Projet LTHC

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1 Definition des différents gradients

- Premier $\mathcal{H}$:

$$\begin{aligned}
\mathcal{H}(u,v) &= \frac{1}{n^2}\|Y - uv^T\|_F^2 \\
&= \frac{1}{n^2}((\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - u_i v_j)^2)^{1/2})^2 \\
&= \frac{1}{n^2}\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - u_i v_j)^2
\end{aligned}$$

$$\nabla_u \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u_1,v)}{\partial u_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u_N,v)}{\partial u_N} \end{pmatrix} \qquad \nabla_v \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u,v_1)}{\partial v_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u,v_M)}{\partial v_M} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{n^2}\sum_{j=1}^{M}(-2v_j)(Y_{1j} - u_1 v_j) \\ \vdots \\ \frac{1}{n^2}\sum_{j=1}^{M}(-2v_j)(Y_{Nj} - u_N v_j) \end{pmatrix} \qquad = \begin{pmatrix} \frac{1}{n^2}\sum_{i=1}^{N}(-2u_i)(Y_{i1} - u_i v_1) \\ \vdots \\ \frac{1}{n^2}\sum_{i=1}^{N}(-2v_M)(Y_{iM} - u_i v_M) \end{pmatrix}$$

$$= \frac{-2}{n^2}\begin{pmatrix} \sum_{j=1}^{M} v_j(Y_{1j} - u_1 v_j) \\ \vdots \\ \sum_{j=1}^{M} v_j(Y_{Nj} - u_N v_j) \end{pmatrix} \qquad = \frac{-2}{n^2}\begin{pmatrix} \sum_{i=1}^{N} u_i(Y_{i1} - u_i v_1) \\ \vdots \\ \sum_{i=1}^{N} u_i(Y_{iM} - u_i v_M) \end{pmatrix}$$

- Second $\mathcal{H}$:

$$\mathcal{H}(u,v) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - \sqrt{\frac{\lambda}{N}}u_i v_j)^2$$

$$\nabla_u \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u_1,v)}{\partial u_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u_N,v)}{\partial u_N} \end{pmatrix} \qquad \nabla_v \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u,v_1)}{\partial v_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u,v_M)}{\partial v_M} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\sum_{j=1}^{M}(-2\sqrt{\frac{\lambda}{N}}v_j)(Y_{1j} - \sqrt{\frac{\lambda}{N}}u_1 v_j) \\ \vdots \\ \frac{1}{2}\sum_{j=1}^{M}(-2\sqrt{\frac{\lambda}{N}}v_j)(Y_{Nj} - \sqrt{\frac{\lambda}{N}}u_N v_j) \end{pmatrix} \qquad = \begin{pmatrix} \frac{1}{2}\sum_{i=1}^{N}(-2\sqrt{\frac{\lambda}{N}}u_i)(Y_{i1} - \sqrt{\frac{\lambda}{N}}u_i v_1) \\ \vdots \\ \frac{1}{2}\sum_{i=1}^{N}(-2\sqrt{\frac{\lambda}{N}}u_i)(Y_{iM} - \sqrt{\frac{\lambda}{N}}u_i v_M) \end{pmatrix}$$

$$= -\sqrt{\frac{\lambda}{N}} \begin{pmatrix} \sum_{j=1}^{M} v_j(Y_{1j} - \sqrt{\frac{\lambda}{N}}u_1 v_j) \\ \vdots \\ \sum_{j=1}^{M} v_j(Y_{Nj} - \sqrt{\frac{\lambda}{N}}u_N v_j) \end{pmatrix} \qquad = -\sqrt{\frac{\lambda}{N}} \begin{pmatrix} \sum_{i=1}^{N} u_i(Y_{i1} - \sqrt{\frac{\lambda}{N}}u_i v_1) \\ \vdots \\ \sum_{i=1}^{N} u_i(Y_{iM} - \sqrt{\frac{\lambda}{N}}u_i v_M) \end{pmatrix}$$

2 Discrétisation:

On veut discrétiser :

$$du(t) = -\frac{1}{\lambda_1}(\mathbb{1} - \frac{u(t)u(t)^\top}{N})\nabla_u \mathcal{H} dt + \sqrt{\frac{2}{\lambda_1 \beta_1}}(\mathbb{1} - \frac{u(t)u(t)^\top}{N}) dW_u(t) - \frac{N-1}{N\lambda_1\beta_1} u(t) dt$$

On pose:
- $du(t) = u_{n+1} - u_n$
- $dt = h$, h est le pas de temps > 0, $t_{n+1} = t_n + h$
- $u(t_n) = u_n$
- $dW_u(t) \approx \mathcal{N}(0, h) = w$

$$u_{n+1} = u_n - \frac{1}{\lambda_1}(\mathbb{1} - \frac{u_n u_n^\top}{N})\nabla_u \mathcal{H}(u_n) h + \sqrt{\frac{2}{\lambda_1 \beta_1}}(\mathbb{1} - \frac{u_n u_n^\top}{N}) w - \frac{N-1}{N\lambda_1\beta_1} u_n h$$