Inflation inequality redistribution under household heterogeneity

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October 5, 2022

Motivation

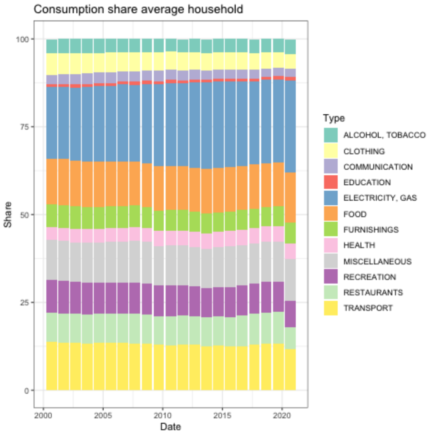
- Inflation is at an all time high since the 80s.
- Increase cost of living does not spread evenly across households.
- Heterogeneous inflation exposure by consumption bundle.
- Desire to update current CPI with characteristics specific one to account for "real" inflation exposure.

- How inflation redistribute welfare across household when price surge is consumed heavily by one category of people?
- Use non-homothetic preferences Comin et al. [2021] to account for characteristic specific choices.

Main results

- ECB, Eurostat data shows pro-rich inflation in which lowest income households experience higher cumulative price index.
- In an endowment model with non homothetic preferences, poorest households are hit twice during a recession.
  - Lose wealth, now less resources
  - Experience subsequent inflation
- Aiyagari model with non homothetic preferences across goods,
  - Households save less on average.
  - People at the bottom of the distribution decide to acquire more assets to consume superior goods in the next period.

Inflation per purpose



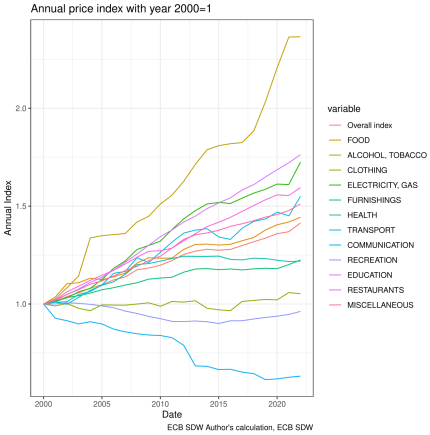
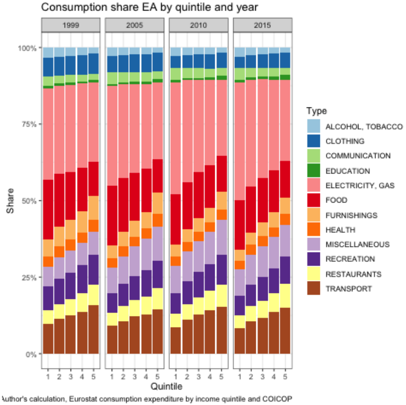
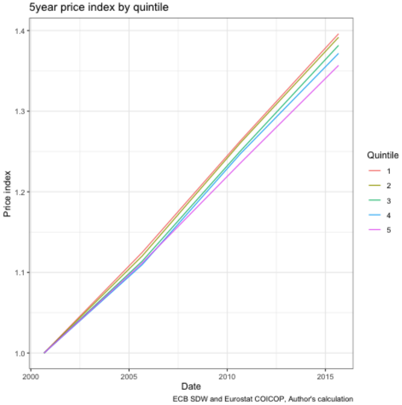


Figure: Heterogeneous consumption bundle and inflation

Different consumption bundle across quintiles





$$\mathcal{P}_q = \sum_{j}^{\mathcal{J}} \frac{P_t^j}{P_0^j} \omega_{q,t}^j .$$

Endowment models

$$\max_{\{C_i\}_{i=1}^I} U(C_1,...,C_I) \quad st \quad E(\mathbf{C},\mathbf{P}) \equiv \sum_{i=1}^I p_i W_i \geq \sum_{i=1}^I p_i C_i .$$

$$\sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_i}{g(U)^{\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 1.$$

