Code Library

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Contents

1 Data Structure

1.1 atlantis

```
#include<cstdio>
#include<algorithm>
#include<map>

#define MAXX 111
#define inf 333
#define MAX inf*5

int mid[MAX],cnt[MAX];
double len[MAX];

int n,i,cas;
double x1,x2,y1,y2;
double ans;
std::map<double,int>map;
std::map<double,int>::iterator it;
double rmap[inf];

void make(int id,int l,int r)
{
    mid[id]=(l+r)>>1;
    if(l!=r)
    {
        make(id<<1,l,mid[id]);
        make(id<<1|1,mid[id]+1,r);
    }
}

void update(int id,int ll,int rr,int l,int r,int val)
{
    if(ll==l && rr==r)
    {
        cnt[id]+=val;
        if(cnt[id])
            len[id]=rmap[r]-rmap[l-1];
        else
            if(l!=r)
                len[id]=len[id<<1]+len[id<<1|1];
            else
                len[id]=0;
        return;
    }
    if(mid[id]>=r)
        update(id<<1,ll,mid[id],l,r,val);
    else
        if(mid[id]<l)
            update(id<<1|1,mid[id]+1,rr,l,r,val);
        else
        {
            update(id<<1,ll,mid[id],l,mid[id],val);
            update(id<<1|1,mid[id]+1,rr,mid[id]+1,r,val);
        }
    if(!cnt[id])
        len[id]=len[id<<1]+len[id<<1|1];
}

struct node
{
    double l,r,h;
    char f;
    inline bool operator<(const node &a)const
    {
        return h<a.h;
    }
    inline void print()
    {
        printf("%lf␣%lf␣%lf␣%d\n",l,r,h,f);
    }
}ln[inf];

int main()
{
    make(1,1,inf);
    while(scanf("%d",&n),n)
    {
        n<<=1;
        map.clear();
        for(i=0;i<n;++i)
        {
            scanf("%lf%lf%lf%lf",&x1,&y1,&x2,&y2);
            if(x1>x2)
                std::swap(x1,x2);
            if(y1>y2)
                std::swap(y1,y2);
            ln[i].l=x1;
            ln[i].r=x2;
            ln[i].h=y1;
            ln[i].f=1;
            ln[++i].l=x1;
            ln[i].r=x2;
            ln[i].h=y2;
```

```
            ln[i].f=-1;
            map[x1]=1;
            map[x2]=1;
        }
        i=1;
        for(it=map.begin();it!=map.end();++it,++i)
        {
            it->second=i;
            rmap[i]=it->first;
        }
        std::sort(ln,ln+n);
        ans=0;
        update(1,1,inf,map[ln[0].l]+1,map[ln[0].r],ln[0].f);
        for(i=1;i<n;++i)
        {
            ans+=len[1]*(ln[i].h-ln[i-1].h);
            update(1,1,inf,map[ln[i].l]+1,map[ln[i].r],ln[i].f)
                ;
        }
        printf("Test case #%d\nTotal explored area: %.2lf\n\n"
             ,++cas,ans);
    }
    return 0;
}
```

1.2 binary indexed tree

```
int tree[MAXX];

inline void update(int pos,const int &val)
{
    while(pos<MAXX)
    {
        tree[pos]+=val;
        pos+=pos&-pos;
    }
}

inline int read(int pos)
{
    int re(0);
    while(pos>0)
    {
        re+=tree[pos];
        pos-=pos&-pos;
    }
    return re;
}

int find_Kth(int k)
{
    int now=0;
    for (char i=20;i>=0;--i)
    {
        now|=(1<<i);
        if (now>MAXX || tree[now]>=k)
            now^=(1<<i);
        else k-=tree[now];
    }
    return now+1;
}
```

1.3 COT

```
#include<cstdio>
#include<algorithm>

#define MAXX 100111
#define MAX (MAXX*23)
#define N 18

int sz[MAX],lson[MAX],rson[MAX],cnt;
int head[MAXX];
int pre[MAXX][N];
int map[MAXX],m;

int edge[MAXX],nxt[MAXX<<1],to[MAXX<<1];
int n,i,j,k,q,l,r,mid;
int num[MAXX],dg[MAXX];

int make(int l,int r)
{
    if(l==r)
        return ++cnt;
    int id(++cnt),mid((l+r)>>1);
    lson[id]=make(l,mid);
    rson[id]=make(mid+1,r);
    return id;
}

inline int update(int id,int pos)
{
    int re(++cnt);
    l=1;
    r=m;
    int nid(re);
    sz[nid]=sz[id]+1;
    while(l<r)
    {
        mid=(l+r)>>1;
        if(pos<=mid)
        {
            lson[nid]=++cnt;
            rson[nid]=rson[id];
            nid=lson[nid];
            id=lson[id];
            r=mid;
        }
        else
        {
            lson[nid]=lson[id];
            rson[nid]=++cnt;
            nid=rson[nid];
            id=rson[id];
            l=mid+1;
        }
        sz[nid]=sz[id]+1;
    }
    return re;
}

void rr(int now,int fa)
{
    dg[now]=dg[fa]+1;
    head[now]=update(head[fa],num[now]);
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa)
        {
            j=1;
            for(pre[to[i]][0]=now;j<N;++j)
                pre[to[i]][j]=pre[pre[to[i]][j-1]][j-1];
            rr(to[i],now);
        }
}

inline int query(int a,int b,int n,int k)
{
    static int tmp,t;
    l=1;
    r=m;
    a=head[a];
    b=head[b];
    t=num[n];
    n=head[n];
    while(l<r)
    {
        mid=(l+r)>>1;
        tmp=sz[lson[a]]+sz[lson[b]]-2*sz[lson[n]]+(l<=t && t<=
            mid);
        if(tmp>=k)
        {
            a=lson[a];
            b=lson[b];
            n=lson[n];
            r=mid;
        }
        else
        {
            k-=tmp;
            a=rson[a];
            b=rson[b];
            n=rson[n];
            l=mid+1;
        }
    }
    return l;
}

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=pre[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(pre[a][i]!=pre[b][i])
        {
            a=pre[a][i];
            b=pre[b][i];
        }
    return pre[a][0];
}

int main()
{
```

```
    scanf("%d␣%d",&n,&q);
    for(i=1;i<=n;++i)
    {
        scanf("%d",num+i);
        map[i]=num[i];
    }
    std::sort(map+1,map+n+1);
    m=std::unique(map+1,map+n+1)-map-1;
    for(i=1;i<=n;++i)
        num[i]=std::lower_bound(map+1,map+m+1,num[i])-map;
    for(i=1;i<n;++i)
    {
        scanf("%d␣%d",&j,&k);
        nxt[++cnt]=edge[j];
        edge[j]=cnt;
        to[cnt]=k;

        nxt[++cnt]=edge[k];
        edge[k]=cnt;
        to[cnt]=j;
    }
    cnt=0;
    head[0]=make(1,m);
    rr(1,0);
    while(q--)
    {
        scanf("%d␣%d␣%d",&i,&j,&k);
        printf("%d\n",map[query(i,j,lca(i,j),k)]);
    }
    return 0;
}
```

1.4 hose

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<cmath>

#define MAXX 50111

struct Q
{
    int l,r,s,w;
    bool operator<(const Q &i)const
    {
        return w==i.w?r<i.r:w<i.w;
    }
}a[MAXX];

int c[MAXX];
long long col[MAXX],sz[MAXX],ans[MAXX];
int n,m,cnt,len;

long long gcd(long long a,long long b)
{
    return a?gcd(b%a,a):b;
}

int i,j,k,now;
long long all,num;

int main()
{
    scanf("%d␣%d",&n,&m);
    for(i=1;i<=n;++i)
        scanf("%d",c+i);
    len=sqrt(m);
    for(i=1;i<=m;++i)
    {
        scanf("%d␣%d",&a[i].l,&a[i].r);
        if(a[i].l>a[i].r)
            std::swap(a[i].l,a[i].r);
        sz[i]=a[i].r-a[i].l+1;
        a[i].w=a[i].l/len+1;
        a[i].s=i;
    }
    std::sort(a+1,a+m+1);
    i=1;
    while(i<=m)
    {
        now=a[i].w;
        memset(col,0,sizeof col);
        for(j=a[i].l;j<=a[i].r;++j)
            ans[a[i].s]+=2*(col[c[j]]++);
        for(++i;a[i].w==now;++i)
        {
            ans[a[i].s]=ans[a[i-1].s];
            for(j=a[i-1].r+1;j<=a[i].r;++j)
                ans[a[i].s]+=2*(col[c[j]]++);
            if(a[i-1].l<a[i].l)
                for(j=a[i-1].l;j<a[i].l;++j)
                    ans[a[i].s]-=2*(--col[c[j]]);
            else
                for(j=a[i].l;j<a[i-1].l;++j)
                    ans[a[i].s]+=2*(col[c[j]]++);
        }
        for(i=1;i<=m;++i)
        {
            if(sz[i]==1)
                all=1ll;
            else
                all=sz[i]*(sz[i]-1);
            num=gcd(ans[i],all);
            printf("%lld/%lld\n",ans[i]/num,all/num);
        }
        return 0;
}
```

1.5 Leftist tree

```
#include<cstdio>
#include<algorithm>

#define MAXX 100111

int val[MAXX],l[MAXX],r[MAXX],d[MAXX];

int set[MAXX];

int merge(int a,int b)
{
    if(!a)
        return b;
    if(!b)
        return a;
    if(val[a]<val[b]) // max-heap
        std::swap(a,b);
    r[a]=merge(r[a],b);
    if(d[l[a]]<d[r[a]])
        std::swap(l[a],r[a]);
    d[a]=d[r[a]]+1;
    set[l[a]]=set[r[a]]=a; // set a as father of its sons
    return a;
}

inline int find(int &a)
{
    while(set[a]) //brute-force to get the index of root
        a=set[a];
    return a;
}

inline void reset(int i) { l[i]=r[i]=d[i]=set[i]=0; }
int n,i,j,k;

int main()
{
    while(scanf("%d",&n)!=EOF)
    {
        for(i=1;i<=n;++i)
        {
            scanf("%d",val+i);
            reset(i);
        }
        scanf("%d",&n);
        while(n--)
        {
            scanf("%d␣%d",&i,&j);
            if(find(i)==find(j))
                puts("-1");
            else
            {
                k=merge(l[i],r[i]);
                val[i]>>=1;
                reset(i);
                set[i=merge(i,k)]=0;

                k=merge(l[j],r[j]);
                val[j]>>=1;
                reset(j);
                set[j=merge(j,k)]=0;

                set[k=merge(i,j)]=0;
                printf("%d\n",val[k]);
            }
        }
    }
    return 0;
}
```

1.6 Link-Cut Tree

```
//记得随手 down 啊……亲……
//debug 时记得优先检查 up/down/select
#define MAXX
#define lson nxt[id][0]
#define rson nxt[id][1]
```

```
int nxt[MAXX][2],fa[MAXX],pre[MAXX];
bool rev[MAXX];

inline void up(int id)
{
}

inline void rot(int id,int tp)
{
    static int k;
    k=pre[id];
    nxt[k][tp^1]=nxt[id][tp];
    if(nxt[id][tp])
        pre[nxt[id][tp]]=k;
    if(pre[k])
        nxt[pre[k]][k==nxt[pre[k]][1]]=id;
    pre[id]=pre[k];
    nxt[id][tp]=k;
    pre[k]=id;
    up(k);
    up(id);
}

inline void down(int id) //记得随手 down 啊……亲……
{
    static int i;
    if(rev[id])
    {
        rev[id]=false;
        for(i=0;i<2;++i)
            if(nxt[id][i])
            {
                rev[nxt[id][i]]^=true;
                std::swap(nxt[nxt[id][i]][0],nxt[nxt[id][i
                    ]][1]);
            }
    }
}

inline void splay(int id)//记得随手 down 啊……亲……
{
    down(id);
    if(!pre[id])
        return;
    static int rt,k,st[MAXX];
    for(rt=id,k=0;rt;rt=pre[rt])
        st[k++]=rt;
    rt=st[k-1];
    while(k)
        down(st[--k]);
    for(std::swap(fa[id],fa[rt]);pre[id];rot(id,id==nxt[pre[id
        ]][0]));
    /* another faster methond:
    std::swap(fa[id],fa[rt]);
    do
    {
        rt=pre[id];
        if(pre[rt])
        {
            k=(nxt[pre[rt]][0]==rt);
            if(nxt[rt][k]==id)
                rot(id,k^1);
            else
                rot(rt,k);
            rot(id,k);
        }
        else
            rot(id,id==nxt[rt][0]);
    }
    while(pre[id]);
    */
}

inline int access(int id)
{
    static int to;
    for(to=0;id;id=fa[id])
    {
        splay(id);
        if(rson)
        {
            pre[rson]=0;
            fa[rson]=id;
        }
        rson=to;
        if(to)
        {
            pre[to]=id;
            fa[to]=0;
        }
        up(to=id);
    }
    return to;
}
```

1.7 Network

```
//HLD……备忘……_(:3JZ)_
#include<cstdio>
#include<algorithm>
#include<cstdlib>

#define MAXX 80111
#define MAXE (MAXX<<1)
#define N 18

int edge[MAXX],nxt[MAXE],to[MAXE],cnt;
int fa[MAXX][N],dg[MAXX];

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=fa[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(fa[a][i]!=fa[b][i])
        {
            a=fa[a][i];
            b=fa[b][i];
        }
    return fa[a][0];
}

inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

int sz[MAXX],pre[MAXX],next[MAXX];

void rr(int now)
{
    sz[now]=1;
    int max,id;
    max=0;
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa[now][0])
        {
            fa[to[i]][0]=now;
            dg[to[i]]=dg[now]+1;
            rr(to[i]);
            sz[now]+=sz[to[i]];
            if(sz[to[i]]>max)
            {
                max=sz[to[i]];
                id=to[i];
            }
        }
    if(max)
    {
        next[now]=id;
        pre[id]=now;
    }
}

#define MAXT (MAXX*N*5)

namespace Treap
{
    int cnt;
    int son[MAXT][2],key[MAXT],val[MAXT],sz[MAXT];

    inline void init()
    {
        key[0]=RAND_MAX;
        val[0]=0xc0c0c0c0;
        cnt=0;
    }

    inline void up(int id)
    {
        sz[id]=sz[son[id][0]]+sz[son[id][1]]+1;
    }
    inline void rot(int &id,int tp)
    {
        static int k;
        k=son[id][tp];
        son[id][tp]=son[k][tp^1];
        son[k][tp^1]=id;
        up(id);
        up(k);
        id=k;
```

```
    }
    void insert(int &id,int v)
    {
        if(id)
        {
            int k(v>=val[id]);
            insert(son[id][k],v);
            if(key[son[id][k]]<key[id])
                rot(id,k);
            else
                up(id);
            return;
        }
        id=++cnt;
        key[id]=rand()-1;
        val[id]=v;
        sz[id]=1;
        son[id][0]=son[id][1]=0;
    }
    void del(int &id,int v)
    {
        if(!id)
            return;
        if(val[id]==v)
        {
            int k(key[son[id][1]]<key[son[id][0]]);
            if(!son[id][k])
            {
                id=0;
                return;
            }
            rot(id,k);
            del(son[id][k^1],v);
        }
        else
            del(son[id][v>val[id]],v);
        up(id);
    }
    int rank(int id,int v)
    {
        if(!id)
            return 0;
        if(val[id]<=v)
            return sz[son[id][0]]+1+rank(son[id][1],v);
        return rank(son[id][0],v);
    }
    void print(int id)
    {
        if(!id)
            return;
        print(son[id][0]);
        printf("%d ",val[id]);
        print(son[id][1]);
    }
}

int head[MAXX],root[MAXX],len[MAXX],pos[MAXX];

#define MAX (MAXX*6)
#define mid (l+r>>1)
#define lc lson[id],l,mid
#define rc rson[id],mid+1,r

int lson[MAX],rson[MAX];
int treap[MAX];

void make(int &id,int l,int r,int *the)
{
    id=++cnt;
    static int k;
    for(k=l;k<=r;++k)
        Treap::insert(treap[id],the[k]);
    if(l!=r)
    {
        make(lc,the);
        make(rc,the);
    }
}

int query(int id,int l,int r,int a,int b,int q)
{
    if(a<=l && r<=b)
        return Treap::rank(treap[id],q);
    int re(0);
    if(a<=mid)
        re=query(lc,a,b,q);
    if(b>mid)
        re+=query(rc,a,b,q);
    return re;
}

inline int query(int a,int b,int v)
{
    static int re;
    for(re=0;root[a]!=root[b];a=fa[root[a]][0])
        re+=query(head[root[a]],1,len[root[a]],1,pos[a],v);
    re+=query(head[root[a]],1,len[root[a]],pos[b],pos[a],v);
    return re;
}

inline void update(int id,int l,int r,int pos,int val,int n)
{
    while(l<=r)
    {
        Treap::del(treap[id],val);
        Treap::insert(treap[id],n);
        if(l==r)
            return;
        if(pos<=mid)
        {
            id=lson[id];
            r=mid;
        }
        else
        {
            id=rson[id];
            l=mid+1;
        }
    }
}

int n,q,i,j,k;
int val[MAXX];

int main()
{
    srand(1e9+7);
    scanf("%d %d",&n,&q);
    for(i=1;i<=n;++i)
        scanf("%d",val+i);
    for(k=1;k<n;++k)
    {
        scanf("%d %d",&i,&j);
        add(i,j);
        add(j,i);
    }
    rr(rand()%n+1);
    for(j=1;j<N;++j)
        for(i=1;i<=n;++i)
            fa[i][j]=fa[fa[i][j-1]][j-1];

    Treap::init();
    cnt=0;
    for(i=1;i<=n;++i)
        if(!pre[i])
        {
            static int tmp[MAXX];
            for(k=1,j=i;j;j=next[j],++k)
            {
                pos[j]=k;
                root[j]=i;
                tmp[k]=val[j];
            }
            --k;
            len[i]=k;
            make(head[i],1,k,tmp);
        }
    while(q--)
    {
        scanf("%d",&k);
        if(k)
        {
            static int a,b,c,d,l,r,ans,m;
            scanf("%d %d",&a,&b);
            c=lca(a,b);
            if(dg[a]+dg[b]-2*dg[c]+1<k)
            {
                puts("invalid request!");
                continue;
            }
            k=dg[a]+dg[b]-2*dg[c]+1-k+1;
            if(dg[a]<dg[b])
                std::swap(a,b);
            l=-1e9;
            r=1e9;
            if(b!=c)
            {
                d=a;
                for(i=0,j=dg[a]-dg[c]-1;j;j>>=1,++i)
                    if(j&1)
                        d=fa[d][i];
                while(l<=r)
                {
                    m=l+r>>1;
                    if(query(a,d,m)+query(b,c,m)>=k)
                    {
                        ans=m;
                        r=m-1;
                    }
                    else
                        l=m+1;
                }
```

```
            }
            else
            {
                while(l<=r)
                {
                    m=l+r>>1;
                    if(query(a,c,m)>=k)
                    {
                        ans=m;
                        r=m-1;
                    }
                    else
                        l=m+1;
                }
                printf("%d\n",ans);
            }
            else
            {
                scanf("%d %d",&i,&j);
                update(head[root[i]],1,len[root[i]],pos[i],val[i],j
                    );
                val[i]=j;
            }
        }
        return 0;
}
```

1.8 picture

```
#include<cstdio>
#include<algorithm>
#include<map>

#define MAXX 5555
#define MAX MAXX<<3
#define inf 10011

int n,i;
int mid[MAX],cnt[MAX],len[MAX],seg[MAX];
bool rt[MAX],lf[MAX];

std::map<int,int>map;
std::map<int,int>::iterator it;
int rmap[inf];
long long sum;
int x1,x2,y1,y2,last;

void make(int id,int l,int r)
{
    mid[id]=(l+r)>>1;
    if(l!=r)
    {
        make(id<<1,l,mid[id]);
        make(id<<1|1,mid[id]+1,r);
    }
}

void update(int id,int ll,int rr,int l,int r,int val)
{
    if(l==ll && rr==r)
    {
        cnt[id]+=val;
        if(cnt[id])
        {
            rt[id]=lf[id]=true;
            len[id]=rmap[r]-rmap[l-1];
            seg[id]=1;
        }
        else
            if(l!=r)
            {
                len[id]=len[id<<1]+len[id<<1|1];
                seg[id]=seg[id<<1]+seg[id<<1|1];
                if(rt[id<<1] && lf[id<<1|1])
                    --seg[id];
                rt[id]=rt[id<<1|1];
                lf[id]=lf[id<<1];
            }
            else
            {
                len[id]=0;
                rt[id]=lf[id]=false;
                seg[id]=0;
            }
        return;
    }
    if(mid[id]>=r)
        update(id<<1,ll,mid[id],l,r,val);
    else
        if(mid[id]<l)
            update(id<<1|1,mid[id]+1,rr,l,r,val);
        else
        {
            update(id<<1,ll,mid[id],l,mid[id],val);
            update(id<<1|1,mid[id]+1,rr,mid[id]+1,r,val);
        }
    if(!cnt[id])
    {
        len[id]=len[id<<1]+len[id<<1|1];
        seg[id]=seg[id<<1]+seg[id<<1|1];
        if(rt[id<<1] && lf[id<<1|1])
            --seg[id];
        rt[id]=rt[id<<1|1];
        lf[id]=lf[id<<1];
    }
}

struct node
{
    int l,r,h;
    char val;
    inline bool operator<(const node &a)const
    {
        return h==a.h?val<a.val:h<a.h;   // trick watch out.
            val<a.val? val>a.val?
    }
    inline void print()
    {
        printf("%d %d %d %d\n",l,r,h,val);
    }
}ln[inf];

int main()
{
    make(1,1,inf);
    scanf("%d",&n);
    n<<=1;
    map.clear();
    for(i=0;i<n;++i)
    {
        scanf("%d%d%d%d",&x1,&y1,&x2,&y2);
        ln[i].l=x1;
        ln[i].r=x2;
        ln[i].h=y1;
        ln[i].val=1;
        ln[++i].l=x1;
        ln[i].r=x2;
        ln[i].h=y2;
        ln[i].val=-1;
        map[x1]=1;
        map[x2]=1;
    }
    i=1;
    for(it=map.begin();it!=map.end();++it,++i)
    {
        it->second=i;
        rmap[i]=it->first;
    }
    i=0;
    std::sort(ln,ln+n);
    update(1,1,inf,map[ln[0].l]+1,map[ln[0].r],ln[0].val);
    sum+=len[1];
    last=len[1];
    for(i=1;i<n;++i)
    {
        sum+=2*seg[1]*(ln[i].h-ln[i-1].h);
        update(1,1,inf,map[ln[i].l]+1,map[ln[i].r],ln[i].val);
        sum+=abs(len[1]-last);
        last=len[1];
    }
    printf("%lld\n",sum);
    return 0;
}
```

1.9 Size Blanced Tree

```
template<class Tp>class sbt
{
    public:
        inline void init() { rt=cnt=l[0]=r[0]=sz[0]=0; }
        inline void ins(const Tp &a) { ins(rt,a); }
        inline void del(const Tp &a) { del(rt,a); }
        inline bool find(const Tp &a) { return find(rt,a); }
        inline Tp pred(const Tp &a) { return pred(rt,a); }
        inline Tp succ(const Tp &a) { return succ(rt,a); }
        inline bool empty() { return !sz[rt]; }
        inline Tp min() { return min(rt); }
        inline Tp max() { return max(rt); }
        inline void delsmall(const Tp &a) { dels(rt,a); }
        inline int rank(const Tp &a) { return rank(rt,a); }
        inline Tp sel(const int &a) { return sel(rt,a); }
        inline Tp delsel(int a) { return delsel(rt,a); }
    private:
        int cnt,rt,l[MAXX],r[MAXX],sz[MAXX];
        Tp val[MAXX];
        inline void rro(int &pos)
        {
            int k(l[pos]);
            l[pos]=r[k];
```

```
    r[k]=pos;
    sz[k]=sz[pos];
    sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
    pos=k;
}
inline void lro(int &pos)
{
    int k(r[pos]);
    r[pos]=l[k];
    l[k]=pos;
    sz[k]=sz[pos];
    sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
    pos=k;
}
inline void mt(int &pos,bool flag)
{
    if(!pos)
        return;
    if(flag)
        if(sz[r[r[pos]]]>sz[l[pos]])
            lro(pos);
        else
            if(sz[l[r[pos]]]>sz[l[pos]])
            {
                rro(r[pos]);
                lro(pos);
            }
            else
                return;
    else
        if(sz[l[l[pos]]]>sz[r[pos]])
            rro(pos);
        else
            if(sz[r[l[pos]]]>sz[r[pos]])
            {
                lro(l[pos]);
                rro(pos);
            }
            else
                return;
    mt(l[pos],false);
    mt(r[pos],true);
    mt(pos,false);
    mt(pos,true);
}
void ins(int &pos,const Tp &a)
{
    if(pos)
    {
        ++sz[pos];
        if(a<val[pos])
            ins(l[pos],a);
        else
            ins(r[pos],a);
        mt(pos,a>=val[pos]);
        return;
    }
    pos=++cnt;
    l[pos]=r[pos]=0;
    val[pos]=a;
    sz[pos]=1;
}
Tp del(int &pos,const Tp &a)
{
    --sz[pos];
    if(val[pos]==a || (a<val[pos] && !l[pos]) || (a>val
         [pos] && !r[pos]))
    {
        Tp ret(val[pos]);
        if(!l[pos] || !r[pos])
            pos=l[pos]+r[pos];
        else
            val[pos]=del(l[pos],val[pos]+1);
        return ret;
    }
    else
        if(a<val[pos])
            return del(l[pos],a);
        else
            return del(r[pos],a);
}
bool find(int &pos,const Tp &a)
{
    if(!pos)
        return false;
    if(a<val[pos])
        return find(l[pos],a);
    else
        return (val[pos]==a || find(r[pos],a));
}
Tp pred(int &pos,const Tp &a)
{
    if(!pos)
        return a;
    if(a>val[pos])
    {
        Tp ret(pred(r[pos],a));
        if(ret==a)
            return val[pos];
        else
            return ret;
    }
    return pred(l[pos],a);
}
Tp succ(int &pos,const Tp &a)
{
    if(!pos)
        return a;
    if(a<val[pos])
    {
        Tp ret(succ(l[pos],a));
        if(ret==a)
            return val[pos];
        else
            return ret;
    }
    return succ(r[pos],a);
}
Tp min(int &pos)
{
    if(l[pos])
        return min(l[pos]);
    return val[pos];
}
Tp max(int &pos)
{
    if(r[pos])
        return max(r[pos]);
    return val[pos];
}
void dels(int &pos,const Tp &v)
{
    if(!pos)
        return;
    if(val[pos]<v)
    {
        pos=r[pos];
        dels(pos,v);
        return;
    }
    dels(l[pos],v);
    sz[pos]=1+sz[l[pos]]+sz[r[pos]];
}
int rank(const int &pos,const Tp &v)
{
    if(val[pos]==v)
        return sz[l[pos]]+1;
    if(v<val[pos])
        return rank(l[pos],v);
    return rank(r[pos],v)+sz[l[pos]]+1;
}
Tp sel(const int &pos,const int &v)
{
    if(sz[l[pos]]+1==v)
        return val[pos];
    if(v>sz[l[pos]])
        return sel(r[pos],v-sz[l[pos]]-1);
    return sel(l[pos],v);
}
Tp delsel(int &pos,int k)
{
    --sz[pos];
    if(sz[l[pos]]+1==k)
    {
        Tp re(val[pos]);
        if(!l[pos] || !r[pos])
            pos=l[pos]+r[pos];
        else
            val[pos]=del(l[pos],val[pos]+1);
        return re;
    }
    if(k>sz[l[pos]])
        return delsel(r[pos],k-1-sz[l[pos]]);
    return delsel(l[pos],k);
}
};
```

1.10 sparse table

```
//normal
int num[MAXX],min[MAXX][20];
int lg[MAXX];

inline int init(int n)
{
    static int i,j,k,l,j_,j__;
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    for(i=1;i<=n;++i)
        min[i][0]=num[i];
    for(j=1;j<=lg[n];++j)
```

```
    {
        l=n+1-(1<<j);
        j_=j-1;
        j__=(1<<j_);
        for(i=1;i<=l;++i)
            min[i][j]=std::min(min[i][j_],min[i+j__][j_]);
    }
}

inline int query(int i,int j)
{
    static int k;
    k=lg[j-i+1];
    return std::min(min[i][k],min[j-(1<<k)+1][k]);
}

//rectangle
int lg[MAXX];
int table[9][9][MAXX][MAXX];
int mat[MAXX][MAXX]

inline void init(int n)
{
    static int i,j,ii,jj;
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
            table[0][0][i][j]=mat[i][j];
    for(i=0;i<=lg[n];++i)
        for(j=0;j<=lg[n];++j)
        {
            if(i==0 && j==0)
                continue;
            for(ii=0;ii+(1<<j)<=n;++ii)
                for(jj=0;jj+(1<<i)<=n;++jj)
                    if(i==0)
                        table[i][j][ii][jj]=std::min(table[i][j
                            -1][ii][jj],table[i][j-1][ii+(1<<(
                            j-1))][jj]);
                    else
                        table[i][j][ii][jj]=std::min(table[i
                            -1][j][ii][jj],table[i-1][j][ii][
                            jj+(1<<(i-1))]);
        }
}

inline int query(int r1,int c1,int r2,int c2)
{
    --r1;
    --c1;
    --r2;
    --c2;
    static int w,h;
    w=lg[c2-c1+1];
    h=lg[r2-r1+1];
    return std::min(table[w][h][r1][c1],std::min(table[w][h][r1
        ][c2-(1<<w)+1],std::min(table[w][h][r2-(1<<h)+1][c1],
        table[w][h][r2-(1<<h)+1][c2-(1<<w)+1])));
}

//square
int num[MAXX][MAXX],max[MAXX][MAXX][10];
int lg[MAXX];

inline void init(int n)
{
    static int i,j,k,l;
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
            max[i][j][0]=num[i][j];
    for(k=1;k<=lg[n];++k)
    {
        l=n+1-(1<<k);
        for(i=0;i<l;++i)
            for(j=0;j<l;++j)
                max[i][j][k]=std::max(std::max(max[i][j][k-1],
                    max[i+(1<<(k-1))][j][k-1]),std::max(max[i
                    ][j+(1<<(k-1))][k-1],max[i+(1<<(k-1))][j
                    +(1<<(k-1))][k-1]));
    }
}

inline int query(int i,int j,int l)
{
    static int k;
    --i;
    --j;
    k=lg[l];
    return std::max(std::max(max[i][j][k],max[i][j+l-(1<<k)][k
        ]),std::max(max[i+l-(1<<k)][j][k],max[i+l-(1<<k)][j+l
        -(1<<k)][k]));
}
```

1.11 treap

```
struct node
{
    node *ch[2];
    int sz,val,key;
    node(){memset(this,0,sizeof(node));}
    node(int a);
}*null;

node::node(int a):sz(1),val(a),key(rand()-1){ch[0]=ch[1]=null;}

class Treap
{
    inline void up(node *pos)
    {
        pos->sz=pos->ch[0]->sz+pos->ch[1]->sz+1;
    }
    inline void rot(node *&pos,int tp)
    {
        node *k(pos->ch[tp]);
        pos->ch[tp]=k->ch[tp^1];
        k->ch[tp^1]=pos;
        up(pos);
        up(k);
        pos=k;
    }

    void insert(node *&pos,int val)
    {
        if(pos!=null)
        {
            int t(val>=pos->val);
            insert(pos->ch[t],val);
            if(pos->ch[t]->key<pos->key)
                rot(pos,t);
            else
                up(pos);
            return;
        }
        pos=new node(val);
    }
    void rec(node *pos)
    {
        if(pos!=null)
        {
            rec(pos->ch[0]);
            rec(pos->ch[1]);
            delete pos;
        }
    }
    inline int sel(node *pos,int k)
    {
        while(pos->ch[0]->sz+1!=k)
            if(pos->ch[0]->sz>=k)
                pos=pos->ch[0];
            else
            {
                k-=pos->ch[0]->sz+1;
                pos=pos->ch[1];
            }
        return pos->val;
    }
    void del(node *&pos,int val)
    {
        if(pos!=null)
        {
            if(pos->val==val)
            {
                int t(pos->ch[1]->key<pos->ch[0]->key);
                if(pos->ch[t]==null)
                {
                    delete pos;
                    pos=null;
                    return;
                }
                rot(pos,t);
                del(pos->ch[t^1],val);
            }
            else
                del(pos->ch[val>pos->val],val);
            up(pos);
        }
    }
    public:
    node *rt;

    Treap():rt(null){}
    inline void insert(int val) { insert(rt,val); }
    inline void reset() { rec(rt); rt=null; }
    inline int sel(int k)
    {
        if(k<1 || k>rt->sz)
            return 0;
        return sel(rt,rt->sz+1-k);
```

```
    }
    inline void del(int val) { del(rt,val); }
    inline int size() { return rt->sz; }
}treap[MAXX];

inline void init()
{
    srand(time(0));
    null=new node();
    null->val=0xc0c0c0c0;
    null->sz=0;
    null->key=RAND_MAX;
    null->ch[0]=null->ch[1]=null;
    for(i=0;i<MAXX;++i)
        treap[i].rt=null;
}
```

2 Geometry

2.1 3D

```
struct pv
{
  double x,y,z;
  pv() {}
  pv(double xx,double yy,double zz):x(xx),y(yy),z(zz) {}
  pv operator -(const pv& b)const
  {
    return pv(x-b.x,y-b.y,z-b.z);
  }
  pv operator *(const pv& b)const
  {
    return pv(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
  }
  double operator &(const pv& b)const
  {
    return x*b.x+y*b.y+z*b.z;
  }
};

//模
double Norm(pv p)
{
  return sqrt(p&p);
}

//计算 cross product U x V
point3 xmult(point3 u,point3 v)
{
    point3 ret;
    ret.x=u.y*v.z-v.y*u.z;
    ret.y=u.z*v.x-u.x*v.z;
    ret.z=u.x*v.y-u.y*v.x;
    return ret;
}
//计算 dot product U . V
double dmult(point3 u,point3 v)
{
    return u.x*v.x+u.y*v.y+u.z*v.z;
}
//矢量差 U - V
point3 subt(point3 u,point3 v)
{
    point3 ret;
    ret.x=u.x-v.x;
    ret.y=u.y-v.y;
    ret.z=u.z-v.z;
    return ret;
}
//取平面法向量
point3 pvec(plane3 s)
{
    return xmult(subt(s.a,s.b),subt(s.b,s.c));
}
point3 pvec(point3 s1,point3 s2,point3 s3)
{
    return xmult(subt(s1,s2),subt(s2,s3));
}
//两点距离，单参数取向量大小
double distance(point3 p1,point3 p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)
        +(p1.z-p2.z)*(p1.z-p2.z));
}
//向量大小
double vlen(point3 p)
{
    return sqrt(p.x*p.x+p.y*p.y+p.z*p.z);
}
```

2.1.1 Geographic

Geographic coordinate system coversion witch Cartesian coordinate system:
$x = r \times \sin(\theta) \times \cos(\alpha)$
$y = r \times \sin(\theta) \times \sin(\alpha)$
$z = r \times \cos(\theta)$

$r = \sqrt{x^2 + y^2 + z^2}$
α=atan(y/x);
θ=acos(z/r);

$r \in [0, \infty)$
$\alpha \in [0, 2\pi]$
$\theta \in [0, \pi]$

$lat \in [-\frac{\pi}{2}, \frac{\pi}{2}]$
$lng \in [-\pi, \pi]$

```
pv getpv(double lat,double lng,double r)
{
  lat += pi/2;
  lng += pi;
  return
    pv(r*sin(lat)*cos(lng),r*sin(lat)*sin(lng),r*cos(lat));
}
```

Distance in the suface of ball:

```
#include<cstdio>
#include<cmath>

#define MAXX 1111

char buf[MAXX];
const double r=6875.0/2,pi=acos(-1.0);
double a,b,c,x1,x2,y2,ans;

int main()
{
    double y1;
    while(gets(buf)!=NULL)
    {
        gets(buf);
        gets(buf);

        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        x1=a+b/60+c/3600;
        x1=x1*pi/180;
        if(buf[0]=='S')
            x1=-x1;

        scanf("%s",buf);
        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        y1=a+b/60+c/3600;
        y1=y1*pi/180;
        if(buf[0]=='W')
            y1=-y1;

        gets(buf);

        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        x2=a+b/60+c/3600;
        x2=x2*pi/180;
        if(buf[0]=='S')
            x2=-x2;

        scanf("%s",buf);
        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        y2=a+b/60+c/3600;
        y2=y2*pi/180;
        if(buf[0]=='W')
            y2=-y2;

        ans=acos(cos(x1)*cos(x2)*cos(y1-y2)+sin(x1)*sin(x2))*r;
        printf("The distance to the iceberg: %.2lf miles.\n",
             ans);
        if(ans+0.005<100)
            puts("DANGER!");

        gets(buf);
    }
    return 0;
}
```

zju:

2.1.2 Checks

```
//判三点共线
int dots_inline(point3 p1,point3 p2,point3 p3)
{
    return vlen(xmult(subt(p1,p2),subt(p2,p3)))<eps;
}
//判四点共面
int dots_onplane(point3 a,point3 b,point3 c,point3 d)
{
    return zero(dmult(pvec(a,b,c),subt(d,a)));
}
//判点是否在线段上，包括端点和共线
int dot_online_in(point3 p,line3 l)
{
    return zero(vlen(xmult(subt(p,l.a),subt(p,l.b))))&&(l.a.x-p
        .x)*(l.b.x-p.x)<eps&&
        (l.a.y-p.y)*(l.b.y-p.y)<eps&&(l.a.z-p.z)*(l.b.z-p.z)<
            eps;
}
int dot_online_in(point3 p,point3 l1,point3 l2)
{
    return zero(vlen(xmult(subt(p,l1),subt(p,l2))))&&(l1.x-p.x)
        *(l2.x-p.x)<eps&&
        (l1.y-p.y)*(l2.y-p.y)<eps&&(l1.z-p.z)*(l2.z-p.z)<eps;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point3 p,line3 l)
{
    return dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a
        .y)||!zero(p.z-l.a.z))&&
        (!zero(p.x-l.b.x)||!zero(p.y-l.b.y)||!zero(p.z-l.b.z));
}
int dot_online_ex(point3 p,point3 l1,point3 l2)
{
    return dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-
        l1.y)||!zero(p.z-l1.z))&&
        (!zero(p.x-l2.x)||!zero(p.y-l2.y)||!zero(p.z-l2.z));
}
//判点是否在空间三角形上，包括边界，三点共线无意义
int dot_inplane_in(point3 p,plane3 s)
{
    return zero(vlen(xmult(subt(s.a,s.b),subt(s.a,s.c)))-vlen(
        xmult(subt(p,s.a),subt(p,s.b)))-
            vlen(xmult(subt(p,s.b),subt(p,s.c)))-vlen(xmult(
                subt(p,s.c),subt(p,s.a))));
}
int dot_inplane_in(point3 p,point3 s1,point3 s2,point3 s3)
{
    return zero(vlen(xmult(subt(s1,s2),subt(s1,s3)))-vlen(xmult
        (subt(p,s1),subt(p,s2)))-
            vlen(xmult(subt(p,s2),subt(p,s3)))-vlen(xmult(subt(
                p,s3),subt(p,s1))));
}
//判点是否在空间三角形上，不包括边界，三点共线无意义
int dot_inplane_ex(point3 p,plane3 s)
{
    return dot_inplane_in(p,s)&&vlen(xmult(subt(p,s.a),subt(p,s
        .b)))>eps&&
        vlen(xmult(subt(p,s.b),subt(p,s.c)))>eps&&vlen(xmult(
            subt(p,s.c),subt(p,s.a)))>eps;
}
int dot_inplane_ex(point3 p,point3 s1,point3 s2,point3 s3)
{
    return dot_inplane_in(p,s1,s2,s3)&&vlen(xmult(subt(p,s1),
        subt(p,s2)))>eps&&
        vlen(xmult(subt(p,s2),subt(p,s3)))>eps&&vlen(xmult(subt
            (p,s3),subt(p,s1)))>eps;
}
//判两点在线段同侧，点在线段上返回 0，不共面无意义
int same_side(point3 p1,point3 p2,line3 l)
{
    return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l
        .a,l.b),subt(p2,l.b)))>eps;
}
int same_side(point3 p1,point3 p2,point3 l1,point3 l2)
{
    return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,
        l2),subt(p2,l2)))>eps;
}
//判两点在线段异侧，点在线段上返回 0，不共面无意义
int opposite_side(point3 p1,point3 p2,line3 l)
{
    return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l
        .a,l.b),subt(p2,l.b)))<-eps;
}
int opposite_side(point3 p1,point3 p2,point3 l1,point3 l2)
{
    return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,
        l2),subt(p2,l2)))<-eps;
}
//判两点在平面同侧，点在平面上返回 0
int same_side(point3 p1,point3 p2,plane3 s)
{
    return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.
        a))>eps;
}
int same_side(point3 p1,point3 p2,point3 s1,point3 s2,point3 s3
    )
{
    return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,
        s3),subt(p2,s1))>eps;
}
//判两点在平面异侧，点在平面上返回 0
int opposite_side(point3 p1,point3 p2,plane3 s)
{
    return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.
        a))<-eps;
}
int opposite_side(point3 p1,point3 p2,point3 s1,point3 s2,
    point3 s3)
{
    return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,
        s3),subt(p2,s1))<-eps;
}
//判两直线平行
int parallel(line3 u,line3 v)
{
    return vlen(xmult(subt(u.a,u.b),subt(v.a,v.b)))<eps;
}
int parallel(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return vlen(xmult(subt(u1,u2),subt(v1,v2)))<eps;
}
//判两平面平行
int parallel(plane3 u,plane3 v)
{
    return vlen(xmult(pvec(u),pvec(v)))<eps;
}
int parallel(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,
    point3 v3)
{
    return vlen(xmult(pvec(u1,u2,u3),pvec(v1,v2,v3)))<eps;
}
//判直线与平面平行
int parallel(line3 l,plane3 s)
{
    return zero(dmult(subt(l.a,l.b),pvec(s)));
}
int parallel(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return zero(dmult(subt(l1,l2),pvec(s1,s2,s3)));
}
//判两直线垂直
int perpendicular(line3 u,line3 v)
{
    return zero(dmult(subt(u.a,u.b),subt(v.a,v.b)));
}
int perpendicular(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return zero(dmult(subt(u1,u2),subt(v1,v2)));
}
//判两平面垂直
int perpendicular(plane3 u,plane3 v)
{
    return zero(dmult(pvec(u),pvec(v)));
}
int perpendicular(point3 u1,point3 u2,point3 u3,point3 v1,
    point3 v2,point3 v3)
{
    return zero(dmult(pvec(u1,u2,u3),pvec(v1,v2,v3)));
}
//判直线与平面平行
int perpendicular(line3 l,plane3 s)
{
    return vlen(xmult(subt(l.a,l.b),pvec(s)))<eps;
}
int perpendicular(point3 l1,point3 l2,point3 s1,point3 s2,
    point3 s3)
{
    return vlen(xmult(subt(l1,l2),pvec(s1,s2,s3)))<eps;
}
//判两线段相交，包括端点和部分重合
int intersect_in(line3 u,line3 v)
{
    if (!dots_onplane(u.a,u.b,v.a,v.b))
        return 0;
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||
        dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point3 u1,point3 u2,point3 v1,point3 v2)
{
    if (!dots_onplane(u1,u2,v1,v2))
        return 0;
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2)
            ;
```

