Ch 5 1 ModelSelect

Analyze the ISLR (Introduction to Statistical Learning with R) data package's baseball 'Hitters' data frame:

```
library(ISLR)
summary(Hitters)
```

```
HmRun
##
        AtBat
                           Hits
                                                           Runs
##
    Min.
           : 16.0
                                    Min.
                                            : 0.00
                                                      Min.
                                                              : 0.00
                     Min.
                             :
    1st Qu.:255.2
                     1st Qu.: 64
                                    1st Qu.: 4.00
                                                      1st Qu.: 30.25
##
##
    Median :379.5
                     Median: 96
                                    Median: 8.00
                                                      Median: 48.00
            :380.9
                                                              : 50.91
##
    Mean
                     Mean
                             :101
                                    Mean
                                            :10.77
                                                      Mean
    3rd Qu.:512.0
##
                     3rd Qu.:137
                                    3rd Qu.:16.00
                                                      3rd Qu.: 69.00
            :687.0
                             :238
                                            :40.00
##
    Max.
                     Max.
                                    Max.
                                                      Max.
                                                              :130.00
##
##
         RBI
                           Walks
                                             Years
                                                               CAtBat
##
    Min.
           :
              0.00
                              :
                                 0.00
                                         Min.
                                                : 1.000
                                                                   :
                                                                       19.0
                      Min.
                                                           Min.
##
    1st Qu.: 28.00
                      1st Qu.: 22.00
                                         1st Qu.: 4.000
                                                           1st Qu.:
                                                                      816.8
##
    Median : 44.00
                      Median : 35.00
                                         Median : 6.000
                                                           Median: 1928.0
##
           : 48.03
                              : 38.74
                                                : 7.444
                                                                   : 2648.7
    Mean
                      Mean
                                         Mean
                                                           Mean
    3rd Qu.: 64.75
                      3rd Qu.: 53.00
##
                                         3rd Qu.:11.000
                                                           3rd Qu.: 3924.2
##
    Max.
           :121.00
                      Max.
                              :105.00
                                         Max.
                                                :24.000
                                                           Max.
                                                                   :14053.0
##
##
        CHits
                           CHmRun
                                             CRuns
                                                                 CRBI
##
                              : 0.00
                                                :
                                                                       0.00
    Min.
                4.0
                      Min.
                                         Min.
                                                     1.0
                                                           Min.
                      1st Qu.: 14.00
                                                                      88.75
##
    1st Qu.: 209.0
                                         1st Qu.: 100.2
                                                           1st Qu.:
##
    Median : 508.0
                      Median: 37.50
                                         Median : 247.0
                                                           Median: 220.50
##
    Mean
           : 717.6
                              : 69.49
                                                : 358.8
                                                                   : 330.12
                      Mean
                                         Mean
                                                           Mean
    3rd Qu.:1059.2
                      3rd Qu.: 90.00
##
                                         3rd Qu.: 526.2
                                                           3rd Qu.: 426.25
            :4256.0
##
    Max.
                      Max.
                              :548.00
                                         Max.
                                                :2165.0
                                                           Max.
                                                                   :1659.00
##
##
        CWalks
                                             PutOuts
                                                               Assists
                       League
                               Division
##
    Min.
           :
                0.00
                       A:175
                                E:157
                                          Min.
                                                  :
                                                      0.0
                                                            Min.
                                                                       0.0
##
    1st Qu.: 67.25
                       N:147
                                W:165
                                          1st Qu.: 109.2
                                                            1st Qu.:
                                                                      7.0
##
    Median: 170.50
                                          Median : 212.0
                                                            Median: 39.5
            : 260.24
                                                  : 288.9
                                                                    :106.9
##
    Mean
                                          Mean
                                                            Mean
##
    3rd Qu.: 339.25
                                          3rd Qu.: 325.0
                                                            3rd Qu.:166.0
            :1566.00
##
    Max.
                                          Max.
                                                 :1378.0
                                                            Max.
                                                                    :492.0
##
##
        Errors
                          Salary
                                        NewLeague
##
           : 0.00
                             : 67.5
                                        A:176
    Min.
                     Min.
##
    1st Qu.: 3.00
                     1st Qu.: 190.0
                                        N:146
    Median: 6.00
                     Median: 425.0
##
            : 8.04
                             : 535.9
    Mean
                     Mean
##
    3rd Qu.:11.00
                     3rd Qu.: 750.0
            :32.00
                             :2460.0
##
    Max.
                     Max.
##
                     NA's
                             :59
```

