	367
Interest rate	4.18%	4.15%
Average asset level	5.638	6.134
Asset 10th Percentile	1.963	2.182
Asset 50th Percentile	5.165	5.605
Asset 99th Percentile	14.336	15.614

Notes:[0.5, 1.5, 3.0] is imposed in both methods. EGP and EE methods have non-homothetic preferences. Both methods reach similar distribution.



(a) Before



(b) After

Figure 10: **Before and after productivity shock**

Notes: Figure 10a shows partial equilibrium asset distribution with endogenous grid point before shock. I set p=[0.5, 1.5, 3.0] as price vectors that clear the market. Figure 10b shows asset distribution after a 5% increase in overall productivity.

4 Policy implications

Using household expenditure survey from Eurostat, I observe a higher inflation exposure for households at the first quintiles in the Euro Area when weighted to consumption bundle belonging to them. This is not new as several authors have used CEX and CPI from US Bureau of Labor Statistics (Mejia and Hartley [2022], Hobijn and Lagos [2005], Jaravel [2021]). Even if it corresponds to five data points the cumulative price index is higher for each year considered. What could be recommended at this stage is to conduct surveys for household expenditure at least once a year. For now it is only done every 5 years with at least 1 year lag to be official figures. As such, we cannot look for instantaneous (1 year lag) consumption bundle composition with respect to household characteristics like age, wealth, income or even degree of urbanisation via publicly available data. Policy decisions to support these household exposure to inflation are then delayed because people are less informed. A good thing is that it has been done in 2020 where section 2 reflects this clear change in consumption bundle composition due to multiple lockdowns. Consumption weights for the average household has been updated to account for more consumption in foods and housing during lockdowns and less for recreation and travels. Consequently, I recommend further effort into constructing these surveys in order to provide additional information publicly available. Price indices are already at the monthly level, why can't we try to coordinate EA country member to conduct these surveys at least on an annual basis like the US Bureau of Labor Statistics?

With different consumption bundle come different inflation exposure. In times where goods are in shortage, those who depend on it should be the first targeted with transfers so that saving is not the only way to insure themselves intertemporally. Indeed in the NH model shown, overall equilibrium saving level is higher on average compared to an Aiyagari model where precautionary saving has already been emphasized for insuring against income risk. In the NH model, saving is even higher because households prefer to save in order to consume income elastic good as it provides them extra utility. For now, I did not try to include a government block but it is worthy effort for future research. Indeed, I expect taxes to distort consumption/ saving in a way that smooth saving so that we do not incur too much saving in the economy. Lump sum transfers would then permit to compensate needs in acquiring luxury goods. In such a case, households would then not be bound intertemporally to their saving decisions, rather they could then form decisions based on taxes and transfers from the government.

Channel not exploited in my model is a preference for goods with stable prices. In such an environment, households are inclined to save not only for precautionary savings but also in insuring themselves against goods with relatively high price fluctuations. Suppose a supply shock in which an economy lose a trade term with a net supplier country. Due to supply shortage, prices surge. In such a case, households relying extensively on this commodity is the most impacted in their daily life. What households could do to avoid such a real term loss is to buy less or substitute with another consumption good less inclined to such a surge in prices. A preference for less price volatile good is then worthy to study in future research, I would call it an elasticity of substitution across goods for household with price stability goal. It measures how much households are willing to exchange their initial quantity of consumption from a price volatile good to another with less price volatility.

In a situation where there is a supply shortage for an essential commodity, policy makers are favorable to preserve purchasing power and prevent price surge with price caps and lump-sum transfers to the most vulnerable. Current situation with soaring energy bills rings a bell to that kind of situation where the most vulnerable are protected by energy bills support (transfers) and energy price caps like the tariff shield imposed until 2023 in France. In the short run, it acts as a shield to prevent soaring prices but it is costly in the long run and won't make people sober in the long run. Indeed, price cap are mainly contract for differences (CfDs) and do not produce the incentives for changes in consumption patterns, rather it could strengthens the demand on energies while costing a lot for the state. In a previous presentation for my PhD project, I have notified an interest in designing policies for shift in consumption patterns. If the richest are less exposed to inflation then I argue that it could great to induce people at the bottom of the distribution to mimic consumption bundles with stable prices. We should then direct consumption change with right incentives. Planner could subsidy preferred consumption good or tax goods that he does not recommend. Goals for the planner extend from health, green energy by encouraging cleaner and sober use or even ask people to buy price stable goods so that their consumption bundle price is stickier.

As a result inflation exposure would be unequal along the distribution.

5 Discussion, Caveats

Analysis provided above is for sure not immune to criticism as it assume a surprise supply shock without any possible adjustment from households side. Indeed I impose fully sticky prices when the supply shock occur and assume revenue channel cannot compensate real term losses incurred by more expensive consumption bundles. In the endowment model presented in section 3.1 if I let the market clear at the price post shock, then the main winners are people who possess high endowments in primary goods. Indeed their goods, consistent with market availability and non-homothetic preferences, is highly asked so the price of it is higher. It translates into an additional revenue for the household that discover that his endowment is now highly valued by the economy.

In the decentralized competitive economy, I cannot say that fully sticky wages is consistent as a supply shock might been offset in the long run by households who ask for an additional return from their work and thus keep welfare constant in real term. However, we can see that if I let the market clear at post shock, people are here not endowed with goods but rather buy with their capital and labor revenue. In this case, when income are rigid, the one who lost the most from the supply shock are the one at the bottom of the distribution in terms of revenue. Due to their lack of resources, they are constrained to consume the less income elastic good and thus have a higher share of their revenue into primary needs. I see that even after the shock, people have a tendency to shift their consumption from necessities to others as it is now more expensive to consume necessities. However the overall wealth effect overtake the substitution effect so that people at the bottom of the distribution are still consuming necessities and are damaged by a higher price in their prefered commodity.

