Nonlinear elasto-plastic Von-Mises material analysis via MFront
−Isotropic linear hardening plasticity−

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Abstract

This document details a tutorial that allows one to use PSD-MFront interface for non linear elasto-plastic problem. MFront is used for general handling the material behaviour, i.e. it it handles the non-linearity updates and material parameters. It is advised to follow this tutorial after tutorial 1 and tutorial 9 of linear-elasticity, which introduce MFront for linear-elasticty (I believe, it is best to walk before we run).

Note that, non linear elasto-plastic merely provides means of getting started with Mfornt, the real potential lies in using different and more complex non linear materials and laws which Mfront provides. So this tutorial should be considered as baptism to the world of non linear PSD-MFront coupling which we believe has a lot of potential to solve some non trivial problems.

This tutorial is concerned with a non-linear problem of the incremental analysis of an elasto-plastic Von-Mises material. A two-dimensional test (in $x − y$) will be considered. The problem of interest is a quarter of a cylinder (with external radius R_e and internal radius R_i) fig. 2. Boundary conditions correspond to symmetry conditions on the bottom horizontal $y = 0$ and left vertical borders $x = 0$ (resp. numbered 1 and 3 as mesh labels). Loading consists of a uniform pressure (traction boundary) on the internal boundary (numbered 4 as mesh label). Load (pressure) q will be progressively increased from 0 to $q_{\lim} = \frac{2}{\sqrt{3}}\sigma_0 \log(\frac{R_e}{R_i})$ which is the analytical collapse load for a perfectly-plastic material (no hardening):
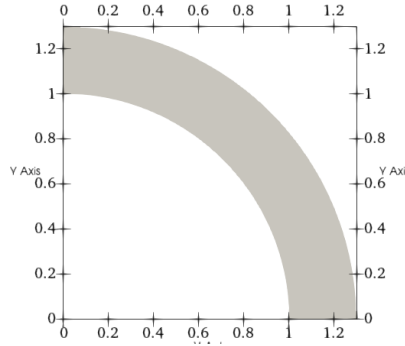

Figure 1: Domain of the non-linear problem.

The material is represented by an isotropic elasto-plastic von Mises yield condition of uniaxial strength σ_0 and with isotropic hardening of modulus H. The yield condition is thus given by:

$$f(\sigma) = \sqrt{\frac{3}{2}s : s} - \sigma_0 - Hp \leq 0$$

where p is the cumulated equivalent plastic strain and s denoting the deviatoric elastic stress $s = \mathrm{dev}\,\sigma_{\mathrm{elas}}$. The hardening modulus can also be related to a tangent elastic modulus $E_t = \frac{EH}{E+H}$.

To compute the structures response an iterative predictor-corrector return mapping algorithm embedded in a Newton-Raphson global loop for restoring equilibrium is used. Due to the simple expression of the von Mises criterion, the return mapping procedure is completely analytical (with linear isotropic hardening). We point out that the two-dimensional nature of the problem will be impose keeping track of the out-of-plane ε_{zz}^p plastic strain and dealing with representations of stress/strain states including the zz component.
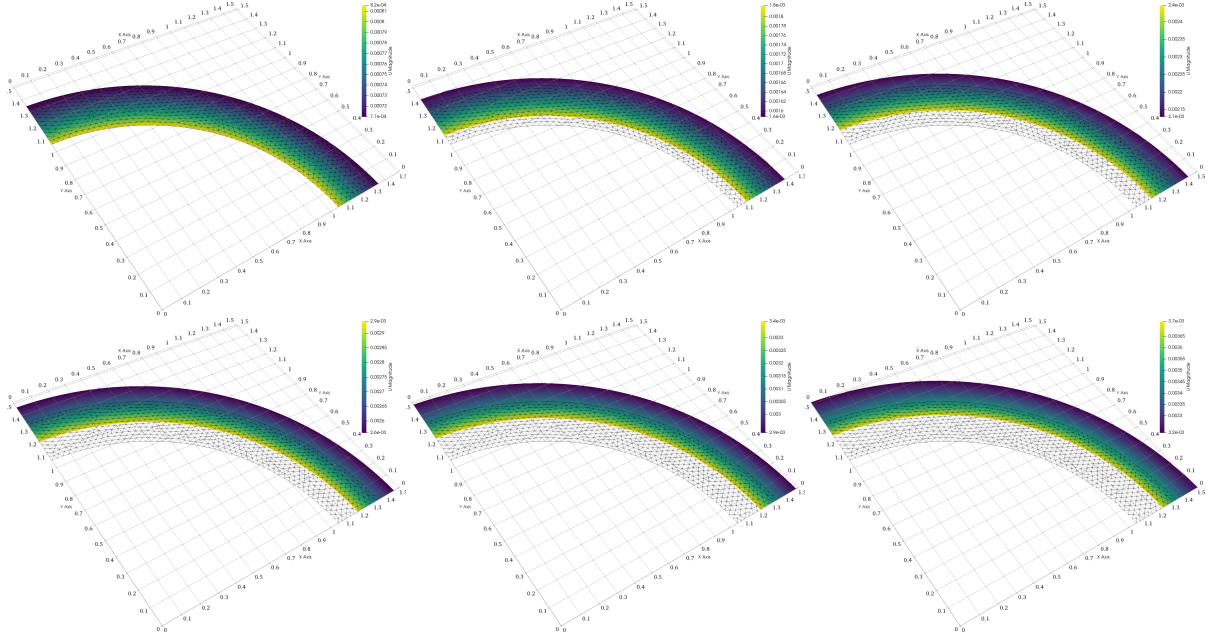
Figure 2: Warped displacement field evolution - from top left $t_0, t_4, t_8, t_{12}, t_{16}, t_{19}$.

