Mathematical Model for Customer Support System Staff Scheduling

In this section we develop a mathematical model to find the optimal workforce assignment to meet the per shift demand and minimize total costs, Monday through Sunday.

The Model

The objective function minimizes the total cost incurred by the customer support system staff. This total cost is composed of fixed expenses such as hiring and training costs, and recurring expenses such as a week's worth of staffs' salaries. Fixed costs are only incurred once for every new hire and vary depending on the type of requests that the hire will be handling in the customer support system: call requests, chat requests, or a mixture of both. Employees handling call requests or chat requests are referred to as single skilled employees, whereas those handling call, and chat requests are referred to as multi skilled employees. Training costs of multi skilled employees are naturally higher than single skilled training costs.

Indices and Sets:

i	index for the set of employees I where $I = \{1, \dots, 8\}$
j	index for the set of two-hour shifts J where $J = \{1, \dots, 12\}$
k	index for the set of days in a week K where $K = \{1, \dots, 7\}$

Parameters:

CE	hiring cost of any employee
CT_{call} , CT_{chat} , CT_{mix}	training cost of employees handling call requests, chat requests, call and chat requests respectively
CH_{call} , CH_{chat} , CH_{mix}	hourly cost of employees handling call requests, chat requests, call and chat requests respectively
D_{jk}^{call} , D_{jk}^{chat}	demand of call (and chat) on shift j, on day k
S	availability of an employee in a shift in seconds where $S = 7200$
p	productivity of an employee where $p = 0.9$
M	number that goes to infinity (big M) where $M = 10000$

Decision Variables:

x_{ijk} , y_{ijk} , z_{ijk}	1 if employee i works on shift j, on day k handling call requests, chat requests, call and chat requests respectively; 0 otherwise
xx_i , yy_i , zz_i	1 if employee i is hired to handle call requests, chat requests, call and chat requests respectively; 0 otherwise
xp_{ik} , yp_{ik} , zp_{ik}	1 if employee i works on day k handling call requests, chat requests, call and chat requests respectively; 0 otherwise

a_{ijk} , b_{ijk} , c_{ijk}	1 if employee i starts working on shift j, on day k handling call requests, chat requests, call and chat requests respectively; 0 otherwise
f_{ijk} , e_{ijk} , d_{ijk}	1 if employee i works on shift j, and worked on shift $j-1$ on day k handling call requests, chat requests, call and chat requests respectively; 0 otherwise

The mathematical model for the customer support system staff scheduling is presented below.
Minimize

$$\sum_{i\in I} CE(xx_i + yy_i + zz_i)$$
$$+ \sum_{i\in I} CT_{call}xx_i + \sum_{i\in I} CT_{chat}yy_i + \sum_{i\in I} CT_{mix}zz_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} 2CH_{call}\, x_{ijk}$$
$$+ \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} 2CH_{chat}\, y_{ijk} + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K} 2CH_{mix}z_{ijk}$$

Subject to

a) Required number of hours worked in a day for full-time and part-time employees

handling call requests, chat requests, call and chat requests

a. Full-time employees

i. For call requests

$$\sum_{j\in J} x_{ijk} + M\,(1 - xp_{ik}) \geq 4 \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$
$$\sum_{j\in J} x_{ijk} \leq M\, xp_{ik} \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$
$$\sum_{j\in J} x_{ijk} \leq 4 \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$

ii. For chat requests

$$\sum_{j\in J} y_{ijk} + M\,(1 - yp_{ik}) \geq 4 \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$
$$\sum_{j\in J} y_{ijk} \leq M\, yp_{ik} \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$
$$\sum_{j\in J} y_{ijk} \leq 4 \quad \forall\, i \in I = \{1, \ldots, 4\}, k \in K$$

iii. For call and chat requests

$$\sum_{j \in J} z_{ijk} + M\,(1 - zp_{ik}) \geq 4 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j \in J} z_{ijk} \leq M\, zp_{ik} \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j \in J} z_{ijk} \leq 4 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$

b. Part-time employees

i. For call requests

$$\sum_{j \in J} x_{ijk} + M\,(1 - xp_{ik}) \geq 2 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$\sum_{j \in J} x_{ijk} \leq M\, xp_{ik} \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$

ii. For chat requests

$$\sum_{j \in J} y_{ijk} + M\,(1 - yp_{ik}) \geq 2 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$\sum_{j \in J} y_{ijk} \leq M\, yp_{ik} \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$

