Code Library

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Contents

1 Data Structure

1.1 atlantis

```
#include<cstdio>
#include<algorithm>
#include<map>

#define MAXX 111
#define inf 333
#define MAX inf*5

int mid[MAX],cnt[MAX];
double len[MAX];

int n,i,cas;
double x1,x2,y1,y2;
double ans;
std::map<double,int>map;
std::map<double,int>::iterator it;
double rmap[inf];

void make(int id,int l,int r)
{
    mid[id]=(l+r)>>1;
    if(l!=r)
    {
        make(id<<1,l,mid[id]);
        make(id<<1|1,mid[id]+1,r);
    }
}

void update(int id,int ll,int rr,int l,int r,int val)
{
    if(ll==l && rr==r)
    {
        cnt[id]+=val;
        if(cnt[id])
            len[id]=rmap[r]-rmap[l-1];
        else
            if(l!=r)
                len[id]=len[id<<1]+len[id<<1|1];
            else
                len[id]=0;
        return;
    }
    if(mid[id]>=r)
        update(id<<1,ll,mid[id],l,r,val);
    else
        if(mid[id]<l)
            update(id<<1|1,mid[id]+1,rr,l,r,val);
        else
        {
            update(id<<1,ll,mid[id],l,mid[id],val);
            update(id<<1|1,mid[id]+1,rr,mid[id]+1,r,val);
        }
    if(!cnt[id])
        len[id]=len[id<<1]+len[id<<1|1];
}

struct node
{
    double l,r,h;
    char f;
    inline bool operator<(const node &a)const
    {
        return h<a.h;
    }
    inline void print()
    {
        printf("%lf␣%lf␣%lf␣%d\n",l,r,h,f);
    }
}ln[inf];

int main()
{
    make(1,1,inf);
    while(scanf("%d",&n),n)
    {
        n<<=1;
        map.clear();
        for(i=0;i<n;++i)
        {
            scanf("%lf%lf%lf%lf",&x1,&y1,&x2,&y2);
            if(x1>x2)
                std::swap(x1,x2);
            if(y1>y2)
                std::swap(y1,y2);
            ln[i].l=x1;
            ln[i].r=x2;
            ln[i].h=y1;
            ln[i].f=1;
            ln[++i].l=x1;
            ln[i].r=x2;
            ln[i].h=y2;
            ln[i].f=-1;
            map[x1]=1;
            map[x2]=1;
        }
        i=1;
        for(it=map.begin();it!=map.end();++it,++i)
        {
            it->second=i;
            rmap[i]=it->first;
        }
        std::sort(ln,ln+n);
        ans=0;
        update(1,1,inf,map[ln[0].l]+1,map[ln[0].r],ln[0].f);
        for(i=1;i<n;++i)
        {
            ans+=len[1]*(ln[i].h-ln[i-1].h);
            update(1,1,inf,map[ln[i].l]+1,map[ln[i].r],ln[i].f);
        }
        printf("Test␣case␣#%d\nTotal␣explored␣area:␣%.2lf\n\n",++cas,ans);
    }
    return 0;
}
```

1.2 binary indexed tree

```
int tree[MAXX];

inline int lowbit(const int &a)
{
    return a&-a;
}

inline void update(int pos,const int &val)
{
    while(pos<MAXX)
    {
        tree[pos]+=val;
        pos+=lowbit(pos);
    }
}

inline int read(int pos)
{
    int re(0);
    while(pos>0)
    {
        re+=tree[pos];
        pos-=lowbit(pos);
    }
    return re;
}

int find_Kth(int k)
{
    int now=0;
    for (char i=20;i>=0;--i)
    {
        now|=(1<<i);
        if (now>MAXX || tree[now]>=k)
            now^=(1<<i);
        else k-=tree[now];
    }
    return now+1;
}
```

1.3 COT

```
#include<cstdio>
#include<algorithm>

#define MAXX 100111
#define MAX (MAXX*23)
#define N 18

int sz[MAX],lson[MAX],rson[MAX],cnt;
int head[MAXX];
int pre[MAXX][N];
int map[MAXX],m;

int edge[MAXX],nxt[MAXX<<1],to[MAXX<<1];
int n,i,j,k,q,l,r,mid;
int num[MAXX],dg[MAXX];

int make(int l,int r)
{
    if(l==r)
        return ++cnt;
    int id(++cnt),mid((l+r)>>1);
    lson[id]=make(l,mid);
    rson[id]=make(mid+1,r);
    return id;
}

inline int update(int id,int pos)
{
    int re(++cnt);
    l=1;
    r=m;
    int nid(re);
    sz[nid]=sz[id]+1;
    while(l<r)
    {
        mid=(l+r)>>1;
        if(pos<=mid)
        {
            lson[nid]=++cnt;
            rson[nid]=rson[id];
            nid=lson[nid];
            id=lson[id];
            r=mid;
        }
        else
        {
            lson[nid]=lson[id];
            rson[nid]=++cnt;
            nid=rson[nid];
            id=rson[id];
            l=mid+1;
        }
        sz[nid]=sz[id]+1;
    }
    return re;
}

void rr(int now,int fa)
{
    dg[now]=dg[fa]+1;
    head[now]=update(head[fa],num[now]);
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa)
        {
            j=1;
            for(pre[to[i]][0]=now;j<N;++j)
                pre[to[i]][j]=pre[pre[to[i]][j-1]][j-1];
            rr(to[i],now);
        }
}

inline int query(int a,int b,int n,int k)
{
    static int tmp,t;
    l=1;
    r=m;
    a=head[a];
    b=head[b];
    t=num[n];
    n=head[n];
    while(l<r)
    {
        mid=(l+r)>>1;
        tmp=sz[lson[a]]+sz[lson[b]]-2*sz[lson[n]]+(l<=t && t<=mid);
        if(tmp>=k)
        {
            a=lson[a];
            b=lson[b];
            n=lson[n];
            r=mid;
        }
        else
        {
            k-=tmp;
            a=rson[a];
            b=rson[b];
            n=rson[n];
            l=mid+1;
        }
    }
    return l;
}

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=pre[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(pre[a][i]!=pre[b][i])
        {
            a=pre[a][i];
            b=pre[b][i];
        }
    return pre[a][0];
}

int main()
{
    scanf("%d␣%d",&n,&q);
    for(i=1;i<=n;++i)
    {
        scanf("%d",num+i);
        map[i]=num[i];
    }
    std::sort(map+1,map+n+1);
    m=std::unique(map+1,map+n+1)-map-1;
    for(i=1;i<=n;++i)
        num[i]=std::lower_bound(map+1,map+m+1,num[i])-map;
    for(i=1;i<n;++i)
    {
        scanf("%d␣%d",&j,&k);
        nxt[++cnt]=edge[j];
        edge[j]=cnt;
        to[cnt]=k;

        nxt[++cnt]=edge[k];
        edge[k]=cnt;
        to[cnt]=j;
    }
    cnt=0;
    head[0]=make(1,m);
    rr(1,0);
    while(q--)
    {
        scanf("%d␣%d␣%d",&i,&j,&k);
        printf("%d\n",map[query(i,j,lca(i,j),k)]);
    }
    return 0;
}
```

1.4 hose

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<cmath>

#define MAXX 50111

struct Q
{
    int l,r,s,w;
    bool operator<(const Q &i)const
    {
        return w==i.w?r<i.r:w<i.w;
    }
}a[MAXX];

int c[MAXX];
long long col[MAXX],sz[MAXX],ans[MAXX];
int n,m,cnt,len;

long long gcd(long long a,long long b)
{
    return a?gcd(b%a,a):b;
}

int i,j,k,now;
long long all,num;

int main()
{
    scanf("%d␣%d",&n,&m);
    for(i=1;i<=n;++i)
        scanf("%d",c+i);
    len=sqrt(m);
    for(i=1;i<=m;++i)
    {
        scanf("%d␣%d",&a[i].l,&a[i].r);
        if(a[i].l>a[i].r)
            std::swap(a[i].l,a[i].r);
        sz[i]=a[i].r-a[i].l+1;
        a[i].w=a[i].l/len+1;
        a[i].s=i;
    }
    std::sort(a+1,a+m+1);
    i=1;
    while(i<=m)
    {
        now=a[i].w;
        memset(col,0,sizeof col);
        for(j=a[i].l;j<=a[i].r;++j)
            ans[a[i].s]+=2*(col[c[j]]++);
        for(++i;a[i].w==now;++i)
        {
            ans[a[i].s]=ans[a[i-1].s];
            for(j=a[i-1].r+1;j<=a[i].r;++j)
                ans[a[i].s]+=2*(col[c[j]]++);
            if(a[i-1].l<a[i].l)
                for(j=a[i-1].l;j<a[i].l;++j)
                    ans[a[i].s]-=2*(--col[c[j]]);
            else
                for(j=a[i].l;j<a[i-1].l;++j)
                    ans[a[i].s]+=2*(col[c[j]]++);
        }
    }
    for(i=1;i<=m;++i)
    {
        if(sz[i]==1)
            all=1ll;
        else
            all=sz[i]*(sz[i]-1);
        num=gcd(ans[i],all);
        printf("%lld/%lld\n",ans[i]/num,all/num);
    }
    return 0;
}
```

1.5 Leftist tree

```
#include<cstdio>
#include<algorithm>

#define MAXX 100111

int val[MAXX],l[MAXX],r[MAXX],d[MAXX];

int set[MAXX];

int merge(int a,int b)
{
    if(!a)
        return b;
    if(!b)
        return a;
    if(val[a]<val[b]) // max-heap
        std::swap(a,b);
    r[a]=merge(r[a],b);
    if(d[l[a]]<d[r[a]])
        std::swap(l[a],r[a]);
    d[a]=d[r[a]]+1;
    set[l[a]]=set[r[a]]=a; // set a as father of its sons
    return a;
}

inline int find(int &a)
{
    while(set[a]) //brute-force to get the index of root
        a=set[a];
    return a;
}

inline void reset(int i)
{
    l[i]=r[i]=d[i]=set[i]=0;
}

int n,i,j,k;

int main()
{
    while(scanf("%d",&n)!=EOF)
    {
        for(i=1;i<=n;++i)
        {
            scanf("%d",val+i);
            reset(i);
        }
        scanf("%d",&n);
        while(n--)
        {
            scanf("%d␣%d",&i,&j);
            if(find(i)==find(j))
                puts("-1");
            else
            {
                k=merge(l[i],r[i]);
                val[i]>>=1;
                reset(i);
                set[i=merge(i,k)]=0;

                k=merge(l[j],r[j]);
                val[j]>>=1;
                reset(j);
                set[j=merge(j,k)]=0;

                set[k=merge(i,j)]=0;
                printf("%d\n",val[k]);
            }
        }
    }
    return 0;
}
```

1.6 Network

```
//HLD……备忘……_(:3JZ)_
#include<cstdio>
#include<algorithm>
#include<cstdlib>

#define MAXX 80111
#define MAXE (MAXX<<1)
#define N 18

int edge[MAXX],nxt[MAXE],to[MAXE],cnt;
int fa[MAXX][N],dg[MAXX];

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=fa[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(fa[a][i]!=fa[b][i])
        {
            a=fa[a][i];
            b=fa[b][i];
        }
    return fa[a][0];
}

inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

int sz[MAXX],pre[MAXX],next[MAXX];

void rr(int now)
{
    sz[now]=1;
    int max,id;
    max=0;
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa[now][0])
        {
            fa[to[i]][0]=now;
            dg[to[i]]=dg[now]+1;
            rr(to[i]);
            sz[now]+=sz[to[i]];
            if(sz[to[i]]>max)
            {
                max=sz[to[i]];
                id=to[i];
            }
        }
    if(max)
    {
        next[now]=id;
        pre[id]=now;
    }
}

#define MAXT (MAXX*N*5)

namespace Treap
{
    int cnt;
    int son[MAXT][2],key[MAXT],val[MAXT],sz[MAXT];

    inline void init()
    {
        key[0]=RAND_MAX;
        val[0]=0xc0c0c0c0;
        cnt=0;
    }

    inline void up(int id)
    {
        sz[id]=sz[son[id][0]]+sz[son[id][1]]+1;
    }
    inline void rot(int &id,int tp)
    {
        static int k;
        k=son[id][tp];
        son[id][tp]=son[k][tp^1];
        son[k][tp^1]=id;
        up(id);
        up(k);
        id=k;
    }
    void insert(int &id,int v)
    {
        if(id)
        {
            int k(v>=val[id]);
            insert(son[id][k],v);
            if(key[son[id][k]]<key[id])
                rot(id,k);
            else
                up(id);
            return;
        }
        id=++cnt;
        key[id]=rand()-1;
        val[id]=v;
        sz[id]=1;
        son[id][0]=son[id][1]=0;
    }
    void del(int &id,int v)
    {
        if(!id)
            return;
        if(val[id]==v)
        {
            int k(key[son[id][1]]<key[son[id][0]]);
            if(!son[id][k])
            {
                id=0;
                return;
            }
            rot(id,k);
            del(son[id][k^1],v);
        }
        else
            del(son[id][v>val[id]],v);
        up(id);
    }
    int rank(int id,int v)
    {
        if(!id)
            return 0;
        if(val[id]<=v)
            return sz[son[id][0]]+1+rank(son[id][1],v);
        return rank(son[id][0],v);
    }
    void print(int id)
    {
        if(!id)
            return;
        print(son[id][0]);
        printf("%d␣",val[id]);
        print(son[id][1]);
    }
}

int head[MAXX],root[MAXX],len[MAXX],pos[MAXX];

#define MAX (MAXX*6)
#define mid (l+r>>1)
#define lc lson[id],l,mid
#define rc rson[id],mid+1,r

int lson[MAX],rson[MAX];
int treap[MAX];

void make(int &id,int l,int r,int *the)
```

```
{
    id=++cnt;
    static int k;
    for(k=l;k<=r;++k)
        Treap::insert(treap[id],the[k]);
    if(l!=r)
    {
        make(lc,the);
        make(rc,the);
    }
}

int query(int id,int l,int r,int a,int b,int q)
{
    if(a<=l && r<=b)
        return Treap::rank(treap[id],q);
    int re(0);
    if(a<=mid)
        re=query(lc,a,b,q);
    if(b>mid)
        re+=query(rc,a,b,q);
    return re;
}

inline int query(int a,int b,int v)
{
    static int re;
    for(re=0;root[a]!=root[b];a=fa[root[a]][0])
        re+=query(head[root[a]],1,len[root[a]],1,pos[a],v);
    re+=query(head[root[a]],1,len[root[a]],pos[b],pos[a],v);
    return re;
}

inline void update(int id,int l,int r,int pos,int val,int n)
{
    while(l<=r)
    {
        Treap::del(treap[id],val);
        Treap::insert(treap[id],n);
        if(l==r)
            return;
        if(pos<=mid)
        {
            id=lson[id];
            r=mid;
        }
        else
        {
            id=rson[id];
            l=mid+1;
        }
    }
}

int n,q,i,j,k;
int val[MAXX];

int main()
{
    srand(1e9+7);
    scanf("%d␣%d",&n,&q);
    for(i=1;i<=n;++i)
        scanf("%d",val+i);
    for(k=1;k<n;++k)
    {
        scanf("%d␣%d",&i,&j);
        add(i,j);
        add(j,i);
    }
    rr(rand()%n+1);
    for(j=1;j<N;++j)
        for(i=1;i<=n;++i)
            fa[i][j]=fa[fa[i][j-1]][j-1];

    Treap::init();
    cnt=0;
    for(i=1;i<=n;++i)
        if(!pre[i])
        {
            static int tmp[MAXX];
            for(k=1,j=i;j;j=next[j],++k)
            {
                pos[j]=k;
                root[j]=i;
                tmp[k]=val[j];
            }
            --k;
            len[i]=k;
            make(head[i],1,k,tmp);
        }
    while(q--)
    {
        scanf("%d",&k);
        if(k)
        {
            static int a,b,c,d,l,r,ans,m;
            scanf("%d␣%d",&a,&b);
            c=lca(a,b);
            if(dg[a]+dg[b]-2*dg[c]+1<k)
            {
                puts("invalid␣request!");
                continue;
            }
            k=dg[a]+dg[b]-2*dg[c]+1-k+1;
            if(dg[a]<dg[b])
                std::swap(a,b);
            l=-1e9;
            r=1e9;
            if(b!=c)
            {
                d=a;
                for(i=0,j=dg[a]-dg[c]-1;j;j>>=1,++i)
                    if(j&1)
                        d=fa[d][i];
                while(l<=r)
                {
                    m=l+r>>1;
                    if(query(a,d,m)+query(b,c,m)>=k)
                    {
                        ans=m;
                        r=m-1;
                    }
                    else
                        l=m+1;
                }
            }
            else
            {
                while(l<=r)
                {
                    m=l+r>>1;
                    if(query(a,c,m)>=k)
                    {
                        ans=m;
                        r=m-1;
                    }
                    else
                        l=m+1;
                }
            }
            printf("%d\n",ans);
        }
        else
        {
            scanf("%d␣%d",&i,&j);
            update(head[root[i]],1,len[root[i]],pos[i],val[i],j);
            val[i]=j;
        }
    }
    return 0;
}
```

1.7 OTOCI

```
//记得随手 down 啊……亲……
//debug 时记得优先检查 up/down/select
#include<cstdio>
#include<algorithm>

#define MAXX 30111
#define lson nxt[id][0]
#define rson nxt[id][1]

int nxt[MAXX][2],fa[MAXX],pre[MAXX],val[MAXX],sum[MAXX];
bool rev[MAXX];

inline void up(int id)
{
    static int i;
    sum[id]=val[id];
    for(i=0;i<2;++i)
        if(nxt[id][i])
            sum[id]+=sum[nxt[id][i]];
}

inline void rot(int id,int tp)
{
    static int k;
    k=pre[id];
    nxt[k][tp^1]=nxt[id][tp];
    if(nxt[id][tp])
        pre[nxt[id][tp]]=k;
    if(pre[k])
        nxt[pre[k]][k==nxt[pre[k]][1]]=id;
    pre[id]=pre[k];
    nxt[id][tp]=k;
    pre[k]=id;
    up(k);
    up(id);
}

inline void down(int id) //记得随手 down 啊……亲……
{
    static int i;
    if(rev[id])
    {
        rev[id]=false;
        for(i=0;i<2;++i)
            if(nxt[id][i])
            {
                rev[nxt[id][i]]^=true;
                std::swap(nxt[nxt[id][i]][0],nxt[nxt[id][i]][1]);
            }
    }
}

inline void splay(int id)//记得随手 down 啊……亲……
{
    down(id);
    if(!pre[id])
        return;
    static int rt,k,st[MAXX];
    for(rt=id,k=0;rt;rt=pre[rt])
        st[k++]=rt;
    rt=st[k-1];
    while(k)
        down(st[--k]);
    for(std::swap(fa[id],fa[rt]);pre[id];rot(id,id==nxt[pre[id]][0]));
    /* another faster methond:
    std::swap(fa[id],fa[rt]);
    do
    {
        rt=pre[id];
        if(pre[rt])
        {
            k=(nxt[pre[rt]][0]==rt);
            if(nxt[rt][k]==id)
                rot(id,k^1);
            else
                rot(rt,k);
            rot(id,k);
        }
        else
            rot(id,id==nxt[rt][0]);
    }
    while(pre[id]);
    */
}

inline int access(int id)
{
    static int to;
    for(to=0;id;id=fa[id])
    {
        splay(id);
        if(rson)
        {
            pre[rson]=0;
            fa[rson]=id;
        }
        rson=to;
        if(to)
        {
            pre[to]=id;
            fa[to]=0;
        }
        up(to=id);
    }
    return to;
}

inline int getrt(int id)
{
    access(id);
    splay(id);
    while(nxt[id][0])
    {
        id=nxt[id][0];
        down(id);
    }
    return id;
}

inline void makert(int id)
{
    access(id);
    splay(id);
    if(nxt[id][0])
    {
        rev[id]^=true;
        std::swap(lson,rson);
    }
}

int n,i,j,k,q;
char buf[11];

int main()
{
    scanf("%d",&n);
    for(i=1;i<=n;++i)
        scanf("%d",val+i);
    scanf("%d",&q);
    while(q--)
    {
        scanf("%s␣%d␣%d",buf,&i,&j);
        switch(buf[0])
        {
            case 'b':
                if(getrt(i)==getrt(j))
                    puts("no");
                else
                {
                    puts("yes");
                    makert(i);
                    fa[i]=j;
                }
                break;
            case 'p':
                access(i);
                splay(i);
                val[i]=j;
                up(i);
                break;
            case 'e':
                if(getrt(i)!=getrt(j))
                    puts("impossible");
                else
                {
                    makert(i);
                    access(j);
                    splay(j);
                    printf("%d\n",sum[j]);
                }
                break;
        }
    }
    return 0;
}
```

1.8 picture

```
#include<cstdio>
#include<algorithm>
#include<map>

#define MAXX 5555
#define MAX MAXX<<3
#define inf 10011

int n,i;
int mid[MAX],cnt[MAX],len[MAX],seg[MAX];
bool rt[MAX],lf[MAX];

std::map<int,int>map;
std::map<int,int>::iterator it;
int rmap[inf];
long long sum;
int x1,x2,y1,y2,last;

void make(int id,int l,int r)
{
    mid[id]=(l+r)>>1;
    if(l!=r)
    {
        make(id<<1,l,mid[id]);
        make(id<<1|1,mid[id]+1,r);
    }
}

void update(int id,int ll,int rr,int l,int r,int val)
{
    if(l==ll && rr==r)
    {
        cnt[id]+=val;
        if(cnt[id])
        {
            rt[id]=lf[id]=true;
            len[id]=rmap[r]-rmap[l-1];
            seg[id]=1;
        }
        else
            if(l!=r)
            {
                len[id]=len[id<<1]+len[id<<1|1];
                seg[id]=seg[id<<1]+seg[id<<1|1];
                if(rt[id<<1] && lf[id<<1|1])
                    --seg[id];
                rt[id]=rt[id<<1|1];
                lf[id]=lf[id<<1];
            }
            else
            {
                len[id]=0;
                rt[id]=lf[id]=false;
                seg[id]=0;
            }
        return;
    }
    if(mid[id]>=r)
        update(id<<1,ll,mid[id],l,r,val);
    else
        if(mid[id]<l)
            update(id<<1|1,mid[id]+1,rr,l,r,val);
        else
        {
            update(id<<1,ll,mid[id],l,mid[id],val);
            update(id<<1|1,mid[id]+1,rr,mid[id]+1,r,val);
        }
    if(!cnt[id])
    {
        len[id]=len[id<<1]+len[id<<1|1];
        seg[id]=seg[id<<1]+seg[id<<1|1];
        if(rt[id<<1] && lf[id<<1|1])
            --seg[id];
        rt[id]=rt[id<<1|1];
        lf[id]=lf[id<<1];
    }
}

struct node
{
    int l,r,h;
    char val;
    inline bool operator<(const node &a)const
    {
        return h==a.h?val<a.val:h<a.h;   // trick watch out. val<a.val? val>a.val?
    }
    inline void print()
    {
        printf("%d␣%d␣%d␣%d\n",l,r,h,val);
    }
}ln[inf];

int main()
{
    make(1,1,inf);
    scanf("%d",&n);
    n<<=1;
    map.clear();
    for(i=0;i<n;++i)
    {
        scanf("%d%d%d%d",&x1,&y1,&x2,&y2);
        ln[i].l=x1;
        ln[i].r=x2;
        ln[i].h=y1;
        ln[i].val=1;
        ln[++i].l=x1;
        ln[i].r=x2;
        ln[i].h=y2;
        ln[i].val=-1;
        map[x1]=1;
        map[x2]=1;
    }
    i=1;
    for(it=map.begin();it!=map.end();++it,++i)
    {
        it->second=i;
        rmap[i]=it->first;
    }
    i=0;
    std::sort(ln,ln+n);
    update(1,1,inf,map[ln[0].l]+1,map[ln[0].r],ln[0].val);
    sum+=len[1];
    last=len[1];
    for(i=1;i<n;++i)
    {
        sum+=2*seg[1]*(ln[i].h-ln[i-1].h);
        update(1,1,inf,map[ln[i].l]+1,map[ln[i].r],ln[i].val);
        sum+=abs(len[1]-last);
        last=len[1];
    }
    printf("%lld\n",sum);
    return 0;
}
```

1.9 Size Blanced Tree

```
template<class Tp>class sbt
{
    public:
        inline void init()
        {
            rt=cnt=l[0]=r[0]=sz[0]=0;
        }
        inline void ins(const Tp &a)
        {
            ins(rt,a);
        }
        inline void del(const Tp &a)
        {
            del(rt,a);
        }
        inline bool find(const Tp &a)
        {
            return find(rt,a);
        }
        inline Tp pred(const Tp &a)
        {
            return pred(rt,a);
        }
        inline Tp succ(const Tp &a)
        {
            return succ(rt,a);
        }
        inline bool empty()
        {
            return !sz[rt];
        }
        inline Tp min()
        {
            return min(rt);
        }
        inline Tp max()
        {
            return max(rt);
        }
        inline void delsmall(const Tp &a)
        {
            dels(rt,a);
        }
        inline int rank(const Tp &a)
        {
            return rank(rt,a);
        }
        inline Tp sel(const int &a)
        {
            return sel(rt,a);
        }
        inline Tp delsel(int a)
        {
            return delsel(rt,a);
        }
    private:
        int cnt,rt,l[MAXX],r[MAXX],sz[MAXX];
        Tp val[MAXX];
        inline void rro(int &pos)
        {
            int k(l[pos]);
            l[pos]=r[k];
            r[k]=pos;
            sz[k]=sz[pos];
            sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
            pos=k;
        }
        inline void lro(int &pos)
        {
            int k(r[pos]);
            r[pos]=l[k];
            l[k]=pos;
            sz[k]=sz[pos];
            sz[pos]=sz[l[pos]]+sz[r[pos]]+1;
            pos=k;
        }
        inline void mt(int &pos,bool flag)
        {
            if(!pos)
                return;
            if(flag)
                if(sz[r[r[pos]]]>sz[l[pos]])
                    lro(pos);
                else
                    if(sz[l[r[pos]]]>sz[l[pos]])
                    {
                        rro(r[pos]);
                        lro(pos);
                    }
                    else
                        return;
            else
                if(sz[l[l[pos]]]>sz[r[pos]])
                    rro(pos);
                else
                    if(sz[r[l[pos]]]>sz[r[pos]])
                    {
                        lro(l[pos]);
                        rro(pos);
                    }
                    else
                        return;
            mt(l[pos],false);
            mt(r[pos],true);
            mt(pos,false);
            mt(pos,true);
        }
        void ins(int &pos,const Tp &a)
        {
            if(pos)
            {
                ++sz[pos];
                if(a<val[pos])
                    ins(l[pos],a);
                else
                    ins(r[pos],a);
                mt(pos,a>=val[pos]);
                return;
            }
            pos=++cnt;
            l[pos]=r[pos]=0;
            val[pos]=a;
            sz[pos]=1;
        }
        Tp del(int &pos,const Tp &a)
        {
            --sz[pos];
            if(val[pos]==a || (a<val[pos] && !l[pos]) ||
                  (a>val[pos] && !r[pos]))
            {
                Tp ret(val[pos]);
                if(!l[pos] || !r[pos])
                    pos=l[pos]+r[pos];
                else
                    val[pos]=del(l[pos],val[pos]+1);
                return ret;
            }
            else
                if(a<val[pos])
                    return del(l[pos],a);
                else
                    return del(r[pos],a);
        }
        bool find(int &pos,const Tp &a)
        {
            if(!pos)
                return false;
            if(a<val[pos])
                return find(l[pos],a);
            else
                return (val[pos]==a || find(r[pos],a));
        }
        Tp pred(int &pos,const Tp &a)
        {
            if(!pos)
                return a;
            if(a>val[pos])
            {
                Tp ret(pred(r[pos],a));
                if(ret==a)
                    return val[pos];
                else
                    return ret;
            }
            return pred(l[pos],a);
        }
        Tp succ(int &pos,const Tp &a)
        {
            if(!pos)
                return a;
            if(a<val[pos])
            {
                Tp ret(succ(l[pos],a));
                if(ret==a)
                    return val[pos];
                else
                    return ret;
            }
            return succ(r[pos],a);
        }
        Tp min(int &pos)
        {
            if(l[pos])
                return min(l[pos]);
            else
                return val[pos];
        }
        Tp max(int &pos)
        {
            if(r[pos])
                return max(r[pos]);
            else
                return val[pos];
        }
        void dels(int &pos,const Tp &v)
        {
            if(!pos)
                return;
            if(val[pos]<v)
            {
                pos=r[pos];
                dels(pos,v);
                return;
            }
            dels(l[pos],v);
            sz[pos]=1+sz[l[pos]]+sz[r[pos]];
        }
        int rank(const int &pos,const Tp &v)
        {
            if(val[pos]==v)
                return sz[l[pos]]+1;
            if(v<val[pos])
                return rank(l[pos],v);
            return rank(r[pos],v)+sz[l[pos]]+1;
        }
        Tp sel(const int &pos,const int &v)
        {
            if(sz[l[pos]]+1==v)
                return val[pos];
            if(v>sz[l[pos]])
                return sel(r[pos],v-sz[l[pos]]-1);
            return sel(l[pos],v);
        }
        Tp delsel(int &pos,int k)
        {
            --sz[pos];
            if(sz[l[pos]]+1==k)
            {
                Tp re(val[pos]);
                if(!l[pos] || !r[pos])
                    pos=l[pos]+r[pos];
                else
                    val[pos]=del(l[pos],val[pos]+1);
                return re;
            }
            if(k>sz[l[pos]])
                return delsel(r[pos],k-1-sz[l[pos]]);
            return delsel(l[pos],k);
        }
};
```

1.10 sparse table - rectangle

```
#include<iostream>
#include<cstdio>
#include<algorithm>

#define MAXX 310

int mat[MAXX][MAXX];
int table[9][9][MAXX][MAXX];
int n;
short lg[MAXX];

int main()
{
    for(int i(2);i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    int T;
    std::cin >> T;
    while (T--)
    {
        std::cin >> n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
            {
                std::cin >> mat[i][j];
                table[0][0][i][j] = mat[i][j];
            }

        // 从小到大计算，保证后来用到的都已经计算过
        for(int i=0;i<=lg[n];++i) // width
        {
            for(int j=0;j<=lg[n];++j) //height
            {
                if(i==0 && j==0)
                    continue;
                for(int ii=0;ii+(1<<j)<=n;++ii)
                    for(int jj=0;jj+(1<<i)<=n;++jj)
                        if(i==0)
                            table[i][j][ii][jj]=std::min(table[i][j-1][ii][jj],table[i][j-1][ii+(1<<(j-1))][jj]);
                        else
                            table[i][j][ii][jj]=std::min(table[i-1][j][ii][jj],table[i-1][j][ii][jj+(1<<(i-1))]);
            }
        }
        long long N;
        std::cin >> N;
        int r1, c1, r2, c2;
        for (int i = 0; i < N; ++i)
        {
            scanf("%d%d%d%d",&r1,&c1,&r2,&c2);
            --r1;
            --c1;
            --r2;
            --c2;
            int w=lg[c2-c1+1];
            int h=lg[r2-r1+1];
            printf("%d\n",std::min(table[w][h][r1][c1],std::min(table[w][h][r1][c2-(1<<w)+1],std::min(table[w][h][r2-(1<<h)+1][c1],table[w][h][r2-(1<<h)+1][c2-(1<<w)+1]))));
        }
    }
    return 0;
}
```

1.11 sparse table - square

```
int num[MAXX][MAXX],max[MAXX][MAXX][10];
short lg[MAXX];

int main()
{
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    scanf("%hd␣%d",&n,&q);
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
        {
            scanf("%d",num[i]+j);
            max[i][j][0]=num[i][j];
        }
    for(k=1;k<=lg[n];++k)
    {
        l=n+1-(1<<k);
        for(i=0;i<l;++i)
            for(j=0;j<l;++j)
                max[i][j][k]=std::max(std::max(max[i][j][k-1],max[i+(1<<(k-1))][j][k-1]),std::max(max[i][j+(1<<(k-1))][k-1],max[i+(1<<(k-1))][j+(1<<(k-1))][k-1]));
    }
    printf("Case␣%hd:\n",t);
    while(q--)
    {
        scanf("%hd␣%hd␣%hd",&i,&j,&l);
        --i;
        --j;
        k=lg[l];
        printf("%d\n",std::max(std::max(max[i][j][k],max[i][j+l-(1<<k)][k]),std::max(max[i+l-(1<<k)][j][k],max[i+l-(1<<k)][j+l-(1<<k)][k])));
    }
}
```

1.12 sparse table

```
int num[MAXX],min[MAXX][20];
int lg[MAXX];


int main()
{
    for(i=2;i<MAXX;++i)
        lg[i]=lg[i>>1]+1;
    scanf("%d␣%d",&n,&q);
    for(i=1;i<=n;++i)
    {
        scanf("%d",num+i);
        min[i][0]=num[i];
    }
    for(j=1;j<=lg[n];++j)
    {
        l=n+1-(1<<j);
        j_=j-1;
        j__=(1<<j_);
        for(i=1;i<=l;++i)
            min[i][j]=std::min(min[i][j_],min[i+j__][j_]);
    }
    printf("Case␣%hd:\n",t);
    while(q--)
    {
        scanf("%d␣%d",&i,&j);
        k=lg[j-i+1];
        printf("%d\n",std::min(min[i][k],min[j-(1<<k)+1][k]));
    }
}
```

1.13 treap

```
#include<cstdlib>
#include<ctime>
#include<cstring>

struct node
{
    node *ch[2];
    int sz,val,key;
    node(){memset(this,0,sizeof(node));}
    node(int a);
}*null;

node::node(int a):sz(1),val(a),key(rand()-1){ch[0]=ch[1]=null;}

class Treap
{
    inline void up(node *pos)
    {
        pos->sz=pos->ch[0]->sz+pos->ch[1]->sz+1;
    }
    inline void rot(node *&pos,int tp)
    {
        node *k(pos->ch[tp]);
        pos->ch[tp]=k->ch[tp^1];
        k->ch[tp^1]=pos;
        up(pos);
        up(k);
        pos=k;
    }

    void insert(node *&pos,int val)
    {
        if(pos!=null)
        {
            int t(val>=pos->val);
            insert(pos->ch[t],val);
            if(pos->ch[t]->key<pos->key)
                rot(pos,t);
            else
                up(pos);
            return;
        }
        pos=new node(val);
    }
    void rec(node *pos)
    {
        if(pos!=null)
        {
            rec(pos->ch[0]);
            rec(pos->ch[1]);
            delete pos;
        }
    }
    inline int sel(node *pos,int k)
    {
        while(pos->ch[0]->sz+1!=k)
            if(pos->ch[0]->sz>=k)
                pos=pos->ch[0];
            else
            {
                k-=pos->ch[0]->sz+1;
                pos=pos->ch[1];
            }
        return pos->val;
    }
    void del(node *&pos,int val)
    {
        if(pos!=null)
        {
            if(pos->val==val)
            {
                int t(pos->ch[1]->key<pos->ch[0]->key);
                if(pos->ch[t]==null)
                {
                    delete pos;
                    pos=null;
                    return;
                }
                rot(pos,t);
                del(pos->ch[t^1],val);
            }
            else
                del(pos->ch[val>pos->val],val);
            up(pos);
        }
    }
    public:
    node *rt;

    Treap():rt(null){}
    inline void insert(int val)
    {
        insert(rt,val);
    }
    inline void reset()
    {
        rec(rt);
        rt=null;
    }
    inline int sel(int k)
    {
        if(k<1 || k>rt->sz)
            return 0;
        return sel(rt,rt->sz+1-k);
    }
    inline void del(int val)
    {
        del(rt,val);
    }
    inline int size()
    {
        return rt->sz;
    }
}treap[MAXX];

init:
{
    srand(time(0));
    null=new node();
    null->val=0xc0c0c0c0;
    null->sz=0;
    null->key=RAND_MAX;
    null->ch[0]=null->ch[1]=null;
    for(i=0;i<MAXX;++i)
        treap[i].rt=null;
}
```

2 Geometry

2.1 3D

```
struct pv
{
  double x,y,z;
  pv() {}
  pv(double xx,double yy,double zz):x(xx),y(yy),z(zz) {}
  pv operator -(const pv& b)const
  {
    return pv(x-b.x,y-b.y,z-b.z);
  }
  pv operator *(const pv& b)const
  {
    return pv(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
  }
  double operator &(const pv& b)const
  {
    return x*b.x+y*b.y+z*b.z;
  }
};

//模
double Norm(pv p)
{
  return sqrt(p&p);
}

//绕单位向量 V 旋转 theta 角度
pv Trans(pv pa,pv V,double theta)
{
    double s = sin(theta);
    double c = cos(theta);
    double x,y,z;
    x = V.x;
    y = V.y;
    z = V.z;
    pv pp =
        pv(
                (x*x*(1-c)+c)*pa.x+(x*y*(1-c)-z*s)*pa.y+(x*z*(1-c)+y*s)*pa.z,
                (y*x*(1-c)+z*s)*pa.x+(y*y*(1-c)+c)*pa.y+(y*z*(1-c)-x*s)*pa.z,
                (x*z*(1-c)-y*s)*pa.x+(y*z*(1-c)+x*s)*pa.y+(z*z*(1-c)+c)*pa.z
          );
    return pp;
}

//经纬度转换

x = r × sin(θ) × cos(α)
y = r × sin(θ) × sin(α)
z = r × cos(θ)

r = √(x × 2 + y × 2 + z × 2)
α=atan(y/x);
θ=acos(z/r);

r ∈ [0, ∞)
α ∈ [0, 2π]
θ ∈ [0, π]

lat ∈ [−π/2, π/2]
lng ∈ [−π, π]

pv getpv(double lat,double lng,double r)
{
  lat += pi/2;
  lng += pi;
  return
    pv(r*sin(lat)*cos(lng),r*sin(lat)*sin(lng),r*cos(lat));
}

//经纬度球面距离

#include<cstdio>
#include<cmath>

#define MAXX 1111

char buf[MAXX];
const double r=6875.0/2,pi=acos(-1.0);
double a,b,c,x1,x2,y2,ans;

int main()
{
    double y1;
    while(gets(buf)!=NULL)
    {
        gets(buf);
        gets(buf);

        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        x1=a+b/60+c/3600;
        x1=x1*pi/180;
        if(buf[0]=='S')
            x1=-x1;

        scanf("%s",buf);
        scanf("%lf^%lf'%lf\" %s\n",&a,&b,&c,buf);
        y1=a+b/60+c/3600;
        y1=y1*pi/180;
        if(buf[0]=='W')
            y1=-y1;

        gets(buf);

        scanf("%lf^%lf'%lf\"␣%s\n",&a,&b,&c,buf);
        x2=a+b/60+c/3600;
        x2=x2*pi/180;
        if(buf[0]=='S')
            x2=-x2;

        scanf("%s",buf);
        scanf("%lf^%lf'%lf\"␣%s\n",&a,&b,&c,buf);
        y2=a+b/60+c/3600;
        y2=y2*pi/180;
        if(buf[0]=='W')
            y2=-y2;

        ans=acos(cos(x1)*cos(x2)*cos(y1-y2)+sin(x1)*sin(x2))*r;
        printf("The␣distance␣to␣the␣iceberg:␣%.2lf␣miles.\n",ans);
        if(ans+0.005<100)
            puts("DANGER!");

        gets(buf);
    }
    return 0;
}

inline bool ZERO(const double &a)
{
    return fabs(a)<eps;
}

//三维向量是否为零
inline bool ZERO(pv p)
{
    return (ZERO(p.x) && ZERO(p.y) && ZERO(p.z));
}

//直线相交
bool LineIntersect(Line3D L1, Line3D L2)
{
    pv s = L1.s-L1.e;
    pv e = L2.s-L2.e;
    pv p  = s*e;
    if (ZERO(p))
        return false;    //是否平行
    p = (L2.s-L1.e)*(L1.s-L1.e);
    return ZERO(p&L2.e);         //是否共面
}

//线段相交
bool inter(pv a,pv b,pv c,pv d)
{
    pv ret = (a-b)*(c-d);
    pv t1 = (b-a)*(c-a);
    pv t2 = (b-a)*(d-a);
    pv t3 = (d-c)*(a-c);
    pv t4 = (d-c)*(b-c);
    return sgn(t1&ret)*sgn(t2&ret) < 0 && sgn(t3&ret)*sgn(t4&ret) < 0;
}

//点在直线上
bool OnLine(pv p, Line3D L)
{
    return ZERO((p-L.s)*(L.e-L.s));
}

//点在线段上
bool OnSeg(pv p, Line3D L)
{
    return (ZERO((L.s-p)*(L.e-p)) && EQ(Norm(p-L.s)+Norm(p-L.e),Norm(L.e-L.s)));
}

