Markov Chain Monte Carlo Methods : Cosmological Parameter Estimation

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Supervisor

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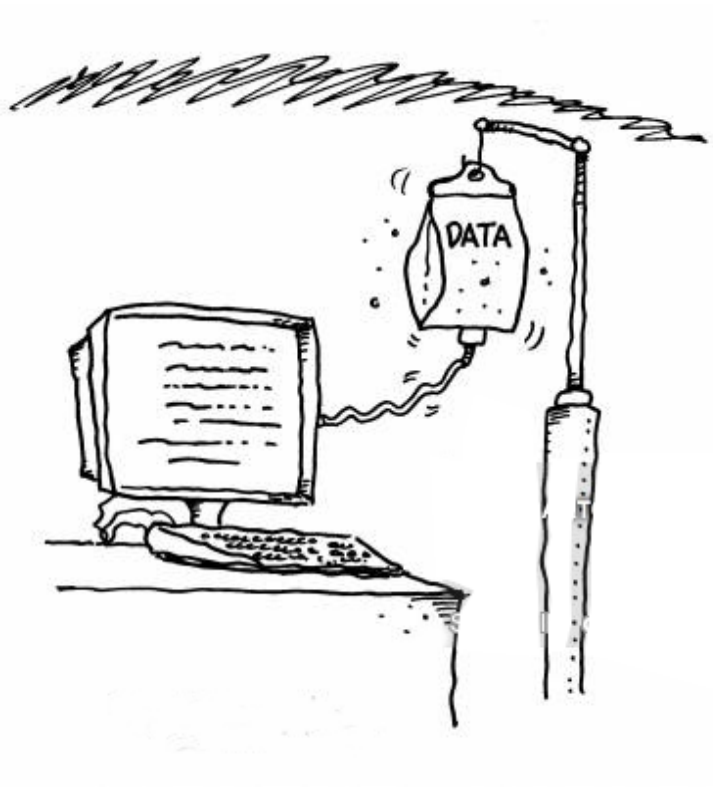
Transition of Science

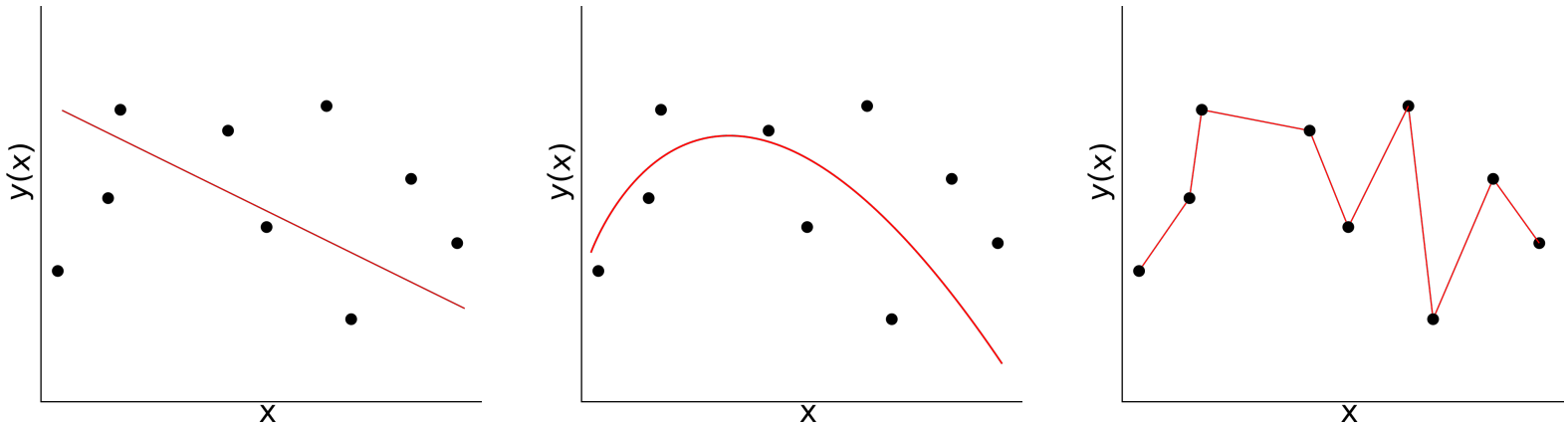
Science before | Science now



From **Data** To **Theory**
Contents

Plenty of Data...! What is the connection...? & Which is best...?

Choosing the method.



Why not choose the method with best fit of data ..?

How well are going to predict future data drawn from the same distribution.

Deducing the **Theory** ◇
Statistical Approach

Bayesian inference

- **Prior probability** : Pdf before evidence is taken into account.
- **Posterior probability** : Pdf conditional on the evidence obtained from observations.
- **Bayes' Theorem**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \quad P(A|B) = P(B|A)$$

So now we have,

$$P(Data|Theory) = P(Theory|Data)$$

Ref pg : 9,10.

Deducing the **Theory** ◇
from **Data** *to* **Theory**

Likelihood

- **Likelihood** is used when describing a function of a parameter(θ) given an outcome(x).Probability is the opposite.
- $L(data|theory) = P(theory|data)$

χ^2 Test

- A statistical method assessing the goodness of fit between a set of observed values and those expected theoretically.

