# Linear Elasticity Tutorial 1

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**Abstract**

This document details a single tutorials of 'linear elasticity' module of PSD in a more verbos manner.

To showcase the usage of Linear elasticity, we shall discuss here an example of a 2D bar, which bends under its own load. The bar $5 \times 1$ m$^2$ in area is made up of material with $\rho = 8 \times 10^3$, $E = 200 \times 10^9$, and $\nu = 0.3$.
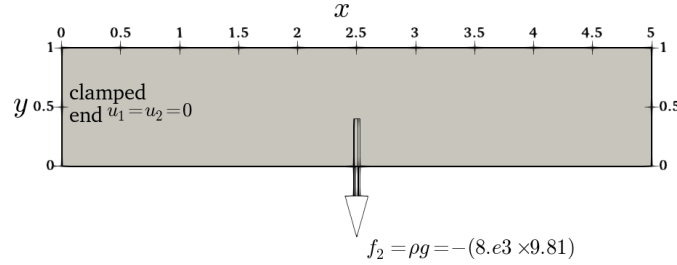


Figure 1: The 2D clamped bar problem.

## Step 1: Preprocessing

First step in a PSD simulation is PSD preprocessing, at this step you tell PSD what kind of physics, boundary conditions, approximations, mesh, etc are you expecting to solve.

In the terminal cd to the folder /home/PSD-tutorials/linear-elasticity. Launch PSD_PreProcess from the terminal, to do so run the following command.

```
1 PSD_PreProcess -problem linear_elasticity -dimension 2 -bodyforceconditions 1 \
2 -dirichletconditions 1 -postprocess u
```

After the PSD_PreProcess runs successfully you should see many .edp files in your current folder.

**What do the arguments mean ?**

- -problem linear_elasticity means that we are solving linear elasticity problem;

- -dimension 2 means it is a 2D simulation;

- -bodyforceconditions 1 with applied body force acting on the domain;

- -dirichletconditions 1 says we have one Dirichlet border;

- -postprocess u means we would like to have ParaView post processing files.

At this stage the input properties $E, \nu$ can be mentioned in ControlParameters.edp, use E = 200.e9, and nu = 0.3;. The volumetric body force condition is mentioned in the same file via variable Fbc0Fy -78480.0, i.e ($\rho * g = 8.e3 * (-9.81) = -78480.0$). One can also provide the mesh to be used in ControlParameters.edp, via ThName = "../Meshes/2D/bar.msh" (*note that mesh can also be provided in the next step*) .In addition variable Fbc0On 1 has to be provided in order to indicate the volume (region) for which the body force is acting, here 1 is the integer volume tag of the mesh. Dirichlet boundary conditions are also provided in ControlParameters.edp. To provide the clamped boundary condition the variables Dbc0On 2, Dbc0Ux 0., and Dbc0Uy 0. are used, which means for Dirichlet border 2 (Dbc0On 2) where 2 is the clamped border label of the mesh Dirichlet constrain is applied and Dbc0Ux 0., Dbc0Uy 0 i.e., the clamped end condition ($u_x = u_y = 0$).

## Step 2: Solving

As PSD is a parallel solver, let us use 4 cores to solve the 2D bar case. To do so enter the following command:

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/2D/bar.msh -v 0
```

Here `-np 4` denote the argument used to enter the number of parallel processes (MPI processes) used while solving. `-mesh ./../Meshes/2D/bar.msh` is used to provide the mesh file to the solver. `-v 0` denotes the verbosity level on screen. `PSD_Solve` is a wrapper around `FreeFem++` or `FreeFem++-mpi`. Note that if your problem is large use more cores. PSD has been tested upto 13,000 parallel processes and problem sizes with billions of unknowns, surely you will now need that many for the 2D bar problem.

## Step 3: Postprocessing

PSD allows postprocessing of results in ParaView. After the step 2 mentioned above finishes. Launch ParaView and have a look at the `.pvd` file in the `VTUs...` folder. Using ParaView for postprocessing the results that are provided in the `VTUs...` folder, results such as those shown in figure~2 can be extracted.
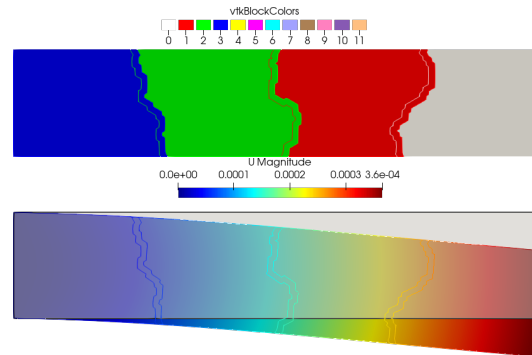


Figure 2: The 2D clamped bar problem: partitioned mesh and displacement field visualization in ParaView.

You are all done with your 2D linear-elasticty simulation.

## 2D bar is ok, but what about 3D ?

3D follows the same logic as 2D, in the preprocessing step

```
1 PSD_PreProcess -problem linear_elasticity -dimension 3 -bodyforceconditions 1 \
2 -dirichletconditions 1 -postprocess u
```

note that all what has changed `-dimension 3` instead of `-dimension 2`

Solving step remains exactly the same with `-mesh` flag now pointing towards the `3D` mesh.

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/3D/bar.msh -v 0
```

Using ParaView for postprocessing the results that are provided in the `VTUs...` folder, results such as those shown in figure~3 can be extracted.