Additionally, I have not covered all income channels through which households can earn, especially in the long run. I have for now focused on telling inflation are the highest for poorer households because their consumption bundle is composed of highly price volatile goods, which is not enough. Doepke and Schneider [2006], Auclert [2019] demonstrate how inflation can redistribute wealth through Fisher credit channel. If we take the fact that young and poor households borrow more than old and rich person. There is then a redistribution in wealth from old and rich to young and poor after a surprise inflation. Indeed, most of the contract are in nominal terms and not adjusted directly to inflation. If rigid contracts are not adjusted to suit inflationary pressure, then the main property holders, old person tend to lose their wealth in the long run. In line with these results are Meh and Terajima [2011], Adam and Zhu [2016], Cardoso et al. [2022]. As a result is not sufficient to conclude that poorest are hit the hardest on the consumption side without taking care of the asset side inflation exposure.

Lastly, my algorithm are not optimized to handle large consumption structure. As shown in the numerical simulation, it has difficulty to converge and find the fixed point for price level when I use Euler equation consistent with non-homothetic preferences. Intuition given would not hold water if I have failed to include non-homothetic preferences in an Aiyagari model.

6 Conclusion

Non-homothetic preferences are useful to account for consumption non linearity. These properties permit to form characteristics dependent consumption bundle and inflation exposure

- here income/wealth level. Data at the Euro Area level first reveals that households at the bottom of the income distribution have a higher cumulative price level from 2000 to 2020. Additionally, I present an endowment model in which preferences are non-homothetic à la Comin et al. [2021]. Shortage in primary goods lead to a price surge but also a drop in the price of other goods. Shock is transmitted to the whole economy and is less rich. Households are then becoming poorer and are, consistent with non-homothetic preferences, more interested in primary goods. As a result, price surge occur on the market. I simulate that with a 20% supply cut on primary goods and obtained a 58% price surge. Consequently poorer households are first hit by a supply shock in which they lose wealth but also by a price surge as a aftermath.

I observe lower average savings in a general equilibrium models with non-homothetic preferences compared to an Aiyagari model. I link it to a intertemporal preference for luxury goods. Poorer households earn extra utilities by saving more and consume income elastic goods in the next period. While richest households benefit low prices linked to mass-dependent wealth distribution, they would then prefer to consume more as saving needs are already satisfied. As a result, savings increase for poor households and drop for the richest leading to an overall drop in saving (drop for the richest is higher). Consistent with non-homothetic preferences, I attribute a 2% drop in overall saving when price are set endogenously with preferences.

References

K. Adam and J. Zhu. Price-Level Changes and the Redistribution of Nominal Wealth across the Euro Area. *Journal of the European Economic Association*, 14(4):871–906, Aug. 2016. ISSN 1542-4766. doi: 10.1111/jeea.12155. URL https://doi.org/10.1111/jeea.12155.

S. R. Aiyagari. Uninsured Idiosyncratic Risk and Aggregate Saving*. *The Quarterly Journal of Economics*, 109(3):659–684, Aug. 1994. ISSN 0033-5533. doi: 10.2307/2118417. URL https://doi.org/10.2307/2118417.

U. Akkoç and B. Kizilirmak. Household Level Inflation Rates and Inflation Inequality in Turkey. *Business and Economics Research Journal*, 12:17–32, Jan. 2021. doi: 10.20409/berj.2021.308.

Argente and Lee. Cost of Living Inequality During the Great Recession | Journal of the European Economic Association | Oxford Academic, 2021. URL https://academic.oup.com/jeea/article-abstract/19/2/913/5863149.

A. Auclert. Monetary Policy and the Redistribution Channel. *American Economic Review*, 109(6):2333–2367, June 2019. ISSN 0002-8282. doi: 10.1257/aer.20160137. URL https://www.aeaweb.org/articles?id=10.1257/aer.20160137.

J. E. Baez, O. K. Inan, and M. Nebiler. *Getting Real? The Uneven Burden of Inflation across Households in Turkey*. Policy Research Working Papers. The World Bank, Nov. 2021. doi: 10.1596/1813-9450-9869. URL http://elibrary.worldbank.org/doi/book/10.1596/1813-9450-9869.

R. Braun and S. M. Lein. Heterogeneity in Inflation and Preferences across Households. page 57, 2020.

M. Cardoso, C. Ferreira, J. M. Leiva, G. Nuño, Ortiz, and T. Rodrigo. The Heterogeneous Impact of Inflation on Households' Balance Sheets. page 21, June 2022.

Clayton, Jaravel, and Schaab. Heterogeneous Price Rigidities and Monetary Policy by Christopher Clayton, Xavier Jaravel, Andreas Schaab :: SSRN, 2018. URL https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3186438.

D. Comin, D. Lashkari, and M. Mestieri. Structural Change With Long-Run Income and Price Effects. *Econometrica*, 89(1):311–374, 2021. ISSN 1468-0262. doi: 10.3982/ECTA16317. URL https://onlinelibrary.wiley.com/doi/abs/10.3982/ECTA16317. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA16317.

J. Cravino and A. A. Levchenko. The Distributional Consequences of Large Devaluations. page 41, 2017.

M. Doepke and M. Schneider. Inflation and the Redistribution of Nominal Wealth, 2006. URL https://www.journals.uchicago.edu/doi/epdf/10.1086/508379.

M. Giarda and D. Romero. Expenditure Heterogeneity, Nominal Rigidities, and Aggregate Fluctuations. page 36, 2022.

R. Gouvêa. Large devaluations and inflation inequality: Replicating Cravino and Levchenko (2017) with evidence from Brazil. *Journal of Applied Econometrics*, 37(3):657–664, 2022. ISSN 1099-1255. doi: 10.1002/jae.2880. URL https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.2880. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1002/jae.2880.

G. Hanoch. Production and Demand Models with Direct or Indirect Implicit Additivity. *Econometrica*, 43(3):395–419, 1975. ISSN 0012-9682. doi: 10.2307/1914273. URL https://www.jstor.org/stable/1914273. Publisher: [Wiley, Econometric Society].

Hobijn and Lagos. INFLATION INEQUALITY IN THE UNITED STATES - Hobijn - 2005 - Review of Income and Wealth - Wiley Online Library, 2005. URL https://onlinelibrary.wiley.com/doi/10.1111/j.1475-4991.2005.00170.x.

P. Hochmuth, M. Pettersson, and C. Weissert. A Nonhomothetic Price Index and Inflation Heterogeneity. page 44, 2022.

Jaravel. Inflation Inequality: Measurement, Causes, and Policy Implications | Annual Review of Economics, 2021. URL https://www.annualreviews.org/doi/abs/10.1146/annurev-economics-091520-082042.