//点到直线距离
double Distance(pv p, Line3D L)
{
    return (Norm((p-L.s)*(L.e-L.s))/Norm(L.e-L.s));
}

//线段夹角
//范围值为[0,π 之间的弧度]
double Inclination(Line3D L1, Line3D L2)
{
    pv u = L1.e - L1.s;
    pv v = L2.e - L2.s;
    return acos( (u & v) / (Norm(u)*Norm(v)) );
}
```

2.2 3DCH

```
#include<cstdio>
#include<cmath>
#include<vector>
#include<algorithm>

#define MAXX 1111
#define eps 1e-8
#define inf 1e20

struct pv
{
    double x,y,z;
    pv(){}
    pv(const double &xx,const double &yy,const double &zz):x(xx),y(yy),z(zz){}
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y,z-i.z);
    }
    inline pv operator+(const pv &i)const
    {
        return pv(x+i.x,y+i.y,z+i.z);
    }
    inline pv operator+=(const pv &i)
    {
        x+=i.x;
        y+=i.y;
        z+=i.z;
        return *this;
    }
    inline pv operator*(const pv &i)const //叉积
    {
        return pv(y*i.z-z*i.y,z*i.x-x*i.z,x*i.y-y*i.x);
    }
    inline pv operator*(const double a)const
    {
        return pv(x*a,y*a,z*a);
    }
    inline double operator^(const pv &i)const //点积
    {
        return x*i.x+y*i.y+z*i.z;
    }
    inline double len()
    {
        return sqrt(x*x+y*y+z*z);
    }
};

struct pla
{
    short a,b,c;
    bool ok;
    pla(){}
    pla(const short &aa,const short &bb,const short &cc):a(aa),b(bb),c(cc),ok(true){}
    inline void set();
    inline void print()
    {
        printf("%hd␣%hd␣%hd\n",a,b,c);
    }
};

pv pnt[MAXX];
std::vector<pla>fac;
int to[MAXX][MAXX];

inline void pla::set()
{
    to[a][b]=to[b][c]=to[c][a]=fac.size();
}

inline double ptof(const pv &p,const pla &f) //点面距离?
{
    return (pnt[f.b]-pnt[f.a])*(pnt[f.c]-pnt[f.a])^(p-pnt[f.a]);
}

inline double vol(const pv &a,const pv &b,const pv &c,const pv &d)//有向体积，即六面体体积*6
{
    return (b-a)*(c-a)^(d-a);
}

inline double ptof(const pv &p,const short &f) //点到号面的距离pf
{
    return fabs(vol(pnt[fac[f].a],pnt[fac[f].b],pnt[fac[f].c],p)/((pnt[fac[f].b]-pnt[fac[f].a])*(pnt[fac[f].c]-pnt[fac[f].a])).len());
}

void dfs(const short&,const short&);

void deal(const short &p,const short &a,const short &b)
{
    if(fac[to[a][b]].ok)
        if(ptof(pnt[p],fac[to[a][b]])>eps)
            dfs(p,to[a][b]);
        else
        {
            pla add(b,a,p);
            add.set();
            fac.push_back(add);
        }
}

void dfs(const short &p,const short &now)
{
    fac[now].ok=false;
    deal(p,fac[now].b,fac[now].a);
    deal(p,fac[now].c,fac[now].b);
    deal(p,fac[now].a,fac[now].c);
}

inline void make(int n)
{
    static int i,j;
    fac.resize(0);
    if(n<4)
        return;

    for(i=1;i<n;++i)
        if((pnt[0]-pnt[i]).len()>eps)
        {
            std::swap(pnt[i],pnt[1]);
            break;
        }
    if(i==n)
        return;

    for(i=2;i<n;++i)
        if(((pnt[0]-pnt[1])*(pnt[1]-pnt[i])).len()>eps)
        {
            std::swap(pnt[i],pnt[2]);
            break;
        }
    if(i==n)
        return;

    for(i=3;i<n;++i)
        if(fabs((pnt[0]-pnt[1])*(pnt[1]-pnt[2])^(pnt[2]-pnt[i]))>eps)
        {
            std::swap(pnt[3],pnt[i]);
            break;
        }
    if(i==n)
        return;

    for(i=0;i<4;++i)
    {
        pla add((i+1)%4,(i+2)%4,(i+3)%4);
        if(ptof(pnt[i],add)>0)
            std::swap(add.c,add.b);
        add.set();
        fac.push_back(add);
    }
    for(;i<n;++i)
        for(j=0;j<fac.size();++j)
            if(fac[j].ok && ptof(pnt[i],fac[j])>eps)
            {
                dfs(i,j);
                break;
            }

    short tmp(fac.size());
    fac.resize(0);
    for(i=0;i<tmp;++i)
        if(fac[i].ok)
            fac.push_back(fac[i]);
}

inline pv gc() //重心
{
    pv re(0,0,0),o(0,0,0);
    double all(0),v;
    for(int i=0;i<fac.size();++i)
    {
        v=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
        re+=(pnt[fac[i].a]+pnt[fac[i].b]+pnt[fac[i].c])*0.25f*v;
        all+=v;
    }
    return re*(1/all);
}

inline bool same(const short &s,const short &t) //两面是否相等
{
    pv &a=pnt[fac[s].a],&b=pnt[fac[s].b],&c=pnt[fac[s].c];
    return fabs(vol(a,b,c,pnt[fac[t].a]))<eps && fabs(vol(a,b,c,pnt[fac[t].b]))<eps && fabs(vol(a,b,c,pnt[fac[t].c]))<eps;
}

//表面多边形数目
inline int facetcnt()
{
    int ans=0;
    static int i,j;
    for(i=0;i<fac.size();++i)
    {
        for(j=0;j<i;++j)
            if(same(i,j))
                break;
        if(j==i)
            ++ans;
    }
    return ans;
}

//表面三角形数目
inline short trianglecnt()
{
    return fac.size();
}

//三点构成的三角形面积*2
inline double area(const pv &a,const pv &b,const pv &c)
{
        return ((b-a)*(c-a)).len();
}

//表面积
inline double area()
{
    double ret(0);
    static int i;
    for(i=0;i<fac.size();++i)
        ret+=area(pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return ret/2;
}

//体积
inline double volume()
{
    pv o(0,0,0);
    double ret(0);
    for(short i(0);i<fac.size();++i)
        ret+=vol(o,pnt[fac[i].a],pnt[fac[i].b],pnt[fac[i].c]);
    return fabs(ret/6);
}
```

2.3 circle's area

```
//去重
{
    for (int i = 0; i < n; i++)
    {
        scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        del[i] = false;
    }
    for (int i = 0; i < n; i++)
        if (del[i] == false)
        {
            if (c[i].r == 0.0)
                del[i] = true;
            for (int j = 0; j < n; j++)
                if (i != j)
                    if (del[j] == false)
                        if (cmp(Point(c[i].c,c[j].c).Len()+c[i].r,c[j].r) <= 0)
                            del[i] = true;
        }
    tn = n;
    n = 0;
    for (int i = 0; i < tn; i++)
        if (del[i] == false)
            c[n++] = c[i];
}

//ans[i表示被覆盖]次的面积i
const double pi = acos(-1.0);
const double eps = 1e-8;
struct Point
{
    double x,y;
    Point(){}
    Point(double _x,double _y)
    {
        x = _x;
        y = _y;
    }
    double Length()
    {
        return sqrt(x*x+y*y);
    }
};
struct Circle
{
    Point c;
    double r;
};
struct Event
{
    double tim;
    int typ;
    Event(){}
    Event(double _tim,int _typ)
    {
        tim = _tim;
        typ = _typ;
    }
};

int cmp(const double& a,const double& b)
{
    if (fabs(a-b) < eps)    return 0;
    if (a < b)  return -1;
    return 1;
}

bool Eventcmp(const Event& a,const Event& b)
{
    return cmp(a.tim,b.tim) < 0;
}

double Area(double theta,double r)
{
    return 0.5*r*r*(theta-sin(theta));
}

double xmult(Point a,Point b)
{
    return a.x*b.y-a.y*b.x;
}

int n,cur,tote;
Circle c[1000];
double ans[1001],pre[1001],AB,AC,BC,theta,fai,a0,a1;
Event e[4000];
Point lab;

int main()
{
    while (scanf("%d",&n) != EOF)
    {
        for (int i = 0;i < n;i++)
            scanf("%lf%lf%lf",&c[i].c.x,&c[i].c.y,&c[i].r);
        for (int i = 1;i <= n;i++)
            ans[i] = 0.0;
        for (int i = 0;i < n;i++)
        {
            tote = 0;
            e[tote++] = Event(-pi,1);
            e[tote++] = Event(pi,-1);
            for (int j = 0;j < n;j++)
                if (j != i)
                {
                    lab = Point(c[j].c.x-c[i].c.x,c[j].c.y-c[i].c.y);
                    AB = lab.Length();
                    AC = c[i].r;
                    BC = c[j].r;
                    if (cmp(AB+AC,BC) <= 0)
                    {
                        e[tote++] = Event(-pi,1);
                        e[tote++] = Event(pi,-1);
                        continue;
                    }
                    if (cmp(AB+BC,AC) <= 0) continue;
                    if (cmp(AB,AC+BC) > 0)  continue;
                    theta = atan2(lab.y,lab.x);
                    fai = acos((AC*AC+AB*AB-BC*BC)/(2.0*AC*AB));
                    a0 = theta-fai;
                    if (cmp(a0,-pi) < 0)    a0 += 2*pi;
                    a1 = theta+fai;
                    if (cmp(a1,pi) > 0)  a1 -= 2*pi;
                    if (cmp(a0,a1) > 0)
                    {
                        e[tote++] = Event(a0,1);
                        e[tote++] = Event(pi,-1);
                        e[tote++] = Event(-pi,1);
                        e[tote++] = Event(a1,-1);
                    }
                    else
                    {
                        e[tote++] = Event(a0,1);
                        e[tote++] = Event(a1,-1);
                    }
                }
            sort(e,e+tote,Eventcmp);
            cur = 0;
            for (int j = 0;j < tote;j++)
            {
                if (cur != 0 && cmp(e[j].tim,pre[cur]) != 0)
                {
                    ans[cur] += Area(e[j].tim-pre[cur],c[i].r);
                    ans[cur] += xmult(Point(c[i].c.x+c[i].r*cos(pre[cur]),c[i].c.y+c[i].r*sin(pre[cur])),
                        Point(c[i].c.x+c[i].r*cos(e[j].tim),c[i].c.y+c[i].r*sin(e[j].tim)))/2.0;
                }
                cur += e[j].typ;
                pre[cur] = e[j].tim;
            }
        }
        for (int i = 1;i < n;i++)
            ans[i] -= ans[i+1];
        for (int i = 1;i <= n;i++)
            printf("[%d]␣=␣%.3f\n",i,ans[i]);
    }
    return 0;
}
```

2.4 circle

```
//单位圆覆盖
#include<cstdio>
#include<cmath>
#include<algorithm>
#include<vector>

#define eps 1e-8
#define MAXX 211
const double pi(acos(-1));
typedef std::pair<double,int> pdi;

struct pv
{
    double x,y;
    pv(double a=0,double b=0):x(a),y(b){}
    pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    double len()
    {
        return hypot(x,y);
    }
}pnt[MAXX];

std::vector<pdi>alpha(MAXX<<1);

inline int solve(double r) //radius
{
    static int ans,sum,i,j;
    sum=ans=0;
    for(i=0;i<n;++i)
    {
        alpha.resize(0);
        static double d,theta,phi;
        static pv vec;
        for(j=0;j<n;++j)
        {
            if(j==i || (d=(vec=pnt[i]-pnt[j]).len())>2*r+eps)
                continue;
            if((theta=atan2(vec.y,vec.x))<-eps)
                theta+=2*pi;
            phi=acos(d/(2*r));
            alpha.push_back(pdi(theta-phi+2*pi,-1));
            alpha.push_back(pdi(theta+phi+2*pi,1));
        }
        std::sort(alpha.begin(),alpha.end());
        for(j=0;j<alpha.size();++j)
        {
            sum-=alpha[j].second;
            if(sum>ans)
                ans=sum;
        }
    }
    return ans+1;
}
```

2.5 closest point pair

```
//演算法笔记1

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* : 把所有點分成左右兩側，點數盡量一樣多。Divide */

    int M = (L + R) / 2;

    /* : 左側、右側分別遞迴求解。Conquer */

    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* : 尋找靠近中線的點，並依座標排序。MergeYO(NlogN)。 */

    int N = 0;  // 靠近中線的點數目
    for (int i=M;   i>=L && p[M].x - p[i].x < d; --i) t[N++] = p[i];
    for (int i=M+1; i<=R && p[i].x - p[M].x < d; ++i) t[N++] = p[i];
    sort(t, t+N, cmpy); // Quicksort O(NlogN)

    /* : 尋找橫跨兩側的最近點對。MergeO(N)。 */

    for (int i=0; i<N-1; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}


//演算法笔记2

struct Point {double x, y;} p[10], t[10];
bool cmpx(const Point& i, const Point& j) {return i.x < j.x;}
bool cmpy(const Point& i, const Point& j) {return i.y < j.y;}

double DnC(int L, int R)
{
    if (L >= R) return 1e9; // 沒有點、只有一個點。

    /* : 把所有點分成左右兩側，點數盡量一樣多。Divide */

    int M = (L + R) / 2;

    // 先把中線的座標記起來，因為待會重新排序之後會跑掉。X
    double x = p[M].x;

    /* : 左側、右側分別遞迴求解。Conquer */

    // 遞迴求解，並且依照座標重新排序。Y
    double d = min(DnC(L,M), DnC(M+1,R));
    //  if (d == 0.0) return d; // 提早結束

    /* : 尋找靠近中線的點，並依座標排序。MergeYO(N)。 */

    // 尋找靠近中線的點，先找左側。各點已照座標排序了。Y
    int N = 0;  // 靠近中線的點數目
    for (int i=0; i<=M; ++i)
        if (x - p[i].x < d)
            t[N++] = p[i];

    // 尋找靠近中線的點，再找右側。各點已照座標排序了。Y
    int P = N;  // 為分隔位置P
    for (int i=M+1; i<=R; ++i)
        if (p[i].x - x < d)
            t[N++] = p[i];

    // 以座標排序。使用YMerge 方式，合併已排序的兩陣列。Sort
    inplace_merge(t, t+P, t+N, cmpy);

    /* : 尋找橫跨兩側的最近點對。MergeO(N)。 */

    for (int i=0; i<N; ++i)
        for (int j=1; j<=2 && i+j<N; ++j)
            d = min(d, distance(t[i], t[i+j]));

    /* : 重新以座標排序所有點。MergeYO(N)。 */

    // 如此一來，更大的子問題就可以直接使用Merge 。Sort
    inplace_merge(p+L, p+M+1, p+R+1, cmpy);

    return d;
}

double closest_pair()
{
    sort(p, p+10, cmpx);
    return DnC(0, N-1);
}

//mzry
//分治
double calc_dis(Point &a ,Point &b) {
  return sqrt((a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y));
}
//别忘了排序
bool operator<(const Point &a ,const Point &b) {
  if(a.y != b.y) return a.x < b.x;
  return a.x < b.x;
}
double Gao(int l ,int r ,Point pnts[]) {
  double ret = inf;
  if(l == r) return ret;
  if(l+1 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    return ret;
  }
  if(l+2 ==r) {
    ret = min(calc_dis(pnts[l],pnts[l+1]) ,ret);
    ret = min(calc_dis(pnts[l],pnts[l+2]) ,ret);
    ret = min(calc_dis(pnts[l+1],pnts[l+2]) ,ret);
    return ret;
  }

  int mid = l+r>>1;
  ret = min (ret ,Gao(l ,mid,pnts));
  ret = min (ret , Gao(mid+1, r,pnts));

  for(int c = l ; c<=r; c++)
    for(int d = c+1; d <=c+7 && d<=r; d++) {
      ret = min(ret , calc_dis(pnts[c],pnts[d]));
    }
  return ret;
}

//增量
#include <iostream>
#include <cstdio>
#include <cstring>
#include <map>
#include <vector>
#include <cmath>
#include <algorithm>
#define Point pair<double,double>
using namespace std;

const int step[9][2] =
    {{-1,-1},{-1,0},{-1,1},{0,-1},{0,0},{0,1},{1,-1},{1,0},{1,1}};

int n,x,y,nx,ny;
map<pair<int,int>,vector<Point > > g;
vector<Point > tmp;
Point p[20000];
double tx,ty,ans,nowans;
vector<Point >::iterator it,op,ed;
pair<int,int> gird;
bool flag;

double Dis(Point p0,Point p1)
{
  return sqrt((p0.first-p1.first)*(p0.first-p1.first)+
        (p0.second-p1.second)*(p0.second-p1.second));
}

double CalcDis(Point p0,Point p1,Point p2)
{
  return Dis(p0,p1)+Dis(p0,p2)+Dis(p1,p2);
}

void build(int n,double w)
{
  g.clear();
  for (int i = 0;i < n;i++)
    g[make_pair((int)floor(p[i].first/w),(int)floor(p[i].second/w))].push_back(p[i]);
}

int main()
{
  int t;
  scanf("%d",&t);
  for (int ft = 1;ft <= t;ft++)
  {
    scanf("%d",&n);
    for (int i = 0;i < n;i++)
    {
      scanf("%lf%lf",&tx,&ty);
      p[i] = make_pair(tx,ty);
    }
    random_shuffle(p,p+n);
    ans = CalcDis(p[0],p[1],p[2]);
    build(3,ans/2.0);
    for (int i = 3;i < n;i++)
    {
      x = (int)floor(2.0*p[i].first/ans);
      y = (int)floor(2.0*p[i].second/ans);
      tmp.clear();
      for (int k = 0;k < 9;k++)
      {
        nx = x+step[k][0];
        ny = y+step[k][1];
        gird = make_pair(nx,ny);
        if (g.find(gird) != g.end())
        {
          op = g[gird].begin();
          ed = g[gird].end();
          for (it = op;it != ed;it++)
            tmp.push_back(*it);
        }
      }
      flag = false;
      for (int j = 0;j < tmp.size();j++)
        for (int k = j+1;k < tmp.size();k++)
        {
          nowans = CalcDis(p[i],tmp[j],tmp[k]);
          if (nowans < ans)
          {
            ans = nowans;
            flag = true;
          }
        }
      if (flag == true)
        build(i+1,ans/2.0);
      else
        g[make_pair((int)floor(2.0*p[i].first/ans),(int)floor(2.0*p[i].second/ans))].push_back(p[i]);
    }
    printf("%.3f\n",ans);
  }
}
```

2.6 half-plane intersection

```
//解析几何方式abc
inline pv ins(const pv &p1,const pv &p2)
{
    u=fabs(a*p1.x+b*p1.y+c);
    v=fabs(a*p2.x+b*p2.y+c);
    return pv((p1.x*v+p2.x*u)/(u+v),(p1.y*v+p2.y*u)/(u+v));
}

inline void get(const pv& p1,const pv& p2,double & a,double & b,double & c)
{
    a=p2.y-p1.y;
    b=p1.x-p2.x;
    c=p2.x*p1.y-p2.y*p1.x;
}

inline pv ins(const pv &x,const pv &y)
{
    get(x,y,d,e,f);
    return pv((b*f-c*e)/(a*e-b*d),(a*f-c*d)/(b*d-a*e));
}

std::vector<pv>p[2];
inline bool go()
{
    k=0;
    p[k].resize(0);
    p[k].push_back(pv(-inf,inf));
    p[k].push_back(pv(-inf,-inf));
    p[k].push_back(pv(inf,-inf));
    p[k].push_back(pv(inf,inf));
    for(i=0;i<n;++i)
    {
        get(pnt[i],pnt[(i+1)%n],a,b,c);
        c+=the*sqrt(a*a+b*b);
        p[!k].resize(0);
        for(l=0;l<p[k].size();++l)
            if(a*p[k][l].x+b*p[k][l].y+c<eps)
                p[!k].push_back(p[k][l]);
            else
            {
                m=(l+p[k].size()-1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
                m=(l+1)%p[k].size();
                if(a*p[k][m].x+b*p[k][m].y+c<-eps)
                    p[!k].push_back(ins(p[k][m],p[k][l]));
            }
        k=!k;
        if(p[k].empty())
            break;
    }
    //结果在p[k中]
    return p[k].empty();
}

//计算几何方式
//本例求多边形核

inline pv ins(const pv &a,const pv &b)
{
    u=fabs(ln.cross(a-pnt[i]));
    v=fabs(ln.cross(b-pnt[i]))+u;
    tl=b-a;
    return pv(u*tl.x/v+a.x,u*tl.y/v+a.y);
}

int main()
{
    j=0;
    for(i=0;i<n;++i)
    {
        ln=pnt[(i+1)%n]-pnt[i];
        p[!j].resize(0);
        for(k=0;k<p[j].size();++k)
            if(ln.cross(p[j][k]-pnt[i])<=0)
                p[!j].push_back(p[j][k]);
            else
            {
                l=(k-1+p[j].size())%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
                l=(k+1)%p[j].size();
                if(ln.cross(p[j][l]-pnt[i])<0)
                    p[!j].push_back(ins(p[j][k],p[j][l]));
            }
        j=!j;
    }
    //结果在p[j中]
}

//mrzy
```

```

bool HPIcmp(Line a, Line b)
{
    if (fabs(a.k - b.k) > eps)
        return a.k < b.k;
    return ((a.s - b.s) * (b.e-b.s)) < 0;
}

Line Q[100];

void HPI(Line line[], int n, Point res[], int &resn)
{
    int tot = n;
    std::sort(line, line + n, HPIcmp);
    tot = 1;
    for (int i = 1; i < n; i++)
        if (fabs(line[i].k - line[i - 1].k) > eps)
            line[tot++] = line[i];
    int head = 0, tail = 1;
    Q[0] = line[0];
    Q[1] = line[1];
    resn = 0;
    for (int i = 2; i < tot; i++)
    {
        if (fabs((Q[tail].e-Q[tail].s)*(Q[tail - 1].e-Q[tail - 1].s)) < eps || fabs((Q[head].e-Q[head].s)*(Q[head + 1].e-Q[head + 1].s)) < eps)
            return;
        while (head < tail && (((Q[tail]&Q[tail - 1]) - line[i].s) * (line[i].e-line[i].s)) > eps)
            --tail;
        while (head < tail && (((Q[head]&Q[head + 1]) - line[i].s) * (line[i].e-line[i].s)) > eps)
            ++head;
        Q[++tail] = line[i];
    }
    while (head < tail && (((Q[tail]&Q[tail - 1]) - Q[head].s) * (Q[head].e-Q[head].s)) > eps)
        tail--;
    while (head < tail && (((Q[head]&Q[head + 1]) - Q[tail].s) * (Q[tail].e-Q[tail].s)) > eps)
        head++;
    if (tail <= head + 1)
        return;
    for (int i = head; i < tail; i++)
        res[resn++] = Q[i] & Q[i + 1];
    if (head < tail + 1)
        res[resn++] = Q[head] & Q[tail];
}
```

2.7 intersection of circle and poly

```
pv c;
double r;

inline double cal(const pv &a,const pv &b)
{
    static double A,B,C,x,y,ts;
    A=(b-c).len();
    B=(a-c).len();
    C=(a-b).len();
    if(A<r && B<r)
        return (a-c).cross(b-c)/2;
    x=((a-b).dot(c-b)+sqrt(r*r*C*C-sqr((a-b).cross(c-b))))/C;
    y=((b-a).dot(c-a)+sqrt(r*r*C*C-sqr((b-a).cross(c-a))))/C;
    ts=(a-c).cross(b-c)/2;

    if(A<r && B>=r)
        return asin(ts*(1-x/C)*2/r/B*(1-eps))*r*r/2+ts*x/C;
    if(A>=r && B<r)
        return asin(ts*(1-y/C)*2/r/A*(1-eps))*r*r/2+ts*y/C;

    if(fabs((a-c).cross(b-c))>=r*C || (b-a).dot(c-a)<=0 || (a-b).dot(c-b)<=0)
    {
        if((a-c).dot(b-c)<0)
        {
            if((a-c).cross(b-c)<0)
                return (-pi-asin((a-c).cross(b-c)/A/B*(1-eps)))*r*r/2;
            return (pi-asin((a-c).cross(b-c)/A/B*(1-eps)))*r*r/2;
        }
        return asin((a-c).cross(b-c)/A/B*(1-eps))*r*r/2;
    }

    return (asin(ts*(1-x/C)*2/r/B*(1-eps))+asin(ts*(1-y/C)*2/r/A*(1-eps)))*r*r/2+ts*((y+x)/C-1);
}

inline double get(pv *the,int n)
{
    double ans=0;
    for(int i=0;i<n;++i)
        ans+=cal(the[i],the[(i+1)%n]);
    return ans;
}
```

2.8 k-d tree

```
/*
有个很关键的剪枝，在计算完与 mid 点的距离后，我们应该先进入左右哪个子树？我们应该先进入对于当前维度，查询点位于的那一边。显然，在查询点所在的子树，更容易查找出正确解。

那么当进入完左或右子树后，以查询点为圆心做圆，如果当前维度，查询点距离 mid 的距离（另一个子树中的点距离查询点的距离肯定大于这个距离）比堆里的最大值还大，那么就不再递归另一个子树。注意一下：如果堆里的元素个数不足 M，仍然还要进入另一棵子树。

说白了就是随便乱搞啦…………………
*/
// hysbz 2626
#include<cstdio>
#include<algorithm>
#include<queue>

inline long long sqr(long long a){ return a*a;}
typedef std::pair<long long,int> pli;

#define MAXX 100111
#define MAX (MAXX<<2)
#define inf 0x3f3f3f3fll
int idx;

struct PNT
{
    long long x[2];
    int lb;
    bool operator<(const PNT &i)const
    {
        return x[idx]<i.x[idx];
    }
    pli dist(const PNT &i)const
    {
        return pli(-(sqr(x[0]-i.x[0])+sqr(x[1]-i.x[1])),lb);
    }
}a[MAXX],the[MAX],p;

#define mid (l+r>>1)
#define lson (id<<1)
#define rson (id<<1|1)
#define lc lson,l,mid-1
#define rc rson,mid+1,r
int n,m;

long long rg[MAX][2][2];

void make(int id=1,int l=1,int r=n,int d=0)
{
    the[id].lb=-1;
    rg[id][0][0]=rg[id][1][0]=inf;
    rg[id][0][1]=rg[id][1][1]=-inf;
    if(l>r)
        return;
    idx=d;
    std::nth_element(a+l,a+mid,a+r+1);
    the[id]=a[mid];
    rg[id][0][0]=rg[id][0][1]=the[id].x[0];
    rg[id][1][0]=rg[id][1][1]=the[id].x[1];
    make(lc,d^1);
    make(rc,d^1);

    rg[id][0][0]=std::min(rg[id][0][0],std::min(rg[lson][0][0],rg[rson][0][0]));
    rg[id][1][0]=std::min(rg[id][1][0],std::min(rg[lson][1][0],rg[rson][1][0]));

    rg[id][0][1]=std::max(rg[id][0][1],std::max(rg[lson][0][1],rg[rson][0][1]));
    rg[id][1][1]=std::max(rg[id][1][1],std::max(rg[lson][1][1],rg[rson][1][1]));
}

inline long long cal(int id)
{
    static long long a[2];
    static int i;
    for(i=0;i<2;++i)
        a[i]=std::max(abs(p.x[i]-rg[id][i][0]),abs(p.x[i]-rg[id][i][1]));
    return sqr(a[0])+sqr(a[1]);
}

std::priority_queue<pli>ans;

void query(const int id=1,const int d=0)
{
    if(the[id].lb<0)
        return;
    pli tmp(the[id].dist(p));
    int a(lson),b(rson);
    if(p.x[d]<=the[id].x[d])
        std::swap(a,b);
    if(ans.size()<m)
        ans.push(tmp);
    else
        if(tmp<ans.top())
        {
            ans.push(tmp);
            ans.pop();
        }
    if(ans.size()<m || cal(a)>=-ans.top().first)
        query(a,d^1);
    if(ans.size()<m || cal(b)>=-ans.top().first)
        query(b,d^1);
}

int q,i,j,k;

int main()
{
    scanf("%d",&n);
    for(i=1;i<=n;++i)
    {
        scanf("%lld␣%lld",&a[i].x[0],&a[i].x[1]);
        a[i].lb=i;
    }
    make();
    scanf("%d",&q);
    while(q--)
    {
        scanf("%lld␣%lld",&p.x[0],&p.x[1]);
        scanf("%d",&m);
        while(!ans.empty())
            ans.pop();
        query();
        printf("%d\n",ans.top().second);
    }
    return 0;
}
```

2.9 Manhattan MST

```
#include<iostream>
#include<cstdio>
#include<cstring>
#include<queue>
#include<cmath>
using namespace std;
const int srange = 10000000;      //坐标范围
const int ra = 131072;      //线段树常量
int c[ ra * 2 ], d[ ra * 2 ];      //线段树
int a[ 100000 ], b[ 100000 ];    //排序临时变量
int order[ 400000 ], torder[ 100000 ]; //排序结果
int Index[ 100000 ];      //排序结果取反（为了在常数时间内取得某数的位置）
int road[ 100000 ][ 8 ];     //每个点连接出去的条边8
int y[ 100000 ], x[ 100000 ];      //点坐标
int n;          //点个数

int swap( int &a, int &b )     //交换两个数
{
    int t = a; a = b; b = t;
}

int insert( int a, int b, int i ) //向线段树中插入一个数
{
    a += ra;
    while ( a != 0 )
    {
        if ( c[ a ] > b )
        {
            c[ a ] = b;
            d[ a ] = i;
        }
        else break;
        a >>= 1;
    }
}

int find( int a )      //从c[0..a中找最小的数，线段树查询]
{
    a += ra;
    int ret = d[ a ], max = c[ a ];
    while ( a > 1 )
    {
        if ( ( a & 1 ) == 1 )
            if ( c[ --a ] < max )
            {
                max = c[ a ];
                ret = d[ a ];
            }
        a >>= 1;
    }
    return ret;
}

int ta[ 65536 ], tb[ 100000 ];    //基数排序临时变量

int radixsort( int *p )     //基数排序，以为基准p
{
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < n; i++ ) ta[ p[ i ] & 0xffff ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = n - 1; i >= 0; i-- ) tb[ --ta[ p[ order[ i ] ] & 0xffff ] ] = order[ i ];
    memmove( order, tb, n * sizeof( int ) );
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < n; i++ ) ta[ p[ i ] >> 16 ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = n - 1; i >= 0; i-- ) tb[ --ta[ p[ order[ i ] ] >> 16 ] ] = order[ i ];
    memmove( order, tb, n * sizeof( int ) );
}

int work( int ii )                //求每个点在一个方向上最近的点
{
    for (int i = 0; i < n; i++ ) //排序前的准备工作
    {
        a[ i ] = y[ i ] - x[ i ] + srange;
        b[ i ] = srange - y[ i ];
        order[ i ] = i;
    }
    radixsort( b );      //排序
    radixsort( a );
    for (int i = 0; i < n; i++ )
    {
        torder[ i ] = order[ i ];
        order[ i ] = i;
    }
    radixsort( a );      //为线段树而做的排序
    radixsort( b );
    for (int i = 0; i < n; i++ )
    {
        Index[ order[ i ] ] = i; //取反，求orderIndex
    }
    for (int i = 1; i < ra + n; i++ ) c[ i ] = 0x7fffffff; //线段树初始化
    memset( d, 0xff, sizeof( d ) );
    for (int i = 0; i < n; i++ ) //线段树插入删除调用
    {
        int tt = torder[ i ];
        road[ tt ][ ii ] = find( Index[ tt ] );
        insert( Index[ tt ], y[ tt ] + x[ tt ], tt );
    }
}

int distanc( int a, int b )       //求两点的距离，之所以少一个是因为编译器不让使用作为函数名edistance
{
    return abs( x[ a ] - x[ b ] ) + abs( y[ a ] - y[ b ] );
}

int ttb[ 400000 ];       //边排序的临时变量
int rx[ 400000 ], ry[ 400000 ], rd[ 400000 ]; //边的存储
int rr = 0;

int radixsort_2( int *p )    //还是基数排序，copy+的产物paste
```

```
{
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < rr; i++ ) ta[ p[ i ] & 0xffff ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = rr - 1; i >= 0; i-- ) ttb[ --ta[ p[ order[ i ] ] & 0xffff ] ] = order[ i ];
    memmove( order, ttb, rr * sizeof( int ) );
    memset( ta, 0, sizeof( ta ) );
    for (int i = 0; i < rr; i++ ) ta[ p[ i ] >> 16 ]++;
    for (int i = 0; i < 65535; i++ ) ta[ i + 1 ] += ta[ i ];
    for (int i = rr - 1; i >= 0; i-- ) ttb[ --ta[ p[ order[ i ] ] >> 16 ] ] = order[ i ];
    memmove( order, ttb, rr * sizeof( int ) );
}

int father[ 100000 ], rank[ 100000 ];    //并查集
int findfather( int x )                  //并查集寻找代表元
{
    if ( father[ x ] != -1 )
        return ( father[ x ] = findfather( father[ x ] ) );
    else return x;
}

long long kruskal()                      //最小生成树
{
    rr = 0;
    int tot = 0;
    long long ans = 0;
    for (int i = 0; i < n; i++ )         //得到边表
    {
        for (int j = 0; j < 4; j++ )
        {
            if ( road[ i ][ j ] != -1 )
            {
                rx[ rr ] = i;
                ry[ rr ] = road[ i ][ j ];
                rd[ rr++ ] = distanc( i, road[ i ][ j ] );
            }
        }
    }
    for (int i = 0; i < rr; i++ ) order[ i ] = i; //排序
    radixsort_2( rd );
    memset( father, 0xff, sizeof( father ) ); //并查集初始化
    memset( rank, 0, sizeof( rank ) );
    for (int i = 0; i < rr; i++ )     //最小生成树标准算法kruskal
    {
        if ( tot == n - 1 ) break;
        int t = order[ i ];
        int x = findfather( rx[ t ] ), y = findfather( ry[ t ] );
        if ( x != y )
        {
            ans += rd[ t ];
            tot++;
            int &rkx = rank[ x ], &rky = rank[ y ];
            if ( rkx > rky ) father[ y ] = x;
            else
            {
                father[ x ] = y;
                if ( rkx == rky ) rky++;
            }
        }
    }
    return ans;
}

int casenum = 0;

int main()
{
    while ( cin >> n )
    {
        if ( n == 0 ) break;
        for (int i = 0; i < n; i++ )
            scanf( "%d␣%d", &x[ i ], &y[ i ] );
        memset( road, 0xff, sizeof( road ) );
        for (int i = 0; i < 4; i++ )              //为了减少编程复杂度，work()函数只写了一种，其他情况用转换坐标的方式类似处理
        {           //为了降低算法复杂度，只求出个方向的边4
            if ( i == 2 )
            {
                for (int j = 0; j < n; j++ ) swap( x[ j ], y[ j ] );
            }
            if ( ( i & 1 ) == 1 )
            {
                for (int j = 0; j < n; j++ ) x[ j ] = srange - x[ j ];
            }
            work( i );
        }
        printf( "Case␣%d:␣Total␣Weight␣=␣", ++casenum );
        cout << kruskal() << endl;
    }
    return 0;
}
```

2.10 rotating caliper

```
//最远点对

inline double go()
{
    l=ans=0;
    for(i=0;i<n;++i)
    {
        tl=pnt[(i+1)%n]-pnt[i];
        while(abs(tl.cross(pnt[(l+1)%n]-pnt[i]))>=abs(tl.cross(pnt[l]-pnt[i])))
            l=(l+1)%n;
        ans=std::max(ans,std::max(dist(pnt[l],pnt[i]),dist(pnt[l],pnt[(i+1)%n])));
    }
    return ans;
}

//两凸包最近距离
double go()
{
    sq=sp=0;
    for(i=1;i<ch[1].size();++i)
        if(ch[1][sq]<ch[1][i])
            sq=i;
    tp=sp;
    tq=sq;
    ans=(ch[0][sp]-ch[1][sq]).len();
    do
    {
        a1=ch[0][sp];
        a2=ch[0][(sp+1)%ch[0].size()];
        b1=ch[1][sq];
        b2=ch[1][(sq+1)%ch[1].size()];
        tpv=b1-(b2-a1);
        tpv.x = b1.x - (b2.x - a1.x);
        tpv.y = b1.y - (b2.y - a1.y);
        len=(tpv-a1).cross(a2-a1);
        if(fabs(len)<eps)
        {
            ans=std::min(ans,p2l(a1,b1,b2));
            ans=std::min(ans,p2l(a2,b1,b2));
            ans=std::min(ans,p2l(b1,a1,a2));
            ans=std::min(ans,p2l(b2,a1,a2));
            sp=(sp+1)%ch[0].size();
            sq=(sq+1)%ch[1].size();
        }
        else
            if(len<-eps)
            {
                ans=std::min(ans,p2l(b1,a1,a2));
                sp=(sp+1)%ch[0].size();
            }
            else
            {
                ans=std::min(ans,p2l(a1,b1,b2));
                sq=(sq+1)%ch[1].size();
            }
    }while(tp!=sp || tq!=sq);
    return ans;
}

//外接矩形 by mzry
inline void solve()
{
    resa = resb = 1e100;
    double dis1,dis2;
    Point xp[4];
    Line l[4];
    int a,b,c,d;
    int sa,sb,sc,sd;
    a = b = c = d = 0;
    sa = sb = sc = sd = 0;
    Point va,vb,vc,vd;
    for (a = 0; a < n; a++)
    {
        va = Point(p[a],p[(a+1)%n]);
        vc = Point(-va.x,-va.y);
        vb = Point(-va.y,va.x);
        vd = Point(-vb.x,-vb.y);
        if (sb < sa)
        {
            b = a;
            sb = sa;
        }
        while (xmult(vb,Point(p[b],p[(b+1)%n])) < 0)
        {
            b = (b+1)%n;
            sb++;
        }
        if (sc < sb)
        {
            c = b;
            sc = sb;
        }
        while (xmult(vc,Point(p[c],p[(c+1)%n])) < 0)
        {
            c = (c+1)%n;
            sc++;
        }
        if (sd < sc)
        {
            d = c;
            sd = sc;
        }
        while (xmult(vd,Point(p[d],p[(d+1)%n])) < 0)
        {
            d = (d+1)%n;
            sd++;
        }

        //卡在 p[a],p[b],p[c],p[d] 上
        sa++;
    }
}

//合并凸包给定凸多边形
 P = { p(1) , ... , p(m) } 和 Q = { q(1) , ... , q(n) , 一个点对} (p(i), q(j)) 形成 P 和 Q 之间的桥当且仅当：

(p(i), q(j)) 形成一个并踵点对。
p(i-1), p(i+1), q(j-1), q(j+1) 都位于由 (p(i), q(j)) 组成的线的同一侧。假设多边形以标准形式给出并且顶点是以顺时针序排列，算法如下：、分别计算



1 P 和 Q 拥有最大 y 坐标的顶点。如果存在不止一个这样的点，取  x 坐标最大的。、构造这些点的遂平切线，
2 以多边形处于其右侧为正方向（因此他们指向 x 轴正方向）。、同时顺时针旋转两条切线直到其中一条与边相交。
3 得到一个新的并踵点对 (p(i), q(j)) 。对于平行边的情况，得到三个并踵点对。、对于所有有效的并踵点对
4 (p(i), q(j))： 判定 p(i-1), p(i+1), q(j-1), q(j+1) 是否都位于连接点 (p(i), q(j)) 形成的线的同一侧。如果是，这个并踵点对就形成了一个桥，并标记他。、重复执行步骤和步骤直到切线回到他们原来的位置。
534、所有可能的桥此时都已经确定了。
6 通过连续连接桥间对应的凸包链来构造合并凸包。上述的结论确定了算法的正确性。运行时间受步骤，，约束。

 156 他们都为 O(N) 运行时间（N 是顶点总数）。因此算法拥有现行的时间复杂度。一个凸多边形间的桥实际上确定了另一个有用的概念：多边形间公切线。同时，桥也是计算凸多边形交的算法核心。



