

Static Variational formulation:

$$\int_{\Omega} c_0(\vec{d}\vec{u} \cdot \vec{v}) + \int_{\Omega} \sigma(\vec{d}\vec{u}) : \epsilon(\vec{v}) + \int_{\Omega} \rho g v_3 = 0$$

with $du_1 = du_2 = du_3 = 0$ at base

$$c_0 = \frac{\rho}{\beta \Delta t^2}$$

$$c_1 = \frac{\rho}{\beta \Delta t}$$

$$c_2 = \frac{\rho(1-2\beta)}{2\beta}$$

dynamic Variational formulation:

$$\begin{aligned} & \int_{\Omega} c_0(\vec{d}\vec{u} \cdot \vec{v}) + \int_{\Omega} \sigma(\vec{d}\vec{u}) : \epsilon(\vec{v}) + \int_{\Omega} c_0(\vec{u}_{old} \cdot \vec{v}) \\ & + \int_{\Omega} c_1(\vec{v}_{old} \cdot \vec{v}) + \int_{\Omega} c_2(\vec{a}_{old} \cdot \vec{v}) + \int_{\Omega} \rho g v_3 = 0 \end{aligned}$$

with $\vec{d}\vec{u} = \vec{u}_{signal}$ at base

— : boundary conditions 'B'

— : main matrix 'A'

↓
in the variational formulation

Algorithm:

- Solve static ($du = A^{-1}b$)
- $u_{old} = du$
- loop on time
 - solve dynamic ($du = A^{-1}b$)
 - update $u_{old}(du)$, $v_{old}(du)$
 $a_{old}(du)$ using Newmark- β
 - parameter u_{old} .