X. Jaravel and M. O'Connell. Real-time price indices: Inflation spike and falling product variety during the Great Lockdown. *J Public Econ*, 191:104270, Nov. 2020. ISSN 0047-2727. doi: 10.1016/j.jpubeco.2020.104270. URL https://www.ncbi.nlm.nih.gov/pmc/articles/PMC7487746/.

Kaplan, Moll, and Violante. Monetary Policy According to HANK - American Economic Association, 2018. URL https://www.aeaweb.org/articles?id=10.1257/aer.20160042.

G. Kaplan, G. L. Violante, and J. Weidner. The Wealthy Hand-to-Mouth, Apr. 2014. URL https://www.nber.org/papers/w20073.

McGranahan and Paulson. Constructing the Chicago Fed Income Based Economic Index – Consumer Price Index: Inflation Experiences by Demographic Group: 1983-2005 - Federal Reserve Bank of Chicago, 2005. URL https://www.chicagofed.org/publications/working-papers/2005/2005-20.

A. McKay, E. Nakamura, and J. Steinsson. The Power of Forward Guidance Revisited. *American Economic Review*, 106(10):3133–3158, Oct. 2016. ISSN 0002-8282. doi: 10.1257/aer.20150063. URL https://www.aeaweb.org/articles?id=10.1257/aer.20150063.

C. A. Meh and Y. Terajima. Inflation, nominal portfolios, and wealth redistribution in Canada. *Canadian Journal of Economics/Revue canadienne d'économique*, 44(4):1369–1402, 2011. ISSN 1540-5982. doi: 10.1111/j.1540-5982.2011.01678.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-5982.2011.01678.x. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-5982.2011.01678.x.

J. Mejia and J. S. Hartley. Estimating Inflation Inequality. page 15, 2022.

E. Nakamura and J. Steinsson. Five Facts about Prices: A Reevaluation of Menu Cost Models*. *The Quarterly Journal of Economics*, 123(4):1415–1464, Nov. 2008. ISSN 0033-5533. doi: 10.1162/qjec.2008.123.4.1415. URL https://doi.org/10.1162/qjec.2008.123.4.1415.

J. Orchard. Cyclical Demand Shifts and Cost of Living Inequality, May 2022. URL https://papers.ssrn.com/abstract=4033572.

R. Sato. Homothetic and Non-Homothetic CES Production Functions. *The American Economic Review*, 67(4):559–569, 1977. ISSN 0002-8282. URL https://www.jstor.org/stable/1813389. Publisher: American Economic Association.

M. van Kints and R. Breunig. Inflation Variability Across Australian Households: Implications for Inequality and Indexation Policy*. *Economic Record*, 97(316):1–23, 2021. ISSN 1475-4932. doi: 10.1111/1475-4932.12575. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/1475-4932.12575. _eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/1475-4932.12575.

C. J. Weissert. Essays in macroeconomics. 2022.

Z. Zhu, L. Lu, X. Yao, W. Zhang, and W. Liu. Who is suffering most from rising inflation? Feb. 2022. URL http://119.78.100.173/C666//handle/2XK7JSWQ/345707. Accepted: 2022-02-16T09:07:01Z.

A Appendix

A.1 Data

A.1.1 Price indices

Here are the following 12 HICP index per purpose from ECB SDW website: Food and non-alcoholic beverages, Alcoholic beverages, tobacco and narcotics, Clothing and footwear, Housing, water, electricity, gas and other fuels, Furnishings, household equipment and routine household maintenance, Health, Transport, Communications, Recreation and culture, Education, Restaurants and hotels. I then take household expenditure share per purpose from Eurostat with same categories as above price index.