//临界切线、计算
1 P 上 y 坐标值最小的顶点（称为 yminP ）和  Q 上 y 坐标值最大的顶点（称为）。 ymaxQ、为多边形在
2 yminP 和 ymaxQ 处构造两条切线 LP 和 LQ 使得他们对应的多边形位于他们的右侧。此时  LP 和 LQ 拥有不同的方向，并且  yminP 和 ymaxQ 成为了多边形间的一个对踵点对。、令
3 p(i)= , yminP q(j)= 。ymaxQ (p(i), q(j)) 构成了多边形间的一个对踵点对。检测是否有  p(i-1),p(i+1) 在线 (p(i), q(j)) 的一侧，并且  q(j-1),q(j+1) 在另一侧。如果成立，  (p(i), q(j)) 确定了一条线。CS、旋转这两条线，
4 直到其中一条和其对应的多边形的边重合。、一个新的对踵点对确定了。
5 如果两条线都与边重合，总共三对对踵点对（原先的顶点和新的顶点的组合）需要考虑。对于所有的对踵点对，执行上面的测试。、重复执行步骤和步骤，
645 直到新的点对为(yminP,ymaxQ)。、输出
7线。CS

//最小最大周长面积外接矩形//、计算全部四个多边形的端点，
1 称之为，  xminP , xmaxP , yminP 。ymaxP、通过四个点构造
2 P 的四条切线。他们确定了两个“卡壳”集合。、如果一条（或两条）线与一条边重合，
3 那么计算由四条线决定的矩形的面积，并且保存为当前最小值。否则将当前最小值定义为无穷大。、顺时针旋转线直到其中一条和多边形的一条边重合。
4、计算新矩形的周长面积，
5/ 并且和当前最小值比较。如果小于当前最小值则更新，并保存确定最小值的矩形信息。、重复步骤和步骤，
645 直到线旋转过的角度大于度。90、输出外接矩形的最小周长。
7
```

2.11 shit

```
struct pv
{
    double x,y;
    pv(double a=0,double b=0):x(a),y(b){}
    inline pv operator+(const pv &i)const
    {
        return pv(x+i.x,y+i.y);
    }
    inline pv operator-(const pv &i)const
    {
        return pv(x-i.x,y-i.y);
    }
    inline bool operator ==(const pv &i)const
    {
        return fabs(x-i.x)<eps && fabs(y-i.y)<eps;
    }
    inline bool operator<(const pv &i)const
    {
        return y==i.y?x<i.x:y<i.y;
    }
    inline double cross(const pv &i)const
    {
        return x*i.y-y*i.x;
    }
    inline double dot(const pv &i)const
    {
        return x*i.x+y*i.y;
    }
    inline double len()
    {
        return hypot(x,y);
    }
    inline pv rotate(pv p,double theta)
    {
        static pv v;
        v=*this-p;
        static double c,s;
        c=cos(theta);
        s=sin(theta);
        return pv(p.x+v.x*c-v.y*s,p.y+v.x*s+v.y*c);
    }
};

pv rotate(pv v,pv p,double theta,double sc=1) // rotate vector v, θ ∈ [0,2π]
{
    static pv re;
    re=p;
    v=v-p;
    p.x=sc*cos(theta);
    p.y=sc*sin(theta);
    re.x+=v.x*p.x-v.y*p.y;
    re.y+=v.x*p.y+v.y*p.x;
    return re;
}

struct line
{
    pv pnt[2];
    line(double a,double b,double c) // a*x + b*y + c = 0
    {
#define maxl 1e2 //preciseness should not be too high ( compare with eps )
        if(fabs(b)>eps)
        {
            pnt[0]=pv(maxl,(c+a*maxl)/(-b));
            pnt[1]=pv(-maxl,(c-a*maxl)/(-b));
        }
        else
        {
            pnt[0]=pv(-c/a,maxl);
            pnt[1]=pv(-c/a,-maxl);
        }
#undef maxl
    }
    pv cross(const line &v)const
    {
        double a=(v.pnt[1]-v.pnt[0]).cross(pnt[0]-v.pnt[0]);
        double b=(v.pnt[1]-v.pnt[0]).cross(pnt[1]-v.pnt[0]);
        return pv((pnt[0].x*b-pnt[1].x*a)/(b-a),(pnt[0].y*b-pnt[1].y*a)/(b-a));
    }
};

inline std::pair<pv,double> getcircle(const pv &a,const pv &b,const pv &c)
{
    static pv ct;
    ct=line(2*(b.x-a.x),2*(b.y-a.y),a.len()-b.len()).cross(line(2*(c.x-b.x),2*(c.y-b.y),b.len()-c.len()));
    return std::make_pair(ct,sqrt((ct-a).len()));
}

//sort with polar angle
inline bool cmp(const Point& a,const Point& b)
{
    if (a.y*b.y <= 0)
    {
        if (a.y > 0 || b.y > 0)
            return a.y < b.y;
        if (a.y == 0 && b.y == 0)
            return a.x < b.x;
    }
    return a.cross(b) > 0;
}

//graham
inline bool com(const pv &a,const pv &b)
{
    if(fabs(t=(a-pnt[0]).cross(b-pnt[0]))>eps)
        return t>0;
    return (a-pnt[0]).len()<(b-pnt[0]).len();
}

inline void graham(std::vector<pv> &ch,const int n)
{
    std::nth_element(pnt,pnt,pnt+n);
    std::sort(pnt+1,pnt+n,com);
    ch.resize(0);
    ch.push_back(pnt[0]);
    ch.push_back(pnt[1]);
    static int i;
    for(i=2;i<n;++i)
        if(fabs((pnt[i]-ch[0]).cross(ch[1]-ch[0]))>eps)
        {
            ch.push_back(pnt[i++]);
            break;
        }
        else
            ch.back()=pnt[i];
    for(;i<n;++i)
    {
        while((ch.back()-ch[ch.size()-2]).cross(pnt[i]-ch[ch.size()-2])<eps)
            ch.pop_back();
        ch.push_back(pnt[i]);
    }
}
```

2.12 other

2.12.1 Pick's theorem

给定顶点座标均是整点（或正方形格点）的简单多边形
A: 面积
i: 内部格点数目
b: 边上格点数目
$A = i + \frac{b}{2} - 1$

取格点的组成图形的面积为一单位。在平行四边形格点，皮克定理依然成立。套用于任意三角形格点，皮克定理则是
$A = 2 \times i + b - 2$

2.12.2 Triangle

Area:
$p = \frac{a+b+c}{2}$
$area = \sqrt{p \times (p-a) \times (p-b) \times (p-c)}$
$area = \frac{a \times b \times \sin(\angle C)}{2}$
$area = \frac{a^2 \times \sin(\angle B) \times \sin(\angle C)}{2 \times \sin(\angle B + \angle C)}$
$area = \frac{a^2}{2 \times (\cot(\angle B) + \cot(\angle C))}$

centroid:
center of mass
intersection of triangle's three triangle medians

Trigonometric conditions:
$\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$
$\sin^2\frac{\alpha}{2} + \sin^2\frac{\beta}{2} + \sin^2\frac{\gamma}{2} + 2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2} = 1$

Circumscribed circle:
$diameter = \frac{abc}{2 \cdot \text{area}} = \frac{|AB||BC||CA|}{2|\Delta ABC|}$

$= \frac{abc}{2\sqrt{s(s-a)(s-b)(s-c)}}$
$= \frac{2abc}{\sqrt{(a+b+c)(-a+b+c)(a-b+c)(a+b-c)}}$
$diameter = \sqrt{\frac{2 \cdot \text{area}}{\sin A \sin B \sin C}}$
$diameter = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Incircle:
$inradius = \frac{2 \times area}{a+b+c}$
coordinates(x,y)=$\left(\frac{ax_a+bx_b+cx_c}{a+b+c}, \frac{ay_a+by_b+cy_c}{a+b+c}\right) =$
$\frac{a}{a+b+c}(x_a, y_a) + \frac{b}{a+b+c}(x_b, y_b) + \frac{c}{a+b+c}(x_c, y_c)$

Excircles:
radius[a]=$\frac{2 \times area}{b+c-a}$
radius[b]=$\frac{2 \times area}{a+c-b}$
radius[c]=$\frac{2 \times area}{a+b-c}$

Steiner circumellipse (least area circumscribed ellipse)
area=$\Delta \times \frac{4\pi}{3\sqrt{3}}$
center is the triangle's centroid.

Steiner inellipse (maximum area inellipse)
area=$\Delta \times \frac{\pi}{3\sqrt{3}}$
center is the triangle's centroid.

Fermat Point:

1. 当有一个内角不小于 120° 时，费马点为此角对应顶点。

2. 当三角形的内角都小于 120°

   (a) 以三角形的每一边为底边，向外做三个正三角形 ΔABC'，ΔBCA'，ΔCAB'。

   (b) 连接 CC'、BB'、AA'，则三条线段的交点就是所求的点。

2.12.3 Ellipse

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$x=h+a\times\cos(t)$$
$$y=k+b\times\sin(t)$$

area=$\pi\times a\times b$
distance from center to focus: $f=\sqrt{a^2-b^2}$
eccentricity: $e=\sqrt{a-\frac{b}{a}^2}=\frac{f}{a}$
focal parameter: $\frac{b^2}{\sqrt{a^2-b^2}}=\frac{b^2}{f}$

```
inline double circumference(double a,double b) // accuracy: pow(0.5,53);
{
    static double digits=53;
    static double tol=sqrt(pow(0.5,digits));
    double x=a;
    double y=b;
    if(x<y)
        std::swap(x,y);
    if(digits*y<tol*x)
        return 4*x;
    double s=0,m=1;
    while(x>(tol+1)*y)
    {
        double tx=x;
        double ty=y;
        x=0.5f*(tx+ty);
        y=sqrt(tx*ty);
        m*=2;
        s+=m*pow(x-y,2);
    }
    return pi*(pow(a+b,2)-s)/(x+y);
}
```

2.12.4 about double

如果 sqrt(a), asin(a), acos(a) 中的 a 是你自己算出来并传进来的，那就得小心了。如果 a 本来应该是 0 的，由于浮点误差，可能实际是一个绝对值很小的负数（比如 -1^{-12}），这样 sqrt(a) 应得 0 的，直接因 a 不在定义域而出错。类似地，如果 a 本来应该是 ±1, 则 asin(a)、acos(a) 也有可能出错。因此，对于此种函数，必需事先对 a 进行校正。

现在考虑一种情况，题目要求输出保留两位小数。有个 case 的正确答案的精确值是 0.005, 按理应该输出 0.01，但你的结果可能是 0.005000000001(恭喜)，也有可能是 0.004999999999(悲剧)，如果按照 printf("%.2lf", a) 输出，那你的遭遇将和括号里的字相同。
如果 a 为正，则输出 a + eps, 否则输出 a - eps。

不要输出 -0.000

注意 double 的数据范围

$a=b$	fabs(a-b)<eps
$a\neq b$	fabs(a-b)>eps
$a<b$	a+eps<b
$a\leq b$	a<b+eps
$a>b$	a>b+eps
$a\geq b$	a+eps>b

2.12.5 trigonometric functions

	input	output
sin	radian	$[-1,+1]$
cos	radian	$[-1,+1]$
tan	radian	$(-\infty,+\infty)$
asin	$[-1,+1]$	$[-\frac{\pi}{2},+\frac{\pi}{2}]$
acos	$[-1,+1]$	[0,π]
atan	$(-\infty,+\infty)$	$[-\frac{\pi}{2},+\frac{\pi}{2}]$
atan2	(y,x)	$\tan(\frac{y}{x})\in[-\pi,+\pi]$ (watch out if x=y=0

exp	x^e
log	ln
log10	log_{10}
ceil	smallest interger $\geq$ x (watch out x<0
floor	greatest interger $\leq$ x (watch out x<0
trunc	nearest integral value close to 0
nearybyint	round to intergral, up to fegetround
round	round with halfway cases rounded away from zer

2.12.6 round

1. cpp: 四舍六入五留双
   (a) 当尾数小于或等于 4 时，直接将尾数舍去
   (b) 当尾数大于或等于 6 时，将尾数舍去并向前一位进位
   (c) 当尾数为 5，而尾数后面的数字均为 0 时，应看尾数“5”的前一位：若前一位数字此时为奇数，就应向前进一位；若前一位数字此时为偶数，则应将尾数舍去。数字“0”在此时应被视为偶数
   (d) 当尾数为 5，而尾数“5”的后面还有任何不是 0 的数字时，无论前一位在此时为奇数还是偶数，也无论“5”后面不为 0 的数字在哪一位上，都应向前进一位
2. java: add 0.5,then floor

2.12.7 rotation matrix

original matrix:
$$\begin{bmatrix}x\\y\end{bmatrix}$$
$$\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}$$
3-dimension:
$$\begin{bmatrix}x\\y\\z\end{bmatrix}$$
$$R_x(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & -\sin\theta\\ 0 & \sin\theta & \cos\theta\end{bmatrix}$$
$$R_y(\theta)=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$$
$$R_z(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$
rotation by unit vector $v=(x,y,z)$:
$$\begin{bmatrix}\cos\theta+(1-\cos\theta)x^2 & (1-\cos\theta)xy-(\sin\theta)z & (1-\cos\\ (1-\cos\theta)yx+(\sin\theta)z & \cos\theta+(1-\cos\theta)y^2 & (1-\cos\\ (1-\cos\theta)zx-(\sin\theta)y & (1-\cos\theta)zy+(\sin\theta)x & \cos\theta-\end{bmatrix}$$

we use transform matrix muliply our original matrix

and we can presetation a transformation as a 4×4 matrix:
$$\begin{bmatrix} a_{11} & a_{12} & a_{12} & a_{14} \\ a_{21} & a_{22} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{32} & a_{34} \\ a_{41} & a_{42} & a_{42} & a_{44} \end{bmatrix}$$
Matrix $\begin{bmatrix} a_{11} & a_{12} & a_{12} \\ a_{21} & a_{22} & a_{22} \\ a_{31} & a_{32} & a_{32} \end{bmatrix}$ presetation the transformation as same as 3×3 matrx.
Matrix $\begin{bmatrix} a_{14} \\ a_{24} \\ a_{34} \end{bmatrix}$ as translation.
Matrix $\begin{bmatrix} a_{41} & a_{42} & a_{43} \end{bmatrix}$ as projection.
Matrix $\begin{bmatrix} a_{44} \end{bmatrix}$ as scale.

original Matrix:
$$\begin{bmatrix} x \\ y \\ z \\ Scale \end{bmatrix}$$

3 Geometry/tmp

3.1 test

```
//三角形:
//1. 半周长 $P = \frac{a+b+c}{2}$
//2. 面积 $S = \frac{aH}{2} = \frac{ab\sin(C)}{2} = \sqrt{P \times (P-a) \times (P-b) \times (P-c)}$
//3. 中线 $Ma = \frac{\sqrt{2(b^2+c^2)-a^2}}{2} = \frac{\sqrt{b^2+c^2+2bc\cos(A)}}{2}$
//4. 角平分线 $Ta = \frac{\sqrt{bc((b+c)^2-a^2)}}{b+c} = \frac{2bc\cos(\frac{A}{2})}{b+c}$
//5. 高线 $Ha = b\sin(C) = c\sin(B) = \sqrt{b^2 - \frac{a^2+b^2-c^2}{2a}^2}$
//6. 内切圆半径 $r = \frac{S}{P} = \frac{\arcsin(\frac{B}{2})\sin(\frac{C}{2})}{\sin(\frac{B+C}{2})} = 4R\sin(\frac{A}{2})\sin(\frac{B}{2})\sin(\frac{C}{2}) = \sqrt{\frac{(P-a)(P-b)(P-c)}{P}} = P\tan(\frac{A}{2})\tan(\frac{B}{2})\tan(\frac{C}{2})$
//7. 外接圆半径 $R = \frac{abc}{4S} = \frac{a}{2\sin(A)} = \frac{b}{2\sin(B)} = \frac{c}{2\sin(C)}$
//四边形:
//D1,D2 为对角线,M 对角线中点连线,A 为对角线夹角
//1. $a^2+b^2+c^2+d^2 = {D_1}^2+{D_2}^2+4M^2$
//2. $S = \frac{D_1 D_2 \sin(A)}{2}$
//(以下对圆的内接四边形)
//3. $ac+bd = D_1 D_2$
//4. $S = \sqrt{(P-a)(P-b)(P-c)(P-d)}$,P 为半周长
//正 n 边形:
//R 为外接圆半径,r 为内切圆半径
//1. 中心角 $A = \frac{2\pi}{n}$
//2. 内角 $C = (n-2)\frac{\pi}{n}$
//3. 边长 $a = 2\sqrt{R^2-r^2} = 2R\sin(\frac{A}{2}) = 2r\tan(\frac{A}{2})$
//4. 面积 $S = \frac{nar}{2} = nr^2\tan(\frac{A}{2}) = \frac{nR^2\sin(A)}{2} = \frac{na^2}{4\tan(\frac{A}{2})}$
//圆:
//1. 弧长 $l = rA$
//2. 弦长 $a = 2\sqrt{2hr-h^2} = 2r\sin(\frac{A}{2})$
//3. 弓形高 $h = r - \sqrt{r^2 - \frac{a^2}{4}} = r(1-\cos(\frac{A}{2})) = \frac{\arctan(\frac{A}{4})}{2}$
//4. 扇形面积 $S1 = \frac{rl}{2} = \frac{r^2A}{2}$
//5. 弓形面积 $S2 = \frac{rl-a(r-h)}{2} = \frac{r^2(A-\sin(A))}{2}$
//棱柱:
//1. 体积 $V = Ah$,A 为底面积,h 为高
//2. 侧面积 $S = lp$,l 为棱长,p 为直截面周长
//3. 全面积 $T = S+2A$
//棱锥:
//1. 体积 $V = \frac{Ah}{3}$,A 为底面积,h 为高
//(以下对正棱锥)
//2. 侧面积 $S = \frac{lp}{2}$,l 为斜高,p 为底面周长
//3. 全面积 $T = S+A$
//棱台:
//1. 体积 $V = (A_1+A_2+\sqrt{A_1A_2})\frac{h}{3}$,A1.A2 为上下底面积,h 为高
//(以下为正棱台)
//2. 侧面积 $S = \frac{(p_1+p_2)l}{2}$,p1.p2 为上下底面周长,l 为斜高
//3. 全面积 $T = S+A_1+A_2$
//圆柱:
//1. 侧面积 $S = 2\pi rh$
//2. 全面积 $T = 2\pi r(h+r)$
//3. 体积 $V = \pi r^2 h$
//圆锥:
//1. 斜高 $l = \sqrt{h^2+r^2}$
//2. 侧面积 $S = \pi rl$
//3. 全面积 $T = \pi r(l+r)$
//4. 体积 $V = \pi r^2 \frac{h}{3}$
//圆台:
//1. 母线 $l = \sqrt{h^2+(r_1-r_2)^2}$
//2. 侧面积 $S = \pi(r_1+r_2)l$
//3. 全面积 $T = \pi r_1(l+r_1)+\pi r_2(l+r_2)$
//4. 体积 $V = \pi({r_1}^2+{r_2}^2+r_1r_2)\frac{h}{3}$
//球:
//1. 全面积 $T = 4\pi r^2$
//2. 体积 $V = \pi r^3 \frac{4}{3}$
//球台:
//1. 侧面积 $S = 2\pi rh$
//2. 全面积 $T = \pi(2rh+{r_1}^2+{r_2}^2)$
//3. 体积 V=$\frac{1}{6}\pi h(3({r_1}^2+{r_2}^2)+h^2)$
//球扇形:
//1. 全面积 $T = \pi r(2h+r_0)$,h 为球冠高,r0 为球冠底面半径
//2. 体积 $V = \frac{2}{3}\pi r^2 h$

//polygon
#include <stdlib.h>
#include <math.h>
#define MAXN 1000
#define offset 10000
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
#define _sign(x) ((x)>eps?1:((x)<-eps?2:0))
struct point{double x,y;};
struct line{point a,b;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
//判定凸多边形，顶点按顺时针或逆时针给出，允许相邻边共线
int is_convex(int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[1]|s[2];
}
//判定凸多边形，顶点按顺时针或逆时针给出，不允许相邻边共线
int is_convex_v2(int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
    return s[0]&&s[1]|s[2];
}
//判点在凸多边形内或多边形边上，顶点按顺时针或逆时针给出
int inside_convex(point q,int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[1]|s[2];
}
//判点在凸多边形内，顶点按顺时针或逆时针给出，在多边形边上返回 0
int inside_convex_v2(point q,int n,point* p)
{
    int i,s[3]={1,1,1};
    for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
        s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
    return s[0]&&s[1]|s[2];
}
//判点在任意多边形内，顶点按顺时针或逆时针给出
//on_edge 表示点在多边形边上时的返回值,offset 为多边形坐标上限
int inside_polygon(point q,int n,point* p,int on_edge=1)
{
    point q2;
    int i=0,count;
    while (i<n)
        for (count=i=0,q2.x=rand()+offset,q2.y=rand()+offset;i<n;i++)
            if
                (zero(xmult(q,p[i],p[(i+1)%n]))&&(p[i].x-q.x)*(p[(i+1)%n].x-q.x)<eps&&(p[i
].y-q.y)*(p[(i+1)%n].y-q.y)<eps)
                    return on_edge;
            else if (zero(xmult(q,q2,p[i])))
                break;
            else if
                (xmult(q,p[i],q2)*xmult(q,p[(i+1)%n],q2)<-eps&&xmult(p[i],q,p[(i+1)%n])*xmult(p[i],q2,p[(i+1)%n])<-eps)
                    count++;
    return count&1;
}
inline int opposite_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}
inline int dot_online_in(point p,point l1,point l2)
{
    return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1.y-p.y)*(l2.y-p.y)<eps;
}
//判线段在任意多边形内，顶点按顺时针或逆时针给出，与边界相交返回 1
int inside_polygon(point l1,point l2,int n,point* p)
{
    point t[MAXN],tt;
    int i,j,k=0;
    if (!inside_polygon(l1,n,p)||!inside_polygon(l2,n,p))
        return 0;
    for (i=0;i<n;i++)
        if (opposite_side(l1,l2,p[i],p[(i+1)%n])&&opposite_side(p[i],p[(i+1)%n],l1,l2))
            return 0;
        else if (dot_online_in(l1,p[i],p[(i+1)%n]))
            t[k++]=l1;
        else if (dot_online_in(l2,p[i],p[(i+1)%n]))
            t[k++]=l2;
        else if (dot_online_in(p[i],l1,l2))
            t[k++]=p[i];
    for (i=0;i<k;i++)
        for (j=i+1;j<k;j++)
        {
            tt.x=(t[i].x+t[j].x)/2;
            tt.y=(t[i].y+t[j].y)/2;
            if (!inside_polygon(tt,n,p))
                return 0;
        }
    return 1;
}
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}
point barycenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b=c;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b=b;
    return intersection(u,v);
}
//多边形重心
point barycenter(int n,point* p)
{
    point ret,t;
    double t1=0,t2;
    int i;
    ret.x=ret.y=0;
    for (i=1;i<n-1;i++)
        if (fabs(t2=xmult(p[0],p[i],p[i+1]))>eps)
        {
            t=barycenter(p[0],p[i],p[i+1]);
            ret.x+=t.x*t2;
            ret.y+=t.y*t2;
            t1+=t2;
        }
    if (fabs(t1)>eps)
        ret.x/=t1,ret.y/=t1;
    return ret;
}


//cut polygon
//多边形切割
//可用于半平面交
#define MAXN 100
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
int same_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//将多边形沿 l1,l2 确定的直线切割在 side 侧切割，保证 l1,l2,side 不共线
void polygon_cut(int& n,point* p,point l1,point l2,point side)
{
    point pp[100];
    int m=0,i;
    for (i=0;i<n;i++)
    {
        if (same_side(p[i],side,l1,l2))
            pp[m++]=p[i];
        if
            (!same_side(p[i],p[(i+1)%n],l1,l2)&&!(zero(xmult(p[i],l1,l2))&&zero(xmult(p[(i+1)%n],l1,l2))))
                pp[m++]=intersection(p[i],p[(i+1)%n],l1,l2);
    }
    for (n=i=0;i<m;i++)
        if (!i||!zero(pp[i].x-pp[i-1].x)||!zero(pp[i].y-pp[i-1].y))
            p[n++]=pp[i];
    if (zero(p[n-1].x-p[0].x)&&zero(p[n-1].y-p[0].y))
        n--;
    if (n<3)
        n=0;
}

//float
//浮点几何函数库
#include <math.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
struct line{point a,b;};
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,double y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算 dot product (P1-P0).(P2-P0)
double dmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
double dmult(double x1,double y1,double x2,double y2,double x0,double y0)
{
    return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}
//两点距离
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}
double distance(double x1,double y1,double x2,double y2)
```

```
{
    return sqrt((x1-x2)*(x1-x2)+(y1-y2)*(y1-y2));
}
//判三点共线
int dots_inline(point p1,point p2,point p3)
{
    return zero(xmult(p1,p2,p3));
}
int dots_inline(double x1,double y1,double x2,double y2,double x3,double y3)
{
    return zero(xmult(x1,y1,x2,y2,x3,y3));
}
//判点是否在线段上，包括端点
int dot_online_in(point p,line l)
{
    return zero(xmult(p,l.a,l.b))&&(l.a.x-p.x)*(l.b.x-p.x)<eps&&(l.a.y-p.y)*(l.b.y-p.y)<eps;
}
int dot_online_in(point p,point l1,point l2)
{
    return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1.y-p.y)*(l2.y-p.y)<eps;
}
int dot_online_in(double x,double y,double x1,double y1,double x2,double y2)
{
    return zero(xmult(x,y,x1,y1,x2,y2))&&(x1-x)*(x2-x)<eps&&(y1-y)*(y2-y)<eps;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point p,line l)
{
    return
        dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a.y))&&(!zero(p.x-l.b.x)||!zero(p.y-l.b.y));
}
int dot_online_ex(point p,point l1,point l2)
{
    return
        dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-l1.y))&&(!zero(p.x-l2.x)||!zero(p.y-l2.y));
}
int dot_online_ex(double x,double y,double x1,double y1,double x2,double y2)
{
    return
        dot_online_in(x,y,x1,y1,x2,y2)&&(!zero(x-x1)||!zero(y-y1))&&(!zero(x-x2)||!zero(y-y2));
}
//判两点在线段同侧，点在线段上返回 0
int same_side(point p1,point p2,line l)
{
    return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)>eps;
}
int same_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}
//判两点在线段异侧，点在线段上返回 0
int opposite_side(point p1,point p2,line l)
{
    return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)<-eps;
}
int opposite_side(point p1,point p2,point l1,point l2)
{
    return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}
//判两直线平行
int parallel(line u,line v)
{
    return zero((u.a.x-u.b.x)*(v.a.y-v.b.y)-(v.a.x-v.b.x)*(u.a.y-u.b.y));
}
int parallel(point u1,point u2,point v1,point v2)
{
    return zero((u1.x-u2.x)*(v1.y-v2.y)-(v1.x-v2.x)*(u1.y-u2.y));
}
//判两直线垂直
int perpendicular(line u,line v)
{
    return zero((u.a.x-u.b.x)*(v.a.x-v.b.x)+(u.a.y-u.b.y)*(v.a.y-v.b.y));
}
int perpendicular(point u1,point u2,point v1,point v2)
{
    return zero((u1.x-u2.x)*(v1.x-v2.x)+(u1.y-u2.y)*(v1.y-v2.y));
}
//判两线段相交，包括端点和部分重合
int intersect_in(line u,line v)
{
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2)
{
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
            2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line u,line v)
{
    return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2)
{
    return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}
//计算两直线交点，注意事先判断直线是否平行！
//线段交点请另外判线段相交（同时还是要判断是否平行！）
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//点到直线上的最近点
point ptoline(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    return intersection(p,t,l.a,l.b);
}
point ptoline(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    return intersection(p,t,l1,l2);
}
//点到直线距离
double disptoline(point p,line l)
{
    return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoline(point p,point l1,point l2)
{
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
double disptoline(double x,double y,double x1,double y1,double x2,double y2)
{
    return fabs(xmult(x,y,x1,y1,x2,y2))/distance(x1,y1,x2,y2);
}
//点到线段上的最近点
point ptoseg(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
```

```
        return distance(p,l.a)<distance(p,l.b)?l.a:l.b;
    return intersection(p,t,l.a,l.b);
}
point ptoseg(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
        return distance(p,l1)<distance(p,l2)?l1:l2;
    return intersection(p,t,l1,l2);
}
//点到线段距离
double disptoseg(point p,line l)
{
    point t=p;
    t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
    if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
        return distance(p,l.a)<distance(p,l.b)?distance(p,l.a):distance(p,l.b);
    return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoseg(point p,point l1,point l2)
{
    point t=p;
    t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
    if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
        return distance(p,l1)<distance(p,l2)?distance(p,l1):distance(p,l2);
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
//矢量 V 以 P 为顶点逆时针旋转 angle 并放大 scale 倍
point rotate(point v,point p,double angle,double scale)
{
    point ret=p;
    v.x-=p.x,v.y-=p.y;
    p.x=scale*cos(angle);
    p.y=scale*sin(angle);
    ret.x+=v.x*p.x-v.y*p.y;
    ret.y+=v.x*p.y+v.y*p.x;
    return ret;
}

//area
#include <math.h>
struct point{double x,y;};
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,double y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算三角形面积, 输入三顶点
double area_triangle(point p1,point p2,point p3)
{
    return fabs(xmult(p1,p2,p3))/2;
}
double area_triangle(double x1,double y1,double x2,double y2,double x3,double y3)
{
    return fabs(xmult(x1,y1,x2,y2,x3,y3))/2;
}
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//计算三角形面积, 输入三边长
double area_triangle(double a,double b,double c)
{
    double s=(a+b+c)/2;
    return sqrt(s*(s-a)*(s-b)*(s-c));
}
//计算多边形面积, 顶点按顺时针或逆时针给出
double area_polygon(int n,point* p)
{
    double s1=0,s2=0;
    int i;
    for (i=0;i<n;i++)
        s1+=p[(i+1)%n].y*p[i].x,s2+=p[(i+1)%n].y*p[(i+2)%n].x;
    return fabs(s1-s2)/2;
}

//surface of ball
#include <math.h>
const double pi=acos(-1);
//计算圆心角 lat 表示纬度,-90<=w<=90,lng 表示经度
//返回两点所在大圆劣弧对应圆心角,0<=angle<=pi
double angle(double lng1,double lat1,double lng2,double lat2)
{
    double dlng=fabs(lng1-lng2)*pi/180;
    while (dlng>=pi+pi)
        dlng-=pi+pi;
    if (dlng>pi)
        dlng=pi+pi-dlng;
    lat1*=pi/180,lat2*=pi/180;
    return acos(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*sin(lat2));
}
//计算距离,r 为球半径
double line_dist(double r,double lng1,double lat1,double lng2,double lat2)
{
    double dlng=fabs(lng1-lng2)*pi/180;
    while (dlng>=pi+pi)
        dlng-=pi+pi;
    if (dlng>pi)
        dlng=pi+pi-dlng;
    lat1*=pi/180,lat2*=pi/180;
    return r*sqrt(2-2*(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*sin(lat2)));
}
//计算球面距离,r 为球半径
inline double sphere_dist(double r,double lng1,double lat1,double lng2,double lat2)
{
    return r*angle(lng1,lat1,lng2,lat2);
}

//triangle
#include <math.h>
struct point{double x,y;};
struct line{point a,b;};
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}
point intersection(line u,line v)
{
    point ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    return ret;
}
//外心
point circumcenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b.x=u.a.x-a.y+b.y;
    u.b.y=u.a.y+a.x-b.x;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b.x=v.a.x-a.y+c.y;
    v.b.y=v.a.y+a.x-c.x;
    return intersection(u,v);
}
//内心
point incenter(point a,point b,point c)
{
    line u,v;
    double m,n;
    u.a=a;
    m=atan2(b.y-a.y,b.x-a.x);
    n=atan2(c.y-a.y,c.x-a.x);
    u.b.x=u.a.x+cos((m+n)/2);
    u.b.y=u.a.y+sin((m+n)/2);
    v.a=b;
    m=atan2(a.y-b.y,a.x-b.x);
    n=atan2(c.y-b.y,c.x-b.x);
    v.b.x=v.a.x+cos((m+n)/2);
    v.b.y=v.a.y+sin((m+n)/2);
    return intersection(u,v);
}
//垂心
point perpencenter(point a,point b,point c)
{
    line u,v;
    u.a=c;
    u.b.x=u.a.x-a.y+b.y;
    u.b.y=u.a.y+a.x-b.x;
    v.a=b;
    v.b.x=v.a.x-a.y+c.y;
    v.b.y=v.a.y+a.x-c.x;
    return intersection(u,v);
```

```
}
//重心
//到三角形三顶点距离的平方和最小的点
//三角形内到三边距离之积最大的点
point barycenter(point a,point b,point c)
{
    line u,v;
    u.a.x=(a.x+b.x)/2;
    u.a.y=(a.y+b.y)/2;
    u.b=c;
    v.a.x=(a.x+c.x)/2;
    v.a.y=(a.y+c.y)/2;
    v.b=b;
    return intersection(u,v);
}
//费马点
//到三角形三顶点距离之和最小的点
point fermentpoint(point a,point b,point c)
{
    point u,v;
    double step=fabs(a.x)+fabs(a.y)+fabs(b.x)+fabs(b.y)+fabs(c.x)+fabs(c.y);
    int i,j,k;
    u.x=(a.x+b.x+c.x)/3;
    u.y=(a.y+b.y+c.y)/3;
    while (step>1e-10)
        for (k=0;k<10;step/=2,k++)
            for (i=-1;i<=1;i++)
                for (j=-1;j<=1;j++)
                {
                    v.x=u.x+step*i;
                    v.y=u.y+step*j;
                    if
                        (distance(u,a)+distance(u,b)+distance(u,c)>distance(v,a)+distance(v,b)+distance(v,c))
                        u=v;
                }
    return u;
}

//3-d
//三维几何函数库
#include <math.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point3{double x,y,z;};
struct line3{point3 a,b;};
struct plane3{point3 a,b,c;};
//计算 cross product U x V
point3 xmult(point3 u,point3 v)
{
    point3 ret;
    ret.x=u.y*v.z-v.y*u.z;
    ret.y=u.z*v.x-u.x*v.z;
    ret.z=u.x*v.y-u.y*v.x;
    return ret;
}
//计算 dot product U . V
double dmult(point3 u,point3 v)
{
    return u.x*v.x+u.y*v.y+u.z*v.z;
}
//矢量差 U - V
point3 subt(point3 u,point3 v)
{
    point3 ret;
    ret.x=u.x-v.x;
    ret.y=u.y-v.y;
    ret.z=u.z-v.z;
    return ret;
}
//取平面法向量
point3 pvec(plane3 s)
{
    return xmult(subt(s.a,s.b),subt(s.b,s.c));
}
point3 pvec(point3 s1,point3 s2,point3 s3)
{
    return xmult(subt(s1,s2),subt(s2,s3));
}
//两点距离，单参数取向量大小
double distance(point3 p1,point3 p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)+(p1.z-p2.z)*(p1.z-p2.z));
}
//向量大小
double vlen(point3 p)
{
    return sqrt(p.x*p.x+p.y*p.y+p.z*p.z);
}
//判三点共线
int dots_inline(point3 p1,point3 p2,point3 p3)
{
    return vlen(xmult(subt(p1,p2),subt(p2,p3)))<eps;
}
//判四点共面
int dots_onplane(point3 a,point3 b,point3 c,point3 d)
{
    return zero(dmult(pvec(a,b,c),subt(d,a)));
}
//判点是否在线段上，包括端点和共线
int dot_online_in(point3 p,line3 l)
{
    return zero(vlen(xmult(subt(p,l.a),subt(p,l.b))))&&(l.a.x-p.x)*(l.b.x-p.x)<eps&&
        (l.a.y-p.y)*(l.b.y-p.y)<eps&&(l.a.z-p.z)*(l.b.z-p.z)<eps;
}
int dot_online_in(point3 p,point3 l1,point3 l2)
{
    return zero(vlen(xmult(subt(p,l1),subt(p,l2))))&&(l1.x-p.x)*(l2.x-p.x)<eps&&
        (l1.y-p.y)*(l2.y-p.y)<eps&&(l1.z-p.z)*(l2.z-p.z)<eps;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point3 p,line3 l)
{
    return dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a.y)||!zero(p.z-l.a.z))&&
        (!zero(p.x-l.b.x)||!zero(p.y-l.b.y)||!zero(p.z-l.b.z));
}
int dot_online_ex(point3 p,point3 l1,point3 l2)
{
    return dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-l1.y)||!zero(p.z-l1.z))&&
        (!zero(p.x-l2.x)||!zero(p.y-l2.y)||!zero(p.z-l2.z));
}
//判点是否在空间三角形上，包括边界，三点共线无意义
int dot_inplane_in(point3 p,plane3 s)
{
    return zero(vlen(xmult(subt(s.a,s.b),subt(s.a,s.c)))-vlen(xmult(subt(p,s.a),subt(p,s.b)))-
        vlen(xmult(subt(p,s.b),subt(p,s.c)))-vlen(xmult(subt(p,s.c),subt(p,s.a))));
}
int dot_inplane_in(point3 p,point3 s1,point3 s2,point3 s3)
{
    return zero(vlen(xmult(subt(s1,s2),subt(s1,s3)))-vlen(xmult(subt(p,s1),subt(p,s2)))-
        vlen(xmult(subt(p,s2),subt(p,s3)))-vlen(xmult(subt(p,s3),subt(p,s1))));
}
//判点是否在空间三角形上，不包括边界，三点共线无意义
int dot_inplane_ex(point3 p,plane3 s)
{
    return dot_inplane_in(p,s)&&vlen(xmult(subt(p,s.a),subt(p,s.b)))>eps&&
        vlen(xmult(subt(p,s.b),subt(p,s.c)))>eps&&vlen(xmult(subt(p,s.c),subt(p,s.a)))>eps;
}
int dot_inplane_ex(point3 p,point3 s1,point3 s2,point3 s3)
{
    return dot_inplane_in(p,s1,s2,s3)&&vlen(xmult(subt(p,s1),subt(p,s2)))>eps&&
        vlen(xmult(subt(p,s2),subt(p,s3)))>eps&&vlen(xmult(subt(p,s3),subt(p,s1)))>eps;
}
//判两点在线段同侧，点在线段上返回 0，不共面无意义
int same_side(point3 p1,point3 p2,line3 l)
{
    return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l.a,l.b),subt(p2,l.b)))>eps;
}
int same_side(point3 p1,point3 p2,point3 l1,point3 l2)
{
    return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,l2),subt(p2,l2)))>eps;
}
//判两点在线段异侧，点在线段上返回 0，不共面无意义
int opposite_side(point3 p1,point3 p2,line3 l)
{
    return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l.a,l.b),subt(p2,l.b)))<-eps;
}
```