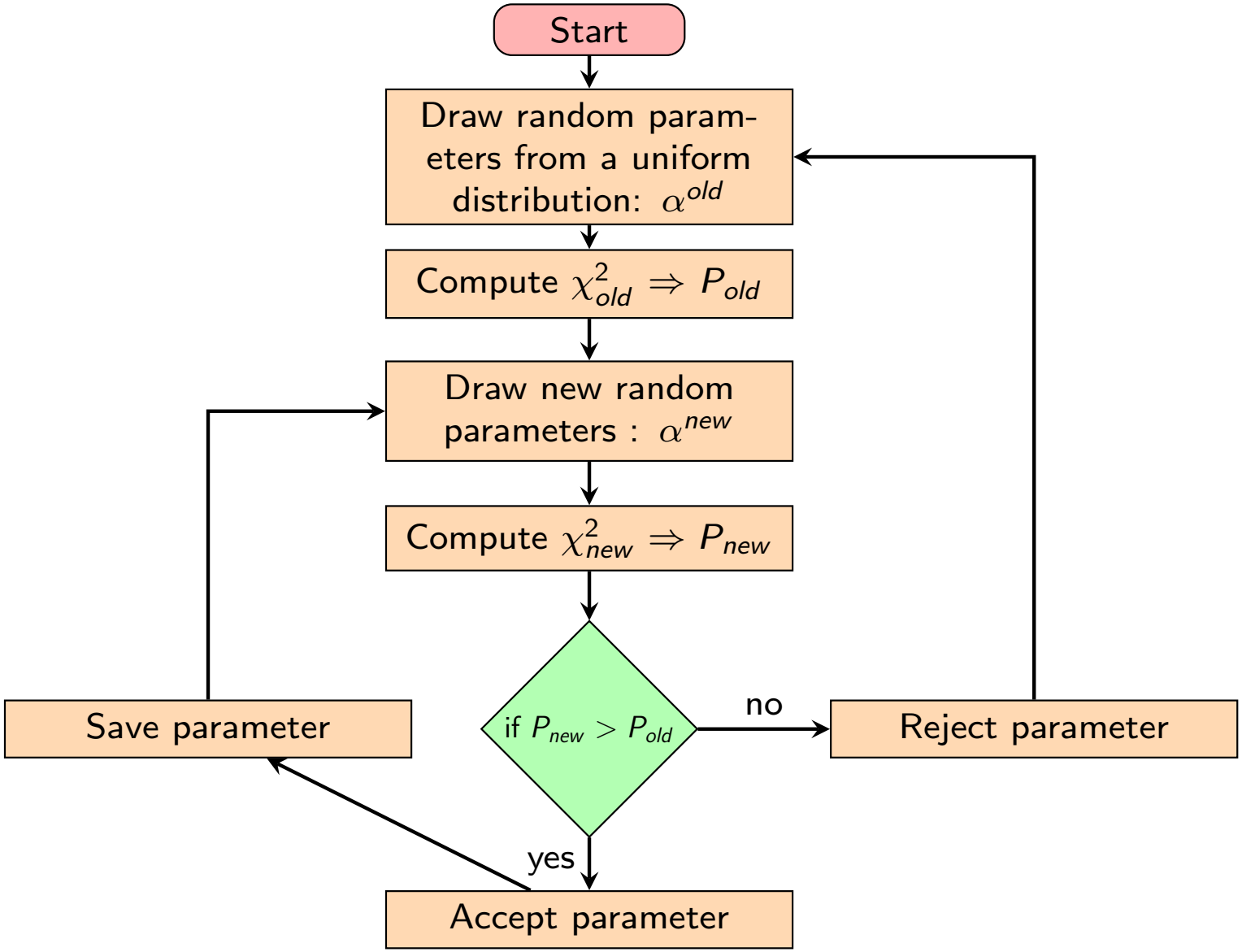
$$\chi^2 = \frac{(Data_{observed} - Data_{theory})^2}{error^2}$$

- Probability $P = e^{-\frac{\chi^2}{2}}$

Ref pg : 10,11,12.

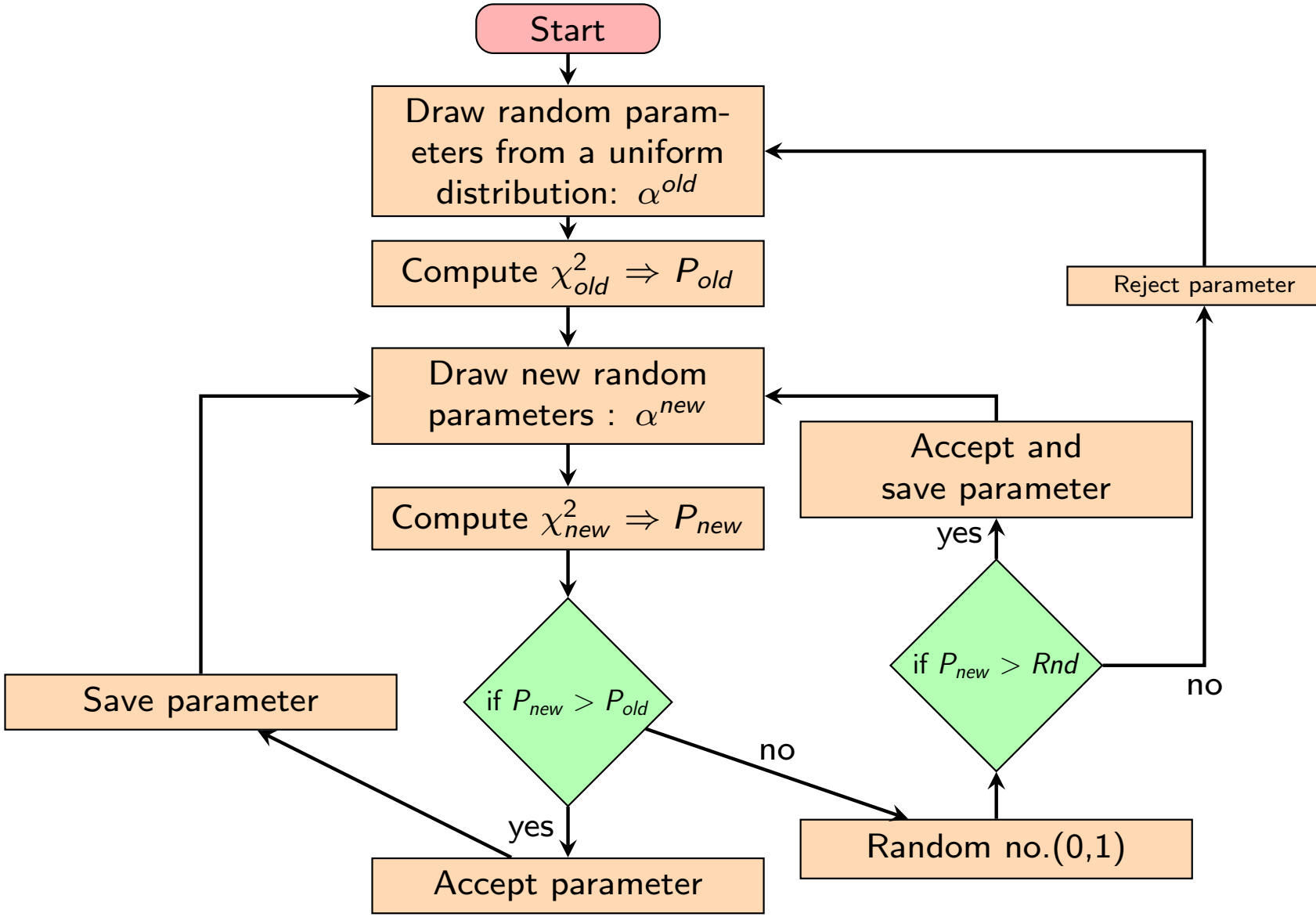
Memoryless | Vague : Inception !

Simple Algorithm



Memoryless | Vague : Inception !

MH Algorithm



The Chain

What is Chain.....?

Chain is the array of parameters saved during the process

What special about Markov Chain.....?

It is the chain generated by **Markov** process.

Markov process

A Markov process is a process in which the next step or iteration of the process only depends upon the current step and not upon any previous steps in the process

Ref pg : 21.