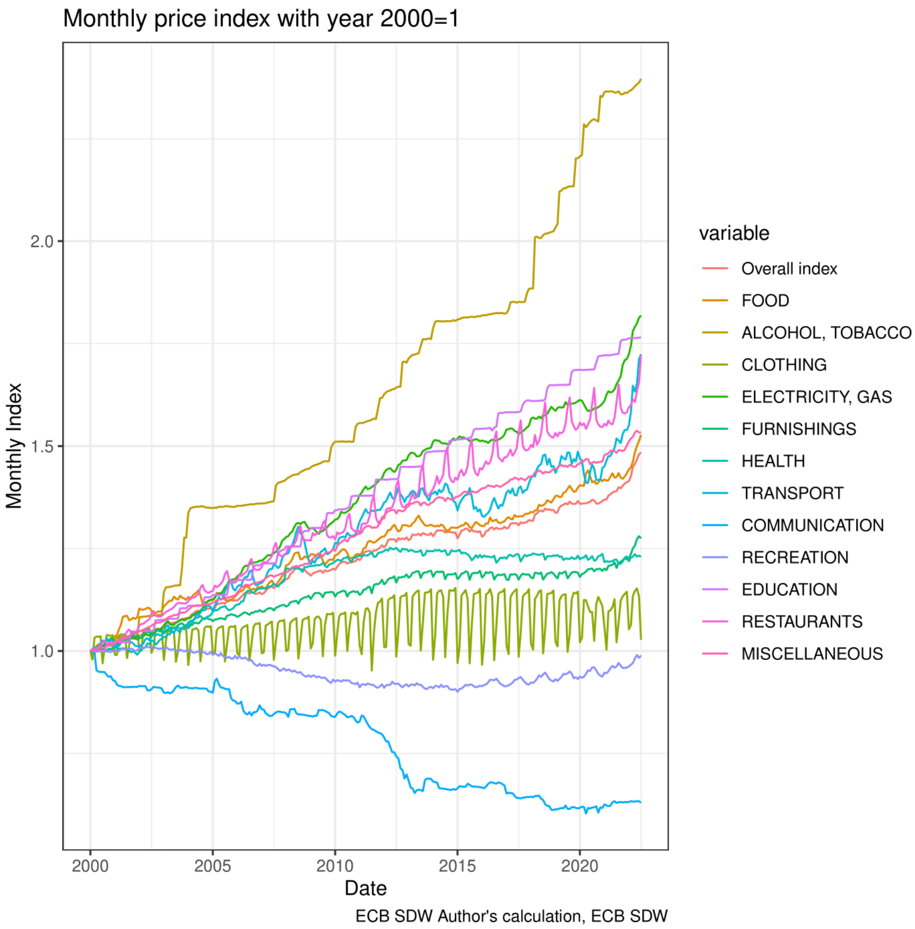
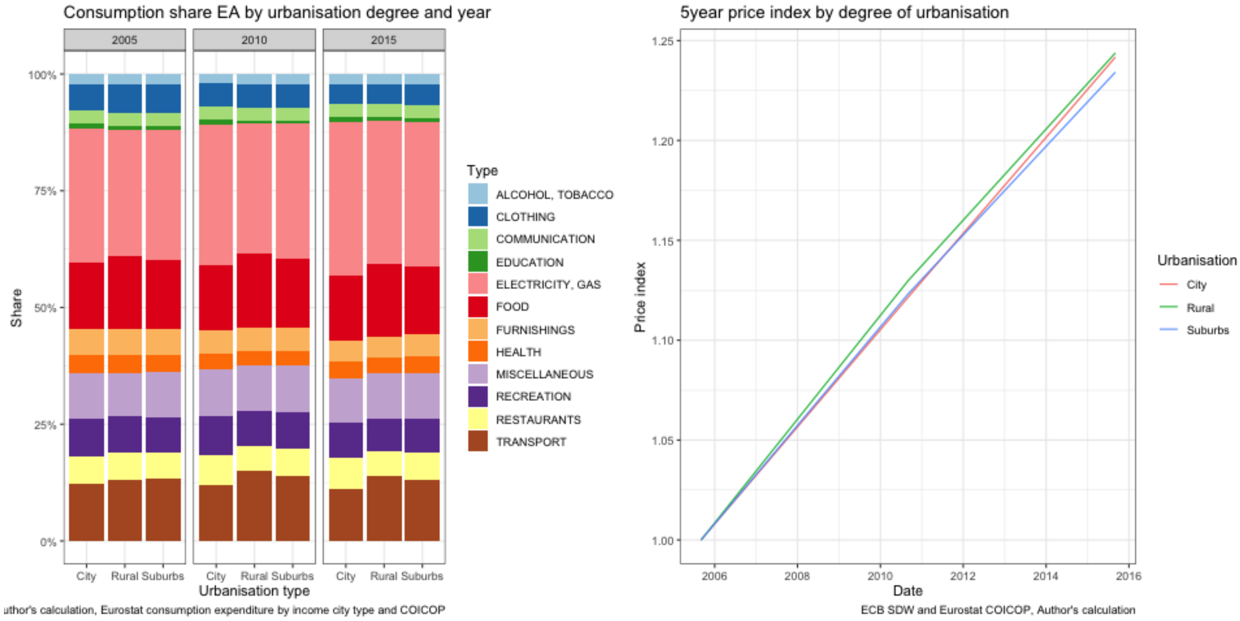


Figure 11: Monthly price indexes from 12 consumption purposes

A.1.2 Other dimensions of heterogeneity in consumption bundle

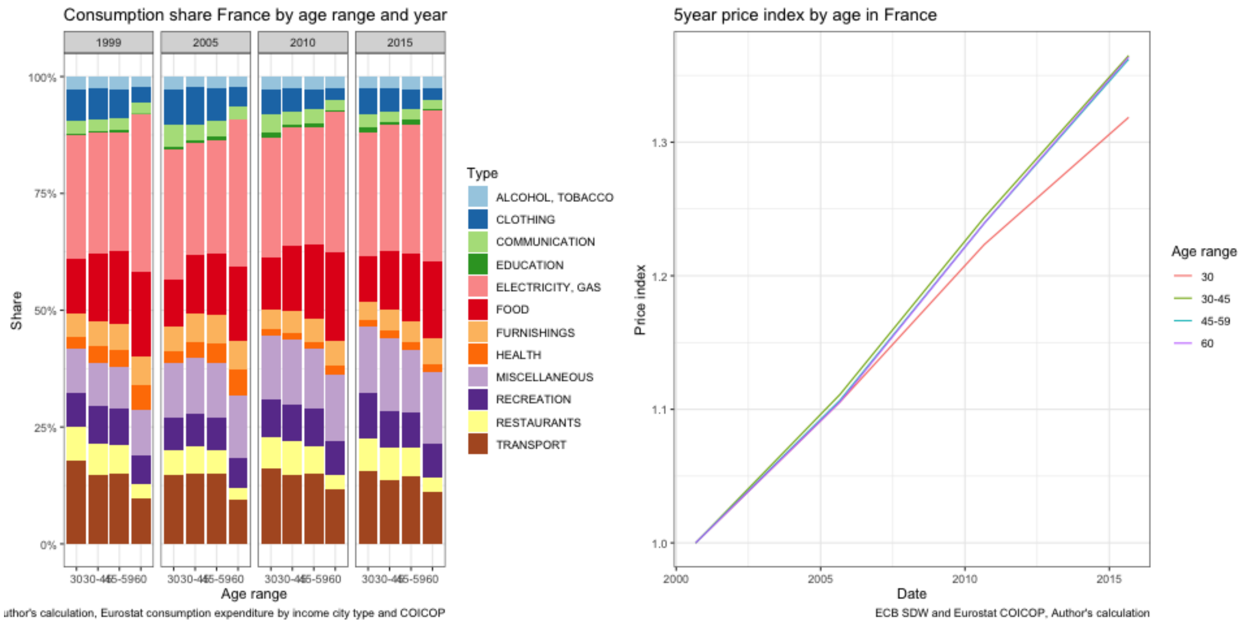
I illustrate how consumption bundle differs across groups of characteristics. I take degree of urbanisation, age and activity status. Due to data restriction, I have only taken France for activity status and age. Urbanisation degree is at the EA level. Please read this file for more details about the sampling. I end up with higher cumulative price level for people in rural areas, professionally active, retired and manual workers. These heterogeneity dimensions are also discussed in Marginal Propensity to Consume literature where people with different characteristics react differently to a sudden windfall in revenue. These different behaviors are also leading to different inflation exposure as consumption bundle at different intensities.