```
int opposite_side(point3 p1,point3 p2,point3 l1,point3 l2)
{
    return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,l2),subt(p2,l2)))<-eps;
}
//判两点在平面同侧，点在平面上返回 0
int same_side(point3 p1,point3 p2,plane3 s)
{
    return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.a))>eps;
}
int same_side(point3 p1,point3 p2,point3 s1,point3 s2,point3 s3)
{
    return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,s3),subt(p2,s1))>eps;
}
//判两点在平面异侧，点在平面上返回 0
int opposite_side(point3 p1,point3 p2,plane3 s)
{
    return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.a))<-eps;
}
int opposite_side(point3 p1,point3 p2,point3 s1,point3 s2,point3 s3)
{
    return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,s3),subt(p2,s1))<-eps;
}
//判两直线平行
int parallel(line3 u,line3 v)
{
    return vlen(xmult(subt(u.a,u.b),subt(v.a,v.b)))<eps;
}
int parallel(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return vlen(xmult(subt(u1,u2),subt(v1,v2)))<eps;
}
//判两平面平行
int parallel(plane3 u,plane3 v)
{
    return vlen(xmult(pvec(u),pvec(v)))<eps;
}
int parallel(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3)
{
    return vlen(xmult(pvec(u1,u2,u3),pvec(v1,v2,v3)))<eps;
}
//判直线与平面平行
int parallel(line3 l,plane3 s)
{
    return zero(dmult(subt(l.a,l.b),pvec(s)));
}
int parallel(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return zero(dmult(subt(l1,l2),pvec(s1,s2,s3)));
}
//判两直线垂直
int perpendicular(line3 u,line3 v)
{
    return zero(dmult(subt(u.a,u.b),subt(v.a,v.b)));
}
int perpendicular(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return zero(dmult(subt(u1,u2),subt(v1,v2)));
}
//判两平面垂直
int perpendicular(plane3 u,plane3 v)
{
    return zero(dmult(pvec(u),pvec(v)));
}
int perpendicular(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3)
{
    return zero(dmult(pvec(u1,u2,u3),pvec(v1,v2,v3)));
}
//判直线与平面平行
int perpendicular(line3 l,plane3 s)
{
    return vlen(xmult(subt(l.a,l.b),pvec(s)))<eps;
}
int perpendicular(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return vlen(xmult(subt(l1,l2),pvec(s1,s2,s3)))<eps;
}
//判两线段相交，包括端点和部分重合
int intersect_in(line3 u,line3 v)
{
    if (!dots_onplane(u.a,u.b,v.a,v.b))
        return 0;
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point3 u1,point3 u2,point3 v1,point3 v2)
{
    if (!dots_onplane(u1,u2,v1,v2))
        return 0;
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
            2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line3 u,line3 v)
{
    return dots_onplane(u.a,u.b,v.a,v.b)&&opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return
        dots_onplane(u1,u2,v1,v2)&&opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}
//判线段与空间三角形相交，包括交于边界和（部分）包含
int intersect_in(line3 l,plane3 s)
{
    return !same_side(l.a,l.b,s)&&!same_side(s.a,s.b,l.a,l.b,s.c)&&
        !same_side(s.b,s.c,l.a,l.b,s.a)&&!same_side(s.c,s.a,l.a,l.b,s.b);
}
int intersect_in(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return !same_side(l1,l2,s1,s2,s3)&&!same_side(s1,s2,l1,l2,s3)&&
        !same_side(s2,s3,l1,l2,s1)&&!same_side(s3,s1,l1,l2,s2);
}
//判线段与空间三角形相交，不包括交于边界和（部分）包含
int intersect_ex(line3 l,plane3 s)
{
    return opposite_side(l.a,l.b,s)&&opposite_side(s.a,s.b,l.a,l.b,s.c)&&
        opposite_side(s.b,s.c,l.a,l.b,s.a)&&opposite_side(s.c,s.a,l.a,l.b,s.b);
}
int intersect_ex(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return opposite_side(l1,l2,s1,s2,s3)&&opposite_side(s1,s2,l1,l2,s3)&&
        opposite_side(s2,s3,l1,l2,s1)&&opposite_side(s3,s1,l1,l2,s2);
}
//计算两直线交点，注意事先判断直线是否共面和平行！
//线段交点请另外判线段相交（同时还是要判断是否平行!）
point3 intersection(line3 u,line3 v)
{
    point3 ret=u.a;
    double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
        /((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
    ret.x+=(u.b.x-u.a.x)*t;
    ret.y+=(u.b.y-u.a.y)*t;
    ret.z+=(u.b.z-u.a.z)*t;
    return ret;
}
point3 intersection(point3 u1,point3 u2,point3 v1,point3 v2)
{
    point3 ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    ret.z+=(u2.z-u1.z)*t;
    return ret;
}
//计算直线与平面交点，注意事先判断是否平行，并保证三点不共线!
//线段和空间三角形交点请另外判断
point3 intersection(line3 l,plane3 s)
{
    point3 ret=pvec(s);
    double t=(ret.x*(s.a.x-l.a.x)+ret.y*(s.a.y-l.a.y)+ret.z*(s.a.z-l.a.z))/
        (ret.x*(l.b.x-l.a.x)+ret.y*(l.b.y-l.a.y)+ret.z*(l.b.z-l.a.z));
    ret.x=l.a.x+(l.b.x-l.a.x)*t;
    ret.y=l.a.y+(l.b.y-l.a.y)*t;
    ret.z=l.a.z+(l.b.z-l.a.z)*t;
    return ret;
}
point3 intersection(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    point3 ret=pvec(s1,s2,s3);
    double t=(ret.x*(s1.x-l1.x)+ret.y*(s1.y-l1.y)+ret.z*(s1.z-l1.z))/
        (ret.x*(l2.x-l1.x)+ret.y*(l2.y-l1.y)+ret.z*(l2.z-l1.z));
    ret.x=l1.x+(l2.x-l1.x)*t;
    ret.y=l1.y+(l2.y-l1.y)*t;
    ret.z=l1.z+(l2.z-l1.z)*t;
    return ret;
}
//计算两平面交线，注意事先判断是否平行，并保证三点不共线!
line3 intersection(plane3 u,plane3 v)
{
    line3 ret;
    ret.a=parallel(v.a,v.b,u.a,u.b,u.c)?intersection(v.b,v.c,u.a,u.b,u.c):intersection(v.a,v.b,u.a,u.b,u.
        c);
    ret.b=parallel(v.c,v.a,u.a,u.b,u.c)?intersection(v.b,v.c,u.a,u.b,u.c):intersection(v.c,v.a,u.a,u.b,u.
        c);
    return ret;
}
line3 intersection(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3)
{
    line3 ret;
    ret.a=parallel(v1,v2,u1,u2,u3)?intersection(v2,v3,u1,u2,u3):intersection(v1,v2,u1,u2,u3);
    ret.b=parallel(v3,v1,u1,u2,u3)?intersection(v2,v3,u1,u2,u3):intersection(v3,v1,u1,u2,u3);
    return ret;
}
//点到直线距离
double ptoline(point3 p,line3 l)
{
    return vlen(xmult(subt(p,l.a),subt(l.b,l.a)))/distance(l.a,l.b);
}
double ptoline(point3 p,point3 l1,point3 l2)
{
    return vlen(xmult(subt(p,l1),subt(l2,l1)))/distance(l1,l2);
}
//点到平面距离
double ptoplane(point3 p,plane3 s)
{
    return fabs(dmult(pvec(s),subt(p,s.a)))/vlen(pvec(s));
}
double ptoplane(point3 p,point3 s1,point3 s2,point3 s3)
{
    return fabs(dmult(pvec(s1,s2,s3),subt(p,s1)))/vlen(pvec(s1,s2,s3));
}
//直线到直线距离
double linetoline(line3 u,line3 v)
{
    point3 n=xmult(subt(u.a,u.b),subt(v.a,v.b));
    return fabs(dmult(subt(u.a,v.a),n))/vlen(n);
}
double linetoline(point3 u1,point3 u2,point3 v1,point3 v2)
{
    point3 n=xmult(subt(u1,u2),subt(v1,v2));
    return fabs(dmult(subt(u1,v1),n))/vlen(n);
}
//两直线夹角 cos 值
double angle_cos(line3 u,line3 v)
{
    return dmult(subt(u.a,u.b),subt(v.a,v.b))/vlen(subt(u.a,u.b))/vlen(subt(v.a,v.b));
}
double angle_cos(point3 u1,point3 u2,point3 v1,point3 v2)
{
    return dmult(subt(u1,u2),subt(v1,v2))/vlen(subt(u1,u2))/vlen(subt(v1,v2));
}
//两平面夹角 cos 值
double angle_cos(plane3 u,plane3 v)
{
    return dmult(pvec(u),pvec(v))/vlen(pvec(u))/vlen(pvec(v));
}
double angle_cos(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3)
{
    return dmult(pvec(u1,u2,u3),pvec(v1,v2,v3))/vlen(pvec(u1,u2,u3))/vlen(pvec(v1,v2,v3));
}
//直线平面夹角 sin 值
double angle_sin(line3 l,plane3 s)
{
    return dmult(subt(l.a,l.b),pvec(s))/vlen(subt(l.a,l.b))/vlen(pvec(s));
}
double angle_sin(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3)
{
    return dmult(subt(l1,l2),pvec(s1,s2,s3))/vlen(subt(l1,l2))/vlen(pvec(s1,s2,s3));
}

//CH
#include <stdlib.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
//graham 算法顺时针构造包含所有共线点的凸包,O(nlogn)
point p1,p2;
int graham_cp(const void* a,const void* b)
{
    double ret=xmult(*((point*)a),*((point*)b),p1);
    return zero(ret)?(xmult(*((point*)a),*((point*)b),p2)>0?1:-1):(ret>0?1:-1);
}
void _graham(int n,point* p,int& s,point* ch)
{
    int i,k=0;
    for (p1=p2=p[0],i=1;i<n;p2.x+=p[i].x,p2.y+=p[i].y,i++)
        if (p1.y-p[i].y>eps||(zero(p1.y-p[i].y)&&p1.x>p[i].x))
            p1=p[k=i];
    p2.x/=n,p2.y/=n;
    p[k]=p[0],p[0]=p1;
    qsort(p+1,n-1,sizeof(point),graham_cp);
    for (ch[0]=p[0],ch[1]=p[1],ch[2]=p[2],s=i=3;i<n;ch[s++]=p[i++])
        for (;s>2&&xmult(ch[s-2],p[i],ch[s-1])<-eps;s--);
}
//构造凸包接口函数，传入原始点集大小 n，点集 p(p 原有顺序被打乱!)
//返回凸包大小，凸包的点在 convex 中
//参数 maxsize 为 1 包含共线点，为 0 不包含共线点，缺省为 1
//参数 clockwise 为 1 顺时针构造，为 0 逆时针构造，缺省为 1
//在输入仅有若干共线点时算法不稳定，可能有此类情况请另行处理!
//不能去掉点集中重合的点
int graham(int n,point* p,point* convex,int maxsize=1,int dir=1)
{
    point* temp=new point[n];
    int s,i;
    _graham(n,p,s,temp);
    for (convex[0]=temp[0],n=1,i=(dir?1:(s-1));dir?(i<s):i;i+=(dir?1:-1))
        if (maxsize||!zero(xmult(temp[i-1],temp[i],temp[(i+1)%s])))
            convex[n++]=temp[i];
    delete []temp;
    return n;
}
```

```c

//Pick's
#define abs(x) ((x)>0?(x):-(x))
struct point{int x,y;};
int gcd(int a,int b)
{
    return b?gcd(b,a%b):a;
}
//多边形上的网格点个数
int grid_onedge(int n,point* p)
{
    int i,ret=0;
    for (i=0;i<n;i++)
        ret+=gcd(abs(p[i].x-p[(i+1)%n].x),abs(p[i].y-p[(i+1)%n].y));
    return ret;
}
//多边形内的网格点个数
int grid_inside(int n,point* p)
{
    int i,ret=0;
    for (i=0;i<n;i++)
        ret+=p[(i+1)%n].y*(p[i].x-p[(i+2)%n].x);
    return (abs(ret)-grid_onedge(n,p))/2+1;
}

//circle
#include <math.h>
#define eps 1e-8
struct point{double x,y;};
double xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double distance(point p1,point p2)
{
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}
double disptoline(point p,point l1,point l2)
{
    return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
point intersection(point u1,point u2,point v1,point v2)
{
    point ret=u1;
    double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
        /((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
    ret.x+=(u2.x-u1.x)*t;
    ret.y+=(u2.y-u1.y)*t;
    return ret;
}
//判直线和圆相交，包括相切
int intersect_line_circle(point c,double r,point l1,point l2)
{
    return disptoline(c,l1,l2)<r+eps;
}
//判线段和圆相交，包括端点和相切
int intersect_seg_circle(point c,double r,point l1,point l2)
{
    double t1=distance(c,l1)-r,t2=distance(c,l2)-r;
    point t=c;
    if (t1<eps||t2<eps)
        return t1>-eps||t2>-eps;
    t.x+=l1.y-l2.y;
    t.y+=l2.x-l1.x;
    return xmult(l1,c,t)*xmult(l2,c,t)<eps&&disptoline(c,l1,l2)-r<eps;
}
//判圆和圆相交，包括相切
int intersect_circle_circle(point c1,double r1,point c2,double r2)
{
    return distance(c1,c2)<r1+r2+eps&&distance(c1,c2)>fabs(r1-r2)-eps;
}
//计算圆上到点 p 最近点，如 p 与圆心重合，返回 p 本身
point dot_to_circle(point c,double r,point p)
{
    point u,v;
    if (distance(p,c)<eps)
        return p;
    u.x=c.x+r*fabs(c.x-p.x)/distance(c,p);
    u.y=c.y+r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)<0?-1:1);
    v.x=c.x-r*fabs(c.x-p.x)/distance(c,p);
    v.y=c.y-r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)<0?-1:1);
    return distance(u,p)<distance(v,p)?u:v;
}
//计算直线与圆的交点，保证直线与圆有交点
//计算线段与圆的交点可用这个函数后判点是否在线段上
void intersection_line_circle(point c,double r,point l1,point l2,point& p1,point& p2)
{
    point p=c;
    double t;
    p.x+=l1.y-l2.y;
    p.y+=l2.x-l1.x;
    p=intersection(p,c,l1,l2);
    t=sqrt(r*r-distance(p,c)*distance(p,c))/distance(l1,l2);
    p1.x=p.x+(l2.x-l1.x)*t;
    p1.y=p.y+(l2.y-l1.y)*t;
    p2.x=p.x-(l2.x-l1.x)*t;
    p2.y=p.y-(l2.y-l1.y)*t;
}
//计算圆与圆的交点，保证圆与圆有交点，圆心不重合
void intersection_circle_circle(point c1,double r1,point c2,double r2,point& p1,point& p2)
{
    point u,v;
    double t;
    t=(1+(r1*r1-r2*r2)/distance(c1,c2)/distance(c1,c2))/2;
    u.x=c1.x+(c2.x-c1.x)*t;
    u.y=c1.y+(c2.y-c1.y)*t;
    v.x=u.x+c1.y-c2.y;
    v.y=u.y-c1.x+c2.x;
    intersection_line_circle(c1,r1,u,v,p1,p2);
}

//integer
//整数几何函数库
//注意某些情况下整数运算会出界！
#define sign(a) ((a)>0?1:(((a)<0?-1:0)))
struct point{int x,y;};
struct line{point a,b;};
//计算 cross product (P1-P0)x(P2-P0)
int xmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
int xmult(int x1,int y1,int x2,int y2,int x0,int y0)
{
    return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}
//计算 dot product (P1-P0).(P2-P0)
int dmult(point p1,point p2,point p0)
{
    return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
int dmult(int x1,int y1,int x2,int y2,int x0,int y0)
{
    return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}
//判三点共线
int dots_inline(point p1,point p2,point p3)
{
    return !xmult(p1,p2,p3);
}
int dots_inline(int x1,int y1,int x2,int y2,int x3,int y3)
{
    return !xmult(x1,y1,x2,y2,x3,y3);
}
//判点是否在线段上，包括端点和部分重合
int dot_online_in(point p,line l)
{
    return !xmult(p,l.a,l.b)&&(l.a.x-p.x)*(l.b.x-p.x)<=0&&(l.a.y-p.y)*(l.b.y-p.y)<=0;
}
int dot_online_in(point p,point l1,point l2)
{
    return !xmult(p,l1,l2)&&(l1.x-p.x)*(l2.x-p.x)<=0&&(l1.y-p.y)*(l2.y-p.y)<=0;
}
int dot_online_in(int x,int y,int x1,int y1,int x2,int y2)
{
    return !xmult(x,y,x1,y1,x2,y2)&&(x1-x)*(x2-x)<=0&&(y1-y)*(y2-y)<=0;
}
//判点是否在线段上，不包括端点
int dot_online_ex(point p,line l)
{
    return dot_online_in(p,l)&&(p.x!=l.a.x||p.y!=l.a.y)&&(p.x!=l.b.x||p.y!=l.b.y);
}
int dot_online_ex(point p,point l1,point l2)
{
    return dot_online_in(p,l1,l2)&&(p.x!=l1.x||p.y!=l1.y)&&(p.x!=l2.x||p.y!=l2.y);
}
int dot_online_ex(int x,int y,int x1,int y1,int x2,int y2)
{
    return dot_online_in(x,y,x1,y1,x2,y2)&&(x!=x1||y!=y1)&&(x!=x2||y!=y2);
}
//判两点在直线同侧，点在直线上返回 0
int same_side(point p1,point p2,line l)
{
    return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)>0;
}
int same_side(point p1,point p2,point l1,point l2)
{
    return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)>0;
}
//判两点在直线异侧，点在直线上返回 0
int opposite_side(point p1,point p2,line l)
{
    return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)<0;
}
int opposite_side(point p1,point p2,point l1,point l2)
{
    return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)<0;
}
//判两直线平行
int parallel(line u,line v)
{
    return (u.a.x-u.b.x)*(v.a.y-v.b.y)==(v.a.x-v.b.x)*(u.a.y-u.b.y);
}
int parallel(point u1,point u2,point v1,point v2)
{
    return (u1.x-u2.x)*(v1.y-v2.y)==(v1.x-v2.x)*(u1.y-u2.y);
}
//判两直线垂直
int perpendicular(line u,line v)
{
    return (u.a.x-u.b.x)*(v.a.x-v.b.x)==-(u.a.y-u.b.y)*(v.a.y-v.b.y);
}
int perpendicular(point u1,point u2,point v1,point v2)
{
    return (u1.x-u2.x)*(v1.x-v2.x)==-(u1.y-u2.y)*(v1.y-v2.y);
}
//判两线段相交，包括端点和部分重合
int intersect_in(line u,line v)
{
    if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
        return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
    return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2)
{
    if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
        return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
    return
        dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
            2);
}
//判两线段相交，不包括端点和部分重合
int intersect_ex(line u,line v)
{
    return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2)
{
    return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}
```

3.2 tmp

```
#include<vector>
#include<list>
#include<map>
#include<set>
#include<deque>
#include<queue>
#include<stack>
#include<bitset>
#include<algorithm>
#include<functional>
#include<numeric>
#include<utility>
#include<iostream>
#include<sstream>
#include<iomanip>
#include<cstdio>
#include<cmath>
#include<cstdlib>
#include<cctype>
#include<string>
#include<cstring>
#include<cstdio>
#include<cmath>
#include<cstdlib>
#include<ctime>
#include<climits>
#include<complex>
#define mp make_pair
#define pb push_back
using namespace std;
const double eps=1e-8;
const double pi=acos(-1.0);
const double inf=1e20;
const int maxp=8;
int dblcmp(double d)
{
    if (fabs(d)<eps)return 0;
    return d>eps?1:-1;
}
inline double sqr(double x){return x*x;}
struct point
{
    double x,y;
    point(){}
    point(double _x,double _y):
        x(_x),y(_y){};
    void input()
    {
        scanf("%lf%lf",&x,&y);
    }
    void output()
    {
        printf("%.2f␣%.2f\n",x,y);
    }
    bool operator==(point a)const
    {
        return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0;
    }
    bool operator<(point a)const
    {
        return dblcmp(a.x-x)==0?dblcmp(y-a.y)<0:x<a.x;
    }
    double len()
    {
        return hypot(x,y);
    }
    double len2()
    {
        return x*x+y*y;
    }
    double distance(point p)
    {
        return hypot(x-p.x,y-p.y);
    }
    point add(point p)
    {
        return point(x+p.x,y+p.y);
    }
    point sub(point p)
    {
        return point(x-p.x,y-p.y);
    }
    point mul(double b)
    {
        return point(x*b,y*b);
    }
    point div(double b)
    {
        return point(x/b,y/b);
    }
    double dot(point p)
    {
        return x*p.x+y*p.y;
    }
    double det(point p)
    {
        return x*p.y-y*p.x;
    }
    double rad(point a,point b)
    {
        point p=*this;
        return fabs(atan2(fabs(a.sub(p).det(b.sub(p))),a
             .sub(p).dot(b.sub(p))));
    }
    point trunc(double r)
    {
        double l=len();
        if (!dblcmp(l))return *this;
        r/=l;
        return point(x*r,y*r);
    }
    point rotleft()
    {
        return point(-y,x);
    }
    point rotright()
    {
        return point(y,-x);
    }
    point rotate(point p,double angle)//绕点逆时针旋转角
         度pangle
    {
        point v=this->sub(p);
        double c=cos(angle),s=sin(angle);
        return point(p.x+v.x*c-v.y*s,p.y+v.x*s+v.y*c);
    }
};
struct line
{
    point a,b;
    line(){}
    line(point _a,point _b)
    {
        a=_a;
        b=_b;
    }
    bool operator==(line v)
    {
        return (a==v.a)&&(b==v.b);
    }
    //倾斜角angle
    line(point p,double angle)
    {
        a=p;
        if (dblcmp(angle-pi/2)==0)
        {
            b=a.add(point(0,1));
        }
        else
        {
            b=a.add(point(1,tan(angle)));
        }
    }
    //ax+by+c=0
    line(double _a,double _b,double _c)
    {
        if (dblcmp(_a)==0)
        {
            a=point(0,-_c/_b);
            b=point(1,-_c/_b);
        }
        else if (dblcmp(_b)==0)
        {
            a=point(-_c/_a,0);
            b=point(-_c/_a,1);
        }
        else
        {
            a=point(0,-_c/_b);
            b=point(1,(-_c-_a)/_b);
        }
    }
    void input()
    {
        a.input();
        b.input();
    }
    void adjust()
    {
        if (b<a)swap(a,b);
    }
    double length()
    {
        return a.distance(b);
    }
    double angle()//直线倾斜角 0<=angle<180
    {
        double k=atan2(b.y-a.y,b.x-a.x);
        if (dblcmp(k)<0)k+=pi;
        if (dblcmp(k-pi)==0)k-=pi;
        return k;
    }
    //点和线段关系
    //1 在逆时针
    //2 在顺时针
    //3 平行
    int relation(point p)
    {
        int c=dblcmp(p.sub(a).det(b.sub(a)));
        if (c<0)return 1;
        if (c>0)return 2;
        return 3;
    }
    bool pointonseg(point p)
    {
        return dblcmp(p.sub(a).det(b.sub(a)))==0&&dblcmp
             (p.sub(a).dot(p.sub(b)))<=0;
    }
    bool parallel(line v)
    {
        return dblcmp(b.sub(a).det(v.b.sub(v.a)))==0;
    }
    //2 规范相交
    //1 非规范相交
    //0 不相交
    int segcrossseg(line v)
    {
        int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
        int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
        int d3=dblcmp(v.b.sub(v.a).det(a.sub(v.a)));
        int d4=dblcmp(v.b.sub(v.a).det(b.sub(v.a)));
        if ((d1^d2)==-2&&(d3^d4)==-2)return 2;
        return (d1==0&&dblcmp(v.a.sub(a).dot(v.a.sub(b))
             )<=0||
                d2==0&&dblcmp(v.b.sub(a).dot(v.b.sub(b))
                     )<=0||
                d3==0&&dblcmp(a.sub(v.a).dot(a.sub(v.b))
                     )<=0||
                d4==0&&dblcmp(b.sub(v.a).dot(b.sub(v.b))
                     )<=0);
    }
    int linecrossseg(line v)//*this seg v line
    {
        int d1=dblcmp(b.sub(a).det(v.a.sub(a)));
        int d2=dblcmp(b.sub(a).det(v.b.sub(a)));
        if ((d1^d2)==-2)return 2;
        return (d1==0||d2==0);
    }
    //0 平行
    //1 重合
    //2 相交
    int linecrossline(line v)
    {
        if ((*this).parallel(v))
        {
            return v.relation(a)==3;
        }
        return 2;
    }
    point crosspoint(line v)
    {
        double a1=v.b.sub(v.a).det(a.sub(v.a));
        double a2=v.b.sub(v.a).det(b.sub(v.a));
        return point((a.x*a2-b.x*a1)/(a2-a1),(a.y*a2-b.y
             *a1)/(a2-a1));
    }
    double dispointtoline(point p)
    {
        return fabs(p.sub(a).det(b.sub(a)))/length();
    }
    double dispointtoseg(point p)
    {
        if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.
             sub(a).dot(b.sub(a)))<0)
        {
            return min(p.distance(a),p.distance(b));
        }
        return dispointtoline(p);
    }
    point lineprog(point p)
    {
        return a.add(b.sub(a).mul(b.sub(a).dot(p.sub(a))
             /b.sub(a).len2()));
    }
    point symmetrypoint(point p)
    {
        point q=lineprog(p);
        return point(2*q.x-p.x,2*q.y-p.y);
    }
};
struct circle
{
    point p;
    double r;
    circle(){}
    circle(point _p,double _r):
        p(_p),r(_r){};
    circle(double x,double y,double _r):
        p(point(x,y)),r(_r){};
    circle(point a,point b,point c)//三角形的外接圆
    {
        p=line(a.add(b).div(2),a.add(b).div(2).add(b.sub(a).rotleft())).crosspoint(line(c.add(b).div(2),c.add(b).div(2).add(b.sub(c).rotleft())));
        r=p.distance(a);
    }
    circle(point a,point b,point c,bool t)//三角形的内切圆
    {
        line u,v;
        double m=atan2(b.y-a.y,b.x-a.x),n=atan2(c.y-a.y,c.x-a.x);
        u.a=a;
        u.b=u.a.add(point(cos((n+m)/2),sin((n+m)/2)));
        v.a=b;
        m=atan2(a.y-b.y,a.x-b.x),n=atan2(c.y-b.y,c.x-b.x);
        v.b=v.a.add(point(cos((n+m)/2),sin((n+m)/2)));
        p=u.crosspoint(v);
        r=line(a,b).dispointtoseg(p);
    }
    void input()
    {
        p.input();
        scanf("%lf",&r);
    }
    void output()
    {
        printf("%.2lf␣%.2lf␣%.2lf\n",p.x,p.y,r);
    }
    bool operator==(circle v)
    {
        return ((p==v.p)&&dblcmp(r-v.r)==0);
    }
    bool operator<(circle v)const
    {
        return ((p<v.p)||(p==v.p)&&dblcmp(r-v.r)<0);
    }
    double area()
    {
        return pi*sqr(r);
    }
    double circumference()
    {
        return 2*pi*r;
    }
    //0 圆外
    //1 圆上
    //2 圆内
    int relation(point b)
    {
        double dst=b.distance(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationseg(line v)
    {
        double dst=v.dispointtoseg(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    int relationline(line v)
    {
        double dst=v.dispointtoline(p);
        if (dblcmp(dst-r)<0)return 2;
        if (dblcmp(dst-r)==0)return 1;
        return 0;
    }
    //过a 两点b 半径的两个圆r
    int getcircle(point a,point b,double r,circle&c1,circle&c2)
    {
        circle x(a,r),y(b,r);
        int t=x.pointcrosscircle(y,c1.p,c2.p);
        if (!t)return 0;
        c1.r=c2.r=r;
        return t;
    }
    //与直线相切u 过点q 半径的圆r1
    int getcircle(line u,point q,double r1,circle &c1,circle &c2)
    {
        double dis=u.dispointtoline(q);
        if (dblcmp(dis-r1*2)>0)return 0;
        if (dblcmp(dis)==0)
        {
            c1.p=q.add(u.b.sub(u.a).rotleft().trunc(r1));
            c2.p=q.add(u.b.sub(u.a).rotright().trunc(r1));
            c1.r=c2.r=r1;
            return 2;
        }
        line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),u.b.add(u.b.sub(u.a).rotleft().trunc(r1)));
        line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1)),u.b.add(u.b.sub(u.a).rotright().trunc(r1)));
        circle cc=circle(q,r1);
        point p1,p2;
        if (!cc.pointcrossline(u1,p1,p2))cc.pointcrossline(u2,p1,p2);
        c1=circle(p1,r1);
        if (p1==p2)
        {
            c2=c1;return 1;
        }
        c2=circle(p2,r1);
        return 2;
    }
    //同时与直线u,相切v 半径的圆r1
    int getcircle(line u,line v,double r1,circle &c1,circle &c2,circle &c3,circle &c4)
    {
        if (u.parallel(v))return 0;
        line u1=line(u.a.add(u.b.sub(u.a).rotleft().trunc(r1)),u.b.add(u.b.sub(u.a).rotleft().trunc(r1)));
        line u2=line(u.a.add(u.b.sub(u.a).rotright().trunc(r1)),u.b.add(u.b.sub(u.a).rotright().trunc(r1)));
        line v1=line(v.a.add(v.b.sub(v.a).rotleft().trunc(r1)),v.b.add(v.b.sub(v.a).rotleft().trunc(r1)));
        line v2=line(v.a.add(v.b.sub(v.a).rotright().trunc(r1)),v.b.add(v.b.sub(v.a).rotright().trunc(r1)));
        c1.r=c2.r=c3.r=c4.r=r1;
        c1.p=u1.crosspoint(v1);
        c2.p=u1.crosspoint(v2);
        c3.p=u2.crosspoint(v1);
        c4.p=u2.crosspoint(v2);
        return 4;
    }
    //同时与不相交圆cx,相切cy 半径为的圆r1
    int getcircle(circle cx,circle cy,double r1,circle&c1,circle&c2)
    {
        circle x(cx.p,r1+cx.r),y(cy.p,r1+cy.r);
        int t=x.pointcrosscircle(y,c1.p,c2.p);
        if (!t)return 0;
        c1.r=c2.r=r1;
        return t;
    }
    int pointcrossline(line v,point &p1,point &p2)//求与线段交要先判断relationseg
    {
        if (!(*this).relationline(v))return 0;
        point a=v.lineprog(p);
        double d=v.dispointtoline(p);
        d=sqrt(r*r-d*d);
        if (dblcmp(d)==0)
        {
            p1=a;
            p2=a;
            return 1;
        }
        p1=a.sub(v.b.sub(v.a).trunc(d));
        p2=a.add(v.b.sub(v.a).trunc(d));
        return 2;
    }
    //5 相离
    //4 外切
    //3 相交
    //2 内切
    //1 内含
    int relationcircle(circle v)
```

```cpp
    {
        double d=p.distance(v.p);
        if (dblcmp(d-r-v.r)>0)return 5;
        if (dblcmp(d-r-v.r)==0)return 4;
        double l=fabs(r-v.r);
        if (dblcmp(d-r-v.r)<0&&dblcmp(d-l)>0)return 3;
        if (dblcmp(d-l)==0)return 2;
        if (dblcmp(d-l)<0)return 1;
    }
    int pointcrosscircle(circle v,point &p1,point &p2)
    {
        int rel=relationcircle(v);
        if (rel==1||rel==5)return 0;
        double d=p.distance(v.p);
        double l=(d+(sqr(r)-sqr(v.r))/d)/2;
        double h=sqrt(sqr(r)-sqr(l));
        p1=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotleft().trunc(h)));
        p2=p.add(v.p.sub(p).trunc(l).add(v.p.sub(p).rotright().trunc(h)));
        if (rel==2||rel==4)
        {
            return 1;
        }
        return 2;
    }
    //过一点做圆的切线 先判断点和圆关系()
    int tangentline(point q,line &u,line &v)
    {
        int x=relation(q);
        if (x==2)return 0;
        if (x==1)
        {
            u=line(q,q.add(q.sub(p).rotleft()));
            v=u;
            return 1;
        }
        double d=p.distance(q);
        double l=sqr(r)/d;
        double h=sqrt(sqr(r)-sqr(l));
        u=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotleft().trunc(h))));
        v=line(q,p.add(q.sub(p).trunc(l).add(q.sub(p).rotright().trunc(h))));
        return 2;
    }
    double areacircle(circle v)
    {
        int rel=relationcircle(v);
        if (rel>=4)return 0.0;
        if (rel<=2)return min(area(),v.area());
        double d=p.distance(v.p);
        double hf=(r+v.r+d)/2.0;
        double ss=2*sqrt(hf*(hf-r)*(hf-v.r)*(hf-d));
        double a1=acos((r*r+d*d-v.r*v.r)/(2.0*r*d));
        a1=a1*r*r;
        double a2=acos((v.r*v.r+d*d-r*r)/(2.0*v.r*d));
        a2=a2*v.r*v.r;
        return a1+a2-ss;
    }
    double areatriangle(point a,point b)
    {
        if (dblcmp(p.sub(a).det(p.sub(b))==0))return 0.0;
        point q[5];
        int len=0;
        q[len++]=a;
        line l(a,b);
        point p1,p2;
        if (pointcrossline(l,q[1],q[2])==2)
        {
            if (dblcmp(a.sub(q[1]).dot(b.sub(q[1])))<0)q[len++]=q[1];
            if (dblcmp(a.sub(q[2]).dot(b.sub(q[2])))<0)q[len++]=q[2];
        }
        q[len++]=b;
        if (len==4&&(dblcmp(q[0].sub(q[1]).dot(q[2].sub(q[1])))>0))swap(q[1],q[2]);
        double res=0;
        int i;
        for (i=0;i<len-1;i++)
        {
            if (relation(q[i])==0||relation(q[i+1])==0)
            {
                double arg=p.rad(q[i],q[i+1]);
                res+=r*r*arg/2.0;
            }
            else
            {
                res+=fabs(q[i].sub(p).det(q[i+1].sub(p))/2.0);
            }
        }
        return res;
    }
};
struct polygon
{
    int n;
    point p[maxp];
    line l[maxp];
    void input()
    {
        n=4;
        p[0].input();
        p[2].input();
        double dis=p[0].distance(p[2]);
        p[1]=p[2].rotate(p[0],pi/4);
        p[1]=p[0].add((p[1].sub(p[0])).trunc(dis/sqrt(2.0)));
        p[3]=p[2].rotate(p[0],2*pi-pi/4);
        p[3]=p[0].add((p[3].sub(p[0])).trunc(dis/sqrt(2.0)));
    }
    void add(point q)
    {
        p[n++]=q;
    }
    void getline()
    {
        for (int i=0;i<n;i++)
        {
            l[i]=line(p[i],p[(i+1)%n]);
        }
    }
    struct cmp
    {
        point p;
        cmp(const point &p0){p=p0;}
        bool operator()(const point &aa,const point &bb)
        {
            point a=aa,b=bb;
            int d=dblcmp(a.sub(p).det(b.sub(p)));
            if (d==0)
            {
                return dblcmp(a.distance(p)-b.distance(p))<0;
            }
            return d>0;
        }
    };
    void norm()
    {
        point mi=p[0];
        for (int i=1;i<n;i++)mi=min(mi,p[i]);
        sort(p,p+n,cmp(mi));
    }
    void getconvex(polygon &convex)
    {
        int i,j,k;
        sort(p,p+n);
        convex.n=n;
        for (i=0;i<min(n,2);i++)
        {
            convex.p[i]=p[i];
        }
        if (n<=2)return;
        int &top=convex.n;
        top=1;
        for (i=2;i<n;i++)
        {
            while (top&&convex.p[top].sub(p[i]).det(convex.p[top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
        int temp=top;
        convex.p[++top]=p[n-2];
        for (i=n-3;i>=0;i--)
        {
            while (top!=temp&&convex.p[top].sub(p[i]).det(convex.p[top-1].sub(p[i]))<=0)
                top--;
            convex.p[++top]=p[i];
        }
    }
    bool isconvex()
    {
        bool s[3];
        memset(s,0,sizeof(s));
        int i,j,k;
        for (i=0;i<n;i++)
        {
            j=(i+1)%n;
            k=(j+1)%n;
            s[dblcmp(p[j].sub(p[i]).det(p[k].sub(p[i])))+1]=1;
            if (s[0]&&s[2])return 0;
        }
        return 1;
    }
    //3 点上
    //2 边上
    //1 内部
    //0 外部
    int relationpoint(point q)
    {
        int i,j;
        for (i=0;i<n;i++)
        {
            if (p[i]==q)return 3;
        }
        getline();
        for (i=0;i<n;i++)
        {
            if (l[i].pointonseg(q))return 2;
        }
        int cnt=0;
        for (i=0;i<n;i++)
        {
            j=(i+1)%n;
            int k=dblcmp(q.sub(p[j]).det(p[i].sub(p[j])));
            int u=dblcmp(p[i].y-q.y);
            int v=dblcmp(p[j].y-q.y);
            if (k>0&&u<0&&v>=0)cnt++;
            if (k<0&&v<0&&u>=0)cnt--;
        }
        return cnt!=0;
    }
    //1 在多边形内长度为正
    //2 相交或与边平行
    //0 无任何交点
    int relationline(line u)
    {
        int i,j,k=0;
        getline();
        for (i=0;i<n;i++)
        {
            if (l[i].segcrossseg(u)==2)return 1;
            if (l[i].segcrossseg(u)==1)k=1;
        }
        if (!k)return 0;
        vector<point>vp;
        for (i=0;i<n;i++)
        {
            if (l[i].segcrossseg(u))
            {
                if (l[i].parallel(u))
                {
                    vp.pb(u.a);
                    vp.pb(u.b);
                    vp.pb(l[i].a);
                    vp.pb(l[i].b);
                    continue;
                }
                vp.pb(l[i].crosspoint(u));
            }
        }
        sort(vp.begin(),vp.end());
        int sz=vp.size();
        for (i=0;i<sz-1;i++)
        {
            point mid=vp[i].add(vp[i+1]).div(2);
            if (relationpoint(mid)==1)return 1;
        }
        return 2;
    }
    //直线切割凸多边形左侧u
    //注意直线方向
    void convexcut(line u,polygon &po)
    {
        int i,j,k;
        int &top=po.n;
        top=0;
        for (i=0;i<n;i++)
        {
            int d1=dblcmp(p[i].sub(u.a).det(u.b.sub(u.a)));
            int d2=dblcmp(p[(i+1)%n].sub(u.a).det(u.b.sub(u.a)));
            if (d1>=0)po.p[top++]=p[i];
            if (d1*d2<0)po.p[top++]=u.crosspoint(line(p[i],p[(i+1)%n]));
```