```
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||
            dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
                2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line3 u,line3 v)
{
    return dots_onplane(u.a,u.b,v.a,v.b)&&opposite_side(u.a,u.b
        ,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return
        dots_onplane(u1,u2,v1,v2)&&opposite_side(u1,u2,v1,v2)&&
            opposite_side(v1,v2,u1,u2);
}
//判线段与空间三角形相交，包括交于边界和（部分）包含
int intersect_in(line3 l,plane3 s)
{
    return !same_side(l.a,l.b,s)&&!same_side(s.a,s.b,l.a,l.b,s.
        c)&&
        !same_side(s.b,s.c,l.a,l.b,s.a)&&!same_side(s.c,s.a,l.a
            ,l.b,s.b);
}
int intersect_in(point3 l1,point3 l2,point3 s1,point3 s2,point3
     s3)
{
    return !same_side(l1,l2,s1,s2,s3)&&!same_side(s1,s2,l1,l2,
        s3)&&
        !same_side(s2,s3,l1,l2,s1)&&!same_side(s3,s1,l1,l2,s2);
}
//判线段与空间三角形相交，不包括交于边界和（部分）包含
int intersect_ex(line3 l,plane3 s)
{
    return opposite_side(l.a,l.b,s)&&opposite_side(s.a,s.b,l.a,
        l.b,s.c)&&
        opposite_side(s.b,s.c,l.a,l.b,s.a)&&opposite_side(s.c,s
            .a,l.a,l.b,s.b);
}
int intersect_ex(point3 l1,point3 l2,point3 s1,point3 s2,point3
     s3)
{
    return opposite_side(l1,l2,s1,s2,s3)&&opposite_side(s1,s2,
        l1,l2,s3)&&
        opposite_side(s2,s3,l1,l2,s1)&&opposite_side(s3,s1,l1,
            l2,s2);
}

//mzry
inline bool ZERO(const double &a)
{
    return fabs(a)<eps;
}

inline bool ZERO(pv p)
{
    return (ZERO(p.x) && ZERO(p.y) && ZERO(p.z));
}

//直线相交
bool LineIntersect(Line3D L1, Line3D L2)
{
    pv s = L1.s-L1.e;
    pv e = L2.s-L2.e;
    pv p  = s*e;
    if (ZERO(p))
        return false;    //是否平行
    p = (L2.s-L1.e)*(L1.s-L1.e);
    return ZERO(p&L2.e);          //是否共面
}

//线段相交
bool inter(pv a,pv b,pv c,pv d)
{
    pv ret = (a-b)*(c-d);
    pv t1 = (b-a)*(c-a);
    pv t2 = (b-a)*(d-a);
    pv t3 = (d-c)*(a-c);
    pv t4 = (d-c)*(b-c);
    return sgn(t1&ret)*sgn(t2&ret) < 0 && sgn(t3&ret)*sgn(t4&
        ret) < 0;
}

//点在直线上
bool OnLine(pv p, Line3D L)
{
    return ZERO((p-L.s)*(L.e-L.s));
}

//点在线段上
bool OnSeg(pv p, Line3D L)
{
    return (ZERO((L.s-p)*(L.e-p)) && EQ(Norm(p-L.s)+Norm(p-L.e)
        ,Norm(L.e-L.s)));
}
```

2.1.3 Intersection

```
//计算两直线交点，注意事先判断直线是否共面和平行!
//线段交点请另外判线段相交（同时还是要判断是否平行!）
point3 intersection(line3 u,line3 v)
{
    point3 ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-
        v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.
            x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    ret.z+=(u.b.z-u.a.z)*t;
    return ret;
}
point3 intersection(point3 u1,point3 u2,point3 v1,point3 v2)
{
    point3 ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    ret.z+=(u2.z-u1.z)*t;
    return ret;
}
//计算直线与平面交点，注意事先判断是否平行，并保证三点不共线!
//线段和空间三角形交点请另外判断
point3 intersection(line3 l,plane3 s)
{
    point3 ret=pvec(s);
    double t=(ret.x*(s.a.x-l.a.x)+ret.y*(s.a.y-l.a.y)+ret.z*(s.
        a.z-l.a.z))/
        (ret.x*(l.b.x-l.a.x)+ret.y*(l.b.y-l.a.y)+ret.z*(l.b.z-l
            .a.z));
    ret.x=l.a.x+(l.b.x-l.a.x)*t;
    ret.y=l.a.y+(l.b.y-l.a.y)*t;
    ret.z=l.a.z+(l.b.z-l.a.z)*t;
    return ret;
}
point3 intersection(point3 l1,point3 l2,point3 s1,point3 s2,
    point3 s3)
{
    point3 ret=pvec(s1,s2,s3);
    double t=(ret.x*(s1.x-l1.x)+ret.y*(s1.y-l1.y)+ret.z*(s1.z-
        l1.z))/
        (ret.x*(l2.x-l1.x)+ret.y*(l2.y-l1.y)+ret.z*(l2.z-l1.z))
            ;
    ret.x=l1.x+(l2.x-l1.x)*t;
    ret.y=l1.y+(l2.y-l1.y)*t;
    ret.z=l1.z+(l2.z-l1.z)*t;
    return ret;
}
//计算两平面交线，注意事先判断是否平行，并保证三点不共线!
line3 intersection(plane3 u,plane3 v)
{
    line3 ret;
    ret.a=parallel(v.a,v.b,u.a,u.b,u.c)?intersection(v.b,v.c,u.
        a,u.b,u.c):intersection(v.a,v.b,u.a,u.b,u.
            c);
    ret.b=parallel(v.c,v.a,u.a,u.b,u.c)?intersection(v.b,v.c,u.
        a,u.b,u.c):intersection(v.c,v.a,u.a,u.b,u.
            c);
    return ret;
}
line3 intersection(point3 u1,point3 u2,point3 u3,point3 v1,
    point3 v2,point3 v3)
{
    line3 ret;
    ret.a=parallel(v1,v2,u1,u2,u3)?intersection(v2,v3,u1,u2,u3)
        :intersection(v1,v2,u1,u2,u3);
    ret.b=parallel(v3,v1,u1,u2,u3)?intersection(v2,v3,u1,u2,u3)
        :intersection(v3,v1,u1,u2,u3);
    return ret;
}
```

2.1.4 Distance

```
//点到直线距离
double ptoline(point3 p,line3 l)
{
    return vlen(xmult(subt(p,l.a),subt(l.b,l.a)))/distance(l.a,
        l.b);
}
double ptoline(point3 p,point3 l1,point3 l2)
{
}

//点到直线距离
double Distance(pv p, Line3D L)
{
    return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
}
```

```
{
    return vlen(xmult(subt(p,l1),subt(l2,l1)))/distance(l1,l2);
}
//点到平面距离
double ptoplane(point3 p,plane3 s)
{
    return fabs(dmult(pvec(s),subt(p,s.a)))/vlen(pvec(s));
}
double ptoplane(point3 p,point3 s1,point3 s2,point3 s3)
{
    return fabs(dmult(pvec(s1,s2,s3),subt(p,s1)))/vlen(pvec(s1,
        s2,s3));
}
//直线到直线距离
double linetoline(line3 u,line3 v)
{
    point3 n=xmult(subt(u.a,u.b),subt(v.a,v.b));
    return fabs(dmult(subt(u.a,v.a),n))/vlen(n);
}
double linetoline(point3 u1,point3 u2,point3 v1,point3 v2)
{
    point3 n=xmult(subt(u1,u2),subt(v1,v2));
    return fabs(dmult(subt(u1,v1),n))/vlen(n);
}
```

2.1.5 Angle

```
//两直线夹角 cos 值
double angle_cos(line3 u,line3 v)
{
    return dmult(subt(u.a,u.b),subt(v.a,v.b))/vlen(subt(u.a,u.b
        ))/vlen(subt(v.a,v.b));
}
double angle_cos(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return dmult(subt(u1,u2),subt(v1,v2))/vlen(subt(u1,u2))/
        vlen(subt(v1,v2));
}
//两平面夹角 cos 值
double angle_cos(plane3 u,plane3 v)
{
    return dmult(pvec(u),pvec(v))/vlen(pvec(u))/vlen(pvec(v));
}
double angle_cos(point3 u1,point3 u2,point3 u3,point3 v1,point3
      v2,point3 v3)
{
    return dmult(pvec(u1,u2,u3),pvec(v1,v2,v3))/vlen(pvec(u1,u2
        ,u3))/vlen(pvec(v1,v2,v3));
}
//直线平面夹角 sin 值
double angle_sin(line3 l,plane3 s)
{
    return dmult(subt(l.a,l.b),pvec(s))/vlen(subt(l.a,l.b))/
        vlen(pvec(s));
}
double angle_sin(point3 l1,point3 l2,point3 s1,point3 s2,point3
      s3)
{
    return dmult(subt(l1,l2),pvec(s1,s2,s3))/vlen(subt(l1,l2))/
        vlen(pvec(s1,s2,s3));
}
```

2.2 3DCH

```
#include<cstdio>
#include<cmath>
#include<vector>
#include<algorithm>

#define MAXX 1111
#define eps 1e-8
#define inf 1e20

struct pv
{
    double x,y,z;
    pv(double a=0,double b=0,double c=0):x(a),y(b),z(c){}
    pv operator-(const pv &i)const { return pv(x-i.x,y-i.y,z-i.
        z); }
    pv operator+(const pv &i)const { return pv(x+i.x,y+i.y,z+i.
        z); }
    pv operator*(double a)const{return pv(x*a,y*a,z*a);}
    pv cross(const pv &i)const{return pv(y*i.z-z*i.y,z*i.x-x*i.
        z,x*i.y-y*i.x);}
    double dot(const pv &i)const{return x*i.x+y*i.y+z*i.z;}
    pv operator*(const pv &i)const{return cross(i);}
    double operator^(const pv &i)const{return dot(i);}
    double len()const{return sqrt(x*x+y*y+z*z);}
};

struct pla
{
    int a,b,c;
    bool ok;
    pla(int aa=0,int bb=0,int cc=0):a(aa),b(bb),c(cc),ok(true)
        {}
    void set();
};

std::vector<pla>fac(MAXX*MAXX);
int to[MAXX][MAXX];

inline void pla::set(){to[a][b]=to[b][c]=to[c][a]=fac.size();}

inline double vol(const pv &a,const pv &b,const pv &c,const pv
    &d)
{
    return (b-a)*(c-a)^(d-a);
}
inline double ptof(const pv &p,const pla &f)
{
    return vol(pnt[f.a],pnt[f.b],pnt[f.c],p);
}
inline double ptof(const pv &p,int f)
{
    return fabs(ptof(p,fac[f])/((pnt[fac[f].b]-pnt[fac[f].a])*(
        pnt[fac[f].c]-pnt[fac[f].a])).len());
}

void dfs(int,int);
void deal(int p,int a,int b)
{
    if(!fac[to[a][b]].ok)
        return;
    if(ptof(pnt[p],fac[to[a][b]])>eps)
        dfs(p,to[a][b]);
    else
    {
        pla add(p,b,a);
        add.set();
        fac.push_back(add);
    }
}
void dfs(int p,int now)
{
    fac[now].ok=false;
    deal(p,fac[now].b,fac[now].a);
    deal(p,fac[now].c,fac[now].b);
    deal(p,fac[now].a,fac[now].c);
}

inline void make(const int n)
{
    static int i,j,m;
    fac.resize(0);
    if(n<4)
        return;

    for(i=1;i<n;++i)
        if((pnt[0]-pnt[i]).len()>eps)
        {
            std::swap(pnt[i],pnt[1]);
            break;
        }
    if(i==n)
        return;
    for(i=2;i<n;++i)
        if(((pnt[0]-pnt[1])*(pnt[1]-pnt[i])).len()>eps)
        {
            std::swap(pnt[i],pnt[2]);
            break;
        }
    if(i==n)
        return;
    for(i=3;i<n;++i)
        if(fabs((pnt[0]-pnt[1])*(pnt[1]-pnt[2])^(pnt[2]-pnt[i])
            )>eps)
        {
            std::swap(pnt[3],pnt[i]);
            break;
        }
    if(i==n)
        return;

    for(i=0;i<4;++i)
    {
        pla add((i+1)%4,(i+2)%4,(i+3)%4);
        if(ptof(pnt[i],add)>0)
            std::swap(add.c,add.b);
        add.set();
        fac.push_back(add);
    }
    for(;i<n;++i)
        for(j=0;j<fac.size();++j)
            if(fac[j].ok && ptof(pnt[i],fac[j])>eps)
            {
                dfs(i,j);
                break;
            }
    m=fac.size();
```

```
    fac.resize(0);
    for(i=0;i<m;++i)
        if(fac[i].ok)
            fac.push_back(fac[i]);
}

inline pv gc() //重心
{
    pv re(0,0,0),o(0,0,0);
    double all(0),v;
    for(int i=0;i<fac.size();++i)
    {
        v=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
        re+=(pnt[fac[i].a]+pnt[fac[i].b]+pnt[fac[i].c])*0.25f*v
            ;
        all+=v;
    }
    return re*(1/all);
}

inline bool same(const short &s,const short &t) //两面是否相等
{
    pv &a=pnt[fac[s].a],&b=pnt[fac[s].b],&c=pnt[fac[s].c];
    return fabs(vol(a,b,c,pnt[fac[t].a]))<eps && fabs(vol(a,b,c
        ,pnt[fac[t].b]))<eps && fabs(vol(a,b,c,pnt[fac[t].c]))
        <eps;
}

//表面多边形数目
inline int facetcnt()
{
    int ans=0;
    static int i,j;
    for(i=0;i<fac.size();++i)
    {
        for(j=0;j<i;++j)
            if(same(i,j))
                break;
        if(j==i)
            ++ans;
    }
    return ans;
}

//表面三角形数目
inline short trianglecnt()
{
    return fac.size();
}

//三点构成的三角形面积*2
inline double area(const pv &a,const pv &b,const pv &c)
{
        return ((b-a)*(c-a)).len();
}

//表面积
inline double area()
{
    double ret(0);
    static int i;
    for(i=0;i<fac.size();++i)
        ret+=area(pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return ret/2;
}

//体积
inline double volume()
{
    pv o(0,0,0);
    double ret(0);
    for(short i(0);i<fac.size();++i)
        ret+=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return fabs(ret/6);
}
```

2.3 circle's area

```
//去重
{
    for (int i = 0; i < n; i++)
    {
        scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        del[i] = false;
    }
    for (int i = 0; i < n; i++)
        if (del[i] == false)
        {
            if (c[i].r == 0.0)
                del[i] = true;
            for (int j = 0; j < n; j++)
                if (i != j)
                    if (del[j] == false)
                        if (cmp(Point(c[i].c,c[j].c).Len()+c[i
                            ].r,c[j].r) <= 0)
                            del[i] = true;
        }
    tn = n;
    n = 0;
    for (int i = 0; i < tn; i++)
        if (del[i] == false)
            c[n++] = c[i];
}

//ans[i] 表示被覆盖 i 次的面积
const double pi = acos(-1.0);
const double eps = 1e-8;
struct Point
{
    double x,y;
    Point(){}
    Point(double _x,double _y)
    {
        x = _x;
        y = _y;
    }
    double Length()
    {
        return sqrt(x*x+y*y);
    }
};
struct Circle
{
    Point c;
    double r;
};
struct Event
{
    double tim;
    int typ;
    Event(){}
    Event(double _tim,int _typ)
    {
        tim = _tim;
        typ = _typ;
    }
};

int cmp(const double& a,const double& b)
{
    if (fabs(a-b) < eps)    return 0;
    if (a < b)  return -1;
    return 1;
}

bool Eventcmp(const Event& a,const Event& b)
{
    return cmp(a.tim,b.tim) < 0;
}

double Area(double theta,double r)
{
    return 0.5*r*r*(theta-sin(theta));
}

double xmult(Point a,Point b)
{
    return a.x*b.y-a.y*b.x;
}

int n,cur,tote;
Circle c[1000];
double ans[1001],pre[1001],AB,AC,BC,theta,fai,a0,a1;
Event e[4000];
Point lab;

int main()
{
    while (scanf("%d",&n) != EOF)
    {
        for (int i = 0;i < n;i++)
            scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        for (int i = 1;i <= n;i++)
            ans[i] = 0.0;
        for (int i = 0;i < n;i++)
        {
            tote = 0;
            e[tote++] = Event(-pi,1);
            e[tote++] = Event(pi,-1);
            for (int j = 0;j < n;j++)
                if (j != i)
                {
                    lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i
                        ].c.y);
                    AB = lab.Length();
                    AC = c[i].r;
                    BC = c[j].r;
                    if (cmp(AB+AC,BC) <= 0)
                    {
                        e[tote++] = Event(-pi,1);
                        e[tote++] = Event(pi,-1);
```

```
                    continue;
                }
                if (cmp(AB+BC,AC) <= 0) continue;
                if (cmp(AB,AC+BC) > 0)  continue;
                theta = atan2(lab.y,lab.x);
                fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB))
                    ;
                a0 = theta-fai;
                if (cmp(a0,-pi) < 0)    a0 += 2*pi;
                a1 = theta+fai;
                if (cmp(a1,pi) > 0)  a1 -= 2*pi;
                if (cmp(a0,a1) > 0)
                {
                    e[tote++] = Event(a0,1);
                    e[tote++] = Event(pi,-1);
                    e[tote++] = Event(-pi,1);
                    e[tote++] = Event(a1,-1);
                }
                else
                {
                    e[tote++] = Event(a0,1);
                    e[tote++] = Event(a1,-1);
                }
            }
        sort(e,e+tote,Eventcmp);
        cur = 0;
        for (int j = 0;j < tote;j++)
        {
            if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0)
            {
                ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
                ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos
                    (pre[cur]),c[i].c.y+c[i].r*sin(pre[cur
                    ])),
                        Point(c[i].c.x+c[i].r*cos(e[j].tim)
                            ,c[i].c.y+c[i].r*sin(e[j].tim)
                            ))/2.0;
            }
            cur += e[j].typ;
            pre[cur] = e[j].tim;
        }
    }
    for (int i = 1;i < n;i++)
        ans[i] -= ans[i+1];
    for (int i = 1;i <= n;i++)
        printf("[%d] = %.3f\n",i,ans[i]);
  }
  return 0;
}
```

2.4 circle

```
//单位圆覆盖
#include<cstdio>
#include<cmath>
#include<algorithm>
#include<vector>

#define eps 1e-8
#define MAXX 211
const double pi(acos(-1));
typedef std::pair<double,int> pdi;

struct pv
{
    double x,y;
    pv(double a=0,double b=0):x(a),y(b){}
    pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    double len()
    {
        return hypot(x,y);
    }
}pnt[MAXX];

std::vector<pdi>alpha(MAXX<<1);

inline int solve(double r) //radius
{
    static int ans,sum,i,j;
    sum=ans=0;
    for(i=0;i<n;++i)
    {
        alpha.resize(0);
        static double d,theta,phi;
        static pv vec;
        for(j=0;j<n;++j)
        {
            if(j==i || (d=(vec=pnt[i]-pnt[j]).len())>2*r+eps)
                continue;
            if((theta=atan2(vec.y,vec.x))<-eps)
                theta+=2*pi;
            phi=acos(d/(2*r));
            alpha.push_back(pdi(theta-phi+2*pi,-1));
            alpha.push_back(pdi(theta+phi+2*pi,1));
        }
        std::sort(alpha.begin(),alpha.end());
        for(j=0;j<alpha.size();++j)
        {
            sum-=alpha[j].second;
            if(sum>ans)
                ans=sum;
        }
    }
    return ans+1;
}
```

2.5 closest point pair

```
//演算法笔记1

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* Divide: 把所有點分成左右兩側，點數盡量一樣多。 */

    int M = (L + R) / 2;

    /* Conquer: 左側、右側分別遞迴求解。 */

    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* Merge: 尋找靠近中線的點，並依 Y 座標排序。O(NlogN)。 */

    int N = 0;  // 靠近中線的點數目
    for (int i=M;   i>=L && p[M].x - p[i].x < d; --i) t[N++] =
         p[i];
    for (int i=M+1; i<=R && p[i].x - p[M].x < d; ++i) t[N++] =
         p[i];
    sort(t, t+N, cmpy); // Quicksort O(NlogN)

    /* Merge: 尋找橫跨兩側的最近點對。O(N)。 */

    for (int i=0; i<N-1; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}


//演算法笔记2

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* Divide: 把所有點分成左右兩側，點數盡量一樣多。 */

    int M = (L + R) / 2;

    // 先把中線的 X 座標記起來，因為待會重新排序之後會跑掉。
    double x = p[M].x;

    /* Conquer: 左側、右側分別遞迴求解。 */

    // 遞迴求解，並且依照 Y 座標重新排序。
    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* Merge: 尋找靠近中線的點，並依 Y 座標排序。O(N)。 */

    // 尋找靠近中線的點，先找左側。各點已照 Y 座標排序了。
    int N = 0;  // 靠近中線的點數目
    for (int i=0; i<=M; ++i)
        if (x - p[i].x < d)
            t[N++] = p[i];

    // 尋找靠近中線的點，再找右側。各點已照 Y 座標排序了。
    int P = N;  // P 為分隔位置
```

```
    for (int i=M+1; i<=R; ++i)
        if (p[i].x - x < d)
            t[N++] = p[i];

    // 以 Y 座標排序。使用 Merge Sort 方式，合併已排序的兩陣列。
    inplace_merge(t, t+P, t+N, cmpy);

    /* Merge: 尋找橫跨兩側的最近點對。O(N)。 */

    for (int i=0; i<N; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    /* Merge: 重新以 Y 座標排序所有點。O(N)。 */

    // 如此一來，更大的子問題就可以直接使用 Merge Sort。
    inplace_merge(p+L, p+M+1, p+R+1, cmpy);

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}

//mzry
//分治
double calc_dis(Point &a ,Point &b) {
  return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
//別忘了排序
bool operator<(const Point &a ,const Point &b) {
  if(a.y != b.y) return a.x < b.x;
  return a.x < b.x;
}
double Gao(int l ,int r ,Point pnts[]) {
  double ret = inf;
  if(l == r) return ret;
  if(l+1 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    return ret;
  }
  if(l+2 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    ret = min(calc_dis(pnts[l],pnts[l+2]) ,ret);
    ret = min(calc_dis(pnts[l+1],pnts[l+2]) ,ret);
    return ret;
  }

  int mid = l+r>>1;
  ret = min (ret ,Gao(l ,mid,pnts));
  ret = min (ret , Gao(mid+1, r,pnts));

  for(int c = l ; c<=r; c++)
    for(int d = c+1; d <=c+7 && d<=r; d++) {
      ret = min(ret , calc_dis(pnts[c],pnts[d]));
    }
  return ret;
}

//增量
#include <iostream>
#include <cstdio>
#include <cstring>
#include <map>
#include <vector>
#include <cmath>
#include <algorithm>
#define Point pair<double,double>
using namespace std;

const int step[9][2] =
     {{-1,-1},{-1,0},{-1,1},{0,-1},{0,0},{0,1},{1,-1},{1,0},{1,1}};

int n,x,y,nx,ny;
map<pair<int,int>,vector<Point > > g;
vector<Point > tmp;
Point p[20000];
double tx,ty,ans,nowans;
vector<Point >::iterator it,op,ed;
pair<int,int> gird;
bool flag;

double Dis(Point p0,Point p1)
{
  return sqrt((p0.first-p1.first)*(p0.first-p1.first)+
        (p0.second-p1.second)*(p0.second-p1.second));
}

double CalcDis(Point p0,Point p1,Point p2)
{
  return Dis(p0,p1)+Dis(p0,p2)+Dis(p1,p2);
}

void build(int n,double w)
{
  g.clear();
  for (int i = 0;i < n;i++)
    g[make_pair((int)floor(p[i].first/w),(int)floor(p[i].second
         /w))].push_back(p[i]);
}

int main()
{
  int t;
  scanf("%d",&t);
  for (int ft = 1;ft <= t;ft++)
  {
    scanf("%d",&n);
    for (int i = 0;i < n;i++)
    {
      scanf("%lf%lf",&tx,&ty);
      p[i] = make_pair(tx,ty);
    }
    random_shuffle(p,p+n);
    ans = CalcDis(p[0],p[1],p[2]);
    build(3,ans/2.0);
    for (int i = 3;i < n;i++)
    {
      x = (int)floor(2.0*p[i].first/ans);
      y = (int)floor(2.0*p[i].second/ans);
      tmp.clear();
      for (int k = 0;k < 9;k++)
      {
        nx = x+step[k][0];
        ny = y+step[k][1];
        gird = make_pair(nx,ny);
        if (g.find(gird) != g.end())
        {
          op = g[gird].begin();
          ed = g[gird].end();
          for (it = op;it != ed;it++)
            tmp.push_back(*it);
        }
      }
      flag = false;
      for (int j = 0;j < tmp.size();j++)
        for (int k = j+1;k < tmp.size();k++)
        {
          nowans = CalcDis(p[i],tmp[j],tmp[k]);
          if (nowans < ans)
          {
            ans = nowans;
            flag = true;
          }
        }
      if (flag == true)
        build(i+1,ans/2.0);
      else
        g[make_pair((int)floor(2.0*p[i].first/ans),(int)floor
             (2.0*p[i].second/ans))].push_back(p[i]);
    }
    printf("%.3f\n",ans);
  }
}
```

2.6 half-plane intersection

```
//解析几何方式abc
inline pv ins(const pv &p1,const pv &p2)
{
    u=fabs(a*p1.x+b*p1.y+c);
    v=fabs(a*p2.x+b*p2.y+c);
    return pv((p1.x*v+p2.x*u)/(u+v),(p1.y*v+p2.y*u)/(u+v));
}

inline void get(const pv& p1,const pv& p2,double & a,double & b
    ,double & c)
{
    a=p2.y-p1.y;
    b=p1.x-p2.x;
    c=p2.x*p1.y-p2.y*p1.x;
}

inline pv ins(const pv &x,const pv &y)
{
    get(x,y,d,e,f);
    return pv((b*f-c*e)/(a*e-b*d),(a*f-c*d)/(b*d-a*e));
}

std::vector<pv>p[2];
inline bool go()
{
    k=0;
    p[k].resize(0);
    p[k].push_back(pv(-inf,inf));
    p[k].push_back(pv(-inf,-inf));
    p[k].push_back(pv(inf,-inf));
```

```
    p[k].push_back(pv(inf,inf));
    for(i=0;i<n;++i)
    {
        get(pnt[i],pnt[(i+1)%n],a,b,c);
        c+=the*sqrt(a*a+b*b);
        p[!k].resize(0);
        for(l=0;l<p[k].size();++l)
            if(a*p[k][l].x+b*p[k][l].y+c<eps)
                p[!k].push_back(p[k][l]);
            else
            {
                m=(l+p[k].size()-1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
                m=(l+1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
            }
        k=!k;
        if(p[k].empty())
            break;
    }
    //结果在 p[k] 中
    return p[k].empty();
}

//计算几何方式
//本例求多边形核

inline pv ins(const pv &a,const pv &b)
{
    u=fabs(ln.cross(a-pnt[i]));
    v=fabs(ln.cross(b-pnt[i]))+u;
    tl=b-a;
    return pv(u*tl.x/v+a.x,u*tl.y/v+a.y);
}

int main()
{
    j=0;
    for(i=0;i<n;++i)
    {
        ln=pnt[(i+1)%n]-pnt[i];
        p[!j].resize(0);
        for(k=0;k<p[j].size();++k)
            if(ln.cross(p[j][k]-pnt[i])<=0)
                p[!j].push_back(p[j][k]);
            else
            {
                l=(k-1+p[j].size())%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
                l=(k+1)%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
            }
        j=!j;
    }
    //结果在 p[j] 中
}

struct hp
{
    pv p,v; // from point p with vector v, left of it
    double k;
    hp(){}
    hp(const pv &i,const pv &j):p(i),v(j),k(atan2(j.y,j.x)){}
    bool operator<(const hp &i)const { return k<i.k; }
    bool onleft(const pv &pnt)const { return v.cross(pnt-p)
        >=0;}//>eps; }
    pv ins(const hp &b)const { return p+v*(b.v.cross(p-b.p)/v.
        cross(b.v)); } //line-line intersection
};
std::vector<hp>ln(MAXX);

inline void hpi(std::vector<hp>&l,std::vector<pv>&ot)
{
    static hp q[MAXX];
    static pv p[MAXX];
    static int i,qh,qt;
    ot.resize(0);
    std::sort(l.begin(),l.end());
    q[qh=qt=0]=l[0];
    for(i=0;i<l.size();++i)
    {
        while(qh<qt && !l[i].onleft(p[qt-1]))
            --qt;
        while(qh<qt && !l[i].onleft(p[qh]))
            ++qh;
        q[++qt]=l[i];
        if(fabs(q[qt].v.cross(q[qt-1].v))<eps)
        {
            --qt;
            if(q[qt].onleft(l[i].p))
                q[qt]=l[i];
        }
        if(qh<qt)
            p[qt-1]=q[qt].ins(q[qt-1]);
    }
    while(qh<qt && !q[qh].onleft(p[qt-1]))
        --qt;
    if(qh>=qt)
        return;
    if(qh<qt)
        p[qt]=q[qh].ins(q[qt]);
    for(i=qh;i<=qt;++i)
        ot.push_back(p[i]);
}
```

2.7 intersection of circle and poly

```
pv c;
double r;

inline double cal(const pv &a,const pv &b)
{
    static double A,B,C,x,y,ts;
    A=(b-c).len();
    B=(a-c).len();
    C=(a-b).len();
    if(A<r && B<r)
        return (a-c).cross(b-c)/2;
    x=((a-b).dot(c-b)+sqrt(r*r*C*C-sqr((a-b).cross(c-b))))/C;
    y=((b-a).dot(c-a)+sqrt(r*r*C*C-sqr((b-a).cross(c-a))))/C;
    ts=(a-c).cross(b-c)/2;

    if(A<r && B>=r)
        return asin(ts*(1-x/C)*2/r/B*(1-eps))*r*r/2+ts*x/C;
    if(A>=r && B<r)
        return asin(ts*(1-y/C)*2/r/A*(1-eps))*r*r/2+ts*y/C;

    if(fabs((a-c).cross(b-c))>=r*C || (b-a).dot(c-a)<=0 || (a-b
        ).dot(c-b)<=0)
    {
        if((a-c).dot(b-c)<0)
        {
            if((a-c).cross(b-c)<0)
                return (-pi-asin((a-c).cross(b-c)/A/B*(1-eps)))
                    *r*r/2;
            return (pi-asin((a-c).cross(b-c)/A/B*(1-eps)))*r*r
                /2;
        }
        return asin((a-c).cross(b-c)/A/B*(1-eps))*r*r/2;
    }

    return (asin(ts*(1-x/C)*2/r/B*(1-eps))+asin(ts*(1-y/C)*2/r/
        A*(1-eps)))*r*r/2+ts*((y+x)/C-1);
}

inline double get(pv *the,int n)
{
    double ans=0;
    for(int i=0;i<n;++i)
        ans+=cal(the[i],the[(i+1)%n]);
    return ans;
}
```

2.8 k-d tree

```
/*
有个很关键的剪枝，在计算完与 mid 点的距离后，我们应该先进入左右哪个子树？我
    们应该先进入对于当前维度，查询点位于的那一边。显然，在查询点所在的子
    树，更容易查找出正确解。

那么当进入完左或右子树后，以查询点为圆心做圆，如果当前维度，查询点距离 mid
    的距离（另一个子树中的点距离查询点的距离肯定大于这个距离）比堆里的最大
    值还大，那么就不再递归另一个子树。注意一下：如果堆里的元素个数不足 M，
    仍然还要进入另一棵子树。

说白了就是随便乱搞啦……
*/
// hysbz 2626
#include<cstdio>
#include<algorithm>
#include<queue>

inline long long sqr(long long a){ return a*a;}
typedef std::pair<long long,int> pli;

#define MAXX 100111
#define MAX (MAXX<<2)
#define inf 0x3f3f3f3fll
int idx;

struct PNT
{
    long long x[2];
    int lb;
```

```
    bool operator<(const PNT &i)const
    {
        return x[idx]<i.x[idx];
    }
    pli dist(const PNT &i)const
    {
        return pli(-(sqr(x[0]-i.x[0])+sqr(x[1]-i.x[1])),lb);
    }
}a[MAXX],the[MAX],p;

#define mid (l+r>>1)
#define lson (id<<1)
#define rson (id<<1|1)
#define lc lson,l,mid-1
#define rc rson,mid+1,r
int n,m;

long long rg[MAX][2][2];

void make(int id=1,int l=1,int r=n,int d=0)
{
    the[id].lb=-1;
    rg[id][0][0]=rg[id][1][0]=inf;
    rg[id][0][1]=rg[id][1][1]=-inf;
    if(l>r)
        return;
    idx=d;
    std::nth_element(a+l,a+mid,a+r+1);
    the[id]=a[mid];
    rg[id][0][0]=rg[id][0][1]=the[id].x[0];
    rg[id][1][0]=rg[id][1][1]=the[id].x[1];
    make(lc,d^1);
    make(rc,d^1);

    rg[id][0][0]=std::min(rg[id][0][0],std::min(rg[lson][0][0],
         rg[rson][0][0]));
    rg[id][1][0]=std::min(rg[id][1][0],std::min(rg[lson][1][0],
         rg[rson][1][0]));

    rg[id][0][1]=std::max(rg[id][0][1],std::max(rg[lson][0][1],
         rg[rson][0][1]));
    rg[id][1][1]=std::max(rg[id][1][1],std::max(rg[lson][1][1],
         rg[rson][1][1]));
}

inline long long cal(int id)
{
    static long long a[2];
    static int i;
    for(i=0;i<2;++i)
        a[i]=std::max(abs(p.x[i]-rg[id][i][0]),abs(p.x[i]-rg[id
             ][i][1]));
    return sqr(a[0])+sqr(a[1]);
}

std::priority_queue<pli>ans;

void query(const int id=1,const int d=0)
{
    if(the[id].lb<0)
        return;
    pli tmp(the[id].dist(p));
    int a(lson),b(rson);
    if(p.x[d]<=the[id].x[d])
        std::swap(a,b);
    if(ans.size()<m)
        ans.push(tmp);
    else
        if(tmp<ans.top())
        {
            ans.push(tmp);
            ans.pop();
        }
    if(ans.size()<m || cal(a)>=-ans.top().first)
        query(a,d^1);
    if(ans.size()<m || cal(b)>=-ans.top().first)
        query(b,d^1);
}

int q,i,j,k;

int main()
{
    scanf("%d",&n);
    for(i=1;i<=n;++i)
    {
        scanf("%lld␣%lld",&a[i].x[0],&a[i].x[1]);
        a[i].lb=i;
    }
    make();
    scanf("%d",&q);
    while(q--)
    {
        scanf("%lld␣%lld",&p.x[0],&p.x[1]);
        scanf("%d",&m);
        while(!ans.empty())
            ans.pop();
        query();
        printf("%d\n",ans.top().second);
    }
    return 0;
}
```

2.9 Manhattan MST

```
#include<iostream>
#include<cstdio>
#include<cstring>
#include<queue>
#include<cmath>
using namespace std;
const int srange = 10000000;      //坐标范围
const int ra = 131072;     //线段树常量
int c[ ra * 2 ], d[ ra * 2 ];     //线段树
int a[ 100000 ], b[ 100000 ];   //排序临时变量
int order[ 400000 ], torder[ 100000 ]; //排序结果
int Index[ 100000 ];     //排序结果取反（为了在常数时间内取得某数的位
    置）
int road[ 100000 ][ 8 ];    //每个点连接出去的条边8
int y[ 100000 ], x[ 100000 ];     //点坐标
int n;         //点个数

int swap( int &a, int &b )    //交换两个数
{
    int t = a; a = b; b = t;
}

int insert( int a, int b, int i ) //向线段树中插入一个数
{
    a += ra;
    while ( a != 0 )
    {
        if ( c[ a ] > b )
        {
            c[ a ] = b;
            d[ a ] = i;
        }
        else break;
        a >>= 1;
    }
}

int find( int a )      //从c[0..a中找最小的数，线段树查询]
{
    a += ra;
    int ret = d[ a ], max = c[ a ];
    while ( a > 1 )
    {
        if ( ( a & 1 ) == 1 )
            if ( c[ --a ] < max )
            {
                max = c[ a ];
                ret = d[ a ];
            }
        a >>= 1;
    }
    return ret;
}

int ta[ 65536 ], tb[ 100000 ];    //基数排序临时变量

int radixsort( int *p )     //基数排序，以为基准p
{
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < n; i++ ) ta[ p[ i ] & 0xffff ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = n - 1; i >= 0; i-- ) tb[ --ta[ p[ order[ i ] ]
          & 0xffff ] ] = order[ i ];
    memmove( order, tb, n * sizeof( int ) );
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < n; i++ ) ta[ p[ i ] >> 16 ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = n - 1; i >= 0; i-- ) tb[ --ta[ p[ order[ i ] ]
          >> 16 ] ] = order[ i ];
    memmove( order, tb, n * sizeof( int ) );
}

int work( int ii )                //求每个点在一个方向上最近的点
{
    for (int i = 0; i < n; i++ ) //排序前的准备工作
    {
        a[ i ] = y[ i ] - x[ i ] + srange;
        b[ i ] = srange - y[ i ];
        order[ i ] = i;
    }
    radixsort( b );      //排序
    radixsort( a );
    for (int i = 0; i < n; i++ )
    {
```