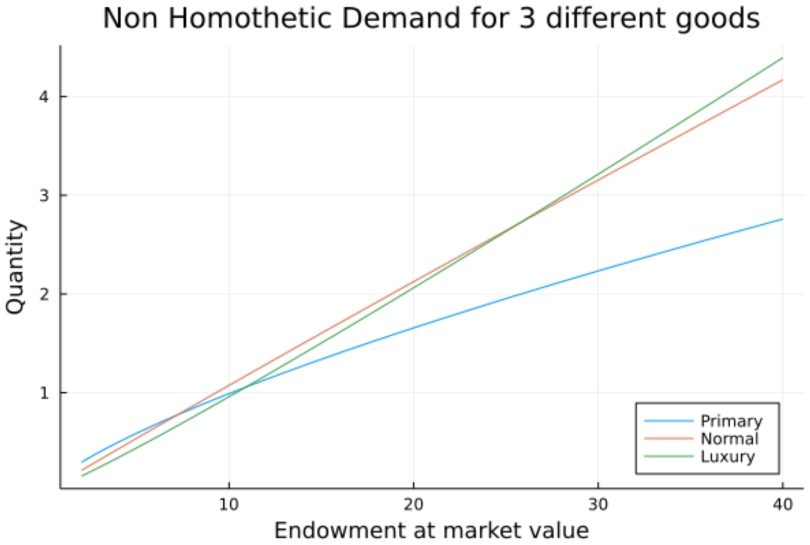
Comin et al. [2021]

With optimal demands across goods, we get:

$$\sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{\left(\frac{p_i}{E(\mathbf{C},\mathbf{P})}\right)^{-\sigma} \gamma_i}{U^{(1-\sigma)\sigma\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 1$$

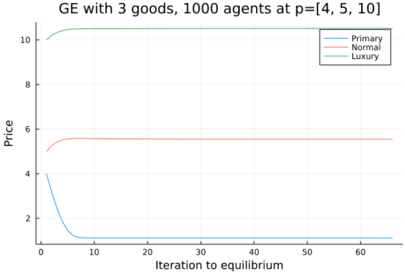
$$\log\left(\frac{C_i}{C_j}\right) = \underbrace{\sigma \log\left(\frac{p_j}{p_i}\right)}_{\text{Relative price Effect}} + \underbrace{(\epsilon_i - \epsilon_j)(1-\sigma)\log g(U)}_{\text{Non-homothetic preferences Income effect}} + \underbrace{\log\left(\frac{\gamma_i}{\gamma_j}\right)}_{\text{Relative Weights}} .$$

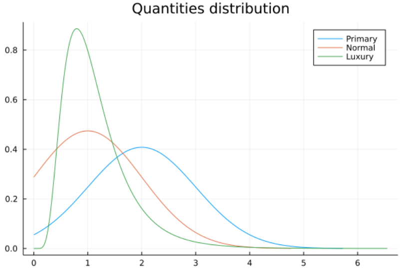
Non linearity in demands



Simulation with market clearing

First guess at p=[4.0, 5.0, 10.0]



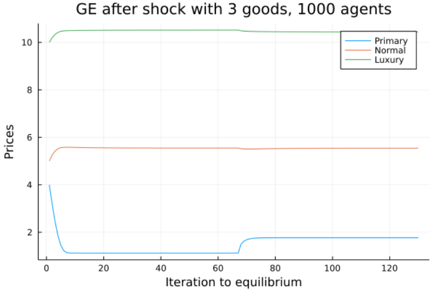


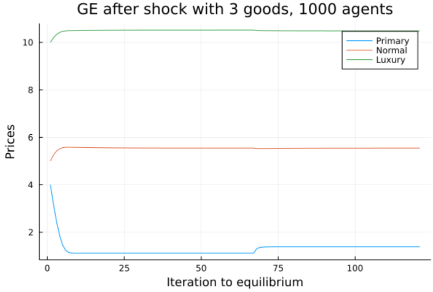
Market clearing overall endowment equal overall demands across households

$$\forall i \in \mathcal{I} \quad \sum_{h=1} W_i^h = \sum_{h=1} C_i^h$$

Shock on primary goods

S1: 20% CUT, S2: 10% CUT in primary goods





Good	Before shock	S1	Change	S2	Change
Primary	1.120	1.772	+ 58%	1.390	+ 24%
Normal	5.546	5.541	- 0.09%	5.546	- 0%
Luxury	10.515	10.438	- 0.74%	10.485	- 0.28%