```
        }
    }
    double getcircumference()
    {
        double sum=0;
        int i;
        for (i=0;i<n;i++)
        {
            sum+=p[i].distance(p[(i+1)%n]);
        }
        return sum;
    }
    double getarea()
    {
        double sum=0;
        int i;
        for (i=0;i<n;i++)
        {
            sum+=p[i].det(p[(i+1)%n]);
        }
        return fabs(sum)/2;
    }
    bool getdir()//代表逆时针1 代表顺时针0
    {
        double sum=0;
        int i;
        for (i=0;i<n;i++)
        {
            sum+=p[i].det(p[(i+1)%n]);
        }
        if (dblcmp(sum)>0)return 1;
        return 0;
    }
    point getbarycentre()
    {
        point ret(0,0);
        double area=0;
        int i;
        for (i=1;i<n-1;i++)
        {
            double tmp=p[i].sub(p[0]).det(p[i+1].sub(p[0]));
            if (dblcmp(tmp)==0)continue;
            area+=tmp;
            ret.x+=(p[0].x+p[i].x+p[i+1].x)/3*tmp;
            ret.y+=(p[0].y+p[i].y+p[i+1].y)/3*tmp;
        }
        if (dblcmp(area))ret=ret.div(area);
        return ret;
    }
    double areaintersection(polygon po)
    {
    }
    double areaunion(polygon po)
    {
        return getarea()+po.getarea()-areaintersection(po);
    }
    double areacircle(circle c)
    {
        int i,j,k,l,m;
        double ans=0;
        for (i=0;i<n;i++)
        {
            int j=(i+1)%n;
            if (dblcmp(p[j].sub(c.p).det(p[i].sub(c.p)))>=0)
            {
                ans+=c.areatriangle(p[i],p[j]);
            }
            else
            {
                ans-=c.areatriangle(p[i],p[j]);
            }
        }
        return fabs(ans);
    }
    //多边形和圆关系
    //0 一部分在圆外
    //1 与圆某条边相切
    //2 完全在圆内
    int relationcircle(circle c)
    {
        getline();
        int i,x=2;
        if (relationpoint(c.p)!=1)return 0;
        for (i=0;i<n;i++)
        {
            if (c.relationseg(l[i])==2)return 0;
            if (c.relationseg(l[i])==1)x=1;
        }
        return x;
    }
    void find(int st,point tri[],circle &c)
    {
        if (!st)
        {
            c=circle(point(0,0),-2);
        }
        if (st==1)
        {
            c=circle(tri[0],0);
        }
        if (st==2)
        {
            c=circle(tri[0].add(tri[1]).div(2),tri[0].distance(tri[1])/2.0);
        }
        if (st==3)
        {
            c=circle(tri[0],tri[1],tri[2]);
        }
    }
    void solve(int cur,int st,point tri[],circle &c)
    {
        find(st,tri,c);
        if (st==3)return;
        int i;
        for (i=0;i<cur;i++)
        {
            if (dblcmp(p[i].distance(c.p)-c.r)>0)
            {
                tri[st]=p[i];
                solve(i,st+1,tri,c);
            }
        }
    }
    circle mincircle()//点集最小圆覆盖
    {
        random_shuffle(p,p+n);
        point tri[4];
        circle c;
        solve(n,0,tri,c);
        return c;
    }
    int circlecover(double r)//单位圆覆盖
    {
        int ans=0,i,j;
        vector<pair<double,int> >v;
        for (i=0;i<n;i++)
        {
            v.clear();
            for (j=0;j<n;j++)if (i!=j)
            {
                point q=p[i].sub(p[j]);
                double d=q.len();
                if (dblcmp(d-2*r)<=0)
                {
                    double arg=atan2(q.y,q.x);
                    if (dblcmp(arg)<0)arg+=2*pi;
                    double t=acos(d/(2*r));
                    v.push_back(make_pair(arg-t+2*pi,-1));
                    v.push_back(make_pair(arg+t+2*pi,1));
                }
            }
            sort(v.begin(),v.end());
            int cur=0;
            for (j=0;j<v.size();j++)
            {
                if (v[j].second==-1)++cur;
                else --cur;
                ans=max(ans,cur);
            }
        }
        return ans+1;
    }
    int pointinpolygon(point q)//点在凸多边形内部的判定
    {
        if (getdir())reverse(p,p+n);
        if (dblcmp(q.sub(p[0]).det(p[n-1].sub(p[0])))==0)
        {
            if (line(p[n-1],p[0]).pointonseg(q))return -1;
            return -1;
        }
        int low=1,high=n-2,mid;
        while (low<=high)
        {
            mid=(low+high)>>1;
            if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>=0&&dblcmp(q.sub(p[0]).det(p[mid+1].sub(p[0])))<0)
            {
                polygon c;
                c.p[0]=p[mid];
                c.p[1]=p[mid+1];
                c.p[2]=p[0];
                c.n=3;
                if (c.relationpoint(q))return mid;
                return -1;
            }
            if (dblcmp(q.sub(p[0]).det(p[mid].sub(p[0])))>0)
            {
                low=mid+1;
            }
            else
            {
                high=mid-1;
            }
        }
        return -1;
    }
};
struct polygons
{
    vector<polygon>p;
    polygons()
    {
        p.clear();
    }
    void clear()
    {
        p.clear();
    }
    void push(polygon q)
    {
        if (dblcmp(q.getarea()))p.pb(q);
    }
    vector<pair<double,int> >e;
    void ins(point s,point t,point X,int i)
    {
        double r=fabs(t.x-s.x)>eps?(X.x-s.x)/(t.x-s.x):(X.y-s.y)/(t.y-s.y);
        r=min(r,1.0);r=max(r,0.0);
        e.pb(mp(r,i));
    }
    double polyareaunion()
    {
        double ans=0.0;
        int c0,c1,c2,i,j,k,w;
        for (i=0;i<p.size();i++)
        {
            if (p[i].getdir()==0)reverse(p[i].p,p[i].p+p[i].n);
        }
        for (i=0;i<p.size();i++)
        {
            for (k=0;k<p[i].n;k++)
            {
                point &s=p[i].p[k],&t=p[i].p[(k+1)%p[i].n];
                if (!dblcmp(s.det(t)))continue;
                e.clear();
                e.pb(mp(0.0,1));
                e.pb(mp(1.0,-1));
                for (j=0;j<p.size();j++)if (i!=j)
                {
                    for (w=0;w<p[j].n;w++)
                    {
                        point a=p[j].p[w],b=p[j].p[(w+1)%p[j].n],c=p[j].p[(w-1+p[j].n)%p[j].n];
                        c0=dblcmp(t.sub(s).det(c.sub(s)));
                        c1=dblcmp(t.sub(s).det(a.sub(s)));
                        c2=dblcmp(t.sub(s).det(b.sub(s)));
                        if (c1*c2<0)ins(s,t,line(s,t).crosspoint(line(a,b)),-c2);
                        else if (!c1&&c0*c2<0)ins(s,t,a,-c2);
                        else if (!c1&&!c2)
                        {
                            int c3=dblcmp(t.sub(s).det(p[j].p[(w+2)%p[j].n].sub(s)));
                            int dp=dblcmp(t.sub(s).dot(b.sub(a)));
                            if (dp&&c0)ins(s,t,a,dp>0?c0*((j>i)^(c0<0)):-(c0<0));
                            if (dp&&c3)ins(s,t,b,dp>0?c3*((j>i)^(c3<0)):c3<0);
                        }
                    }
                }
                sort(e.begin(),e.end());
                int ct=0;
                double tot=0.0,last;
                for (j=0;j<e.size();j++)
                {
                    if (ct==p.size())tot+=e[j].first-last;
                    ct+=e[j].second;
                    last=e[j].first;
                }
                ans+=s.det(t)*tot;
            }
        }
        return fabs(ans)*0.5;
    }
};
const int maxn=500;
struct circles
{
    circle c[maxn];
    double ans[maxn];//ans[i表示被覆盖了]次的面积i
    double pre[maxn];
    int n;
    circles(){}
    void add(circle cc)
    {
        c[n++]=cc;
    }
    bool inner(circle x,circle y)
    {
        if (x.relationcircle(y)!=1)return 0;
        return dblcmp(x.r-y.r)<=0?1:0;
    }
    void init_or()//圆的面积并去掉内含的圆
    {
        int i,j,k=0;
        bool mark[maxn]={0};
        for (i=0;i<n;i++)
        {
            for (j=0;j<n;j++)if (i!=j&&!mark[j])
            {
                if ((c[i]==c[j])||inner(c[i],c[j]))break;
            }
            if (j<n)mark[i]=1;
        }
        for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
        n=k;
    }
    void init_and()//圆的面积交去掉内含的圆
    {
        int i,j,k=0;
        bool mark[maxn]={0};
        for (i=0;i<n;i++)
        {
            for (j=0;j<n;j++)if (i!=j&&!mark[j])
            {
                if ((c[i]==c[j])||inner(c[j],c[i]))break;
            }
            if (j<n)mark[i]=1;
        }
        for (i=0;i<n;i++)if (!mark[i])c[k++]=c[i];
        n=k;
    }
    double areaarc(double th,double r)
    {
        return 0.5*sqr(r)*(th-sin(th));
    }
    void getarea()
    {
        int i,j,k;
        memset(ans,0,sizeof(ans));
        vector<pair<double,int> >v;
        for (i=0;i<n;i++)
        {
            v.clear();
            v.push_back(make_pair(-pi,1));
            v.push_back(make_pair(pi,-1));
            for (j=0;j<n;j++)if (i!=j)
            {
                point q=c[j].p.sub(c[i].p);
                double ab=q.len(),ac=c[i].r,bc=c[j].r;
                if (dblcmp(ab+ac-bc)<=0)
                {
                    v.push_back(make_pair(-pi,1));
                    v.push_back(make_pair(pi,-1));
                    continue;
                }
                if (dblcmp(ab+bc-ac)<=0)continue;
                if (dblcmp(ab-ac-bc)>0) continue;
                double th=atan2(q.y,q.x),fai=acos((ac*ac+ab*ab-bc*bc)/(2.0*ac*ab));
                double a0=th-fai;
                if (dblcmp(a0+pi)<0)a0+=2*pi;
                double a1=th+fai;
                if (dblcmp(a1-pi)>0)a1-=2*pi;
                if (dblcmp(a0-a1)>0)
                {
                    v.push_back(make_pair(a0,1));
                    v.push_back(make_pair(pi,-1));
                    v.push_back(make_pair(-pi,1));
                    v.push_back(make_pair(a1,-1));
                }
                else
                {
                    v.push_back(make_pair(a0,1));
                    v.push_back(make_pair(a1,-1));
                }
            }
            sort(v.begin(),v.end());
            int cur=0;
            for (j=0;j<v.size();j++)
            {
                if (cur&&dblcmp(v[j].first-pre[cur]))
                {
                    ans[cur]+=areaarc(v[j].first-pre[cur],c[i].r);
                    ans[cur]+=0.5*point(c[i].p.x+c[i].r*cos(pre[cur]),c[i].p.y+c[i].r*sin(pre[cur])).det(point(c[i].p.x+c[i].r*cos(v[j].first),c[i].p.y+c[i].r*sin(v[j].first)));
                }
                cur+=v[j].second;
                pre[cur]=v[j].first;
            }
        }
        for (i=1;i<=n;i++)
        {
            ans[i]-=ans[i+1];
        }
    }
};
struct halfplane:public line
{
    double angle;
    halfplane(){}
    //表示向量 a->逆时针b左侧()的半平面
    halfplane(point _a,point _b)
    {
        a=_a;
        b=_b;
    }
    halfplane(line v)
    {
        a=v.a;
        b=v.b;
    }
    void calcangle()
    {
        angle=atan2(b.y-a.y,b.x-a.x);
    }
    bool operator<(const halfplane &b)const
    {
        return angle<b.angle;
    }
};
struct halfplanes
{
    int n;
    halfplane hp[maxp];
    point p[maxp];
    int que[maxp];
    int st,ed;
    void push(halfplane tmp)
    {
        hp[n++]=tmp;
    }
    void unique()
    {
        int m=1,i;
        for (i=1;i<n;i++)
        {
            if (dblcmp(hp[i].angle-hp[i-1].angle))hp[m++]=hp[i];
            else if (dblcmp(hp[m-1].b.sub(hp[m-1].a).det(hp[i].a.sub(hp[m-1].a))>0))hp[m-1]=hp[
                i];
        }
        n=m;
    }
    bool halfplaneinsert()
    {
        int i;
        for (i=0;i<n;i++)hp[i].calcangle();
        sort(hp,hp+n);
        unique();
        que[st=0]=0;
        que[ed=1]=1;
        p[1]=hp[0].crosspoint(hp[1]);
        for (i=2;i<n;i++)
        {
            while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[ed].sub(hp[i].a))))<0)ed--;
            while (st<ed&&dblcmp((hp[i].b.sub(hp[i].a).det(p[st+1].sub(hp[i].a))))<0)st++;
            que[++ed]=i;
            if (hp[i].parallel(hp[que[ed-1]]))return false;
            p[ed]=hp[i].crosspoint(hp[que[ed-1]]);
        }
        while (st<ed&&dblcmp(hp[que[st]].b.sub(hp[que[st]].a).det(p[ed].sub(hp[que[st]].a)))<0)ed--;
        while (st<ed&&dblcmp(hp[que[ed]].b.sub(hp[que[ed]].a).det(p[st+1].sub(hp[que[ed]].a)))<0)st++;
        if (st+1>=ed)return false;
        return true;
    }
    void getconvex(polygon &con)
    {
        p[st]=hp[que[st]].crosspoint(hp[que[ed]]);
        con.n=ed-st+1;
        int j=st,i=0;
        for (;j<=ed;i++,j++)
        {
            con.p[i]=p[j];
        }
    }
};
struct point3
{
    double x,y,z;
    point3(){}
    point3(double _x,double _y,double _z):
        x(_x),y(_y),z(_z){};
    void input()
    {
        scanf("%lf%lf%lf",&x,&y,&z);
    }
    void output()
    {
        printf("%.2lf␣%.2lf␣%.2lf\n",x,y,z);
    }
    bool operator==(point3 a)
    {
        return dblcmp(a.x-x)==0&&dblcmp(a.y-y)==0&&dblcmp(a.z-z)==0;
    }
    bool operator<(point3 a)const
    {
        return dblcmp(a.x-x)==0?dblcmp(y-a.y)==0?dblcmp(z-a.z)<0:y<a.y:x<a.x;
    }
    double len()
    {
        return sqrt(len2());
    }
    double len2()
    {
        return x*x+y*y+z*z;
    }
    double distance(point3 p)
    {
        return sqrt((p.x-x)*(p.x-x)+(p.y-y)*(p.y-y)+(p.z-z)*(p.z-z));
    }
    point3 add(point3 p)
    {
        return point3(x+p.x,y+p.y,z+p.z);
    }
    point3 sub(point3 p)
    {
        return point3(x-p.x,y-p.y,z-p.z);
    }
    point3 mul(double d)
    {
        return point3(x*d,y*d,z*d);
    }
    point3 div(double d)
    {
        return point3(x/d,y/d,z/d);
    }
    double dot(point3 p)
    {
        return x*p.x+y*p.y+z*p.z;
    }
    point3 det(point3 p)
    {
        return point3(y*p.z-p.y*z,p.x*z-x*p.z,x*p.y-p.x*y);
    }
    double rad(point3 a,point3 b)
    {
        point3 p=(*this);
        return acos(a.sub(p).dot(b.sub(p))/(a.distance(p)*b.distance(p)));
    }
    point3 trunc(double r)
    {
        r/=len();
        return point3(x*r,y*r,z*r);
    }
    point3 rotate(point3 o,double r)
    {
    }
};
struct line3
{
    point3 a,b;
    line3(){}
    line3(point3 _a,point3 _b)
    {
        a=_a;
        b=_b;
    }
    bool operator==(line3 v)
    {
        return (a==v.a)&&(b==v.b);
    }
    void input()
    {
        a.input();
        b.input();
    }
    double length()
    {
        return a.distance(b);
    }
    bool pointonseg(point3 p)
    {
        return dblcmp(p.sub(a).det(p.sub(b)).len())==0&&dblcmp(a.sub(p).dot(b.sub(p)))<=0;
    }
    double dispointtoline(point3 p)
    {
        return b.sub(a).det(p.sub(a)).len()/a.distance(b);
    }
    double dispointtoseg(point3 p)
    {
        if (dblcmp(p.sub(b).dot(a.sub(b)))<0||dblcmp(p.sub(a).dot(b.sub(a)))<0)
        {
            return min(p.distance(a),p.distance(b));
        }
        return dispointtoline(p);
    }
    point3 lineprog(point3 p)
    {
        return a.add(b.sub(a).trunc(b.sub(a).dot(p.sub(a))/b.distance(a)));
    }
    point3 rotate(point3 p,double ang)//绕此向量逆时针角度parg
    {
        if (dblcmp((p.sub(a).det(p.sub(b)).len()))==0)
            return p;
        point3 f1=b.sub(a).det(p.sub(a));
        point3 f2=b.sub(a).det(f1);
        double len=fabs(a.sub(p).det(b.sub(p)).len()/a.distance(b));
        f1=f1.trunc(len);f2=f2.trunc(len);
        point3 h=p.add(f2);
        point3 pp=h.add(f1);
        return h.add((p.sub(h)).mul(cos(ang*1.0))).add((pp.sub(h)).mul(sin(ang*1.0)));
    }
};
struct plane
```

```
{
    point3 a,b,c,o;
    plane(){}
    plane(point3 _a,point3 _b,point3 _c)
    {
        a=_a;
        b=_b;
        c=_c;
        o=pvec();
    }
    plane(double _a,double _b,double _c,double _d)
    {
        //ax+by+cz+d=0
        o=point3(_a,_b,_c);
        if (dblcmp(_a)!=0)
        {
            a=point3((-_d-_c-_b)/_a,1,1);
        }
        else if (dblcmp(_b)!=0)
        {
            a=point3(1,(-_d-_c-_a)/_b,1);
        }
        else if (dblcmp(_c)!=0)
        {
            a=point3(1,1,(-_d-_a-_b)/_c);
        }
    }
    void input()
    {
        a.input();
        b.input();
        c.input();
        o=pvec();
    }
    point3 pvec()
    {
        return b.sub(a).det(c.sub(a));
    }
    bool pointonplane(point3 p)//点是否在平面上
    {
        return dblcmp(p.sub(a).dot(o))==0;
    }
    //0 不在
    //1 在边界上
    //2 在内部
    int pointontriangle(point3 p)//点是否在空间三角形上abc
    {
        if (!pointonplane(p))return 0;
        double s=a.sub(b).det(c.sub(b)).len();
        double s1=p.sub(a).det(p.sub(b)).len();
        double s2=p.sub(a).det(p.sub(c)).len();
        double s3=p.sub(b).det(p.sub(c)).len();
        if (dblcmp(s-s1-s2-s3))return 0;
        if (dblcmp(s1)&&dblcmp(s2)&&dblcmp(s3))return 2;
        return 1;
    }
    //判断两平面关系
    //0 相交
    //1 平行但不重合
    //2 重合
    bool relationplane(plane f)
    {
        if (dblcmp(o.det(f.o).len()))return 0;
        if (pointonplane(f.a))return 2;
        return 1;
    }
    double angleplane(plane f)//两平面夹角
    {
        return acos(o.dot(f.o)/(o.len()*f.o.len()));
    }
    double dispoint(point3 p)//点到平面距离
    {
        return fabs(p.sub(a).dot(o)/o.len());
    }
    point3 pttoplane(point3 p)//点到平面最近点
    {
        line3 u=line3(p,p.add(o));
        crossline(u,p);
        return p;
    }
    int crossline(line3 u,point3 &p)//平面和直线的交点
    {
        double x=o.dot(u.b.sub(a));
        double y=o.dot(u.a.sub(a));
        double d=x-y;
        if (dblcmp(fabs(d))==0)return 0;
        p=u.a.mul(x).sub(u.b.mul(y)).div(d);
        return 1;
    }
    int crossplane(plane f,line3 &u)//平面和平面的交线
    {
        point3 oo=o.det(f.o);
        point3 v=o.det(oo);
        double d=fabs(f.o.dot(v));
        if (dblcmp(d)==0)return 0;
        point3 q=a.add(v.mul(f.o.dot(f.a.sub(a))/d));
        u=line3(q,q.add(oo));
        return 1;
    }
};
```

4 Graph

4.1 2SAT

```
/*
x & y == true:
~x -> x
~y -> y

x & y == false:
x -> ~y
y -> ~x

x | y == true:
~x -> y
~y -> x

x | y == false:
x -> ~x
y -> ~y

x ^ y == true:
~x -> y
y -> ~x
x -> ~y
~y -> x

x ^ y == false:
x -> y
y -> x
~x -> ~y
~y -> ~x
*/
#include<cstdio>
#include<cstring>

#define MAXX 16111
#define MAXE 200111
#define v to[i]

int edge[MAXX],to[MAXE],nxt[MAXE],cnt;
inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

bool done[MAXX];
int st[MAXX];

bool dfs(const int now)
{
    if(done[now^1])
        return false;
    if(done[now])
        return true;
    done[now]=true;
    st[cnt++]=now;
    for(int i(edge[now]);i;i=nxt[i])
        if(!dfs(v))
            return false;
    return true;
}

int n,m;
int i,j,k;

inline bool go()
{
    memset(done,0,sizeof done);
    for(i=0;i<n;i+=2)
        if(!done[i] && !done[i^1])
        {
            cnt=0;
            if(!dfs(i))
            {
                while(cnt)
                    done[st[--cnt]]=false;
                if(!dfs(i^1))
```

```
                    return false;
            }
        }
    return true;
}
//done array will be a solution with minimal lexicographical order
// or maybe we can solve it with dual SCC method, and get a solution by reverse the edges of DAG then product a topsort
```

4.2 Articulation

```
void dfs(int now,int fa)  // now 从 1 开始
{
    int p(0);
    dfn[now]=low[now]=cnt++;
    for(std::list<int>::const_iterator it(edge[now].begin());it!=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it,now);
            ++p;
            low[now]=std::min(low[now],low[*it]);
            if((now==1 && p>1) || (now!=1 && low[*it]>=dfn[now])) // 如果从出发点出发的子节点不能由兄弟节点到达，那么出发点为割点。如果现节点不是出发点，但是其子孙节点不能达到祖先节点，那么该节点为割点
                ans.insert(now);
        }
        else
            if(*it!=fa)
                low[now]=std::min(low[now],dfn[*it]);
}
```

4.3 Augmenting Path Algorithm for Maximum Cardinality Bipartite Matching

```
#include<cstdio>
#include<cstring>

#define MAXX 111

bool Map[MAXX][MAXX],visit[MAXX];
int link[MAXX],n,m;
bool dfs(int t)
{
    for (int i=0; i<m; i++)
        if (!visit[i] && Map[t][i]){
            visit[i] = true;
            if (link[i]==-1 || dfs(link[i])){
                link[i] = t;
                return true;
            }
        }
    return false;
}
int main()
{
    int k,a,b,c;
    while (scanf("%d",&n),n){
        memset(Map,false,sizeof(Map));
        scanf("%d%d",&m,&k);
        while (k--){
            scanf("%d%d%d",&a,&b,&c);
            if (b && c)
                Map[b][c] = true;
        }
        memset(link,-1,sizeof(link));
        int ans = 0;
        for (int i=0; i<n; i++){
            memset(visit,false,sizeof(visit));
            if (dfs(i))
                ans++;
        }
        printf("%d\n",ans);
    }
}
```

4.4 Biconnected Component - Edge

```
// hdu 4612
#include<cstdio>
#include<algorithm>
#include<set>
#include<cstring>
#include<stack>
#include<queue>

#define MAXX 200111
#define MAXE (1000111*2)
#pragma comment(linker, "/STACK:16777216")

int edge[MAXX],to[MAXE],nxt[MAXE],cnt;
#define v to[i]
inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

int dfn[MAXX],low[MAXX],col[MAXX],belong[MAXX];
int idx,bcnt;
std::stack<int>st;

void tarjan(int now,int last)
{
    col[now]=1;
    st.push(now);
    dfn[now]=low[now]=++idx;
    bool flag(false);
    for(int i(edge[now]);i;i=nxt[i])
    {
        if(v==last && !flag)
        {
            flag=true;
            continue;
        }
        if(!col[v])
        {
            tarjan(v,now);
            low[now]=std::min(low[now],low[v]);
            /*
            if(low[v]>dfn[now])
            then this is a bridge
            */
        }
        else
            if(col[v]==1)
                low[now]=std::min(low[now],dfn[v]);
    }
    col[now]=2;
    if(dfn[now]==low[now])
    {
        ++bcnt;
        static int x;
        do
        {
            x=st.top();
            st.pop();
            belong[x]=bcnt;
        }while(x!=now);
    }
}

std::set<int>set[MAXX];

int dist[MAXX];
std::queue<int>q;
int n,m,i,j,k;

inline int go(int s)
{
    static std::set<int>::const_iterator it;
    memset(dist,0x3f,sizeof dist);
    dist[s]=0;
    q.push(s);
    while(!q.empty())
    {
        s=q.front();
        q.pop();
        for(it=set[s].begin();it!=set[s].end();++it)
            if(dist[*it]>dist[s]+1)
            {
                dist[*it]=dist[s]+1;
                q.push(*it);
            }
    }
    return std::max_element(dist+1,dist+1+bcnt)-dist;
}

int main()
{
    while(scanf("%d␣%d",&n,&m),(n||m))
    {
        cnt=0;
        memset(edge,0,sizeof edge);
        while(m--)
        {
            scanf("%d␣%d",&i,&j);
            add(i,j);
            add(j,i);
        }

        memset(dfn,0,sizeof dfn);
        memset(belong,0,sizeof belong);
        memset(low,0,sizeof low);
        memset(col,0,sizeof col);
        bcnt=idx=0;
        while(!st.empty())
            st.pop();

        tarjan(1,-1);
        for(i=1;i<=bcnt;++i)
            set[i].clear();
        for(i=1;i<=n;++i)
            for(j=edge[i];j;j=nxt[j])
                set[belong[i]].insert(belong[to[j]]);
        for(i=1;i<=bcnt;++i)
            set[i].erase(i);
        /*
        printf("%d\n",dist[go(go(1))]);
        for(i=1;i<=bcnt;++i)
            printf("%d\n",dist[i]);
        puts("");
        */
        printf("%d\n",bcnt-1-dist[go(go(1))]);
    }
    return 0;
}
```

4.5 Biconnected Component

```
#include<cstdio>
#include<cstring>
#include<stack>
#include<queue>
#include<algorithm>

const int MAXN=100000*2;
const int MAXM=200000;

//0-based

struct edges
{
    int to,next;
    bool cut,visit;
} edge[MAXM<<1];

int head[MAXN],low[MAXN],dpt[MAXN],L;
bool visit[MAXN],cut[MAXN];
int idx;
std::stack<int> st;
int bcc[MAXM];

void init(int n)
{
    L=0;
    memset(head,-1,4*n);
    memset(visit,0,n);
}

void add_edge(int u,int v)
{
    edge[L].cut=edge[L].visit=false;
    edge[L].to=v;
    edge[L].next=head[u];
    head[u]=L++;
}

void dfs(int u,int fu,int deg)
{
    cut[u]=false;
    visit[u]=true;
    low[u]=dpt[u]=deg;
    int tot=0;
    for (int i=head[u]; i!=-1; i=edge[i].next)
    {
        int v=edge[i].to;
        if (edge[i].visit)
            continue;
        st.push(i/2);
        edge[i].visit=edge[i^1].visit=true;
        if (visit[v])
        {
            low[u]=dpt[v]>low[u]?low[u]:dpt[v];
            continue;
        }
        dfs(v,u,deg+1);
        edge[i].cut=edge[i^1].cut=(low[v]>dpt[u] || edge[i].cut);
        if (u!=fu) cut[u]=low[v]>=dpt[u]?1:cut[u];
        if (low[v]>=dpt[u] || u==fu)
        {
            while (st.top()!=i/2)
            {
                int x=st.top()*2,y=st.top()*2+1;
                bcc[st.top()]=idx;
                st.pop();
            }
            bcc[i/2]=idx++;
            st.pop();
        }
        low[u]=low[v]>low[u]?low[u]:low[v];
        tot++;
    }
    if (u==fu && tot>1)
        cut[u]=true;
}

int main()
{
    int n,m;
    while (scanf("%d%d",&n,&m)!=EOF)
    {
        init(n);
        for (int i=0; i<m; i++)
        {
            int u,v;
            scanf("%d%d",&u,&v);
            add_edge(u,v);
            add_edge(v,u);
        }
        idx=0;
        for (int i=0; i<n; i++)
            if (!visit[i])
                dfs(i,i,0);
    }
    return 0;
}
```

4.6 Blossom algorithm

```
#include<cstdio>
#include<vector>
#include<cstring>
#include<algorithm>

#define MAXX 233

bool map[MAXX][MAXX];
std::vector<int>p[MAXX];
int m[MAXX];
int vis[MAXX];
int q[MAXX],*qf,*qb;

int n;

inline void label(int x,int y,int b)
{
    static int i,z;
    for(i=b+1;i<p[x].size();++i)
        if(vis[z=p[x][i]]==1)
        {
            p[z]=p[y];
            p[z].insert(p[z].end(),p[x].rbegin(),p[x].rend()-i);
            vis[z]=0;
            *qb++=z;
        }
}

inline bool bfs(int now)
{
    static int i,x,y,z,b;
    for(i=0;i<n;++i)
        p[i].resize(0);
    p[now].push_back(now);
    memset(vis,-1,sizeof vis);
    vis[now]=0;
    qf=qb=q;
    *qb++=now;

    while(qf<qb)
        for(x=*qf++,y=0;y<n;++y)
            if(map[x][y] && m[y]!=y && vis[y]!=1)
            {
                if(vis[y]==-1)
                    if(m[y]==-1)
                    {
                        for(i=0;i+1<p[x].size();i+=2)
                        {
                            m[p[x][i]]=p[x][i+1];
                            m[p[x][i+1]]=p[x][i];
                        }
                        m[x]=y;
                        m[y]=x;
                        return true;
                    }
                    else
                    {
                        p[z=m[y]]=p[x];
                        p[z].push_back(y);
                        p[z].push_back(z);
                        vis[y]=1;
                        vis[z]=0;
                        *qb++=z;
                    }
                else
                {
                    for(b=0;b<p[x].size() && b<p[y].size() && p[x][b]==p[y][b];++b);
                    --b;
                    label(x,y,b);
                    label(y,x,b);
                }
            }
    return false;
}

int i,j,k;
int ans;

int main()
{
    scanf("%d",&n);
    for(i=0;i<n;++i)
        p[i].reserve(n);
    while(scanf("%d␣%d",&i,&j)!=EOF)
    {
        --i;
        --j;
        map[i][j]=map[j][i]=true;
    }
    memset(m,-1,sizeof m);
    for(i=0;i<n;++i)
        if(m[i]==-1)
        {
            if(bfs(i))
                ++ans;
            else
                m[i]=i;
        }
    printf("%d\n",ans<<1);
    for(i=0;i<n;++i)
        if(i<m[i])
            printf("%d␣%d\n",i+1,m[i]+1);
    return 0;
}
```

4.7 Bridge

```
void dfs(const short &now,const short &fa)
{
    dfn[now]=low[now]=cnt++;
    for(int i(0);i<edge[now].size();++i)
        if(dfn[edge[now][i]]==-1)
        {
            dfs(edge[now][i],now);
            low[now]=std::min(low[now],low[edge[now][i]]);
            if(low[edge[now][i]]>dfn[now]) //如果子节点不能够走到父节点之前去，那么该边为桥
            {
                if(edge[now][i]<now)
                {
                    j=edge[now][i];
                    k=now;
                }
                else
                {
                    j=now;
                    k=edge[now][i];
                }
                ans.push_back(node(j,k));
            }
        }
        else
            if(edge[now][i]!=fa)
                low[now]=std::min(low[now],low[edge[now][i]]);
}
```

4.8 Chu-Liu:Edmonds' Algorithm

```
#include<cstdio>
#include<cstring>
#include<vector>

#define MAXX 1111
#define MAXE 10111
#define inf 0x3f3f3f3f

int n,m,i,j,k,ans,u,v,tn,rt,sum,on,om;
int pre[MAXX],id[MAXX],in[MAXX],vis[MAXX];

struct edge
{
    int a,b,c;
    edge(){}
    edge(int aa,int bb,int cc):a(aa),b(bb),c(cc){}
};
std::vector<edge>ed(MAXE);

int main()
{
    while(scanf("%d␣%d",&n,&m)!=EOF)
    {
        on=n;
        om=m;
        ed.resize(0);
        sum=1;
        while(m--)
        {
            scanf("%d␣%d␣%d",&i,&j,&k);
            if(i!=j)
            {
                ed.push_back(edge(i,j,k));
                sum+=k;
            }
        }
        ans=0;
        rt=n;
        for(i=0;i<n;++i)
            ed.push_back(edge(n,i,sum));
        ++n;
        while(true)
        {
            memset(in,0x3f,sizeof in);
            for(i=0;i<ed.size();++i)
                if(ed[i].a!=ed[i].b && in[ed[i].b]>ed[i].c)
                {
                    in[ed[i].b]=ed[i].c;
                    pre[ed[i].b]=ed[i].a;
                    if(ed[i].a==rt)
                        j=i;
                }
            for(i=0;i<n;++i)
                if(i!=rt && in[i]==inf)
                    goto ot;
            memset(id,-1,sizeof id);
            memset(vis,-1,sizeof vis);
            tn=in[rt]=0;
            for(i=0;i<n;++i)
            {
                ans+=in[i];
                for(v=i;vis[v]!=i && id[v]==-1 && v!=rt;v=pre[v])
                    vis[v]=i;
                if(v!=rt && id[v]==-1)
                {
                    for(u=pre[v];u!=v;u=pre[u])
                        id[u]=tn;
                    id[v]=tn++;
                }
            }
            if(!tn)
                break;
            for(i=0;i<n;++i)
                if(id[i]==-1)
                    id[i]=tn++;
            for(i=0;i<ed.size();++i)
            {
                v=ed[i].b;
                ed[i].a=id[ed[i].a];
                ed[i].b=id[ed[i].b];
                if(ed[i].a!=ed[i].b)
                    ed[i].c-=in[v];
            }
            n=tn;
            rt=id[rt];
        }
        if(ans>=2*sum)
ot:            puts("impossible");
```

```
        else
            printf("%d␣%d\n",ans-sum,j-om);
        puts("");
    }
    return 0;
}
```

4.9 Count MST

```
//hdu 4408
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 111

long long mod;
long long a[MAXX][MAXX];

inline long long det(int n)
{
    static int i,j,k;
    static long long re,t;
    for(i=0;i<n;++i)
        for(j=0;j<n;++j)
            a[i][j]%=mod;
    re=1ll;
    for(i=0;i<n;++i)
    {
        for(j=i+1;j<n;++j)
            while(a[j][i])
            {
                t=a[i][i]/a[j][i];
                for(k=i;k<n;++k)
                    a[i][k]=(a[i][k]-a[j][k]*t)%mod;
                for(k=i;k<n;++k)
                    std::swap(a[i][k],a[j][k]);
                re=-re;
            }
        if(!a[i][i])
            return 0ll;
        re=re*a[i][i]%mod;
    }
    return (re+mod)%mod;
}

struct E
{
    int a,b,c;
    bool operator<(const E &i)const
    {
        return c<i.c;
    }
}edge[1111];

int set[2][MAXX];
int find(int a,int t)
{
    return set[t][a]?set[t][a]=find(set[t][a],t):a;
}

int id[MAXX],dg[MAXX];
int map[MAXX][MAXX];
int n,m,i,j,k;
long long ans;
int cnt;

int main()
{
    while(scanf("%d␣%d␣%lld",&n,&m,&mod),(n||m||mod))
    {
        for(i=0;i<m;++i)
            scanf("%d␣%d␣%d",&edge[i].a,&edge[i].b,&edge[i].c);
        std::sort(edge,edge+m);
        memset(set[0],0,sizeof set[0]);
        ans=cnt=1;
        for(i=0;i<m;i=j)
        {
            for(j=i;j<m;++j)
                if(edge[i].c!=edge[j].c)
                    break;
            memset(dg,0,sizeof dg);
            memset(map,0,sizeof map);
            memset(set[1],0,sizeof set[0]);
            static int t,x,y;
            t=0;
            for(k=i;k<j;++k)
            {
                x=find(edge[k].a,0);
                y=find(edge[k].b,0);
                if(x!=y)
                {
                    ++map[x][y];
                    ++map[y][x];
                    ++dg[x];
                    ++dg[y];
                    x=find(x,1);
                    y=find(y,1);
                    if(x!=y)
                        set[1][x]=y;
                    ++t;
                }
            }
            for(k=i;k<j;++k)
            {
                x=find(edge[k].a,0);
                y=find(edge[k].b,0);
                if(x!=y)
                {
                    ++cnt;
                    set[0][x]=y;
                }
            }
            if(t)
            {
                for(k=1;k<=n;++k)
                    if(dg[k] && find(k,1)==k)
                    {
                        memset(a,0,sizeof a);
                        t=0;
                        static int ii,jj;
                        for(ii=1;ii<=n;++ii)
                            if(dg[ii] && find(ii,1)==k)
                                id[ii]=t++;
                        for(ii=1;ii<=n;++ii)
                            if(dg[ii] && find(ii,1)==k)
                            {
                                a[id[ii]][id[ii]]=dg[ii];
                                for(jj=1;jj<=n;++jj)
                                {
                                    if(!dg[jj] || ii==jj || find(jj,1)!=k)
                                        continue;
                                    if(map[ii][jj])
                                    {
                                        static long long cnt;
                                        cnt=-map[ii][jj];
                                        a[id[ii]][id[jj]]=(cnt%mod+mod)%mod;
                                    }
                                }
                            }
                        ans=(ans*det(t-1))%mod;
                    }
            }
        }
        if(cnt!=n)
            puts("0");
        else
            printf("%lld\n",(ans%mod+mod)%mod);
    }
    return 0;
}
```

4.10 Covering problems

最大团以及相关知识

独立集：独立集是指图的顶点集的一个子集，该子集的导出子图的点互不相邻．如果一个独立集不是任何一个独立集的子集，那么称这个独立集是一个极大独立集．一个图中包含顶点数目最多的独立集称为最大独立集。最大独立集一定是极大独立集，但是极大独立集不一定是最大的独立集。

支配集：与独立集相对应的就是支配集，支配集也是图顶点集的一个子集，设 S 是图 G 的一个支配集，则对于图中的任意一个顶点 u，要么属于集合 s，要么与 s 中的顶点相邻。在 s 中除去任何元素后 s 不再是支配集，则支配集 s 是极小支配集。称 G 的所有支配集中顶点个数最少的支配集为最小支配集，最小支配集中的顶点个数成为支配数。

最小点 (对边) 的覆盖：最小点的覆盖也是图的顶点集的一个子集，如果我们选中一个点，则称这个点将以他为端点的所有边都覆盖了。将图中所有的边都覆盖所用顶点数最少，这个集合就是最小的点的覆盖。

最大团：图 G 的顶点的子集，设 D 是最大团，则 D 中任意两点相邻。若 u, v 是最大团，则 u,v 有边相连，其补图 u,v 没有边相连，所以 图 G 的最大团 = 其补图的最大独立集。给定无向图 G = (V;E),

```
    如果 U 属于 V，并且对于任意 u,v 包含于 U 有 < u; v > 包含于 E，则称 U 是 G 的完全子图，G 的完全子图 U 是 G 的团，当且仅当 U 不包含在 G 的更大的完全子图中，G 的最大团是指 G 中所含顶点数目最多的团。如果 U 属于 V，并且对于任意 u; v 包含于 U 有 < u; v > 不包含于 E，则称 U 是 G 的空子图，G 的空子图 U 是 G 的独立集，当且仅当 U 不包含在 G 的更大的独立集，G 的最大团是指 G 中所含顶点数目最多的独立集。

性质：
最大独立集 + 最小覆盖集 = V
最大团 = 补图的最大独立集
最小覆盖集 = 最大匹配

minimum cover:
vertex cover vertex bipartite graph = maximum cardinality bipartite matching
找完最大二分匹配後，有三種情況要分別處理：
甲、X 側未匹配點的交錯樹們。
乙、Y 側未匹配點的交錯樹們。
丙、層層疊疊的交錯環們（包含單獨的匹配邊）。
這三個情況互不干涉。用 Graph Traversal 建立甲、乙的交錯樹們，剩下部分就是丙。
要找點覆蓋，甲、乙是取盡奇數距離的點，丙是取盡偶數距離的點、或者是取盡奇數距離的點，每塊連通分量可以各自為政。另外，小心處理的話，是可以印出字典順序最小的點覆蓋的。
已經有最大匹配時，求點覆蓋的時間複雜度等同於一次 Graph Traversal 的時間。

vertex cover edge

edge cover vertex
首先在圖上求得一個 Maximum Matching 之後，對於那些單身的點，都由匹配點連過去。如此便形成了 Minimum Edge Cover 。

edge cover edge

path cover vertex
general graph: NP-H
tree: DP
DAG: 将每个节点拆分为入点和出点,ans= 节点数 -匹配数

path cover edge
minimize the count of euler path ( greedy is ok? )
dg[i] 表示每个点的 id-od,$ans = \sum dg[i], \forall dg[i] > 0$

cycle cover vertex
general: NP-H
weighted: do like path cover vertex, with KM algorithm

cycle cover edge
NP-H
```

4.11 difference constraints

```
for a - b <= c
    add(b,a,c);

最短路得最远解
最长路得最近解
//根据情况反转边?(反转方向及边权)

全 0 点得普通解
```

4.12 Dinitz's algorithm

```cpp
#include<cstdio>
#include<algorithm>
#include<cstring>

#define MAXX 111
#define MAXM (MAXX*MAXX*4)
#define inf 0x3f3f3f3f

int n;
int w[MAXX],h[MAXX],q[MAXX];
int edge[MAXX],to[MAXM],cap[MAXM],nxt[MAXM],cnt;
int source,sink;

inline void add(int a,int b,int c)
{
    nxt[cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    ++cnt;
}

inline bool bfs()
{
    static int *qf,*qb;
    static int i;
    memset(h,-1,sizeof h);
    qf=qb=q;
    h[*qb++=source]=0;
    for(;qf!=qb;++qf)
        for(i=edge[*qf];i!=-1;i=nxt[i])
            if(cap[i] && h[to[i]]==-1)
                h[*qb++=to[i]]=h[*qf]+1;
    return h[sink]!=-1;
}

int dfs(int now,int maxcap)
{
    if(now==sink)
        return maxcap;
    int flow(maxcap),d;
    for(int &i(w[now]);i!=-1;i=nxt[i])
        if(cap[i] && h[to[i]]==h[now]+1)// && (flow=dfs(to[i],std::min(maxcap,cap[i]))))
        {
            d=dfs(to[i],std::min(flow,cap[i]));
            cap[i]-=d;
            cap[i^1]+=d;
            flow-=d;
            if(!flow)
                return maxcap;
        }
    return maxcap-flow;
}

int nc,np,m,i,j,k;
int ans;

int main()
{
    while(scanf("%d␣%d␣%d␣%d",&n,&np,&nc,&m)!=EOF)
    {
        cnt=0;
        memset(edge,-1,sizeof edge);
        while(m--)
        {
            while(getchar()!='(');
            scanf("%d",&i);
            while(getchar()!=',');
            scanf("%d",&j);
            while(getchar()!=')');
            scanf("%d",&k);
            if(i!=j)
            {
                ++i;
                ++j;
                add(i,j,k);
                add(j,i,0);
            }
        }
        source=++n;
        while(np--)
        {
            while(getchar()!='(');
            scanf("%d",&i);
            while(getchar()!=')');
            scanf("%d",&j);
            ++i;
            add(source,i,j);
            add(i,source,0);
        }
        sink=++n;
        while(nc--)
        {
            while(getchar()!='(');
            scanf("%d",&i);
            while(getchar()!=')');
            scanf("%d",&j);
            ++i;
            add(i,sink,j);
            add(sink,i,0);
        }
        ans=0;
        while(bfs())
        {
            memcpy(w,edge,sizeof edge);
            ans+=dfs(source,inf);
            /*
            while((k=dfs(source,inf)))
                ans+=k;
                */
        }
        printf("%d\n",ans);
    }
    return 0;
```

```
}
```

4.13 Flow network

```
Maximum weighted closure of a graph:

所有由这个子图中的点出发的边都指向这个子图，那么这个子图为原图的一个 closure（闭合子图）

每个节点向其所有依赖节点连边，容量 inf
源点向所有正权值节点连边，容量为该权值
所有负权值节点向汇点连边，容量为该权值绝对值
以上均为有向边
最大权为 sum{正权值}-{新图的最小割}
残量图中所有由源点可达的点即为所选子图