iii. For call and chat requests

$$\sum_{j \in J} z_{ijk} + M\,(1 - zp_{ik}) \geq 2 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$\sum_{j \in J} z_{ijk} \leq M\, zp_{ik} \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$

b) Maximum number of days worked in a week for employees handling any type of requests

$$\sum_{k \in K} xp_{ik} \leq 5 \quad \forall\, i \in I$$
$$\sum_{k \in K} yp_{ik} \leq 5 \quad \forall\, i \in I$$

$$\sum_{k \in K} zp_{ik} \leq 5 \quad \forall\, i \in I$$

c) Maximum number of shifts worked in a week for full-time and part-time employees

a. Full-time employees

$$\sum_{j \in J} \sum_{k \in K} x_{ijk} \leq 20 \quad \forall\, i \in I = \{1, \dots, 4\}$$
$$\sum_{j \in J} \sum_{k \in K} y_{ijk} \leq 20 \quad \forall\, i \in I = \{1, \dots, 4\}$$
$$\sum_{j \in J} \sum_{k \in K} z_{ijk} \leq 20 \quad \forall\, i \in I = \{1, \dots, 4\}$$

b. Part-time employees

$$\sum_{j \in J} \sum_{k \in K} x_{ijk} \leq 12 \quad \forall\, i \in I = \{5, \dots, 8\}$$
$$\sum_{j \in J} \sum_{k \in K} y_{ijk} \leq 12 \quad \forall\, i \in I = \{5, \dots, 8\}$$
$$\sum_{j \in J} \sum_{k \in K} z_{ijk} \leq 12 \quad \forall\, i \in I = \{5, \dots, 8\}$$

d) Consecutive shifts constraint for full-time and part-time employees

a. For call requests

$$f_{ijk} \geq x_{ijk} + x_{ij-1k} - 1 \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$2f_{ijk} \leq x_{ijk} + x_{ij-1k} \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$a_{ijk} = x_{ijk} - f_{ijk} \quad \forall\, i \in I, j \in J, k \in K$$
$$a_{i1k} = x_{i1k} \quad \forall\, i \in I, j \in J, k \in K$$

i. Full-time employees

$$\sum_{q=0}^{3} x_{ij+3-qk} - 3 \leq \; a_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$
$$\sum_{q=0}^{3} x_{ij+3-qk} - 3 \geq 4\, a_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$

ii. Part-time employees

$$\sum_{q=0}^{1} x_{ij+1-qk} - 1 \leq \; a_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$
$$\sum_{q=0}^{1} x_{ij+1-qk} - 1 \geq 2\, a_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$

b. For chat requests

$$e_{ijk} \geq y_{ijk} + y_{ij-1k} - 1 \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$2e_{ijk} \leq y_{ijk} + y_{ij-1k} \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$b_{ijk} = y_{ijk} - e_{ijk} \quad \forall\, i \in I, j \in J, k \in K$$
$$b_{i1k} = y_{i1k} \quad \forall\, i \in I, j \in J, k \in K$$

i. Full-time employees

$$\sum_{q=0}^{3} y_{ij+3-qk} - 3 \leq b_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$
$$\sum_{q=0}^{3} y_{ij+3-qk} - 3 \geq 4\, b_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$

ii. Part-time employees

$$\sum_{q=0}^{1} y_{ij+1-qk} - 1 \leq b_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$
$$\sum_{q=0}^{1} y_{ij+1-qk} - 1 \geq 2\, b_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$

c. For call and chat requests

$$d_{ijk} \geq z_{ijk} + z_{ij-1k} - 1 \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$2d_{ijk} \leq z_{ijk} + z_{ij-1k} \quad \forall\, i \in I, j \in J = \{2, \dots, 12\}, k \in K$$
$$c_{ijk} = z_{ijk} - d_{ijk} \quad \forall\, i \in I, j \in J, k \in K$$
$$c_{i1k} = z_{i1k} \quad \forall\, i \in I, j \in J, k \in K$$

i. Full-time employees

$$\sum_{q=0}^{3} z_{ij+3-qk} - 3 \leq c_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$
$$\sum_{q=0}^{3} z_{ij+3-qk} - 3 \geq 4\, c_{ijk} \quad \forall\, i \in I = \{1, \dots, 4\}, j \in J = \{1, \dots, 9\}, k \in K$$

ii. Part-time employees

$$\sum_{q=0}^{1} z_{ij+1-qk} - 1 \leq c_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$
$$\sum_{q=0}^{1} z_{ij+1-qk} - 1 \geq 2\, c_{ijk} \quad \forall\, i \in I = \{5, \dots, 8\}, j \in J = \{1, \dots, 11\}, k \in K$$