Trial Run

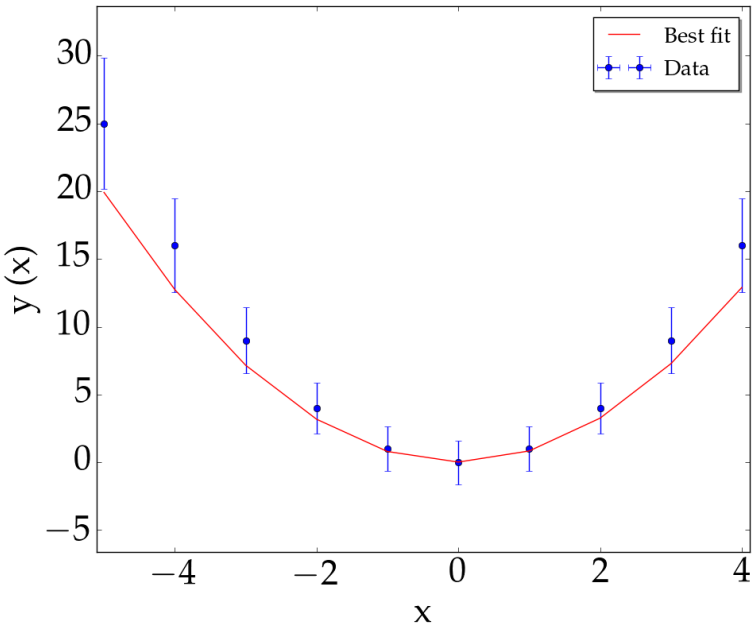
Parabolic Function

$$y(x) = ax^2 + bx + x$$

with parameter values,
$a = 1, b = 0, c = 0.$

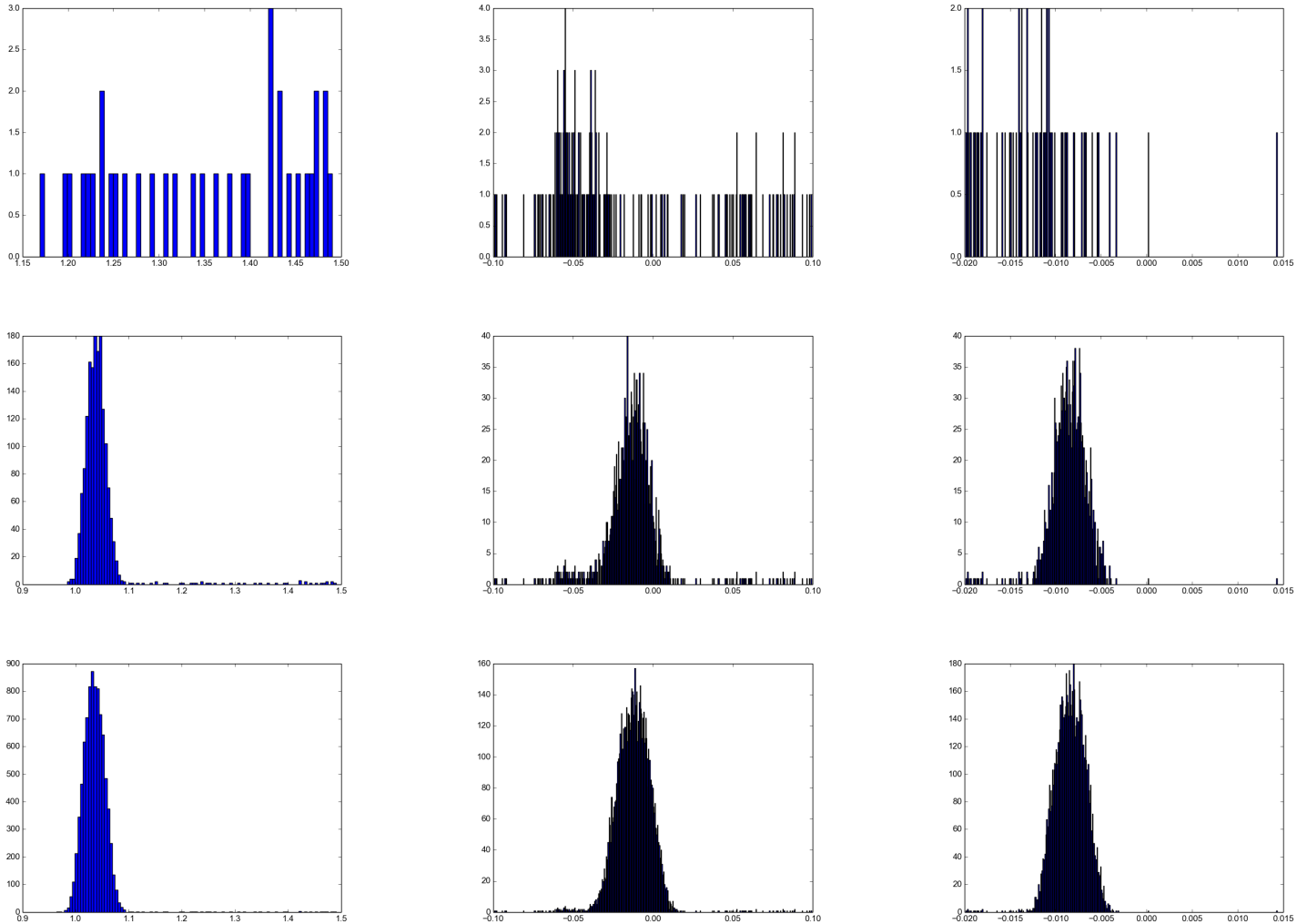
Fitted Parameters

- $a = 0.89$
- $b = 0.0261$
- $c = 0.014$



Gaussian Alarm

Chain convergence dependants on no. of iterations.



Should we go beyond ?
Potential Reduction Factor **R**

Gelman-Rubin Diagnostics

Let I represent the point in parameter space in position I of chain J.

$$\overline{\theta} \equiv \frac{1}{N}\sum_{I=1}^{N}\theta_I^J$$

The chain-to-chain variance B is

$$B = \frac{1}{(M-1)}\sum_{J=1}^{M}(\overline{\theta}^J - \overline{\theta})^2$$

and the average variance of each chain is

$$W = \frac{1}{M(N-1)}\sum_{I=1}^{N}\sum_{J=1}^{M}(\overline{\theta}_I^J - \overline{\theta}^J)^2$$

