Projet LTHC

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1 Definition des différents gradients

- Premier $\mathcal{H}$:

$$\begin{aligned}\mathcal{H}(u,v) &= \frac{1}{n^2}\|Y - uv^T\|_F^2 \\ &= \frac{1}{n^2}((\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - u_i v_j)^2)^{1/2})^2 \\ &= \frac{1}{n^2}\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - u_i v_j)^2\end{aligned}$$

$$\nabla_v \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u,v_1)}{\partial v_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u,v_N)}{\partial v_N} \end{pmatrix} \qquad \nabla_u \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u_1,v)}{\partial u_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u_N,v)}{\partial u_N} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{n^2}\sum_{i=1}^{N}(-2u_i)(Y_{i1} - u_i v_1) \\ \vdots \\ \frac{1}{n^2}\sum_{i=1}^{N}(-2v_M)(Y_{iM} - u_i v_M) \end{pmatrix} \qquad = \begin{pmatrix} \frac{1}{n^2}\sum_{j=1}^{M}(-2v_j)(Y_{1j} - u_1 v_j) \\ \vdots \\ \frac{1}{n^2}\sum_{j=1}^{M}(-2v_j)(Y_{Nj} - u_N v_j) \end{pmatrix}$$

$$= \frac{-2}{n^2}\begin{pmatrix} \sum_{i=1}^{N} u_i(Y_{i1} - u_i v_1) \\ \vdots \\ \sum_{i=1}^{N} u_i(Y_{iM} - u_i v_M) \end{pmatrix} \qquad = \frac{-2}{n^2}\begin{pmatrix} \sum_{j=1}^{M} v_j(Y_{1j} - u_1 v_j) \\ \vdots \\ \sum_{j=1}^{M} v_j(Y_{Nj} - u_N v_j) \end{pmatrix}$$

- Second $\mathcal{H}$:

$$\mathcal{H}(u,v) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{M}(Y_{ij} - \sqrt{\frac{\lambda}{N}}u_i v_j)^2$$

$$\nabla_u \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u_1,v)}{\partial u_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u_N,v)}{\partial u_N} \end{pmatrix} \qquad \nabla_v \mathcal{H}(u,v) = \begin{pmatrix} \frac{\partial \mathcal{H}(u,v_1)}{\partial v_1} \\ \vdots \\ \frac{\partial \mathcal{H}(u,v_N)}{\partial v_N} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2}\sum_{j=1}^{M}(-2\sqrt{\frac{\lambda}{N}}v_j)(Y_{1j} - \sqrt{\frac{\lambda}{N}}u_1 v_j) \\ \vdots \\ \frac{1}{2}\sum_{j=1}^{M}(-2\sqrt{\frac{\lambda}{N}}v_j)(Y_{Nj} - \sqrt{\frac{\lambda}{N}}u_N v_j) \end{pmatrix} \qquad = \begin{pmatrix} \frac{1}{2}\sum_{i=1}^{N}(-2\sqrt{\frac{\lambda}{N}}u_i)(Y_{i1} - \sqrt{\frac{\lambda}{N}}u_i v_1) \\ \vdots \\ \frac{1}{2}\sum_{i=1}^{N}(-2\sqrt{\frac{\lambda}{N}}u_i)(Y_{iM} - \sqrt{\frac{\lambda}{N}}u_i v_M) \end{pmatrix}$$

$$= -\sqrt{\frac{\lambda}{N}} \begin{pmatrix} \sum_{j=1}^{M} v_j(Y_{1j} - \sqrt{\frac{\lambda}{N}}u_1 v_j) \\ \vdots \\ \sum_{j=1}^{M} v_j(Y_{Nj} - \sqrt{\frac{\lambda}{N}}u_N v_j) \end{pmatrix} \qquad = -\sqrt{\frac{\lambda}{N}} \begin{pmatrix} \sum_{i=1}^{N} u_i(Y_{i1} - \sqrt{\frac{\lambda}{N}}u_i v_1) \\ \vdots \\ \sum_{i=1}^{N} u_i(Y_{iM} - \sqrt{\frac{\lambda}{N}}u_i v_M) \end{pmatrix}$$