Idiosyncratic income risk and non homothetic preferences

HOUSEHOLDS.

$$\max_{\{\mathbf{C}_t, B_{it+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$\text{s.t.} \quad \begin{aligned} w_t L_{it} + (1+r_t)B_{it} - B_{it+1} - E_t(\mathbf{C_t}, \mathbf{P_t}) \geq 0 \\ E_t(\mathbf{C_t}, \mathbf{P_t}) - \sum_{i=1}^{I} P_{it} C_{it} \geq 0, B_{it} \geq 0 \\ \sum_i^I \gamma_i^{\frac{1}{\sigma}} \left[\frac{C_{it}}{g(C_t)^{\epsilon_i}}\right]^{\frac{\sigma-1}{\sigma}} = 1. \end{aligned} \tag{1}$$

with $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$ and $C_t = \mathcal{U}_t(\mathbf{E_t}, \mathbf{C_t}, \mathbf{P_t})$:

$$\frac{C_{t+1}}{C_t} = \left[\beta(1+r_t)\frac{\varepsilon_{t+1}^{G/E}}{\varepsilon_t^{G/E}}\frac{\mathcal{U}_{t+1}}{\mathcal{U}_t}\frac{E_t}{E_{t+1}}\right]^{\frac{1}{\theta}} \tag{2}$$

$$\lim_{t\to\infty} \beta^t (1+r_t) B_{it} C_t^{-\theta} \varepsilon_t^{G/E} \frac{\mathcal{U}_t}{E_t} = 0$$

Idiosyncratic income with non homothetic preferences

FIRMS.

$$\forall i \in \mathcal{I}, \quad Y_{it} = A_i K_{it}^{\alpha} L_{it}^{1-\alpha} \tag{3}$$

$$\forall i \in \mathcal{I}, \quad \max_{K_{it}, L_{it} \geq 0} \Pi_{i,t} = P_{it} Y_{it} - R_t K_{it} - w_t L_{it} \tag{4}$$

$$\begin{gathered} r_t = A_t \alpha \left(\frac{\mathbf{L_t}}{\mathbf{K_t}} \right)^{1-\alpha} - \delta \\ w_t(r) = A_t (1-\alpha) \left(\frac{A\alpha}{r+\delta} \right)^{\frac{\alpha}{1-\alpha}} \end{gathered} \tag{5}$$

$$\begin{gathered} \sum_{h=1}^{H} \sum_{i=1}^{I} L_{it} = \mathbf{L_t} \\ \sum_{i=1}^{I} B_{it} = \mathbf{B_t} \equiv \sum_{i=1}^{I} K_{it} = \mathbf{K_t} \end{gathered} \tag{6}$$

Market clearing

Decentralized competitive market equilibrium in our economy $\mathcal{E}_d$ consist of:

- $\{\mathbf{C_t}, \mathbf{B_t}, \mathbf{L_t}, \mathbf{K_t}, \mathbf{P_t}, R_t, w_t\}_{t=0}^{\infty}$ such that $r_t = R_t - \delta$ at each date t
- $\{\mathbf{C_t}, \mathbf{B_t}\}_{t=0}^{\infty}$ solves households intertemporal optimization problems (1) given B_0 and $\{\mathbf{P_t}, r_t, w_t\}_{t=0}^{\infty}$
- $\{\mathbf{K_t}, \mathbf{L_t}\}_{t=0}^{\infty}$ solves firms profit maximization problems (4) given $\{\mathbf{P_t}, R_t, w_t\}_{t=0}^{\infty}$
- Markets must clear with aggregate non-homothetic demands equalizing aggregate production level for all our sectors with $\{\mathbf{P_t}, R_t, r_t, w_t\}_{t=0}^{\infty}$, so:
    - $\mathbf{L_t} \sim \mathcal{N}(\mu, \sigma)$ which is iid for labor efficiency and income risk
    - $\mathbf{K_t} = \mathbf{B_t}$ Saving provided are used by firms to produce goods.
    - $\mathbf{Y_t} = \mathbf{C_t}$, Non-homothetic demands equal Production.