The displacement field evolution during the cylinder expansion will look like this the one presented in fig. 2:

The return mapping procedure consists in finding a new stress σ_{n+1} and internal variable p_{n+1} state verifying the current plasticity condition from a previous stress σ_n and internal variable p_n state and an increment of total deformation $\Delta\varepsilon$. All this is handled by MFront, where MFront requests PSD for the previous n time-step strains ε_n. In order to use a Newton-Raphson procedure to resolve global equilibrium, we also need to derive the algorithmic consistent tangent matrix which is also given by MFront.

Within MFront an elastic trial stress $\sigma_{\text{elas}} = \sigma_n + \mathbf{M}\Delta\varepsilon$ is first computed. Then the plasticity criterion is then evaluated with the previous plastic strain $f_{\text{elas}} = \sigma_{\text{elas}}^{\text{eq}} - \sigma_0 - Hp_n$, where $\sigma_{\text{elas}}^{\text{eq}} = \sqrt{\frac{3}{2}s : s}$. Consequently, if $f_{\text{elas}} \leq 0$ no plasticity occurs during this time increment and $\Delta\varepsilon^p = \Delta p = 0$. Otherwise if $f_{\text{elas}} > 0$, plasticity occurs and the increment of plastic strain is given by

$$\Delta p = \frac{f_{\text{elas}}}{3\mu + H}$$

The final stress state is corrected by the plastic strain as follows

$$\sigma_{n+1} = \sigma_{\text{elas}} - \beta s \quad \text{with} \quad \beta = \frac{3\mu}{\sigma_{\text{elas}}^{\text{eq}}}\Delta p$$
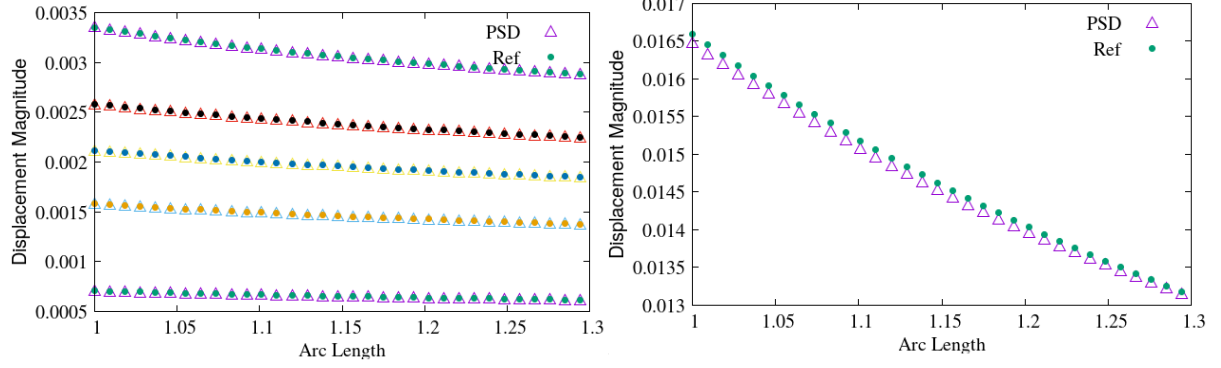
Bibliography

Figure 3: Validation of the displacement field obtained by PSD and another reference code. The displacement magnitude is plotted on the central line which bisects the geometry into two. On the left time steps - $t_0, t_4, t_8, t_{12}, t_{16}$ are plotted and on the right t_{19}. Reference results used for comparison were obtained by installing and running the FEniCS Solid Mechanics library [Garth N. Wells (2021)].
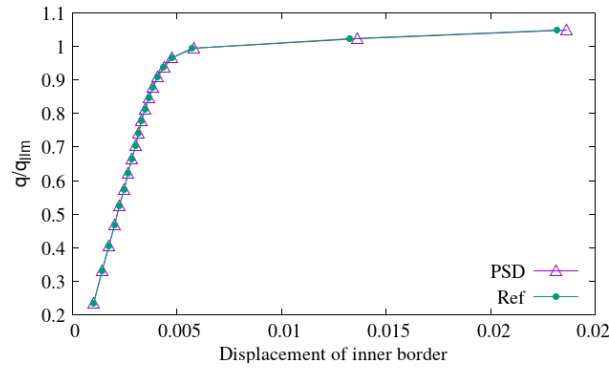