```
        torder[ i ] = order[ i ];
        order[ i ] = i;
    }
    radixsort( a );      //为线段树而做的排序
    radixsort( b );
    for (int i = 0; i < n; i++ )
    {
        Index[ order[ i ] ] = i; //取反，求orderIndex
    }
    for (int i = 1; i < ra + n; i++ ) c[ i ] = 0x7fffffff; //线
        段树初始化
    memset( d, 0xff, sizeof( d ) );
    for (int i = 0; i < n; i++ ) //线段树插入删除调用
    {
        int tt = torder[ i ];
        road[ tt ][ ii ] = find( Index[ tt ] );
        insert( Index[ tt ], y[ tt ] + x[ tt ], tt );
    }
}

int distanc( int a, int b )       //求两点的距离，之所以少一个是因为
    编译器不让使用作为函数名edistance
{
    return abs( x[ a ] - x[ b ] ) + abs( y[ a ] - y[ b ] );
}

int ttb[ 400000 ];      //边排序的临时变量
int rx[ 400000 ], ry[ 400000 ], rd[ 400000 ]; //边的存储
int rr = 0;

int radixsort_2( int *p )    //还是基数排序，copy+的产物paste
{
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < rr; i++ ) ta[ p[ i ] & 0xffff ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = rr - 1; i >= 0; i-- ) ttb[ --ta[ p[ order[ i ]
        ] & 0xffff ] ] = order[ i ];
    memmove( order, ttb, rr * sizeof( int ) );
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < rr; i++ ) ta[ p[ i ] >> 16 ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = rr - 1; i >= 0; i-- ) ttb[ --ta[ p[ order[ i ]
        ] >> 16 ] ] = order[ i ];
    memmove( order, ttb, rr * sizeof( int ) );
}

int father[ 100000 ], rank[ 100000 ];    //并查集
int findfather( int x )                  //并查集寻找代表元
{
    if ( father[ x ] != -1 )
        return ( father[ x ] = findfather( father[ x ] ) );
    else return x;
}

long long kruskal()                      //最小生成树
{
    rr = 0;
    int tot = 0;
    long long ans = 0;
    for (int i = 0; i < n; i++ )         //得到边表
    {
        for (int j = 0; j < 4; j++ )
        {
            if ( road[ i ][ j ] != -1 )
            {
                rx[ rr ] = i;
                ry[ rr ] = road[ i ][ j ];
                rd[ rr++ ] = distanc( i, road[ i ][ j ] );
            }
        }
    }
    for (int i = 0; i < rr; i++ ) order[ i ] = i; //排序
    radixsort_2( rd );
    memset( father, 0xff, sizeof( father ) ); //并查集初始化
    memset( rank, 0, sizeof( rank ) );
    for (int i = 0; i < rr; i++ )     //最小生成树标准算法kruskal
    {
        if ( tot == n - 1 ) break;
        int t = order[ i ];
        int x = findfather( rx[ t ] ), y = findfather( ry[ t ]
            );
        if ( x != y )
        {
            ans += rd[ t ];
            tot++;
            int &rkx = rank[ x ], &rky = rank[ y ];
            if ( rkx > rky ) father[ y ] = x;
            else
            {
                father[ x ] = y;
                if ( rkx == rky ) rky++;
            }
        }
    }
    return ans;
}

int casenum = 0;

int main()
{
    while ( cin >> n )
    {
        if ( n == 0 ) break;
        for (int i = 0; i < n; i++ )
            scanf( "%d %d", &x[ i ], &y[ i ] );
        memset( road, 0xff, sizeof( road ) );
        for (int i = 0; i < 4; i++ )              //为了减少编程复
            杂度，work()函数只写了一种，其他情况用转换坐标的方式类似处
            理
        {           //为了降低算法复杂度，只求出个方向的边4
            if ( i == 2 )
            {
                for (int j = 0; j < n; j++ ) swap( x[ j ], y[ j
                    ] );
            }
            if ( ( i & 1 ) == 1 )
            {
                for (int j = 0; j < n; j++ ) x[ j ] = srange -
                    x[ j ];
            }
            work( i );
        }
        printf( "Case %d: Total Weight = ", ++casenum );
        cout << kruskal() << endl;
    }
    return 0;
}
```

2.10 other

2.10.1 Pick's theorem

给定顶点座标均是整点（或正方形格点）的简单多边形
A: 面积
i: 内部格点数目
b: 边上格点数目
$A = i + \frac{b}{2} - 1$

取格点的组成图形的面积为一单位。在平行四边形格点，皮克定理依然成立。套用于任意三角形格点，皮克定理则是
$A = 2 \times i + b - 2$

2.10.2 Triangle

Area:
$p = \frac{a+b+c}{2}$
$area = \sqrt{p \times (p-a) \times (p-b) \times (p-c)}$
$area = \frac{a \times b \times \sin(\angle C)}{2}$
$area = \frac{a^2 \times \sin(\angle B) \times \sin(\angle C)}{2 \times \sin(\angle B + \angle C)}$
$area = \frac{a^2}{2 \times (\cot(\angle B) + \cot(\angle C))}$

centroid:
center of mass
intersection of triangle's three triangle medians

Trigonometric conditions:
$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
$\sin^2\frac{\alpha}{2} + \sin^2\frac{\beta}{2} + \sin^2\frac{\gamma}{2} + 2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} = 1$

Circumscribed circle:
$diameter = \frac{abc}{2 \cdot \text{area}} = \frac{|AB||BC||CA|}{2|\triangle ABC|}$

$= \frac{abc}{2\sqrt{s(s-a)(s-b)(s-c)}}$

$= \frac{2abc}{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}$

$diameter = \sqrt{\frac{2 \cdot \text{area}}{\sin A \sin B \sin C}}$

$diameter = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Incircle:
$inradius = \frac{2\times area}{a+b+c}$
coordinates(x,y)=$\left(\frac{ax_a+bx_b+cx_c}{a+b+c}, \frac{ay_a+by_b+cy_c}{a+b+c}\right)$ = $\frac{a}{a+b+c}(x_a,y_a) + \frac{b}{a+b+c}(x_b,y_b) + \frac{c}{a+b+c}(x_c,y_c)$

Excircles:
radius[a]=$\frac{2\times area}{b+c-a}$
radius[b]=$\frac{2\times area}{a+c-b}$
radius[c]=$\frac{2\times area}{a+b-c}$

Steiner circumellipse (least area circumscribed ellipse)
area=$\triangle \times \frac{4\pi}{3\sqrt{3}}$
center is the triangle's centroid.

Steiner inellipse (maximum area inellipse)
area=$\triangle \times \frac{\pi}{3\sqrt{3}}$
center is the triangle's centroid.

Fermat Point:

- 当有一个内角不小于 120° 时，费马点为此角对应顶点。
- 当三角形的内角都小于 120°
  1. 以三角形的每一边为底边，向外做三个正三角形 $\triangle ABC'$，$\triangle BCA'$，$\triangle CAB'$。
  2. 连接 CC'、BB'、AA'，则三条线段的交点就是所求的点。

2.10.3 Ellipse

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$x = h + a \times \cos(t)$
$y = k + b \times \sin(t)$

area=$\pi \times a \times b$
distance from center to focus: $f = \sqrt{a^2 - b^2}$
eccentricity: $e = \sqrt{a - \frac{b}{a}^2} = \frac{f}{a}$
focal parameter: $\frac{b^2}{\sqrt{a^2-b^2}} = \frac{b^2}{f}$

```
inline double circumference(double a,double b) // accuracy: pow
    (0.5,53);
{
    static double digits=53;
    static double tol=sqrt(pow(0.5,digits));
    double x=a;
    double y=b;
    if(x<y)
        std::swap(x,y);
    if(digits*y<tol*x)
        return 4*x;
    double s=0,m=1;
    while(x>(tol+1)*y)
    {
        double tx=x;
        double ty=y;
        x=0.5f*(tx+ty);
        y=sqrt(tx*ty);
        m*=2;
        s+=m*pow(x-y,2);
    }
    return pi*(pow(a+b,2)-s)/(x+y);
}
```

2.10.4 Summaries

- 三角形
  - 半周长 $P = \frac{a+b+c}{2}$
  - 面积 $S = \frac{aH}{2} = \frac{ab\sin(C)}{2} = \sqrt{P\times(P-a)\times(P-b)\times(P-c)}$
  - 中线 $Ma = \frac{\sqrt{2(b^2+c^2)-a^2}}{2} = \frac{\sqrt{b^2+c^2+2bc\cos(A)}}{2}$
  - 角平分线 $Ta = \frac{\sqrt{bc((b+c)^2-a^2)}}{b+c} = \frac{2bc\cos(\frac{A}{2})}{b+c}$
  - 高线 $Ha = b\sin(C) = c\sin(B) = \sqrt{b^2 - \frac{a^2+b^2-c^2}{2a}^2}$
  - 内切圆半径 $r = \frac{S}{P} = \frac{\arcsin(\frac{B}{2})\sin(\frac{C}{2})}{\sin(\frac{B+C}{2})} = 4R\sin(\frac{A}{2})\sin(\frac{B}{2})\sin(\frac{C}{2}) = \sqrt{\frac{(P-a)(P-b)(P-c)}{P}} = P\tan(\frac{A}{2})\tan(\frac{B}{2})\tan(\frac{C}{2})$
  - 外接圆半径 $R = \frac{abc}{4S} = \frac{a}{2\sin(A)} = \frac{b}{2\sin(B)} = \frac{c}{2\sin(C)}$
- 四边形
  D1,D2 为对角线,M 对角线中点连线,A 为对角线夹角
  - $a^2+b^2+c^2+d^2 = {D_1}^2 + {D_2}^2 + 4M^2$
  - $S = \frac{D_1 D_2 \sin(A)}{2}$

  (以下对圆的内接四边形)
  - $ac + bd = D_1 D_2$
  - $S = \sqrt{(P-a)(P-b)(P-c)(P-d)}$,P 为半周长
- 正 n 边形
  R 为外接圆半径,r 为内切圆半径
  - 中心角 $A = \frac{2\pi}{n}$
  - 内角 $C = (n-2)\frac{\pi}{n}$
  - 边长 $a = 2\sqrt{R^2-r^2} = 2R\sin(\frac{A}{2}) = 2r\tan(\frac{A}{2})$
  - 面积 $S = \frac{nar}{2} = nr^2\tan(\frac{A}{2}) = \frac{nR^2\sin(A)}{2} = \frac{na^2}{4\tan(\frac{A}{2})}$
- 圆
  - 弧长 $l = rA$
  - 弦长 $a = 2\sqrt{2hr-h^2} = 2r\sin(\frac{A}{2})$
  - 弓形高 $h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1-\cos(\frac{A}{2})) = \frac{\arctan(\frac{A}{4})}{2}$
  - 扇形面积 $S1 = \frac{rl}{2} = \frac{r^2A}{2}$
  - 弓形面积 $S2 = \frac{rl-a(r-h)}{2} = \frac{r^2(A-\sin(A))}{2}$
- 棱柱
  - 体积 $V = Ah$,A 为底面积,h 为高
  - 侧面积 $S = lp$,l 为棱长,p 为直截面周长
  - 全面积 $T = S + 2A$
- 棱锥
  - 体积 $V = \frac{Ah}{3}$,A 为底面积,h 为高

  (以下对正棱锥)
  - 侧面积 $S = \frac{lp}{2}$,l 为斜高,p 为底面周长
  - 全面积 $T = S + A$

- 棱台
  - 体积 $V=(A_1+A_2+\sqrt{A_1A_2})\frac{h}{3}$,A1.A2 为上下底面积,h 为高

  (以下为正棱台)
  - 侧面积 $S=\frac{(p_1+p_2)l}{2}$,p1,p2 为上下底面周长,l 为斜高
  - 全面积 $T=S+A_1+A_2$
- 圆柱
  - 侧面积 $S=2\pi rh$
  - 全面积 $T=2\pi r(h+r)$
  - 体积 $V=\pi r^2h$
- 圆锥
  - 斜高 $l=\sqrt{h^2+r^2}$
  - 侧面积 $S=\pi rl$
  - 全面积 $T=\pi r(l+r)$
  - 体积 $V=\pi r^2\frac{h}{3}$
- 圆台
  - 母线 $l=\sqrt{h^2+(r_1-r_2)^2}$
  - 侧面积 $S=\pi(r_1+r_2)l$
  - 全面积 $T=\pi r_1(l+r_1)+\pi r_2(l+r_2)$
  - 体积 $V=\pi({r_1}^2+{r_2}^2+r_1r_2)\frac{h}{3}$
- 球
  - 全面积 $T=4\pi r^2$
  - 体积 $V=\pi r^3\frac{4}{3}$
- 球台
  - 侧面积 $S=2\pi rh$
  - 全面积 $T=\pi(2rh+{r_1}^2+{r_2}^2)$
  - 体积 V=$\frac{1}{6}\pi h(3({r_1}^2+{r_2}^2)+h^2)$
- 球扇形
  - 全面积 $T=\pi r(2h+r_0)$,h 为球冠高,r0 为球冠底面半径
  - 体积 $V=\frac{2}{3}\pi r^2h$

2.10.5 about double

如果 sqrt(a), asin(a), acos(a) 中的 a 是你自己算出来并传进来的，那就得小心了。如果 a 本来应该是 0 的，由于浮点误差，可能实际是一个绝对值很小的负数（比如 -1^{-12}），这样 sqrt(a) 应得 0 的，直接因 a 不在定义域而出错。类似地，如果 a 本来应该是 ±1, 则 asin(a)、acos(a) 也有可能出错。因此，对于此种函数，必需事先对 a 进行校正。

现在考虑一种情况，题目要求输出保留两位小数。有个 case 的正确答案的精确值是 0.005, 按理应该输出 0.01，但你的结果可能是 0.005000000001(恭喜)，也有可能是 0.004999999999(悲剧)，如果按照 printf("%.2lf", a) 输出，那你的遭遇将和括号里的字相同。
如果 a 为正，则输出 a + eps, 否则输出 a - eps。

不要输出 -0.000

注意 double 的数据范围

$a=b$	fabs(a-b)<eps
$a\neq b$	fabs(a-b)>eps
$a<b$	a+eps<b
$a\leq b$	a<b+eps
$a>b$	a>b+eps
$a\geq b$	a+eps>b

exp	x^e
log	ln
log10	log_{10}
ceil	smallest interger $\geq$ x (watch out x<0
floor	greatest interger $\leq$ x (watch out x<0
trunc	nearest integral value close to 0
nearybyint	round to intergral, up to fegetround
round	round with halfway cases rounded away from zero

2.10.6 trigonometric functions

	input	output
sin	radian	$[-1,+1]$
cos	radian	$[-1,+1]$
tan	radian	$(-\infty,+\infty)$
asin	$[-1,+1]$	$[-\frac{\pi}{2},+\frac{\pi}{2}]$
acos	$[-1,+1]$	$[0,\pi]$
atan	$(-\infty,\infty)$	$[-\frac{\pi}{2},+\frac{\pi}{2}]$
atan2	(y,x)	$\tan(\frac{y}{x})\in[-\pi,+\pi]$ (watch out if x=y=0)

2.10.7 round

- cpp: 四舍六入五留双
  1. 当尾数小于或等于 4 时，直接将尾数舍去
  2. 当尾数大于或等于 6 时，将尾数舍去并向前一位进位
  3. 当尾数为 5，而尾数后面的数字均为 0 时，应看尾数“5”的前一位：若前一位数字此时为奇数，就应向前进一位；若前一位数字此时为偶数，则应将尾数舍去。数字“0”在此时应被视为偶数
  4. 当尾数为 5，而尾数“5”的后面还有任何不是 0 的数字时，无论前一位在此时为奇数还是偶数，也无论“5”后面不为 0 的数字在哪一位上，都应向前进一位
- java: add 0.5,then floor

2.10.8 rotation matrix

original matrix:
$\begin{bmatrix}x\\y\end{bmatrix}$
$\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$
3-dimension:
$\begin{bmatrix}x\\y\\z\end{bmatrix}$

$$R_x(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$

$$R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation by unit vector $v = (x, y, z)$:

$$\begin{bmatrix} \cos\theta + (1-\cos\theta)x^2 & (1-\cos\theta)xy - (\sin\theta)z & (1-\cos\theta)xz + (\sin\theta)y \\ (1-\cos\theta)yx + (\sin\theta)z & \cos\theta + (1-\cos\theta)y^2 & (1-\cos\theta)yz - (\sin\theta)x \\ (1-\cos\theta)zx - (\sin\theta)y & (1-\cos\theta)zy + (\sin\theta)x & \cos\theta + (1-\cos\theta)z^2 \end{bmatrix}$$

we use transform matrix muliply our original matrix

and we can presetation a transformation as a 4×4 matrix:

$$\begin{bmatrix} a_{11} & a_{12} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{42} & a_{44} \end{bmatrix}$$

Matrix $\begin{bmatrix} a_{11} & a_{12} & a_{12} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{32} \end{bmatrix}$ presetation the transformation as same as 3×3 matrx.

Matrix $\begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{bmatrix}$ as translation.

Matrix $\begin{bmatrix} a_{41} & a_{42} & a_{43} \end{bmatrix}$ as projection.

Matrix $\begin{bmatrix} a_{44} \end{bmatrix}$ as scale.

original Matrix:

$$\begin{bmatrix} x \\ y \\ z \\ Scale \end{bmatrix}$$

2.11 rotating caliper

```
//最远点对

inline double go()
{
    l=ans=0;
    for(i=0;i<n;++i)
    {
        tl=pnt[(i+1)%n]-pnt[i];
        while(abs(tl.cross(pnt[(l+1)%n]-pnt[i]))>=abs(tl.cross(
             pnt[l]-pnt[i])))
            l=(l+1)%n;
        ans=std::max(ans,std::max(dist(pnt[l],pnt[i]),dist(pnt[
             l],pnt[(i+1)%n])));
    }
    return ans;
}

//两凸包最近距离
double go()
{
    sq=sp=0;
    for(i=1;i<ch[1].size();++i)
        if(ch[1][sq]<ch[1][i])
            sq=i;
    tp=sp;
    tq=sq;
    ans=(ch[0][sp]-ch[1][sq]).len();
    do
    {
        a1=ch[0][sp];
        a2=ch[0][(sp+1)%ch[0].size()];
        b1=ch[1][sq];
        b2=ch[1][(sq+1)%ch[1].size()];
        tpv=b1-(b2-a1);
        tpv.x = b1.x - (b2.x - a1.x);
        tpv.y = b1.y - (b2.y - a1.y);
        len=(tpv-a1).cross(a2-a1);
        if(fabs(len)<eps)
        {
            ans=std::min(ans,p2l(a1,b1,b2));
            ans=std::min(ans,p2l(a2,b1,b2));
            ans=std::min(ans,p2l(b1,a1,a2));
            ans=std::min(ans,p2l(b2,a1,a2));
            sp=(sp+1)%ch[0].size();
            sq=(sq+1)%ch[1].size();
        }
        else
        if(len<-eps)
        {
            ans=std::min(ans,p2l(b1,a1,a2));
            sp=(sp+1)%ch[0].size();
        }
        else
        {
            ans=std::min(ans,p2l(a1,b1,b2));
            sq=(sq+1)%ch[1].size();
        }
    }while(tp!=sp || tq!=sq);
    return ans;
}

//外接矩形 by mzry
inline void solve()
{
    resa = resb = 1e100;
    double dis1,dis2;
    Point xp[4];
    Line l[4];
    int a,b,c,d;
    int sa,sb,sc,sd;
    a = b = c = d = 0;
    sa = sb = sc = sd = 0;
    Point va,vb,vc,vd;
    for (a = 0; a < n; a++)
    {
        va = Point(p[a],p[(a+1)%n]);
        vc = Point(-va.x,-va.y);
        vb = Point(-va.y,va.x);
        vd = Point(-vb.x,-vb.y);
        if (sb < sa)
        {
            b = a;
            sb = sa;
        }
        while (xmult(vb,Point(p[b],p[(b+1)%n])) < 0)
        {
            b = (b+1)%n;
            sb++;
        }
        if (sc < sb)
        {
            c = b;
            sc = sb;
        }
        while (xmult(vc,Point(p[c],p[(c+1)%n])) < 0)
        {
            c = (c+1)%n;
            sc++;
        }
        if (sd < sc)
        {
            d = c;
            sd = sc;
        }
        while (xmult(vd,Point(p[d],p[(d+1)%n])) < 0)
        {
            d = (d+1)%n;
            sd++;
        }

        //卡在 p[a],p[b],p[c],p[d] 上
        sa++;
    }
}
```

合并凸包

给定凸多边形 P = { p(1) , ... , p(m) } 和 Q = { q(1) , ... , q(n) }，一个点对 (p(i), q(j)) 形成 P 和 Q 之间的桥当且仅当：

- (p(i), q(j)) 形成一个并踵点对。
- p(i-1), p(i+1), q(j-1), q(j+1) 都位于由 (p(i), q(j)) 组成的线的同一侧。

假设多边形以标准形式给出并且顶点是以顺时针序排列，算法如下：

1. 分别计算 P 和 Q 拥有最大 y 坐标的顶点。如果存在不止一个这样的点，取 x 坐标最大的。
2. 构造这些点的遂平切线，以多边形处于其右侧为正方向（因此他们指向 x 轴正方向）。
3. 同时顺时针旋转两条切线直到其中一条与边相交。得到一个新的并踵点对 (p(i), q(j)) 。对于平行边的情况，得到三个并踵点对。

4. 对于所有有效的并踵点对 (p(i), q(j))：判定 p(i-1), p(i+1), q(j-1), q(j+1) 是否都位于连接点 (p(i), q(j)) 形成的线的同一侧。如果是，这个并踵点对就形成了一个桥，并标记他。

5. 重复执行步骤 3 和步骤 4 直到切线回到他们原来的位置。

6. 所有可能的桥此时都已经确定了。通过连续连接桥间对应的凸包链来构造合并凸包。

上述的结论确定了算法的正确性。运行时间受步骤 1，5，6 约束。他们都为 O(N) 运行时间（N 是顶点总数）。因此算法拥有线性的时间复杂度。

一个凸多边形间的桥实际上确定了另一个有用的概念：多边形间公切线。同时，桥也是计算凸多边形交的算法核心。

临界切线

1. 计算 P 上 y 坐标值最小的顶点（称为 yminP ）和 Q 上 y 坐标值最大的顶点（称为 ymaxQ）。

2. 为多边形在 yminP 和 ymaxQ 处构造两条切线 LP 和 LQ 使得他们对应的多边形位于他们的右侧。此时 LP 和 LQ 拥有不同的方向，并且 yminP 和 ymaxQ 成为了多边形间的一个对踵点对。

3. 令 p(i)= yminP，q(j)= ymaxQ。(p(i), q(j)) 构成了多边形间的一个对踵点对。检测是否有 p(i-1),p(i+1) 在线 (p(i), q(j)) 的一侧，并且 q(j-1),q(j+1) 在另一侧。如果成立，(p(i), q(j)) 确定了一条 CS 线。

4. 旋转这两条线，直到其中一条和其对应的多边形的边重合。

5. 一个新的对踵点对确定了。如果两条线都与边重合，总共三对对踵点对（原先的顶点和新的顶点的组合）需要考虑。对于所有的对踵点对，执行上面的测试。

6. 重复执行步骤 4 和步骤 5，直到新的点对为 (yminP,ymaxQ)。

7. 输出 CS 线。

最小/最大周长/面积外接矩形

1. 计算全部四个多边形的端点，称之为 xminP，xmaxP，yminP，ymaxP。

2. 通过四个点构造 P 的四条切线。他们确定了两个“卡壳”集合。

3. 如果一条（或两条）线与一条边重合，那么计算由四条线决定的矩形的面积，并且保存为当前最小值。否则将当前最小值定义为无穷大。

4. 顺时针旋转线直到其中一条和多边形的一条边重合。

5. 计算新矩形的周长/面积，并且和当前最小值比较。如果小于当前最小值则更新，并保存确定最小值的矩形信息。

6. 重复步骤 4 和步骤 5，直到线旋转过的角度大于 90 度。

7. 输出外接矩形的最小周长。

2.12 shit

```
struct pv
{
    double x,y;
    pv(double a=0,double b=0):x(a),y(b){}
    inline pv operator+(const pv &i)const
    {
        return pv(x+i.x,y+i.y);
    }
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    inline bool operator ==(const pv &i)const
    {
        return fabs(x-i.x)<eps && fabs(y-i.y)<eps;
    }
    inline bool operator<(const pv &i)const
    {
        return y==i.y?x<i.x:y<i.y;
    }
    inline double cross(const pv &i)const
    {
        return x*i.y-y*i.x;
    }
    inline double dot(const pv &i)const
    {
        return x*i.x+y*i.y;
    }
    inline double len()
    {
        return hypot(x,y);
    }
};

struct line
{
    pv pnt[2];
    line(double a,double b,double c) // a*x + b*y + c = 0
    {
#define maxl 1e2 //preciseness should not be too high ( compare
      with eps )
        if(fabs(b)>eps)
        {
            pnt[0]=pv(maxl,(c+a*maxl)/(-b));
            pnt[1]=pv(-maxl,(c-a*maxl)/(-b));
        }
        else
        {
            pnt[0]=pv(-c/a,maxl);
            pnt[1]=pv(-c/a,-maxl);
        }
#undef maxl
    }
    pv cross(const line &v)const
    {
        double a=(v.pnt[1]-v.pnt[0]).cross(pnt[0]-v.pnt[0]);
        double b=(v.pnt[1]-v.pnt[0]).cross(pnt[1]-v.pnt[0]);
        return pv((pnt[0].x*b-pnt[1].x*a)/(b-a),(pnt[0].y*b-pnt
             [1].y*a)/(b-a));
    }
};

inline std::pair<pv,double> getcircle(const pv &a,const pv &b,
     const pv &c)
{
    static pv ct;
    ct=line(2*(b.x-a.x),2*(b.y-a.y),a.len()-b.len()).cross(line
         (2*(c.x-b.x),2*(c.y-b.y),b.len()-c.len()));
    return std::make_pair(ct,sqrt((ct-a).len()));
}

//sort with polar angle
inline bool cmp(const Point& a,const Point& b)
{
    if (a.y*b.y <= 0)
    {
        if (a.y > 0 || b.y > 0)
            return a.y < b.y;
        if (a.y == 0 && b.y == 0)
            return a.x < b.x;
    }
    return a.cross(b) > 0;
}

//graham
inline bool com(const pv &a,const pv &b)
{
    static double t;
    if(fabs(t=(a-pnt[0]).cross(b-pnt[0]))>eps)
        return t>0;
    return (a-pnt[0]).len()<(b-pnt[0]).len();
}
```

```
inline void graham(std::vector<pv> &ch,const int n)
{
    std::nth_element(pnt,pnt,pnt+n);
    std::sort(pnt+1,pnt+n,com);
    ch.resize(0);
    ch.push_back(pnt[0]);
    ch.push_back(pnt[1]);
    static int i;
    for(i=2;i<n;++i)
        if(fabs((pnt[i]-ch[0]).cross(ch[1]-ch[0]))>eps)
        {
            ch.push_back(pnt[i++]);
            break;
        }
        else
            ch.back()=pnt[i];
    for(;i<n;++i)
    {
        while((ch.back()-ch[ch.size()-2]).cross(pnt[i]-ch[ch.
             size()-2])<eps)
            ch.pop_back();
        ch.push_back(pnt[i]);
    }
}
```

3 Geometry/tmp

3.1 test

```
//polygon
#include <stdlib.h>
#include <math.h>
#define MAXN 1000
#define offset 10000
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
#define _sign(x) ((x)>eps?1:((x)<-eps?2:0))
struct point{double x,y;};
struct line{point a,b;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
//判定凸多边形，顶点按顺时针或逆时针给出，允许相邻边共线
int is_convex(int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[1]|s[2];
}
//判定凸多边形，顶点按顺时针或逆时针给出，不允许相邻边共线
int is_convex_v2(int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[0]&&s[1]|s[2];
}
//判点在凸多边形内或多边形边上，顶点按顺时针或逆时针给出
int inside_convex(point q,int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[1]|s[2];
}
//判点在凸多边形内，顶点按顺时针或逆时针给出，在多边形边上返回 0
int inside_convex_v2(point q,int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[0]&&s[1]|s[2];
}
//判点在任意多边形内，顶点按顺时针或逆时针给出
//on_edge 表示点在多边形边上时的返回值,offset 为多边形坐标上限
int inside_polygon(point q,int n,point* p,int on_edge=1)
{
    point q2;
    int i=0,count;
    while (i<n)
        for (count=i=0,q2.x=rand()+offset,q2.y=rand()+offset;i<
             n;i++)
            if
                (zero(xmult(q,p[i],p[(i+1)%n]))&&(p[i].x-q.x)*(
                     p[(i+1)%n].x-q.x)<eps&&(p[i].y-q.y)*(p[(i
                     +1)%n].y-q.y)<eps)
                    return on_edge;
            else if (zero(xmult(q,q2,p[i])))
                break;
            else if
                (xmult(q,p[i],q2)*xmult(q,p[(i+1)%n],q2)<-eps&&
                     xmult(p[i],q,p[(i+1)%n])*xmult(p[i],q2,p[(
                          i+1)%n])<-eps)
                        count++;
    return count&1;
}
inline int opposite_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}
inline int dot_online_in(point p,point l1,point l2)
{
    return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1
         .y-p.y)*(l2.y-p.y)<eps;
}
//判线段在任意多边形内，顶点按顺时针或逆时针给出，与边界相交返回 1
int inside_polygon(point l1,point l2,int n,point* p)
{
    point t[MAXN],tt;
    int i,j,k=0;
    if (!inside_polygon(l1,n,p)||!inside_polygon(l2,n,p))
        return 0;
    for (i=0;i<n;i++)
        if (opposite_side(l1,l2,p[i],p[(i+1)%n])&&opposite_side
             (p[i],p[(i+1)%n],l1,l2))
            return 0;
        else if (dot_online_in(l1,p[i],p[(i+1)%n]))
            t[k++]=l1;
        else if (dot_online_in(l2,p[i],p[(i+1)%n]))
            t[k++]=l2;
        else if (dot_online_in(p[i],l1,l2))
            t[k++]=p[i];
    for (i=0;i<k;i++)
        for (j=i+1;j<k;j++)
        {
            tt.x=(t[i].x+t[j].x)/2;
            tt.y=(t[i].y+t[j].y)/2;
            if (!inside_polygon(tt,n,p))
                return 0;
        }
    return 1;
}
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-
         v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.
             x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}
point barycenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b=c;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b=b;
    return intersection(u,v);
}
//多边形重心
point barycenter(int n,point* p)
{
    point ret,t;
    double t1=0,t2;
    int i;
    ret.x=ret.y=0;
    for (i=1;i<n-1;i++)
        if (fabs(t2=xmult(p[0],p[i],p[i+1]))>eps)
        {
            t=barycenter(p[0],p[i],p[i+1]);
            ret.x+=t.x*t2;
            ret.y+=t.y*t2;
            t1+=t2;
        }
    if (fabs(t1)>eps)
        ret.x/=t1,ret.y/=t1;
    return ret;
}


//cut polygon
//多边形切割
//可用于半平面交
#define MAXN 100
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
int same_side(point p1,point p2,point l1,point l2)
```

```
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//将多边形沿 l1,l2 确定的直线切割在 side 侧切割，保证 l1,l2,side 不共线
void polygon_cut(int& n,point* p,point l1,point l2,point side)
{
    point pp[100];
    int m=0,i;
    for (i=0;i<n;i++)
    {
        if (same_side(p[i],side,l1,l2))
            pp[m++]=p[i];
        if
            (!same_side(p[i],p[(i+1)%n],l1,l2)&&!(zero(xmult(p[
                i],l1,l2))&&zero(xmult(p[(i+1)%n],l1,l2))))
                pp[m++]=intersection(p[i],p[(i+1)%n],l1,l2);
    }
    for (n=i=0;i<m;i++)
        if (!i||!zero(pp[i].x-pp[i-1].x)||!zero(pp[i].y-pp[i
            -1].y))
            p[n++]=pp[i];
    if (zero(p[n-1].x-p[0].x)&&zero(p[n-1].y-p[0].y))
        n--;
    if (n<3)
        n=0;
}

//float
//浮点几何函数库
#include <math.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
struct line{point a,b;};
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,
    double y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算 dot product (P1-P0).(P2-P0)
double dmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
double dmult(double x1,double y1,double x2,double y2,double x0,
    double y0)
{
    return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}
//两点距离
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)
        );
}
double distance(double x1,double y1,double x2,double y2)
{
    return sqrt((x1-x2)*(x1-x2)+(y1-y2)*(y1-y2));
}
//判三点共线
int dots_inline(point p1,point p2,point p3)
{
    return zero(xmult(p1,p2,p3));
}
int dots_inline(double x1,double y1,double x2,double y2,double
    x3,double y3)
{
    return zero(xmult(x1,y1,x2,y2,x3,y3));
}
//判点是否在线段上，包括端点
int dot_online_in(point p,line l)
{
    return zero(xmult(p,l.a,l.b))&&(l.a.x-p.x)*(l.b.x-p.x)<eps
        &&(l.a.y-p.y)*(l.b.y-p.y)<eps;
}
int dot_online_in(point p,point l1,point l2)
{
    return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1
        .y-p.y)*(l2.y-p.y)<eps;
}
int dot_online_in(double x,double y,double x1,double y1,double
    x2,double y2)
{
    return zero(xmult(x,y,x1,y1,x2,y2))&&(x1-x)*(x2-x)<eps&&(y1
        -y)*(y2-y)<eps;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point p,line l)
{
    return
        dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a.y)
            )&&(!zero(p.x-l.b.x)||!zero(p.y-l.b.y));
}
int dot_online_ex(point p,point l1,point l2)
{
    return
        dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-l1.
            y))&&(!zero(p.x-l2.x)||!zero(p.y-l2.y));
}
int dot_online_ex(double x,double y,double x1,double y1,double
    x2,double y2)
{
    return
        dot_online_in(x,y,x1,y1,x2,y2)&&(!zero(x-x1)||!zero(y-
            y1))&&(!zero(x-x2)||!zero(y-y2));
}
//判两点在线段同侧，点在线段上返回 0
int same_side(point p1,point p2,line l)
{
    return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)>eps;
}
int same_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}
//判两点在线段异侧，点在线段上返回 0
int opposite_side(point p1,point p2,line l)
{
    return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)<-eps;
}
int opposite_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}
//判两直线平行
int parallel(line u,line v)
{
    return zero((u.a.x-u.b.x)*(v.a.y-v.b.y)-(v.a.x-v.b.x)*(u.a.
        y-u.b.y));
}
int parallel(point u1,point u2,point v1,point v2)
{
    return zero((u1.x-u2.x)*(v1.y-v2.y)-(v1.x-v2.x)*(u1.y-u2.y)
        );
}
//判两直线垂直
int perpendicular(line u,line v)
{
    return zero((u.a.x-u.b.x)*(v.a.x-v.b.x)+(u.a.y-u.b.y)*(v.a.
        y-v.b.y));
}
int perpendicular(point u1,point u2,point v1,point v2)
{
    return zero((u1.x-u2.x)*(v1.x-v2.x)+(u1.y-u2.y)*(v1.y-v2.y)
        );
}
//判两线段相交，包括端点和部分重合
int intersect_in(line u,line v)
{
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||
        dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2)
{
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2)
            ;
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||
            dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
                2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line u,line v)
{
    return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2)
{
    return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,
        u2);
}
//计算两直线交点，注意事先判断直线是否平行!
```

```
//线段交点请另外判线段相交 (同时还是要判断是否平行!)
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-
         v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.
            x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//点到直线上的最近点
point ptoline(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    return intersection(p,t,l.a,l.b);
}
point ptoline(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    return intersection(p,t,l1,l2);
}
//点到直线距离
double disptoline(point p,line l)
{
    return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoline(point p,point l1,point l2)
{
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
double disptoline(double x,double y,double x1,double y1,double
     x2,double y2)
{
    return fabs(xmult(x,y,x1,y1,x2,y2))/distance(x1,y1,x2,y2);
}
//点到线段上的最近点
point ptoseg(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
        return distance(p,l.a)<distance(p,l.b)?l.a:l.b;
    return intersection(p,t,l.a,l.b);
}
point ptoseg(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
        return distance(p,l1)<distance(p,l2)?l1:l2;
    return intersection(p,t,l1,l2);
}
//点到线段距离
double disptoseg(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
        return distance(p,l.a)<distance(p,l.b)?distance(p,l.a):
            distance(p,l.b);
    return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoseg(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
        return distance(p,l1)<distance(p,l2)?distance(p,l1):
            distance(p,l2);
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
//矢量 V 以 P 为顶点逆时针旋转 angle 并放大 scale 倍
point rotate(point v,point p,double angle,double scale)
{
    point ret=p;
    v.x-=p.x,v.y-=p.y;
    p.x=scale*cos(angle);
    p.y=scale*sin(angle);
    ret.x+=v.x*p.x-v.y*p.y;
    ret.y+=v.x*p.y+v.y*p.x;
    return ret;
}
```

```
//area
#include <math.h>
struct point{double x,y;};
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,
     double y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算三角形面积, 输入三顶点
double area_triangle(point p1,point p2,point p3)
{
    return fabs(xmult(p1,p2,p3))/2;
}
double area_triangle(double x1,double y1,double x2,double y2,
     double x3,double y3)
{
    return fabs(xmult(x1,y1,x2,y2,x3,y3))/2;
}
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//计算三角形面积, 输入三边长
double area_triangle(double a,double b,double c)
{
    double s=(a+b+c)/2;
    return sqrt(s*(s-a)*(s-b)*(s-c));
}
//计算多边形面积, 顶点按顺时针或逆时针给出
double area_polygon(int n,point* p)
{
    double s1=0,s2=0;
    int i;
    for (i=0;i<n;i++)
        s1+=p[(i+1)%n].y*p[i].x,s2+=p[(i+1)%n].y*p[(i+2)%n].x;
    return fabs(s1-s2)/2;
}

//surface of ball
#include <math.h>
const double pi=acos(-1);
//计算圆心角 lat 表示纬度,-90<=w<=90,lng 表示经度
//返回两点所在大圆劣弧对应圆心角,0<=angle<=pi
double angle(double lng1,double lat1,double lng2,double lat2)
{
    double dlng=fabs(lng1-lng2)*pi/180;
    while (dlng>=pi+pi)
        dlng-=pi+pi;
    if (dlng>pi)
        dlng=pi+pi-dlng;
    lat1*=pi/180,lat2*=pi/180;
    return acos(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*sin(
         lat2));
}
//计算距离,r 为球半径
double line_dist(double r,double lng1,double lat1,double lng2,
     double lat2)
{
    double dlng=fabs(lng1-lng2)*pi/180;
    while (dlng>=pi+pi)
        dlng-=pi+pi;
    if (dlng>pi)
        dlng=pi+pi-dlng;
    lat1*=pi/180,lat2*=pi/180;
    return r*sqrt(2-2*(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*
         sin(lat2)));
}
//计算球面距离,r 为球半径
inline double sphere_dist(double r,double lng1,double lat1,
     double lng2,double lat2)
{
    return r*angle(lng1,lat1,lng2,lat2);
}

//triangle
#include <math.h>
struct point{double x,y;};
struct line{point a,b;};
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)
         );
}
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-
         v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.
            x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
```

```
    return ret;
}
//外心
point circumcenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b.x=u.a.x-a.y+b.y;
    u.b.y=u.a.y+a.x-b.x;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b.x=v.a.x-a.y+c.y;
    v.b.y=v.a.y+a.x-c.x;
    return intersection(u,v);
}
//内心
point incenter(point a,point b,point c)
{
    line u,v;
    double m,n;
    u.a=a;
    m=atan2(b.y-a.y,b.x-a.x);
    n=atan2(c.y-a.y,c.x-a.x);
    u.b.x=u.a.x+cos((m+n)/2);
    u.b.y=u.a.y+sin((m+n)/2);
    v.a=b;
    m=atan2(a.y-b.y,a.x-b.x);
    n=atan2(c.y-b.y,c.x-b.x);
    v.b.x=v.a.x+cos((m+n)/2);
    v.b.y=v.a.y+sin((m+n)/2);
    return intersection(u,v);
}
//垂心
point perpencenter(point a,point b,point c)
{
    line u,v;
    u.a=c;
    u.b.x=u.a.x-a.y+b.y;
    u.b.y=u.a.y+a.x-b.x;
    v.a=b;
    v.b.x=v.a.x-a.y+c.y;
    v.b.y=v.a.y+a.x-c.x;
    return intersection(u,v);
}
//重心
//到三角形三顶点距离的平方和最小的点
//三角形内到三边距离之积最大的点
point barycenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b=c;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b=b;
    return intersection(u,v);
}
//费马点
//到三角形三顶点距离之和最小的点
point fermentpoint(point a,point b,point c)
{
    point u,v;
    double step=fabs(a.x)+fabs(a.y)+fabs(b.x)+fabs(b.y)+fabs(c.
        x)+fabs(c.y);
    int i,j,k;
    u.x=(a.x+b.x+c.x)/3;
    u.y=(a.y+b.y+c.y)/3;
    while (step>1e-10)
        for (k=0;k<10;step/=2,k++)
            for (i=-1;i<=1;i++)
                for (j=-1;j<=1;j++)
                {
                    v.x=u.x+step*i;
                    v.y=u.y+step*j;
                    if
                        (distance(u,a)+distance(u,b)+distance(u
                            ,c)>distance(v,a)+distance(v,b)+
                            distance(v,c))
                            u=v;
                }
    return u;
}

//Pick's
#define abs(x) ((x)>0?(x):-(x))
struct point{int x,y;};
int gcd(int a,int b)
{
    return b?gcd(b,a%b):a;
}
//多边形上的网格点个数
int grid_onedge(int n,point* p)
{
    int i,ret=0;
    for (i=0;i<n;i++)
        ret+=gcd(abs(p[i].x-p[(i+1)%n].x),abs(p[i].y-p[(i+1)%n
            ].y));
    return ret;
}
//多边形内的网格点个数
int grid_inside(int n,point* p)
{
    int i,ret=0;
    for (i=0;i<n;i++)
        ret+=p[(i+1)%n].y*(p[i].x-p[(i+2)%n].x);
    return (abs(ret)-grid_onedge(n,p))/2+1;
}

