# Abstract

One of the main task in statistical analysis of cosmological models is the estimation of unknown cosmological parameters. In this project, we are interested in determining these unknown parameters especially in cases where the model contains a large number of parameters, using Markov Chain Monte Carlo (MCMC) methods. In paticular we have developed a multi thread MCMC algorithm developed in C++ which can be used for parallel applcations. We have tested the algorithm with several test problems. These problems include finding parameters of linear, parabola, quintic funtions and minimum of rosenbrock function. Gelman Rubin diagnostics is used for chains convergence test.We then used our algorithm to estimate cosmological parameteres for a flat ΛCDM universe using observations of Hubble constant at different redshifts [13][12][9][7][16] [17]. Best fit parameter values for curved universe are $\Omega_m = 0.29 \pm 0.012$, $\Omega_\Lambda = 0.69 \pm 0.033$ and $H_0$(km/s/Mpc) $= 71.23 \pm 1.43$. For a flat universe $\Omega_m = 0.28 \pm 0.012$ and $H_0$(km/s/Mpc)$= 70.98 \pm 0.95$. We have also obtained 2d marginalized distributions of these parameters.For this we have used python. Our results are consistent with other estimates from literature.

# Chapter 7

# Result and Conclusion

In chapter 5, we generated data with putting parameter values, then using that data, program is executed. Chain generated in parameter space converged to values which are very consistent with known parameters. So it confirms that our program is ready for addressing real problems. Chapter 6 introduced concept of cosmological models.In this chapter we are going to fit Lambda Cold Dark-matter Model($\Lambda$CDM)

## 7.1 Parameter estimation of Curved $\Lambda$CDM model with parameter set $p = (H_0, \Omega_{m0}, \Omega_\Lambda)$

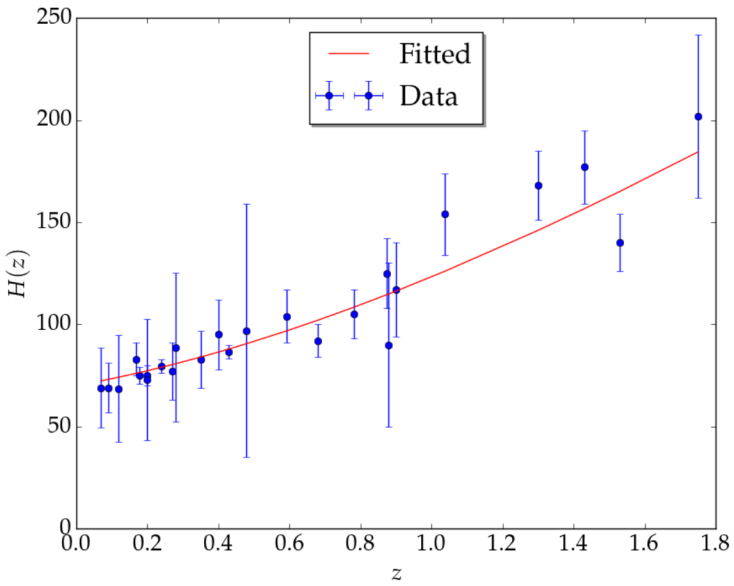


Figure 7.1: Data and fitted plot of OHD

Data were collected from different articles, all available OHD is tabulated in Table 7.1. In Figure 7.1 data points is plotted with corresponding errorbars. 2D Confidence region of parameters $\Omega_m, \Omega_\Lambda, \Omega_k$ and $H_0$ is shown in figure 7.2

| $z$ | $H(z)$ | $\sigma_{H(z)}$ | $Reference$ | $z$ | $H(z)$ | $\sigma_{H(z)}$ | $Reference$ |
|---|---|---|---|---|---|---|---|
| 0.090 | 69 | 12 | [7] | 0.352 | 83 | 14 | [9] |
| 0.170 | 83 | 8 | [16] | 0.593 | 104 | 13 | [9] |
| 0.270 | 77 | 14 | [16] | 0.680 | 92 | 8 | [9] |
| 0.400 | 95 | 17 | [16] | 0.781 | 105 | 12 | [9] |
| 0.900 | 117 | 23 | [16] | 0.875 | 125 | 17 | [9] |
| 1.300 | 168 | 17 | [16] | 1.037 | 154 | 20 | [9] |
| 1.430 | 177 | 18 | [16] | 0.24 | 79.69 | 3.32 | [12] |
| 1.530 | 140 | 14 | [16] | 0.43 | 86.45 | 3.27 | [12] |
| 1.750 | 202 | 40 | [16] | 0.07 | 69.0 | 19.6 | [17] |
| 0.480 | 97 | 62 | [13] | 0.12 | 68.6 | 26.2 | [17] |
| 0.880 | 90 | 40 | [13] | 0.20 | 72.9 | 29.6 | [17] |
| 0.179 | 75 | 4 | [9] | 0.28 | 88.8 | 36.6 | [17] |
| 0.199 | 75 | 5 | [9] | | | | |