Eulerian circuit:
计入度和出度之差
无向边任意定向
出入度之差为奇数则无解
然后构图:
原图有向边不变，容量 1 // 好像需要在新图中忽略有向边?
无向边按之前认定方向，容量 1
源点向所有度数为正的点连边，容量 abs(度数/2)
所有度数为负的点向汇点连边，容量 abs(度数/2)
两侧均满流则有解
相当于规约为可行流问题
注意连通性的 trick

终点到起点加一条有向边即可将 path 问题转为 circuit 问题



Feasible flow problem:
由超级源点出发的边全部满流则有解
有源汇时，由汇点向源点连边，下界 0 上界 inf 即可转化为无源无汇上下界流

对于每条边 <a->b cap{u,d}>，建边 <ss->b cap(u)>、<a->st cap(u)>、<a->b cap(d-u)>

Maximum flow: //好像也可以二分
//将流量还原至原图后，在残量网络上继续完成最大流
直接把 source 和 sink 设为原来的 st，此时输出的最大流即是答案
不需要删除或者调整 t->s 弧
Minimum flow: //好像也可以二分
建图时先不连汇点到源点的边，新图中完成最大流之后再连原汇至原源的边完成第二次最大流，此时 t->s 这条弧的流量即为最小流
判断可行流存在还是必须连原汇 -> 原源的边之后查看满流
所以可以使用跑流 -> 加 ts 弧 -> 跑流，最后检查超级源点满流情况来一步搞定
tips:
合并流量、减少边数来加速



Minimum cost feasible flow problem:
TODO
看起来像是在上面那样跑费用流就行了……



Minimum weighted vertex cover edge for bipartite graph:
for all vertex in X:
edge < s->x cap(weight(x)) >
for all vertex in Y:
edge < y->t cap(weight(y)) >
for original edges
edge < x->y cap(inf) >

ans={maximum flow}={minimum cut}
残量网络中的所有简单割（（源点可达 && 汇点不可达）||（源点不可达 && 汇点可达））对应着解



Maximum weighted vertex independent set for bipartite graph:
ans=Sum 点权 -valueMinimum weighted vertex cover edge
解应该就是最小覆盖集的补图吧……



方格取数: // refer: hdu 3820 golden eggs
取方格获得收益
当取了相邻方格时付出边的代价

必取的方格到源/汇的边的容量 inf
相邻方格之间的边的容量为 {代价}*2
ans=sum{方格收益}-{最大流}



最小割的唯一性: // refer: 关键边。有向边起点为 s 集，终点为 t 集
从源和汇分别能够到的点集是所有点时，最小割唯一
也就是每一条增广路径都仅有一条边满流
注意查看的是实际的网络，不是残量网络

具体来说

void rr(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i] && !done[v])
            rr(v);
}

void dfs(int now)
{
    done[now]=true;
    ++cnt;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i^1] && !done[v])
            dfs(v);
}

memset(done,0,sizeof done);
cnt=0;
rr(source);
dfs(sink);
puts(cnt==n?"UNIQUE":"AMBIGUOUS");



Tips:
两点间可以不止有一种边，也可以不止有一条边，无论有向无向;
两点间容量 inf 则可以设法化简为一个点;
点权始终要转化为边权;
不参与决策的边权设为 inf 来排除掉;
贪心一个初始不合法情况，然后通过可行流调整; // refer: 混合图欧拉回路存在性、有向/无向图中国邮差问题（遍历所有边至少一次后回到原点）
按时间拆点（时间层……？）;
```

4.14 Hamiltonian circuit

```
//if every point connect with not less than [(N+1)/2] points
#include<cstdio>
#include<algorithm>
#include<cstring>

#define MAXX 177
#define MAX (MAXX*MAXX)

int edge[MAXX],nxt[MAX],to[MAX],cnt;

inline void add(int a,int b)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
}

bool done[MAXX];
int n,m,i,j,k;

inline int find(int a)
{
    static int i;
    for(i=edge[a];i;i=nxt[i])
        if(!done[to[i]])
        {
            edge[a]=nxt[i];
            return to[i];
        }
    return 0;
}

int a,b;
int next[MAXX],pre[MAXX];
bool mat[MAXX][MAXX];
```

```

int main()
{
    while(scanf("%d␣%d",&n,&m)!=EOF)
    {
        for(i=1;i<=n;++i)
            next[i]=done[i]=edge[i]=0;
        memset(mat,0,sizeof mat);
        cnt=0;
        while(m--)
        {
            scanf("%d␣%d",&i,&j);
            add(i,j);
            add(j,i);
            mat[i][j]=mat[j][i]=true;
        }
        a=1;
        b=to[edge[a]];
        cnt=2;
        done[a]=done[b]=true;
        next[a]=b;
        while(cnt<n)
        {
            while(i=find(a))
            {
                next[i]=a;
                done[a=i]=true;
                ++cnt;
            }
            while(i=find(b))
            {
                next[b]=i;
                done[b=i]=true;
                ++cnt;
            }
            if(!mat[a][b])
                for(i=next[a];next[i]!=b;i=next[i])
                    if(mat[a][next[i]] && mat[i][b])
                    {
                        for(j=next[i];j!=b;j=next[j])
                            pre[next[j]]=j;
                        for(j=b;j!=next[i];j=pre[j])
                            next[j]=pre[j];
                        std::swap(next[i],b);
                        break;
                    }
            next[b]=a;
            for(i=a;i!=b;i=next[i])
                if(find(i))
                {
                    a=next[b=i];
                    break;
                }
        }
        while(a!=b)
        {
            printf("%d␣",a);
            a=next[a];
        }
        printf("%d\n",b);
    }
    return 0;
}
```

4.15 Hopcroft-Karp algorithm

```
#include<cstdio>
#include<cstring>

#define MAXX 50111
#define MAX 150111

int nx,p;
int i,j,k;
int x,y;
int ans;
bool flag;

int edge[MAXX],nxt[MAX],to[MAX],cnt;

int cx[MAXX],cy[MAXX];
int px[MAXX],py[MAXX];

int q[MAXX],*qf,*qb;

bool ag(int i)
{
    int j,k;
    for(k=edge[i];k;k=nxt[k])
        if(py[j=to[k]]==px[i]+1)
        {
            py[j]=0;
            if(cy[j]==-1 || ag(cy[j]))
            {
                cx[i]=j;
                cy[j]=i;
                return true;
            }
        }
    return false;
}

int main()
{
    scanf("%d␣%*d␣%d",&nx,&p);
    while(p--)
    {
        scanf("%d␣%d",&i,&j);
        nxt[++cnt]=edge[i];
        edge[i]=cnt;
        to[cnt]=j;
    }
    memset(cx,-1,sizeof cx);
    memset(cy,-1,sizeof cy);
    while(true)
    {
        memset(px,0,sizeof(px));
        memset(py,0,sizeof(py));
        qf=qb=q;
        flag=false;

        for(i=1;i<=nx;++i)
            if(cx[i]==-1)
                *qb++=i;
        while(qf!=qb)
            for(k=edge[i=*qf++];k;k=nxt[k])
                if(!py[j=to[k]])
                {
                    py[j]=px[i]+1;
                    if(cy[j]==-1)
                        flag=true;
                    else
                    {
                        px[cy[j]]=py[j]+1;
                        *qb++=cy[j];
                    }
                }
        if(!flag)
            break;
        for(i=1;i<=nx;++i)
            if(cx[i]==-1 && ag(i))
                ++ans;
    }
    printf("%d\n",ans);
    return 0;
}
```

4.16 Improved Shortest Augmenting Path Algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 5111
#define MAXM (30111*4)
#define inf 0x3f3f3f3f3f3f3f3fll

int edge[MAXX],to[MAXM],nxt[MAXM],cnt;
#define v to[i]
long long cap[MAXM];

int n;
int h[MAXX],gap[MAXX],pre[MAXX],w[MAXX];

inline void add(int a,int b,long long c)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
}

int source,sink;

inline long long go(const int N=sink)
{
    static int now,i;
    static long long min,mf;
    memset(gap,0,sizeof gap);
    memset(h,0,sizeof h);
    memcpy(w,edge,sizeof w);
    gap[0]=N;
    mf=0;

    pre[now=source]=-1;
    while(h[source]<N)
    {
rep:
        if(now==sink)
        {
            min=inf;
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
                if(min>=cap[i])
                {
                    min=cap[i];
                    now=to[i^1];
                }
            for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            {
                cap[i]-=min;
                cap[i^1]+=min;
            }
            mf+=min;
        }
        for(int &i(w[now]);i!=-1;i=nxt[i])
            if(cap[i] && h[v]+1==h[now])
            {
                pre[now=v]=i;
                goto rep;
            }
        if(!--gap[h[now]])
            return mf;
        min=N;
        for(i=w[now]=edge[now];i!=-1;i=nxt[i])
            if(cap[i])
                min=std::min(min,(long long)h[v]);
        ++gap[h[now]=min+1];
        if(now!=source)
            now=to[pre[now]^1];
    }
    return mf;
}

int m,i,j,k;
long long ans;

int main()
{
    scanf("%d␣%d",&n,&m);
    source=1;
    sink=n;
    cnt=-1;
    memset(edge,-1,sizeof edge);
    while(m--)
    {
        scanf("%d␣%d␣%lld",&i,&j,&ans);
        add(i,j,ans);
        add(j,i,ans);
    }
    printf("%lld\n",go());
    return 0;
}
```

4.17 k Shortest Path

```
#include<cstdio>
#include<cstring>
#include<queue>
#include<vector>

int K;

class states
{
    public:
        int cost,id;
};

int dist[1000];

class cmp
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost>j.cost;
        }
};

class cmp2
{
    public:
        bool operator ()(const states &i,const states &j)
        {
            return i.cost+dist[i.id]>j.cost+dist[j.id];
        }
};

struct edges
{
    int to,next,cost;
} edger[100000],edge[100000];

int headr[1000],head[1000],Lr,L;

void dijkstra(int s)
{
    states u;
    u.id=s;
    u.cost=0;
    dist[s]=0;
    std::priority_queue<states,std::vector<states>,cmp> q;
    q.push(u);
    while (!q.empty())
    {
        u=q.top();
        q.pop();
        if (u.cost!=dist[u.id])
            continue;
        for (int i=headr[u.id]; i!=-1; i=edger[i].next)
        {
            states v=u;
            v.id=edger[i].to;
            if (dist[v.id]>dist[u.id]+edger[i].cost)
            {
                v.cost=dist[v.id]=dist[u.id]+edger[i].cost;
                q.push(v);
            }
        }
    }
}

int num[1000];

inline void init(int n)
{
    Lr=L=0;
    memset(head,-1,4*n);
    memset(headr,-1,4*n);
    memset(dist,63,4*n);
    memset(num,0,4*n);
}

void add_edge(int u,int v,int x)
{
    edge[L].to=v;
    edge[L].cost=x;
    edge[L].next=head[u];
    head[u]=L++;
    edger[Lr].to=u;
    edger[Lr].cost=x;
    edger[Lr].next=headr[v];
    headr[v]=Lr++;
}

inline int a_star(int s,int t)
{
    if (dist[s]==0x3f3f3f3f)
        return -1;
    std::priority_queue<states,std::vector<states>,cmp2> q;
    states tmp;
    tmp.id=s;
    tmp.cost=0;
    q.push(tmp);
    while (!q.empty())
    {
        states u=q.top();
        q.pop();
        num[u.id]++;
        if (num[t]==K)
            return u.cost;
        for (int i=head[u.id]; i!=-1; i=edge[i].next)
        {
            int v=edge[i].to;
            tmp.id=v;
            tmp.cost=u.cost+edge[i].cost;
            q.push(tmp);
        }
    }
    return -1;
}

int main()
{
    int n,m;
    scanf("%d%d",&n,&m);
    init(n);
    for (int i=0; i<m; i++)
    {
        int u,v,x;
        scanf("%d%d%d",&u,&v,&x);
        add_edge(u-1,v-1,x);
    }
    int s,t;
    scanf("%d%d%d",&s,&t,&K);
    if (s==t)
        ++K;
    dijkstra(t-1);
    printf("%d\n",a_star(s-1,t-1));
    return 0;
}
```

4.18 Kariv-Hakimi Algorithm

```
//Absolute Center of a graph, not only a tree
#include<cstdio>
#include<algorithm>
#include<vector>
#include<cstring>
#include<set>

#define MAXX 211
#define inf 0x3f3f3f3f

int e[MAXX][MAXX],dist[MAXX][MAXX];
double dp[MAXX],ta;
int ans,d;
int n,m,a,b;
int i,j,k;
typedef std::pair<int,int> pii;
std::vector<pii>vt[2];
bool done[MAXX];
typedef std::pair<double,int> pdi;
std::multiset<pdi>q;
int pre[MAXX];

int main()
{
    vt[0].reserve(MAXX);
    vt[1].reserve(MAXX);
    scanf("%d␣%d",&n,&m);
    memset(e,0x3f,sizeof(e));
    while(m--)
    {
        scanf("%d␣%d␣%d",&i,&j,&k);
        e[i][j]=e[j][i]=std::min(e[i][j],k);
    }
    for(i=1;i<=n;++i)
        e[i][i]=0;
    memcpy(dist,e,sizeof(dist));
    for(k=1;k<=n;++k)
        for(i=1;i<=n;++i)
            for(j=1;j<=n;++j)
                dist[i][j]=std::min(dist[i][j],dist[i][k]+dist[k][j]);
    ans=inf;
    for(i=1;i<=n;++i)
        for(j=i;j<=n;++j)
            if(e[i][j]!=inf)
            {
                vt[0].resize(0);
                vt[1].resize(0);
                static int i;
                for(i=1;i<=n;++i)
                    vt[0].push_back(pii(dist[::i][i],dist[j][i]));
                std::sort(vt[0].begin(),vt[0].end());
                for(i=0;i<vt[0].size();++i)
                {
                    while(!vt[1].empty() && vt[1].back().second<=vt[0][i].second)
                        vt[1].pop_back();
                    vt[1].push_back(vt[0][i]);
                }
                d=inf;
                if(vt[1].size()==1)
                    if(vt[1][0].first<vt[1][0].second)
                    {
                        ta=0;
                        d=(vt[1][0].first<<1);
                    }
                    else
                    {
                        ta=e[::i][j];
                        d=(vt[1][0].second<<1);
                    }
                else
                    for(i=1;i<vt[1].size();++i)
                        if(d>e[::i][j]+vt[1][i-1].first+vt[1][i].second)
                        {
                            ta=(e[::i][j]+vt[1][i].second-vt[1][i-1].first)/(double)2.0f;
                            d=e[::i][j]+vt[1][i-1].first+vt[1][i].second;
                        }
                if(d<ans)
                {
                    ans=d;
                    a=::i;
                    b=j;
                    dp[::i]=ta;
                    dp[j]=e[::i][j]-ta;
                }
            }
    printf("%d\n",ans);
    for(i=1;i<=n;++i)
        if(i!=a && i!=b)
            dp[i]=1e20;
    q.insert(pdi(dp[a],a));
    if(a!=b)
        q.insert(pdi(dp[b],b));
    if(a!=b)
        pre[b]=a;
    while(!q.empty())
    {
        k=q.begin()->second;
        q.erase(q.begin());
        if(done[k])
            continue;
        done[k]=true;
        for(i=1;i<=n;++i)
            if(e[k][i]!=inf && dp[k]+e[k][i]<dp[i])
            {
                dp[i]=dp[k]+e[k][i];
                q.insert(pdi(dp[i],i));
                pre[i]=k;
            }
    }
    vt[0].resize(0);
    for(i=1;i<=n;++i)
        if(pre[i])
            if(i<pre[i])
                printf("%d␣%d\n",i,pre[i]);
            else
                printf("%d␣%d\n",pre[i],i);
    return 0;
}
```

4.19 Kuhn-Munkres algorithm

```
bool match(int u)//匈牙利
{
    vx[u]=true;
    for(int i=1;i<=n;++i)
        if(lx[u]+ly[i]==g[u][i]&&!vy[i])
        {
            vy[i]=true;
            if(!d[i]||match(d[i]))
            {
                d[i]=u;
                return true;
            }
        }
    return false;
}
inline void update()//
{
    int i,j;
    int a=1<<30;
    for(i=1;i<=n;++i)if(vx[i])
        for(j=1;j<=n;++j)if(!vy[j])
            a=min(a,lx[i]+ly[j]-g[i][j]);
    for(i=1;i<=n;++i)
    {
        if(vx[i])lx[i]-=a;
        if(vy[i])ly[i]+=a;
    }
}
void km()
{
    int i,j;
    for(i=1;i<=n;++i)
    {
        lx[i]=ly[i]=d[i]=0;
        for(j=1;j<=n;++j)
            lx[i]=max(lx[i],g[i][j]);
    }
    for(i=1;i<=n;++i)
    {
        while(true)
        {
            memset(vx,0,sizeof(vx));
            memset(vy,0,sizeof(vy));
            if(match(i))
                break;
            update();
        }
    }
    int ans=0;
    for(i=1;i<=n;++i)
        if(d[i]!=0)
            ans+=g[d[i]][i];
    printf("%d\n",ans);
}
int main()
{
    while(scanf("%d\n",&n)!=EOF)
    {
        for(int i=1;i<=n;++i)gets(s[i]);
        memset(g,0,sizeof(g));
        for(int i=1;i<=n;++i)
            for(int j=1;j<=n;++j)
                if(i!=j) g[i][j]=cal(s[i],s[j]);
        km();
    }
    return 0;
}


//bupt

//算法：求二分图最佳匹配km n复杂度^3
int dfs(int u)//匈牙利求增广路
{
    int v;
    sx[u]=1;
    for ( v=1; v<=n; v++)
        if (!sy[v] && lx[u]+ly[v]==map[u][v])
        {
            sy[v]=1;
            if (match[v]==-1 || dfs(match[v]))
            {
                match[v]=u;
                return 1;
            }
        }
    return 0;
}

int bestmatch(void)//求最佳匹配km
{
    int i,j,u;
    for (i=1; i<=n; i++)//初始化顶标
    {
        lx[i]=-1;
        ly[i]=0;
        for (j=1; j<=n; j++)
            if (lx[i]<map[i][j])
                lx[i]=map[i][j];
    }
    memset(match, -1, sizeof(match));
    for (u=1; u<=n; u++)
    {
        while (true)
        {
            memset(sx,0,sizeof(sx));
            memset(sy,0,sizeof(sy));
            if (dfs(u))
                break;
            int dx=Inf;//若找不到增广路，则修改顶标~~
            for (i=1; i<=n; i++)
            {
                if (sx[i])
                    for (j=1; j<=n; j++)
                        if(!sy[j] && dx>lx[i]+ly[j]-map[i][j])
                            dx=lx[i]+ly[j]-map[i][j];
            }
            for (i=1; i<=n; i++)
            {
                if (sx[i])
                    lx[i]-=dx;
                if (sy[i])
                    ly[i]+=dx;
            }
        }
    }
    int sum=0;
    for (i=1; i<=n; i++)
        sum+=map[match[i]][i];
    return sum;
}
```

4.20 LCA - DA

```
int edge[MAXX],nxt[MAXX<<1],to[MAXX<<1],cnt;
int pre[MAXX][N],dg[MAXX];

inline void add(int j,int k)
{
    nxt[++cnt]=edge[j];
    edge[j]=cnt;
    to[cnt]=k;
}

void rr(int now,int fa)
{
    dg[now]=dg[fa]+1;
    for(int i(edge[now]);i;i=nxt[i])
        if(to[i]!=fa)
        {
            static int j;
            j=1;
            for(pre[to[i]][0]=now;j<N;++j)
                pre[to[i]][j]=pre[pre[to[i]][j-1]][j-1];
            rr(to[i],now);
        }
}

inline int lca(int a,int b)
{
    static int i,j;
    j=0;
    if(dg[a]<dg[b])
        std::swap(a,b);
    for(i=dg[a]-dg[b];i;i>>=1,++j)
        if(i&1)
            a=pre[a][j];
    if(a==b)
        return a;
    for(i=N-1;i>=0;--i)
        if(pre[a][i]!=pre[b][i])
        {
            a=pre[a][i];
            b=pre[b][i];
        }
    return pre[a][0];

// looks like above is a wrong version

    static int i,log;
    for(log=0;(1<<(log+1))<=dg[a];++log);
    for(i=log;i>=0;--i)
        if(dg[a]-(1<<i)>=dg[b])
            a=pre[a][i];
    if(a==b)
        return a;
    for(i=log;i>=0;--i)
        if(pre[a][i]!=-1 && pre[a][i]!=pre[b][i])
            a=pre[a][i],b=pre[b][i];
    return pre[a][0];
}
```

4.21 LCA - tarjan - minmax

```
#include<cstdio>
#include<list>
#include<algorithm>
#include<cstring>

#define MAXX 100111
#define inf 0x5fffffff

short T,t;
int set[MAXX],min[MAXX],max[MAXX],ans[2][MAXX];
bool done[MAXX];
std::list<std::pair<int,int> >edge[MAXX];
std::list<std::pair<int,int> >q[MAXX];
int n,i,j,k,l,m;

struct node
{
    int a,b,id;
    node() {}
    node(const int &aa,const int &bb,const int &idd): a(aa),b(bb),id(idd){}
};

std::list<node>to[MAXX];

int find(const int &a)
{
    if(set[a]==a)
        return a;
    int b(set[a]);
    set[a]=find(set[a]);
    max[a]=std::max(max[a],max[b]);
    min[a]=std::min(min[a],min[b]);
    return set[a];
}

void tarjan(const int &now)
{
    done[now]=true;
    for(std::list<std::pair<int,int> >::const_iterator it(q[now].begin());it!=q[now].end();++it)
        if(done[it->first])
            if(it->second>0)
                to[find(it->first)].push_back(node(now,it->first,it->second));
            else
                to[find(it->first)].push_back(node(it->first,now,-it->second));
    for(std::list<std::pair<int,int> >::const_iterator it(edge[now].begin());it!=edge[now].end();++it)
        if(!done[it->first])
        {
            tarjan(it->first);
            set[it->first]=now;
            min[it->first]=it->second;
            max[it->first]=it->second;
        }
    for(std::list<node>::const_iterator it(to[now].begin());it!=to[now].end();++it)
    {
        find(it->a);
        find(it->b);
        ans[0][it->id]=std::min(min[it->b],min[it->a]);
        ans[1][it->id]=std::max(max[it->a],max[it->b]);
    }
}

int main()
{
    scanf("%hd",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d",&n);
        for(i=1;i<=n;++i)
        {
            edge[i].clear();
            q[i].clear();
            to[i].clear();
            done[i]=false;
            set[i]=i;
            min[i]=inf;
            max[i]=0;
        }
        for(i=1;i<n;++i)
        {
            scanf("%d%d%d",&j,&k,&l);
            edge[j].push_back(std::make_pair(k,l));
            edge[k].push_back(std::make_pair(j,l));
        }
        scanf("%d",&m);
        for(i=0;i<m;++i)
        {
            scanf("%d␣%d",&j,&k);
            q[j].push_back(std::make_pair(k,i));
            q[k].push_back(std::make_pair(j,-i));
        }
        tarjan(1);
        printf("Case␣%hd:\n",t);
        for(i=0;i<m;++i)
            printf("%d␣%d\n",ans[0][i],ans[1][i]);
    }
    return 0;
}
```

4.22 Minimum Ratio Spanning Tree

```
#include<cstdio>
#include<cstring>
#include<cmath>

#define MAXX 1111

struct
{
    int x,y;
    double z;
} node[MAXX];

struct
{
    double l,c;
} map[MAXX][MAXX];

int n,l,f[MAXX],pre[MAXX];
double dis[MAXX];

double mst(double x)
{
    int i,j,tmp;
    double min,s=0,t=0;
    memset(f,0,sizeof(f));
    f[1]=1;
    for (i=2; i<=n; i++)
    {
        dis[i]=map[1][i].c-map[1][i].l*x;
        pre[i]=1;
    }
    for (i=1; i<n; i++)
    {
        min=1e10;
        for (j=1; j<=n; j++)
            if (!f[j] && min>dis[j])
            {
                min=dis[j];
                tmp=j;
            }
        f[tmp]=1;
        t+=map[pre[tmp]][tmp].l;
        s+=map[pre[tmp]][tmp].c;
        for (j=1; j<=n; j++)
            if (!f[j] && map[tmp][j].c-map[tmp][j].l*x<dis[j])
            {
                dis[j]=map[tmp][j].c-map[tmp][j].l*x;
                pre[j]=tmp;
            }
    }
    return s/t;
}

int main()
{
    int i,j;
    double a,b;
    while (scanf("%d",&n),n);
    {
        for (i=1; i<=n; i++)
            scanf("%d%d%lf",&node[i].x,&node[i].y,&node[i].z);
        for (i=1; i<=n; i++)
            for (j=i+1; j<=n; j++)
            {
                map[j][i].l=map[i][j].l=sqrt(1.0*(node[i].x-node[j].x)*(node[i].x-node[j].x)+(node[i].y-node[j].y)*(node[i].y-node[j].y));
                map[j][i].c=map[i][j].c=fabs(node[i].z-node[j].z);
            }
        a=0,b=mst(a);
        while (fabs(b-a)>1e-8)
        {
            a=b;
            b=mst(a);
        }
        printf("%.3lf\n",b);
    }
    return 0;

}
```

4.23 Minimum Steiner Tree

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<queue>

#define MAXX 211
#define MAXE 10111
#define inf 0x3f3f3f3f

int edge[MAXX],nxt[MAXE],to[MAXE],wg[MAXE],cnt;
```

```
inline void add(int a,int b,int c)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    wg[cnt]=c;
}

int dp[1<<8];
int s[MAXX];
int d[1<<8][MAXX];
int S[MAXX],P[MAXX];
int fac[8];

struct node
{
    int a,b,dist;
    node(){}
    node(int i,int j,int k):a(i),b(j),dist(k){}
    bool operator<(const node &i)const
    {
        return dist>i.dist;
    }
    int &get()
    {
        return d[b][a];
    }
}now;

std::priority_queue<node>q;

int n,m,nn,i,j,k;
int cs,cf,x,y;
int ans,cst;

inline bool check(int x)
{
    static int re,i;
    for(i=re=0;x;x>>=1,++i)
        re+=(x&1)*(i<cf?fac[i]:-1);
    return re>=0;
}

inline int count(int x)
{
    static int i,re;
    x>>=cf;
    for(re=0;x;x>>=1)
        re+=(x&1);
    return re;
}

int main()
{
    while(scanf("%d",&n)!=EOF)
    {
        memset(s,0,sizeof s);
        memset(d,0x3f,sizeof d);
        memset(dp,0x3f,sizeof dp);
        ans=cnt=cf=cs=0;
        memset(edge,0,sizeof edge);
        for(i=1;i<=n;++i)
        {
            scanf("%d␣%d",P+i,S+i);
            if(S[i] && P[i])
            {
                ++ans;
                --P[i];
                S[i]=0;
            }
            if(P[i])
            {
                s[i]=1<<cf;
                fac[cf]=P[i];
                d[s[i]][i]=0;
                ++cf;
            }
        }
        for(i=1;i<=n;++i)
            if(S[i])
            {
                s[i]=1<<(cf+cs);
                d[s[i]][i]=0;
                ++cs;
            }
        nn=1<<(cf+cs);
        scanf("%d",&m);
        while(m--)
        {
            scanf("%d␣%d␣%d",&i,&j,&k);
            add(i,j,k);
            add(j,i,k);
        }
        for(y=1;y<nn;++y)
        {
            for(x=1;x<=n;++x)
            {
                if(s[x] && !(s[x]&y))
                    continue;
                for(i=(y-1)&y;i;i=(i-1)&y)
                    d[y][x]=std::min(d[y][x],d[i|s[x]][x]+d[(y^i)|s[x]][x]);
                if(d[y][x]!=inf)
                    q.push(node(x,y,d[y][x]));
            }
            while(!q.empty())
            {
                now=q.top();
                q.pop();
                if(now.dist!=now.get())
                    continue;
                static int x,y,a,b;
                x=now.a;
                y=now.b;
                for(i=edge[x];i;i=nxt[i])
                {
                    a=to[i];
                    b=y|s[a];
                    if(d[b][a]>now.get()+wg[i])
                    {
                        d[b][a]=now.get()+wg[i];
                        if(b==y)
                            q.push(node(a,b,d[b][a]));
                    }
                }
            }
        }
        for(j=0;j<nn;++j)
            dp[j]=*std::min_element(d[j]+1,d[j]+1+n);
        cnt=cst=0;
        for(i=1;i<nn;++i)
            if(check(i))
            {
                for(j=(i-1)&i;j;j=(j-1)&i)
                    if(check(j) && check(i^j))
                        dp[i]=std::min(dp[i],dp[j]+dp[i^j]);
                k=count(i);
                if(dp[i]!=inf && (k>cnt || (k==cnt && dp[i]<cst)))
                {
                    cnt=k;
                    cst=dp[i];
                }
            }
        printf("%d␣%d\n",ans+cnt,cst);
    }
    return 0;
}
```

4.24 Minimum-cost flow problem

```
// like Edmonds-Karp Algorithm
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<queue>

#define MAXX 5011
#define MAXE (MAXX*10*2)
#define inf 0x3f3f3f3f

int edge[MAXX],nxt[MAXE],to[MAXE],cap[MAXE],cst[MAXE],cnt;
#define v to[i]
inline void adde(int a,int b,int c,int d)
{
    nxt[++cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    cst[cnt]=d;
}
inline void add(int a,int b,int c,int d)
{ adde(a,b,c,d);adde(b,a,0,-d);}

int dist[MAXX],pre[MAXX];
int source,sink;
std::queue<int>q;
bool in[MAXX];

inline bool go()
{
    static int now,i;
    memset(dist,0x3f,sizeof dist);
    dist[source]=0;
    pre[source]=-1;
    q.push(source);
    in[source]=true;
    while(!q.empty())
    {
        in[now=q.front()]=false;
        q.pop();
        for(i=edge[now];i!=-1;i=nxt[i])
            if(cap[i] && dist[v]>dist[now]+cst[i])
            {
                dist[v]=dist[now]+cst[i];
                pre[v]=i;
                if(!in[v])
                {
                    q.push(v);
                    in[v]=true;
                }
            }
    }
    return dist[sink]!=inf;
}

inline int mcmf(int &flow)
{
    static int ans,i;
    flow=ans=0;
    while(go())
    {
        static int min;
        min=inf;
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
            min=std::min(min,cap[i]);
        flow+=min;
        ans+=min*dist[sink];
        for(i=pre[sink];i!=-1;i=pre[to[i^1]])
        {
            cap[i]-=min;
            cap[i^1]+=min;
        }
    }
    return ans;
}
```

4.25 Second-best MST

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXN 511
#define MAXM 2500111
#define v to[i]

int set[MAXN];
int find(int a)
{
    return set[a]?set[a]=find(set[a]):a;
}

int n,m,i,j,k,ans;

struct edge
{
    int a,b,c;
    bool in;
    bool operator<(const edge &i)const
    {
        return c<i.c;
    }
}ed[MAXM];

int map[MAXN][MAXN];
bool done[MAXN];

int head[MAXN],to[MAXN<<1],nxt[MAXN<<1],wg[MAXN<<1],cnt;
inline void add(int a,int b,int c)
{
    nxt[++cnt]=head[a];
    head[a]=cnt;
    to[cnt]=b;
    wg[cnt]=c;
}

void dfs(const int now,const int fa)
{
    done[now]=true;
    for(int i(head[now]);i;i=nxt[i])
        if(v!=fa)
        {
            for(int j(1);j<=n;++j)
                if(done[j])
                    map[v][j]=map[j][v]=std::max(map[j][now],wg[i]);
            dfs(v,now);
        }
}

int main()
{
    scanf("%d␣%d",&n,&m);
    for(i=0;i<m;++i)
        scanf("%d␣%d␣%d",&ed[i].a,&ed[i].b,&ed[i].c);
    std::sort(ed,ed+m);
    for(i=0;i<m;++i)
        if(find(ed[i].a)!=find(ed[i].b))
        {
            j+=ed[i].c;
            ++k;
            set[find(ed[i].a)]=find(ed[i].b);
            ed[i].in=true;
            add(ed[i].a,ed[i].b,ed[i].c);
            add(ed[i].b,ed[i].a,ed[i].c);
        }
    if(k+1!=n)
        puts("Cost:␣-1\nCost:␣-1");
    else
    {
        printf("Cost:␣%d\n",j);
        if(m==n-1)
        {
            puts("Cost:␣-1");
            return 0;
        }
        ans=0x3f3f3f3f;
        memset(map,0x3f,sizeof map);
        for(i=1;i<=n;++i)
            map[i][i]=0;
        dfs(1,0);
        for(i=0;i<m;++i)
            if(!ed[i].in)
                ans=std::min(ans,j+ed[i].c-map[ed[i].a][ed[i].b]);
        printf("Cost:␣%d\n",ans);
    }
    return 0;
}
```

4.26 Spanning tree

```
Minimum Bottleneck Spanning Tree:
Kruscal

All-pairs vertexes' Minimum Bottleneck Path:
DP in the Kruscal's MST
O(n^2)*O(1)

Minimum Diameter Spanning Tree:
Kariv-Hakimi Algorithm

Directed MST:-
ChuLiu/Edmonds' Algorithm

Second-best MST:
get All-pairs vertexes' Minimum Bottleneck Path, then enumerate all no-tree-edges to replace the longest edge between two vertexes to get a worse MST

Degree-constrained MST:
remove the vertex from the whole graph,then add edges to increase degrees and connect different connected components together ( O(mlogm + n) with kruscal )
if we can't connect all connected components together, there exists no any spanning tree
next step is add edges to root vertex greedily, increase degrees, and decrease our answer ( O(k*n) )
need all vertexes' minimum bottleneck path to root vertex

Minimum Ratio Spanning Tree:
Binary search

Manhattan MST:
combining line sweep with divide-and-conquer algorithm

Minimum Steiner Tree:
the MST contain all k vertexes
bit-mask with dijkstra O( (1<<k)*( {dijkstra} ) )
then run a bit-mask DP( O( n*(1<<k) ) )

Count Spanning Trees:
Kirchhoff's theorem
```

```
simply calculate the minor of (degree Matrix - edge Matrix)

k-best MST:
do like second-best MST for k times
```

4.27 Stable Marriage

```
//对于每个预备队列中的对象，及被匹配对象，先按照喜好程度排列匹配对象

while(!g.empty())  // 预备匹配队列
{
    if(dfn[edge[g.front()].front()]==-1)
        dfn[edge[g.front()].front()]=g.front(); // 如果目前还没尝试匹配过的对象没有被任何别的对象占据
    else
    {
        for(it=edge[edge[g.front()].front()].begin();it!=edge[edge[g.front()].front()].end();++it)
            if(*it==dfn[edge[g.front()].front()] || *it==g.front()) //如果被匹配对象更喜欢正在被匹配的人或现在准备匹配的对象
                break;
        if(*it==g.front()) //如果更喜欢新的
        {
            g.push_back(dfn[edge[g.front()].front()]);
            dfn[edge[g.front()].front()]=g.front();
        }
        else
            g.push_back(g.front()); //否则放到队尾，重新等待匹配
    }
    edge[g.front()].pop_front(); //每组匹配最多只考虑一次
    g.pop_front();
}
```

4.28 Stoer-Wagner Algorithm

```
#include<cstdio>
#include<cstring>

const int maxn=510;

int map[maxn][maxn];
int n;

void contract(int x,int y)//合并两个点
{
    int i,j;
    for (i=0; i<n; i++)
        if (i!=x)
        {
            map[x][i]+=map[y][i];
            map[i][x]+=map[i][y];
        }
    for (i=y+1; i<n; i++)
        for (j=0; j<n; j++)
        {
            map[i-1][j]=map[i][j];
            map[j][i-1]=map[j][i];
        }
    n--;
}

int w[maxn],c[maxn];
int sx,tx;

int mincut() //求最大生成树，计算最后一个点的割，并保存最后一条边的两个顶点
{
    static int i,j,k,t;
    memset(c,0,sizeof(c));
    c[0]=1;
    for (i=0; i<n; i++)
        w[i]=map[0][i];
    for (i=1; i+1<n; i++)
    {
        t=k=-1;
        for (j=0; j<n; j++)
            if (c[j]==0&&w[j]>k)
                k=w[t=j];
        c[sx=t]=1;
        for (j=0; j<n; j++)
            w[j]+=map[t][j];
    }
    for (i=0; i<n; i++)
        if (c[i]==0)
            return w[tx=i];
}
int main()
{
    int i,j,k,m;
    while (scanf("%d%d",&n,&m)!=EOF)
    {
        memset(map,0,sizeof(map));
        while (m--)
        {
            scanf("%d%d%d",&i,&j,&k);
            map[i][j]+=k;
            map[j][i]+=k;
        }
        int mint=999999999;
        while (n>1)
        {
            k=mincut();
            if (k<mint) mint=k;
            contract(sx,tx);
        }
        printf("%d\n",mint);
    }
    return 0;
}
```

4.29 Strongly Connected Component

```
//缩点后注意自环
void dfs(const short &now)
{
    dfn[now]=low[now]=cnt++;
    st.push(now);
    for(std::list<short>::const_iterator it(edge[now].begin());it!=edge[now].end();++it)
        if(dfn[*it]==-1)
        {
            dfs(*it);
            low[now]=std::min(low[now],low[*it]);
        }
        else
            if(sc[*it]==-1)
                low[now]=std::min(low[now],dfn[*it]);
    if(dfn[now]==low[now])
    {
        while(sc[now]==-1)
        {
            sc[st.top()]=p;
            st.pop();
        }
        ++p;
    }
}
```

4.30 ZKW's Minimum-cost flow

```
#include<cstdio>
#include<algorithm>
#include<cstring>
#include<vector>
#include<deque>

#define MAXX 111
#define MAXN 211
#define MAXE (MAXN*MAXN*3)
#define inf 0x3f3f3f3f

char buf[MAXX];

int edge[MAXN],nxt[MAXE],to[MAXE],cap[MAXE],cst[MAXE],cnt;

inline void adde(int a,int b,int c,int k)
{
    nxt[cnt]=edge[a];
    edge[a]=cnt;
    to[cnt]=b;
    cap[cnt]=c;
    cst[cnt]=k;
    ++cnt;
}

inline void add(int a,int b,int c,int k)
{
    adde(a,b,c,k);
    adde(b,a,0,-k);
}

int n,mf,cost,pi1;
int source,sink;
bool done[MAXN];

```

```
int aug(int now,int maxcap)
{
    if(now==sink)
    {
        mf+=maxcap;
        cost+=maxcap*pi1;
        return maxcap;
    }
    done[now]=true;
    int l=maxcap;
    for(int i(edge[now]);i!=-1;i=nxt[i])
        if(cap[i] && !cst[i] && !done[to[i]])
        {
            int d(aug(to[i],std::min(l,cap[i])));
            cap[i]-=d;
            cap[i^1]+=d;
            l-=d;
            if(!l)
                return maxcap;
        }
    return maxcap-l;
}

inline bool label()
{
    static int d,i,j;
    d=inf;
    for(i=1;i<=n;++i)
        if(done[i])
            for(j=edge[i];j!=-1;j=nxt[j])
                if(cap[j] && !done[to[j]] && cst[j]<d)
                    d=cst[j];
    if(d==inf)
        return false;
    for(i=1;i<=n;++i)
        if(done[i])
            for(j=edge[i];j!=-1;j=nxt[j])
            {
                cst[j]-=d;
                cst[j^1]+=d;
            }
    pi1+=d;
    return true;
    /* primal-dual approach
    static int d[MAXN],i,j;
    static std::deque<int>q;
    memset(d,0x3f,sizeof d);
    d[sink]=0;
    q.push_back(sink);
    while(!q.empty())
    {
        static int dt,now;
        now=q.front();
        q.pop_front();
        for(i=edge[now];i!=-1;i=nxt[i])
            if(cap[i^1] && (dt=d[now]-cst[i])<d[to[i]])
                if((d[to[i]]=dt)<=d[q.empty()?0:q.front()])
                    q.push_front(to[i]);
                else
                    q.push_back(to[i]);
    }
    for(i=1;i<=n;++i)
        for(j=edge[i];j!=-1;j=nxt[j])
            cst[j]+=d[to[j]]-d[i];
    pi1+=d[source];
    return d[source]!=inf;
    */
}

int m,i,j,k;
typedef std::pair<int,int> pii;
std::vector<pii>M(MAXN),H(MAXN);

int main()
{
    while(scanf("%d␣%d",&n,&m),(n||m))
    {
        M.resize(0);
        H.resize(0);
        for(i=0;i<n;++i)
        {
            scanf("%s",buf);
            for(j=0;j<m;++j)
                if(buf[j]=='m')
                    M.push_back(pii(i,j));
                else
                    if(buf[j]=='H')
                        H.push_back(pii(i,j));
        }
        n=M.size()+H.size();
        source=++n;
        sink=++n;
        memset(edge,-1,sizeof edge);
        cnt=0;
        for(i=0;i<M.size();++i)
            for(j=0;j<H.size();++j)
                add(i+1,j+1+M.size(),1,abs(M[i].first-H[j].first)+abs(M[i].second-H[j].second));
        for(i=0;i<M.size();++i)
            add(source,i+1,1,0);
        for(i=0;i<H.size();++i)
            add(i+1+M.size(),sink,1,0);
        mf=cost=pi1=0;
        do
            do
                memset(done,0,sizeof done);
            while(aug(source,inf));
        while(label());
        /* primal-dual approach
        while(label())
            do
                memset(done,0,sizeof done);
            while(aug(source,inf));
        */
        printf("%d\n",cost);
    }
    return 0;
}
```