//circle
#include <math.h>
#define eps 1e-8
struct point{double x,y;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)
        );
}
double disptoline(point p,point l1,point l2)
{
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//判直线和圆相交，包括相切
int intersect_line_circle(point c,double r,point l1,point l2)
{
    return disptoline(c,l1,l2)<r+eps;
}
//判线段和圆相交，包括端点和相切
int intersect_seg_circle(point c,double r,point l1,point l2)
{
    double t1=distance(c,l1)-r,t2=distance(c,l2)-r;
    point t=c;
    if (t1<eps||t2<eps)
        return t1>-eps||t2>-eps;
    t.x+=l1.y-l2.y;
    t.y+=l2.x-l1.x;
    return xmult(l1,c,t)*xmult(l2,c,t)<eps&&disptoline(c,l1,l2)
        -r<eps;
}
//判圆和圆相交，包括相切
int intersect_circle_circle(point c1,double r1,point c2,double
    r2)
{
    return distance(c1,c2)<r1+r2+eps&&distance(c1,c2)>fabs(r1-
        r2)-eps;
}
//计算圆上到点 p 最近点，如 p 与圆心重合，返回 p 本身
point dot_to_circle(point c,double r,point p)
{
    point u,v;
    if (distance(p,c)<eps)
        return p;
    u.x=c.x+r*fabs(c.x-p.x)/distance(c,p);
    u.y=c.y+r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)
        <0?-1:1);
    v.x=c.x-r*fabs(c.x-p.x)/distance(c,p);
    v.y=c.y-r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)
        <0?-1:1);
    return distance(u,p)<distance(v,p)?u:v;
}
//计算直线与圆的交点，保证直线与圆有交点
//计算线段与圆的交点可用这个函数后判点是否在线段上
void intersection_line_circle(point c,double r,point l1,point
    l2,point& p1,point& p2)
{
    point p=c;
    double t;
    p.x+=l1.y-l2.y;
    p.y+=l2.x-l1.x;
    p=intersection(p,c,l1,l2);
    t=sqrt(r*r-distance(p,c)*distance(p,c))/distance(l1,l2);
    p1.x=p.x+(l2.x-l1.x)*t;
    p1.y=p.y+(l2.y-l1.y)*t;
    p2.x=p.x-(l2.x-l1.x)*t;
    p2.y=p.y-(l2.y-l1.y)*t;
}
```

```
//计算圆与圆的交点，保证圆与圆有交点，圆心不重合
void intersection_circle_circle(point c1,double r1,point c2,
     double r2,point& p1,point& p2)
{
    point u,v;
    double t;
    t=(1+(r1*r1-r2*r2)/distance(c1,c2)/distance(c1,c2))/2;
    u.x=c1.x+(c2.x-c1.x)*t;
    u.y=c1.y+(c2.y-c1.y)*t;
    v.x=u.x+c1.y-c2.y;
    v.y=u.y-c1.x+c2.x;
    intersection_line_circle(c1,r1,u,v,p1,p2);
}

//integer
//整数几何函数库
//注意某些情况下整数运算会出界!
#define sign(a) ((a)>0?1:(((a)<0?-1:0)))
struct point{int x,y;};
struct line{point a,b;};
//计算 cross product (P1-P0)x(P2-P0)
int xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
int xmult(int x1,int y1,int x2,int y2,int x0,int y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算 dot product (P1-P0).(P2-P0)
int dmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
int dmult(int x1,int y1,int x2,int y2,int x0,int y0)
{
    return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}
//判三点共线
int dots_inline(point p1,point p2,point p3)
{
    return !xmult(p1,p2,p3);
}
int dots_inline(int x1,int y1,int x2,int y2,int x3,int y3)
{
    return !xmult(x1,y1,x2,y2,x3,y3);
}
//判点是否在线段上，包括端点和部分重合
int dot_online_in(point p,line l)
{
    return !xmult(p,l.a,l.b)&&(l.a.x-p.x)*(l.b.x-p.x)<=0&&(l.a.
        y-p.y)*(l.b.y-p.y)<=0;
}
int dot_online_in(point p,point l1,point l2)
{
    return !xmult(p,l1,l2)&&(l1.x-p.x)*(l2.x-p.x)<=0&&(l1.y-p.y
        )*(l2.y-p.y)<=0;
}
int dot_online_in(int x,int y,int x1,int y1,int x2,int y2)
{
    return !xmult(x,y,x1,y1,x2,y2)&&(x1-x)*(x2-x)<=0&&(y1-y)*(
        y2-y)<=0;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point p,line l)
{
    return dot_online_in(p,l)&&(p.x!=l.a.x||p.y!=l.a.y)&&(p.x!=
        l.b.x||p.y!=l.b.y);
}
int dot_online_ex(point p,point l1,point l2)
{
    return dot_online_in(p,l1,l2)&&(p.x!=l1.x||p.y!=l1.y)&&(p.x
        !=l2.x||p.y!=l2.y);
}
int dot_online_ex(int x,int y,int x1,int y1,int x2,int y2)
{
    return dot_online_in(x,y,x1,y1,x2,y2)&&(x!=x1||y!=y1)&&(x!=
        x2||y!=y2);
}
//判两点在直线同侧，点在直线上返回 0
int same_side(point p1,point p2,line l)
{
    return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)>0;
}
int same_side(point p1,point p2,point l1,point l2)
{
    return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)>0;
}
//判两点在直线异侧，点在直线上返回 0
int opposite_side(point p1,point p2,line l)
{
    return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)<0;
}
int opposite_side(point p1,point p2,point l1,point l2)
{
    return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)<0;
}
//判两直线平行
int parallel(line u,line v)
{
    return (u.a.x-u.b.x)*(v.a.y-v.b.y)==(v.a.x-v.b.x)*(u.a.y-u.
        b.y);
}
int parallel(point u1,point u2,point v1,point v2)
{
    return (u1.x-u2.x)*(v1.y-v2.y)==(v1.x-v2.x)*(u1.y-u2.y);
}
//判两直线垂直
int perpendicular(line u,line v)
{
    return (u.a.x-u.b.x)*(v.a.x-v.b.x)==-(u.a.y-u.b.y)*(v.a.y-v
        .b.y);
}
int perpendicular(point u1,point u2,point v1,point v2)
{
    return (u1.x-u2.x)*(v1.x-v2.x)==-(u1.y-u2.y)*(v1.y-v2.y);
}
//判两线段相交，包括端点和部分重合
int intersect_in(line u,line v)
{
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||
        dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2)
{
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2)
            ;
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||
            dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
                2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line u,line v)
{
    return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2)
{
    return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,
        u2);
}
```

3.2 tmp

```
const double inf=1e20;
const int maxp=1111;
int dblcmp(double d)
{
    if (fabs(d)<eps)return 0;
    return d>eps?1:-1;
}
inline double sqr(double x){return x*x;}
struct point
{
    double x,y;
    point(){}
    point(double _x,double _y):
        x(_x),y(_y){};
    void input()
    {
        scanf("%lf%lf",&x,&y);
    }
    void output()
    {
        printf("%.2f␣%.2f\n",x,y);
    }
    bool operator==(point a)const
    {
        return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0;
    }
    bool operator<(point a)const
    {
        return dblcmp(a.x-x)==0?dblcmp(y-a.y)<0:x<a.x;
    }
    double len()
    {
        return hypot(x,y);
    }
    double len2()
    {
        return x*x+y*y;
    }
    double distance(point p)
    {
        return hypot(x-p.x,y-p.y);
    }
```

```
    point add(point p)
    {
        return point(x+p.x,y+p.y);
    }
    point sub(point p)
    {
        return point(x-p.x,y-p.y);
    }
    point mul(double b)
    {
        return point(x*b,y*b);
    }
    point div(double b)
    {
        return point(x/b,y/b);
    }
    double dot(point p)
    {
        return x*p.x+y*p.y;
    }
    double det(point p)
    {
        return x*p.y-y*p.x;
    }
    double rad(point a,point b)
    {
        point p=*this;
        return fabs(atan2(fabs(a.sub(p).det(b.sub(p))),a.sub(p)
             .dot(b.sub(p))));
    }
    point trunc(double r)
    {
        double l=len();
        if (!dblcmp(l))return *this;
        r/=l;
        return point(x*r,y*r);
    }
    point rotleft()
    {
        return point(-y,x);
    }
    point rotright()
    {
        return point(y,-x);
    }
    point rotate(point p,double angle)//绕点逆时针旋转角度pangle
    {
        point v=this->sub(p);
        double c=cos(angle),s=sin(angle);
        return point(p.x+v.x*c-v.y*s,p.y+v.x*s+v.y*c);
    }
};
struct line
{
    point a,b;
    line(){}
    line(point _a,point _b)
    {
        a=_a;
        b=_b;
    }
    bool operator==(line v)
    {
        return (a==v.a)&&(b==v.b);
    }
    //倾斜角angle
    line(point p,double angle)
    {
        a=p;
        if (dblcmp(angle-pi/2)==0)
        {
            b=a.add(point(0,1));
        }
        else
        {
            b=a.add(point(1,tan(angle)));
        }
    }
    //ax+by+c=0
    line(double _a,double _b,double _c)
    {
        if (dblcmp(_a)==0)
        {
            a=point(0,-_c/_b);
            b=point(1,-_c/_b);
        }
        else if (dblcmp(_b)==0)
        {
            a=point(-_c/_a,0);
            b=point(-_c/_a,1);
        }
        else
        {
            a=point(0,-_c/_b);
            b=point(1,(-_c-_a)/_b);
        }
    }
    void input()
    {
        a.input();
        b.input();
    }
    void adjust()
    {
        if (b<a)swap(a,b);
    }
    double length()
    {
        return a.distance(b);
    }
    double angle()//直线倾斜角 0<=angle<180
    {
        double k=atan2(b.y-a.y,b.x-a.x);
        if (dblcmp(k)<0)k+=pi;
        if (dblcmp(k-pi)==0)k-=pi;
        return k;
    }
    //点和线段关系
    //1 在逆时针
    //2 在顺时针
    //3 平行
    int relation(point p)
    {
        int c=dblcmp(p.sub(a).det(b.sub(a)));
        if (c<0)return 1;
        if (c>0)return 2;
        return 3;
    }
    bool pointonseg(point p)
    {
        return dblcmp(p.sub(a).det(b.sub(a)))==0&&dblcmp(p.sub(
            a).dot(p.sub(b)))<=0;
    }
    bool parallel(line v)
    {
        return dblcmp(b.sub(a).det(v.b.sub(v.a)))==0;
    }
    //2 规范相交
    //1 非规范相交
    //0 不相交
    int segcrossseg(line v)
    {
        int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
        int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
        int d3=dblcmp(v.b.sub(v.a).det(a.sub(v.a)));
        int d4=dblcmp(v.b.sub(v.a).det(b.sub(v.a)));
        if ((d1^d2)==-2&&(d3^d4)==-2)return 2;
        return (d1==0&&dblcmp(v.a.sub(a).dot(v.a.sub(b)))<=0||
                d2==0&&dblcmp(v.b.sub(a).dot(v.b.sub(b)))<=0||
                d3==0&&dblcmp(a.sub(v.a).dot(a.sub(v.b)))<=0||
                d4==0&&dblcmp(b.sub(v.a).dot(b.sub(v.b)))<=0);
    }
    int linecrossseg(line v)//*this seg v line
    {
        int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
        int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
        if ((d1^d2)==-2)return 2;
        return (d1==0||d2==0);
    }
    //0 平行
    //1 重合
    //2 相交
    int linecrossline(line v)
    {
        if ((*this).parallel(v))
        {
            return v.relation(a)==3;
        }
        return 2;
    }
    point crosspoint(line v)
    {
        double a1=v.b.sub(v.a).det(a.sub(v.a));
        double a2=v.b.sub(v.a).det(b.sub(v.a));
        return point((a.x*a2-b.x*a1)/(a2-a1),(a.y*a2-b.y*a1)/(
            a2-a1));
    }
    double dispointtoline(point p)
    {
        return fabs(p.sub(a).det(b.sub(a)))/length();
    }
    double dispointtoseg(point p)
    {
        if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.sub(a).
            dot(b.sub(a)))<0)
        {
            return min(p.distance(a),p.distance(b));
        }
        return dispointtoline(p);
    }
```

```
    point lineprog(point p)
    {
        return a.add(b.sub(a).mul(b.sub(a).dot(p.sub(a))/b.sub(
            a).len2()));
    }
    point symmetrypoint(point p)
    {
        point q=lineprog(p);
        return point(2*q.x-p.x,2*q.y-p.y);
    }
};
struct circle
{
    point p;
    double r;
    circle(){}
    circle(point _p,double _r):
        p(_p),r(_r){};
    circle(double x,double y,double _r):
        p(point(x,y)),r(_r){};
    circle(point a,point b,point c)//三角形的外接圆
    {
        p=line(a.add(b).div(2),a.add(b).div(2).add(b.sub(a).
            rotleft())).crosspoint(line(c.add(b).div(2),c.add(
            b).div(2).add(b.sub(c).rotleft())));
        r=p.distance(a);
    }
    circle(point a,point b,point c,bool t)//三角形的内切圆
    {
        line u,v;
        double m=atan2(b.y-a.y,b.x-a.x),n=atan2(c.y-a.y,c.x-a.x
            );
        u.a=a;
        u.b=u.a.add(point(cos((n+m)/2),sin((n+m)/2)));
        v.a=b;
        m=atan2(a.y-b.y,a.x-b.x),n=atan2(c.y-b.y,c.x-b.x);
        v.b=v.a.add(point(cos((n+m)/2),sin((n+m)/2)));
        p=u.crosspoint(v);
        r=line(a,b).dispointtoseg(p);
    }
    void input()
    {
        p.input();
        scanf("%lf",&r);
    }
    void output()
    {
        printf("%.2lf %.2lf %.2lf\n",p.x,p.y,r);
    }
    bool operator==(circle v)
    {
        return ((p==v.p)&&dblcmp(r-v.r)==0);
    }
    bool operator<(circle v)const
    {
        return ((p<v.p)||(p==v.p)&&dblcmp(r-v.r)<0);
    }
    double area()
    {
        return pi*sqr(r);
    }
    double circumference()
    {
        return 2*pi*r;
    }
    //0 圆外
    //1 圆上
    //2 圆内
    int relation(point b)
    {
        double dst=b.distance(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationseg(line v)
    {
        double dst=v.dispointtoseg(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationline(line v)
    {
        double dst=v.dispointtoline(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    //过a 两点b 半径的两个圆r
    int getcircle(point a,point b,double r,circle&c1,circle&c2)
    {
        circle x(a,r),y(b,r);
        int t=x.pointcrosscircle(y,c1.p,c2.p);
        if (!t)return 0;
        c1.r=c2.r=r;
        return t;
    }
    //与直线相切u 过点q 半径的圆r1
    int getcircle(line u,point q,double r1,circle &c1,circle &
        c2)
    {
        double dis=u.dispointtoline(q);
        if (dblcmp(dis-r1*2)>0)return 0;
        if (dblcmp(dis)==0)
        {
            c1.p=q.add(u.b.sub(u.a).rotleft().trunc(r1));
            c2.p=q.add(u.b.sub(u.a).rotright().trunc(r1));
            c1.r=c2.r=r1;
            return 2;
        }
        line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),
            u.b.add(u.b.sub(u.a).rotleft().trunc(r1)));
        line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1))
            ,u.b.add(u.b.sub(u.a).rotright().trunc(r1)));
        circle cc=circle(q,r1);
        point p1,p2;
        if (!cc.pointcrossline(u1,p1,p2))cc.pointcrossline(u2,
            p1,p2);
        c1=circle(p1,r1);
        if (p1==p2)
        {
            c2=c1;return 1;
        }
        c2=circle(p2,r1);
        return 2;
    }
    //同时与直线u,相切v 半径的圆r1
    int getcircle(line u,line v,double r1,circle &c1,circle &c2
        ,circle &c3,circle &c4)
    {
        if (u.parallel(v))return 0;
        line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),
            u.b.add(u.b.sub(u.a).rotleft().trunc(r1)));
        line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1))
            ,u.b.add(u.b.sub(u.a).rotright().trunc(r1)));
        line v1=line(v.a.add(v.b.sub(v.a).rotleft().trunc(r1)),
            v.b.add(v.b.sub(v.a).rotleft().trunc(r1)));
        line v2=line(v.a.add(v.b.sub(v.a).rotright().trunc(r1))
            ,v.b.add(v.b.sub(v.a).rotright().trunc(r1)));
        c1.r=c2.r=c3.r=c4.r=r1;
        c1.p=u1.crosspoint(v1);
        c2.p=u1.crosspoint(v2);
        c3.p=u2.crosspoint(v1);
        c4.p=u2.crosspoint(v2);
        return 4;
    }
    //同时与不相交圆cx,相切cy 半径为的圆r1
    int getcircle(circle cx,circle cy,double r1,circle&c1,
        circle&c2)
    {
        circle x(cx.p,r1+cx.r),y(cy.p,r1+cy.r);
        int t=x.pointcrosscircle(y,c1.p,c2.p);
        if (!t)return 0;
        c1.r=c2.r=r1;
        return t;
    }
    int pointcrossline(line v,point &p1,point &p2)//求与线段交要
        先判断relationseg
    {
        if (!(*this).relationline(v))return 0;
        point a=v.lineprog(p);
        double d=v.dispointtoline(p);
        d=sqrt(r*r-d*d);
        if (dblcmp(d)==0)
        {
            p1=a;
            p2=a;
            return 1;
        }
        p1=a.sub(v.b.sub(v.a).trunc(d));
        p2=a.add(v.b.sub(v.a).trunc(d));
        return 2;
    }
    //5 相离
    //4 外切
    //3 相交
    //2 内切
    //1 内含
    int relationcircle(circle v)
    {
        double d=p.distance(v.p);
        if (dblcmp(d-r-v.r)>0)return 5;
        if (dblcmp(d-r-v.r)==0)return 4;
        double l=fabs(r-v.r);
        if (dblcmp(d-r-v.r)<0&&dblcmp(d-l)>0)return 3;
        if (dblcmp(d-l)==0)return 2;
        if (dblcmp(d-l)<0)return 1;
    }
    int pointcrosscircle(circle v,point &p1,point &p2)
```

```
    {
        int rel=relationcircle(v);
        if (rel==1||rel==5)return 0;
        double d=p.distance(v.p);
        double l=(d+(sqr(r)-sqr(v.r))/d)/2;
        double h=sqrt(sqr(r)-sqr(l));
        p1=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotleft().
             trunc(h)));
        p2=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotright().
             trunc(h)));
        if (rel==2||rel==4)
        {
            return 1;
        }
        return 2;
    }
    //过一点做圆的切线 先判断点和圆关系()
    int tangentline(point q,line &u,line &v)
    {
        int x=relation(q);
        if (x==2)return 0;
        if (x==1)
        {
            u=line(q,q.add(q.sub(p).rotleft()));
            v=u;
            return 1;
        }
        double d=p.distance(q);
        double l=sqr(r)/d;
        double h=sqrt(sqr(r)-sqr(l));
        u=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotleft()
             .trunc(h))));
        v=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotright
             ().trunc(h))));
        return 2;
    }
    double areacircle(circle v)
    {
        int rel=relationcircle(v);
        if (rel>=4)return 0.0;
        if (rel<=2)return min(area(),v.area());
        double d=p.distance(v.p);
        double hf=(r+v.r+d)/2.0;
        double ss=2*sqrt(hf*(hf-r)*(hf-v.r)*(hf-d));
        double a1=acos((r*r+d*d-v.r*v.r)/(2.0*r*d));
        a1=a1*r*r;
        double a2=acos((v.r*v.r+d*d-r*r)/(2.0*v.r*d));
        a2=a2*v.r*v.r;
        return a1+a2-ss;
    }
    double areatriangle(point a,point b)
    {
        if (dblcmp(p.sub(a).det(p.sub(b))==0))return 0.0;
        point q[5];
        int len=0;
        q[len++]=a;
        line l(a,b);
        point p1,p2;
        if (pointcrossline(l,q[1],q[2])==2)
        {
            if (dblcmp(a.sub(q[1]).dot(b.sub(q[1])))<0)q[len
                 ++]=q[1];
            if (dblcmp(a.sub(q[2]).dot(b.sub(q[2])))<0)q[len
                 ++]=q[2];
        }
        q[len++]=b;
        if (len==4&&(dblcmp(q[0].sub(q[1]).dot(q[2].sub(q[1])))
             >0))swap(q[1],q[2]);
        double res=0;
        int i;
        for (i=0;i<len-1;i++)
        {
            if (relation(q[i])==0||relation(q[i+1])==0)
            {
                double arg=p.rad(q[i],q[i+1]);
                res+=r*r*arg/2.0;
            }
            else
            {
                res+=fabs(q[i].sub(p).det(q[i+1].sub(p))/2.0);
            }
        }
        return res;
    }
};
struct polygon
{
    int n;
    point p[maxp];
    line l[maxp];
    void input()
    {
        n=4;
        for (int i=0;i<n;i++)
        {
            p[i].input();
        }
    }
    void add(point q)
    {
        p[n++]=q;
    }
    void getline()
    {
        for (int i=0;i<n;i++)
        {
            l[i]=line(p[i],p[(i+1)%n]);
        }
    }
    struct cmp
    {
        point p;
        cmp(const point &p0){p=p0;}
        bool operator()(const point &aa,const point &bb)
        {
            point a=aa,b=bb;
            int d=dblcmp(a.sub(p).det(b.sub(p)));
            if (d==0)
            {
                return dblcmp(a.distance(p)-b.distance(p))<0;
            }
            return d>0;
        }
    };
    void norm()
    {
        point mi=p[0];
        for (int i=1;i<n;i++)mi=min(mi,p[i]);
        sort(p,p+n,cmp(mi));
    }
    void getconvex(polygon &convex)
    {
        int i,j,k;
        sort(p,p+n);
        convex.n=n;
        for (i=0;i<min(n,2);i++)
        {
            convex.p[i]=p[i];
        }
        if (n<=2)return;
        int &top=convex.n;
        top=1;
        for (i=2;i<n;i++)
        {
            while (top&&convex.p[top].sub(p[i]).det(convex.p[
                 top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
        int temp=top;
        convex.p[++top]=p[n-2];
        for (i=n-3;i>=0;i--)
        {
            while (top!=temp&&convex.p[top].sub(p[i]).det(
                 convex.p[top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
    }
    bool isconvex()
    {
        bool s[3];
        memset(s,0,sizeof(s));
        int i,j,k;
        for (i=0;i<n;i++)
        {
            j=(i+1)%n;
            k=(j+1)%n;
            s[dblcmp(p[j].sub(p[i]).det(p[k].sub(p[i])))+1]=1;
            if (s[0]&&s[2])return 0;
        }
        return 1;
    }
    //3 点上
    //2 边上
    //1 内部
    //0 外部
    int relationpoint(point q)
    {
        int i,j;
        for (i=0;i<n;i++)
        {
            if (p[i]==q)return 3;
        }
        getline();
        for (i=0;i<n;i++)
        {
            if (l[i].pointonseg(q))return 2;
        }
        int cnt=0;
        for (i=0;i<n;i++)
```

```
    {
        j=(i+1)%n;
        int k=dblcmp(q.sub(p[j]).det(p[i].sub(p[j])));
        int u=dblcmp(p[i].y-q.y);
        int v=dblcmp(p[j].y-q.y);
        if (k>0&&u<0&&v>=0)cnt++;
        if (k<0&&v<0&&u>=0)cnt--;
    }
    return cnt!=0;
}
//1 在多边形内长度为正
//2 相交或与边平行
//0 无任何交点
int relationline(line u)
{
    int i,j,k=0;
    getline();
    for (i=0;i<n;i++)
    {
        if (l[i].segcrossseg(u)==2)return 1;
        if (l[i].segcrossseg(u)==1)k=1;
    }
    if (!k)return 0;
    vector<point>vp;
    for (i=0;i<n;i++)
    {
        if (l[i].segcrossseg(u))
        {
            if (l[i].parallel(u))
            {
                vp.pb(u.a);
                vp.pb(u.b);
                vp.pb(l[i].a);
                vp.pb(l[i].b);
                continue;
            }
            vp.pb(l[i].crosspoint(u));
        }
    }
    sort(vp.begin(),vp.end());
    int sz=vp.size();
    for (i=0;i<sz-1;i++)
    {
        point mid=vp[i].add(vp[i+1]).div(2);
        if (relationpoint(mid)==1)return 1;
    }
    return 2;
}
//直线切割凸多边形左侧u
//注意直线方向
void convexcut(line u,polygon &po)
{
    int i,j,k;
    int &top=po.n;
    top=0;
    for (i=0;i<n;i++)
    {
        int d1=dblcmp(p[i].sub(u.a).det(u.b.sub(u.a)));
        int d2=dblcmp(p[(i+1)%n].sub(u.a).det(u.b.sub(u.a))
            );
        if (d1>=0)po.p[top++]=p[i];
        if (d1*d2<0)po.p[top++]=u.crosspoint(line(p[i],p[(i
            +1)%n]));
    }
}
double getcircumference()
{
    double sum=0;
    int i;
    for (i=0;i<n;i++)
    {
        sum+=p[i].distance(p[(i+1)%n]);
    }
    return sum;
}
double getarea()
{
    double sum=0;
    int i;
    for (i=0;i<n;i++)
    {
        sum+=p[i].det(p[(i+1)%n]);
    }
    return fabs(sum)/2;
}
bool getdir()//代表逆时针1 代表顺时针0
{
    double sum=0;
    int i;
    for (i=0;i<n;i++)
    {
        sum+=p[i].det(p[(i+1)%n]);
    }
    if (dblcmp(sum)>0)return 1;
    return 0;
}
point getbarycentre()
{
    point ret(0,0);
    double area=0;
    int i;
    for (i=1;i<n-1;i++)
    {
        double tmp=p[i].sub(p[0]).det(p[i+1].sub(p[0]));
        if (dblcmp(tmp)==0)continue;
        area+=tmp;
        ret.x+=(p[0].x+p[i].x+p[i+1].x)/3*tmp;
        ret.y+=(p[0].y+p[i].y+p[i+1].y)/3*tmp;
    }
    if (dblcmp(area))ret=ret.div(area);
    return ret;
}
double areaintersection(polygon po)
{
}
double areaunion(polygon po)
{
    return getarea()+po.getarea()-areaintersection(po);
}
double areacircle(circle c)
{
    int i,j,k,l,m;
    double ans=0;
    for (i=0;i<n;i++)
    {
        int j=(i+1)%n;
        if (dblcmp(p[j].sub(c.p).det(p[i].sub(c.p)))>=0)
        {
            ans+=c.areatriangle(p[i],p[j]);
        }
        else
        {
            ans-=c.areatriangle(p[i],p[j]);
        }
    }
    return fabs(ans);
}
//多边形和圆关系
//0 一部分在圆外
//1 与圆某条边相切
//2 完全在圆内
int relationcircle(circle c)
{
    getline();
    int i,x=2;
    if (relationpoint(c.p)!=1)return 0;
    for (i=0;i<n;i++)
    {
        if (c.relationseg(l[i])==2)return 0;
        if (c.relationseg(l[i])==1)x=1;
    }
    return x;
}
void find(int st,point tri[],circle &c)
{
    if (!st)
    {
        c=circle(point(0,0),-2);
    }
    if (st==1)
    {
        c=circle(tri[0],0);
    }
    if (st==2)
    {
        c=circle(tri[0].add(tri[1]).div(2),tri[0].distance(
            tri[1])/2.0);
    }
    if (st==3)
    {
        c=circle(tri[0],tri[1],tri[2]);
    }
}
void solve(int cur,int st,point tri[],circle &c)
{
    find(st,tri,c);
    if (st==3)return;
    int i;
    for (i=0;i<cur;i++)
    {
        if (dblcmp(p[i].distance(c.p)-c.r)>0)
        {
            tri[st]=p[i];
            solve(i,st+1,tri,c);
        }
    }
}
circle mincircle()//点集最小圆覆盖
{
    random_shuffle(p,p+n);
```

```
        point tri[4];
        circle c;
        solve(n,0,tri,c);
        return c;
    }
    int circlecover(double r)//单位圆覆盖
    {
        int ans=0,i,j;
        vector<pair<double,int> >v;
        for (i=0;i<n;i++)
        {
            v.clear();
            for (j=0;j<n;j++)if (i!=j)
            {
                point q=p[i].sub(p[j]);
                double d=q.len();
                if (dblcmp(d-2*r)<=0)
                {
                    double arg=atan2(q.y,q.x);
                    if (dblcmp(arg)<0)arg+=2*pi;
                    double t=acos(d/(2*r));
                    v.push_back(make_pair(arg-t+2*pi,-1));
                    v.push_back(make_pair(arg+t+2*pi,1));
                }
            }
            sort(v.begin(),v.end());
            int cur=0;
            for (j=0;j<v.size();j++)
            {
                if (v[j].second==-1)++cur;
                else --cur;
                ans=max(ans,cur);
            }
        }
        return ans+1;
    }
    int pointinpolygon(point q)//点在凸多边形内部的判定
    {
        if (getdir())reverse(p,p+n);
        if (dblcmp(q.sub(p[0]).det(p[n-1].sub(p[0])))==0)
        {
            if (line(p[n-1],p[0]).pointonseg(q))return n-1;
            return -1;
        }
        int low=1,high=n-2,mid;
        while (low<=high)
        {
            mid=(low+high)>>1;
            if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>=0&&
                dblcmp(q.sub(p[0]).det(p[mid+1].sub(p[0])))<0)
            {
                polygon c;
                c.p[0]=p[mid];
                c.p[1]=p[mid+1];
                c.p[2]=p[0];
                c.n=3;
                if (c.relationpoint(q))return mid;
                return -1;
            }
            if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>0)
            {
                low=mid+1;
            }
            else
            {
                high=mid-1;
            }
        }
        return -1;
    }
};
struct polygons
{
    vector<polygon>p;
    polygons()
    {
        p.clear();
    }
    void clear()
    {
        p.clear();
    }
    void push(polygon q)
    {
        if (dblcmp(q.getarea()))p.pb(q);
    }
    vector<pair<double,int> >e;
    void ins(point s,point t,point X,int i)
    {
        double r=fabs(t.x-s.x)>eps?(X.x-s.x)/(t.x-s.x):(X.y-s.y
            )/(t.y-s.y);
        r=min(r,1.0);r=max(r,0.0);
        e.pb(mp(r,i));
    }
    double polyareaunion()
    {
        double ans=0.0;
        int c0,c1,c2,i,j,k,w;
        for (i=0;i<p.size();i++)
        {
            if (p[i].getdir()==0)reverse(p[i].p,p[i].p+p[i].n);
        }
        for (i=0;i<p.size();i++)
        {
            for (k=0;k<p[i].n;k++)
            {
                point &s=p[i].p[k],&t=p[i].p[(k+1)%p[i].n];
                if (!dblcmp(s.det(t)))continue;
                e.clear();
                e.pb(mp(0.0,1));
                e.pb(mp(1.0,-1));
                for (j=0;j<p.size();j++)if (i!=j)
                {
                    for (w=0;w<p[j].n;w++)
                    {
                        point a=p[j].p[w],b=p[j].p[(w+1)%p[j].n
                            ],c=p[j].p[(w-1+p[j].n)%p[j].n];
                        c0=dblcmp(t.sub(s).det(c.sub(s)));
                        c1=dblcmp(t.sub(s).det(a.sub(s)));
                        c2=dblcmp(t.sub(s).det(b.sub(s)));
                        if (c1*c2<0)ins(s,t,line(s,t).
                            crosspoint(line(a,b)),-c2);
                        else if (!c1&&c0*c2<0)ins(s,t,a,-c2);
                        else if (!c1&&!c2)
                        {
                            int c3=dblcmp(t.sub(s).det(p[j].p[(
                                w+2)%p[j].n].sub(s)));
                            int dp=dblcmp(t.sub(s).dot(b.sub(a)
                                ));
                            if (dp&&c0)ins(s,t,a,dp>0?c0*((j>i)
                                ^(c0<0)):-(c0<0));
                            if (dp&&c3)ins(s,t,b,dp>0?-c3*((j>i
                                )^(c3<0)):c3<0);
                        }
                    }
                }
                sort(e.begin(),e.end());
                int ct=0;
                double tot=0.0,last;
                for (j=0;j<e.size();j++)
                {
                    if (ct==2)tot+=e[j].first-last;
                    ct+=e[j].second;
                    last=e[j].first;
                }
                ans+=s.det(t)*tot;
            }
        }
        return fabs(ans)*0.5;
    }
};
const int maxn=500;
struct circles
{
    circle c[maxn];
    double ans[maxn];//ans[i表示被覆盖了]次的面积i
    double pre[maxn];
    int n;
    circles(){}
    void add(circle cc)
    {
        c[n++]=cc;
    }
    bool inner(circle x,circle y)
    {
        if (x.relationcircle(y)!=1)return 0;
        return dblcmp(x.r-y.r)<=0?1:0;
    }
    void init_or()//圆的面积并去掉内含的圆
    {
        int i,j,k=0;
        bool mark[maxn]={0};
        for (i=0;i<n;i++)
        {
            for (j=0;j<n;j++)if (i!=j&&!mark[j])
            {
                if ((c[i]==c[j])||inner(c[i],c[j]))break;
            }
            if (j<n)mark[i]=1;
        }
        for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
        n=k;
    }
    void init_and()//圆的面积交去掉内含的圆
    {
        int i,j,k=0;
        bool mark[maxn]={0};
        for (i=0;i<n;i++)
        {
            for (j=0;j<n;j++)if (i!=j&&!mark[j])
            {
                if ((c[i]==c[j])||inner(c[j],c[i]))break;
```

```
            }
            if (j<n)mark[i]=1;
        }
        for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
        n=k;
    }
    double areaarc(double th,double r)
    {
        return 0.5*sqr(r)*(th-sin(th));
    }
    void getarea()
    {
        int i,j,k;
        memset(ans,0,sizeof(ans));
        vector<pair<double,int> >v;
        for (i=0;i<n;i++)
        {
            v.clear();
            v.push_back(make_pair(-pi,1));
            v.push_back(make_pair(pi,-1));
            for (j=0;j<n;j++)if (i!=j)
            {
                point q=c[j].p.sub(c[i].p);
                double ab=q.len(),ac=c[i].r,bc=c[j].r;
                if (dblcmp(ab+ac-bc)<=0)
                {
                    v.push_back(make_pair(-pi,1));
                    v.push_back(make_pair(pi,-1));
                    continue;
                }
                if (dblcmp(ab+bc-ac)<=0)continue;
                if (dblcmp(ab-ac-bc)>0) continue;
                double th=atan2(q.y,q.x),fai=acos((ac*ac+ab*ab-
                    bc*bc)/(2.0*ac*ab));
                double a0=th-fai;
                if (dblcmp(a0+pi)<0)a0+=2*pi;
                double a1=th+fai;
                if (dblcmp(a1-pi)>0)a1-=2*pi;
                if (dblcmp(a0-a1)>0)
                {
                    v.push_back(make_pair(a0,1));
                    v.push_back(make_pair(pi,-1));
                    v.push_back(make_pair(-pi,1));
                    v.push_back(make_pair(a1,-1));
                }
                else
                {
                    v.push_back(make_pair(a0,1));
                    v.push_back(make_pair(a1,-1));
                }
            }
            sort(v.begin(),v.end());
            int cur=0;
            for (j=0;j<v.size();j++)
            {
                if (cur&&dblcmp(v[j].first-pre[cur]))
                {
                    ans[cur]+=areaarc(v[j].first-pre[cur],c[i].
                        r);
                    ans[cur]+=0.5*point(c[i].p.x+c[i].r*cos(pre
                        [cur]),c[i].p.y+c[i].r*sin(pre[cur])).
                        det(point(c[i].p.x+c[i].r*cos(v[j].
                        first),c[i].p.y+c[i].r*sin(v[j].first)
                        ));
                }
                cur+=v[j].second;
                pre[cur]=v[j].first;
            }
        }
        for (i=1;i<=n;i++)
        {
            ans[i]-=ans[i+1];
        }
    }
};
struct halfplane:public line
{
    double angle;
    halfplane(){}
    //表示向量 a->逆时针b左侧()的半平面
    halfplane(point _a,point _b)
    {
        a=_a;
        b=_b;
    }
    halfplane(line v)
    {
        a=v.a;
        b=v.b;
    }
    void calcangle()
    {
        angle=atan2(b.y-a.y,b.x-a.x);
    }
    bool operator<(const halfplane &b)const
    {
        return angle<b.angle;
    }
};
struct halfplanes
{
    int n;
    halfplane hp[maxp];
    point p[maxp];
    int que[maxp];
    int st,ed;
    void push(halfplane tmp)
    {
        hp[n++]=tmp;
    }
    void unique()
    {
        int m=1,i;
        for (i=1;i<n;i++)
        {
            if (dblcmp(hp[i].angle-hp[i-1].angle))hp[m++]=hp[i
                ];
            else if (dblcmp(hp[m-1].b.sub(hp[m-1].a).det(hp[i].
                a.sub(hp[m-1].a))>0))hp[m-1]=hp[i];
        }
        n=m;
    }
    bool halfplaneinsert()
    {
        int i;
        for (i=0;i<n;i++)hp[i].calcangle();
        sort(hp,hp+n);
        unique();
        que[st=0]=0;
        que[ed=1]=1;
        p[1]=hp[0].crosspoint(hp[1]);
        for (i=2;i<n;i++)
        {
            while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[ed
                ].sub(hp[i].a))))<0)ed--;
            while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[st
                +1].sub(hp[i].a))))<0)st++;
            que[++ed]=i;
            if (hp[i].parallel(hp[que[ed-1]]))return false;
            p[ed]=hp[i].crosspoint(hp[que[ed-1]]);
        }
        while (st<ed&&dblcmp(hp[que[st]].b.sub(hp[que[st]].a).
            det(p[ed].sub(hp[que[st]].a)))<0)ed--;
        while (st<ed&&dblcmp(hp[que[ed]].b.sub(hp[que[ed]].a).
            det(p[st+1].sub(hp[que[ed]].a)))<0)st++;
        if (st+1>=ed)return false;
        return true;
    }
    void getconvex(polygon &con)
    {
        p[st]=hp[que[st]].crosspoint(hp[que[ed]]);
        con.n=ed-st+1;
        int j=st,i=0;
        for (;j<=ed;i++,j++)
        {
            con.p[i]=p[j];
        }
    }
};
struct point3
{
    double x,y,z;
    point3(){}
    point3(double _x,double _y,double _z):
        x(_x),y(_y),z(_z){};
    void input()
    {
        scanf("%lf%lf%lf",&x,&y,&z);
    }
    void output()
    {
        printf("%.2lf␣%.2lf␣%.2lf\n",x,y,z);
    }
    bool operator==(point3 a)
    {
        return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0&&dblcmp(a.z-z
            )==0;
    }
    bool operator<(point3 a)const
    {
        return dblcmp(a.x-x)==0?dblcmp(y-a.y)==0?dblcmp(z-a.z)
            <0:y<a.y:x<a.x;
    }
    double len()
    {
        return sqrt(len2());
    }
    double len2()
    {
        return x*x+y*y+z*z;
    }
    double distance(point3 p)
```

```
    {
        return sqrt((p.x-x)*(p.x-x)+(p.y-y)*(p.y-y)+(p.z-z)*(p.
            z-z));
    }
    point3 add(point3 p)
    {
        return point3(x+p.x,y+p.y,z+p.z);
    }
    point3 sub(point3 p)
    {
        return point3(x-p.x,y-p.y,z-p.z);
    }
    point3 mul(double d)
    {
        return point3(x*d,y*d,z*d);
    }
    point3 div(double d)
    {
        return point3(x/d,y/d,z/d);
    }
    double dot(point3 p)
    {
        return x*p.x+y*p.y+z*p.z;
    }
    point3 det(point3 p)
    {
        return point3(y*p.z-p.y*z,p.x*z-x*p.z,x*p.y-p.x*y);
    }
    double rad(point3 a,point3 b)
    {
        point3 p=(*this);
        return acos(a.sub(p).dot(b.sub(p))/(a.distance(p)*b.
            distance(p)));
    }
    point3 trunc(double r)
    {
        r/=len();
        return point3(x*r,y*r,z*r);
    }
    point3 rotate(point3 o,double r)
    {
    }
};
struct line3
{
    point3 a,b;
    line3(){}
    line3(point3 _a,point3 _b)
    {
        a=_a;
        b=_b;
    }
    bool operator==(line3 v)
    {
        return (a==v.a)&&(b==v.b);
    }
    void input()
    {
        a.input();
        b.input();
    }
    double length()
    {
        return a.distance(b);
    }
    bool pointonseg(point3 p)
    {
        return dblcmp(p.sub(a).det(p.sub(b)).len())==0&&dblcmp(
            a.sub(p).dot(b.sub(p)))<=0;
    }
    double dispointtoline(point3 p)
    {
        return b.sub(a).det(p.sub(a)).len()/a.distance(b);
    }
    double dispointtoseg(point3 p)
    {
        if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.sub(a).
            dot(b.sub(a)))<0)
        {
            return min(p.distance(a),p.distance(b));
        }
        return dispointtoline(p);
    }
    point3 lineprog(point3 p)
    {
        return a.add(b.sub(a).trunc(b.sub(a).dot(p.sub(a))/b.
            distance(a)));
    }
    point3 rotate(point3 p,double ang)//绕此向量逆时针角度parg
    {
        if (dblcmp((p.sub(a).det(p.sub(b)).len()))==0)return p;
        point3 f1=b.sub(a).det(p.sub(a));
        point3 f2=b.sub(a).det(f1);
        double len=fabs(a.sub(p).det(b.sub(p)).len()/a.distance
            (b));
        f1=f1.trunc(len);f2=f2.trunc(len);
        point3 h=p.add(f2);
        point3 pp=h.add(f1);
        return h.add((p.sub(h)).mul(cos(ang*1.0))).add((pp.sub(
            h)).mul(sin(ang*1.0)));
    }
};
struct plane
{
    point3 a,b,c,o;
    plane(){}
    plane(point3 _a,point3 _b,point3 _c)
    {
        a=_a;
        b=_b;
        c=_c;
        o=pvec();
    }
    plane(double _a,double _b,double _c,double _d)
    {
        //ax+by+cz+d=0
        o=point3(_a,_b,_c);
        if (dblcmp(_a)!=0)
        {
            a=point3((-_d-_c-_b)/_a,1,1);
        }
        else if (dblcmp(_b)!=0)
        {
            a=point3(1,(-_d-_c-_a)/_b,1);
        }
        else if (dblcmp(_c)!=0)
        {
            a=point3(1,1,(-_d-_a-_b)/_c);
        }
    }
    void input()
    {
        a.input();
        b.input();
        c.input();
        o=pvec();
    }
    point3 pvec()
    {
        return b.sub(a).det(c.sub(a));
    }
    bool pointonplane(point3 p)//点是否在平面上
    {
        return dblcmp(p.sub(a).dot(o))==0;
    }
    //0 不在
    //1 在边界上
    //2 在内部
    int pointontriangle(point3 p)//点是否在空间三角形上abc
    {
        if (!pointonplane(p))return 0;
        double s=a.sub(b).det(c.sub(b)).len();
        double s1=p.sub(a).det(p.sub(b)).len();
        double s2=p.sub(a).det(p.sub(c)).len();
        double s3=p.sub(b).det(p.sub(c)).len();
        if (dblcmp(s-s1-s2-s3))return 0;
        if (dblcmp(s1)&&dblcmp(s2)&&dblcmp(s3))return 2;
        return 1;
    }
    //判断两平面关系
    //0 相交
    //1 平行但不重合
    //2 重合
    bool relationplane(plane f)
    {
        if (dblcmp(o.det(f.o).len()))return 0;
        if (pointonplane(f.a))return 2;
        return 1;
    }
    double angleplane(plane f)//两平面夹角
    {
        return acos(o.dot(f.o)/(o.len()*f.o.len()));
    }
    double dispoint(point3 p)//点到平面距离
    {
        return fabs(p.sub(a).dot(o)/o.len());
    }
    point3 pttoplane(point3 p)//点到平面最近点
    {
        line3 u=line3(p,p.add(o));
        crossline(u,p);
        return p;
    }
    int crossline(line3 u,point3 &p)//平面和直线的交点
    {
        double x=o.dot(u.b.sub(a));
        double y=o.dot(u.a.sub(a));
        double d=x-y;
        if (dblcmp(fabs(d))==0)return 0;
        p=u.a.mul(x).sub(u.b.mul(y)).div(d);
```