e) Employees can only start working on specific shifts on a day

a. Full-time employees

$$\sum_{j=1}^{9} a_{ijk} \leq 1 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j=10}^{12} a_{ijk} = 0 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j=1}^{9} b_{ijk} \leq 1 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j=10}^{12} b_{ijk} = 0 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j=1}^{9} c_{ijk} \leq 1 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$
$$\sum_{j=10}^{12} c_{ijk} = 0 \quad \forall\, i \in I = \{1, \dots, 4\}, k \in K$$

b. Part-time employees

$$\sum_{j=1}^{11} a_{ijk} \leq 1 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$a_{i12k} = 0 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$\sum_{j=1}^{11} b_{ijk} \leq 1 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$b_{i12k} = 0 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$\sum_{j=1}^{11} c_{ijk} \leq 1 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$
$$c_{i12k} = 0 \quad \forall\, i \in I = \{5, \dots, 8\}, k \in K$$

f) Employees working should get at least 12 hours of break before their next start shift

a. Full-time employees

i. For call requests

$$\sum_{j=1}^{6} a_{ijk+1} \leq M(1 - a_{i9k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{5} a_{ijk+1} \leq M(1 - a_{i8k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{4} a_{ijk+1} \leq M(1 - a_{i7k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{3} a_{ijk+1} \leq M(1 - a_{i6k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{2} a_{ijk+1} \leq M(1 - a_{i5k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$a_{i1k+1} \leq M(1 - a_{i4k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

ii. For chat requests

$$\sum_{j=1}^{6} b_{ijk+1} \leq M(1 - b_{i9k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{5} b_{ijk+1} \leq M(1 - b_{i8k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{4} b_{ijk+1} \leq M(1 - b_{i7k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{3} b_{ijk+1} \leq M(1 - b_{i6k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{2} b_{ijk+1} \leq M(1 - b_{i5k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$b_{i1k+1} \leq M(1 - b_{i4k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

iii. For call and chat requests

$$\sum_{j=1}^{6} c_{ijk+1} \leq M(1 - c_{i9k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{5} c_{ijk+1} \leq M(1 - c_{i8k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{4} c_{ijk+1} \leq M(1- c_{i7k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{3} c_{ijk+1} \leq M(1- c_{i6k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{2} c_{ijk+1} \leq M(1- c_{i5k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

$$c_{i1k+1} \leq M(1 - c_{i4k}) \qquad \forall\, i \in I = \{1, \dots, 4\}, k \in K = \{1, \dots, 6\}$$

b. Part-time employees

i. For call requests

$$\sum_{j=1}^{6} a_{ijk+1} \leq M(1- a_{i11k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{5} a_{ijk+1} \leq M(1- a_{i10k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{4} a_{ijk+1} \leq M(1- a_{i9k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{3} a_{ijk+1} \leq M(1- a_{i8k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{2} a_{ijk+1} \leq M(1- a_{i7k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$a_{i1k+1} \leq M(1 - a_{i6k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

ii. For call requests

$$\sum_{j=1}^{6} b_{ijk+1} \leq M(1- b_{i11k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{5} b_{ijk+1} \leq M(1- b_{i10k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{4} b_{ijk+1} \leq M(1- b_{i9k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{3} b_{ijk+1} \leq M(1- b_{i8k}) \qquad \forall\, i \in I = \{5, \dots, 8\}, k \in K = \{1, \dots, 6\}$$

$$\sum_{j=1}^{2} b_{ijk+1} \leq M(1- b_{i7k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$b_{i1k+1} \leq M(1 - b_{i6k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

i. For call requests

$$\sum_{j=1}^{6} c_{ijk+1} \leq M(1- c_{i11k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$\sum_{j=1}^{5} c_{ijk+1} \leq M(1- c_{i10k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$\sum_{j=1}^{4} c_{ijk+1} \leq M(1- c_{i9k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$\sum_{j=1}^{3} c_{ijk+1} \leq M(1- c_{i8k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$\sum_{j=1}^{2} c_{ijk+1} \leq M(1- c_{i7k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

$$c_{i1k+1} \leq M(1 - c_{i6k}) \qquad \forall\, i \in I = \{5,\dots,8\}, k \in K = \{1,\dots,6\}$$

g) Number of call and chat requests must be met in a shift by employees working on said shift