(a) Annual price index per purpose

(b) Consumption share over time

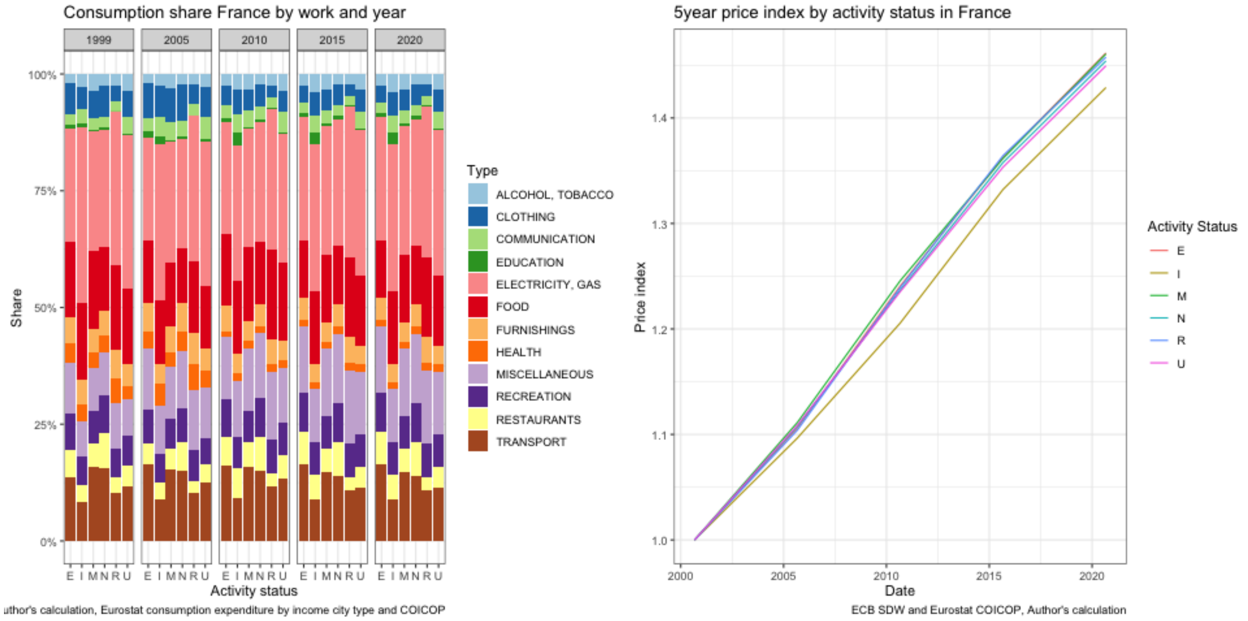
Figure 12: **Consumption bundle share and price indexes by degree of urbanisation**
Notes: Panel (a) shows consumption share for 2005, 2010 and 2015 for 3 type of urbanisation degree : Rural Areas, City, Towns and suburbs. Panel (b) urbanisation degree specific price index available at date 2005 2010 and 2015



(a) Annual price index per purpose

(b) Consumption share over time

Figure 13: **Consumption bundle share and price indexes by age**
Notes: Panel (a) shows consumption share for 2005, 2010 and 2015 for different age range: less than 30, 30-44, 45-59, 60 plus. Panel (b) shows age degree specific price index available at date 1999, 2005, 2010 and 2015



(a) Annual price index per purpose (b) Consumption share over time

Figure 14: **Consumption bundle share and price indexes by activity status**
Notes: Panel (a) shows consumption share for 1999, 2005, 2010, 2015 and 2020 for different activity status range: E: Employed, I: Inactive, M: Manual work, N: Non manual work, R: Retired, U: Unemployed. Panel (b) shows activity status specific price index available at date 1999, 2005, 2010, 2015 and 2020

A.2 Theory

To add an income effect in consumer consumption behavior, an implicit form has to be adopted. I review first the standard homothetic CES preferences and its properties. After that we will look for a model with non homothetic preferences and derives its main implications.

A.2.1 Homothetic CES preferences

Suppose utility defined as a consumption bundle with CES form. Let good be indexed by $i \in \mathcal{I}$. Homothetic preferences will take the following form such that σ correspond to the elasticity of substitution across goods and is designed to be constant. Analoguous to a CES production function, our household utility function is composed of how household choose their consumption among a set of consumption goods in $\mathcal{I}$

$$\mathcal{U}(C_1, C_2, ...C_I) = \left[\sum_i^I \gamma_i C_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

Our utility is then monotonic increasing and concave. It enables a well defined utility function over a consumption bundle $\mathcal{C} = (C_1, ..., C_I)$. γ_i captures the relative weight in household consumption and $\sum_i^I \gamma_i = 1$ so that it accounts for constant return to scale. Consumption bundle consist of having $C_i \geq 1$ when goods are consumed and 0 when not. Utility increases by diversity. In this case, the relative demand across goods depend on the relative price between goods and thus the marginal rate of substitution. By optimizing under constraint, the lagrangian take the following form:

$$\mathcal{L} = \mathcal{U}(C_1, ...C_I) + \lambda[E - \sum_{i=1}^{I} P_i C_i]$$

FOC with respect to $C_i, \forall i \in \mathcal{I}$

$$\mathcal{U}'(C_i) = P_i$$

$$C_i = \mathcal{U}^{-1'}(P_i)$$

When $\sigma = 1$, we get

$$C_i = \frac{\gamma_i}{p_i}\mathcal{U}(C_1, ..., C_n)$$

The demand in good i is then linear in U that is highly linked to household expenditure level E. A share of it is then asked for good i and are asked in relative terms. Namely, we have for all $i, j \in \mathcal{I}$:

$$\log \frac{C_i}{C_j} = \sigma \log \frac{P_j}{P_i}$$

Equivalently with $\frac{C_i}{C_j} = D_{ij}$ and $\frac{P_i}{P_j} = \mathbf{P}_{ij}$

$$\log D_{ij} = \sigma \log \mathbf{P}_{ij}.$$

When $\sigma = 1$ we have our Cobb Douglas utility function of the following form:

$$\mathcal{U}(C_1, C_2, ..., C_I) = \prod_{i}^{I} C_i^{\gamma_i}.$$

This form presents constant return to scale and insures our elasticity of substitution across good is equal to 1 ($\sigma = 1$). Consequently, the relative demand between good i and j is independent of the level of utility and don't tackle preferences concerns. Intuitively, someone with a higher revenue will just consume X times more goods than someone with a poor revenue over the same consumption bundle. This relative demand would then only be defined by the relative product prices and thus the marginal rate of substitution between goods.