```
        return 1;
    }
    int crossplane(plane f,line3 &u)//平面和平面的交线
    {
        point3 oo=o.det(f.o);
        point3 v=o.det(oo);
        double d=fabs(f.o.dot(v));
        if (dblcmp(d)==0)return 0;
        point3 q=a.add(v.mul(f.o.dot(f.a.sub(a))/d));
        u=line3(q,q.add(oo));
        return 1;
    }
};
```

4 Graph

4.1 2SAT

```
/*
x & y == true:
~x -> x
~y -> y

x & y == false:
x -> ~y
y -> ~x

x | y == true:
~x -> y
~y -> x

x | y == false:
x -> ~x
y -> ~y

x ^ y == true:
~x -> y
y -> ~x
x -> ~y
~y -> x

x ^ y == false:
x -> y
y -> x
~x -> ~y
~y -> ~x
*/
#define MAXX 16111
#define MAXE 200111
#define v to[i]

int edge[MAXX],to[MAXE],nxt[MAXE],cnt;
inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

bool done[MAXX];
int st[MAXX];

bool dfs(const int now)
{
    if(done[now^1])
        return false;
    if(done[now])
        return true;
    done[now]=true;
    st[cnt++]=now;
    for(int i(edge[now]);i;i=nxt[i])
        if(!dfs(v))
            return false;
    return true;
}

inline bool go(const int n)
{
    static int i;
    memset(done,0,sizeof done);
    for(i=0;i<n;i+=2)
        if(!done[i] && !done[i^1])
        {
            cnt=0;
            if(!dfs(i))
            {
                while(cnt)
                    done[st[--cnt]]=false;
                if(!dfs(i^1))
                    return false;
            }
        }
    return true;
}
```

```
//done array will be a solution with minimal lexicographical
    order
// or maybe we can solve it with dual SCC method, and get a
    solution by reverse the edges of DAG then product a
    topsort
```

4.2 Articulation

```
void dfs(int now,int fa)  // now 从 1 开始
{
    int p(0);
    dfn[now]=low[now]=cnt++;
    for(std::list<int>::const_iterator it(edge[now].begin());it
        !=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it,now);
            ++p;
            low[now]=std::min(low[now],low[*it]);
            if((now==1 && p>1) || (now!=1 && low[*it]>=dfn[now
                ])) // 如果从出发点出发的子节点不能由兄弟节点到达，那
                么出发点为割点。如果现节点不是出发点，但是其子孙节点不
                能达到祖先节点，那么该节点为割点
                ans.insert(now);
        }
        else
            if(*it!=fa)
                low[now]=std::min(low[now],dfn[*it]);
}
```

4.3 Augmenting Path Algorithm for Maximum Cardinality Bipartite Matching

```
bool map[MAXX][MAXX],done[MAXX];
int in[MAXX],n,m;

bool dfs(int now)
{
    for(int i=0;i<m;i++)
        if(!done[i] && map[now][i])
        {
            done[i] = true;
            if(in[i]==-1 || dfs(in[i]))
            {
                in[i]=now;
                return true;
            }
        }
    return false;
}

inline int go()
{
    memset(in,-1,sizeof(in));
    static int ans,i;
    for(ans=i=0;i<n;i++)
    {
        memset(done,false,sizeof done);
        if (dfs(i))
            ++ans;
    }
    return ans;
}
```

4.4 Biconnected Component - Edge

```
// hdu 4612
#include<cstdio>
#include<algorithm>
#include<set>
#include<cstring>
#include<stack>
#include<queue>

#define MAXX 200111
#define MAXE (1000111*2)
#pragma comment(linker, "/STACK:16777216")

int edge[MAXX],to[MAXE],nxt[MAXE],cnt;
#define v to[i]
inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

int dfn[MAXX],low[MAXX],col[MAXX],belong[MAXX];
int idx,bcnt;
std::stack<int>st;

void tarjan(int now,int last)
```

```
{
    col[now]=1;
    st.push(now);
    dfn[now]=low[now]=++idx;
    bool flag(false);
    for(int i(edge[now]);i;i=nxt[i])
    {
        if(v==last && !flag)
        {
            flag=true;
            continue;
        }
        if(!col[v])
        {
            tarjan(v,now);
            low[now]=std::min(low[now],low[v]);
            /*
            if(low[v]>dfn[now])
            then this is a bridge
            */
        }
        else
            if(col[v]==1)
                low[now]=std::min(low[now],dfn[v]);
    }
    col[now]=2;
    if(dfn[now]==low[now])
    {
        ++bcnt;
        static int x;
        do
        {
            x=st.top();
            st.pop();
            belong[x]=bcnt;
        }while(x!=now);
    }
}

std::set<int>set[MAXX];

int dist[MAXX];
std::queue<int>q;
int n,m,i,j,k;

inline int go(int s)
{
    static std::set<int>::const_iterator it;
    memset(dist,0x3f,sizeof dist);
    dist[s]=0;
    q.push(s);
    while(!q.empty())
    {
        s=q.front();
        q.pop();
        for(it=set[s].begin();it!=set[s].end();++it)
            if(dist[*it]>dist[s]+1)
            {
                dist[*it]=dist[s]+1;
                q.push(*it);
            }
    }
    return std::max_element(dist+1,dist+1+bcnt)-dist;
}

int main()
{
    while(scanf("%d %d",&n,&m),(n||m))
    {
        cnt=0;
        memset(edge,0,sizeof edge);
        while(m--)
        {
            scanf("%d %d",&i,&j);
            add(i,j);
            add(j,i);
        }

        memset(dfn,0,sizeof dfn);
        memset(belong,0,sizeof belong);
        memset(low,0,sizeof low);
        memset(col,0,sizeof col);
        bcnt=idx=0;
        while(!st.empty())
            st.pop();

        tarjan(1,-1);
        for(i=1;i<=bcnt;++i)
            set[i].clear();
        for(i=1;i<=n;++i)
            for(j=edge[i];j;j=nxt[j])
                set[belong[i]].insert(belong[to[j]]);
        for(i=1;i<=bcnt;++i)
            set[i].erase(i);
        printf("%d\n",bcnt-1-dist[go(go(1))]);
    }
    return 0;
}
```

4.5 Biconnected Component - Vertex

```
/* hit 1789 Electricity
 * the graph may not connected
 * cnt records the number of BCC, it's an cut P if != 0*/
const int V = 10000;
vector<int> adj[V];
int low[V], dfn[V], cnt[V], depth;
void initialize(int n)
{
    REP(i, 0, n) adj[i].clear();
    CC(cnt, 0);CC(dfn, 0);
    depth = 0;
}
void dfs(int x, const int ROOT)
{
    low[x] = dfn[x] = ++depth;
    int s = adj[x].size(), w, num = 0;
    REP(i, 0, s)
    {
        w = adj[x][i];
        if (!dfn[w])
        {
            num++;
            dfs(w, ROOT);
            low[x] = min(low[w], low[x]);
            if (x == ROOT && num >= 2)
                cnt[x]++;
            if (x != ROOT && dfn[x] <= low[w])
                cnt[x]++;
        }
        else low[x] = min(low[x], dfn[w]);
    }
}
int solve(int n)
{
    int cc = 0;
    REP(i, 0, n)
    {
        if (dfn[i] == 0)
        {
            dfs(i, i);
            cc++;
        }
    }
    return cc;
}
int main()
{
    int n, m, x, y;
    while (scanf("%d %d", &n, &m) == 2 && n + m)
    {
        initialize(n);
        REP(i, 0, m)
        {
            scanf("%d %d", &x, &y);
            adj[x].push_back(y);
            adj[y].push_back(x);
        }
        int ans = solve(n);
        if (m == 0) printf("%d\n", n - 1);
        else printf("%d\n", ans + *max_element(cnt, cnt + n));
    }
    return 0;
}
```

4.6 Biconnected Component

```
#include<cstdio>
#include<cstring>
#include<stack>
#include<queue>
#include<algorithm>

const int MAXN=100000*2;
const int MAXM=200000;

//0-based

struct edges
{
    int to,next;
    bool cut,visit;
} edge[MAXM<<1];

int head[MAXN],low[MAXN],dpt[MAXN],L;
bool visit[MAXN],cut[MAXN];
int idx;
std::stack<int> st;
int bcc[MAXM];
```

```
void init(int n)
{
    L=0;
    memset(head,-1,4*n);
    memset(visit,0,n);
}

void add_edge(int u,int v)
{
    edge[L].cut=edge[L].visit=false;
    edge[L].to=v;
    edge[L].next=head[u];
    head[u]=L++;
}

void dfs(int u,int fu,int deg)
{
    cut[u]=false;
    visit[u]=true;
    low[u]=dpt[u]=deg;
    int tot=0;
    for (int i=head[u]; i!=-1; i=edge[i].next)
    {
        int v=edge[i].to;
        if (edge[i].visit)
            continue;
        st.push(i/2);
        edge[i].visit=edge[i^1].visit=true;
        if (visit[v])
        {
            low[u]=dpt[v]>low[u]?low[u]:dpt[v];
            continue;
        }
        dfs(v,u,deg+1);
        edge[i].cut=edge[i^1].cut=(low[v]>dpt[u] || edge[i].cut
             );
        if (u!=fu) cut[u]=low[v]>=dpt[u]?1:cut[u];
        if (low[v]>=dpt[u] || u==fu)
        {
            while (st.top()!=i/2)
            {
                int x=st.top()*2,y=st.top()*2+1;
                bcc[st.top()]=idx;
                st.pop();
            }
            bcc[i/2]=idx++;
            st.pop();
        }
        low[u]=low[v]>low[u]?low[u]:low[v];
        tot++;
    }
    if (u==fu && tot>1)
        cut[u]=true;
}

int main()
{
    int n,m;
    while (scanf("%d%d",&n,&m)!=EOF)
    {
        init(n);
        for (int i=0; i<m; i++)
        {
            int u,v;
            scanf("%d%d",&u,&v);
            add_edge(u,v);
            add_edge(v,u);
        }
        idx=0;
        for (int i=0; i<n; i++)
            if (!visit[i])
                dfs(i,i,0);
    }
    return 0;
}
```

4.7 Blossom algorithm

```
#include<cstdio>
#include<vector>
#include<cstring>
#include<algorithm>

#define MAXX 233

bool map[MAXX][MAXX];
std::vector<int>p[MAXX];
int m[MAXX];
int vis[MAXX];
int q[MAXX],*qf,*qb;

int n;

inline void label(int x,int y,int b)
{
    static int i,z;
    for(i=b+1;i<p[x].size();++i)
        if(vis[z=p[x][i]]==1)
        {
            p[z]=p[y];
            p[z].insert(p[z].end(),p[x].rbegin(),p[x].rend()-i)
                ;
            vis[z]=0;
            *qb++=z;
        }
}

inline bool bfs(int now)
{
    static int i,x,y,z,b;
    for(i=0;i<n;++i)
        p[i].resize(0);
    p[now].push_back(now);
    memset(vis,-1,sizeof vis);
    vis[now]=0;
    qf=qb=q;
    *qb++=now;

    while(qf<qb)
        for(x=*qf++,y=0;y<n;++y)
            if(map[x][y] && m[y]!=y && vis[y]!=1)
            {
                if(vis[y]==-1)
                    if(m[y]==-1)
                    {
                        for(i=0;i+1<p[x].size();i+=2)
                        {
                            m[p[x][i]]=p[x][i+1];
                            m[p[x][i+1]]=p[x][i];
                        }
                        m[x]=y;
                        m[y]=x;
                        return true;
                    }
                    else
                    {
                        p[z=m[y]]=p[x];
                        p[z].push_back(y);
                        p[z].push_back(z);
                        vis[y]=1;
                        vis[z]=0;
                        *qb++=z;
                    }
                else
                {
                    for(b=0;b<p[x].size() && b<p[y].size() && p
                         [x][b]==p[y][b];++b);
                    --b;
                    label(x,y,b);
                    label(y,x,b);
                }
            }
    return false;
}

int i,j,k;
int ans;

int main()
{
    scanf("%d",&n);
    for(i=0;i<n;++i)
        p[i].reserve(n);
    while(scanf("%d %d",&i,&j)!=EOF)
    {
        --i;
        --j;
        map[i][j]=map[j][i]=true;
    }
    memset(m,-1,sizeof m);
    for(i=0;i<n;++i)
        if(m[i]==-1)
        {
            if(bfs(i))
                ++ans;
            else
                m[i]=i;
        }
    printf("%d\n",ans<<1);
    for(i=0;i<n;++i)
        if(i<m[i])
            printf("%d %d\n",i+1,m[i]+1);
    return 0;
}
```

4.8 Bridge

```
void dfs(const short &now,const short &fa)
{
    dfn[now]=low[now]=cnt++;
```

```
    for(int i(0);i<edge[now].size();++i)
        if(dfn[edge[now][i]]==-1)
        {
            dfs(edge[now][i],now);
            low[now]=std::min(low[now],low[edge[now][i]]);
            if(low[edge[now][i]]>dfn[now]) //如果子节点不能够走到
                 父节点之前去，那么该边为桥
            {
                if(edge[now][i]<now)
                {
                    j=edge[now][i];
                    k=now;
                }
                else
                {
                    j=now;
                    k=edge[now][i];
                }
                ans.push_back(node(j,k));
            }
        }
        else
            if(edge[now][i]!=fa)
                low[now]=std::min(low[now],low[edge[now][i]]);
}
```

4.9 Chu-Liu:Edmonds' Algorithm

```
#include<cstdio>
#include<cstring>
#include<vector>

#define MAXX 1111
#define MAXE 10111
#define inf 0x3f3f3f3f

int n,m,i,j,k,ans,u,v,tn,rt,sum,on,om;
int pre[MAXX],id[MAXX],in[MAXX],vis[MAXX];

struct edge
{
    int a,b,c;
    edge(){}
    edge(int aa,int bb,int cc):a(aa),b(bb),c(cc){}
};
std::vector<edge>ed(MAXE);

int main()
{
    while(scanf("%d␣%d",&n,&m)!=EOF)
    {
        on=n;
        om=m;
        ed.resize(0);
        sum=1;
        while(m--)
        {
            scanf("%d␣%d␣%d",&i,&j,&k);
            if(i!=j)
            {
                ed.push_back(edge(i,j,k));
                sum+=k;
            }
        }
        ans=0;
        rt=n;
        for(i=0;i<n;++i)
            ed.push_back(edge(n,i,sum));
        ++n;
        while(true)
        {
            memset(in,0x3f,sizeof in);
            for(i=0;i<ed.size();++i)
                if(ed[i].a!=ed[i].b && in[ed[i].b]>ed[i].c)
                {
                    in[ed[i].b]=ed[i].c;
                    pre[ed[i].b]=ed[i].a;
                    if(ed[i].a==rt)
                        j=i;
                }
            for(i=0;i<n;++i)
                if(i!=rt && in[i]==inf)
                    goto ot;
            memset(id,-1,sizeof id);
            memset(vis,-1,sizeof vis);
            tn=in[rt]=0;
            for(i=0;i<n;++i)
            {
                ans+=in[i];
                for(v=i;vis[v]!=i && id[v]==-1 && v!=rt;v=pre[v
                    ])
                    vis[v]=i;
                if(v!=rt && id[v]==-1)
                {
                    for(u=pre[v];u!=v;u=pre[u])
                        id[u]=tn;
                    id[v]=tn++;
                }
            }
            if(!tn)
                break;
            for(i=0;i<n;++i)
                if(id[i]==-1)
                    id[i]=tn++;
            for(i=0;i<ed.size();++i)
            {
                v=ed[i].b;
                ed[i].a=id[ed[i].a];
                ed[i].b=id[ed[i].b];
                if(ed[i].a!=ed[i].b)
                    ed[i].c-=in[v];
            }
            n=tn;
            rt=id[rt];
        }
        if(ans>=2*sum)
ot:             puts("impossible");
        else
            printf("%d␣%d\n",ans-sum,j-om);
        puts("");
    }
    return 0;
}
```

4.10 Count MST

```
//hdu 4408
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 111

long long mod;
long long a[MAXX][MAXX];

inline long long det(int n)
{
    static int i,j,k;
    static long long re,t;
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
            a[i][j]%=mod;
    re=1ll;
    for(i=0;i<n;++i)
    {
        for(j=i+1;j<n;++j)
            while(a[j][i])
            {
                t=a[i][i]/a[j][i];
                for(k=i;k<n;++k)
                    a[i][k]=(a[i][k]-a[j][k]*t)%mod;
                for(k=i;k<n;++k)
                    std::swap(a[i][k],a[j][k]);
                re=-re;
            }
        if(!a[i][i])
            return 0ll;
        re=re*a[i][i]%mod;
    }
    return (re+mod)%mod;
}

struct E
{
    int a,b,c;
    bool operator<(const E &i)const
    {
        return c<i.c;
    }
}edge[1111];

int set[2][MAXX];
int find(int a,int t)
{
    return set[t][a]?set[t][a]=find(set[t][a],t):a;
}

int id[MAXX],dg[MAXX];
int map[MAXX][MAXX];
int n,m,i,j,k;
long long ans;
int cnt;

int main()
{
    while(scanf("%d␣%d␣%lld",&n,&m,&mod),(n||m||mod))
    {
        for(i=0;i<m;++i)
            scanf("%d␣%d␣%d",&edge[i].a,&edge[i].b,&edge[i].c);
```

```
        std::sort(edge,edge+m);
        memset(set[0],0,sizeof set[0]);
        ans=cnt=1;
        for(i=0;i<m;i=j)
        {
            for(j=i;j<m;++j)
                if(edge[i].c!=edge[j].c)
                    break;
            memset(dg,0,sizeof dg);
            memset(map,0,sizeof map);
            memset(set[1],0,sizeof set[0]);
            static int t,x,y;
            t=0;
            for(k=i;k<j;++k)
            {
                x=find(edge[k].a,0);
                y=find(edge[k].b,0);
                if(x!=y)
                {
                    ++map[x][y];
                    ++map[y][x];
                    ++dg[x];
                    ++dg[y];
                    x=find(x,1);
                    y=find(y,1);
                    if(x!=y)
                        set[1][x]=y;
                    ++t;
                }
            }
            for(k=i;k<j;++k)
            {
                x=find(edge[k].a,0);
                y=find(edge[k].b,0);
                if(x!=y)
                {
                    ++cnt;
                    set[0][x]=y;
                }
            }
            if(t)
            {
                for(k=1;k<=n;++k)
                    if(dg[k] && find(k,1)==k)
                    {
                        memset(a,0,sizeof a);
                        t=0;
                        static int ii,jj;
                        for(ii=1;ii<=n;++ii)
                            if(dg[ii] && find(ii,1)==k)
                                id[ii]=t++;
                        for(ii=1;ii<=n;++ii)
                            if(dg[ii] && find(ii,1)==k)
                            {
                                a[id[ii]][id[ii]]=dg[ii];
                                for(jj=1;jj<=n;++jj)
                                {
                                    if(!dg[jj] || ii==jj ||
                                        find(jj,1)!=k)
                                        continue;
                                    if(map[ii][jj])
                                    {
                                        static long long cnt;
                                        cnt=-map[ii][jj];
                                        a[id[ii]][id[jj]]=(cnt%
                                            mod+mod)%mod;
                                    }
                                }
                            }
                        ans=(ans*det(t-1))%mod;
                    }
            }
        }
        if(cnt!=n)
            puts("0");
        else
            printf("%lld\n",(ans%mod+mod)%mod);
    }
    return 0;
}
```

4.11 Covering Problems

最大团以及相关知识

独立集：独立集是指图的顶点集的一个子集, 该子集的导出子图的点互不相邻. 如果一个独立集不是任何一个独立集的子集, 那么称这个独立集是一个极大独立集. 一个图中包含顶点数目最多的独立集称为最大独立集。最大独立集一定是极大独立集，但是极大独立集不一定是最大的独立集。

支配集：与独立集相对应的就是支配集，支配集也是图顶点集的一个子集，设 S 是图 G 的一个支配集，则对于图中的任意一个顶点 u，要么属于集合 s, 要么与 s 中的顶点相邻。在 s 中除去任何元素后 s 不再是支配集，则支配集 s 是极小支配集。称 G 的所有支配集中顶点个数最少的支配集为最小支配集，最小支配集中的顶点个数成为支配数。

最小点 (对边) 的覆盖：最小点的覆盖也是图的顶点集的一个子集，如果我们选中一个点，则称这个点将以他为端点的所有边都覆盖了。将图中所有的边都覆盖所用顶点数最少，这个集合就是最小的点的覆盖。

最大团：图 G 的顶点的子集，设 D 是最大团，则 D 中任意两点相邻。若 u，v 是最大团，则 u,v 有边相连，其补图 u,v 没有边相连，所以图 G 的最大团 = 其补图的最大独立集。给定无向图 G = (V;E)，如果 U 属于 V，并且对于任意 u,v 包含于 U 有 < u; v > 包含于 E，则称 U 是 G 的完全子图，G 的完全子图 U 是 G 的团，当且仅当 U 不包含在 G 的更大的完全子图中，G 的最大团是指 G 中所含顶点数目最多的团。如果 U 属于 V，并且对于任意 u; v 包含于 U 有 < u; v > 不包含于 E，则称 U 是 G 的空子图，G 的空子图 U 是 G 的独立集，当且仅当 U 不包含在 G 的更大的独立集，G 的最大团是指 G 中所含顶点数目最多的独立集。

性质：
最大独立集 + 最小覆盖集 = V
最大团 = 补图的最大独立集
最小覆盖集 = 最大匹配

minimum cover:
edge cover vertex bipartite graph = maximum cardinality bipartite matching
找完最大二分匹配後，有三種情況要分別處理：
甲、X 側未匹配點的交錯樹們。
乙、Y 側未匹配點的交錯樹們。
丙、層層疊疊的交錯環們（包含單獨的匹配邊）。
這三個情況互不干涉。用 Graph Traversal 建立甲、乙的交錯樹們，剩下部分就是丙。
要找點覆蓋，甲、乙是取盡奇數距離的點，丙是取盡偶數距離的點、或者是取盡奇數距離的點，每塊連通分量可以各自為政。另外，小心處理的話，是可以印出字典順序最小的點覆蓋的。
已經有最大匹配時，求點覆蓋的時間複雜度等同於一次 Graph Traversal 的時間。

vertex cover edge

edge cover vertex
首先在圖上求得一個 Maximum Matching 之後，對於那些單身的點，都由匹配點連過去。如此便形成了 Minimum Edge Cover 。

edge cover edge

path cover vertex
general graph: NP-H
tree: DP
DAG: 将每个节点拆分为入点和出点,ans= 节点数 -匹配数

path cover edge
minimize the count of euler path (greedy is ok?)
dg[i] 表示每个点的 id-od,$ans = \sum dg[i], \forall dg[i] > 0$

cycle cover vertex
general: NP-H

weighted: do like path cover vertex, with KM algorithm

cycle cover edge
NP-H

4.12 Difference Constraints

$\forall a-b <= c, add(b,a,c);$
最短路得最远解
最长路得最近解
//根据情况反转边?(反转方向及边权)

全 0 点得普通解

4.13 Dinitz's algorithm

```
#define inf 0x3f3f3f3f

int n;
int w[MAXX],h[MAXX],q[MAXX];
int edge[MAXX],to[MAXM],cap[MAXM],nxt[MAXM],cnt;
int source,sink;

inline void add(int a,int b,int c)
{
    nxt[cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    ++cnt;
}

inline bool bfs()
{
    static int *qf,*qb;
    static int i;
    memset(h,-1,sizeof h);
    qf=qb=q;
    h[*qb++=source]=0;
    for(;qf!=qb;++qf)
        for(i=edge[*qf];i!=-1;i=nxt[i])
            if(cap[i] && h[to[i]]==-1)
                h[*qb++=to[i]]=h[*qf]+1;
    return h[sink]!=-1;
}

int dfs(int now,int maxcap)
{
    if(now==sink)
        return maxcap;
    int flow(maxcap),d;
    for(int &i(w[now]);i!=-1;i=nxt[i])
        if(cap[i] && h[to[i]]==h[now]+1)// && (flow=dfs(to[i],
             std::min(maxcap,cap[i])))
        {
            d=dfs(to[i],std::min(flow,cap[i]));
            cap[i]-=d;
            cap[i^1]+=d;
            flow-=d;
            if(!flow)
                return maxcap;
        }
    return maxcap-flow;
}

inline int go()
{
    static int ans;
    ans=0;
    while(bfs())
    {
        memcpy(w,edge,sizeof edge);
        ans+=dfs(source,inf);
        /*
           while((k=dfs(source,inf)))
           ans+=k;
           */
    }
    return ans;
}
```

4.14 Eulerian path

```
void solve(int x)
{
    int i;
    if (!match[x])
    {
        path[++l]=x;
        return ;
    }
    for (i=1; i<=n; i++)
        if (b[x][i])
        {
            b[x][i]--;
            match[x]--;
            solve(i);
        }
    path[++l]=x;
}

int stack[M], top, len;
void dfs(int v)
{
    stack[++top] = v;
    while (top)
    {
        int x = stack[top];
        bool flag = true;
        for (int i = p[x]; i != -1; i = e[i].next)
            if (e[i].f)
            {
                e[i].f = false; // order here
                stack[++top] = e[i].v;
                flag = false;
                break;
            }
        if (flag) stack[top--] = 0;
    }
}
```

4.15 Flow Network

4.15.1 Maximum weighted closure of a graph

所有由这个子图中的点出发的边都指向这个子图，那么这个子图为原图的一个 closure（闭合子图）

每个节点向其所有依赖节点连边，容量 inf
源点向所有正权值节点连边，容量为该权值
所有负权值节点向汇点连边，容量为该权值绝对值
以上均为有向边
最大权为 sum{正权值}-{新图的最小割}
残量图中所有由源点可达的点即为所选子图

4.15.2 Eulerian circuit

计入度和出度之差
无向边任意定向
出入度之差为奇数则无解
然后构图:
原图有向边不变，容量 1 // 好像需要在新图中忽略有向边？
无向边按之前认定方向，容量 1
源点向所有度数为正的点连边，容量 abs(度数/2)
所有度数为负的点向汇点连边，容量 abs(度数/2)
两侧均满流则有解
相当于规约为可行流问题
注意连通性的 trick

终点到起点加一条有向边即可将 path 问题转为 circuit 问题

4.15.3 Feasible flow problem

由超级源点出发的边全部满流则有解
有源汇时，由汇点向源点连边，下界 0 上界 inf 即可转化为无源无汇上下界流

对于每条边 <a->b cap{u,d}>，建边 <ss->b cap(u)>、<a->st cap(u)>、<a->b cap(d-u)>

- Maximum flow
  //将流量还原至原图后，在残量网络上继续完成最大流

直接把 source 和 sink 设为原来的 st，此时输出的最大流即是答案
不需要删除或者调整 t->s 弧

- Minimum flow
  建图时先不连汇点到源点的边，新图中完成最大流之后再连原汇至原源的边完成第二次最大流，此时 t->s 这条弧的流量即为最小流
  判断可行流存在还是必须连原汇 -> 原源的边之后查看满流
  所以可以使用跑流 -> 加 ts 弧 -> 跑流，最后检查超级源点满流情况来一步搞定

- tips
  合并流量、减少边数来加速

4.15.4 Minimum cost feasible flow problem

TODO
看起来像是在上面那样跑费用流就行了……

4.15.5 Minimum weighted vertex cover edge for bipartite graph

for all vertex in X:
edge < s->x cap(weight(x)) >
for all vertex in Y:
edge < y->t cap(weight(y)) >
for original edges
edge < x->y cap(inf) >

ans={maximum flow}={minimum cut}
残量网络中的所有简单割 ((源点可达 && 汇点不可达) || (源点不可达 && 汇点可达)) 对应着解

4.15.6 Maximum weighted vertex independent set for bipartite graph

ans=Sum{点权}-value{Minimum weighted vertex cover edge}
解应该就是最小覆盖集的补图吧……

4.15.7 方格取数

refer: hdu 3820 golden eggs
取方格获得收益
当取了相邻方格时付出边的代价

必取的方格到源/汇的边的容量 inf
相邻方格之间的边的容量为 {代价}*2
ans=sum{方格收益}-{最大流}

4.15.8 Uniqueness of min-cut

refer: 关键边。有向边起点为 s 集，终点为 t 集
从源和汇分别能够到的点集是所有点时，最小割唯一
也就是每一条增广路径都仅有一条边满流
注意查看的是实际的网络，不是残量网络

具体来说

```
void rr(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i] && !done[v])
            rr(v);
}

void dfs(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i^1] && !done[v])
            dfs(v);
}

memset(done,0,sizeof done);
cnt=0;
rr(source);
dfs(sink);
puts(cnt==n?"UNIQUE":"AMBIGUOUS");
```

4.15.9 Tips

- 两点间可以不止有一种边，也可以不止有一条边，无论有向无向
- 两点间容量 inf 则可以设法化简为一个点
- 点权始终要转化为边权
- 不参与决策的边权设为 inf 来排除掉
- 贪心一个初始不合法情况，然后通过可行流调整
  - 混合图欧拉回路存在性
  - 有向/无向图中国邮差问题 (遍历所有边至少一次后回到原点)
- 按时间拆点 (时间层……？)
- 当边权的分布很集中的时候，应当选择 zkw 费用流而不是 ek 贪心

4.16 Hamiltonian circuit

```
//if every point connect with not less than [(N+1)/2] points
#include<cstdio>
#include<algorithm>
#include<cstring>

#define MAXX 177
#define MAX (MAXX*MAXX)

int edge[MAXX],nxt[MAX],to[MAX],cnt;

inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

bool done[MAXX];
int n,m,i,j,k;

inline int find(int a)
{
    static int i;
    for(i=edge[a];i;i=nxt[i])
        if(!done[to[i]])
        {
            edge[a]=nxt[i];
            return to[i];
        }
    return 0;
}

int a,b;
int next[MAXX],pre[MAXX];
bool mat[MAXX][MAXX];

int main()
{
    while(scanf("%d␣%d",&n,&m)!=EOF)
    {
        for(i=1;i<=n;++i)
            next[i]=done[i]=edge[i]=0;
        memset(mat,0,sizeof mat);
        cnt=0;
        while(m--)
        {
            scanf("%d␣%d",&i,&j);
            add(i,j);
            add(j,i);
```

```
            mat[i][j]=mat[j][i]=true;
        }
        a=1;
        b=to[edge[a]];
        cnt=2;
        done[a]=done[b]=true;
        next[a]=b;
        while(cnt<n)
        {
            while(i=find(a))
            {
                next[i]=a;
                done[a=i]=true;
                ++cnt;
            }
            while(i=find(b))
            {
                next[b]=i;
                done[b=i]=true;
                ++cnt;
            }
            if(!mat[a][b])
                for(i=next[a];next[i]!=b;i=next[i])
                    if(mat[a][next[i]] && mat[i][b])
                    {
                        for(j=next[i];j!=b;j=next[j])
                            pre[next[j]]=j;
                        for(j=b;j!=next[i];j=pre[j])
                            next[j]=pre[j];
                        std::swap(next[i],b);
                        break;
                    }
            next[b]=a;
            for(i=a;i!=b;i=next[i])
                if(find(i))
                {
                    a=next[b=i];
                    break;
                }
        }
        while(a!=b)
        {
            printf("%d␣",a);
            a=next[a];
        }
        printf("%d\n",b);
    }
    return 0;
}
```

4.17 Hopcroft-Karp algorithm

```
int edge[MAXX],nxt[MAX],to[MAX],cnt;

int cx[MAXX],cy[MAXX];
int px[MAXX],py[MAXX];

int q[MAXX],*qf,*qb;

bool ag(int i)
{
    int j,k;
    for(k=edge[i];k;k=nxt[k])
        if(py[j=to[k]]==px[i]+1)
        {
            py[j]=0;
            if(cy[j]==-1 || ag(cy[j]))
            {
                cx[i]=j;
                cy[j]=i;
                return true;
            }
        }
    return false;
}

inline int go(int nx)
{
    static int i,j,k;
    static int x,y;
    static int ans;
    static bool flag;

    memset(cx,-1,sizeof cx);
    memset(cy,-1,sizeof cy);
    while(true)
    {
        memset(px,0,sizeof(px));
        memset(py,0,sizeof(py));
        qf=qb=q;
        flag=false;

        for(i=1;i<=nx;++i)
            if(cx[i]==-1)
                *qb++=i;
        while(qf!=qb)
            for(k=edge[i=*qf++];k;k=nxt[k])
                if(!py[j=to[k]])
                {
                    py[j]=px[i]+1;
                    if(cy[j]==-1)
                        flag=true;
                    else
                    {
                        px[cy[j]]=py[j]+1;
                        *qb++=cy[j];
                    }
                }
        if(!flag)
            break;
        for(i=1;i<=nx;++i)
            if(cx[i]==-1 && ag(i))
                ++ans;
    }
    return ans;
}
```

4.18 Improved Shortest Augmenting Path Algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 5111
#define MAXM (30111*4)
#define inf 0x3f3f3f3f3f3f3f3fll

int edge[MAXX],to[MAXM],nxt[MAXM],cnt;
#define v to[i]
long long cap[MAXM];

int n;
int h[MAXX],gap[MAXX],pre[MAXX],w[MAXX];

inline void add(int a,int b,long long c)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
}

int source,sink;

inline long long go(const int N=sink)
{
    static int now,i;
    static long long min,mf;
    memset(gap,0,sizeof gap);
    memset(h,0,sizeof h);
    memcpy(w,edge,sizeof w);
    gap[0]=N;
    mf=0;

    pre[now=source]=-1;
    while(h[source]<N)
    {
rep:
        if(now==sink)
        {
            min=inf;
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
                if(min>=cap[i])
                {
                    min=cap[i];
                    now=to[i^1];
                }
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            {
                cap[i]-=min;
                cap[i^1]+=min;
            }
            mf+=min;
        }
        for(int &i(w[now]);i!=-1;i=nxt[i])
            if(cap[i] && h[v]+1==h[now])
            {
                pre[now=v]=i;
                goto rep;
            }
        if(!--gap[h[now]])
            return mf;
        min=N;
        for(i=w[now]=edge[now];i!=-1;i=nxt[i])
            if(cap[i])
                min=std::min(min,(long long)h[v]);
        ++gap[h[now]=min+1];
        if(now!=source)
            now=to[pre[now]^1];
```

```
    }
    return mf;
}

int m,i,j,k;
long long ans;

int main()
{
    scanf("%d␣%d",&n,&m);
    source=1;
    sink=n;
    cnt=-1;
    memset(edge,-1,sizeof edge);
    while(m--)
    {
        scanf("%d␣%d␣%lld",&i,&j,&ans);
        add(i,j,ans);
        add(j,i,ans);
    }
    printf("%lld\n",go());
    return 0;
}
```

4.19 k Shortest Path

```
#include<cstdio>
#include<cstring>
#include<queue>
#include<vector>

int K;

class states
{
    public:
        int cost,id;
};

int dist[1000];

class cmp
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost>j.cost;
        }
};

class cmp2
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost+dist[i.id]>j.cost+dist[j.id];
        }
};

struct edges
{
    int to,next,cost;
} edger[100000],edge[100000];

int headr[1000],head[1000],Lr,L;

void dijkstra(int s)
{
    states u;
    u.id=s;
    u.cost=0;
    dist[s]=0;
    std::priority_queue<states,std::vector<states>,cmp> q;
    q.push(u);
    while (!q.empty())
    {
        u=q.top();
        q.pop();
        if (u.cost!=dist[u.id])
            continue;
        for (int i=headr[u.id]; i!=-1; i=edger[i].next)
        {
            states v=u;
            v.id=edger[i].to;
            if (dist[v.id]>dist[u.id]+edger[i].cost)
            {
                v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
                q.push(v);
            }
        }
    }
}

int num[1000];

inline void init(int n)
{
    Lr=L=0;
    memset(head,-1,4*n);
    memset(headr,-1,4*n);
    memset(dist,63,4*n);
    memset(num,0,4*n);
}

void add_edge(int u,int v,int x)
{
    edge[L].to=v;
    edge[L].cost=x;
    edge[L].next=head[u];
    head[u]=L++;
    edger[Lr].to=u;
    edger[Lr].cost=x;
    edger[Lr].next=headr[v];
    headr[v]=Lr++;
}

inline int a_star(int s,int t)
{
    if (dist[s]==0x3f3f3f3f)
        return -1;
    std::priority_queue<states,std::vector<states>,cmp2> q;
    states tmp;
    tmp.id=s;
    tmp.cost=0;
    q.push(tmp);
    while (!q.empty())
    {
        states u=q.top();
        q.pop();
        num[u.id]++;
        if (num[t]==K)
            return u.cost;
        for (int i=head[u.id]; i!=-1; i=edge[i].next)
        {
            int v=edge[i].to;
            tmp.id=v;
            tmp.cost=u.cost+edge[i].cost;
            q.push(tmp);
        }
    }
    return -1;
}

int main()
{
    int n,m;
    scanf("%d%d",&n,&m);
    init(n);
    for (int i=0; i<m; i++)
    {
        int u,v,x;
        scanf("%d%d%d",&u,&v,&x);
        add_edge(u-1,v-1,x);
    }
    int s,t;
    scanf("%d%d%d",&s,&t,&K);
    if (s==t)
        ++K;
    dijkstra(t-1);
    printf("%d\n",a_star(s-1,t-1));
    return 0;
}
```