Should we go beyond ? *Off course, No !*
Potential Reduction Factor **R**

Gelman-Rubin Diagnostics

The weighted estimate of the variance,

$$\sigma^2 = \frac{(N-1)}{N} W + \frac{B}{N}$$

Accounting for the variance of the means gives an estimator of the variance

$$V = \sigma^2 + \frac{B}{NM}$$

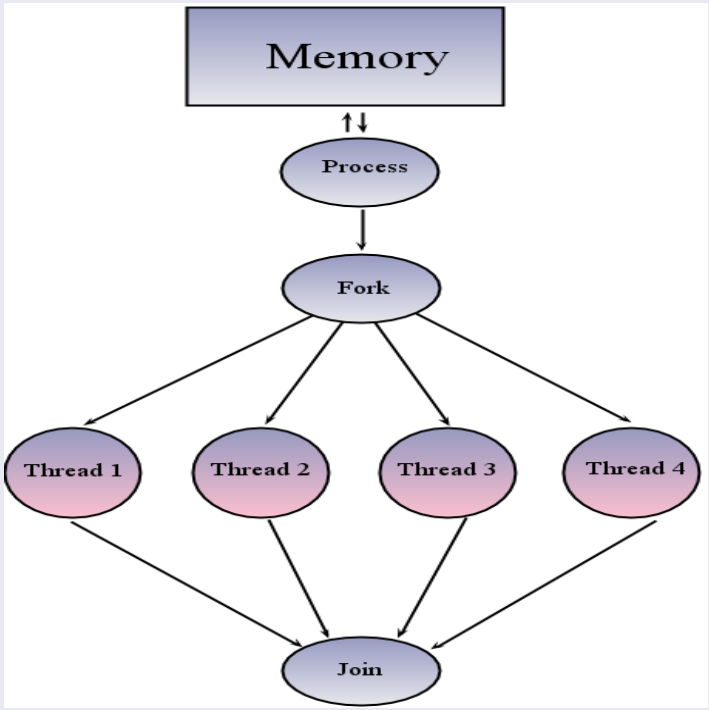
The ratio of the two estimates is

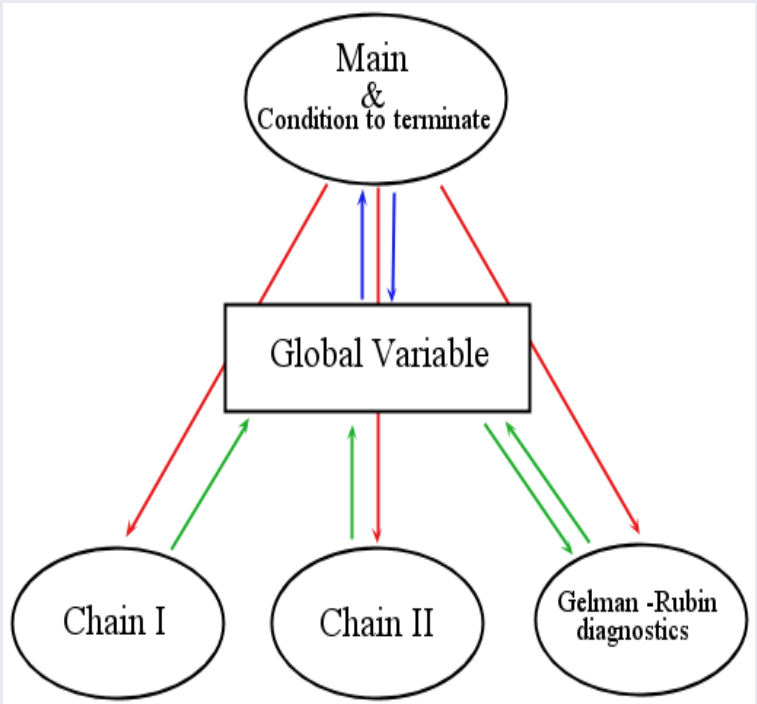
$$\hat{R} = \frac{\frac{(N-1)}{N} W + B(1+\frac{1}{M})}{W}$$

Convergence : How close R to unity? It is suggested to run the chain until the values of $\hat{R}$ are always < 1.03, or <1.02.

Divide & Conquer
Using all the 4 cores of CPU

Parallel Computing or Multi threading





There's no testimony without the test
Linear Function

Linear Function

$$y(x) = ax + b$$

with parameter values,
$a = 0.7, b = -0.1$

Best Fit Parameters

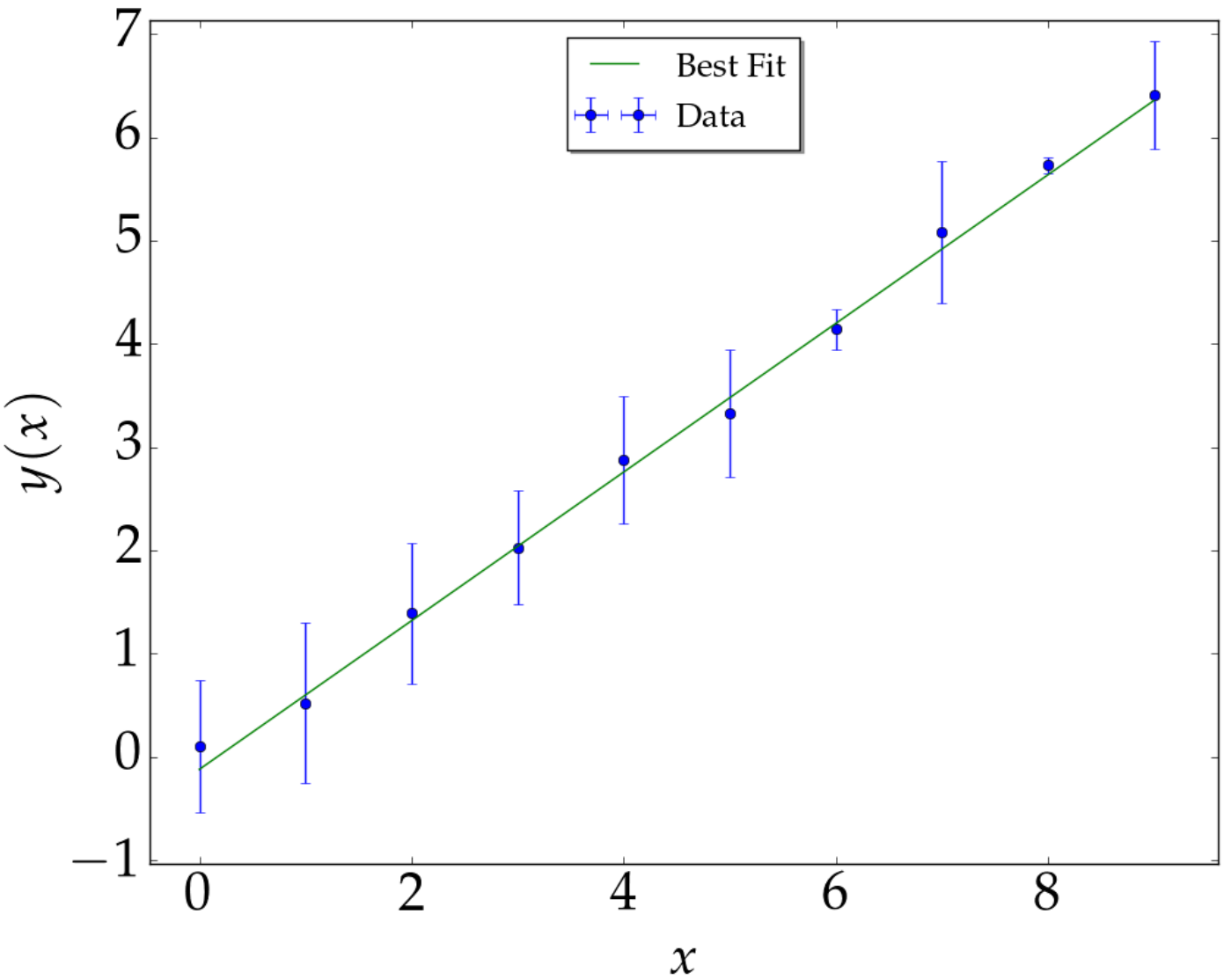
- $a = \quad 0.7296 \pm 0.026$
- $b = \quad -0.1201 \pm 0.010$