5 Math

5.1 cantor

```
const int PermSize = 12;
int fac[PermSize] = {1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800};

inline int Cantor(int a[])
{
    int i, j, cnt;
    int res = 0;
    for (i = 0; i < PermSize; ++i)
    {
        cnt = 0;
        for (j = i + 1; j < PermSize; ++j)
            if (a[i] > a[j])
                ++cnt;
        res = res + cnt * fac[PermSize - i - 1];
    }
    return res;
}

bool h[13];

inline void UnCantor(int x, int res[])
{
    int i,j,l,t;
    for (i = 1;i <= 12;i++)
        h[i] = false;
    for (i = 1; i <= 12; i++)
    {
        t = x / fac[12 - i];
        x -= t * fac[12 - i];
        for (j = 1, l = 0; l <= t; j++)
            if (!h[j])
                l++;
        j--;
        h[j] = true;
        res[i - 1] = j;
    }
}
```

5.2 discrete logarithms - BSGS

```
//The running time of BSGS and the space complexity is O($\sqrt{n}$)
//Pollard's rho algorithm for logarithms' running time is approximately O($\sqrt{p}$) where p is n's largest prime factor.
#include<cstdio>
#include<cmath>
#include<cstring>

struct Hash // std::map is bad. clear() 时会付出巨大的代价
{
    static const int mod=100003; // prime is good
    static const int MAXX=47111; // bigger than $\sqrt{c}$
    int hd[mod],nxt[MAXX],cnt;
    long long v[MAXX],k[MAXX]; // $a^k \equiv v \pmod{c}$
    inline void init()
    {
        memset(hd,0,sizeof hd);
        cnt=0;
    }
    inline long long find(long long v)
    {
        static int now;
        for(now=hd[v%mod];now;now=nxt[now])
            if(this->v[now]==v)
                return k[now];
        return -1ll;
    }
    inline void insert(long long k,long long v)
    {
        if(find(v)!=-1ll)
            return;
        nxt[++cnt]=hd[v%mod];
        hd[v%mod]=cnt;
        this->v[cnt]=v;
        this->k[cnt]=k;
    }
}hash;

long long gcd(long long a,long long b)
{
    return b?gcd(b,a%b):a;
}

long long exgcd(long long a,long long b,long long &x,long long &y)
{
    if(b)
    {
        long long re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1ll;
    y=0ll;
    return a;
}

inline long long bsgs(long long a,long long b,long long c) // a^x ≡ b (mod c)
{
    static long long x,y,d,g,m,am,k;
    static int i,cnt;
    a%=c;
    b%=c;
    x=1ll%c; // if c==1....
    for(i=0;i<100;++i)
    {
        if(x==b)
            return i;
        x=(x*a)%c;
    }
    d=1ll%c;
    cnt=0;
    while((g=gcd(a,c))!=1ll)
    {
        if(b%g)
            return -1ll;
        ++cnt;
        c/=g;
        b/=g;
        d=a/g*d%c;
    }
    hash.init();
    m=sqrt((double)c); // maybe need a ceil
    am=1ll%c;
    hash.insert(0,am);
    for(i=1;i<=m;++i)
    {
        am=am*a%c;
        hash.insert(i,am);
    }
    for(i=0;i<=m;++i)
    {
        g=exgcd(d,c,x,y);
        x=(x*b/g%c+c)%c;
        k=hash.find(x);
        if(k!=-1ll)
            return i*m+k+cnt;
        d=d*am%c;
    }
    return -1ll;
}

long long k,p,n;

int main()
{
    while(scanf("%lld␣%lld␣%lld",&k,&p,&n)!=EOF)
    {
        if(n>p || (k=bsgs(k,n,p))==-1ll)
            puts("Orz,I␣’ cant␣find␣D!");
        else
            printf("%lld\n",k);
    }
    return 0;
}
```

5.3 extended euclidean algorithm

```
//返回ax+by=gcd(a,b)的一组解
long long ex_gcd(long long a,long long b,long long &x,long long &y)
{
    if (b)
    {
        long long ret = ex_gcd(b,a%b,x,y),tmp = x;
        x = y;
        y = tmp-(a/b)*y;
        return ret;
    }
    else
    {
        x = 1;
        y = 0;
        return a;
    }
}
```

5.4 Fast Fourier Transform

```
#include<cstdio>
#include<cstring>
#include<complex>
#include<vector>
#include<algorithm>

#define MAXX 100111
#define MAXN (MAXX<<2)

int T;
int n,i,j,k;

typedef std::complex<long double> com;
std::vector<com>x(MAXN);
int a[MAXX];
long long pre[MAXN],cnt[MAXN];
long long ans;

inline void fft(std::vector<com> &y,int sign)
{
    static int i,j,k,h;
    static com u,t,w,wn;
    for(i=1,j=y.size()/2;i+1<y.size();++i)
    {
        if(i<j)
            std::swap(y[i],y[j]);
        k=y.size()/2;
        while(j>=k)
        {
            j-=k;
            k/=2;
        }
        if(j<k)
            j+=k;
    }
    for(h=2;h<=y.size();h<<=1)
    {
        wn=com(cos(-sign*2*M_PI/h),sin(-sign*2*M_PI/h));
        for(j=0;j<y.size();j+=h)
        {
            w=com(1,0);
            for(k=j;k<j+h/2;++k)
            {
                u=y[k];
                t=w*y[k+h/2];
                y[k]=u+t;
                y[k+h/2]=u-t;
                w*=wn;
            }
        }
    }
    if(sign==-1)
        for(i=0;i<y.size();++i)
            y[i]=com(y[i].real()/y.size(),y[i].imag());
}
```

```

int main()
{
    scanf("%d",&T);
    while(T--)
    {
        memset(cnt,0,sizeof cnt);
        scanf("%d",&n);
        for(i=0;i<n;++i)
        {
            scanf("%d",a+i);
            ++cnt[a[i]];
        }
        std::sort(a,a+n);
        k=a[n-1]+1;
        for(j=1;j<(k<<1);j<<=1);// size must be such many
        x.resize(0);
        for(i=0;i<k;++i)
            x.push_back(com(cnt[i],0));
        x.insert(x.end(),j-k,com(0,0));

        fft(x,1);
        for(i=0;i<x.size();++i)
            x[i]=x[i]*x[i];
        fft(x,-1);
        /*
        if we need to combine 2 arrays
        fft(x,1);
        fft(y,1);
        for(i=0;i<x.size();++i)
            x[i]=x[i]*y[i];
        fft(x,-1);
        */

        for(i=0;i<x.size();++i)
            cnt[i]=ceil(x[i].real()); //  maybe we need (x[i].real()+0.5f) or nearbyint(x[i].real())
        x.resize(2*a[n-1]); // result here
    }
    return 0;
}
```

5.5 Gaussian elimination

```
#define N

inline int ge(int a[N][N],int n) // 返回系数矩阵的秩
{
    static int i,j,k,l;
    for(j=i=0;j<n;++j) //第 i 行, 第 j 列
    {
        for(k=i;k<n;++k)
            if(a[k][j])
                break;
        if(k==n)
            continue;
        for(l=0;l<=n;++l)
            std::swap(a[i][l],a[k][l]);
        for(l=0;l<=n;++l)
            if(l!=i && a[l][j])
                for(k=0;k<=n;++k)
                    a[l][k]^=a[i][k];
        ++i;
    }
    for(j=i;j<n;++j)
        if(a[j][n])
            return -1; //无解
    return i;
}
/*
 */

void dfs(int v)
{
    if(v==n)
    {
        static int x[MAXX],ta[MAXX][MAXX];
        static int tmp;
        memcpy(x,ans,sizeof(x));
        memcpy(ta,a,sizeof(ta));
        for(i=l-1;i>=0;--i)
        {
            for(j=i+1;j<n;++j)
                ta[i][n]^=(x[j]&&ta[i][j]); //迭代消元求解
            x[i]=ta[i][n];
        }
        for(tmp=i=0;i<n;++i)
            if(x[i])
                ++tmp;
        cnt=std::min(cnt,tmp);
        return;
    }
    ans[v]=0;
    dfs(v+1);
    ans[v]=1;
    dfs(v+1);
}

inline int ge(int a[N][N],int n)
{
    static int i,j,k,l;
    for(i=j=0;j<n;++j)
    {
        for(k=i;k<n;++k)
            if(a[k][i])
                break;
        if(k<n)
        {
            for(l=0;l<=n;++l)
                std::swap(a[i][l],a[k][l]);
            for(k=0;k<n;++k)
                if(k!=i && a[k][i])
                    for(l=0;l<=n;++l)
                        a[k][l]^=a[i][l];
            ++i;
        }
        else //将不定元交换到后面去
        {
            l=n-1-j+i;
            for(k=0;k<n;++k)
                std::swap(a[k][l],a[k][i]);
        }
    }
    if(i==n)
    {
        for(i=cnt=0;i<n;++i)
            if(a[i][n])
                ++cnt;
        printf("%d\n",cnt);
        continue;
    }
    for(j=i;j<n;++j)
        if(a[j][n])
            break;
    if(j<n)
        puts("impossible");
    else
    {
        memset(ans,0,sizeof(ans));
        cnt=111;
        dfs(l=i);
        printf("%d\n",cnt);
    }
}

/*
 */
inline int ge(int n,int m)
{
    static int i,j,r,c;
    static double mv;
    for(r=c=0;r<n && c<m;++r,++c)
    {
        for(mv=0,i=r;i<n;++i)
            if(fabs(mv)<fabs(a[i][c]))
                mv=a[j=i][c];
        if(fabs(mv)<eps) // important
        {
            --r;
            continue;
        }
        for(i=0;i<=m;++i)
            std::swap(a[r][i],a[j][i]);
        for(j=c+1;j<=m;++j)
        {
            a[r][j]/=mv;
            for(i=r+1;i<n;++i)
                a[i][j]-=a[i][c]*a[r][j];
        }
    }
    for(i=r;i<n;++i)
        if(fabs(a[i][m])>eps)
            return -1;
    if(r<m) // rank
        return m-r;
    for(i=m-1;i>=0;--i)
        for(j=i+1;j<m;++j)
            a[i][m]-=a[i][j]*a[j][m];  // answer will be a[i][m]
    return 0;
}
```

5.6 Integration

```
// simpson 公式用到的函数
double F(double x) {
  return sqrt(1 + 4*a*a*x*x);
}

// 三点 simpson 法。这里要求 F 是一个全局函数
double simpson(double a, double b) {
  double c = a + (b-a)/2;
  return (F(a)+4*F(c)+F(b))*(b-a)/6;
}

// 自适应 Simpson 公式（递归过程）。已知整个区间 [a,b] 上的三点 simpson 值 A
double asr(double a, double b, double eps, double A) {
  double c = a + (b-a)/2;
  double L = simpson(a, c), R = simpson(c, b);
  if(fabs(L+R-A) <= 15*eps)
      return L+R+(L+R-A)/15.0;
  return asr(a, c, eps/2, L) + asr(c, b, eps/2, R);
}

// 自适应 Simpson 公式（主过程）
double asr(double a, double b, double eps)
{
  return asr(a, b, eps, simpson(a, b));
}

// 用自适应 Simpson 公式计算宽度为 w，高度为 h 的抛物线长
double parabola_arc_length(double w, double h)
{
  a = 4.0*h/(w*w); // 修改全局变量 a，从而改变全局函数 F 的行为
  return asr(0, w/2, 1e-5)*2;
}

// thx for mzry
inline double f(double)
{
    /*
    define the function
    */
}

inline double simp(double l,double r)
{
    double h = (r-l)/2.0;
    return h*(f(l)+4*f((l+r)/2.0)+f(r))/3.0;
}

inline double rsimp(double l,double r) // call here
{
    double mid = (l+r)/2.0;
    if(fabs((simp(l,r)-simp(l,mid)-simp(mid,r)))/15 < eps)
        return simp(l,r);
    else
        return rsimp(l,mid)+rsimp(mid,r);
}

//Romberg

/* Romberg 求定积分
 * 输入：积分区间 [a,b]，被积函数 f(x,y,z)
 * 输出：积分结果
 * f(x,y,z) 示例：
 * double f0( double x, double l, double t )
 * {
 * return sqrt(1.0+l*l*t*t*cos(t*x)*cos(t*x));
 * }
 */
double Integral(double a, double b, double (*f)(double x, double y, double z), double eps, double l, double t);

inline double Romberg (double a, double b, double (*f)(double x, double y, double z), double eps, double l, double t)
{
#define MAX_N 1000
    int i, j, temp2, min;
    double h, R[2][MAX_N], temp4;
    for (i=0; i<MAX_N; i++)
    {
        R[0][i] = 0.0;
        R[1][i] = 0.0;
    }
    h = b-a;
    min = (int)(log(h*10.0)/log(2.0)); //h should be at most 0.1
    R[0][0] = ((*f)(a, l, t)+(*f)(b, l, t))*h*0.50;
    i = 1;
    temp2 = 1;
    while (i<MAX_N)
    {
        i++;
        R[1][0] = 0.0;
        for (j=1; j<=temp2; j++)
            R[1][0] += (*f)(a+h*((double)j-0.50), l, t);
        R[1][0] = (R[0][0] + h*R[1][0])*0.50;
        temp4 = 4.0;
        for (j=1; j<i; j++)
        {
            R[1][j] = R[1][j-1] + (R[1][j-1]-R[0][j-1])/(temp4-1.0);
            temp4 *= 4.0;
        }
        if ((fabs(R[1][i-1]-R[0][i-2])<eps) && (i>min))
            return R[1][i-1];
        h *= 0.50;
        temp2 *= 2;
        for (j=0; j<i; j++)
            R[0][j] = R[1][j];
    }
    return R[1][MAX_N-1];
}

inline double Integral(double a, double b, double (*f)(double x, double y, double z), double eps, double l, double t)
{
    const double pi(acos(-1.0f));
    int n;
    double R, p, res;
    n = (int)(floor)(b * t * 0.50 / pi);
    p = 2.0 * pi / t;
    res = b - (double)n * p;
    if (n)
        R = Romberg (a, p, f0, eps/(double)n, l, t);
    R = R * (double)n + Romberg( 0.0, res, f0, eps, l, t);
    return R/100.0;
}

//
inline double romberg(double a,double b)
{
#define MAXN 111
    double t[MAXN][MAXN];
    int n,k,i,m;
    double h,g,p;
    h=(double)(b-a)/2;
    t[0][0]=h*(func(a)+func(b));
    k=n=1;
    do
    {
        g=0;
        for(i=1;i<=n;i++)
            g+=func((a+((2*i-1)*h)));
        t[k][0]=(t[k-1][0]/2)+(h*g);
        p = 1.0;
        for(m=1;m<=k;m++)
        {
            p=p*4.0f;
            t[k-m][m]=(p*t[k-m+1][m-1]-t[k-m][m-1])/(p-1);
        }
        m-=1;
        h/=2;
        n*=2;
        k+=1;

    }
    while (fabs(t[0][m]-t[0][m-1])>eps);
    return t[0][m];
}
```

5.7 inverse element

```
inline void getInv2(int x,int mod)
{
    inv[1]=1;
    for (int i=2; i<=x; i++)
        inv[i]=(mod-(mod/i)*inv[mod%i]%mod)%mod;
}

long long inv(long long x)// likes above one
{
    return x <= 1ll ? x : (mod - mod / x) * inv(mod % x) % mod;
}
```

```

inline long long power(long long x,long long y,int mod)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

inline int getInv(int x,int mod)//mod 为素数
{
    return power(x,mod-2,mod);
}

//谨慎来说，用 exgcd 更靠谱
void gcd(int n,int k,int &x,int &y)
{
    if(k)
    {
        gcd(k,n%k,x,y);
        int t=x;
        x=y;
        y=t-(n/k)*y;
        return;
    }
    x=1;
    y=0;
}

inline int inv(int b,int mod)
{
    static int x,y;
    gcd(b,mod,x,y);
    if(x<0)
        x+=mod;
    return x;
}
```

5.8 Linear programming

```
#include<cstdio>
#include<cstring>
#include<cmath>
#include<algorithm>

#define MAXN 33
#define MAXM 33
#define eps 1e-8

double a[MAXN][MAXM],b[MAXN],c[MAXM];
double x[MAXM],d[MAXN][MAXM];
int ix[MAXN+MAXM];
double ans;
int n,m;
int i,j,k,r,s;
double D;

inline bool simplex()
{
    r=n;
    s=m++;
    for(i=0;i<n+m;++i)
        ix[i]=i;
    memset(d,0,sizeof d);
    for(i=0;i<n;++i)
    {
        for(j=0;j+1<m;++j)
            d[i][j]=-a[i][j];
        d[i][m-1]=1;
        d[i][m]=b[i];
        if(d[r][m]>d[i][m])
            r=i;
    }
    for(j=0;j+1<m;++j)
        d[n][j]=c[j];
    d[n+1][m-1]=-1;
    while(true)
    {
        if(r<n)
        {
            std::swap(ix[s],ix[r+m]);
            d[r][s]=1./d[r][s];
            for(j=0;j<=m;++j)
                if(j!=s)
                    d[r][j]*=-d[r][s];
            for(i=0;i<=n+1;++i)
                if(i!=r)
                {
                    for(j=0;j<=m;++j)
                        if(j!=s)
                            d[i][j]+=d[r][j]*d[i][s];
                    d[i][s]*=d[r][s];
                }
        }
        r=-1;
        s=-1;
        for(j=0;j<m;++j)
            if((s<0 || ix[s]>ix[j]) && (d[n+1][j]>eps || (d[n+1][j]>-eps && d[n][j]>eps)))
                s=j;
        if(s<0)
            break;
        for(i=0;i<n;++i)
            if(d[i][s]<-eps && (r<0 || (D=(d[r][m]/d[r][s]-d[i][m]/d[i][s]))<-eps || (D<eps && ix[r+m]>ix[i+m])))
                r=i;
        if(r<0)
            return false;
    }
    if(d[n+1][m]<-eps)
        return false;
    for(i=m;i<n+m;++i)
        if(ix[i]+1<m)
            x[ix[i]]=d[i-m][m]; // answer
    ans=d[n][m]; // maxium value
    return true;
}

int main()
{
    while(scanf("%d␣%d",&m,&n)!=EOF)
    {
        for(i=0;i<m;++i)
            scanf("%lf",c+i);  // max{ sum{c[i]*x[i]} }
        for(i=0;i<n;++i)
        {
            for(j=0;j<m;++j)
                scanf("%lf",a[i]+j); // sum{ a[i]*x[i] } <= b
            scanf("%lf",b+i);
            b[i]*=n;
        }
        simplex();
        printf("Nasa␣can␣spend␣%.0lf␣taka.\n",ceil(ans));
    }
    return 0;
}

/*
Simplex C(n+m)(n)
maximize:
    $\sum_{i=1}^{n} (c[i] \times x[i])$
subject to
    $\forall i \in [1,m]$
    $\sum_{j=1}^{n} (a[i][j] \times x[j]) \leq rhs[i]$
限制:
    传入的矩阵必须是标准形式的.
sample:
3 3
15 17 20
0 1 -1 2
3 3 5 15
3 2 1 8
out:
OPTIMAL
76.00000
x[ 1 ] = 0.333333
x[ 2 ] = 3.000000
x[ 3 ] = 1.000000
*/

#include <cstdio>
#include <cstring>
#include <cmath>

#define eps 1e-8
#define inf 1e15
#define OPTIMAL -1 //最优解
#define UNBOUNDED -2 //无边界的
#define FEASIBLE -3 //可行的
#define INFEASIBLE -4 //无解
#define PIVOT_OK 1 //还可以松弛

#define N 45 //变量个数
#define M 45 //约束个数

int basic[N],row[M],col[N];
```

```
double c0[N];

inline double dcmp(double x)
{
    if(x>eps)
        return 1;
    if(x<-eps)
        return -1;
    return 0;
}

inline int Pivot(int n,int m,double *c,double a[M][N], double *rhs,int &i,int &j)
{
    double min=inf;
    int k=-1;
    for(j=0;j<=n;j++)
        if(!basic[j] && dcmp(c[j])>0)
            if(k<0 || dcmp(c[j]-c[k])>0)
                k=j;
    j=k;
    if(k<0)
        return OPTIMAL;
    for(k=-1,i=1;i<=m;i++)
        if(dcmp(a[i][j])>0 && dcmp(rhs[i]/a[i][j]-min) <0)
        {
            min=rhs[i]/a[i][j];
            k=i;
        }
    i=k;
    if(k<0)
        return UNBOUNDED;
    return PIVOT_OK;
}

inline int PhaseII(int n,int m,double *c,double a[M][N], double *rhs,double &ans,int PivotIndex)
{
    static int i,j,k,l;
    static double tmp;
    while((k=Pivot(n,m,c,a,rhs,i,j))==PIVOT_OK || PivotIndex)
    {
        if(PivotIndex)
        {
            i=PivotIndex;
            j=PivotIndex=0;
        }
        basic[row[i]]=0;
        col[row[i]]=0;
        basic[j]=1;
        col[j]=i;
        row[i]=j;
        tmp=a[i][j];
        for(k=0;k<=n;k++)
            a[i][k]/=tmp;
        rhs[i]/=tmp;
        for(k=1;k<=m;k++)
            if(k!=i && dcmp(a[k][j]))
            {
                tmp=-a[k][j];
                for(l=0;l<=n;l++)
                    a[k][l]+=tmp*a[i][l];
                rhs[k]+=tmp*rhs[i];
            }
        tmp=-c[j];
        for(l=0;l<=n;l++)
            c[l]+=a[i][l]*tmp;
        ans-=tmp*rhs[i];
    }
    return k;
}

inline int PhaseI(int n,int m,double *c,double a[M][N], double *rhs,double &ans)
{
    int i,j,k=-1;
    double tmp,min=0,ans0=0;
    for(i=1;i<=m;i++)
        if(dcmp(rhs[i]-min)<0)
        {
            min=rhs[i];
            k=i;
        }
    if(k<0)
        return FEASIBLE;
    for(i=1;i<=m;i++)
        a[i][0]=-1;
    for(j=1;j<=n;j++)
        c0[j]=0;
    c0[0]=-1;
    PhaseII(n,m,c0,a,rhs,ans0,k);
    if(dcmp(ans0)<0)
        return INFEASIBLE;
    for(i=1;i<=m;i++)
        a[i][0]=0;
    for(j=1;j<=n;j++)
        if(dcmp(c[j]) && basic[j])
        {
            tmp=c[j];
            ans+=rhs[col[j]]*tmp;
            for(i=0;i<=n;i++)
                c[i]-=tmp*a[col[j]][i];
        }
    return FEASIBLE;
}
inline int simplex(int n,int m,double *c,double a[M][N], double *rhs,double &ans,double *x)
{
    int i,j,k;
    for(i=1;i<=m;i++)
    {
        for(j=n+1;j<=n+m;j++)
            a[i][j]=0;
        a[i][n+i]=1;
        a[i][0]=0;
        row[i]=n+i;
        col[n+i]=i;
    }
    k=PhaseI(n+m,m,c,a,rhs,ans);
    if(k==INFEASIBLE)
        return k; //无解
    k=PhaseII(n+m,m,c,a,rhs,ans,0);
    for(j=0;j<=n+m;j++)
        x[j] = 0;
    for(i=1;i<=m;i++)
        x[row[i]] = rhs[i];
    return k;
}

double c[M],ans,a[M][N],rhs[M],x[N];

int main()
{
    int i,j,n,m;
    while(scanf("%d%d",&n,&m)!=EOF)
    {
        for(int i=0;i<=n+m;i++)
        {
            for(int j=0;j<=n+m;j++)
                a[i][j]=0;
            basic[i]=0;
            row[i]=0;
            col[i]=0;
            c[i]=0;
            rhs[i]=0;
        }
        ans=0;

        for(j=1;j<=n;++j)
            scanf("%lf",c+j);
        for(i=1;i<=m;++i)
        {
            for(j=1;j<=n;++j)
                scanf("%lf",a[i]+j);
            scanf("%lf",rhs+i);
        }

        switch(simplex(n,m,c,a,rhs,ans,x))
        {
            case OPTIMAL:
                printf("Nasa␣can␣spend␣%.0f␣taka.\n", ceil(m*ans));
                //for(j=1;j<=n;j++)
                //    printf("x[ %2d ] = %10lf\n",j,x[j]);
                break;
            case UNBOUNDED:
                puts("UNBOUNDED");
                break;
            case INFEASIBLE:
                puts("INFEASIBLE");
                break;
        }
    }
    return 0;
}
```

5.9 Lucas' theorem(2)

```
#include<cstdio>
#include<cstring>
#include<iostream>
```

```

int mod;
long long num[100000];
int ni[100],mi[100];
int len;

void init(int p)
{
    mod=p;
    num[0]=1;
    for (int i=1; i<p; i++)
        num[i]=i*num[i-1]%p;
}

void get(int n,int ni[],int p)
{
    for (int i = 0; i < 100; i++)
        ni[i] = 0;
    int tlen = 0;
    while (n != 0)
    {
        ni[tlen++] = n%p;
        n /= p;
    }
    len = tlen;
}

long long power(long long x,long long y)
{
    long long ret=1;
    for (long long a=x%mod; y; y>>=1,a=a*a%mod)
        if (y&1)
            ret=ret*a%mod;
    return ret;
}

long long getInv(long long x)//mod 为素数
{
    return power(x,mod-2);
}

long long calc(int n,int m,int p)//C(n,m)%p
{
    init(p);
    long long ans=1;
    for (; n && m && ans; n/=p,m/=p)
    {
        if (n%p>=m%p)
            ans = ans*num[n%p]%p *getInv(num[m%p]%p)%p *
                getInv(num[n%p-m%p])%p;
        else
            ans=0;
    }
    return ans;
}

int main()
{
    int t;
    scanf("%d",&t);
    while (t--)
    {
        int n,m,p;
        scanf("%d%d%d",&n,&m,&p);
        printf("%lld\n",calc(n+m,m,p));
    }
    return 0;
}
```

5.10 Lucas' theorem

```
#include <cstdio>
/*
   Lucas 快速求解C(n,m)%p
   */
void gcd(int n,int k,int &x,int &y)
{
    if(k)
    {
        gcd(k,n%k,x,y);
        int t=x;
        x=y;
        y=t-(n/k)*y;
        return;
    }
    x=1;
    y=0;
}

int CmodP(int n,int k,int p)
{
    if(k>n)
        return 0;
    int a,b,flag=0,x,y;
    a=b=1;
    for(int i=1;i<=k;i++)
    {
        x=n-i+1;
        y=i;
        while(x%p==0)
        {
            x/=p;
            ++flag;
        }
        while(y%p==0)
        {
            y/=p;
            --flag;
        }
        x%=p;
        y%=p;

        a*=x;
        b*=y;

        b%=p;
        a%=p;
    }
    if(flag)
        return 0;
    gcd(b,p,x,y);
    if(x<0)
        x+=p;
    a*=x;
    a%=p;
    return a;
}

//用Lucas 定理求解 C(n,m) % p ,p 是素数
long long Lucas(long long n, long long m, long long p)
{
    long long ans=1;
    while(m && n && ans)
    {
        ans*=(CmodP(n%p,m%p,p));
        ans=ans%p;
        n=n/p;
        m=m/p;
    }
    return ans;
}
int main()
{
    long long n,k,p,ans;
    int cas=0;
    while(scanf("%I64d%I64d%I64d",&n,&k,&p)!=EOF)
    {
        if(k>n-k)
            k=n-k;
        ans=Lucas(n+1,k,p)+n-k;
        printf("Case␣#%d:␣%I64d\n",++cas,ans%p);
    }
    return 0;
}
```

5.11 matrix

```
template<int n>class Matrix
{
    long long a[n][n];
    inline Matrix<n> operator*(const Matrix<n> &b)const
          //比照着公式来会快一点常数……nmlgb 的 zoj3289
          ……
    {
        //别忘了矩阵乘法虽然满足结合律但是不满足交换律……
        static Matrix<n> re;
        static int i,j,k;
        for(i=0;i<n;++i)
            for(j=0;j<n;++j)
                re.a[i][j]=0;
        for(k=0;k<n;++k)
            for(i=0;i<n;++i)
                if(a[i][k])
                    for(j=0;j<n;++j)
                        if(b.a[k][j])
                            re.a[i][j]=(re.a[i][j]+a[i][
                                k]*b.a[k][j])%mod;
        return re;
    }
    inline Matrix<n> operator^(int y)const
    {
        static Matrix<n> re,x;
        static int i,j;
        for(i=0;i<n;++i)
        {
            for(j=0;j<n;++j)
            {
                re.a[i][j]=0;
                x.a[i][j]=a[i][j];
            }
            re.a[i][i]=1;
        }
        for(;y;y>>=1,x=x*x)
            if(y&1)
                re=re*x;
        return re;
    }
    long long det()
    {
        static int i,j,k;
        static long long ret,t;
        ret=1ll;
        for(i=0;i<n;++i)
            for(j=0;j<n;++j)
                a[i][j]%=mod;
        for(i=0;i<n;++i)
        {
            for(j=i+1;j<n;++j)
                while(a[j][i])
                {
                    t=a[i][i]/a[j][i];
                    for(k=i;k<n;++k)
                        a[i][k]=(a[i][k]-a[j][k]*t)%mod;
                    for(k=i;k<n;++k)
                        std::swap(a[i][k],a[j][k]);
                    ret=-ret;
                }
            if(!a[i][i])
                return 0ll;
            ret=ret*a[i][i]%mod;
        }
        return (ret+mod)%mod;
    }
};

/*
Fibonacci Matrix
1   1
1   0

org[0][j], trans[i][j]
 means
transform(org,1 times) -> org[0][j]=$\sum_{i=0}^{n} org[0][i] \times trans[i][j]$
      */
```

5.12 Pell's equation

```
/*
find the (x,y)pair that $x^2 - n \times y^2 = 1$
these is not solution if and only if n is a square number.

solution:
simply brute-force search the integer y, get (x1,y1). ( toooo slow in some situation )
or we can enumerate the continued fraction of $\sqrt{n}$, as $\frac{x}{y}$, it will be much more faster

other solution pairs' matrix:
x1   n×y1
y1    x1
k-th solution is $\{matrix\}^k$
*/

import java.util.*;
import java.math.*;

public class Main
{
    static BigInteger p,q,p1,p2,p3,q1,q2,q3,a1,a2,a0,h1,h2,g1,g2,n0;
    static int n,t;
    static void solve()
    {
        p2=BigInteger.ONE;
        p1=BigInteger.ZERO;
        q2=BigInteger.ZERO;
        q1=BigInteger.ONE;
        a0=a1=BigInteger.valueOf((long)Math.sqrt(n));
        g1=BigInteger.ZERO;
        h1=BigInteger.ONE;
        n0=BigInteger.valueOf(n);
        while(true)
        {
            g2=a1.multiply(h1).subtract(g1);
            h2=(n0.subtract(g2.multiply(g2))).divide(h1);
            a2=(g2.add(a0)).divide(h2);
            p=p2.multiply(a1).add(p1);
            q=q2.multiply(a1).add(q1);
            if(p.multiply(p).subtract(n0.multiply(q.multiply(q))).equals(BigInteger.ONE))
                return ;
            a1=a2;
            g1=g2;
            h1=h2;
            p1=p2;
            p2=p;
            q1=q2;
            q2=q;
        }
    }
    public static void main(String[] args)
    {
        Scanner in=new Scanner(System.in);
        t=in.nextInt();
        for(int i=0;i<t;++i)
        {
            n=in.nextInt();
            solve();
            System.out.println(p+"␣"+q);
        }
    }
}
```

5.13 Pollard's rho algorithm

```
#include<cstdio>
#include<cstdlib>
#include<list>

short T;
unsigned long long a;
std::list<unsigned long long>fac;

inline unsigned long long multi_mod(const unsigned long long &a,unsigned long long b,const unsigned long long &n)
{
    unsigned long long exp(a%n),tmp(0);
    while(b)
    {
        if(b&1)
        {
            tmp+=exp;
            if(tmp>n)
                tmp-=n;
        }
        exp<<=1;
        if(exp>n)
            exp-=n;
        b>>=1;
    }
    return tmp;
}

inline unsigned long long exp_mod(unsigned long long a,unsigned long long b,const unsigned long long &c)
{
    unsigned long long tmp(1);
    while(b)
    {
        if(b&1)
            tmp=multi_mod(tmp,a,c);
        a=multi_mod(a,a,c);
        b>>=1;
    }
    return tmp;
}

inline bool miller_rabbin(const unsigned long long &n,short T)
{
    if(n==2)
        return true;
    if(n<2 || !(n&1))
        return false;
    unsigned long long a,u(n-1),x,y;
    short t(0),i;
    while(!(u&1))
    {
        ++t;
        u>>=1;
```

```cpp
    }
    while(T--)
    {
        a=rand()%(n-1)+1;
        x=exp_mod(a,u,n);
        for(i=0;i<t;++i)
        {
            y=multi_mod(x,x,n);
            if(y==1 && x!=1 && x!=n-1)
                return false;
            x=y;
        }
        if(y!=1)
            return false;
    }
    return true;
}

unsigned long long gcd(const unsigned long long &a,const unsigned long long &b)
{
    return b?gcd(b,a%b):a;
}

inline unsigned long long pollar_rho(const unsigned long long n,const unsigned long long &c)
{
    unsigned long long x(rand()%(n-1)+1),y,d,i(1),k(2);
    y=x;
    while(true)
    {
        ++i;
        x=(multi_mod(x,x,n)+c)%n;
        d=gcd((x-y+n)%n,n);
        if(d>1 && d<n)
            return d;
        if(x==y)
            return n;
        if(i==k)
        {
            k<<=1;
            y=x;
        }
    }
}

void find(const unsigned long long &n,short c)
{
    if(n==1)
        return;
    if(miller_rabbin(n,6))
    {
        fac.push_back(n);
        return;
    }
    unsigned long long p(n);
    short k(c);
    while(p>=n)
        p=pollar_rho(p,c--);
    find(p,k);
    find(n/p,k);
}

int main()
{
    scanf("%hd",&T);
    while(T--)
    {
        scanf("%llu",&a);
        fac.clear();
        find(a,120);
        if(fac.size()==1)
            puts("Prime");
        else
        {
            fac.sort();
            printf("%llu\n",fac.front());
        }
    }
    return 0;
}
```

5.14 System of linear congruences

```cpp
// minimal val that for all (m,a) , val%m == a
#include<cstdio>

#define MAXX 11

int T,t;
int m[MAXX],a[MAXX];
int n,i,j,k;
int x,y,c,d;
int lcm;

int exgcd(int a,int b,int &x,int &y)
{
    if(b)
    {
        int re(exgcd(b,a%b,x,y)),tmp(x);
        x=y;
        y=tmp-(a/b)*y;
        return re;
    }
    x=1;
    y=0;
    return a;
}

int main()
{
    scanf("%d",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d",&n);
        lcm=1;
        for(i=0;i<n;++i)
        {
            scanf("%d",m+i);
            lcm*=m[i]/exgcd(lcm,m[i],x,y);
        }
        for(i=0;i<n;++i)
            scanf("%d",a+i);
        for(i=1;i<n;++i)
        {
            c=a[i]-a[0];
            d=exgcd(m[0],m[i],x,y);
            if(c%d)
                break;
            y=m[i]/d;
            c/=d;
            x=(x*c%y+y)%y;
            a[0]+=m[0]*x;
            m[0]*=y;
        }
        //标程用的步长可能是最终的 m[0] 而不是 lcm。枚举一下标程
        printf("Case␣%d:␣%d\n",t,i<n?-1:(a[0]?a[0]:lcm));
    }
    return 0;
}
```

5.15 Combinatorics

5.15.1 Subfactorial

!n =number of permutations of n elements with no fixed points

from !0:
1, 0, 1, 2, 9, 44, 265, 1854, 14833, 133496, 1334961, 14684570
$!n = (n-1)(!(n-1)+!(n-2))$
PS:$n! = (n-1)((n-1)!+(n-2)!)$
$!n = n \times n! + (-1)^n$

Rencontres numbers:
$D_{n,k}$ is the number of permutations of { 1, ..., n } that have exactly k fixed points.
$D_{n,0}$ =!n
$D_{n,k} = \binom{n}{k} \times !(n-k)$

5.15.2 Ménage numbers

Ménage numbers:
number of permutations s of [0, ..., n-1] such that.
$\forall i, s(i) \neq i$ and $s(i) \not\equiv i+1 \pmod{n}$.

from A(0):
1, 0, 0, 1, 2, 13, 80, 579, 4738, 43387, 439792, 4890741

$A_n = \sum\limits_{k=0}^{n} (-1)^k \frac{2n}{2n-k} \binom{2n-k}{k} (n-k)!$
$A_n = nA_{n-1} + \frac{n}{n-2}A_{n-2} + \frac{4(-1)^{n-1}}{n-2}$
$A_n = nA_{n-1} + 2A_{n-2} - (n-4)A_{n-3} - A_{n-4}$

5.15.3 Multiset

Permutation:
MultiSet S={1 m,4 s,4 i,2 p}
$P(S) = \frac{(1+4+4+2)!}{1!4!4!2!}$

Combination:
MultiSet S=$\{\infty a1, \infty a2, ... \infty ak\}$
$\binom{S}{r} = \frac{(r+k-1)!}{r!(k-1)!} = \binom{r+k-1}{r}$

if(r>min{count(element[i])})
you have to resolve this problem with inclusion-exclusion principle.

MS T={3 a,4 b,5 c}
MS $T_* = \{\infty a, \infty b, \infty c\}$
$A1 = \{\binom{T_*}{10} | count(a) > 3\} // \binom{8}{6}$
$A2 = \{\binom{T_*}{10} | count(b) > 4\} // \binom{7}{5}$
$A3 = \{\binom{T_*}{10} | count(c) > 5\} // \binom{6}{4}$

$\binom{T}{10} = \binom{T_*}{10} - (|A_1| + |A_2| + |A_3|) + (|A_1 \cap A_2| + |A_1 \cap A_3| + |A_2 \cap A_3|) - |A_1 \cap A_2 \cap A_3|$
ans=C(10,12)-(C(6,8)+C(5,7)+C(4,6))+(C(1,3)+C(0,2)+0)-0=6

5.15.4 Distributing Balls into Boxes

Distributing m Balls into n Boxes.

balls	boxes	empty	counts
diff	diff	empty	n^m
diff	diff	full	$n! \times S(m,n) = \sum\limits_{i=0}^{n} (-1)^n \binom{n}{i} (n-i)^m$(inclusion-exclusionprinciple)
diff	same	empty	$\sum\limits_{k=1}^{min\{n,m\}} s(m,k) = \frac{1}{n!} \sum\limits_{k=1}^{min\{n,m\}} \sum\limits_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^m$
diff	same	full	S(m,n) (Stirling numbers of the second kind)
same	diff	empty	$\binom{n+m-1}{n-1}$
same	diff	full	$\binom{m-1}{n-1}$

same	same	empty	dp[0][0..n]=dp[1..m][1]=1;
			if(m≥n)
			dp[m][n]=dp[m][n-1]+dp[m-n][n];
			else
			dp[m][n]=dp[m][n-1];

same	same	full	g[m][n]=dp[m-n][n];

5.15.5 Combinatorial Game Theory

Wythoff's game:

- There are two piles of counters.
- Players take turns removing counters (at least 1 counter) from one or both piles; in the latter case, the numbers of counters removed from each pile must be equal.
- The player who removes the last counter wins.

consider the counters of status as pair (a,b) ($a \leq b$)
{first player loses} $\iff a = \lfloor (b-a) \times \phi \rfloor, \phi = \frac{\sqrt{5}+1}{2}$

Fibonacci Nim:

- There is one pile of n counters.
- The first player may remove any positive number of counters, but not the whole pile.
- Thereafter, each player may remove at most twice the number of counters his opponent took on the previous move.
- The player who removes the last counter wins.