4.20 Kariv-Hakimi Algorithm

```
//Absolute Center of a graph, not only a tree
#include<cstdio>
#include<algorithm>
#include<vector>
#include<cstring>
#include<set>

#define MAXX 211
#define inf 0x3f3f3f3f

int e[MAXX][MAXX],dist[MAXX][MAXX];
double dp[MAXX],ta;
int ans,d;
int n,m,a,b;
int i,j,k;
typedef std::pair<int,int> pii;
std::vector<pii>vt[2];
bool done[MAXX];
typedef std::pair<double,int> pdi;
std::multiset<pdi>q;
int pre[MAXX];

int main()
{
    vt[0].reserve(MAXX);
```

```
vt[1].reserve(MAXX);
scanf("%d␣%d",&n,&m);
memset(e,0x3f,sizeof(e));
while(m--)
{
    scanf("%d␣%d␣%d",&i,&j,&k);
    e[i][j]=e[j][i]=std::min(e[i][j],k);
}
for(i=1;i<=n;++i)
    e[i][i]=0;
memcpy(dist,e,sizeof(dist));
for(k=1;k<=n;++k)
    for(i=1;i<=n;++i)
        for(j=1;j<=n;++j)
            dist[i][j]=std::min(dist[i][j],dist[i][k]+dist[
                k][j]);
ans=inf;
for(i=1;i<=n;++i)
    for(j=i;j<=n;++j)
        if(e[i][j]!=inf)
        {
            vt[0].resize(0);
            vt[1].resize(0);
            static int i;
            for(i=1;i<=n;++i)
                vt[0].push_back(pii(dist[::i][i],dist[j][i
                    ]));
            std::sort(vt[0].begin(),vt[0].end());
            for(i=0;i<vt[0].size();++i)
            {
                while(!vt[1].empty() && vt[1].back().second
                    <=vt[0][i].second)
                    vt[1].pop_back();
                vt[1].push_back(vt[0][i]);
            }
            d=inf;
            if(vt[1].size()==1)
                if(vt[1][0].first<vt[1][0].second)
                {
                    ta=0;
                    d=(vt[1][0].first<<1);
                }
                else
                {
                    ta=e[::i][j];
                    d=(vt[1][0].second<<1);
                }
            else
                for(i=1;i<vt[1].size();++i)
                    if(d>e[::i][j]+vt[1][i-1].first+vt[1][i
                        ].second)
                    {
                        ta=(e[::i][j]+vt[1][i].second-vt
                            [1][i-1].first)/(double)2.0f;
                        d=e[::i][j]+vt[1][i-1].first+vt[1][
                            i].second;
                    }
            if(d<ans)
            {
                ans=d;
                a=::i;
                b=j;
                dp[::i]=ta;
                dp[j]=e[::i][j]-ta;
            }
        }
printf("%d\n",ans);
for(i=1;i<=n;++i)
    if(i!=a && i!=b)
        dp[i]=1e20;
q.insert(pdi(dp[a],a));
if(a!=b)
    q.insert(pdi(dp[b],b));
if(a!=b)
    pre[b]=a;
while(!q.empty())
{
    k=q.begin()->second;
    q.erase(q.begin());
    if(done[k])
        continue;
    done[k]=true;
    for(i=1;i<=n;++i)
        if(e[k][i]!=inf && dp[k]+e[k][i]<dp[i])
        {
            dp[i]=dp[k]+e[k][i];
            q.insert(pdi(dp[i],i));
            pre[i]=k;
        }
}
vt[0].resize(0);
for(i=1;i<=n;++i)
    if(pre[i])
        if(i<pre[i])
            printf("%d␣%d\n",i,pre[i]);
        else
            printf("%d␣%d\n",pre[i],i);
    return 0;
}
```

4.21 Kuhn-Munkres algorithm

```
bool match(int u)//匈牙利
{
    vx[u]=true;
    for(int i=1;i<=n;++i)
        if(lx[u]+ly[i]==g[u][i]&&!vy[i])
        {
            vy[i]=true;
            if(!d[i]||match(d[i]))
            {
                d[i]=u;
                return true;
            }
        }
    return false;
}
inline void update()//
{
    int i,j;
    int a=1<<30;
    for(i=1;i<=n;++i)if(vx[i])
        for(j=1;j<=n;++j)if(!vy[j])
            a=min(a,lx[i]+ly[j]-g[i][j]);
    for(i=1;i<=n;++i)
    {
        if(vx[i])lx[i]-=a;
        if(vy[i])ly[i]+=a;
    }
}
void km()
{
    int i,j;
    for(i=1;i<=n;++i)
    {
        lx[i]=ly[i]=d[i]=0;
        for(j=1;j<=n;++j)
            lx[i]=max(lx[i],g[i][j]);
    }
    for(i=1;i<=n;++i)
    {
        while(true)
        {
            memset(vx,0,sizeof(vx));
            memset(vy,0,sizeof(vy));
            if(match(i))
                break;
            update();
        }
    }
    int ans=0;
    for(i=1;i<=n;++i)
        if(d[i]!=0)
            ans+=g[d[i]][i];
    printf("%d\n",ans);
}
int main()
{
    while(scanf("%d\n",&n)!=EOF)
    {
        for(int i=1;i<=n;++i)gets(s[i]);
        memset(g,0,sizeof(g));
        for(int i=1;i<=n;++i)
            for(int j=1;j<=n;++j)
                if(i!=j) g[i][j]=cal(s[i],s[j]);
        km();
    }
    return 0;
}


//bupt

//算法：求二分图最佳匹配km n复杂度^3
int dfs(int u)//匈牙利求增广路
{
    int v;
    sx[u]=1;
    for ( v=1; v<=n; v++)
        if (!sy[v] && lx[u]+ly[v]==map[u][v])
        {
            sy[v]=1;
            if (match[v]==-1 || dfs(match[v]))
            {
                match[v]=u;
                return 1;
            }
        }
    return 0;
}
```

```
int bestmatch(void)//求最佳匹配km
{
    int i,j,u;
    for (i=1; i<=n; i++)//初始化顶标
    {
        lx[i]=-1;
        ly[i]=0;
        for (j=1; j<=n; j++)
            if (lx[i]<map[i][j])
                lx[i]=map[i][j];
    }
    memset(match, -1, sizeof(match));
    for (u=1; u<=n; u++)
    {
        while (true)
        {
            memset(sx,0,sizeof(sx));
            memset(sy,0,sizeof(sy));
            if (dfs(u))
                break;
            int dx=Inf;//若找不到增广路，则修改顶标~~
            for (i=1; i<=n; i++)
            {
                if (sx[i])
                    for (j=1; j<=n; j++)
                        if(!sy[j] && dx>lx[i]+ly[j]-map[i][j])
                            dx=lx[i]+ly[j]-map[i][j];
            }
            for (i=1; i<=n; i++)
            {
                if (sx[i])
                    lx[i]-=dx;
                if (sy[i])
                    ly[i]+=dx;
            }
        }
    }
    int sum=0;
    for (i=1; i<=n; i++)
        sum+=map[match[i]][i];
    return sum;
}
```

4.22 LCA - DA

```
int edge[MAXX],nxt[MAXX<<1],to[MAXX<<1],cnt;
int pre[MAXX][N],dg[MAXX];

inline void add(int j,int k)
{
    nxt[++cnt]=edge[j];
    edge[j]=cnt;
    to[cnt]=k;
}

void rr(int now,int fa)
{
    dg[now]=dg[fa]+1;
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa)
        {
            static int j;
            j=1;
            for(pre[to[i]][0]=now;j<N;++j)
                pre[to[i]][j]=pre[pre[to[i]][j-1]][j-1];
            rr(to[i],now);
        }
}

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=pre[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(pre[a][i]!=pre[b][i])
        {
            a=pre[a][i];
            b=pre[b][i];
        }
    return pre[a][0];

// looks like above is a wrong version

    static int i,log;
    for(log=0;(1<<(log+1))<=dg[a];++log);
    for(i=log;i>=0;--i)
        if(dg[a]-(1<<i)>=dg[b])
            a=pre[a][i];
    if(a==b)
        return a;
    for(i=log;i>=0;--i)
        if(pre[a][i]!=-1 && pre[a][i]!=pre[b][i])
            a=pre[a][i],b=pre[b][i];
    return pre[a][0];
}
```

4.23 LCA - tarjan - minmax

```
#include<cstdio>
#include<list>
#include<algorithm>
#include<cstring>

#define MAXX 100111
#define inf 0x5fffffff

short T,t;
int set[MAXX],min[MAXX],max[MAXX],ans[2][MAXX];
bool done[MAXX];
std::list<std::pair<int,int> >edge[MAXX];
std::list<std::pair<int,int> >q[MAXX];
int n,i,j,k,l,m;

struct node
{
    int a,b,id;
    node() {}
    node(const int &aa,const int &bb,const int &idd): a(aa),b(
        bb),id(idd){}
};

std::list<node>to[MAXX];

int find(const int &a)
{
    if(set[a]==a)
        return a;
    int b(set[a]);
    set[a]=find(set[a]);
    max[a]=std::max(max[a],max[b]);
    min[a]=std::min(min[a],min[b]);
    return set[a];
}

void tarjan(const int &now)
{
    done[now]=true;
    for(std::list<std::pair<int,int> >::const_iterator it(q[now
        ].begin());it!=q[now].end();++it)
        if(done[it->first])
            if(it->second>0)
                to[find(it->first)].push_back(node(now,it->
                    first,it->second));
            else
                to[find(it->first)].push_back(node(it->first,
                    now,-it->second));
    for(std::list<std::pair<int,int> >::const_iterator it(edge[
        now].begin());it!=edge[now].end();++it)
        if(!done[it->first])
        {
            tarjan(it->first);
            set[it->first]=now;
            min[it->first]=it->second;
            max[it->first]=it->second;
        }
    for(std::list<node>::const_iterator it(to[now].begin());it
        !=to[now].end();++it)
    {
        find(it->a);
        find(it->b);
        ans[0][it->id]=std::min(min[it->b],min[it->a]);
        ans[1][it->id]=std::max(max[it->a],max[it->b]);
    }
}

int main()
{
    scanf("%hd",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d",&n);
        for(i=1;i<=n;++i)
        {
            edge[i].clear();
            q[i].clear();
            to[i].clear();
            done[i]=false;
            set[i]=i;
            min[i]=inf;
            max[i]=0;
        }
        for(i=1;i<n;++i)
        {
```

```
            scanf("%d%d%d",&j,&k,&l);
            edge[j].push_back(std::make_pair(k,l));
            edge[k].push_back(std::make_pair(j,l));
        }
        scanf("%d",&m);
        for(i=0;i<m;++i)
        {
            scanf("%d␣%d",&j,&k);
            q[j].push_back(std::make_pair(k,i));
            q[k].push_back(std::make_pair(j,-i));
        }
        tarjan(1);
        printf("Case␣%hd:\n",t);
        for(i=0;i<m;++i)
            printf("%d␣%d\n",ans[0][i],ans[1][i]);
    }
    return 0;
}
```

4.24 Minimum Ratio Spanning Tree

```
#include<cstdio>
#include<cstring>
#include<cmath>

#define MAXX 1111

struct
{
    int x,y;
    double z;
} node[MAXX];

struct
{
    double l,c;
} map[MAXX][MAXX];

int n,l,f[MAXX],pre[MAXX];
double dis[MAXX];

double mst(double x)
{
    int i,j,tmp;
    double min,s=0,t=0;
    memset(f,0,sizeof(f));
    f[1]=1;
    for (i=2; i<=n; i++)
    {
        dis[i]=map[1][i].c-map[1][i].l*x;
        pre[i]=1;
    }
    for (i=1; i<n; i++)
    {
        min=1e10;
        for (j=1; j<=n; j++)
            if (!f[j] && min>dis[j])
            {
                min=dis[j];
                tmp=j;
            }
        f[tmp]=1;
        t+=map[pre[tmp]][tmp].l;
        s+=map[pre[tmp]][tmp].c;
        for (j=1; j<=n; j++)
            if (!f[j] && map[tmp][j].c-map[tmp][j].l*x<dis[j])
            {
                dis[j]=map[tmp][j].c-map[tmp][j].l*x;
                pre[j]=tmp;
            }
    }
    return s/t;
}

int main()
{
    int i,j;
    double a,b;
    while (scanf("%d",&n),n);
    {
        for (i=1; i<=n; i++)
            scanf("%d%d%lf",&node[i].x,&node[i].y,&node[i].z);
        for (i=1; i<=n; i++)
            for (j=i+1; j<=n; j++)
            {
                map[j][i].l=map[i][j].l=sqrt(1.0*(node[i].x-
                    node[j].x)*(node[i].x-node[j].x)+(node[i].
                    y-node[j].y)*(node[i].y-node[j].y));
                map[j][i].c=map[i][j].c=fabs(node[i].z-node[j].
                    z);
            }
        a=0,b=mst(a);
        while (fabs(b-a)>1e-8)
        {
            a=b;
            b=mst(a);
        }
        printf("%.3lf\n",b);
    }
    return 0;

}
```

4.25 Minimum Steiner Tree

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<queue>

#define MAXX 211
#define MAXE 10111
#define inf 0x3f3f3f3f

int edge[MAXX],nxt[MAXE],to[MAXE],wg[MAXE],cnt;
inline void add(int a,int b,int c)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    wg[cnt]=c;
}

int dp[1<<8];
int s[MAXX];
int d[1<<8][MAXX];
int S[MAXX],P[MAXX];
int fac[8];

struct node
{
    int a,b,dist;
    node(){}
    node(int i,int j,int k):a(i),b(j),dist(k){}
    bool operator<(const node &i)const
    {
        return dist>i.dist;
    }
    int &get()
    {
        return d[b][a];
    }
}now;

std::priority_queue<node>q;

int n,m,nn,i,j,k;
int cs,cf,x,y;
int ans,cst;

inline bool check(int x)
{
    static int re,i;
    for(i=re=0;x;x>>=1,++i)
        re+=(x&1)*(i<cf?fac[i]:-1);
    return re>=0;
}

inline int count(int x)
{
    static int i,re;
    x>>=cf;
    for(re=0;x;x>>=1)
        re+=(x&1);
    return re;
}

int main()
{
    while(scanf("%d",&n)!=EOF)
    {
        memset(s,0,sizeof s);
        memset(d,0x3f,sizeof d);
        memset(dp,0x3f,sizeof dp);
        ans=cnt=cf=cs=0;
        memset(edge,0,sizeof edge);
        for(i=1;i<=n;++i)
        {
            scanf("%d␣%d",P+i,S+i);
            if(S[i] && P[i])
            {
                ++ans;
                --P[i];
                S[i]=0;
            }
            if(P[i])
            {
                s[i]=1<<cf;
                fac[cf]=P[i];
                d[s[i]][i]=0;
```

```
                ++cf;
            }
        }
        for(i=1;i<=n;++i)
            if(S[i])
            {
                s[i]=1<<(cf+cs);
                d[s[i]][i]=0;
                ++cs;
            }
        nn=1<<(cf+cs);
        scanf("%d",&m);
        while(m--)
        {
            scanf("%d %d %d",&i,&j,&k);
            add(i,j,k);
            add(j,i,k);
        }
        for(y=1;y<nn;++y)
        {
            for(x=1;x<=n;++x)
            {
                if(s[x] && !(s[x]&y))
                    continue;
                for(i=(y-1)&y;i;i=(i-1)&y)
                    d[y][x]=std::min(d[y][x],d[i|s[x]][x]+d[(y^
                        i)|s[x]][x]);
                if(d[y][x]!=inf)
                    q.push(node(x,y,d[y][x]));
            }
            while(!q.empty())
            {
                now=q.top();
                q.pop();
                if(now.dist!=now.get())
                    continue;
                static int x,y,a,b;
                x=now.a;
                y=now.b;
                for(i=edge[x];i;i=nxt[i])
                {
                    a=to[i];
                    b=y|s[a];
                    if(d[b][a]>now.get()+wg[i])
                    {
                        d[b][a]=now.get()+wg[i];
                        if(b==y)
                            q.push(node(a,b,d[b][a]));
                    }
                }
            }
        }
        for(j=0;j<nn;++j)
            dp[j]=*std::min_element(d[j]+1,d[j]+1+n);
        cnt=cst=0;
        for(i=1;i<nn;++i)
            if(check(i))
            {
                for(j=(i-1)&i;j;j=(j-1)&i)
                    if(check(j) && check(i^j))
                        dp[i]=std::min(dp[i],dp[j]+dp[i^j]);
                k=count(i);
                if(dp[i]!=inf && (k>cnt || (k==cnt && dp[i]<cst
                    )))
                {
                    cnt=k;
                    cst=dp[i];
                }
            }
        printf("%d %d\n",ans+cnt,cst);
    }
    return 0;
}
```

4.26 Minimum-cost flow problem

```
// like Edmonds-Karp Algorithm
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<queue>

#define MAXX 5011
#define MAXE (MAXX*10*2)
#define inf 0x3f3f3f3f

int edge[MAXX],nxt[MAXE],to[MAXE],cap[MAXE],cst[MAXE],cnt;
#define v to[i]
inline void adde(int a,int b,int c,int d)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    cst[cnt]=d;
}
inline void add(int a,int b,int c,int d)
{ adde(a,b,c,d);adde(b,a,0,-d);}

int dist[MAXX],pre[MAXX];
int source,sink;
std::queue<int>q;
bool in[MAXX];

inline bool go()
{
    static int now,i;
    memset(dist,0x3f,sizeof dist);
    dist[source]=0;
    pre[source]=-1;
    q.push(source);
    in[source]=true;
    while(!q.empty())
    {
        in[now=q.front()]=false;
        q.pop();
        for(i=edge[now];i!=-1;i=nxt[i])
            if(cap[i] && dist[v]>dist[now]+cst[i])
            {
                dist[v]=dist[now]+cst[i];
                pre[v]=i;
                if(!in[v])
                {
                    q.push(v);
                    in[v]=true;
                }
            }
    }
    return dist[sink]!=inf;
}

inline int mcmf(int &flow)
{
    static int ans,i;
    flow=ans=0;
    while(go())
    {
        static int min;
        min=inf;
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            min=std::min(min,cap[i]);
        flow+=min;
        ans+=min*dist[sink];
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
        {
            cap[i]-=min;
            cap[i^1]+=min;
        }
    }
    return ans;
}
```

4.27 Second-best MST

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXN 511
#define MAXM 2500111
#define v to[i]

int set[MAXN];
int find(int a)
{
    return set[a]?set[a]=find(set[a]):a;
}

int n,m,i,j,k,ans;

struct edge
{
    int a,b,c;
    bool in;
    bool operator<(const edge &i)const
    {
        return c<i.c;
    }
}ed[MAXM];

int map[MAXN][MAXN];
bool done[MAXN];

int head[MAXN],to[MAXN<<1],nxt[MAXN<<1],wg[MAXN<<1],cnt;
inline void add(int a,int b,int c)
{
    nxt[++cnt]=head[a];
    head[a]=cnt;
    to[cnt]=b;
    wg[cnt]=c;
```

```
}

void dfs(const int now,const int fa)
{
    done[now]=true;
    for(int i(head[now]);i;i=nxt[i])
        if(v!=fa)
        {
            for(int j(1);j<=n;++j)
                if(done[j])
                    map[v][j]=map[j][v]=std::max(map[j][now],wg
                        [i]);
            dfs(v,now);
        }
}

int main()
{
    scanf("%d␣%d",&n,&m);
    for(i=0;i<m;++i)
        scanf("%d␣%d␣%d",&ed[i].a,&ed[i].b,&ed[i].c);
    std::sort(ed,ed+m);
    for(i=0;i<m;++i)
        if(find(ed[i].a)!=find(ed[i].b))
        {
            j+=ed[i].c;
            ++k;
            set[find(ed[i].a)]=find(ed[i].b);
            ed[i].in=true;
            add(ed[i].a,ed[i].b,ed[i].c);
            add(ed[i].b,ed[i].a,ed[i].c);
        }
    if(k+1!=n)
        puts("Cost:␣-1\nCost:␣-1");
    else
    {
        printf("Cost:␣%d\n",j);
        if(m==n-1)
        {
            puts("Cost:␣-1");
            return 0;
        }
        ans=0x3f3f3f3f;
        memset(map,0x3f,sizeof map);
        for(i=1;i<=n;++i)
            map[i][i]=0;
        dfs(1,0);
        for(i=0;i<m;++i)
            if(!ed[i].in)
                ans=std::min(ans,j+ed[i].c-map[ed[i].a][ed[i].b
                    ]);
        printf("Cost:␣%d\n",ans);
    }
    return 0;
}
```

4.28 Spanning Tree

- Minimum Bottleneck Spanning Tree
  Kruscal
- All-pairs vertexes' Minimum Bottleneck Path
  DP in the Kruscal's MST
  O(n^2)*O(1)
- Minimum Diameter Spanning Tree
  Kariv-Hakimi Algorithm
- Directed MST
  Chu–Liu/Edmonds' Algorithm
- Second-best MST
  get All-pairs vertexes' Minimum Bottleneck Path, then enumerate all no-tree-edges to replace the longest edge between two vertexes to get a worse MST
- Degree-constrained MST
  1. remove the vertex from the whole graph,then add edges to increase degrees and connect different connected components together (O(mlogm + n) with kruscal)
  2. if we can't connect all connected components together, there exists no any spanning tree
  3. next step is add edges to root vertex greedily, increase degrees, and decrease our answer (O(k*n))
  4. need all vertexes' minimum bottleneck path to root vertex
- Minimum Ratio Spanning Tree
  Binary search
- Manhattan MST
  combining line sweep with divide-and-conquer algorithm
- Minimum Steiner Tree
  the MST contain all k vertexes
  1. bit-mask with dijkstra O($2^k \times \{dijkstra\}$)
  2. then run a bit-mask DP(O(n*(2^k)))
- Count Spanning Trees
  Kirchhoff's theorem
  simply calculate the minor of (degree Matrix - edge Matrix)
- k-best MST
  do like second-best MST for k times

4.29 Stable Marriage

```
//对于每个预备队列中的对象，及被匹配对象，先按照喜好程度排列匹配对象

while(!g.empty())  // 预备匹配队列
{
    if(dfn[edge[g.front()].front()]==-1)
        dfn[edge[g.front()].front()]=g.front(); // 如果目前还没尝
            试匹配过的对象没有被任何别的对象占据
    else
    {
        for(it=edge[edge[g.front()].front()].begin();it!=edge[
            edge[g.front()].front()].end();++it)
            if(*it==dfn[edge[g.front()].front()] || *it==g.
                front()) //如果被匹配对象更喜欢正在被匹配的人或现在准
                备匹配的对象
                break;
        if(*it==g.front()) //如果更喜欢新的
        {
            g.push_back(dfn[edge[g.front()].front()]);
            dfn[edge[g.front()].front()]=g.front();
        }
        else
            g.push_back(g.front()); //否则放到队尾，重新等待匹配
    }
    edge[g.front()].pop_front(); //每组匹配最多只考虑一次
    g.pop_front();
}
```

4.30 Stoer-Wagner Algorithm

```
#include<cstdio>
#include<cstring>

const int maxn=510;

int map[maxn][maxn];
int n;

void contract(int x,int y)//合并两个点
{
    int i,j;
    for (i=0; i<n; i++)
        if (i!=x)
        {
            map[x][i]+=map[y][i];
            map[i][x]+=map[i][y];
        }
    for (i=y+1; i<n; i++)
        for (j=0; j<n; j++)
        {
            map[i-1][j]=map[i][j];
            map[j][i-1]=map[j][i];
        }
    n--;
}

int w[maxn],c[maxn];
int sx,tx;

int mincut() //求最大生成树，计算最后一个点的割，并保存最后一条边的两个顶
    点
```

```
{
    static int i,j,k,t;
    memset(c,0,sizeof(c));
    c[0]=1;
    for (i=0; i<n; i++)
        w[i]=map[0][i];
    for (i=1; i+1<n; i++)
    {
        t=k=-1;
        for (j=0; j<n; j++)
            if (c[j]==0&&w[j]>k)
                k=w[t=j];
        c[sx=t]=1;
        for (j=0; j<n; j++)
            w[j]+=map[t][j];
    }
    for (i=0; i<n; i++)
        if (c[i]==0)
            return w[tx=i];
}
int main()
{
    int i,j,k,m;
    while (scanf("%d%d",&n,&m)!=EOF)
    {
        memset(map,0,sizeof(map));
        while (m--)
        {
            scanf("%d%d%d",&i,&j,&k);
            map[i][j]+=k;
            map[j][i]+=k;
        }
        int mint=999999999;
        while (n>1)
        {
            k=mincut();
            if (k<mint) mint=k;
            contract(sx,tx);
        }
        printf("%d\n",mint);
    }
    return 0;
}
```

4.31 Strongly Connected Component

```
//缩点后注意自环
void dfs(const short &now)
{
    dfn[now]=low[now]=cnt++;
    st.push(now);
    for(std::list<short>::const_iterator it(edge[now].begin());
         it!=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it);
            low[now]=std::min(low[now],low[*it]);
        }
        else
            if(sc[*it]==-1)
                low[now]=std::min(low[now],dfn[*it]);
    if(dfn[now]==low[now])
    {
        while(sc[now]==-1)
        {
            sc[st.top()]=p;
            st.pop();
        }
        ++p;
    }
}
```

4.32 ZKW's Minimum-cost flow

```
#include<cstdio>
#include<algorithm>
#include<cstring>
#include<vector>
#include<deque>

#define MAXX 111
#define MAXN 211
#define MAXE (MAXN*MAXN*3)
#define inf 0x3f3f3f3f

char buf[MAXX];

int edge[MAXN],nxt[MAXE],to[MAXE],cap[MAXE],cst[MAXE],cnt;

inline void adde(int a,int b,int c,int k)
{
    nxt[cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    cst[cnt]=k;
    ++cnt;
}

inline void add(int a,int b,int c,int k)
{
    adde(a,b,c,k);
    adde(b,a,0,-k);
}

int n,mf,cost,pi1;
int source,sink;
bool done[MAXN];

int aug(int now,int maxcap)
{
    if(now==sink)
    {
        mf+=maxcap;
        cost+=maxcap*pi1;
        return maxcap;
    }
    done[now]=true;
    int l=maxcap;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i] && !cst[i] && !done[to[i]])
        {
            int d(aug(to[i],std::min(l,cap[i])));
            cap[i]-=d;
            cap[i^1]+=d;
            l-=d;
            if(!l)
                return maxcap;
        }
    return maxcap-l;
}

inline bool label()
{
    static int d,i,j;
    d=inf;
    for(i=1;i<=n;++i)
        if(done[i])
            for(j=edge[i];j!=-1;j=nxt[j])
                if(cap[j] && !done[to[j]] && cst[j]<d)
                    d=cst[j];
    if(d==inf)
        return false;
    for(i=1;i<=n;++i)
        if(done[i])
            for(j=edge[i];j!=-1;j=nxt[j])
            {
                cst[j]-=d;
                cst[j^1]+=d;
            }
    pi1+=d;
    return true;
    /* primal-dual approach
    static int d[MAXN],i,j;
    static std::deque<int>q;
    memset(d,0x3f,sizeof d);
    d[sink]=0;
    q.push_back(sink);
    while(!q.empty())
    {
        static int dt,now;
        now=q.front();
        q.pop_front();
        for(i=edge[now];i!=-1;i=nxt[i])
            if(cap[i^1] && (dt=d[now]-cst[i])<d[to[i]])
                if((d[to[i]]=dt)<=d[q.empty()?0:q.front()])
                    q.push_front(to[i]);
                else
                    q.push_back(to[i]);
    }
    for(i=1;i<=n;++i)
        for(j=edge[i];j!=-1;j=nxt[j])
            cst[j]+=d[to[j]]-d[i];
    pi1+=d[source];
    return d[source]!=inf;
    */
}

int m,i,j,k;
typedef std::pair<int,int> pii;
std::vector<pii>M(MAXN),H(MAXN);

int main()
{
    while(scanf("%d %d",&n,&m),(n||m))
    {
        M.resize(0);
        H.resize(0);
        for(i=0;i<n;++i)
        {
```

```
            scanf("%s",buf);
            for(j=0;j<m;++j)
                if(buf[j]=='m')
                    M.push_back(pii(i,j));
                else
                    if(buf[j]=='H')
                        H.push_back(pii(i,j));
        }
        n=M.size()+H.size();
        source=++n;
        sink=++n;
        memset(edge,-1,sizeof edge);
        cnt=0;
        for(i=0;i<M.size();++i)
            for(j=0;j<H.size();++j)
                add(i+1,j+1+M.size(),1,abs(M[i].first-H[j].
                    first)+abs(M[i].second-H[j].second));
        for(i=0;i<M.size();++i)
            add(source,i+1,1,0);
        for(i=0;i<H.size();++i)
            add(i+1+M.size(),sink,1,0);
        mf=cost=pi1=0;
        do
            do
                memset(done,0,sizeof done);
            while(aug(source,inf));
        while(label());
        /* primal-dual approach
        while(label())
            do
                memset(done,0,sizeof done);
            while(aug(source,inf));
        */
        printf("%d\n",cost);
    }
    return 0;
}
```

5 Math

5.1 cantor

```
const int PermSize = 12;
int fac[PermSize] = {1, 1, 2, 6, 24, 120, 720, 5040, 40320,
     362880, 3628800, 39916800};

inline int Cantor(int a[])
{
    int i, j, cnt;
    int res = 0;
    for (i = 0; i < PermSize; ++i)
    {
        cnt = 0;
        for (j = i + 1; j < PermSize; ++j)
            if (a[i] > a[j])
                ++cnt;
        res = res + cnt * fac[PermSize - i - 1];
    }
    return res;
}

bool h[13];

inline void UnCantor(int x, int res[])
{
    int i,j,l,t;
    for (i = 1;i <= 12;i++)
        h[i] = false;
    for (i = 1; i <= 12; i++)
    {
        t = x / fac[12 - i];
        x -= t * fac[12 - i];
        for (j = 1, l = 0; l <= t; j++)
            if (!h[j])
                l++;
        j--;
        h[j] = true;
        res[i - 1] = j;
    }
}
```

5.2 discrete logarithms - BSGS

```
//The running time of BSGS and the space complexity is O(√n)
//Pollard's rho algorithm for logarithms' running time is
     approximately O(√p) where p is n's largest prime factor.
#include<cstdio>
#include<cmath>
#include<cstring>

struct Hash // std::map is bad. clear() 时会付出巨大的代价
{
    static const int mod=100003; // prime is good
    static const int MAXX=47111; // bigger than √c
    int hd[mod],nxt[MAXX],cnt;
    long long v[MAXX],k[MAXX]; // a^k ≡ v (mod c)
    inline void init()
    {
        memset(hd,0,sizeof hd);
        cnt=0;
    }
    inline long long find(long long v)
    {
        static int now;
        for(now=hd[v%mod];now;now=nxt[now])
            if(this->v[now]==v)
                return k[now];
        return -1ll;
    }
    inline void insert(long long k,long long v)
    {
        if(find(v)!=-1ll)
            return;
        nxt[++cnt]=hd[v%mod];
        hd[v%mod]=cnt;
        this->v[cnt]=v;
        this->k[cnt]=k;
    }
}hash;

long long gcd(long long a,long long b)
{
    return b?gcd(b,a%b):a;
}

long long exgcd(long long a,long long b,long long &x,long long
     &y)
{
    if(b)
    {
        long long re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1ll;
    y=0ll;
    return a;
}

inline long long bsgs(long long a,long long b,long long c) //
     a^x ≡ b
     (mod c)
{
    static long long x,y,d,g,m,am,k;
    static int i,cnt;
    a%=c;
    b%=c;
    x=1ll%c; // if c==1....
    for(i=0;i<100;++i)
    {
        if(x==b)
            return i;
        x=(x*a)%c;
    }
    d=1ll%c;
    cnt=0;
    while((g=gcd(a,c))!=1ll)
    {
        if(b%g)
            return -1ll;
        ++cnt;
        c/=g;
        b/=g;
        d=a/g*d%c;
    }
    hash.init();
    m=sqrt((double)c); // maybe need a ceil
    am=1ll%c;
    hash.insert(0,am);
    for(i=1;i<=m;++i)
    {
        am=am*a%c;
        hash.insert(i,am);
    }
    for(i=0;i<=m;++i)
    {
        g=exgcd(d,c,x,y);
        x=(x*b/g%c+c)%c;
        k=hash.find(x);
        if(k!=-1ll)
            return i*m+k+cnt;
        d=d*am%c;
    }
    return -1ll;
}

long long k,p,n;
```

```
int main()
{
    while(scanf("%lld␣%lld␣%lld",&k,&p,&n)!=EOF)
    {
        if(n>p || (k=bsgs(k,n,p))==-1ll)
            puts("Orz,I␣' cant␣find␣D!");
        else
            printf("%lld\n",k);
    }
    return 0;
}
```

5.3 extended euclidean algorithm

```
//返回ax+by=gcd(a,b)的一组解
long long ex_gcd(long long a,long long b,long long &x,long long
     &y)
{
    if (b)
    {
        long long ret = ex_gcd(b,a%b,x,y),tmp = x;
        x = y;
        y = tmp-(a/b)*y;
        return ret;
    }
    else
    {
        x = 1;
        y = 0;
        return a;
    }
}
```

5.4 Fast Fourier Transform

```
#include<cstdio>
#include<cstring>
#include<complex>
#include<vector>
#include<algorithm>

#define MAXX 100111
#define MAXN (MAXX<<2)

int T;
int n,i,j,k;

typedef std::complex<long double> com;
std::vector<com>x(MAXN);
int a[MAXX];
long long pre[MAXN],cnt[MAXN];
long long ans;

inline void fft(std::vector<com> &y,int sign)
{
    static int i,j,k,h;
    static com u,t,w,wn;
    for(i=1,j=y.size()/2;i+1<y.size();++i)
    {
        if(i<j)
            std::swap(y[i],y[j]);
        k=y.size()/2;
        while(j>=k)
        {
            j-=k;
            k/=2;
        }
        if(j<k)
            j+=k;
    }
    for(h=2;h<=y.size();h<<=1)
    {
        wn=com(cos(-sign*2*M_PI/h),sin(-sign*2*M_PI/h));
        for(j=0;j<y.size();j+=h)
        {
            w=com(1,0);
            for(k=j;k<j+h/2;++k)
            {
                u=y[k];
                t=w*y[k+h/2];
                y[k]=u+t;
                y[k+h/2]=u-t;
                w*=wn;
            }
        }
    }
    if(sign==-1)
        for(i=0;i<y.size();++i)
            y[i]=com(y[i].real()/y.size(),y[i].imag());
}

int main()
{
    scanf("%d",&T);
    while(T--)
    {
        memset(cnt,0,sizeof cnt);
        scanf("%d",&n);
        for(i=0;i<n;++i)
        {
            scanf("%d",a+i);
            ++cnt[a[i]];
        }
        std::sort(a,a+n);
        k=a[n-1]+1;
        for(j=1;j<(k<<1);j<<=1);// size must be such many
        x.resize(0);
        for(i=0;i<k;++i)
            x.push_back(com(cnt[i],0));
        x.insert(x.end(),j-k,com(0,0));

        fft(x,1);
        for(i=0;i<x.size();++i)
            x[i]=x[i]*x[i];
        fft(x,-1);
        /*
        if we need to combine 2 arrays
        fft(x,1);
        fft(y,1);
        for(i=0;i<x.size();++i)
            x[i]=x[i]*y[i];
        fft(x,-1);
        */

        for(i=0;i<x.size();++i)
            cnt[i]=ceil(x[i].real()); //  maybe we need (x[i].
                 real()+0.5f) or nearbyint(x[i].real())
        x.resize(2*a[n-1]); // result here
    }
    return 0;
}
```

5.5 Gaussian elimination

```
#define N

inline int ge(int a[N][N],int n) // 返回系数矩阵的秩
{
    static int i,j,k,l;
    for(j=i=0;j<n;++j) //第 i 行, 第 j 列
    {
        for(k=i;k<n;++k)
            if(a[k][j])
                break;
        if(k==n)
            continue;
        for(l=0;l<=n;++l)
            std::swap(a[i][l],a[k][l]);
        for(l=0;l<=n;++l)
            if(l!=i && a[l][j])
                for(k=0;k<=n;++k)
                    a[l][k]^=a[i][k];
        ++i;
    }
    for(j=i;j<n;++j)
        if(a[j][n])
            return -1; //无解
    return i;
}
/*
 */

void dfs(int v)
{
    if(v==n)
    {
        static int x[MAXX],ta[MAXX][MAXX];
        static int tmp;
        memcpy(x,ans,sizeof(x));
        memcpy(ta,a,sizeof(ta));
        for(i=l-1;i>=0;--i)
        {
            for(j=i+1;j<n;++j)
                ta[i][n]^=(x[j]&&ta[i][j]); //迭代消元求解
            x[i]=ta[i][n];
        }
        for(tmp=i=0;i<n;++i)
            if(x[i])
                ++tmp;
        cnt=std::min(cnt,tmp);
        return;
    }
    ans[v]=0;
    dfs(v+1);
    ans[v]=1;
    dfs(v+1);
}

inline int ge(int a[N][N],int n)
```

```
{
    static int i,j,k,l;
    for(i=j=0;j<n;++j)
    {
        for(k=i;k<n;++k)
            if(a[k][i])
                break;
        if(k<n)
        {
            for(l=0;l<=n;++l)
                std::swap(a[i][l],a[k][l]);
            for(k=0;k<n;++k)
                if(k!=i && a[k][i])
                    for(l=0;l<=n;++l)
                        a[k][l]^=a[i][l];
            ++i;
        }
        else //将不定元交换到后面去
        {
            l=n-1-j+i;
            for(k=0;k<n;++k)
                std::swap(a[k][l],a[k][i]);
        }
    }
    if(i==n)
    {
        for(i=cnt=0;i<n;++i)
            if(a[i][n])
                ++cnt;
        printf("%d\n",cnt);
        continue;
    }
    for(j=i;j<n;++j)
        if(a[j][n])
            break;
    if(j<n)
        puts("impossible");
    else
    {
        memset(ans,0,sizeof(ans));
        cnt=111;
        dfs(l=i);
        printf("%d\n",cnt);
    }
}

/*
 */
inline int ge(int n,int m)
{
    static int i,j,r,c;
    static double mv;
    for(r=c=0;r<n && c<m;++r,++c)
    {
        for(mv=0,i=r;i<n;++i)
            if(fabs(mv)<fabs(a[i][c]))
                mv=a[j=i][c];
        if(fabs(mv)<eps) // important
        {
            --r;
            continue;
        }
        for(i=0;i<=m;++i)
            std::swap(a[r][i],a[j][i]);
        for(j=c+1;j<=m;++j)
        {
            a[r][j]/=mv;
            for(i=r+1;i<n;++i)
                a[i][j]-=a[i][c]*a[r][j];
        }
    }
    for(i=r;i<n;++i)
        if(fabs(a[i][m])>eps)
            return -1;
    if(r<m) // rank
        return m-r;
    for(i=m-1;i>=0;--i)
        for(j=i+1;j<m;++j)
            a[i][m]-=a[i][j]*a[j][m];  // answer will be a[i][m
                ]
    return 0;
}
```

5.6 Integration

//平滑曲线 f(x) 在 [a,b] 区间的长度 $\int_a^b \sqrt{[f'(x)]^2+1}\,\mathrm{d}x$

```
// simpson 公式用到的函数
double F(double x) {
  return sqrt(1 + 4*a*a*x*x);
}

// 三点 simpson 法。这里要求 F 是一个全局函数
double simpson(double a, double b) {
  double c = a + (b-a)/2;
  return (F(a)+4*F(c)+F(b))*(b-a)/6;
}

// 自适应 Simpson 公式（递归过程）。已知整个区间 [a,b] 上的三点 simpson
    值 A
double asr(double a, double b, double eps, double A) {
  double c = a + (b-a)/2;
  double L = simpson(a, c), R = simpson(c, b);
  if(fabs(L+R-A) <= 15*eps)
      return L+R+(L+R-A)/15.0;
  return asr(a, c, eps/2, L) + asr(c, b, eps/2, R);
}

// 自适应 Simpson 公式（主过程）
double asr(double a, double b, double eps)
{
  return asr(a, b, eps, simpson(a, b));
}

// 用自适应 Simpson 公式计算宽度为 w，高度为 h 的抛物线长
double parabola_arc_length(double w, double h)
{
  a = 4.0*h/(w*w); // 修改全局变量 a，从而改变全局函数 F 的行为
  return asr(0, w/2, 1e-5)*2;
}

// thx for mzry
inline double f(double)
{
    /*
    define the function
    */
}

inline double simp(double l,double r)
{
    double h = (r-l)/2.0;
    return h*(f(l)+4*f((l+r)/2.0)+f(r))/3.0;
}

inline double rsimp(double l,double r) // call here
{
    double mid = (l+r)/2.0;
    if(fabs((simp(l,r)-simp(l,mid)-simp(mid,r)))/15 < eps)
        return simp(l,r);
    else
        return rsimp(l,mid)+rsimp(mid,r);
}

//Romberg

/* Romberg 求定积分
 * 输入：积分区间 [a,b]，被积函数 f(x,y,z)
 * 输出：积分结果
 * f(x,y,z) 示例：
 * double f0( double x, double l, double t )
 * {
 * return sqrt(1.0+l*l*t*t*cos(t*x)*cos(t*x));
 * }
 */
double Integral(double a, double b, double (*f)(double x,
     double y, double z), double eps, double l, double t);

inline double Romberg (double a, double b, double (*f)(double x
     , double y, double z), double eps, double l, double t)
{
#define MAX_N 1000
    int i, j, temp2, min;
    double h, R[2][MAX_N], temp4;
    for (i=0; i<MAX_N; i++)
    {
        R[0][i] = 0.0;
        R[1][i] = 0.0;
    }
    h = b-a;
    min = (int)(log(h*10.0)/log(2.0)); //h should be at most
         0.1
    R[0][0] = ((*f)(a, l, t)+(*f)(b, l, t))*h*0.50;
    i = 1;
    temp2 = 1;
    while (i<MAX_N)
    {
        i++;
        R[1][0] = 0.0;
        for (j=1; j<=temp2; j++)
            R[1][0] += (*f)(a+h*((double)j-0.50), l, t);
        R[1][0] = (R[0][0] + h*R[1][0])*0.50;
        temp4 = 4.0;
        for (j=1; j<i; j++)
        {
            R[1][j] = R[1][j-1] + (R[1][j-1]-R[0][j-1])/(temp4
                 -1.0);
            temp4 *= 4.0;
        }
```

```
        if ((fabs(R[1][i-1]-R[0][i-2])<eps) && (i>min))
            return R[1][i-1];
        h *= 0.50;
        temp2 *= 2;
        for (j=0; j<i; j++)
            R[0][j] = R[1][j];
    }
    return R[1][MAX_N-1];
}

inline double Integral(double a, double b, double (*f)(double x
     , double y, double z), double eps, double l, double t)
{
    const double pi(acos(-1.0f));
    int n;
    double R, p, res;
    n = (int)(floor)(b * t * 0.50 / pi);
    p = 2.0 * pi / t;
    res = b - (double)n * p;
    if (n)
        R = Romberg (a, p, f0, eps/(double)n, l, t);
    R = R * (double)n + Romberg( 0.0, res, f0, eps, l, t );
    return R/100.0;
}

//
inline double romberg(double a,double b)
{
#define MAXN 111
    double t[MAXN][MAXN];
    int n,k,i,m;
    double h,g,p;
    h=(double)(b-a)/2;
    t[0][0]=h*(func(a)+func(b));
    k=n=1;
    do
    {
        g=0;
        for(i=1;i<=n;i++)
            g+=func((a+((2*i-1)*h)));
        t[k][0]=(t[k-1][0]/2)+(h*g);
        p = 1.0;
        for(m=1;m<=k;m++)
        {
            p=p*4.0f;
            t[k-m][m]=(p*t[k-m+1][m-1]-t[k-m][m-1])/(p-1);
        }
        m-=1;
        h/=2;
        n*=2;
        k+=1;

    }
    while (fabs(t[0][m]-t[0][m-1])>eps);
    return t[0][m];
}
```