R Values

Ra	Rb
2.47157	1.84745
1.13047	1.36417
1.05914	1.2735
0.999527	1.06028
0.999002	1.01988

There's no testimony without the test

Linear Function



There's no testimony without the test

Parabolic Function

Parabolic Function

$$y(x) = ax^2 + bx + c$$

with parameter values,
$a = 1, b = 0 and c = 0$

Best Fit Parameters

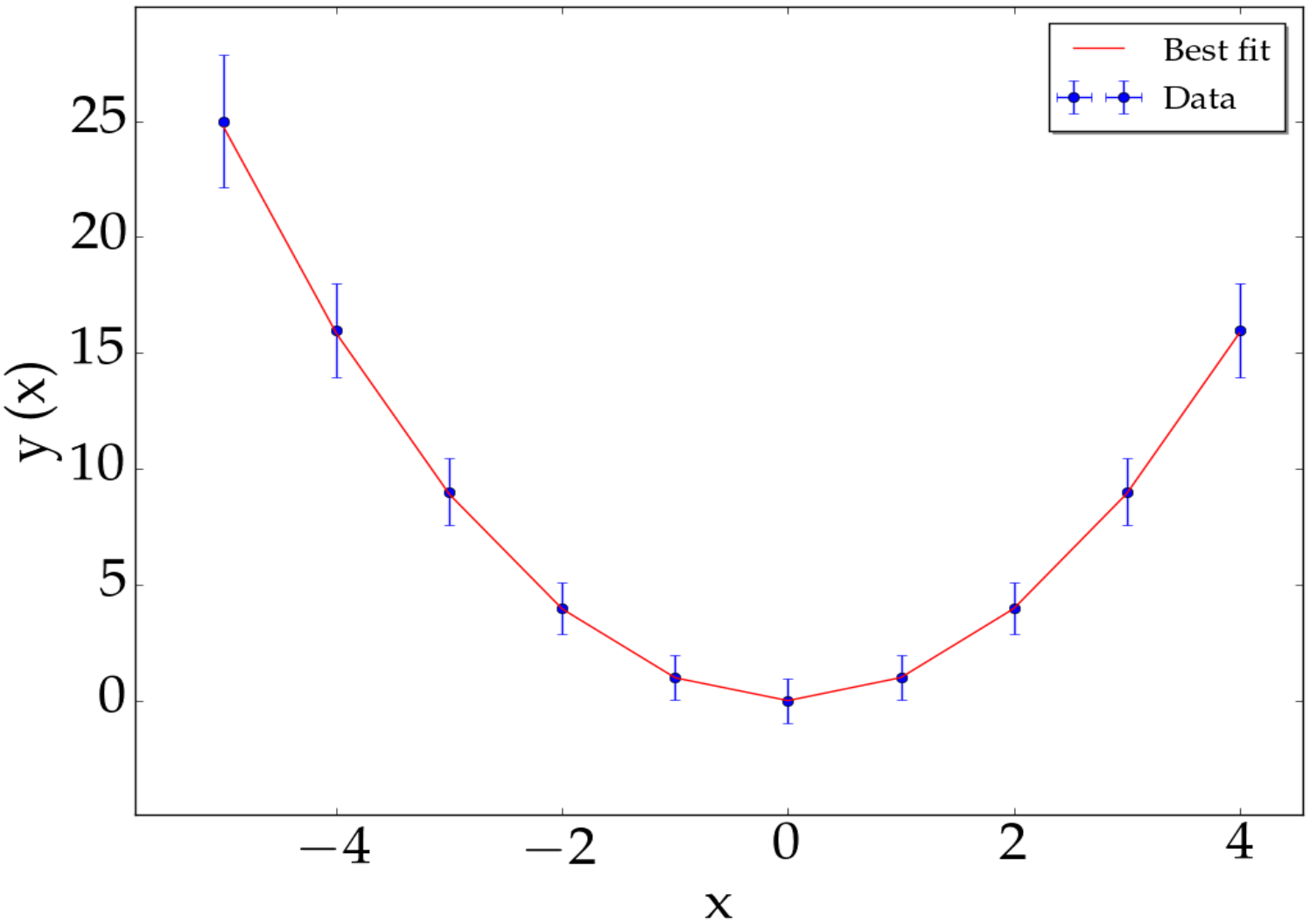
- $a = 0.9994 \pm 0.02199$
- $b = 0.00461 \pm 0.00115$
- $c = 0.0082 \pm 0.00292$

R Values

Ra	Rb	Rc
961235	4593.96	22548.9
⋮	⋮	⋮
18.4961	1.45216	9.93018
⋮	⋮	⋮
1.00463	1.00051	1.02966