A.2.2 Non-homothetic CES preferences

As standard as it can be, I am still maximizing the utility of a given household given his resource constraints. $(C_1, ..., C_{\mathcal{I}})$ correspond to a consumption bundle, E is its expenditure level that coincide with its endowment level at given prices p_i. Finally C_i correspond to the amount of good i consumed by the household.

$$\max_{\{C_i\}_{i=1}^{\mathcal{I}}} \mathcal{U}(C_1, ..., C_{\mathcal{I}}) \quad st \quad E \geq \sum_{i=1}^{I} p_i C_i. \tag{15}$$

To add up an income effect, we are interested in supplying an utility form that takes into account change in consumption behavior across the income ladder. Depending on income/wealth level, people exhibit different preferences over goods. In other words a parameter that captures income elasticity of relative demand would be welcomed. Some goods may not be bought by some individuals due to revenue constraints that cannot satisfy this consumption. Some would be heavily consumed by some income groups, some not at all. Consequently, I permit different consumption bundle to be formed according to their income level. Following Comin et al. [2021], Sato [1977], Hanoch [1975] non homothetic CES utility function adopts an implicit form in which an utility level is reached with minimal cost. Constraints are thus included. Last constraint indicates that consumption is feasible under budget constraint.

Households are then facing an implicit form of non-homothetic utility where this equation holds:

$$\sum_{i}^{I} \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{g(\mathcal{U})^{\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 1$$

where σ stands for the elasticity of substitution and $g(.)$ is a positive-valued, continuously differentiable, and monotonically increasing function. U is an utility formed by a collection of good $\mathbf{C}$ such that $\mathcal{U} = F(\mathbf{C})$. When $g(\mathcal{U})^{\epsilon_i} = U$, we go back to homothetic CES preference. When it is not we have an implicit additive non homothetic CES preference utility function that takes all its ϵ_i as main parameters. It stands for income elasticity regarding relative demand. We assume that $g(\mathcal{U}) = \mathcal{U}^{1-\sigma}$. Finally, when $\sigma < 1$, goods produced by different sectors are complements.

By maximising household utility under budget constraint and satisfying above implicit non homothetic utility equation, we end up with this optimal (refer to Appendix A.2.3 for more details):

$$\forall i \quad C_i = \left(\frac{p_i}{E} \right)^{-\sigma} \gamma_i g(\mathcal{U})^{\epsilon_i (1-\sigma)}.$$

Let $\omega_i = \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{g(\mathcal{U})^{\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}}$ corresponding to the share of good i in household expenditure, then $E\omega_i = p_i C_i$. Total expenditure share and expenditure level for given prices by household h is then

$$\omega_i = \gamma_i \left[\frac{p_i}{E} g(\mathcal{U})^{\epsilon_i} \right]^{(1-\sigma)}. \tag{16}$$

$$E \equiv \sum_{i=1}^{I} p_i C_i = \left[\sum_{i=1}^{I} \gamma_i \left(p_i g(\mathcal{U})^{\epsilon_i} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{17}$$

Relative demand $\forall i, j \in \mathcal{I}$ for household h is then found by taking the log of $\frac{C_i}{C_j}$ from (17) and delivers us:

$$\log \left(\frac{C_i}{C_j} \right) = \sigma \log \left(\frac{p_j}{p_i} \right) + (\epsilon_i - \epsilon_j)(1-\sigma) \log g(\mathcal{U}) + \log \left(\frac{\gamma_i}{\gamma_j} \right). \tag{18}$$

relative demand elasticity with respect to a monotonic transformation of $g(.)$ and prices equal

$$\frac{\partial \log \left(\frac{C_i}{C_j} \right)}{\partial \log g(\mathcal{U})} = (1-\sigma)(\epsilon_i - \epsilon_j) \quad \frac{\partial \log \left(\frac{C_i}{C_j} \right)}{\partial \log \left(\frac{p_j}{p_i} \right)} = \sigma \tag{19}$$

This setup permits to show the income effect behind a consumer decision choice. As shown in (19), I permit now the first term to account for income effect. Quantity wise, the demand for good i will be higher if non homothetic parameter ϵ is higher for good i compared to good j. As usual, price elasticity show the degree of substitutability between good i and j, I now make $\sigma < 1$ so that good are complementary.

A.2.3 Intratemporal Non Homothetic Preferences Derivation

Intratemporal non homothetic preferences household problem 15 takes the following Lagrangian form:

$$\mathcal{L}(\mathcal{U}, C_1, ..., C_I) = \mathcal{U} + \Upsilon \left[1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}}\right] + \lambda \left[E - \sum_i^I p_i C_i\right]$$

FOCs of $\mathcal{L}$ with respect to $\mathcal{U}, (C_1, ..., C_I), \Upsilon, \lambda$ gives:

$$\frac{\partial \mathcal{L}}{\partial \mathcal{U}} = 1 - \Upsilon \frac{(1-\sigma)^2}{\sigma} \sum_{i=1}^I \gamma_i^{\frac{1}{\sigma}} \epsilon_i \frac{1}{\mathcal{U}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0$$

$$\forall i \in \mathcal{I}, \quad \frac{\partial \mathcal{L}}{\partial C_i} = \Upsilon \frac{1-\sigma}{\sigma} \left[\gamma_i^{\frac{1}{\sigma}} \frac{1}{C_i} \left(\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right)^{\frac{\sigma-1}{\sigma}}\right] - \lambda p_i = 0 \tag{20}$$

Then

$$\forall i, j \in \mathcal{I}, \quad \frac{\gamma_i^{\frac{1}{\sigma}} \frac{1}{C_i} \left(\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right)^{\frac{\sigma-1}{\sigma}}}{p_i} = \frac{\gamma_j^{\frac{1}{\sigma}} \frac{1}{C_j} \left(\frac{C_j}{\mathcal{U}^{(1-\sigma)\epsilon_j}}\right)^{\frac{\sigma-1}{\sigma}}}{p_j}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = E - \sum_i^I p_i C_i = 0$$

$$\frac{\partial \mathcal{L}}{\partial \Upsilon} = 1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0$$