There are missing values, before we proceed we will remove them:

```
with(Hitters, sum(is.na(Salary)))
## [1] 59
Hitters=na.omit(Hitters)
with(Hitters, sum(is.na(Salary)))
## [1] 0
```

Best Subset regression

3 (1)""

4 (1)""

5 (1)"*"

6 (1) "*"

7 (1)""

8 (1)"*"

"*"

"*"

11 11

11 11

11 11

11 11

11 11

11 11

11 11 11 11

" " "*"

" " "*"

We will now use the package leaps to evaluate all the best-subset models.

```
library(leaps)
regfit.full = regsubsets(Salary~., data=Hitters)
summary(regfit.full)
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters)
## 19 Variables (and intercept)
             Forced in Forced out
##
## AtBat
                 FALSE
                             FALSE
## Hits
                 FALSE
                             FALSE
## HmRun
                 FALSE
                             FALSE
## Runs
                 FALSE
                             FALSE
## RBI
                 FALSE
                             FALSE
## Walks
                 FALSE
                             FALSE
## Years
                 FALSE
                             FALSE
## CAtBat
                 FALSE
                             FALSE
## CHits
                 FALSE
                             FALSE
## CHmRun
                 FALSE
                             FALSE
                             FALSE
## CRuns
                 FALSE
## CRBI
                 FALSE
                             FALSE
## CWalks
                 FALSE
                             FALSE
                 FALSE
                             FALSE
## LeagueN
## DivisionW
                 FALSE
                             FALSE
## PutOuts
                 FALSE
                             FALSE
## Assists
                 FALSE
                             FALSE
## Errors
                 FALSE
                             FALSE
## NewLeagueN
                 FALSE
                             FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
            AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
##
                             11 11
## 1 (1)""
                 11 11
                                                         11 11
                                                                       11 11
## 2 (1)""
                  "*"
                       11 11
                             11 11
                                 11 11
                                                  11 11
                                                         11 11
                                                               11 11
                                            11 11
```

11 11

11 11

11 11

11 11

11 11

"*"

"*"

"*"

"*"

.. ..

"*"

```
CRBI CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
##
      (1)"*"
                          11 11
                                   11 11
                                              11 11
                                                       11 11
                                                                        11 11
## 1
                                   11 11
                                                                        11 11
      (1)"*"
## 2
## 3
      (1
                                   "*"
                                              "*"
## 4
                                   "*"
                                              "*"
## 5
      (1
                                   "*"
                                              "*"
                                   "*"
      (1)
                                              "*"
## 7
                                   "*"
                                              "*"
## 8
      (1)
```

By default, it gives the first 8 variables best-subset models. Let's do it again for all the variables:

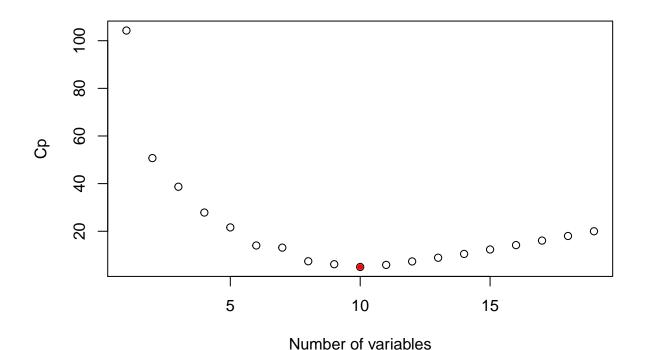
```
regfit.full = regsubsets(Salary~., data=Hitters, nvmax=19)
reg.summary = summary(regfit.full)
names(reg.summary)

## [1] "which" "rsq" "rss" "adjr2" "cp" "bic" "outmat" "obj"

plot(reg.summary$cp, xlab="Number of variables", ylab="Cp")
which.min(reg.summary$cp)

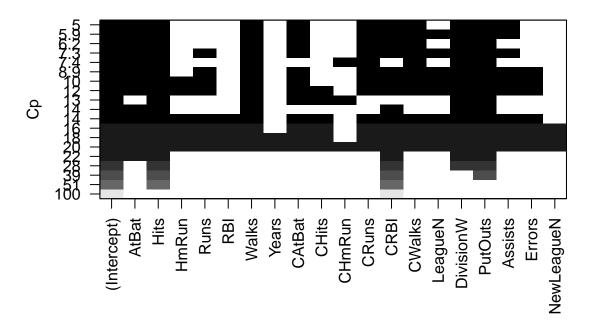
## [1] 10

points(10, reg.summary$cp[10], pch=20, col="red")
```