There's no testimony without the test

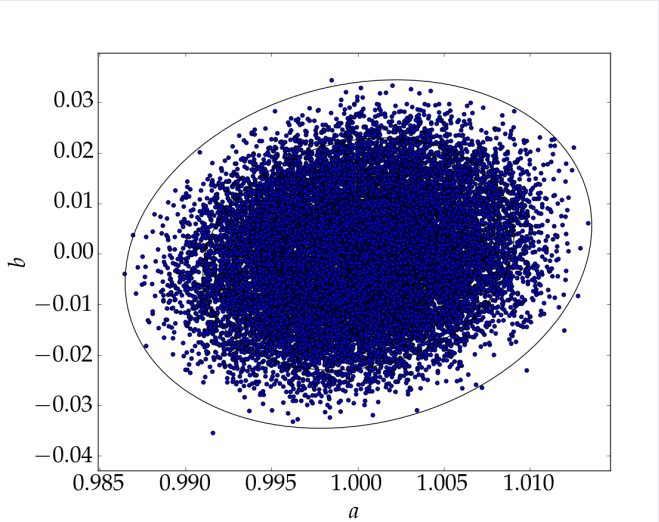
Parabolic Function

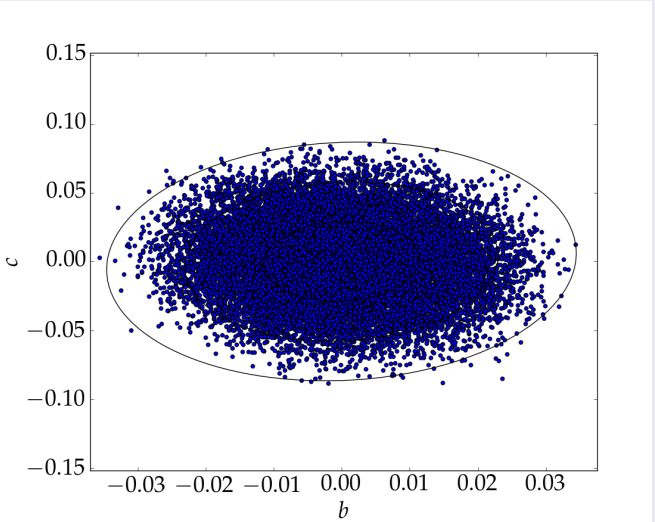


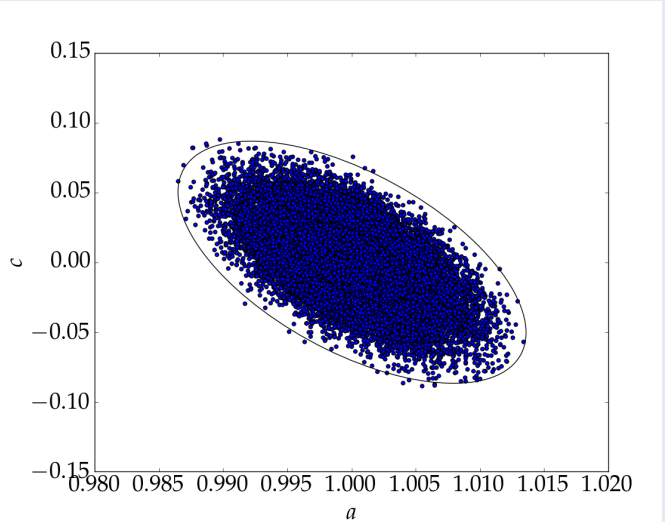
There's no testimony without the test

Parabolic Function

confidence plot







There's no testimony without the test

Quintic Function

Quintic Function

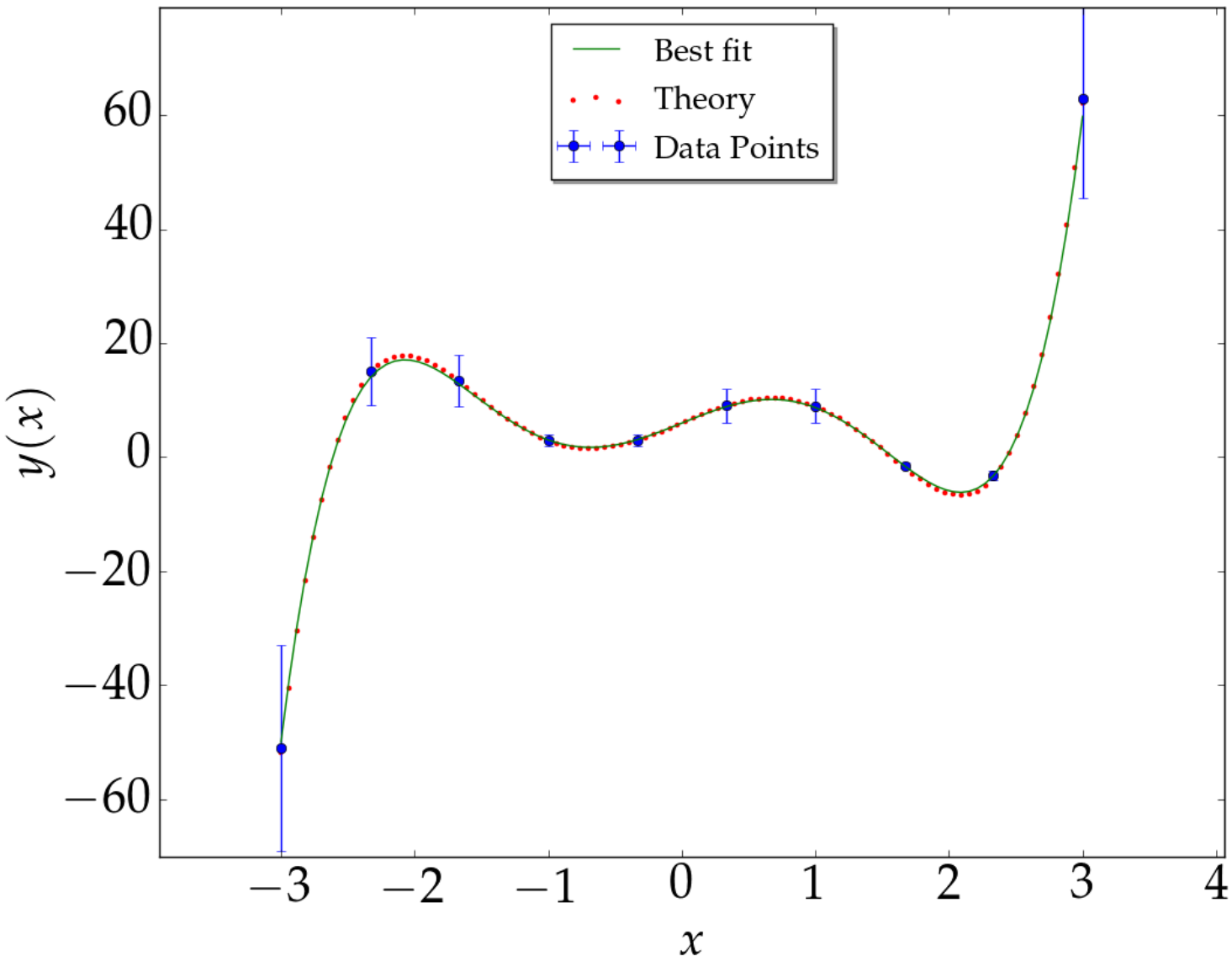
$y(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ with parameter values, $a = 1, b = -8, c = -0.1, d = 10, e = 6$

Best Fit Parameters

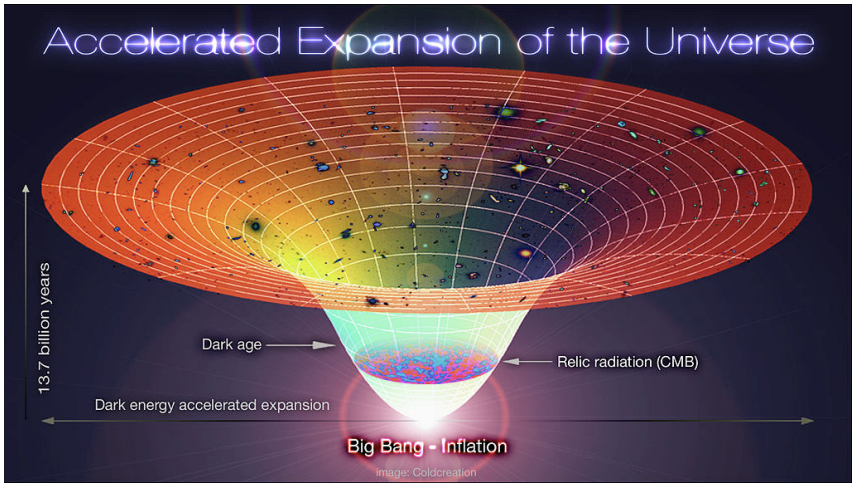
- $a = \quad 0.9569 \pm 0.0409$
- $b = \quad -7.6268 \pm 0..0412$
- $c = -0.1251 \pm 0.0279$
- $d = 9.5185 \pm 0.3456$
- $e = \quad 5.9936 \pm 0.0200$