5.7 inverse element

```
inline void getInv2(int x,int mod)
{
    inv[1]=1;
    for (int i=2; i<=x; i++)
        inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
}

long long inv(long long x)// likes above one
{
    return x <= 1ll ? x : (mod - mod / x) * inv(mod % x) % mod;
}

inline long long power(long long x,long long y,int mod)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

inline int getInv(int x,int mod)//mod 为素数
{
    return power(x,mod-2,mod);
}

//谨慎来说，用 exgcd 更靠谱
void gcd(int n,int k,int &x,int &y)
{
    if(k)
    {
        gcd(k,n%k,x,y);
        int t=x;
        x=y;
        y=t-(n/k)*y;
        return;
    }
    x=1;
    y=0;
}

inline int inv(int b,int mod)
{
    static int x,y;
    gcd(b,mod,x,y);
    if(x<0)
        x+=mod;
    return x;
}
```

5.8 Linear programming

```
#include<cstdio>
#include<cstring>
#include<cmath>
#include<algorithm>

#define MAXN 33
#define MAXM 33
#define eps 1e-8

double a[MAXN][MAXM],b[MAXN],c[MAXM];
double x[MAXM],d[MAXN][MAXM];
int ix[MAXN+MAXM];
double ans;
int n,m;
int i,j,k,r,s;
double D;

inline bool simplex()
{
    r=n;
    s=m++;
    for(i=0;i<n+m;++i)
        ix[i]=i;
    memset(d,0,sizeof d);
    for(i=0;i<n;++i)
    {
        for(j=0;j+1<m;++j)
            d[i][j]=-a[i][j];
        d[i][m-1]=1;
        d[i][m]=b[i];
        if(d[r][m]>d[i][m])
            r=i;
    }
    for(j=0;j+1<m;++j)
        d[n][j]=c[j];
    d[n+1][m-1]=-1;
    while(true)
    {
        if(r<n)
        {
            std::swap(ix[s],ix[r+m]);
            d[r][s]=1./d[r][s];
            for(j=0;j<=m;++j)
                if(j!=s)
                    d[r][j]*=-d[r][s];
            for(i=0;i<=n+1;++i)
                if(i!=r)
                {
                    for(j=0;j<=m;++j)
                        if(j!=s)
                            d[i][j]+=d[r][j]*d[i][s];
                    d[i][s]*=d[r][s];
                }
        }
        r=-1;
        s=-1;
        for(j=0;j<m;++j)
            if((s<0 || ix[s]>ix[j]) && (d[n+1][j]>eps || (d[n
                 +1][j]>-eps && d[n][j]>eps)))
                s=j;
        if(s<0)
            break;
        for(i=0;i<n;++i)
            if(d[i][s]<-eps && (r<0 || (D=(d[r][m]/d[r][s]-d[i
                 ][m]/d[i][s]))<-eps || (D<eps && ix[r+m]>ix[i+
                 m])))
                r=i;
        if(r<0)
            return false;
    }
    if(d[n+1][m]<-eps)
        return false;
    for(i=m;i<n+m;++i)
        if(ix[i]+1<m)
            x[ix[i]]=d[i-m][m]; // answer
    ans=d[n][m]; // maxium value
    return true;
}
```

```
int main()
{
    while(scanf("%d␣%d",&m,&n)!=EOF)
    {
        for(i=0;i<m;++i)
            scanf("%lf",c+i);  // max{ sum{c[i]*x[i]} }
        for(i=0;i<n;++i)
        {
            for(j=0;j<m;++j)
                scanf("%lf",a[i]+j); // sum{ a[i]*x[i] } <= b
            scanf("%lf",b+i);
            b[i]*=n;
        }
        simplex();
        printf("Nasa␣can␣spend␣%.0lf␣taka.\n",ceil(ans));
    }
    return 0;
}

/*
Simplex C(n+m)(n)
maximize:
    \sum_{i=1}^{n} (c[i] \times x[i])
subject to
    \forall i \in [1,m]
    \sum_{j=1}^{n} (a[i][j] \times x[j]) \leq rhs[i]
限制:
    传入的矩阵必须是标准形式的.
sample:
3 3
15 17 20
0 1 -1 2
3 3 5 15
3 2 1 8
out:
OPTIMAL
76.00000
x[ 1 ] = 0.333333
x[ 2 ] = 3.000000
x[ 3 ] = 1.000000
*/

#include <cstdio>
#include <cstring>
#include <cmath>

#define eps 1e-8
#define inf 1e15
#define OPTIMAL -1 //最优解
#define UNBOUNDED -2 //无边界的
#define FEASIBLE -3 //可行的
#define INFEASIBLE -4 //无解
#define PIVOT_OK 1 //还可以松弛

#define N 45 //变量个数
#define M 45 //约束个数

int basic[N],row[M],col[N];
double c0[N];

inline double dcmp(double x)
{
    if(x>eps)
        return 1;
    if(x<-eps)
        return -1;
    return 0;
}

inline int Pivot(int n,int m,double *c,double a[M][N],double *
    rhs,int &i,int &j)
{
    double min=inf;
    int k=-1;
    for(j=0;j<=n;j++)
        if(!basic[j] && dcmp(c[j])>0)
            if(k<0 || dcmp(c[j]-c[k])>0)
                k=j;
    j=k;
    if(k<0)
        return OPTIMAL;
    for(k=-1,i=1;i<=m;i++)
        if(dcmp(a[i][j])>0 && dcmp(rhs[i]/a[i][j]-min)<0)
        {
            min=rhs[i]/a[i][j];
            k=i;
        }
    i=k;
    if(k<0)
        return UNBOUNDED;
    return PIVOT_OK;
}

inline int PhaseII(int n,int m,double *c,double a[M][N],double
    *rhs,double &ans,int PivotIndex)
{
    static int i,j,k,l;
    static double tmp;
    while((k=Pivot(n,m,c,a,rhs,i,j))==PIVOT_OK || PivotIndex)
    {
        if(PivotIndex)
        {
            i=PivotIndex;
            j=PivotIndex=0;
        }
        basic[row[i]]=0;
        col[row[i]]=0;
        basic[j]=1;
        col[j]=i;
        row[i]=j;
        tmp=a[i][j];
        for(k=0;k<=n;k++)
            a[i][k]/=tmp;
        rhs[i]/=tmp;
        for(k=1;k<=m;k++)
            if(k!=i && dcmp(a[k][j]))
            {
                tmp=-a[k][j];
                for(l=0;l<=n;l++)
                    a[k][l]+=tmp*a[i][l];
                rhs[k]+=tmp*rhs[i];
            }
        tmp=-c[j];
        for(l=0;l<=n;l++)
            c[l]+=a[i][l]*tmp;
        ans-=tmp*rhs[i];
    }
    return k;
}

inline int PhaseI(int n,int m,double *c,double a[M][N],double *
    rhs,double &ans)
{
    int i,j,k=-1;
    double tmp,min=0,ans0=0;
    for(i=1;i<=m;i++)
        if(dcmp(rhs[i]-min)<0)
        {
            min=rhs[i];
            k=i;
        }
    if(k<0)
        return FEASIBLE;
    for(i=1;i<=m;i++)
        a[i][0]=-1;
    for(j=1;j<=n;j++)
        c0[j]=0;
    c0[0]=-1;
    PhaseII(n,m,c0,a,rhs,ans0,k);
    if(dcmp(ans0)<0)
        return INFEASIBLE;
    for(i=1;i<=m;i++)
        a[i][0]=0;
    for(j=1;j<=n;j++)
        if(dcmp(c[j]) && basic[j])
        {
            tmp=c[j];
            ans+=rhs[col[j]]*tmp;
            for(i=0;i<=n;i++)
                c[i]-=tmp*a[col[j]][i];
        }
    return FEASIBLE;
}
inline int simplex(int n,int m,double *c,double a[M][N],double
    *rhs,double &ans,double *x)
{
    int i,j,k;
    for(i=1;i<=m;i++)
    {
        for(j=n+1;j<=n+m;j++)
            a[i][j]=0;
        a[i][n+i]=1;
        a[i][0]=0;
        row[i]=n+i;
        col[n+i]=i;
    }
    k=PhaseI(n+m,m,c,a,rhs,ans);
    if(k==INFEASIBLE)
        return k; //无解
    k=PhaseII(n+m,m,c,a,rhs,ans,0);
    for(j=0;j<=n+m;j++)
        x[j] = 0;
    for(i=1;i<=m;i++)
        x[row[i]] = rhs[i];
    return k;
}

double c[M],ans,a[M][N],rhs[M],x[N];
```

```
int main()
{
    int i,j,n,m;
    while(scanf("%d%d",&n,&m)!=EOF)
    {
        for(int i=0;i<=n+m;i++)
        {
            for(int j=0;j<=n+m;j++)
                a[i][j]=0;
            basic[i]=0;
            row[i]=0;
            col[i]=0;
            c[i]=0;
            rhs[i]=0;
        }
        ans=0;

        for(j=1;j<=n;++j)
            scanf("%lf",c+j);
        for(i=1;i<=m;++i)
        {
            for(j=1;j<=n;++j)
                scanf("%lf",a[i]+j);
            scanf("%lf",rhs+i);
        }

        switch(simplex(n,m,c,a,rhs,ans,x))
        {
            case OPTIMAL:
                printf("Nasa␣can␣spend␣%.0f␣taka.\n",ceil(m*ans
                    ));
                //for(j=1;j<=n;j++)
                //    printf("x[ %2d ] = %10lf\n",j,x[j]);
                break;
            case UNBOUNDED:
                puts("UNBOUNDED");
                break;
            case INFEASIBLE:
                puts("INFEASIBLE");
                break;
        }
    }
    return 0;
}
```

5.9 Lucas' theorem(2)

```
#include<cstdio>
#include<cstring>
#include<iostream>

int mod;
long long num[100000];
int ni[100],mi[100];
int len;

void init(int p)
{
    mod=p;
    num[0]=1;
    for (int i=1; i<p; i++)
        num[i]=i*num[i-1]%p;
}

void get(int n,int ni[],int p)
{
    for (int i = 0; i < 100; i++)
        ni[i] = 0;
    int tlen = 0;
    while (n != 0)
    {
        ni[tlen++] = n%p;
        n /= p;
    }
    len = tlen;
}

long long power(long long x,long long y)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

long long getInv(long long x)//mod 为素数
{
    return power(x,mod-2);
}

long long calc(int n,int m,int p)//C(n,m)%p
{
    init(p);
    long long ans=1;
    for (; n && m && ans; n/=p,m/=p)
    {
        if (n%p>=m%p)
            ans = ans*num[n%p]%p *getInv(num[m%p]%p)%p *getInv(
                num[n%p-m%p])%p;
        else
            ans=0;
    }
    return ans;
}

int main()
{
    int t;
    scanf("%d",&t);
    while (t--)
    {
        int n,m,p;
        scanf("%d%d%d",&n,&m,&p);
        printf("%lld\n",calc(n+m,m,p));
    }
    return 0;
}
```

5.10 Lucas' theorem

```
#include <cstdio>
/*
   Lucas 快速求解C(n,m)%p
   */
void gcd(int n,int k,int &x,int &y)
{
    if(k)
    {
        gcd(k,n%k,x,y);
        int t=x;
        x=y;
        y=t-(n/k)*y;
        return;
    }
    x=1;
    y=0;
}

int CmodP(int n,int k,int p)
{
    if(k>n)
        return 0;
    int a,b,flag=0,x,y;
    a=b=1;
    for(int i=1;i<=k;i++)
    {
        x=n-i+1;
        y=i;
        while(x%p==0)
        {
            x/=p;
            ++flag;
        }
        while(y%p==0)
        {
            y/=p;
            --flag;
        }
        x%=p;
        y%=p;

        a*=x;
        b*=y;

        b%=p;
        a%=p;
    }
    if(flag)
        return 0;
    gcd(b,p,x,y);
    if(x<0)
        x+=p;
    a*=x;
    a%=p;
    return a;
}

//用Lucas 定理求解 C(n,m) % p ,p 是素数
long long Lucas(long long n, long long m, long long p)
{
    long long ans=1;
    while(m && n && ans)
    {
        ans*=(CmodP(n%p,m%p,p));
        ans=ans%p;
        n=n/p;
        m=m/p;
    }
```

```
    return ans;
}
int main()
{
    long long n,k,p,ans;
    int cas=0;
    while(scanf("%I64d%I64d%I64d",&n,&k,&p)!=EOF)
    {
        if(k>n-k)
            k=n-k;
        ans=Lucas(n+1,k,p)+n-k;
        printf("Case #%d: %I64d\n",++cas,ans%p);
    }
    return 0;
}
```

5.11 Matrix

```
template<int n>class Matrix
{
    long long a[n][n];
    inline Matrix<n> operator*(const Matrix<n> &b)const //比照着
        公式来会快一点常数……nmlgb 的 zoj3289……
    {
        //别忘了矩阵乘法虽然满足结合律但是不满足交换律……
        static Matrix<n> re;
        static int i,j,k;
        for(i=0;i<n;++i)
            for(j=0;j<n;++j)
                re.a[i][j]=0;
        for(k=0;k<n;++k)
            for(i=0;i<n;++i)
                if(a[i][k])
                    for(j=0;j<n;++j)
                        if(b.a[k][j])
                            re.a[i][j]=(re.a[i][j]+a[i][k]*b.a[
                                k][j])%mod;
        return re;
    }
    inline Matrix<n> operator^(int y)const
    {
        static Matrix<n> re,x;
        static int i,j;
        for(i=0;i<n;++i)
        {
            for(j=0;j<n;++j)
            {
                re.a[i][j]=0;
                x.a[i][j]=a[i][j];
            }
            re.a[i][i]=1;
        }
        for(;y;y>>=1,x=x*x)
            if(y&1)
                re=re*x;
        return re;
    }
    long long det()
    {
        static int i,j,k;
        static long long ret,t;
        ret=1ll;
        for(i=0;i<n;++i)
            for(j=0;j<n;++j)
                a[i][j]%=mod;
        for(i=0;i<n;++i)
        {
            for(j=i+1;j<n;++j)
                while(a[j][i])
                {
                    t=a[i][i]/a[j][i];
                    for(k=i;k<n;++k)
                        a[i][k]=(a[i][k]-a[j][k]*t)%mod;
                    for(k=i;k<n;++k)
                        std::swap(a[i][k],a[j][k]);
                    ret=-ret;
                }
            if(!a[i][i])
                return 0ll;
            ret=ret*a[i][i]%mod;
        }
        return (ret+mod)%mod;
    }
};

int inv(mat& a)
{
    int i,j,k,is[MAXN],js[MAXN];
    double t;
    if (a.n!=a.m)
        return 0;
    for (k=0;k<a.n;k++)
    {
        for (t=0,i=k;i<a.n;i++)
            for (j=k;j<a.n;j++)
                if (fabs(a.data[i][j])>t)
                    t=fabs(a.data[is[k]=i][js[k]=j]);
        if (zero(t))
            return 0;
        if (is[k]!=k)
            for (j=0;j<a.n;j++)
                t=a.data[k][j],a.data[k][j]=a.data[is[k]][j],a.
                    data[is[k]][j]=t;
        if (js[k]!=k)
            for (i=0;i<a.n;i++)
                t=a.data[i][k],a.data[i][k]=a.data[i][js[k]],a.
                    data[i][js[k]]=t;
        a.data[k][k]=1/a.data[k][k];
        for (j=0;j<a.n;j++)
            if (j!=k)
                a.data[k][j]*=a.data[k][k];
        for (i=0;i<a.n;i++)
            if (i!=k)
                for (j=0;j<a.n;j++)
                    if (j!=k)
                        a.data[i][j]-=a.data[i][k]*a.data[k][j
                            ];
        for (i=0;i<a.n;i++)
            if (i!=k)
                a.data[i][k]*=-a.data[k][k];
    }
    for (k=a.n-1;k>=0;k--)
    {
        for (j=0;j<a.n;j++)
            if (js[k]!=k)
                t=a.data[k][j],a.data[k][j]=a.data[js[k]][j],a.
                    data[js[k]][j]=t;
        for (i=0;i<a.n;i++)
            if (is[k]!=k)
                t=a.data[i][k],a.data[i][k]=a.data[i][is[k]],a.
                    data[i][is[k]]=t;
    }
    return 1;
}
double det(const mat& a)
{
    int i,j,k,sign=0;
    double b[MAXN][MAXN],ret=1,t;
    if (a.n!=a.m)
        return 0;
    for (i=0;i<a.n;i++)
        for (j=0;j<a.m;j++)
            b[i][j]=a.data[i][j];
    for (i=0;i<a.n;i++)
    {
        if (zero(b[i][i]))
        {
            for (j=i+1;j<a.n;j++)
                if (!zero(b[j][i]))
                    break;
            if (j==a.n)
                return 0;
            for (k=i;k<a.n;k++)
                t=b[i][k],b[i][k]=b[j][k],b[j][k]=t;
            sign++;
        }
        ret*=b[i][i];
        for (k=i+1;k<a.n;k++)
            b[i][k]/=b[i][i];
        for (j=i+1;j<a.n;j++)
            for (k=i+1;k<a.n;k++)
                b[j][k]-=b[j][i]*b[i][k];
    }
    if (sign&1)
        ret=-ret;
    return ret;
}

/*
   Fibonacci Matrix
   1  1
   1  0

   org[0][j], trans[i][j]
   means

   transform(org,1 times) -> org[0][j]=\sum_{i=0}^{n} org[0][i] \times trans[i][j]

   */
```

5.12 Pell's equation

```
/*
find the (x,y)pair that x^2 - n \times y^2 = 1
these is not solution if and only if n is a square number.

solution:
simply brute-force search the integer y, get (x1,y1). ( toooo
    slow in some situation )
```

```
or we can enumerate the continued fraction of √n, as x/y, it will
     be much more faster

other solution pairs' matrix:
x1   n×y1
y1    x1
k-th solution is {matrix}^k
*/

import java.util.*;
import java.math.*;

public class Main
{
    static BigInteger p,q,p1,p2,p3,q1,q2,q3,a1,a2,a0,h1,h2,g1,
         g2,n0;
    static int n,t;
    static void solve()
    {
        p2=BigInteger.ONE;
        p1=BigInteger.ZERO;
        q2=BigInteger.ZERO;
        q1=BigInteger.ONE;
        a0=a1=BigInteger.valueOf((long)Math.sqrt(n));
        g1=BigInteger.ZERO;
        h1=BigInteger.ONE;
        n0=BigInteger.valueOf(n);
        while(true)
        {
            g2=a1.multiply(h1).subtract(g1);
            h2=(n0.subtract(g2.multiply(g2))).divide(h1);
            a2=(g2.add(a0)).divide(h2);
            p=p2.multiply(a1).add(p1);
            q=q2.multiply(a1).add(q1);
            if(p.multiply(p).subtract(n0.multiply(q.multiply(q)
                 )).equals(BigInteger.ONE))
                return ;
            a1=a2;
            g1=g2;
            h1=h2;
            p1=p2;
            p2=p;
            q1=q2;
            q2=q;
        }
    }
    public static void main(String[] args)
    {
        Scanner in=new Scanner(System.in);
        t=in.nextInt();
        for(int i=0;i<t;++i)
        {
            n=in.nextInt();
            solve();
            System.out.println(p+" "+q);
        }
    }
}
```

5.13 Pollard's rho algorithm

```
#include<cstdio>
#include<cstdlib>
#include<list>

short T;
unsigned long long a;
std::list<unsigned long long>fac;

inline unsigned long long multi_mod(const unsigned long long &a
     ,unsigned long long b,const unsigned long long &n)
{
    unsigned long long exp(a%n),tmp(0);
    while(b)
    {
        if(b&1)
        {
            tmp+=exp;
            if(tmp>n)
                tmp-=n;
        }
        exp<<=1;
        if(exp>n)
            exp-=n;
        b>>=1;
    }
    return tmp;
}

inline unsigned long long exp_mod(unsigned long long a,unsigned
      long long b,const unsigned long long &c)
{
    unsigned long long tmp(1);
    while(b)
    {
        if(b&1)
            tmp=multi_mod(tmp,a,c);
        a=multi_mod(a,a,c);
        b>>=1;
    }
    return tmp;
}

inline bool miller_rabbin(const unsigned long long &n,short T)
{
    if(n==2)
        return true;
    if(n<2 || !(n&1))
        return false;
    unsigned long long a,u(n-1),x,y;
    short t(0),i;
    while(!(u&1))
    {
        ++t;
        u>>=1;
    }
    while(T--)
    {
        a=rand()%(n-1)+1;
        x=exp_mod(a,u,n);
        for(i=0;i<t;++i)
        {
            y=multi_mod(x,x,n);
            if(y==1 && x!=1 && x!=n-1)
                return false;
            x=y;
        }
        if(y!=1)
            return false;
    }
    return true;
}

unsigned long long gcd(const unsigned long long &a,const
     unsigned long long &b)
{
    return b?gcd(b,a%b):a;
}

inline unsigned long long pollar_rho(const unsigned long long n
     ,const unsigned long long &c)
{
    unsigned long long x(rand()%(n-1)+1),y,d,i(1),k(2);
    y=x;
    while(true)
    {
        ++i;
        x=(multi_mod(x,x,n)+c)%n;
        d=gcd((x-y+n)%n,n);
        if(d>1 && d<n)
            return d;
        if(x==y)
            return n;
        if(i==k)
        {
            k<<=1;
            y=x;
        }
    }
}

void find(const unsigned long long &n,short c)
{
    if(n==1)
        return;
    if(miller_rabbin(n,6))
    {
        fac.push_back(n);
        return;
    }
    unsigned long long p(n);
    short k(c);
    while(p>=n)
        p=pollar_rho(p,c--);
    find(p,k);
    find(n/p,k);
}

int main()
{
    scanf("%hd",&T);
    while(T--)
    {
        scanf("%llu",&a);
        fac.clear();
        find(a,120);
        if(fac.size()==1)
            puts("Prime");
        else
        {
            fac.sort();
```

```
            printf("%llu\n",fac.front());
        }
    }
    return 0;
}
```

5.14 System of linear congruences

```
// minimal val that for all (m,a) , val%m == a
#include<cstdio>

#define MAXX 11

int T,t;
int m[MAXX],a[MAXX];
int n,i,j,k;
int x,y,c,d;
int lcm;

int exgcd(int a,int b,int &x,int &y)
{
    if(b)
    {
        int re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1;
    y=0;
    return a;
}

int main()
{
    scanf("%d",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d",&n);
        lcm=1;
        for(i=0;i<n;++i)
        {
            scanf("%d",m+i);
            lcm*=m[i]/exgcd(lcm,m[i],x,y);
        }
        for(i=0;i<n;++i)
            scanf("%d",a+i);
        for(i=1;i<n;++i)
        {
            c=a[i]-a[0];
            d=exgcd(m[0],m[i],x,y);
            if(c%d)
                break;
            y=m[i]/d;
            c/=d;
            x=(x*c%y+y)%y;
            a[0]+=m[0]*x;
            m[0]*=y;
        }
        //标程用的步长可能是最终的 m[0] 而不是 lcm。枚举一下标程
        printf("Case␣%d:␣%d\n",t,i<n?-1:(a[0]?a[0]:lcm));
    }
    return 0;
}
```

5.15 Combinatorics

5.15.1 Subfactorial

$!n$ =number of permutations of n elements with no fixed points

from !0:
1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496, 1334961, 14684570
$!n = (n-1)(!(n-1)+!(n-2))$
PS:$n! = (n-1)((n-1)! + (n-2)!)$
$!n = n \times n! + (-1)^n$

Rencontres numbers:
$D_{n,k}$ is the number of permutations of { 1, ..., n } that have exactly k fixed points.
$D_{n,0} = !n$
$D_{n,k} = \binom{n}{k} \times !(n-k)$

5.15.2 Ménage numbers

Ménage numbers:
number of permutations s of [0, ..., n-1] such that.
$\forall i, s(i) \neq i$ and $s(i) \not\equiv i+1 \pmod n$.

from A(0):
1, 0, 0, 1, 2, 13, 80, 579, 4738, 43387, 439792, 4890741

$A_n = \sum_{k=0}^{n} (-1)^k \frac{2n}{2n-k} \binom{2n-k}{k} (n-k)!$
$A_n = nA_{n-1} + \frac{n}{n-2} A_{n-2} + \frac{4(-1)^{n-1}}{n-2}$
$A_n = nA_{n-1} + 2A_{n-2} - (n-4)A_{n-3} - A_{n-4}$

5.15.3 Multiset

Permutation:
MultiSet S={1 m,4 s,4 i,2 p}
$P(S) = \frac{(1+4+4+2)!}{1!4!4!2!}$

Combination:
MultiSet S=$\{\infty a1, \infty a2, ...\infty ak\}$
$\binom{S}{r} = \frac{(r+k-1)!}{r!(k-1)!} = \binom{r+k-1}{r}$

if(r>min{count(element[i])})
you have to resolve this problem with inclusion-exclusion principle.

MS T={3 a,4 b,5 c}
MS $T_* = \{\infty a, \infty b, \infty c\}$
$A1 = \{\binom{T_*}{10} | count(a) > 3\} // \binom{8}{6}$
$A2 = \{\binom{T_*}{10} | count(b) > 4\} // \binom{7}{5}$
$A3 = \{\binom{T_*}{10} | count(c) > 5\} // \binom{6}{4}$

$\binom{T}{10} = \binom{T_*}{10} - (|A_1| + |A_2| + |A_3|) + (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|) - |A_1 \cap A_2 \cap A_3|$
ans=C(10,12)-(C(6,8)+C(5,7)+C(4,6))+(C(1,3)+C(0,2)+0)-0=6

5.15.4 Distributing Balls into Boxes

Distributing m Balls into n Boxes.

balls	boxes	empty	counts
diff	diff	empty	n^m
diff	diff	full	$n! \times S(m,n) = \sum_{i=0}^{n} (-1)^n \binom{n}{i} (n-i)^m$(inclu
diff	same	empty	$\sum_{k=1}^{min\{n,m\}} s(m,k) = \frac{1}{n!} \sum_{k=1}^{min\{n,m\}} \sum_{i=0}^{k} (-1)^i \binom{k}{i}$(
diff	same	full	S(m,n) (Stirling numbers of the second kind)
same	diff	empty	$\binom{n+m-1}{n-1}$
same	diff	full	$\binom{m-1}{n-1}$

same	same	empty	dp[0][0..n]=dp[1..m][1]=1;
			if(m≥n)
			dp[m][n]=dp[m][n-1]+dp[m-n][n];
			else
			dp[m][n]=dp[m][n-1];

same	same	full	g[m][n]=dp[m-n][n];

5.15.5 Combinatorial Game Theory

Wythoff's game:

- There are two piles of counters.

- Players take turns removing counters (at least 1 counter) from one or both piles; in the latter case, the numbers of counters removed from each pile must be equal.
- The player who removes the last counter wins.

consider the counters of status as pair (a,b) ($a \leq b$)
{first player loses} $\iff a = \lfloor (b-a) \times \phi \rfloor, \phi = \frac{\sqrt{5}+1}{2}$

Fibonacci Nim:

- There is one pile of n counters.
- The first player may remove any positive number of counters, but not the whole pile.
- Thereafter, each player may remove at most twice the number of counters his opponent took on the previous move.
- The player who removes the last counter wins.

{first player wins} $\iff n \notin$ {Fibonacci number}

poj 1740:

- There are n piles of stones.
- At each step of the game,the player choose a pile,remove at least one stones,then freely move stones from this pile to any other pile that still has stones.
- The player who removes the last counter wins.

{first player lose} $\iff$ n is even && $(a_1, a_2, ..., a_k)(a_1 \leq a_2 \leq ... \leq a_{2k})$ satisfy $a_{2i-1} = a_{2i}\{\forall i \in [1,k]\}$

Staircase Nim:

- A staircase of n steps contains coins on some of the steps.
- A move of staircase nim consists of moving any positive number of coins from any step j , to the next lower step, j – 1.
- Coins reaching the ground (step 0) are removed from play.
- The player who removes the last counter wins.

Even steps are unusefull.
$SG = x_1 \oplus x_3 \oplus x_5 ...$

Anti-SG:

- Everything is likes SG.
- The player who removes the last counter loses.

{first player wins} $\iff$
SGsum=0,&& {all piles is 1}
SGsum$\neq$0,&& {some piles ars larger than 1}

Every-SG:

- Everything is likes SG.
- For each turns, player have to move all of sub-games if the sub-game was not ended yet.

{first player wins} $\iff$ max(steps of all sub-games) is odd.

Coin Game:

- Given a horizontal line of N coins with some coins showing heads and some tails.
- Each turn, a player have to follow some rules, flip some coins. But the most right coin he fliped has to be fliped from head to tail.
- The player who can not flip coin loses.

game{THHTTH} = game{TH}$\oplus$game{TTH}$\oplus$game{TTTTTH}

Tree Game:

- There is a rooted tree.
- Each turn, a player has to remove a edge from the tree. The parts can not connect with root with also are removed.
- The player who removes the last edge wins.

$\forall node(x)$,
$SG(x) = (SG(i_1)+1) \oplus (SG(i_2)+1) \oplus ...$($\forall$ i are childnodes of x)

Undirectional Graph Game:

- There is a rooted undirectional graph.
- Other rules are likes Tree Game.

Odd Circle's SG value is 1.
Even Circel's SG value is 0.
turn the graph to a tree.

5.15.6 Catalan number

from C_0
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420
$C_0 = 1$
$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$
$C_{n+1} = \frac{2(2n+1)}{n+1} C_n$

$C_n = \binom{2n}{n} - \binom{2n}{n+1} = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$
$C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$
Applications:

1. C_n counts the number of expressions containing n pairs of parentheses which are correctly matched.
2. C_n is the number of full binary trees with n + 1 leaves.
3. C_n is the number of non-isomorphic ordered trees with n+1 vertices. (An ordered tree is a rooted tree in which the children of each vertex are given a fixed left-to-right order.)
4. C_n is the number of monotonic paths along the edges of a grid with n × n square cells, which do not pass above the diagonal.($x \leq y$ for C_n, $x < y$ for $C_n - 1$)
   (a) for the rectangle (p,q),($x < y$), $ans = \binom{p+q-1}{p} - \binom{p+q-1}{p-1} = \frac{q-p}{q+p}\binom{p+q}{q}$
   (b) for the rectangle (p,q),($x \leq y$),$ans = \binom{p+q}{p} - \binom{p+q}{p-1} = \frac{q-p+1}{q+1}\binom{p+q}{q}$

5. C_n is the number of different ways a convex polygon with n + 2 sides can be cut into triangles by connecting vertices with straight lines.

6. C_n is the number of permutations of {1, ..., n} that avoid the pattern 123.

7. C_n is the number of ways to tile a stairstep shape of height n with n rectangles.

5.15.7 Stirling number

First kind:
Stirling numbers of the first kind is signed.
The unsigned Stirling numbers of the first kind are denoted by s(n,k).
s(4,2)=11
s(n,k) count the number of permutations of n elements with k disjoint cycles.
s(n,0)=s(1,1)=1
s(n+1,k)=s(n,k-1)+n s(n,k)

Second kind:
S(n,k) count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.
S(4,2)=7
S(n,n)=S(n,1)=1
S(n,k)=S(n-1,k-1)+k S(n-1,k)
$S(n,n-1)=\binom{n}{2}=\frac{n(n-1)}{2}$
$S(n,2)=2^{n-1}-1$

5.15.8 Delannoy number

Delannoy number D describes the number of paths from (0, 0) to (m, n), using only single steps north, northeast, or east.
D(0,0)=1
D(m,n)=D(m-1,n)+D(m-1,n-1)+D(m,n-1)

central Delannoy numbers D(n) = D(n,n)
D(n) from 0:
1, 3, 13, 63, 321, 1683, 8989, 48639, 265729
$nD(n)=3(2n-1)D(n-1)-(n-1)D(n-2)$

5.15.9 Schröder number

Large:
Describes the number of paths from (0, 0) to (m, n), using only single steps north, northeast, or east, for all (x,y), ($x \leq y$).
for(n==m),from 0:
1, 2, 6, 22, 90, 394, 1806, 8558, 41586, 206098
$S(n)=S(n-1)+\sum_{k=0}^{n-1}S(k)S(n-1-k)$

Little: (aka. super-Catalan numbers, Hipparchus numbers)

1. the number of different trees with n leaves and with all internal vertices having two or more children.

2. the number of ways of inserting brackets into a sequence.

3. the number of ways of dissecting a convex polygon into smaller polygons by inserting diagonals.

from 0:
1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049
s(n)=S(n)/2
s(0)=s(1)=1
ns(n)=(6n-9)s(n-1)-(n-3)s(n-2)
$a(n+1)=-a(n)+2\sum_{k=1}^{n}a(k)\times a(n+1-k)$
$a(n+1)=\sum_{k=0}^{(n-1)/2}2^k\times 3^{n-1-2k}\binom{n-1}{2k}$

5.15.10 Bell number

Number of partitions of a set of n labeled elements.
from 0:
1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975
$B_{n+1}=\sum_{k=0}^{n}\binom{n}{k}B_k$
$B_{p+n}\equiv B_n+B_{n+1}\pmod p$ (p for prime)
$B_{p^m+n}\equiv mB_n+B_{n+1}\pmod p$ (p for prime)
$B_n=\sum_{k=1}^{n}S(n,k)$(S for Stirling second kind)

5.15.11 Eulerian number

First kind:
the number of permutations of the numbers 1 to n in which exactly m elements are greater than the previous element
A(n,0)=1
A(n,m)=(n-m)A(n-1,m-1)+(m+1)A(n-1,m)
A(n,m)=(n-m+1)A(n-1,m-1)+mA(n-1,m)
A(n,m)=A(n,n-1-m)

Second kind:
count the permutations of the multiset {1,1,2,2,...,n,n} with k ascents with the restriction that for all m
T(n,0)=1
T(n,m)=(2n-m-1)T(n-1,m-1)+(m+1)T(n-1,m)

5.15.12 Motzkin number

1. the number of different ways of drawing non-intersecting chords on a circle between n points

2. Number of sequences of length n-1 consisting of positive integers such that the opening and ending elements are 1 or 2 and the absolute difference between any 2 consecutive elements is 0 or 1

3. paths from (0,0) to (n,0) in an n X n grid using only steps U = (1,1), F = (1,0) and D = (1,-1)

from 0:
1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835, 113634, 310572, 853467
$M_{n+1}=M_n+\sum_{i=0}^{n-1}M_iM_{n-1-i}=\frac{2n+3}{n+3}M_n+\frac{3n}{n+3}M_{n-1}$
$M_n=\sum_{k=0}^{\lfloor n/2\rfloor}\binom{n}{2k}C_k$(C for catalan)

5.15.13 Narayana number

1. the number of expressions containing n pairs of brackets which are correctly matched and which contain k pairs of ().
2. the number of paths from (0, 0) to (2n, 0), with steps only northeast and southeast, not straying below the x-axis, with k peaks.

N(n,0)=0
$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k-1}$
$N(n,k) = \frac{1}{k}\binom{n-1}{k-1}\binom{n}{k-1}$
$\sum_{k=1}^{n} N(n,k) = C_n$(C for catalan)

5.16 Number theory

5.16.1 Divisor Fuction

$n = p_1^{a_1} \times p_2^{a_2} \times ... \times p_s^{a_s}$
sum of positive divisors function
$\sigma(n) = \prod_{j=1}^{s} \frac{p_j^{a_j+1}-1}{p_j-1}$
number of postive diversors function
$\tau(n) = \prod_{j=1}^{s} (a_j+1)$

5.16.2 Reduced Residue System

Euler's totient function:

对正整数 n，欧拉函数 φ 是小于或等于 n 的数中与 n 互质的数的数目，也就是对 n 的简化剩余系的大小。
φ(2)=1（唯一和 1 互质的数就是 1 本身）。
若 m,n 互质，$\varphi(m \times n) = \varphi(m) \times \varphi(n)$。
对于 n 来说，所有这样的数的和为 $\frac{n \times \varphi(n)}{2}$ 。
$gcd(k,n) = d, k \in [1,n]$，这样的 k 有 $\varphi(\frac{n}{d})$

```
inline int phi(int n)
{
    static int i;
    static int re;
    re=n;
    for(i=0;prm[i]*prm[i]<=n;++i)
        if(n%prm[i]==0)
        {
            re-=re/prm[i];
            do
                n/=prm[i];
            while(n%prm[i]==0);
        }
    if(n!=1)
        re-=re/n;
    return re;
}

inline void Euler()
{
    static int i,j;
    phi[1]=1;
    for(i=2;i<MAXX;++i)
        if(!phi[i])
            for(j=i;j<MAXX;j+=i)
            {
                if(!phi[j])
                    phi[j]=j;
                phi[j]=phi[j]/i*(i-1);
            }
}
```

Multiplicative order:

the multiplicative order of a modulo n is the smallest positive integer k with

$a^k \equiv 1 \pmod{n}$

对 m 的简化剩余系中的所有 x,ord(x) 都一定是 φ(m) 的一个约数 (aka. Euler's totient theorem)

求:
method 1、根据定义，对 φ(m) 分解素因子之后暴力寻找最小的一个 $d\{d|\varphi(m)\}$，满足 $x^d \equiv 1 \pmod{m}$;
method 2、

```
inline long long ord(long long x,long long m)
{
    static long long ans;
    static int i,j;
    ans=phi(m);
    for(i=0;i<fac.size();++i)
        for(j=0;j<fac[i].second && pow(x,ans/fac[i].first,m)==1
            ll;++j)
            ans/=fac[i].first;
    return ans;
}
```

Primitive root:

若 ord(x)==φ(m)，则 x 为 m 的一个原根
因此只需检查所有 x^d $\{d|\varphi(m)\}$ 找到使 $x^d \equiv 1 \pmod{m}$ 的所有 d，当且仅当这样的 d 只有一个，并且为 φ(m) 的时候，x 是 m 的一个原根

当且仅当 m= 1,2,4,p^n,2 × p^n {p 为奇质数,n 为正整数} 时，m 存在原根 // 应该是指存在对于完全剩余系的原根……？

当 m 存在原根时，原根数目为 $\varphi(\varphi(m))$

求：
枚举每一个简化剩余系中的数 i，若对于 i 的每一个质因子 p[j],$i^{\frac{\varphi(m)}{p[j]}} \not\equiv 1 \pmod{m}$，那么 i 为 m 的一个原根。也就是说，ord(i)==φ(m)。
最小原根通常极小。

Carmichael function:

λ(n) is defined as the smallest positive integer m such that
$a^m \equiv 1 \pmod{n}\{\forall a! = 1 \&\& gcd(a,n) == 1\}$
也就是简化剩余系 (完全剩余系中存在乘法群中无法得到 1 的数) 中所有 x 的 lcm{ord(x)}

if n=$p[0]^{a[0]} \times p[1]^{a[1]} \times ... \times p[m-1]^{a[m-1]}$
then λ(n)=lcm($\lambda(p[0]^{a[0]})$,$\lambda(p[1]^{a[1]})$,...,$\lambda(p[m-1]^{a[m-1]})$);

if n=$2^c \times p[0]^{a[0]} \times p[1]^{a[1]} \times ... \times p[m-1]^{a[m-1]}$
then λ(n)=lcm(2^c,$\varphi(p[0]^{a[0]})$,$\varphi(p[1]^{a[1]})$,...,$\varphi(p[m-1]^{a[m-1]})$);
c=0 if a<2; c=1 if a==2; c=a-2 if a>3;

Carmichael's theorem:
if gcd(a,n)==1
then $\lambda(n) \equiv 1 \pmod{n}$

5.16.3 Prime

Prime number theorem:
Let π(x) be the prime-counting function that gives the number of primes less than or equal to x, for any real number x.
$\lim_{x\to\infty} \frac{\pi(x)}{x/\ln(x)} = 1$
known as the asymptotic law of distribution of prime numbers.
$\pi(x) \sim \frac{x}{\ln x}$.

```
#include<vector>

std::vector<int>prm;
bool flag[MAXX];

int main()
{
    prm.reserve(MAXX); // pi(x)=x/ln(x);
    for(i=2;i<MAXX;++i)
    {
        if(!flag[i])
            prm.push_back(i);
        for(j=0;j<prm.size() && i*prm[j]<MAXX;++j)
        {
            flag[i*prm[j]]=true;
            if(i%pmr[j]==0)
                break;
        }
    }
    return 0;
}
```

5.16.4 Möbius Fuction

- μ(n) = 1 if n is a square-free positive integer with an even number of prime factors.
- μ(n) = −1 if n is a square-free positive integer with an odd number of prime factors.
- μ(n) = 0 if n has a squared prime factor.

$$\sum_{d|n}\mu(d)=\begin{cases}1 & \text{if } n=1\\ 0 & \text{if } n>1.\end{cases}$$

```
int fac[MAXX],mob[MAXX];
bool flag[MAXX];
std::vector<int>prm(MAXX);
void go()
{
    static int i,j,k;
    prm.resize(0);
    mob[1]=1;
    for(i=2;i<MAXX;++i)
    {
        if(!flag[i])
        {
            prm.push_back(i);
            mob[i]=-1;
            fac[i]=1;
        }
        for(j=0;j<prm.size() && (k=i*prm[j])<MAXX;++j)
        {
            flag[k]=true;
            fac[k]=fac[i]+1;
            if(i%prm[j]==0)
            {
                mob[k]=0;
                break;
            }
            mob[k]=-mob[i];
        }
    }
}
inline ll cal(int n,int m)// [1,n] [1,m]之间互质的数的对数
{
    static int i,j,k;
    static ll re;
    re=0;
    for(i=1;i<=n;i=j+1)
    {
        j=std::min(n/(n/i),m/(m/i));
        re+=(mob[j]-mob[i-1])*(n/i)*(ll)(m/i); //mob 表示前缀和
    }
    return re;
}
```

5.16.5 Euler–Mascheroni constant

$$\gamma=\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k}-\ln(n)\right)=\int_1^{\infty}\left(\frac{1}{\lfloor x\rfloor}-\frac{1}{x}\right)dx$$

0.57721566490153286060651209008240243104215933593992...