Figure 4: Validation of the displacement movement of inner border movement obtained by PSD and another reference code. Reference results used for comparison were obtained by installing and running the FENICS solid mechanics codes https://bitbucket.org/fenics-apps/fenics-solid-mechanics.
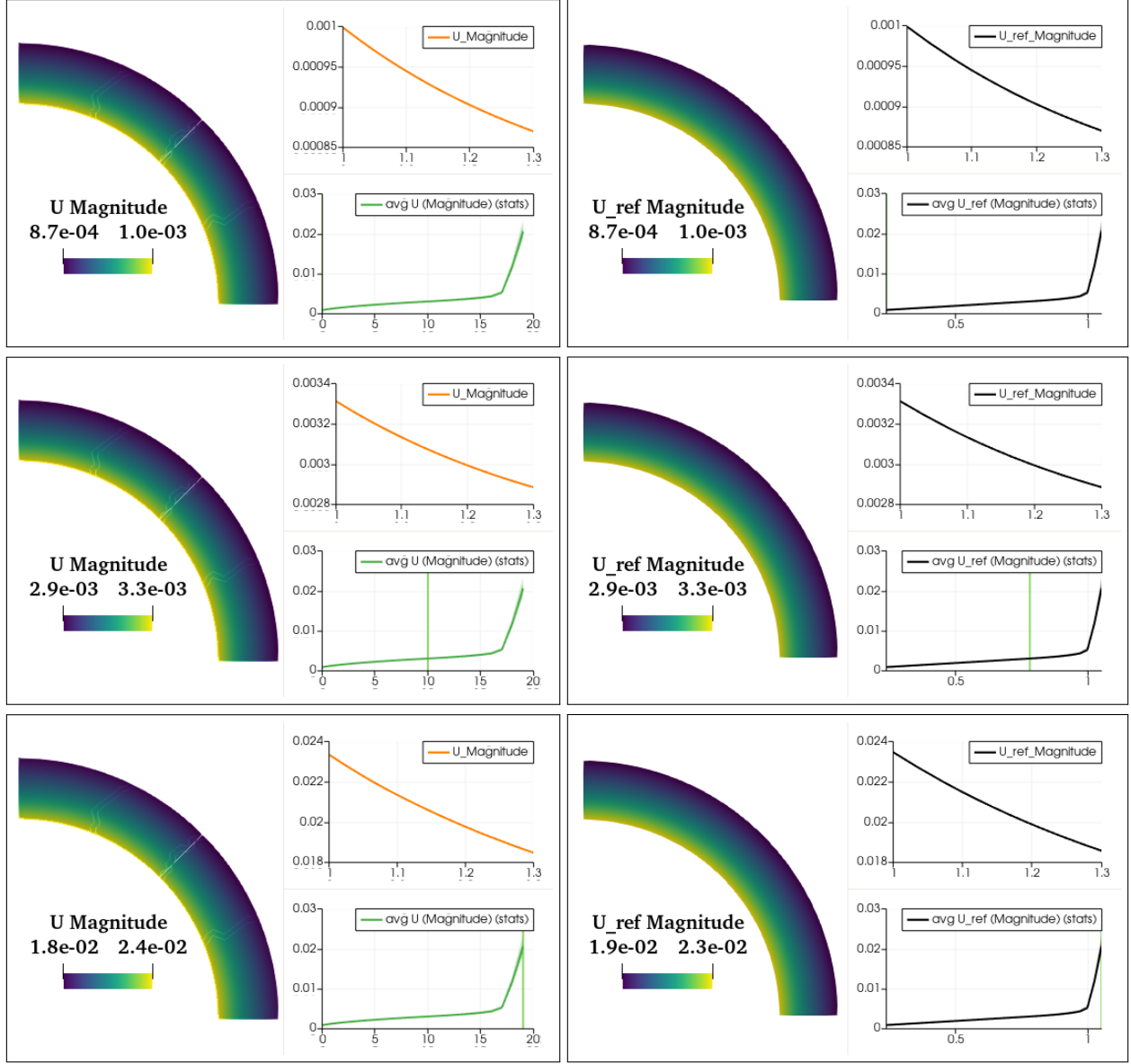
Figure 5: Validation results comparison of PSD (left column) and reference code (right column) at different timesteps (t_0, t_{10}, t_{19}). Reference results used for comparison were obtained by installing and running the FEniCS Solid Mechanics library [Garth N. Wells (2021)].