There's no testimony without the test

Quintic Function



The Big Bang
Hubble's Law



Hubble's Law

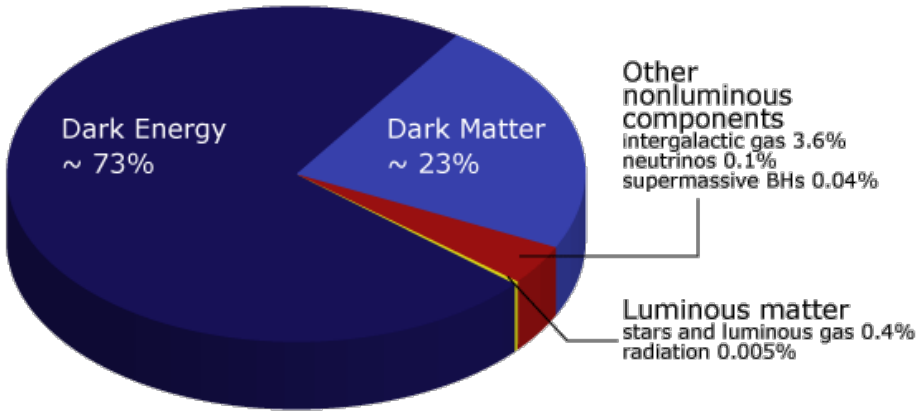
$$v_r = H_0 D$$

- v_r is the recessional velocity, typically expressed in km/s.
- H_0 is Hubble's constant and corresponds to the value of H.
- D is the proper distance from the galaxy to the observer, measured in mega parsecs (Mpc)

Lambda Cold Dark Matter - Model

It is frequently referred to as the standard model of Big Bang cosmology, because it is the simplest model that provides a reasonably good account of the following properties of the cosmos:

- The existence and structure of the cosmic microwave background.
- The large-scale structure in the distribution of galaxies.
- The abundances of hydrogen (including deuterium), helium, and lithium.
- The accelerating expansion of the universe observed in the light from distant galaxies and supernovae.



cout $\ll$ "Hello, Friedmann !" ;

Curved ΛCDM model

The Friedmann equation of the ΛCDM model with spatial curvature($k \neq 0$), $H^2(z, H_0, p)$ is

$$H_0^2 \left[\Omega_{m0}(1+z)^3 + \Omega_\Lambda + (1 - \Omega_{m0} - \Omega_\Lambda)(1+z)^2\right]$$

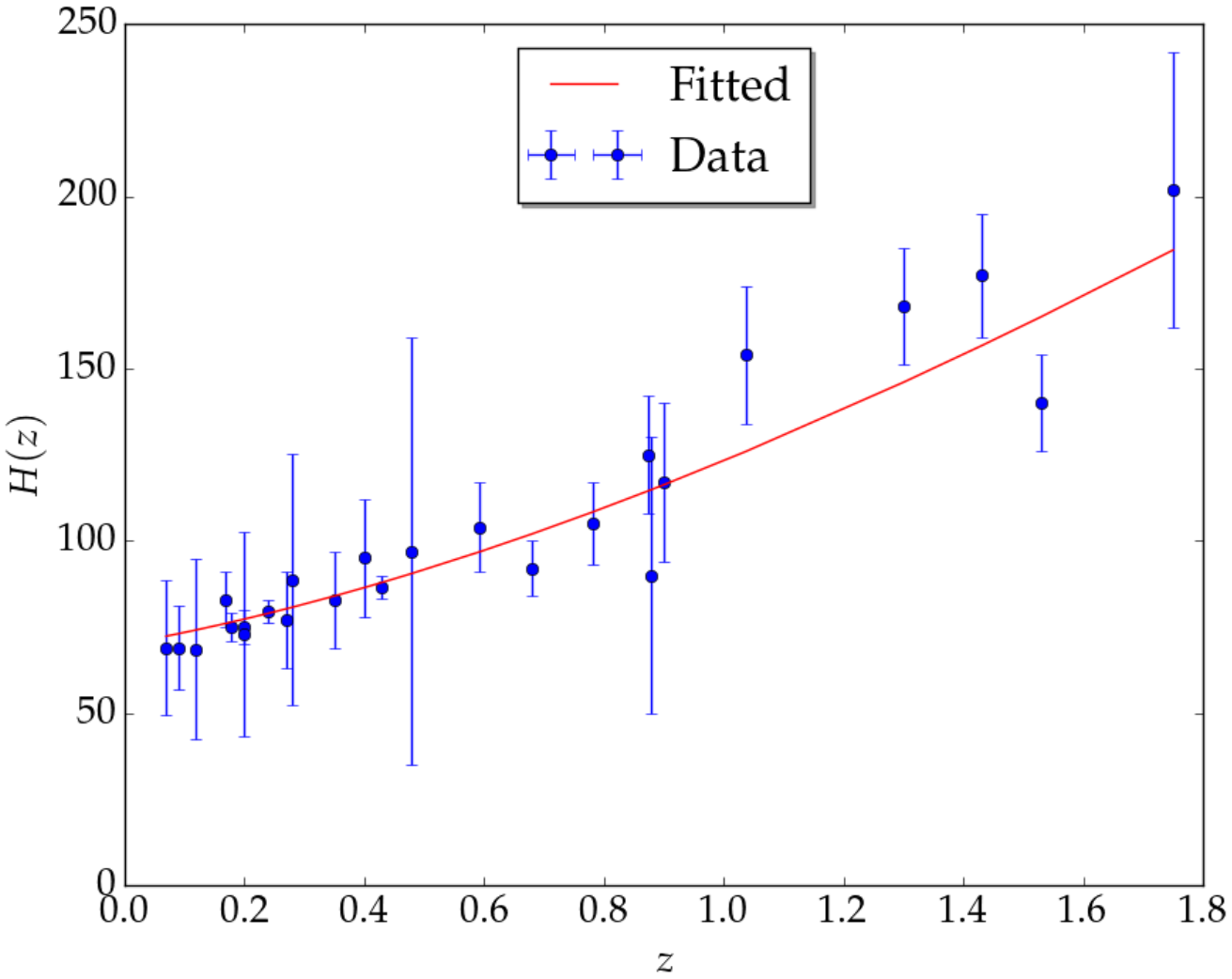
Flat ΛCDM model

For the Friedmann equation of the ΛCDM model without spatial curvature($k = 0$) For $k = 0$, $\Omega_k(\Omega_\Lambda + \Omega_{m0}) = 1$,and the last term in above equation get vanished and hence,