There is a method for the regsubset object:

```
plot(regfit.full,scale="Cp")
```



coef(regfit.full, 10)

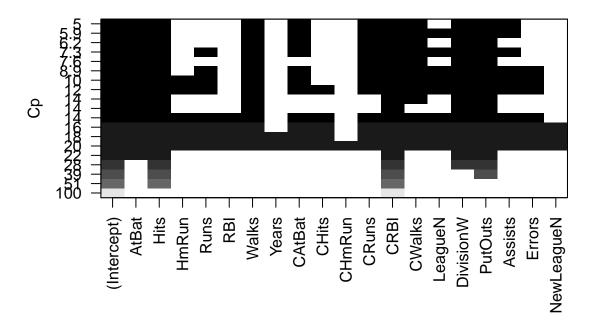
```
##
    (Intercept)
                         AtBat
                                        Hits
                                                     Walks
                                                                  CAtBat
    162.5354420
                                                5.7732246
##
                   -2.1686501
                                  6.9180175
                                                              -0.1300798
##
          CRuns
                          CRBI
                                      CWalks
                                                DivisionW
                                                                 PutOuts
##
      1.4082490
                    0.7743122
                                 -0.8308264 -112.3800575
                                                               0.2973726
##
        Assists
##
      0.2831680
```

Forward Stepwise Selection

We use regsubset again, but specify the method="forward" option.

```
regfit.fwd = regsubsets(Salary~., data=Hitters, nvmax=19, method="forward")
summary(regfit.fwd)
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = Hitters, nvmax = 19, method = "forward")
## 19 Variables (and intercept)
##
              Forced in Forced out
## AtBat
                  FALSE
                             FALSE
                             FALSE
## Hits
                  FALSE
## HmRun
                  FALSE
                             FALSE
## Runs
                  FALSE
                             FALSE
## RBI
                  FALSE
                             FALSE
## Walks
                  FALSE
                             FALSE
## Years
                  FALSE
                             FALSE
## CAtBat
                  FALSE
                             FALSE
## CHits
                  FALSE
                             FALSE
## CHmRun
                  FALSE
                             FALSE
## CRuns
                             FALSE
                  FALSE
## CRBI
                  FALSE
                             FALSE
```

```
## CWalks
                    FALSE
                                 FALSE
                                 FALSE
## LeagueN
                    FALSE
                    FALSE
## DivisionW
                                 FALSE
## PutOuts
                    FALSE
                                 FALSE
## Assists
                    FALSE
                                 FALSE
## Errors
                    FALSE
                                 FALSE
## NewLeagueN
                    FALSE
                                 FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##
               AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns
##
  1
      (1)
##
   2
      (1)
                      "*"
                                                                                  .. ..
##
       (1
##
      ( 1
## 5
       (1)
## 6
       (1
           )
               "*"
##
       (1
           )
               "*"
## 8
       (1)
               "*"
##
  9
       (1)
                                                                                  "*"
               "*"
                                                           11 * 11
## 10
       (1)
                                                                   11 11
                                                                                  "*"
##
   11
        (1
               "*"
                                                                                  "*"
## 12
        (1
                                                           11 * 11
## 13
        (1
            )
               "*"
                                   "*"
                                                           "*"
                                                                                  "*"
                                                                                  "*"
##
   14
        (1
            )
                                   "*"
                                                           "*"
               "*"
                            "*"
                                  "*"
                                                           "*"
                                                                                  "*"
## 15
        ( 1
            )
   16
        (1
                                  "*"
                                                           "*"
                                                                                  "*"
                                                                                  "*"
##
   17
        (1
            )
                            "*"
                                   "*"
                                                           "*"
                                                                   "*"
                                                                                  "*"
##
   18
        (1
                                   "*"
                                        "*" "*"
                                                    11 * 11
                                                           "*"
                                                                   "*"
                      "*"
                            "*"
                                   "*"
                                        "*" "*"
                                                           "*"
                                                                   "*"
                                                                          "*"
                                                                                  "*"
##
        (1)
   19
##
               CRBI
                    CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
                                      11 11
## 1
       (1)
                                      11 11
                                                  11 11
                                                                    11 11
                                                                            11 11
##
   2
       (1
##
   3
      ( 1
                                                  "*"
           )
##
      (1
           )
                                      "*"
                                                  "*"
## 5
       (1
           )
                                                  "*"
##
   6
       (1
                                      "*"
                                                  "*"
##
       ( 1
## 8
      ( 1
                                      "*"
                                                  "*"
                     11 * 11
                                      "*"
                                                  "*"
## 9
       (1
                                                  "*"
## 10
       (1)
              "*"
                     "*"
                                                  "*"
##
        ( 1
                                      "*"
                                                           "*"
                     "*"
                             "*"
                                      "*"
                                                  "*"
                                                           "*"
## 12
        ( 1
            )
##
   13
        (1
                     "*"
                                      "*"
                                                  "*"
                                                           "*"
                                                                    "*"
##
   14
        (1
                                      "*"
                                      "*"
                                                  "*"
                                                                            . .
##
  15
        (1
                                                           "*"
                                                                    " * "
                     "*"
                             "*"
                                      "*"
                                                  "*"
                                                           "*"
                                                                    "*"
## 16
        (1
            )
                                                  "*"
##
   17
        (
          1
            )
                     "*"
                             "*"
                                      "*"
                                                           "*"
                                                                    "*"
                                                                            "*"
                     "*"
                             "*"
                                      "*"
                                                  "*"
                                                           "*"
                                                                    "*"
                                                                            "*"
## 18
        (1
            )
              "*"
        (1)
              "*"
                                      "*"
                                                           "*"
                                                                    "*"
## 19
plot(regfit.fwd, scale="Cp")
```