## What else should you try to become an advanced user

Optionally try using `-withmaterialtensor` flag with `PSD_PreProcess`, and run the simulation. You are encouraged to have a look at `ControlParameters.edp` and `VariationalFormulations.edp` file produced with `-withmaterialtensor` flag and without this flag.
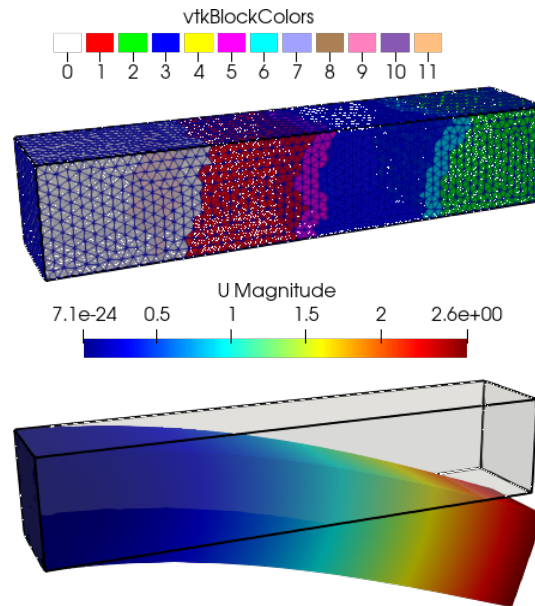
Figure 3: The 3D clamped bar problem: partitioned mesh and displacement field visualization in ParaView.

Add -sequential flag to PSD_PreProcess for sequential solver, but remember to use PSD_Solve_Seq instead of PSD_Solve and no -np flag.

```
1 PSD_PreProcess -problem linear_elasticity -dimension 2 -sequential \
2 -bodyforceconditions 1 -dirichletconditions 2 -postprocess u
```

```
1 PSD_Solve_Seq Main.edp -mesh ./../Meshes/2D/bar.msh -v 0
```

Do the same simulation in 3D.

You are encouraged to use more complex meshes for this same problem, but do not forget to update the ControlParameters.edp file.

## Advance exercise 1

There is a solver run level flag for mesh refinement [1]. This flag is called -split [int] which splits the triangles (resp. tetrahedrons) of your mesh into four smaller triangles (resp. tetrahedrons). As such -split 2 will produce a mesh with 4 times the elements of the input mesh. Similarly, -split n where $n$ is a positive integer produces $2^n$ times more elements than the input mesh. You are encouraged to use this -split flag to produce refined meshes and check, mesh convergence of a problem, computational time, etc. Use of parallel computing is recommended. You could try it out with PSD_Solve or PSD_Solve_Seq, for example:

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/2D/bar.msh -v 0 -split 2
```

for splitting each triangle of the mesh bar.msh into 4.

## Advance exercise 2

There is a preprocess level flag -debug, which as the name suggests should be used for debug proposes by developers. However, this flag will activate OpenGL live visualization of the problems displacement field. You are encouraged to try it out

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[1]Mesh refinement is performed after partitioning.

```
1 PSD_PreProcess -problem linear_elasticity -dimension 2 -bodyforceconditions 1 \
2 -dirichletconditions 1 -postprocess u -timelog -debug
```

Then to run the problem we need additional -wg flag

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/2D/bar.msh -v 0 -wg
```

## Exercise 3

One interesting way of solving a linear Elasticity problem is to solve it via a pseudo nonlinear model. There is a preprocess level flag -model pseudo_nonlinear, which introduces pseudo nonlinearity into the finite element variational formulation of linear elasticity. You are encouraged to use this flag and see how the solver performs. Indeed, now you should see some nonlinear iterations (1 or 2) are taken for convergence.

```
1 PSD_PreProcess -problem linear_elasticity -dimension 2 -bodyforceconditions 1 \
2 -dirichletconditions 1 -postprocess u -timelog -model pseudo_nonlinear
```

Then to run the problem

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/2D/bar.msh -v 0
```

To understand what the flag does, try to find out the difference between the files created by PSD_PreProcess when used with and without -model pseudo_nonlinear flag. Especially, compare LinearFormBuilderAndSolver.edp and VariationalFormulations.edp files produced by PSD_PreProcess step. You will see Newton–Raphsons iterations are performed for solving the linear problem. However, the nonlinear iterations loop converges very rapidly (in 1 iteration) due to linear nature of the problem. **Note:** This flag is exclusive for parallel solver.

## Advance exercise 4

There is a preprocess level flag -withmaterialtensor, which introduces the full material tensor into the finite element variational formulation. You are encouraged to use this flag and see how the solver performs.

```
1 PSD_PreProcess -problem linear_elasticity -dimension 2 -bodyforceconditions 1 \
2 -dirichletconditions 1 -postprocess u -timelog -withmaterialtensor
```

Then to run the problem

```
1 PSD_Solve -np 4 Main.edp -mesh ./../Meshes/2D/bar.msh -v 0
```

To understand what the flag does, try to find out the difference between the files created by PSD_PreProcess when used with and without -withmaterialtensor flag. Especially, compare FemParameters.edp, MeshAndFeSpace and VariationalFormulations.edp files produced by PSD_PreProcess step.