a. For call requests

$$\sum_{i\in I} pS(x_{ijk} + 0.6z_{ijk}) \geq D_{jk}^{call} \qquad \forall\, j \in J, k \in K$$

b. For chat requests

$$\sum_{i\in I} pS(y_{ijk} + 0.4z_{ijk}) \geq D_{jk}^{chat} \qquad \forall\, j \in J, k \in K$$

h) Employees are hired if they work at least for one day

$$\sum_{k\in K} xp_{ik} + M(1 - xx_i) \geq 1 \qquad \forall\, i \in I$$

$$\sum_{k\in K} xp_{ik} \leq Mxx_i \qquad \forall\, i \in I$$

$$\sum_{k\in K} yp_{ik} + M(1 - yy_i) \geq 1 \qquad \forall\, i \in I$$

$$\sum_{k\in K} yp_{ik} \leq Myy_i \qquad \forall\, i \in I$$

$$\sum_{k\in K} zp_{ik} + M(1 - zz_i) \geq 1 \qquad \forall\, i \in I$$

$$\sum_{k\in K} zp_{ik} \leq Mzz_i \qquad \forall\, i \in I$$

Constraint a) ensures that full-time employees with indices $i \in I = \{1, \dots, 4\}$, work for four two-hour shifts for a total of eight hours on day k, handling call requests, chat requests and call and chat requests. Part-time employees with indices $i \in I = \{5, \dots, 8\}$, work for at least two two-hour shifts on day k, handling call requests, chat requests and call and chat requests.

Constraint b) sets an upper bound for the number of days worked in a week for full-time and part-time employees while taking into account the “two days off a week” requirement set by company policy.

Constraint c) restricts full-time employees with indices $i \in I = \{1, \dots, 4\}$ to work not more than forty hours per week which is the equivalent of twenty two-hour shifts, handling call requests, chat requests and call and chat requests. Part-time employees with indices $i \in I = \{5, \dots, 8\}$ cannot work more than twenty-four hours per week which is the equivalent of twelve two-hour shifts, handling call requests, chat requests and call and chat requests.

Constraint d) ensures that the model assigns consecutive shifts for full-time and part-time employees handling any type of requests. The first set of bounds tracks the first shift j when employee i starts working on day k. The second set of bounds make sure that if full-time employee i where $i \in I = \{1, \dots, 4\}$, starts working on shift j on day k, then he/she should keep working for the next three two-hour shifts only, for a total of four two-hour shifts, the equivalent of eight hours on day k. The third set of bounds make sure that if part-time employee i where $i \in I = \{5, \dots, 8\}$, starts working on shift j on day k, then he/she should keep working for the next two-hour shift only, for a total of two two-hour shifts, the equivalent of four hours on day k.

Constraint e) is used to restrict full-time employees from starting work at any of the last three two-hour shifts that is for $j \in J = \{10, \dots, 12\}$, on day k. On the other hand, part-time employees are not allowed to start working at the last two-hour shift that is for $j = 12$, on day k.

This constraint also restricts full-time and part-time employees from starting work more than once on day k , in other words, if employee i comes to work on day k, he/she is not allowed to finish his/her hours and then come back later for another set of shifts on the same day.

Constraint f) is derived to assign twelve-hour breaks to full-time and part-time employees. To account for breaks, six constraints are needed to track when full-time employee i starts working on day k. The constraint will then assign the next nine a's to 0, in other words, once an employee starts working on day k, then he/she cannot start working again on any of the nine shifts that follow. This ensures that if a full-time employee starts working on day k, he/she will have to work for next three two-hour shifts which is restricted by constraint d), for a total of eight hours a day. Constraint f) will then ensure that the following six two-hour shifts are 0, giving the full-time employee a twelve-hour break before starting work again. Same logic applies for part-time employees.

Constraint g) ensures that the call and chat demand on shift j, day k are met by single skilled and/or multi skilled employees. Since multi skilled employees are able to serve call and/or chat demand, then we had to allocate 60% of their total available working time – available time S multiplied by productivity p – to serve call demand D_{jk}^{call}, and 40% for chat demand D_{jk}^{chat}. The proportions were set based on the analysis of the call and chat historical demand data, where call seemed to be higher and hence require more employees to serve it than chat. Single skilled employees are 100% dedicated to serve the demand of the channel they are assigned to.

Constraint h) is used to track the hiring of employees. If an employee works at least one day, then we consider him/her to be hired. With every new hire, there are hiring, and training costs incurred and represented in the objective function.