5.16.6 Fibonacci

gcd(fib[i],fib[j])=fib[gcd(i,j)]

$$a_n=\frac{\sqrt{5}}{5}\cdot\left[\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

5.16.7 Hensel's lemma

设 f(x) 为有整系数多项式，k 为不少于 2 的整数，p 为质数。若整数 r 满足:
$f(r)\equiv 0\pmod{p^{k-1}}$
对于 $f(r+tp^{k-1})\equiv 0\pmod{p^k}$
则有

- 若 $f'(r)\not\equiv 0\pmod{p}$，则存在唯一的整数 $0\le t\le p-1$ 使上式成立
- 若 $f'(r)\equiv 0\pmod{p}$，且 $f(r)\equiv 0\pmod{p^k}$，则任意正整数 t 都成立
- 若 $f'(r)\equiv 0\pmod{p}$，但 $f(r)\not\equiv 0\pmod{p^k}$，则 t 不存在整数解

5.17 Probability theory

5.17.1 Dice

- m 面体骰子连续 n 次掷出同一数字的投掷次数期望: $\frac{m^n-1}{m-1}$
- m 面体骰子连续 n 次掷出不同数字的投掷次数期望:
  $ans=\sum d_i$
  $d_{i+1}=d_i\times\frac{m}{m-i}, d_1=1$

6 String

6.1 Aho-Corasick Algorithm

```
//trie graph
#include<cstring>
#include<queue>

#define MAX 1000111
#define N 26

int nxt[MAX][N],fal[MAX],cnt;
bool ed[MAX];
char buf[MAX];

inline void init(int a)
{
    memset(nxt[a],0,sizeof(nxt[0]));
    fal[a]=0;
    ed[a]=false;
}

inline void insert()
{
    static int i,p;
    for(i=p=0;buf[i];++i)
    {
        if(!nxt[p][map[buf[i]]])
            init(nxt[p][map[buf[i]]]=++cnt);
        p=nxt[p][map[buf[i]]];
    }
    ed[p]=true;
}

inline void make()
{
    static std::queue<int>q;
    int i,now,p;
    q.push(0);
    while(!q.empty())
    {
        now=q.front();
        q.pop();
        for(i=0;i<N;++i)
            if(nxt[now][i])
            {
                q.push(p=nxt[now][i]);
                if(now)
```

```
                fal[p]=nxt[fal[now]][i];
            ed[p]|=ed[fal[p]];
        }
        else
            nxt[now][i]=nxt[fal[now]][i]; // 使用本身的 trie
                存串的时候注意 nxt 已被重载
    }
}

// normal version

#define N 128

char buf[MAXX];
int cnt[1111];

struct node
{
    node *fal,*nxt[N];
    int idx;
    node() { memset(this,0,sizeof node); }
}*rt;
std::queue<node*>Q;

void free(node *p)
{
    for(int i(0);i<N;++i)
        if(p->nxt[i])
            free(p->nxt[i]);
    delete p;
}

inline void add(char *s,int idx)
{
    static node *p;
    for(p=rt;*s;++s)
    {
        if(!p->nxt[*s])
            p->nxt[*s]=new node();
        p=p->nxt[*s];
    }
    p->idx=idx;
}

inline void make()
{
    Q.push(rt);
    static node *p,*q;
    static int i;
    while(!Q.empty())
    {
        p=Q.front();
        Q.pop();
        for(i=0;i<N;++i)
            if(p->nxt[i])
            {
                q=p->fal;
                while(q)
                {
                    if(q->nxt[i])
                    {
                        p->nxt[i]->fal=q->nxt[i];
                        break;
                    }
                    q=q->fal;
                }
                if(!q)
                    p->nxt[i]->fal=rt;
                Q.push(p->nxt[i]);
            }
    }
}

inline void match(const char *s)
{
    static node *p,*q;
    for(p=rt;*s;++s)
    {
        while(p!=rt && !p->nxt[*s])
            p=p->fal;
        p=p->nxt[*s];
        if(!p)
            p=rt;
        for(q=p;q!=rt && q->idx;q=q->fal) // why q->idx ? looks
              like not necessary at all, I delete it in an
             other solution
            ++cnt[q->idx];
    }
}

//可以考虑 dfs 一下，拉直 fal 指针来跳过无效的匹配
//所有 DFA 的 fal 指针都会构成一颗树
```

6.2 Gusfield's Z Algorithm

```
inline void make(int *z,char *buf)
{
    int i,j,l,r;
    l=0;
    r=1;
    z[0]=strlen(buf);
    for(i=1;i<z[0];++i)
        if(r<=i || z[i-l]>=r-i)
        {
            j=std::max(i,r);
            while(j<z[0] && buf[j]==buf[j-i])
                ++j;
            z[i]=j-i;
            if(i<j)
            {
                l=i;
                r=j;
            }
        }
        else
            z[i]=z[i-l];
}

for(i=1;i<len && i+z[i]<len;++i); //i= 可能最小循环节长度
```

6.3 Manacher's Algorithm

```
inline int match(const int a,const int b,const std::vector<int>
     &str)
{
    static int i;
    i=0;
    while(a-i>=0 && b+i<str.size() && str[a-i]==str[b+i])//注意
         是 i 不是 1，打错过很多次了
        ++i;
    return i;
}

inline void go(int *z,const std::vector<int> &str)
{
    static int c,l,r,i,ii,n;
    z[0]=1;
    c=l=r=0;
    for(i=1;i<str.size();++i)
    {
        ii=(l<<1)-i;
        n=r+1-i;

        if(i>r)
        {
            z[i]=match(i,i,str);
            l=i;
            r=i+z[i]-1;
        }
        else
            if(z[ii]==n)
            {
                z[i]=n+match(i-n,i+n,str);
                l=i;
                r=i+z[i]-1;
            }
            else
                z[i]=std::min(z[ii],n);
        if(z[i]>z[c])
            c=i;
    }
}

inline bool check(int *z,int a,int b) //检查子串 [a,b] 是否回文
{
    a=a*2-1;
    b=b*2-1;
    int m=(a+b)/2;
    return z[m]>=b-m+1;
}
```

6.4 Morris-Pratt Algorithm

```
inline void make(char *buf,int *fal)
{
    static int i,j;
    fal[0]=-1;
    for(i=1,j=-1;buf[i];++i)
    {
        while(j>=0 && buf[j+1]!=buf[i])
            j=fal[j];
        if(buf[j+1]==buf[i])
            ++j;
        fal[i]=j;
    }
}

inline int match(char *p,char *t,int* fal)
{
```

```
    static int i,j,re;
    re=0;
    for(i=0,j=-1;t[i];++i)
    {
        while(j>=0 && p[j+1]!=t[i])
            j=fal[j];
        if(p[j+1]==t[i])
            ++j;
        if(!p[j+1])
        {
            ++re;
            j=fal[j];
        }
    }
    return re;
}

inline void make(char *buf,int *fal) // knuth-morris-pratt, not
     tested yet
{
    static int i,j;
    fal[0]=-1;
    for(i=1,j=-1;buf[i];++i)
    {
        while(j>=0 && buf[j+1]!=buf[i])
            j=fal[j];
        if(buf[j+1]==buf[i])
            ++j;
        fal[i]=j;
    }
    for(i-=2;i>=0;--i)
    {
        for(j=fal[i];j!=-1 && buf[j+1]!=buf[i+1];j=fal[j]);
        fal[i]=j;
    }
}
```

6.5 smallest representation

```
int min(char a[],int len)
{
    int i = 0,j = 1,k = 0;
    while (i < len && j < len && k < len)
    {
        int cmp = a[(j+k)%len]-a[(i+k)%len];
        if (cmp == 0)
            k++;
        else
        {
            if (cmp > 0)
                j += k+1;
            else
                i += k+1;
            if (i == j) j++;
            k = 0;
        }
    }
    return std::min(i,j);
}
```

6.6 Suffix Array - DC3 Algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 1111
#define F(x) ((x)/3+((x)%3==1?0:tb))
#define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)

int wa[MAXX],wb[MAXX],wv[MAXX],ws[MAXX];

inline bool c0(const int *str,const int &a,const int &b)
{
    return str[a]==str[b] && str[a+1]==str[b+1] && str[a+2]==
         str[b+2];
}

inline bool c12(const int *str,const int &k,const int &a,const
     int &b)
{
    if(k==2)
        return str[a]<str[b] || str[a]==str[b] && c12(str,1,a
             +1,b+1);
    else
        return str[a]<str[b] || str[a]==str[b] && wv[a+1]<wv[b
             +1];
}

inline void sort(int *str,int *a,int *b,const int &n,const int
     &m)
{
    memset(ws,0,sizeof(ws));
    int i;
    for(i=0;i<n;++i)
        ++ws[wv[i]=str[a[i]]];
    for(i=1;i<m;++i)
        ws[i]+=ws[i-1];
    for(i=n-1;i>=0;--i)
        b[--ws[wv[i]]]=a[i];
}

inline void dc3(int *str,int *sa,const int &n,const int &m)
{
    int *strn(str+n);
    int *san(sa+n),tb((n+1)/3),ta(0),tbc(0),i,j,k;
    str[n]=str[n+1]=0;
    for(i=0;i<n;++i)
        if(i%3)
            wa[tbc++]=i;
    sort(str+2,wa,wb,tbc,m);
    sort(str+1,wb,wa,tbc,m);
    sort(str,wa,wb,tbc,m);
    for(i=j=1,strn[F(wb[0])]=0;i<tbc;++i)
        strn[F(wb[i])]=c0(str,wb[i-1],wb[i])?j-1:j++;
    if(j<tbc)
        dc3(strn,san,tbc,j);
    else
        for(i=0;i<tbc;++i)
            san[strn[i]]=i;
    for(i=0;i<tbc;++i)
        if(san[i]<tb)
            wb[ta++]=san[i]*3;
    if(n%3==1)
        wb[ta++]=n-1;
    sort(str,wb,wa,ta,m);
    for(i=0;i<tbc;++i)
        wv[wb[i]=G(san[i])]=i;
    for(i=j=k=0;i<ta && j<tbc;)
        sa[k++]=c12(str,wb[j]%3,wa[i],wb[j])?wa[i++]:wb[j++];
    while(i<ta)
        sa[k++]=wa[i++];
    while(j<tbc)
        sa[k++]=wb[j++];
}

int rk[MAXX],lcpa[MAXX],sa[MAXX*3];
int str[MAXX*3]; //必须int

int main()
{
    scanf("%d␣%d",&n,&j);
    for(i=0;i<n;++i)
    {
        scanf("%d",&k);
        num[i]=k-j+100;
        j=k;
    }
    num[n]=0;

    dc3(num,sa,n+1,191); //191: str 中取值范围，桶排序

    for(i=1;i<=n;++i) // rank 数组
        rk[sa[i]]=i;
    for(i=k=0;i<n;++i) // lcp 数组
        if(!rk[i])
            lcpa[0]=0;
        else
        {
            j=sa[rk[i]-1];
            if(k>0)
                --k;
            while(num[i+k]==num[j+k])
                ++k;
            lcpa[rk[i]]=k;
        }


    for(i=1;i<=n;++i)
        sptb[0][i]=i;
    for(i=1;i<=lg[n];++i)  //sparse table RMQ
    {
        k=n+1-(1<<i);
        for(j=1;j<=k;++j)
        {
            a=sptb[i-1][j];
            b=sptb[i-1][j+(1<<(i-1))];
            sptb[i][j]=lcpa[a]<lcpa[b]?a:b;
        }
    }
}

inline int ask(int l,int r)
{
    a=lg[r-l+1];
    r-=(1<<a)-1;
    l=sptb[a][l];
    r=sptb[a][r];
    return lcpa[l]<lcpa[r]?l:r;
}
```

```
inline int lcp(int l,int r) // 字符串上 [l,r] 区间的 rmq
{
    l=rk[l];
    r=rk[r];
    if(l>r)
        std::swap(l,r);
    return lcpa[ask(l+1,r)];
}
```

6.7 Suffix Array – Prefix-doubling Algorithm

```
int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];

bool cmp(int *r,int n,int a,int b,int l)
{
    return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
}
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
    int *s = str;
    int *x=wx,*y=wy,*t,p;
    int i,j;
    for(i=0; i<m; i++)
        wss[i]=0;
    for(i=0; i<n; i++)
        wss[x[i]=s[i]]++;
    for(i=1; i<m; i++)
        wss[i]+=wss[i-1];
    for(i=n-1; i>=0; i--)
        sa[--wss[x[i]]]=i;
    for(j=1,p=1; p<n && j<n; j*=2,m=p)
    {
        for(i=n-j,p=0; i<n; i++)
            y[p++]=i;
        for(i=0; i<n; i++)
            if(sa[i]-j>=0)
                y[p++]=sa[i]-j;
        for(i=0; i<n; i++)
            wv[i]=x[y[i]];
        for(i=0; i<m; i++)
            wss[i]=0;
        for(i=0; i<n; i++)
            wss[wv[i]]++;
        for(i=1; i<m; i++)
            wss[i]+=wss[i-1];
        for(i=n-1; i>=0; i--)
            sa[--wss[wv[i]]]=y[i];
        for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
            x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
    }
    for(int i=0; i<n; i++)
        rank[sa[i]]=i;
    for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
        if(rank[i]>0)
            for(k?k--:0,j=sa[rank[i]-1]; i+k < n && j+k < n &&
                str[i+k]==str[j+k]; ++k);
}
```

6.8 Suffix Automaton

```
/*
length(s) ∈ [ min(s), max(s) ] = [ val[fal[s]]+1, val[s] ]
 */
#define MAXX 90111
#define MAXN (MAXX<<1)

int fal[MAXN],nxt[MAXN][26],val[MAXN],cnt,rt,last;

inline int neww(int v=0)
{
    val[++cnt]=v;
    fal[cnt]=0;
    memset(nxt[cnt],0,sizeof nxt[0]);
    return cnt;
}

inline void add(int w)
{
    static int p,np,q,nq;
    p=last;
    last=np=neww(val[p]+1);
    while(p && !nxt[p][w])
    {
        nxt[p][w]=np;
        p=fal[p];
    }
    if(!p)
        fal[np]=rt;
    else
    {
        q=nxt[p][w];
        if(val[p]+1==val[q])
            fal[np]=q;
        else
        {
            nq=neww(val[p]+1);
            memcpy(nxt[nq],nxt[q],sizeof nxt[0]);
            fal[nq]=fal[q];

            fal[q]=fal[np]=nq;
            while(p && nxt[p][w]==q)
            {
                nxt[p][w]=nq;
                p=fal[p];
            }
        }
    }
}

int v[MAXN],the[MAXN];

inline void make(char *str)
{
    cnt=0;
    rt=last=neww();
    static int i,len,now;
    for(i=0;str[i];++i)
        add(str[i]-'a');
    len=i;
    memset(v,0,sizeof v);
    for(i=1;i<=cnt;++i)
        ++v[val[i]];
    for(i=1;i<=len;++i)
        v[i]+=v[i-1];
    for(i=1;i<=cnt;++i)
        the[v[val[i]]--]=i;
    for(i=cnt;i;--i)
    {
        now=the[i];
        // topsort already
    }
}
/*
sizeof right(s):
    init:
        for all np:
            count[np]=1;
    process:
        for all status s:
            count[fal[s]]+=count[s];
*/
```

7 Dynamic Programming

7.1 knapsack problem

```
multiple-choice knapsack problem:

for 所有的组k
    for v=V..0
 for 所有的 i 属于组 k
            f[v]=max{f[v],f[v-c[i]]+w[i]}
```

7.2 LCIS

```
#include<cstdio>
#include<cstring>
#include<vector>

#define MAXX 1111

int T;
int n,m,p,i,j,k;
std::vector<int>the[2];
int dp[MAXX],path[MAXX];
int ans[MAXX];

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    {
        scanf("%d",&n);
        the[0].resize(n);
        for(i=0;i<n;++i)
            scanf("%d",&the[0][i]);
        scanf("%d",&m);
        the[1].resize(m);
        for(i=0;i<m;++i)
            scanf("%d",&the[1][i]);
        memset(dp,0,sizeof dp);
        for(i=0;i<the[0].size();++i)
        {
            n=0;
            p=-1;
            for(j=0;j<the[1].size();++j)
```

```
            {
                if(the[0][i]==the[1][j] && n+1>dp[j])
                {
                    dp[j]=n+1;
                    path[j]=p;
                }
                if(the[1][j]<the[0][i] && n<dp[j])
                {
                    n=dp[j];
                    p=j;
                }
            }
        }
        n=0;
        p=-1;
        for(i=0;i<the[1].size();++i)
            if(dp[i]>n)
                n=dp[p=i];
        printf("%d\n",n);
        for(i=n-1;i>=0;--i)
        {
            ans[i]=the[1][p];
            p=path[p];
        }
        for(i=0;i<n;++i)
            printf("%d ",ans[i]);
        puts("");
    }
    return 0;
}
```

7.3 LCS

```
#include<cstdio>
#include<algorithm>
#include<vector>

#define MAXX 111
#define N 128

std::vector<char>the[2];
std::vector<int>dp(MAXX),p[N];

int i,j,k;
char buf[MAXX];
int t;

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    while(gets(buf),buf[0]!='#')
    {
        the[0].resize(0);
        for(i=0;buf[i];++i)
            the[0].push_back(buf[i]);
        the[1].resize(0);
        gets(buf);
        for(i=0;buf[i];++i)
            the[1].push_back(buf[i]);
        for(i=0;i<N;++i)
            p[i].resize(0);
        for(i=0;i<the[1].size();++i)
            p[the[1][i]].push_back(i);
        dp.resize(1);
        dp[0]=-1;
        for(i=0;i<the[0].size();++i)
            for(j=p[the[0][i]].size()-1;j>=0;--j)
            {
                k=p[the[0][i]][j];
                if(k>dp.back())
                    dp.push_back(k);
                else
                    *std::lower_bound(dp.begin(),dp.end(),k)=k;
            }
        printf("Case #%d: you can visit at most %ld cities.\n"
             ,++t,dp.size()-1);
    }
    return 0;
}
```

7.4 sequence partitioning

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<set>

#define MAXX 40111

int a[MAXX],b[MAXX];
int n,R;
std::multiset<int>set;

inline bool check(const int g)
{
    static int i,j,k;
    static long long sum;
    static int l,r,q[MAXX],dp[MAXX];
    set.clear();
    q[0]=dp[0]=l=r=sum=0;
    for(j=i=1;i<=n;++i)
    {
        sum+=b[i];
        while(sum>g)
            sum-=b[j++];
        if(j>i)
            return false;
        while(l<r && q[l]<j)
        {
            ++l;
            if(l<r && set.count(dp[q[l-1]]+a[q[l]]))
                set.erase(set.find(dp[q[l-1]]+a[q[l]]));
        }
        while(l<r && a[q[r-1]]<=a[i])
        {
            --r;
            if(l<r && set.count(dp[q[r-1]]+a[q[r]]))
                set.erase(set.find(dp[q[r-1]]+a[q[r]]));
        }
        if(l<r)
            set.insert(dp[q[r-1]]+a[i]);
        q[r++]=i;
        dp[i]=dp[j-1]+a[q[l]];
        if(r-l>1)
            dp[i]=std::min(dp[i],*set.begin());
    }
    return dp[n]<=R;
}

int i,j,k;
long long l,r,mid,ans;

int main()
{
    while(scanf("%d %d",&n,&R)!=EOF)
    {
        l=r=0;
        for(i=1;i<=n;++i)
        {
            scanf("%d %d",a+i,b+i);
            r+=b[i];
        }
        ans=-1;
        while(l<=r)
        {
            mid=l+r>>1;
            if(check(mid))
            {
                ans=mid;
                r=mid-1;
            }
            else
                l=mid+1;
        }
        printf("%lld\n",ans);
    }
    return 0;
}
```

8 Search

8.1 dlx - exact cover

```
#define MAXN (N*22) // row
#define MAXM (N*10) // col
#define MAXX (MAXN*MAXM)

int cnt;
int l[MAXX],r[MAXX],u[MAXX],d[MAXX],rh[MAXX],ch[MAXX];
int sz[MAXM],hd[MAXN];
bool done[MAXN]; //solution

inline void init(const int m)
{
    static int i;
    for(i=0;i<=m;++i)
    {
        l[i+1]=i;
        r[i]=i+1;
        u[i]=d[i]=i;
        sz[i]=0;
    }
    r[m]=0;
    cnt=m+1;
}

inline void link(int x,int y)
{
```

```
    d[cnt]=d[y];
    u[cnt]=y;
    u[d[y]]=cnt;
    d[y]=cnt;
    if(hd[x]<0)  // set the val to -1 when you init a new line
    {
        hd[x]=l[cnt]=r[cnt]=cnt;
        done[x]=false;
    }
    else
    {
        l[cnt]=hd[x];
        r[cnt]=r[hd[x]];
        l[r[hd[x]]]=cnt;
        r[hd[x]]=cnt;
    }
    ++sz[y];
    rh[cnt]=x;
    ch[cnt]=y;
    ++cnt;
}

inline void rm(int c)
{
    l[r[c]]=l[c];
    r[l[c]]=r[c];
    static int i,j;
    for(i=d[c];i!=c;i=d[i])
        for(j=r[i];j!=i;j=r[j])
        {
            u[d[j]]=u[j];
            d[u[j]]=d[j];
            --sz[ch[j]];
        }
}

inline void add(int c)
{
    l[r[c]]=c;
    r[l[c]]=c;
    static int i,j;
    for(i=d[c];i!=c;i=d[i])
        for(j=r[i];j!=i;j=r[j])
        {
            u[d[j]]=j;
            d[u[j]]=j;
            ++sz[ch[j]];
        }
}

bool dlx()
{
    if(!r[0])
        return true;
    int i,j,c;
    for(i=c=r[0];i;i=r[i])
        if(sz[i]<sz[c])
            c=i;
    rm(c);
    for(i=d[c];i!=c;i=d[i])
    {
        done[rh[i]]=true;
        for(j=r[i];j!=i;j=r[j])
            rm(ch[j]);
        if(dlx())
            return true;
        for(j=l[i];j!=i;j=l[j])
            add(ch[j]);
        done[rh[i]]=false;
    }
    add(c);
    return false;
}
```

8.2 dlx - repeat cover

```
#define MAXN 55
#define MAXM 55
#define MAXX (MAXN*MAXM)

int cnt;
int l[MAXX],r[MAXX],u[MAXX],d[MAXX],ch[MAXX];
int hd[MAXN],sz[MAXM];

inline void init(int m)
{
    static int i;
    for(i=0;i<=m;++i)
    {
        r[i]=i+1;
        l[i+1]=i;
        u[i]=d[i]=i;
        sz[i]=0;
    }
    r[m]=0;
    cnt=m;
}

inline void link(int x,int y)
{
    ++cnt;
    d[cnt]=d[y];
    u[cnt]=y;
    u[d[y]]=cnt;
    d[y]=cnt;
    if(hd[x]==-1)
        hd[x]=l[cnt]=r[cnt]=cnt;
    else
    {
        l[cnt]=hd[x];
        r[cnt]=r[hd[x]];
        l[r[hd[x]]]=cnt;
        r[hd[x]]=cnt;
    }
    ++sz[y];
    ch[cnt]=y;
}

inline void rm(int c)
{
    static int i;
    for(i=d[c];i!=c;i=d[i])
    {
        r[l[i]]=r[i];
        l[r[i]]=l[i];
    }
}

inline void add(int c)
{
    static int i;
    for(i=d[c];i!=c;i=d[i])
        r[l[i]]=l[r[i]]=i;
}

int K; // can't select more than K rows

inline int A()
{
    static int i,j,k,re;
    static bool done[MAXM];
    re=0;
    memset(done,0,sizeof done);
    for(i=r[0];i;i=r[i])
        if(!done[i])
        {
            ++re;
            for(j=d[i];j!=i;j=d[j])
                for(k=r[j];k!=j;k=r[k])
                    done[ch[k]]=true;
        }
    return re;
}

bool dlx(int now)
{
    if(!r[0])
        return true;
    if(now+A()<=K)
    {
        int i,j,c;
        for(i=c=r[0];i;i=r[i])
            if(sz[i]<sz[c])
                c=i;
        for(i=d[c];i!=c;i=d[i])
        {
            rm(i);
            for(j=r[i];j!=i;j=r[j])
                rm(j);
            if(dlx(now+1))
                return true;
            for(j=l[i];j!=i;j=l[j])
                add(j);
            add(i);
        }
    }
    return false;
}
```

8.3 fibonacci knapsack

```
#include<stdio.h>
#include<stdlib.h>
#include<algorithm>

#define MAXX 71

struct mono
{
    long long weig,cost;
```

```
}goods[MAXX];

int n,T,t,i;
long long carry,sumw,sumc;
long long ans,las[MAXX];

bool comp(const struct mono a,const struct mono b)
{
    if(a.weig!=b.weig)
        return a.weig<b.weig;
    return b.cost<a.cost;
}

void dfs(int i,long long cost_n,long long carry_n,int last)
{
    if(ans<cost_n)
        ans=cost_n;
    if(i==n || goods[i].weig>carry_n || cost_n+las[i]<=ans)
        return;
    if(last || (goods[i].weig!=goods[i-1].weig && goods[i].cost
        >goods[i-1].cost))
        dfs(i+1,cost_n+goods[i].cost,carry_n-goods[i].weig,1);
    dfs(i+1,cost_n,carry_n,0);
}

int main()
{
    scanf("%d",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d %lld",&n,&carry);
        sumw=0;
        sumc=0;
        ans=0;
        for(i=0;i<n;++i)
        {
            scanf("%lld %lld",&goods[i].weig,&goods[i].cost);
            sumw+=goods[i].weig;
            sumc+=goods[i].cost;
        }
        if(sumw<=carry)
        {
            printf("Case %d: %lld\n",t,sumc);
            continue;
        }
        std::sort(goods,goods+n,comp);
        for(i=0;i<n;++i)
        {
            las[i]=sumc;
            sumc-=goods[i].cost;
        }
        dfs(0,0,carry,1);
        printf("Case %d: %lld\n",t,ans);
    }
    return 0;
}
```

9 Others

9.1 .vimrc

```
set number
set history=1000000
set autoindent
set smartindent
set tabstop=4
set shiftwidth=4
set expandtab
set showmatch

set nocp
filetype plugin indent on

filetype on
syntax on
```

9.2 bigint

```
// header files
#include <cstdio>
#include <string>
#include <algorithm>
#include <iostream>

struct Bigint
{
    // representations and structures
    std::string a; // to store the digits
    int sign; // sign = -1 for negative numbers, sign = 1
        otherwise
    // constructors
    Bigint() {} // default constructor
    Bigint( std::string b ) { (*this) = b; } // constructor for
        std::string
    // some helpful methods
    int size() // returns number of digits
    {
        return a.size();
    }
    Bigint inverseSign() // changes the sign
    {
        sign *= -1;
        return (*this);
    }
    Bigint normalize( int newSign ) // removes leading 0, fixes
        sign
    {
        for( int i = a.size() - 1; i > 0 && a[i] == '0'; i-- )
            a.erase(a.begin() + i);
        sign = ( a.size() == 1 && a[0] == '0' ) ? 1 : newSign;
        return (*this);
    }
    // assignment operator
    void operator = ( std::string b ) // assigns a std::string
        to Bigint
    {
        a = b[0] == '-' ? b.substr(1) : b;
        reverse( a.begin(), a.end() );
        this->normalize( b[0] == '-' ? -1 : 1 );
    }
    // conditional operators
    bool operator < ( const Bigint &b ) const // less than
        operator
    {
        if( sign != b.sign )
            return sign < b.sign;
        if( a.size() != b.a.size() )
            return sign == 1 ? a.size() < b.a.size() : a.size()
                > b.a.size();
        for( int i = a.size() - 1; i >= 0; i-- )
            if( a[i] != b.a[i] )
                return sign == 1 ? a[i] < b.a[i] : a[i] > b.a[i
                    ];
        return false;
    }
    bool operator == ( const Bigint &b ) const // operator for
        equality
    {
        return a == b.a && sign == b.sign;
    }

    // mathematical operators
    Bigint operator + ( Bigint b ) // addition operator
        overloading
    {
        if( sign != b.sign )
            return (*this) - b.inverseSign();
        Bigint c;
        for(int i = 0, carry = 0; i<a.size() || i<b.size() ||
            carry; i++ )
        {
            carry+=(i<a.size() ? a[i]-48 : 0)+(i<b.a.size() ? b
                .a[i]-48 : 0);
            c.a += (carry % 10 + 48);
            carry /= 10;
        }
        return c.normalize(sign);
    }

    Bigint operator - ( Bigint b ) // subtraction operator
        overloading
    {
        if( sign != b.sign )
            return (*this) + b.inverseSign();
        int s = sign; sign = b.sign = 1;
        if( (*this) < b )
            return ((b - (*this)).inverseSign()).normalize(-s);
        Bigint c;
        for( int i = 0, borrow = 0; i < a.size(); i++ )
        {
            borrow = a[i] - borrow - (i < b.size() ? b.a[i] :
                48);
            c.a += borrow >= 0 ? borrow + 48 : borrow + 58;
            borrow = borrow >= 0 ? 0 : 1;
        }
        return c.normalize(s);
    }
    Bigint operator * ( Bigint b ) // multiplication operator
        overloading
    {
        Bigint c("0");
        for( int i = 0, k = a[i] - 48; i < a.size(); i++, k = a
            [i] - 48 )
        {
            while(k--)
                c = c + b; // ith digit is k, so, we add k
                    times
            b.a.insert(b.a.begin(), '0'); // multiplied by 10
        }
        return c.normalize(sign * b.sign);
```

```
    }
    Bigint operator / ( Bigint b ) // division operator
         overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0"), d;
        for( int j = 0; j < a.size(); j++ )
            d.a += "0";
        int dSign = sign * b.sign;
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
            {
                c = c - b;
                d.a[i]++;
            }
        }
        return d.normalize(dSign);
    }
    Bigint operator % ( Bigint b ) // modulo operator
         overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0");
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
                c = c - b;
        }
        return c.normalize(sign);
    }

    // output method
    void print()
    {
        if( sign == -1 )
            putchar('-');
        for( int i = a.size() - 1; i >= 0; i-- )
            putchar(a[i]);
    }
};



int main()
{
    Bigint a, b, c; // declared some Bigint variables
    ////////////////////////
    // taking Bigint input //
    ////////////////////////

    std::string input; // std::string to take input
    std::cin >> input; // take the Big integer as std::string
    a = input; // assign the std::string to Bigint a

    std::cin >> input; // take the Big integer as std::string
    b = input; // assign the std::string to Bigint b

    ///////////////////////////////////
    // Using mathematical operators //
    ///////////////////////////////////

    c = a + b; // adding a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a - b; // subtracting b from a
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a * b; // multiplying a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a / b; // dividing a by b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a % b; // a modulo b
    c.print(); // printing the Bigint
    puts(""); // newline

    ///////////////////////////////////
    // Using conditional operators //
    ///////////////////////////////////

    if( a == b )
        puts("equal"); // checking equality
    else
        puts("not equal");

    if( a < b )
        puts("a is smaller than b"); // checking less than
             operator

    return 0;
}
```

9.3 Binary Search

```
//[0,n)
inline int go(int A[],int n,int x) // return the least i that
     make A[i]==x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
            l=mid+1;
        else
        {
            r=mid-1;
            if(A[mid]==x)
                re=mid;
        }
    }
    return re;
}

inline int go(int A[],int n,int x) // return the largest i that
      make A[i]==x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
        {
            l=mid+1;
            if(A[mid]==x)
                re=mid;
        }
        else
            r=mid-1;
    }
    return re;
}

inline int go(int A[],int n,int x) // retrun the largest i that
      make A[i]<x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
        {
            l=mid+1;
            re=mid;
        }
        else
            r=mid-1;
    }
    return re;
}

inline int go(int A[],int n,int x)// return the largest i that
     make A[i]<=x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
        {
            l=mid+1;
            re=mid;
        }
        else
            r=mid-1;
```

```
    }
    return re;
}

inline int go(int A[],int n,int x)// return the least i that
     make A[i]>x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
            l=mid+1;
        else
        {
            r=mid-1;
            re=mid;
        }
    }
    return re;
}

inline int go(int A[],int n,int x)// upper_bound();
{
    static int l,r,mid;
    l=0;
    r=n-1;
    while(l<r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
            l=mid+1;
        else
            r=mid;
    }
    return r;
}

inline int go(int A[],int n,int x)// lower_bound();
{
    static int l,r,mid,;
    l=0;
    r=n-1;
    while(l<r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
            l=mid+1;
        else
            r=mid;
    }
    return r;
}
```

9.4 java

```
//Scanner

Scanner in=new Scanner(new FileReader("asdf"));
PrintWriter pw=new PrintWriter(new Filewriter("out"));
boolean        in.hasNext();
String         in.next();
BigDecimal     in.nextBigDecimal();
BigInteger     in.nextBigInteger();
BigInteger     in.nextBigInteger(int radix);
double         in.nextDouble();
int            in.nextInt();
int            in.nextInt(int radix);
String         in.nextLine();
long           in.nextLong();
long           in.nextLong(int radix);
short          in.nextShort();
short          in.nextShort(int radix);
int            in.radix(); //Returns this scanner's default
     radix.
Scanner        in.useRadix(int radix);// Sets this scanner's
     default radix to the specified radix.
void           in.close();//Closes this scanner.

//String

char           str.charAt(int index);
int            str.compareTo(String anotherString); // <0 if
     less. ==0 if equal. >0 if greater.
int            str.compareToIgnoreCase(String str);
String         str.concat(String str);
boolean        str.contains(CharSequence s);
boolean        str.endsWith(String suffix);
boolean        str.startsWith(String preffix);
boolean        str.startsWith(String preffix,int toffset);
int            str.hashCode();
int            str.indexOf(int ch);
int            str.indexOf(int ch,int fromIndex);
int            str.indexOf(String str);
int            str.indexOf(String str,int fromIndex);
int            str.lastIndexOf(int ch);
int            str.lastIndexOf(int ch,int fromIndex);
//(ry
int            str.length();
String         str.substring(int beginIndex);
String         str.substring(int beginIndex,int endIndex);
String         str.toLowerCase();
String         str.toUpperCase();
String         str.trim();// Returns a copy of the string, with
     leading and trailing whitespace omitted.

//StringBuilder
StringBuilder str.insert(int offset,...);
StringBuilder str.reverse();
void           str.setCharAt(int index,int ch);

//BigInteger
compareTo(); equals(); doubleValue(); longValue(); hashCode();
     toString(); toString(int radix); max(); min(); mod();
     modPow(BigInteger exp,BigInteger m); nextProbablePrime();
     pow();
andNot(); and(); xor(); not(); or(); getLowestSetBit();
     bitCount(); bitLength(); setBig(int n); shiftLeft(int n);
     shiftRight(int n);
add(); divide(); divideAndRemainder(); remainder(); multiply();
      subtract(); gcd(); abs(); signum(); negate();

//BigDecimal
movePointLeft(); movePointRight(); precision();
     stripTrailingZeros(); toBigInteger(); toPlainString();

import java.util.*;

//sort
class pii implements Comparable
{
    public int a,b;
    public int compareTo(Object i)
    {
        pii c=(pii)i;
        return a==c.a?c.b-b:c.a-a;
    }
}

class Main
{
    public static void main(String[] args)
    {
        pii[] the=new pii[2];
        the[0]=new pii();
        the[1]=new pii();
        the[0].a=1;
        the[0].b=1;
        the[1].a=1;
        the[1].b=2;
        Arrays.sort(the);
        for(int i=0;i<2;++i)
            System.out.printf("%d␣%d\n",the[i].a,the[i].b);
    }
}

//fraction
class frac
{
    public BigInteger a,b;
    public frac(long aa,long bb)
    {
        a=BigInteger.valueOf(aa);
        b=BigInteger.valueOf(bb);
        BigInteger c=a.gcd(b);
        a=a.divide(c);
        b=b.divide(c);
    }
    public frac(BigInteger aa,BigInteger bb)
    {
        BigInteger c=aa.gcd(bb);
        a=aa.divide(c);
        b=bb.divide(c);
    }
    public frac mul(frac i)
    {
        return new frac(a.multiply(i.a),b.multiply(i.b));
    }
    public frac mul(long i)
    {
        return new frac(a.multiply(BigInteger.valueOf(i)),b);
    }
    public frac div(long i)
    {
        return new frac(a,b.multiply(BigInteger.valueOf(i)));
    }
    public frac add(frac i)
    {
```

```
        return new frac((a.multiply(i.b)).add(i.a.multiply(b)),
            b.multiply(i.b));
    }
    public void print()
    {
        System.out.println(a+"/"+b); //printf 会 PE 啊尼玛死……
    }
}

//regex
import java.util.*;
import java.util.regex.*;

class Main
{
    public static void main(String[] args)
    {
        Scanner in=new Scanner(System.in);
        int T=in.nextInt();
        while(T--!=0)
            System.out.println(Pattern.compile("\\s*[+|-]?\\d
                +(\\.\\d+)?([E|e][+|-]?\\d+)?").matcher(in.
                next()).matches()?"LEGAL":"ILLEGAL");
    }
}
```

9.5 Others

god damn it windows:

```
#pragma comment(linker, "/STACK:16777216")
#pragma comment(linker,"/STACK:102400000,102400000")

int main()
{
    static const int stk_sz = 10000000;
    static int stack[stk_sz], esp_bak;
    __asm__ __volatile__ ( "movl␣%%esp,␣%0\n\t" "movl␣%1,␣%%esp
        \n\t" : "=g"(esp_bak) : "g"(stack + stk_sz - 1));
    //sol
    __asm__ __volatile__ ( "movl␣%0,␣%%esp\n\t" : : "g"(esp_bak
        )); //也可以不恢复，直
        接espexit(0);
    return 0;
}

char *MyStack=new char[33554432];
int main()
{
    char *SysStack=NULL;
    MyStack+=33554432-1048576;//32M
    __asm__
    (
     "movl␣%%esp,%%eax\n\t"
     "movl␣%1,%%esp\n\t"
     :"=a"(SysStack)
     :"m"(MyStack)
    );
    //sol
    __asm__
    (
     "movl␣%0,%%esp\n\t"
     ::"m"(SysStack)
    );
    return 0;
}

chmod +x [filename]

while true; do
./gen > input
./sol < input > output.sol
./bf < input > output.bf

diff output.sol output.bf
if [ $? -ne 0 ]; then break; fi
done
```

enumerate all $\binom{n}{k}$:

```
inline void enum(int k,int n)
{
    static int s,cut,j;
    cut=(1<<n);
    for(s=(1<<k)-1;s<cut;)
    {
        /*do anything, status in s*/

        j=s&-s;
        s=(s+j)|(((s^(s+j))>>2)/j);
    }
}
```

- nothing to be afraid of, 'cause you love it. isn't it?
- calm_down();calm_down();calm_down();
- 读完题目读完题目读完题目
  - 认真读题、认真读题、认真读题、认真读题、
  - 不盲目跟版
  - 换题/换想法
- 对数/离线/hash/观察问题本身/点 ↔ 区间互转
  - 对数调整精度 or 将乘法转换成加法
  - 点化区间，区间化点
- 数组大小……
- 写解释器/编译器的时候别忘了负数
  - 还有 istringstream in <sstream>
  - 指令/函数名也可能是变量名
- vector 比 array 慢很多
- modPow 比手写快速幂慢很多
- 对于 bool 数组，memset 快 8 倍