Numerically solve

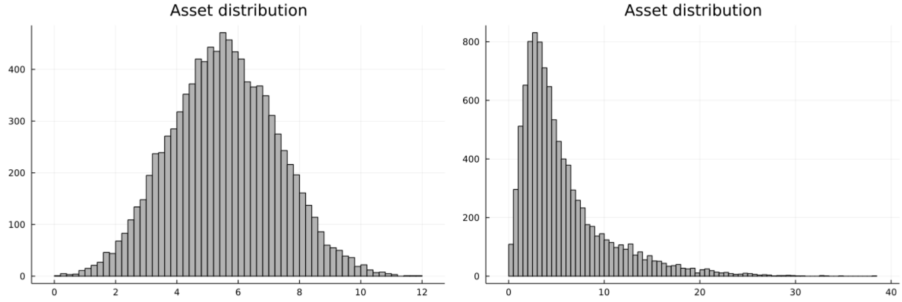
- Consider 3 sectors: Primary, Normal, Luxury with their size primary (35%), normal (35%) and luxury (30%).
- Give initial guess for the price level.
- Set an income distribution following $\mathbf{L_t} \sim \mathcal{N}(1, 0.2)$ with 10 000 households, horizon T=500
- Solve saving decisions with Endogenous Grid Point and non linear solver since non homothetic preferences.
- Iterate until convergence and prices consistency with production level and demands.

NH model vs Aiyagari model

Table: NH model vs Aiyagari model

Components/Methods	market clearing price NH	Aiyagari model
Price	[0.05, 0.267, 2.950]	1
Computation seconds	30400	80
Interest rate	4.31%	4.15%
Average asset level	5.528	5.652
Asset 10th Percentile	3.281	1.571
Asset 50th Percentile	5.507	4.205
Asset 99th Percentile	9.647	22.132

Asset distribution



(a) NH Preferences model with 3 sectors

(b) Aiyagari model

Reasons

- 3 prices for NH model and income elasticity.
- Some people may prefer to consume a lot by benefiting low prices and thus save less : Rich.
- People save in the Aiyagari model for precautionary savings.
- $\rightarrow$ In NH model, poor save to consume income elastic good.
- Save because higher intertemporal value as saving is subsequently used to acquire additional revenue in the next period and consume more income elastic good.

Productivity shock

- Static comparative analysis with a 5% increase in overall productivity
- Partial equilibrium: prices does not clear market.
- Compute asset distribution.

Table: NH model before and after productivity shock.

Comp/Methods	NH EGP before	NH EGP post shock	Change
Price	[0.5, 1.5, 3.0]	[0.5, 1.5, 3.0]	
Time	181	367	
Interest rate	4.18%	4.15%	-0.7%
Average asset level	5.638	6.134	+ 8.8%
Asset 10th Percentile	1.963	2.182	+11.1%
Asset 50th Percentile	5.165	5.605	+8.5%
Asset 99th Percentile	14.336	15.614	+8.9%

Policy Implications/ Conclusion

- Instantaneous loss in purchasing power higher for the bottom deciles
  - surveys could be more frequent at the EA level
- My NH model does not include a government, consumption saving could highly differ especially at the bottom.
  $\rightarrow$ Redistribution to make precautionary saving and intertemporal value of saving in acquiring luxury good gone
- Provide with the right incentives to consume stable price good.
  $\rightarrow$ Stable price good > Price cap
- Be careful of the credit channel which can be huge Auclert [2019], Doepke and Schneider [2006], Meh and Terajima [2011], Adam and Zhu [2016], Cardoso et al. [2022]

Thank you.

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