Table 7.1: Observational Hubble Data

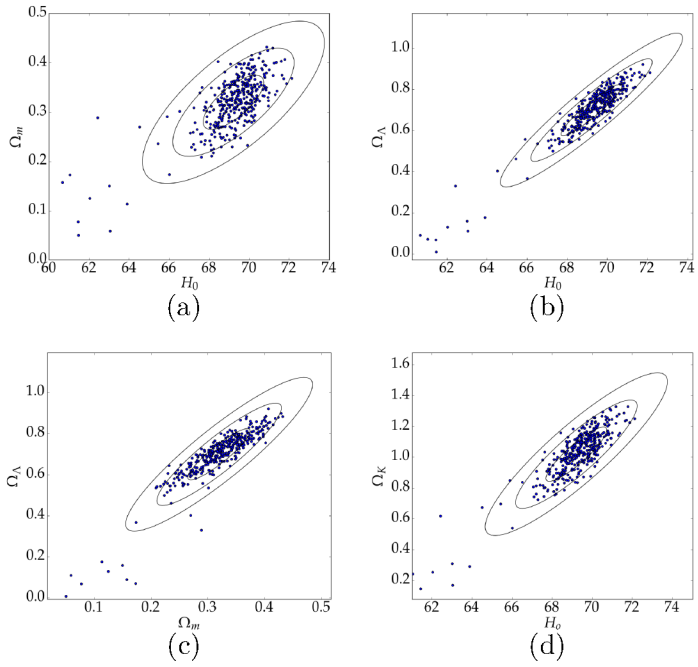


Figure 7.2: Confidence Region of parameters in $1\sigma, 2\sigma$ and $3\sigma$ contours. (a)-$(\Omega_m, H_0)$, (b)-$(\Omega_\Lambda, H_0)$, (c)-$(\Omega_\Lambda, \Omega_m)$ and (d)-$(\Omega_k, H_0)$

Obtained mean values of parameters from Markov chains are

| *Parameter* | *Mean* | $\sigma$ |
|---|---|---|
| $\Omega_m$ | 0.29 | $\pm 0.012$ |
| $\Omega_\Lambda$ | 0.69 | $\pm 0.033$ |
| $H_0(km/s/Mpc)$ | 71.23 | $\pm 1.43$ |

## 7.2 Parameter estimation of flat ΛCDM model with parameter set $p = (H_0, \Omega_{m0})$

Data used for estimation is same we used in previous one. For a flat universe $k = 0$, So Eq 6.28 is fitted.Estimated parameters from Markov chain are:

| *Parameter* | *Mean* | $\sigma$ |
|---|---|---|
| $\Omega_m$ | 0.28 | $\pm 0.012$ |
| $H_0(km/s/Mpc)$ | 70.98 | $\pm 0.95$ |

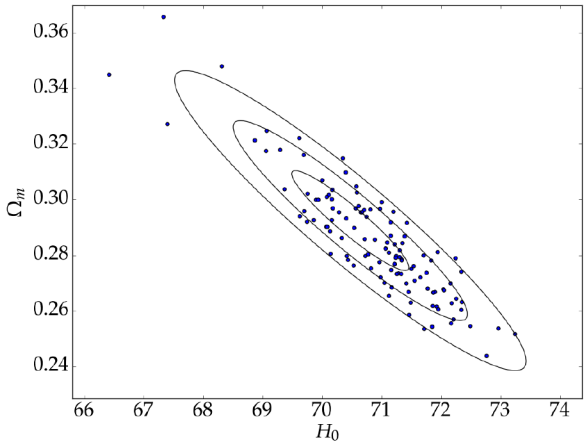


Figure 7.3: Confidence region of $\Omega_m$ and $H_0$

## 7.3 Conclusion

In this dissertation we present our calculations of hubble's constant and present cosmological density parameters.For that we developed a computational program to generate parameter chains using Markov Chain Monte Carlo algorithm. Implemented multi threading method

for chain generations with Gelman Rubin diagnostics as convergence alarm. Program is tested with different multi parameter functions. Data of OHD [7][9][12][13][16][17] are used to find cosmological parameters,from our calculations we found that, for a curved universe $\Omega_m = 0.29 \pm 0.012$, $\Omega_\Lambda = 0.69 \pm 0.033$ and $H_0(km/s/Mpc) = 71.23 \pm 1.43$. For a flat universe $\Omega_m = 0.28 \pm 0.012$ and $H_0(km/s/Mpc) = 70.98 \pm 0.95$.

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