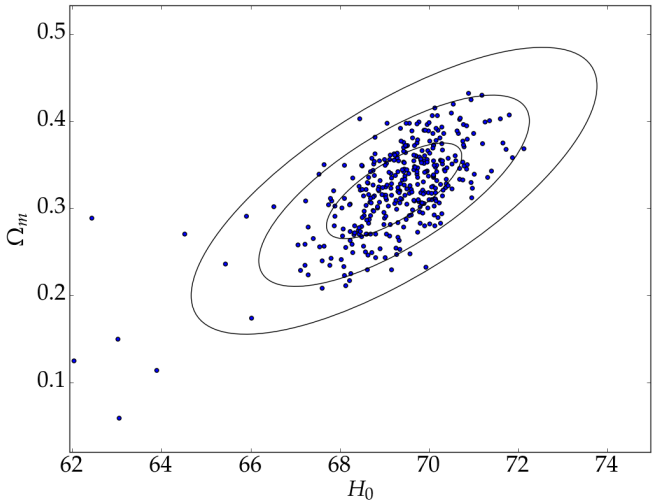
$$H^2(z, H_0, p) = H_0^2 \left[\Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})\right]$$

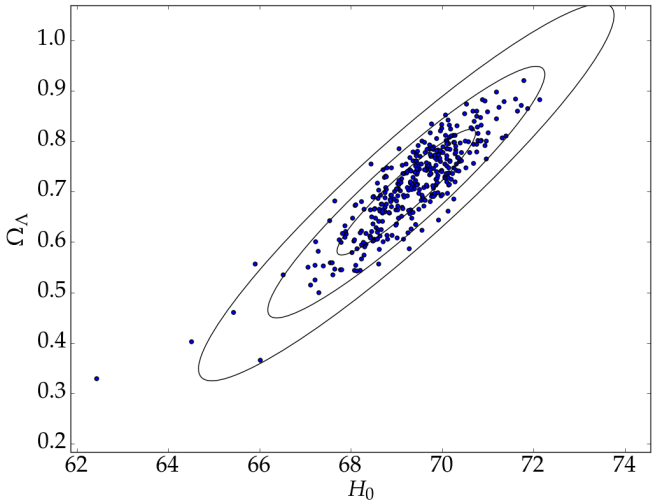
Hello, Friedmann !

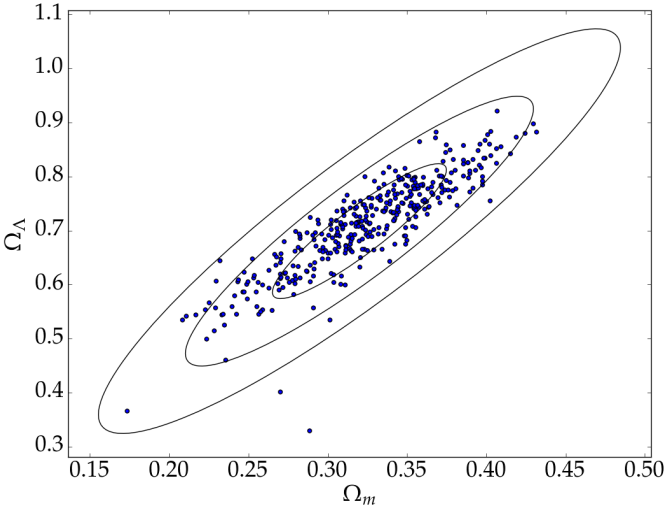
$H_0 = 70.4 \pm 1.4 km/s/Mpc, \Omega_m = 0.3089 \pm 0.0062, \Omega_\Lambda = 0.6911 \pm 0.0062$

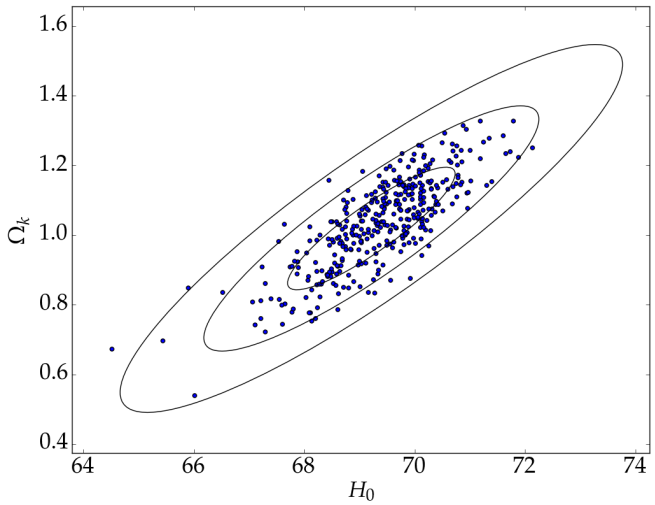


Curved ΛCDM model
Confidence Plot





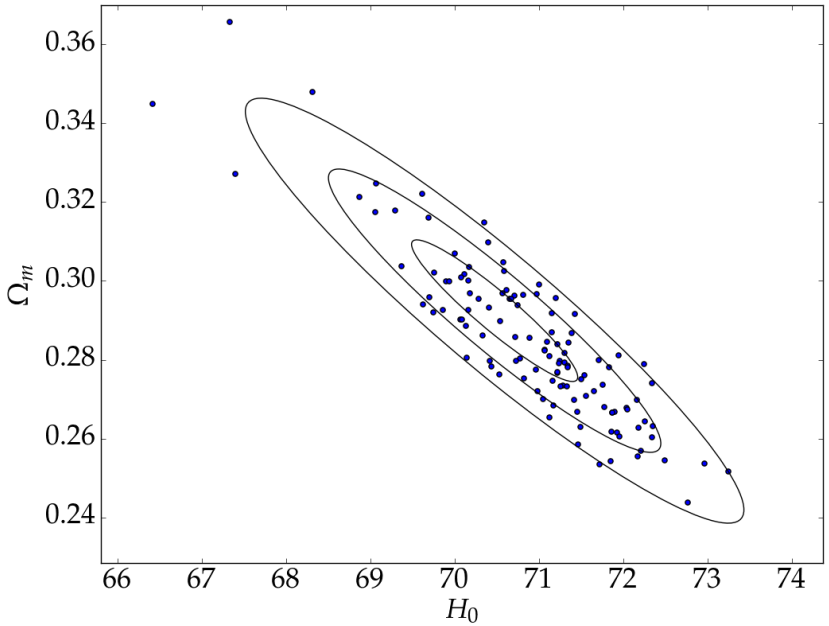




Curved ΛCDM model
Estimated Parameters

Parameter	*Mean*	σ
Ω_m	0.29	± 0.012
Ω_Λ	0.69	± 0.033
$H_0(km/s/Mpc)$	71.23	± 1.43

Flat ΛCDM model
Confidence Plot & Estimated Parameters



Parameter	*Mean*	σ
Ω_m	0.28	± 0.012
$H_0(km/s/Mpc)$	70.98	± 0.95

if(questions = 0) { break; }

THANK YOU