Model Selection Using a Validation Set

Let's make a training and validation set, so that we can choose a good subset model.

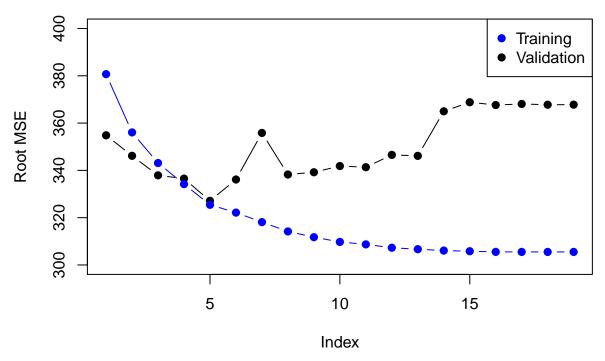
```
dim(Hitters)

## [1] 263 20

set.seed(1)
train = sample(seq(263),180,replace=FALSE)
regfit.fwd = regsubsets(Salary~., data=Hitters[train,], nvmax=19, method="forward")
```

Now, we separate the data to two parts, one for training and another for test/validation to make prediction. There is no prediction method for regsubsets, so we need to write that part. We also create a vector to store the results of the 19 different models.

```
val.errors = rep(NA, 19)
x.test = model.matrix(Salary~., data=Hitters[-train,])
for(i in 1:19){
   coefi = coef(regfit.fwd, id=i)
    pred = x.test[,names(coefi)]%*%coefi
   val.errors[i] = mean((Hitters$Salary[-train]-pred)^2)
}
plot(sqrt(val.errors), ylab="Root MSE", ylim=c(300,400), pch=19, type="b")
points(sqrt(regfit.fwd$rss[-1]/180),col="blue",pch=19,type="b") # -1 excludes null model
legend("topright", legend=c("Training","Validation"),col=c("blue","black"),pch=19)
```



As expected, the model error goes down monotonically as the model gets bigges, but not so for the validation error.

This was a little tedious - not having a predict method regsubsets. So we will write one!