System of equation $\mathcal{F}$ should then hold at the optimum such that $\mathcal{F}(.) = 0$.

$$\begin{cases} \frac{\partial \mathcal{L}}{\partial \mathcal{U}} = 1 - \Upsilon \frac{(1-\sigma)^2}{\sigma} \sum_{i=1}^I \gamma_i^{\frac{1}{\sigma}} \epsilon_i \frac{1}{\mathcal{U}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0 \\ \forall i, j \in \mathcal{I}, \quad \frac{\gamma_i^{\frac{1}{\sigma}} \frac{1}{C_i} \left(\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right)^{\frac{\sigma-1}{\sigma}}}{p_i} = \frac{\gamma_j^{\frac{1}{\sigma}} \frac{1}{C_j} \left(\frac{C_j}{\mathcal{U}^{(1-\sigma)\epsilon_j}}\right)^{\frac{\sigma-1}{\sigma}}}{p_j} \\ \frac{\partial \mathcal{L}}{\partial \lambda} = E - \sum_i^I p_i C_i = 0 \\ \frac{\partial \mathcal{L}}{\partial \Upsilon} = 1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0 \end{cases} \tag{21}$$

Optimal demands from resource constraints are equal to:

$$\forall i, j \in \mathcal{I}, \quad C_i = \left(\frac{p_i}{E}\right)^{-\sigma} \gamma_i \mathcal{U}^{\epsilon_i (1-\sigma)^2}, \quad C_j = \left(\frac{p_j}{E}\right)^{-\sigma} \gamma_j \mathcal{U}^{\epsilon_j (1-\sigma)^2}$$

By substituting this demand into $\frac{\partial \mathcal{L}}{\partial \Upsilon} = 1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0$
We end up with:

$$\frac{\partial \mathcal{L}}{\partial \Upsilon} = 1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{\left(\frac{p_i}{E}\right)^{-\sigma} \gamma_i}{\mathcal{U}^{(1-\sigma)\sigma\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 0 \tag{22}$$

We know $\gamma_i, p_i, E, \sigma, \epsilon_i, \forall i \in \mathcal{I}$, then we can solve for U by having the optimal demand given endowment level.

System of equation $\mathcal{F}$ is then resumed by will:

$$
\begin{cases}
\frac{\partial \mathcal{L}}{\partial \mathcal{U}} = 1 - \Upsilon \frac{(1-\sigma)^2}{\sigma} \sum_{i=1}^{I} \gamma_i^{\frac{1}{\sigma}} \epsilon_i \frac{1}{\mathcal{U}} \left[\frac{C_i}{\mathcal{U}^{(1-\sigma)\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 0 \\
\forall i, j \in \mathcal{I}, \quad C_i = \left(\frac{p_i}{E}\right)^{-\sigma} \gamma_i \mathcal{U}^{\epsilon_i (1-\sigma)^2}, \quad C_j = \left(\frac{p_j}{E}\right)^{-\sigma} \gamma_j \mathcal{U}^{\epsilon_j (1-\sigma)^2} \\
\frac{\partial \mathcal{L}}{\partial \Upsilon} = 1 - \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{\left(\frac{p_i}{E}\right)^{-\sigma} \gamma_i}{\mathcal{U}^{(1-\sigma)\sigma\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 0 \\
Solve \quad for \quad \mathcal{U}
\end{cases}
\tag{23}
$$

A.3 Endowment level flexible prices

Section 3.1 supply shock will redistribute differently if I permit instantaneous price adjustment. People ranked at the lowest in term of endowment value benefit the most as their good is now on high demand.

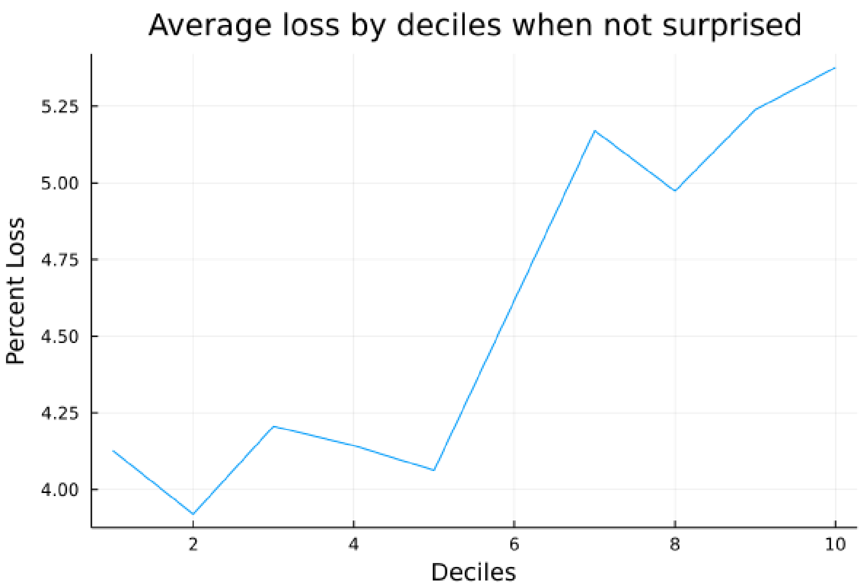
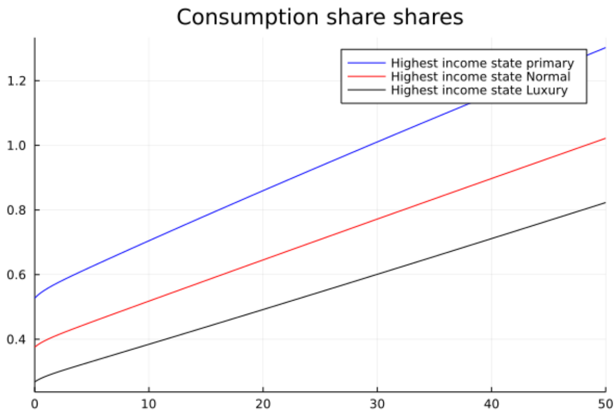


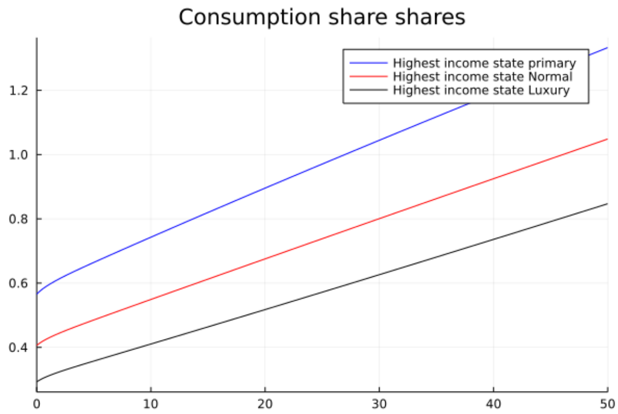
Figure 15: **Post shock welfare loss**

Notes: This figure plots welfare loss across deciles after a supply shock. I group people by deciles to obtain a mean loss.

A.4 Productivity shock

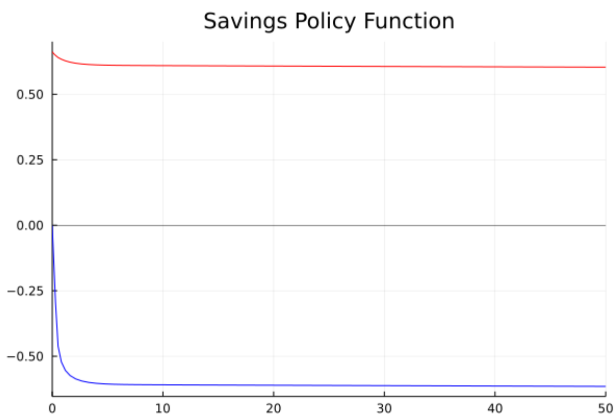


(a) Before

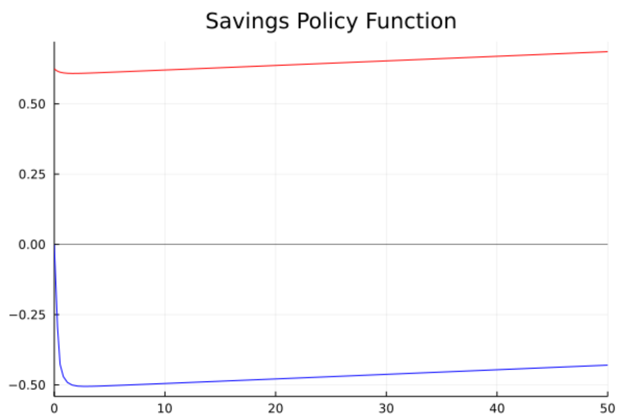


(b) After

Figure 16: **Before and after productivity shock**
Notes: Figure 16a shows partial equilibrium consumption in quantity with endogenous grid point before shock. I set p=[0.5, 1.5, 3.0] as price vectors that clear the market. Figure 16b shows consumption decisions after a 5% increase in overall productivity.



(a) Before



(b) After

Figure 17: **Before and after productivity shock**
Notes: Figure 17a shows partial equilibrium saving decision with endogenous grid point before shock. I set p=[0.5, 1.5, 3.0] as price vectors that clear the market. Figure 17b shows saving decision after a 5% increase in overall productivity.

A.5 Continuous time competitive equilibrium

I now introduce this problem into a dynamic setup with risk free bond yielding r_t with $R_t = 1 + r_t$ in the next period. As usual, household h maximizes its utility given resource at each time $t \in \mathcal{T}$.

I consider a household maximizing his stream of utility by consuming goods produced by different sectors defined as

$$V(B_0) = \max_{\{C_{i,t}\}_{i,t=1}^{\mathcal{I},\mathcal{T}}} \int_0^{\infty} e^{-\rho t} \mathcal{U}(C_{1,t}, ..., C_{I,t}) dt \tag{24}$$

where $\rho > 0$ catches the discount rate and U is the instantaneous utility from consuming a bundle of good $i \in \mathcal{I}$. Over time, the flow of budget constraint satisfy

$$\dot{B} = w_t L_t + r_t B_t - E(\mathbf{C}, \mathbf{P}). \tag{25}$$

where $E(\mathbf{C}, \mathbf{P}) = \sum_{i,t=1}^{I} p_{i,t} C_{i,t}$ and represents the sum of household expenditure across sectors. w_t, r_t denote the wage and risk free rate from their available labor and capital units L_t, B_t. Lastly, I introduce a non homothetic utility demand where household would decide to consume more income elastic good as his income is high defined as

$$\forall t \in \mathcal{T}, \quad \sum_{i}^{I} \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_{i,t}}{g(\mathcal{U})^{\epsilon_i}} \right]^{\frac{\sigma-1}{\sigma}} = 1 \qquad . \tag{26}$$

ϵ_i are the non homothetic parameter that impact household demand. Hence, with given r_t, w_t, p_{it} we can solve the competitive household problem with the following Hamiltonian:

$$\mathcal{H}(\mathbf{C}_t, B_t, \lambda_t) = \mathcal{U}(C_{1,t}, ..., C_{I,t}) + \lambda \left[w_t L_t + r_t B_t - E(\mathbf{C}, \mathbf{P}) \right] \tag{27}$$

Necessary and sufficient conditions give us the following non linear system:

$$\frac{\partial \mathcal{H}}{\partial C} = \mathcal{U}'(\mathbf{C}) - \lambda_t \frac{\partial E}{\partial C} = 0$$

$$\frac{\partial \mathcal{H}}{\partial B} = \lambda_t (r_t - \rho) = \dot{\lambda}$$

$$\dot{B} = w_t L_t + r_t B_t - E(\mathbf{C}, \mathbf{P})$$

$$\lim_{T \to \infty} e^{-\rho T} \mathcal{U}'(\mathbf{C}) B_T = 0$$

The Euler equation thus verify the following term:

$$\frac{\dot{C}}{C} = \frac{r_t - \rho + \frac{\dot{e}}{e} + \frac{\dot{\eta}_E^C}{\eta_E^C}}{1 + \eta_{\mathcal{U}'}^C}$$

We have thus cross term coming from the elasticity and relationship with U (income/wealth) that goes into household intertemporal consumption decision.