{first player wins} $\iff n \notin${Fibonacci number}

poj 1740:

- There are n piles of stones.
- At each step of the game,the player choose a pile,remove at least one stones,then freely move stones from this pile to any other pile that still has stones.
- The player who removes the last counter wins.

{first player lose} $\iff$ n is even && $(a_1, a_2, ..., a_k)(a_1 \leq a_2 \leq ... \leq a_{2k})$ satisfy $a_{2i-1} = a_{2i} \{\forall i \in [1,k]\}$

Staircase Nim:

- A staircase of n steps contains coins on some of the steps.
- A move of staircase nim consists of moving any positive number of coins from any step j , to the next lower step, j − 1.
- Coins reaching the ground (step 0) are removed from play.
- The player who removes the last counter wins.

Even steps are unusefull.
$SG = x_1 \oplus x_3 \oplus x_5 ...$

Anti-SG:

- Everything is likes SG.
- The player who removes the last counter loses.

{first player wins} $\iff$
SGsum=0,&& {all piles is 1}
SGsum≠0,&& {some piles ars larger than 1}

Every-SG:

- Everything is likes SG.
- For each turns, player have to move all of sub-games if the sub-game was not ended yet.

{first player wins} $\iff$ max(steps of all sub-games) is odd.

Coin Game:

- Given a horizontal line of N coins with some coins showing heads and some tails.
- Each turn, a player have to follow some rules, flip some coins. But the most right coin he fliped has to be fliped from head to tail.
- The player who can not flip coin loses.

game{THHTTH} = game{TH}⊕game{TTH}⊕game{TTTTTH}

Tree Game:

- There is a rooted tree.
- Each turn, a player has to remove a edge from the tree. The parts can not connect with root with also are removed.
- The player who removes the last edge wins.

$\forall node(x)$,
$SG(x) = (SG(i_1) + 1) \oplus (SG(i_2) + 1) \oplus$ ...(∀ i are childnodes of x)

Undirectional Graph Game:

- There is a rooted undirectional graph.
- Other rules are likes Tree Game.

Odd Circle's SG value is 1.
Even Circel's SG value is 0.
turn the graph to a tree.

5.15.6 Catalan number

from C_0
1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420

$C_0 = 1$

$C_{n+1} = \sum\limits_{i=0}^{n} C_i C_{n-i}$

$C_{n+1} = \frac{2(2n+1)}{n+1} C_n$

$C_n = \binom{2n}{n} - \binom{2n}{n+1} = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{(n+1)!n!}$

$C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$

Applications:

1. C_n counts the number of expressions containing n pairs of parentheses which are correctly matched.
2. C_n is the number of full binary trees with n + 1 leaves.
3. C_n is the number of non-isomorphic ordered trees with n+1 vertices. (An ordered tree is a rooted tree in which the children of each vertex are given a fixed left-to-right order.)
4. C_n is the number of monotonic paths along the edges of a grid with n × n square cells, which do not pass above the diagonal.($x \leq y$ for C_n, $x < y$ for $C_n - 1$)
   (a) for the rectangle (p,q),($x < y$), $ans = \binom{p+q-1}{p} - \binom{p+q-1}{p-1} = \frac{q-p}{q+p}\binom{p+q}{q}$
   (b) for the rectangle (p,q),($x \leq y$),$ans = \binom{p+q}{p} - \binom{p+q}{p-1} = \frac{q-p+1}{q+1}\binom{p+q}{q}$
5. C_n is the number of different ways a convex polygon with n + 2 sides can be cut into triangles by connecting vertices with straight lines.
6. C_n is the number of permutations of {1, ..., n} that avoid the pattern 123.
7. C_n is the number of ways to tile a stairstep shape of height n with n rectangles.

5.15.7 Stirling number

First kind:
Stirling numbers of the first kind is signed.
The unsigned Stirling numbers of the first kind are denoted by s(n,k).
s(4,2)=11
s(n,k) count the number of permutations of n elements with k disjoint cycles.
s(n,0)=s(1,1)=1
s(n+1,k)=s(n,k-1)+n s(n,k)

Second kind:
S(n,k) count the number of ways to partition a set of n labelled objects into k nonempty unlabelled subsets.
S(4,2)=7
S(n,n)=S(n,1)=1
S(n,k)=S(n-1,k-1)+k S(n-1,k)
$S(n, n-1) = \binom{n}{2} = \frac{n(n-1)}{2}$
$S(n, 2) = 2^{n-1} - 1$

5.15.8 Delannoy number

Delannoy number D describes the number of paths from (0, 0) to (m, n), using only single steps north, northeast, or east.
D(0,0)=1
D(m,n)=D(m-1,n)+D(m-1,n-1)+D(m,n-1)

central Delannoy numbers D(n) = D(n,n)
D(n) from 0:
1, 3, 13, 63, 321, 1683, 8989, 48639, 265729
$nD(n) = 3(2n-1)D(n-1) - (n-1)D(n-2)$

5.15.9 Schröder number

Large:
Describes the number of paths from (0, 0) to (m, n), using only single steps north, northeast, or east, for all (x,y), ($x \leq y$).
for(n==m),from 0:

1, 2, 6, 22, 90, 394, 1806, 8558, 41586, 206098

$S(n) = S(n-1) + \sum_{k=0}^{n-1} S(k)S(n-1-k)$

Little: (aka. super-Catalan numbers, Hipparchus numbers)

1. the number of different trees with n leaves and with all internal vertices having two or more children.
2. the number of ways of inserting brackets into a sequence.
3. the number of ways of dissecting a convex polygon into smaller polygons by inserting diagonals.

from 0:
1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049
s(n)=S(n)/2
s(0)=s(1)=1
ns(n)=(6n-9)s(n-1)-(n-3)s(n-2)

$a(n+1) = -a(n) + 2\sum_{k=1}^{n} a(k) \times a(n+1-k)$

$a(n+1) = \sum_{k=0}^{(n-1)/2} 2^k \times 3^{n-1-2k} \binom{n-1}{2k}$

5.15.10 Bell number

Number of partitions of a set of n labeled elements.
from 0:
1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975

$B_{n+1} = \sum_{k=0}^{n} \binom{n}{k} B_k$

$B_{p+n} \equiv B_n + B_{n+1} \pmod{p}$ (p for prime)

$B_{p^m+n} \equiv mB_n + B_{n+1} \pmod{p}$ (p for prime)

$B_n = \sum_{k=1}^{n} S(n,k)$(S for Stirling second kind)

5.15.11 Eulerian number

First kind:
the number of permutations of the numbers 1 to n in which exactly m elements are greater than the previous element
A(n,0)=1
A(n,m)=(n-m)A(n-1,m-1)+(m+1)A(n-1,m)
A(n,m)=(n-m+1)A(n-1,m-1)+mA(n-1,m)
A(n,m)=A(n,n-1-m)

Second kind:
count the permutations of the multiset {1,1,2,2,...,n,n} with k ascents with the restriction that for all m
T(n,0)=1
T(n,m)=(2n-m-1)T(n-1,m-1)+(m+1)T(n-1,m)

5.15.12 Motzkin number

1. the number of different ways of drawing non-intersecting chords on a circle between n points
2. Number of sequences of length n-1 consisting of positive integers such that the opening and ending elements are 1 or 2 and the absolute difference between any 2 consecutive elements is 0 or 1
3. paths from (0,0) to (n,0) in an n X n grid using only steps U = (1,1), F = (1,0) and D = (1,-1)

from 0:
1, 1, 2, 4, 9, 21, 51, 127, 323, 835, 2188, 5798, 15511, 41835, 113634, 310572, 853467

$M_{n+1} = M_n + \sum_{i=0}^{n-1} M_i M_{n-1-i} = \frac{2n+3}{n+3} M_n + \frac{3n}{n+3} M_{n-1}$

$M_n = \sum_{k=0}^{\lfloor n/2 \rfloor} \binom{n}{2k} C_k$(C for catalan)

5.15.13 Narayana number

1. the number of expressions containing n pairs of brackets which are correctly matched and which contain k pairs of ().
2. the number of paths from (0, 0) to (2n, 0), with steps only northeast and southeast, not straying below the x-axis, with k peaks.

N(n,0)=0

$N(n,k) = \frac{1}{n}\binom{n}{k}\binom{n}{k-1}$

$N(n,k) = \frac{1}{k}\binom{n-1}{k-1}\binom{n}{k-1}$

$\sum_{k=1}^{n} N(n,k) = C_n$(C for catalan)

5.16 Number theory

5.16.1 Divisor Fuction

$n = p_1^{a_1} \times p_2^{a_2} \times ... \times p_s^{a_s}$
sum of positive divisors function

$\sigma(n) = \prod_{j=1}^{s} \frac{p_j^{a_j+1}-1}{p_j-1}$

number of postive diversors function

$\tau(n) = \prod_{j=1}^{s} (a_j + 1)$

5.16.2 Reduced Residue System

Euler's totient function:

对正整数 n，欧拉函数 φ 是小于或等于 n 的数中与 n 互质的数的数目，也就是对 n 的简化剩余系的大小。
$\varphi(2)$=1（唯一和 1 互质的数就是 1 本身）。
若 m,n 互质，$\varphi(m \times n) = \varphi(m) \times \varphi(n)$。
对于 n 来说，所有这样的数的和为 $\frac{n \times \varphi(n)}{2}$ 。
$gcd(k,n) = d, k \in [1,n]$，这样的 k 有 $\varphi(\frac{n}{d})$

```
inline int phi(int n)
{
    static int i;
    static int re;
    re=n;
    for(i=0;prm[i]*prm[i]<=n;++i)
        if(n%prm[i]==0)
        {
            re-=re/prm[i];
            do
                n/=prm[i];
            while(n%prm[i]==0);
        }
    if(n!=1)
        re-=re/n;
    return re;
}

inline void Euler()
{
    static int i,j;
    phi[1]=1;
    for(i=2;i<MAXX;++i)
        if(!phi[i])
            for(j=i;j<MAXX;j+=i)
            {
                if(!phi[j])
                    phi[j]=j;
                phi[j]=phi[j]/i*(i-1);
            }
}
```

Multiplicative order:

the multiplicative order of a modulo n is the smallest positive integer k with
$a^k \equiv 1 \pmod n$

对 m 的简化剩余系中的所有 x,ord(x) 都一定是 φ(m) 的一个约数 (aka. Euler's totient theorem)

求:
method 1、根据定义，对 φ(m) 分解素因子之后暴力寻找最小的一个 $d\{d|\varphi(m)\}$，满足 $x^d \equiv 1 \pmod m$;
method 2、

```
inline long long ord(long long x,long long m)
{
    static long long ans;
    static int i,j;
    ans=phi(m);
    for(i=0;i<fac.size();++i)
        for(j=0;j<fac[i].second && pow(x,ans/fac[i].
             first,m)==1ll;++j)
            ans/=fac[i].first;
    return ans;
}
```

Primitive root:

若 ord(x)==φ(m)，则 x 为 m 的一个原根
因此只需检查所有 x^d $\{d|\varphi(m)\}$ 找到使 $x^d \equiv 1 \pmod m$ 的所有 d，当且仅当这样的 d 只有一个，并且为 φ(m) 的时候，x 是 m 的一个原根

当且仅当 m= 1,2,4,p^n,2 × p^n {p 为奇质数,n 为正整数} 时，m 存在原根 // 应该是指存在对于完全剩余系的原根……？

当 m 存在原根时，原根数目为 φ(φ(m))

求：
枚举每一个简化剩余系中的数 i，若对于 i 的每一个质因子 p[j],$i^{\frac{\varphi(m)}{p[j]}} \not\equiv 1 \pmod m$，那么 i 为 m 的一个原根。也就是说，ord(i)==φ(m)。
最小原根通常极小。

Carmichael function:

λ(n) is defined as the smallest positive integer m such that
$a^m \equiv 1 \pmod n \{\forall a! = 1\&\&gcd(a,n) == 1\}$
也就是简化剩余系 (完全剩余系中存在乘法群中无法得到 1 的数) 中所有 x 的 lcm{ord(x)}

if n=$p[0]^{a[0]} \times p[1]^{a[1]} \times ... \times p[m-1]^{a[m-1]}$
then λ(n)=lcm($\lambda(p[0]^{a[0]})$,$\lambda(p[1]^{a[1]})$,...,$\lambda(p[m-1]^{a[m-1]})$);

if n=$2^c \times p[0]^{a[0]} \times p[1]^{a[1]} \times ... \times p[m-1]^{a[m-1]}$
then λ(n)=lcm(2^c,$\varphi(p[0]^{a[0]})$,$\varphi(p[1]^{a[1]})$,...,$\varphi(p[m-1]^{a[m-1]})$);
c=0 if a<2; c=1 if a==2; c=a-2 if a>3;

Carmichael's theorem:
if gcd(a,n)==1
then $\lambda(n) \equiv 1 \pmod n$

5.16.3 Prime

Prime number theorem:
Let π(x) be the prime-counting function that gives the number of primes less than or equal to x, for any real number x.
$\lim_{x\to\infty} \frac{\pi(x)}{x/\ln(x)} = 1$
known as the asymptotic law of distribution of prime numbers.
$\pi(x) \sim \frac{x}{\ln x}$.

```
#include<vector>

std::vector<int>prm;
bool flag[MAXX];

int main()
{
    prm.reserve(MAXX); // pi(x)=x/ln(x);
    for(i=2;i<MAXX;++i)
    {
        if(!flag[i])
            prm.push_back(i);
        for(j=0;j<prm.size() && i*prm[j]<MAXX;++j)
        {
            flag[i*prm[j]]=true;
            if(i%pmr[j]==0)
                break;
        }
    }
    return 0;
}
```

5.16.4 Euler–Mascheroni constant

$\gamma = \lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{1}{k} - \ln(n)\right) = \int_{1}^{\infty} \left(\frac{1}{\lfloor x \rfloor} - \frac{1}{x}\right) dx$
0.57721566490153286060651209008240243104215933593992…

5.16.5 Fibonacci

gcd(fib[i],fib[j])=fib[gcd(i,j)]

6 String

6.1 Aho-Corasick Algorithm

```
//trie graph
#include<cstring>
#include<queue>

#define MAX 1000111
#define N 26

int nxt[MAX][N],fal[MAX],cnt;
bool ed[MAX];
char buf[MAX];

inline void init(int a)
{
    memset(nxt[a],0,sizeof(nxt[0]));
    fal[a]=0;
    ed[a]=false;
}

inline void insert()
{
    static int i,p;
    for(i=p=0;buf[i];++i)
    {
        if(!nxt[p][map[buf[i]]])
            init(nxt[p][map[buf[i]]]=++cnt);
        p=nxt[p][map[buf[i]]];
    }
    ed[p]=true;
}

inline void make()
{
    static std::queue<int>q;
    int i,now,p;
    q.push(0);
    while(!q.empty())
    {
        now=q.front();
        q.pop();
        for(i=0;i<N;++i)
            if(nxt[now][i])
            {
                q.push(p=nxt[now][i]);
                if(now)
                    fal[p]=nxt[fal[now]][i];
                ed[p]|=ed[fal[p]];
            }
            else
                nxt[now][i]=nxt[fal[now]][i]; // 使用本身的 trie 存串的时候注意 nxt 已被重载
    }
}

// normal version

#define N 128

char buf[MAXX];
int cnt[1111];

struct node
{
    node *fal,*nxt[N];
    int idx;
    node() { memset(this,0,sizeof node); }
}*rt;
std::queue<node*>Q;

void free(node *p)
{
    for(int i(0);i<N;++i)
        if(p->nxt[i])
            free(p->nxt[i]);
    delete p;
}

inline void add(char *s,int idx)
{
    static node *p;
    for(p=rt;*s;++s)
    {
        if(!p->nxt[*s])
            p->nxt[*s]=new node();
        p=p->nxt[*s];
    }
    p->idx=idx;
}

inline void make()
{
    Q.push(rt);
    static node *p,*q;
    static int i;
    while(!Q.empty())
    {
        p=Q.front();
        Q.pop();
        for(i=0;i<N;++i)
            if(p->nxt[i])
            {
                q=p->fal;
                while(q)
                {
                    if(q->nxt[i])
                    {
                        p->nxt[i]->fal=q->nxt[i];
                        break;
                    }
                    q=q->fal;
                }
                if(!q)
                    p->nxt[i]->fal=rt;
                Q.push(p->nxt[i]);
            }
    }
}

inline void match(const char *s)
{
    static node *p,*q;
    for(p=rt;*s;++s)
    {
        while(p!=rt && !p->nxt[*s])
            p=p->fal;
        p=p->nxt[*s];
        if(!p)
            p=rt;
        for(q=p;q!=rt && q->idx;q=q->fal) // why q->idx ? looks like not necessary at all, I delete it in an other solution
            ++cnt[q->idx];
    }
}

//可以考虑 dfs 一下，拉直 fal 指针来跳过无效的匹配
//在线调整关键字存在性的时候，可以考虑欧拉序压扁之后使用 BIT 或者线段树进行区间修改
//fal 指针构成的是一颗树，从匹配到的节点到根都数一次
```

6.2 Gusfield's Z Algorithm

```
inline void make(int *z,char *buf)
{
    int i,j,l,r;
    l=0;
    r=1;
    z[0]=strlen(buf);
    for(i=1;i<z[0];++i)
        if(r<=i || z[i-l]>=r-i)
        {
            j=std::max(i,r);
            while(j<z[0] && buf[j]==buf[j-i])
                ++j;
            z[i]=j-i;
            if(i<j)
            {
                l=i;
                r=j;
            }
        }
        else
            z[i]=z[i-l];
}

for(i=1;i<len && i+z[i]<len;++i); //i= 可能最小循环节长度
```

6.3 Manacher's Algorithm

```
inline int match(const int a,const int b,const std::vector<int> &str)
{
    static int i;
    i=0;
    while(a-i>=0 && b+i<str.size() && str[a-i]==str[b+i])//注意是 i 不是 1，打错过很多次了
        ++i;
    return i;
}

inline void go(int *z,const std::vector<int> &str)
{
    static int c,l,r,i,ii,n;
    z[0]=1;
    c=l=r=0;
    for(i=1;i<str.size();++i)
    {
        ii=(l<<1)-i;
        n=r+1-i;

        if(i>r)
        {
            z[i]=match(i,i,str);
            l=i;
            r=i+z[i]-1;
        }
        else
            if(z[ii]==n)
            {
                z[i]=n+match(i-n,i+n,str);
                l=i;
                r=i+z[i]-1;
            }
            else
                z[i]=std::min(z[ii],n);
        if(z[i]>z[c])
            c=i;
    }
}

inline bool check(int *z,int a,int b) //检查子串 [a,b] 是否回文
{
    a=a*2-1;
    b=b*2-1;
    int m=(a+b)/2;
    return z[m]>=b-m+1;
}
```

6.4 Morris-Pratt Algorithm

```
inline void make(char *buf,int *fal)
{
    static int i,j;
    fal[0]=-1;
    for(i=1,j=-1;buf[i];++i)
    {
        while(j>=0 && buf[j+1]!=buf[i])
            j=fal[j];
        if(buf[j+1]==buf[i])
            ++j;
        fal[i]=j;
    }
}

inline int match(char *p,char *t,int* fal)
{
    static int i,j,re;
    re=0;
    for(i=0,j=-1;t[i];++i)
    {
        while(j>=0 && p[j+1]!=t[i])
            j=fal[j];
        if(p[j+1]==t[i])
            ++j;
        if(!p[j+1])
        {
            ++re;
            j=fal[j];
        }
    }
    return re;
}

inline void make(char *buf,int *fal) // knuth-morris-pratt, not tested yet
{
    static int i,j;
    fal[0]=-1;
    for(i=1,j=-1;buf[i];++i)
    {
        while(j>=0 && buf[j+1]!=buf[i])
            j=fal[j];
        if(buf[j+1]==buf[i])
            ++j;
        fal[i]=j;
    }
    for(i-=2;i>=0;--i)
    {
        for(j=fal[i];j!=-1 && buf[j+1]!=buf[i+1];j=fal[j]]);
        fal[i]=j;
    }
}
```

6.5 smallest representation

```
int min(char a[],int len)
{
    int i = 0,j = 1,k = 0;
    while (i < len && j < len && k < len)
    {
        int cmp = a[(j+k)%len]-a[(i+k)%len];
        if (cmp == 0)
            k++;
        else
        {
            if (cmp > 0)
                j += k+1;
            else
                i += k+1;
            if (i == j) j++;
            k = 0;
        }
    }
    return std::min(i,j);
}
```

6.6 Suffix Array - DC3 Algorithm

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXX 1111
#define F(x) ((x)/3+((x)%3==1?0:tb))
#define G(x) ((x)<tb?(x)*3+1:((x)-tb)*3+2)

int wa[MAXX],wb[MAXX],wv[MAXX],ws[MAXX];

inline bool c0(const int *str,const int &a,const int &b)
{
    return str[a]==str[b] && str[a+1]==str[b+1] && str[a+2]==str[b+2];
}

inline bool c12(const int *str,const int &k,const int &a,const int &b)
{
    if(k==2)
        return str[a]<str[b] || str[a]==str[b] && c12(str,1,a+1,b+1);
    else
        return str[a]<str[b] || str[a]==str[b] && wv[a+1]<wv[b+1];
}

inline void sort(int *str,int *a,int *b,const int &n,const int &m)
{
    memset(ws,0,sizeof(ws));
    int i;
    for(i=0;i<n;++i)
        ++ws[wv[i]=str[a[i]]];
    for(i=1;i<m;++i)
        ws[i]+=ws[i-1];
    for(i=n-1;i>=0;--i)
        b[--ws[wv[i]]]=a[i];
}

inline void dc3(int *str,int *sa,const int &n,const int &m)
{
    int *strn(str+n);
    int *san(sa+n),tb((n+1)/3),ta(0),tbc(0),i,j,k;
    str[n]=str[n+1]=0;
    for(i=0;i<n;++i)
        if(i%3)
            wa[tbc++]=i;
    sort(str+2,wa,wb,tbc,m);
    sort(str+1,wb,wa,tbc,m);
    sort(str,wa,wb,tbc,m);
    for(i=j=1,strn[F(wb[0])]=0;i<tbc;++i)
        strn[F(wb[i])]=c0(str,wb[i-1],wb[i])?j-1:j++;
    if(j<tbc)
        dc3(strn,san,tbc,j);
    else
        for(i=0;i<tbc;++i)
            san[strn[i]]=i;
    for(i=0;i<tbc;++i)
        if(san[i]<tb)
            wb[ta++]=san[i]*3;
    if(n%3==1)
```

```
        wb[ta++]=n-1;
    sort(str,wb,wa,ta,m);
    for(i=0;i<tbc;++i)
        wv[wb[i]=G(san[i])]=i;
    for(i=j=k=0;i<ta && j<tbc;)
        sa[k++]=c12(str,wb[j]%3,wa[i],wb[j])?wa[i++]:wb[j++];
    while(i<ta)
        sa[k++]=wa[i++];
    while(j<tbc)
        sa[k++]=wb[j++];
}

int rk[MAXX],lcpa[MAXX],sa[MAXX*3];
int str[MAXX*3]; //必须int

int main()
{
    scanf("%d␣%d",&n,&j);
    for(i=0;i<n;++i)
    {
        scanf("%d",&k);
        num[i]=k-j+100;
        j=k;
    }
    num[n]=0;

    dc3(num,sa,n+1,191); //191: str 中取值范围，桶排序

    for(i=1;i<=n;++i) // rank 数组
        rk[sa[i]]=i;
    for(i=k=0;i<n;++i) // lcp 数组
        if(!rk[i])
            lcpa[0]=0;
        else
        {
            j=sa[rk[i]-1];
            if(k>0)
                --k;
            while(num[i+k]==num[j+k])
                ++k;
            lcpa[rk[i]]=k;
        }


    for(i=1;i<=n;++i)
        sptb[0][i]=i;
    for(i=1;i<=lg[n];++i)  //sparse table RMQ
    {
        k=n+1-(1<<i);
        for(j=1;j<=k;++j)
        {
            a=sptb[i-1][j];
            b=sptb[i-1][j+(1<<(i-1))];
            sptb[i][j]=lcpa[a]<lcpa[b]?a:b;
        }
    }
}

inline int ask(int l,int r)
{
    a=lg[r-l+1];
    r-=(1<<a)-1;
    l=sptb[a][l];
    r=sptb[a][r];
    return lcpa[l]<lcpa[r]?l:r;
}

inline int lcp(int l,int r) // 字符串上 [l,r] 区间的 rmq
{
    l=rk[l];
    r=rk[r];
    if(l>r)
        std::swap(l,r);
    return lcpa[ask(l+1,r)];
}
```

6.7 Suffix Array - Prefix-doubling Algorithm

```
int wx[maxn],wy[maxn],*x,*y,wss[maxn],wv[maxn];

bool cmp(int *r,int n,int a,int b,int l)
{
    return a+l<n && b+l<n && r[a]==r[b]&&r[a+l]==r[b+l];
}
void da(int str[],int sa[],int rank[],int height[],int n,int m)
{
    int *s = str;
    int *x=wx,*y=wy,*t,p;
    int i,j;
    for(i=0; i<m; i++)
        wss[i]=0;
    for(i=0; i<n; i++)
        wss[x[i]=s[i]]++;
    for(i=1; i<m; i++)
        wss[i]+=wss[i-1];
    for(i=n-1; i>=0; i--)
        sa[--wss[x[i]]]=i;
    for(j=1,p=1; p<n && j<n; j*=2,m=p)
    {
        for(i=n-j,p=0; i<n; i++)
            y[p++]=i;
        for(i=0; i<n; i++)
            if(sa[i]-j>=0)
                y[p++]=sa[i]-j;
        for(i=0; i<n; i++)
            wv[i]=x[y[i]];
        for(i=0; i<m; i++)
            wss[i]=0;
        for(i=0; i<n; i++)
            wss[wv[i]]++;
        for(i=1; i<m; i++)
            wss[i]+=wss[i-1];
        for(i=n-1; i>=0; i--)
            sa[--wss[wv[i]]]=y[i];
        for(t=x,x=y,y=t,p=1,i=1,x[sa[0]]=0; i<n; i++)
            x[sa[i]]=cmp(y,n,sa[i-1],sa[i],j)?p-1:p++;
    }
    for(int i=0; i<n; i++)
        rank[sa[i]]=i;
    for(int i=0,j=0,k=0; i<n; height[rank[i++]]=k)
        if(rank[i]>0)
            for(k?k--:0,j=sa[rank[i]-1]; i+k < n && j+k < n && str[i+k]==str[j+k]; ++k);
}
```

6.8 Suffix Automaton

```
/*
length(s) ∈ [ min(s), max(s) ] = [ val[fal[s]]+1, val[s] ]
 */
#define MAXX 90111
#define MAXN (MAXX<<1)

int fal[MAXN],nxt[MAXN][26],val[MAXN],cnt,rt,last;

inline int neww(int v=0)
{
    val[++cnt]=v;
    fal[cnt]=0;
    memset(nxt[cnt],0,sizeof nxt[0]);
    return cnt;
}

inline void add(int w)
{
    static int p,np,q,nq;
    p=last;
    last=np=neww(val[p]+1);
    while(p && !nxt[p][w])
    {
        nxt[p][w]=np;
        p=fal[p];
    }
    if(!p)
        fal[np]=rt;
    else
    {
        q=nxt[p][w];
        if(val[p]+1==val[q])
            fal[np]=q;
        else
        {
            nq=neww(val[p]+1);
            memcpy(nxt[nq],nxt[q],sizeof nxt[0]);
            fal[nq]=fal[q];

            fal[q]=fal[np]=nq;
            while(p && nxt[p][w]==q)
            {
                nxt[p][w]=nq;
                p=fal[p];
            }
        }
    }
}

int v[MAXN],the[MAXN];

inline void make(char *str)
```

```
{
    cnt=0;
    rt=last=neww();
    static int i,len,now;
    for(i=0;str[i];++i)
        add(str[i]-'a');
    len=i;
    memset(v,0,sizeof v);
    for(i=1;i<=cnt;++i)
        ++v[val[i]];
    for(i=1;i<=len;++i)
        v[i]+=v[i-1];
    for(i=1;i<=cnt;++i)
        the[v[val[i]]--]=i;
    for(i=cnt;i;--i)
    {
        now=the[i];
        // topsort already
    }
}
/*
sizeof right(s):
    init:
        for all np:
            count[np]=1;
    process:
        for all status s:
            count[fal[s]]+=count[s];
*/
```

7 Dynamic Programming

7.1 knapsack problem

```
multiple-choice knapsack problem:

for 所有的组k
    for v=V..0
 for 所有的 i 属于组 k
            f[v]=max{f[v],f[v-c[i]]+w[i]}
```

7.2 LCIS

```
#include<cstdio>
#include<cstring>
#include<vector>

#define MAXX 1111

int T;
int n,m,p,i,j,k;
std::vector<int>the[2];
int dp[MAXX],path[MAXX];
int ans[MAXX];

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    {
        scanf("%d",&n);
        the[0].resize(n);
        for(i=0;i<n;++i)
            scanf("%d",&the[0][i]);
        scanf("%d",&m);
        the[1].resize(m);
        for(i=0;i<m;++i)
            scanf("%d",&the[1][i]);
        memset(dp,0,sizeof dp);
        for(i=0;i<the[0].size();++i)
        {
            n=0;
            p=-1;
            for(j=0;j<the[1].size();++j)
            {
                if(the[0][i]==the[1][j] && n+1>dp[j])
                {
                    dp[j]=n+1;
                    path[j]=p;
                }
                if(the[1][j]<the[0][i] && n<dp[j])
                {
                    n=dp[j];
                    p=j;
                }
            }
        }
        n=0;
        p=-1;
        for(i=0;i<the[1].size();++i)
            if(dp[i]>n)
                n=dp[p=i];
        printf("%d\n",n);
        for(i=n-1;i>=0;--i)
        {
            ans[i]=the[1][p];
            p=path[p];
        }
        for(i=0;i<n;++i)
            printf("%d␣",ans[i]);
        puts("");
    }
    return 0;
}
```

7.3 LCS

```
#include<cstdio>
#include<algorithm>
#include<vector>

#define MAXX 111
#define N 128

std::vector<char>the[2];
std::vector<int>dp(MAXX),p[N];

int i,j,k;
char buf[MAXX];
int t;

int main()
{
    the[0].reserve(MAXX);
    the[1].reserve(MAXX);
    while(gets(buf),buf[0]!='#')
    {
        the[0].resize(0);
        for(i=0;buf[i];++i)
            the[0].push_back(buf[i]);
        the[1].resize(0);
        gets(buf);
        for(i=0;buf[i];++i)
            the[1].push_back(buf[i]);
        for(i=0;i<N;++i)
            p[i].resize(0);
        for(i=0;i<the[1].size();++i)
            p[the[1][i]].push_back(i);
        dp.resize(1);
        dp[0]=-1;
        for(i=0;i<the[0].size();++i)
            for(j=p[the[0][i]].size()-1;j>=0;--j)
            {
                k=p[the[0][i]][j];
                if(k>dp.back())
                    dp.push_back(k);
                else
                    *std::lower_bound(dp.begin(),dp.end(),k)=k;
            }
        printf("Case␣#%d:␣you␣can␣visit␣at␣most␣%ld␣cities.\n",++t,dp.size()-1);
    }
    return 0;
}
```

7.4 sequence partitioning

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<set>

#define MAXX 40111

int a[MAXX],b[MAXX];
int n,R;
std::multiset<int>set;

inline bool check(const int g)
{
    static int i,j,k;
    static long long sum;
    static int l,r,q[MAXX],dp[MAXX];
    set.clear();
    q[0]=dp[0]=l=r=sum=0;
    for(j=i=1;i<=n;++i)
    {
        sum+=b[i];
        while(sum>g)
            sum-=b[j++];
        if(j>i)
            return false;
        while(l<r && q[l]<j)
        {
            ++l;
            if(l<r && set.count(dp[q[l-1]]+a[q[l]]))
                set.erase(set.find(dp[q[l-1]]+a[q[l]]));
        }
        while(l<r && a[q[r-1]]<=a[i])
        {
            --r;
            if(l<r && set.count(dp[q[r-1]]+a[q[r]]))
                set.erase(set.find(dp[q[r-1]]+a[q[r]]));
        }
        if(l<r)
            set.insert(dp[q[r-1]]+a[i]);
        q[r++]=i;
        dp[i]=dp[j-1]+a[q[l]];
        if(r-l>1)
            dp[i]=std::min(dp[i],*set.begin());
    }
    return dp[n]<=R;
}

int i,j,k;
long long l,r,mid,ans;

int main()
{
    while(scanf("%d␣%d",&n,&R)!=EOF)
    {
        l=r=0;
        for(i=1;i<=n;++i)
        {
            scanf("%d␣%d",a+i,b+i);
            r+=b[i];
        }
        ans=-1;
        while(l<=r)
        {
            mid=l+r>>1;
            if(check(mid))
            {
                ans=mid;
                r=mid-1;
            }
            else
                l=mid+1;
        }
        printf("%lld\n",ans);
    }
    return 0;
}
```

8 Search

8.1 dlx

精确覆盖：给定一个 01 矩阵，现在要选择一些行，使得每一列有且仅有一个 1。

每次选定一个元素个数最少的列，从该列中选择一行加入答案，删除该行所有的列以及与该行冲突的行。

重复覆盖：给定一个 01 矩阵，现在要选择一些行，使得每一列至少有一个 1。

每次选定一个元素个数最少的列，从该列中选择一行加入答案，删除该行所有的列。与该行冲突的行可能满足重复覆盖。

8.2 dlx - exact cover

```
#include<cstdio>
#include<cstring>
#include<algorithm>
#include<vector>

#define N 256
#define MAXN N*22
#define MAXM N*5
#define inf 0x3f3f3f3f
const int MAXX(MAXN*MAXM);

bool mat[MAXN][MAXM];

int u[MAXX],d[MAXX],l[MAXX],r[MAXX],ch[MAXX],rh[MAXX];
int sz[MAXM];
std::vector<int>ans(MAXX);
int hd,cnt;

inline int node(int up,int down,int left,int right)
{
    u[cnt]=up;
    d[cnt]=down;
    l[cnt]=left;
    r[cnt]=right;
    u[down]=d[up]=l[right]=r[left]=cnt;
    return cnt++;
}

inline void init(int n,int m)
{
    cnt=0;
    hd=node(0,0,0,0);
    static int i,j,k,r;
    for(j=1;j<=m;++j)
    {
        ch[j]=node(cnt,cnt,l[hd],hd);
        sz[j]=0;
    }
    for(i=1;i<=n;++i)
    {
        r=-1;
        for(j=1;j<=m;++j)
            if(mat[i][j])
            {
                if(r==-1)
                {
                    r=node(u[ch[j]],ch[j],cnt,cnt);
                    rh[r]=i;
                    ch[r]=ch[j];
                }
                else
                {
                    k=node(u[ch[j]],ch[j],l[r],r);
                    rh[k]=i;
                    ch[k]=ch[j];
                }
                ++sz[j];
            }
    }
}

inline void rm(int c)
{
    l[r[c]]=l[c];
    r[l[c]]=r[c];
    static int i,j;
    for(i=d[c];i!=c;i=d[i])
        for(j=r[i];j!=i;j=r[j])
        {
            u[d[j]]=u[j];
            d[u[j]]=d[j];
            --sz[ch[j]];
        }
}

inline void add(int c)
{
    static int i,j;
    for(i=u[c];i!=c;i=u[i])
        for(j=l[i];j!=i;j=l[j])
        {
            ++sz[ch[j]];
            u[d[j]]=d[u[j]]=j;
        }
    l[r[c]]=r[l[c]]=c;
}

bool dlx(int k)
{
    if(hd==r[hd])
    {
        ans.resize(k);
        return true;
    }
    int s=inf,c;
    int i,j;
    for(i=r[hd];i!=hd;i=r[i])
        if(sz[i]<s)
        {
            s=sz[i];
            c=i;
        }
    rm(c);
    for(i=d[c];i!=c;i=d[i])
    {
        ans[k]=rh[i];
        for(j=r[i];j!=i;j=r[j])
            rm(ch[j]);
        if(dlx(k+1))
            return true;
        for(j=l[i];j!=i;j=l[j])
```

```
            add(ch[j]);
    }
    add(c);
    return false;
}

#include <cstdio>
#include <cstring>

#define N 1024
#define M 1024*110
using namespace std;

int l[M], r[M], d[M], u[M], col[M], row[M], h[M], res[N], cntcol[N];
int dcnt = 0;
//初始化一个节点
inline void addnode(int &x)
{
    ++x;
    r[x] = l[x] = u[x] = d[x] = x;
}
//将加入到后xrowx
inline void insert_row(int rowx, int x)
{
    r[l[rowx]] = x;
    l[x] = l[rowx];
    r[x] = rowx;
    l[rowx] = x;
}
//将加入到后xcolx
inline void insert_col(int colx, int x)
{
    d[u[colx]] = x;
    u[x] = u[colx];
    d[x] = colx;
    u[colx] = x;
}
//全局初始化
inline void dlx_init(int cols)
{
    memset(h, -1, sizeof(h));
    memset(cntcol, 0, sizeof(cntcol));
    dcnt = -1;
    addnode(dcnt);
    for (int i = 1; i <= cols; ++i)
    {
        addnode(dcnt);
        insert_row(0, dcnt);
    }
}
//删除一列以及相关的所有行
inline void remove(int c)
{
    l[r[c]] = l[c];
    r[l[c]] = r[c];
    for (int i = d[c]; i != c; i = d[i])
        for (int j = r[i]; j != i; j = r[j])
        {
            u[d[j]] = u[j];
            d[u[j]] = d[j];
            cntcol[col[j]]--;
        }
}
//恢复一列以及相关的所有行
inline void resume(int c)
{
    for (int i = u[c]; i != c; i = u[i])
        for (int j = l[i]; j != i; j = l[j])
        {
            u[d[j]] = j;
            d[u[j]] = j;
            cntcol[col[j]]++;
        }
    l[r[c]] = c;
    r[l[c]] = c;
}
//搜索部分
bool DLX(int deep)
{
    if (r[0] == 0)
    {
//Do anything you want to do here
        printf("%d", deep);
        for (int i = 0; i < deep; ++i) printf("␣%d", res[i]);
        puts("");
        return true;
    }
    int min = INT_MAX, tempc;
    for (int i = r[0]; i != 0; i = r[i])
        if (cntcol[i] < min)
        {
            min = cntcol[i];
            tempc = i;
        }
    remove(tempc);
    for (int i = d[tempc]; i != tempc; i = d[i])
    {
        res[deep] = row[i];
        for (int j = r[i]; j != i; j = r[j]) remove(col[j]);
        if (DLX(deep + 1)) return true;
        for (int j = l[i]; j != i; j = l[j]) resume(col[j]);
    }
    resume(tempc);
    return false;
}
//插入矩阵中的节点"1"
inline void insert_node(int x, int y)
{
    cntcol[y]++;
    addnode(dcnt);
    row[dcnt] = x;
    col[dcnt] = y;
    insert_col(y, dcnt);
    if (h[x] == -1) h[x] = dcnt;
    else insert_row(h[x], dcnt);
}
int main()
{
    int n, m;
    while (~scanf("%d%d", &n, &m))
    {
        dlx_init(m);
        for (int i = 1; i <= n; ++i)
        {
            int k, x;
            scanf("%d", &k);
            while (k--)
            {
                scanf("%d", &x);
                insert_node(i, x);
            }
        }
        if (!DLX(0))
            puts("NO");
    }
    return 0;
}
```

8.3 dlx - repeat cover

```
#include<cstdio>
#include<cstring>
#include<algorithm>

#define MAXN 110
#define MAXM 1000000
#define INF 0x7FFFFFFF

using namespace std;

int G[MAXN][MAXN];
int L[MAXM], R[MAXM], U[MAXM], D[MAXM];
int size, ans, S[MAXM], H[MAXM], C[MAXM];
bool vis[MAXN * 100];
void Link(int r, int c)
{
    U[size] = c;
    D[size] = D[c];
    U[D[c]] = size;
    D[c] = size;
    if (H[r] < 0)
        H[r] = L[size] = R[size] = size;
    else
    {
        L[size] = H[r];
        R[size] = R[H[r]];
        L[R[H[r]]] = size;
        R[H[r]] = size;
    }
    S[c]++;
    C[size++] = c;
}
void Remove(int c)
{
    int i;
    for (i = D[c]; i != c; i = D[i])
    {
        L[R[i]] = L[i];
        R[L[i]] = R[i];
    }
}
void Resume(int c)
{
    int i;
    for (i = D[c]; i != c; i = D[i])
        L[R[i]] = R[L[i]] = i;
}
int A()
{
    int i, j, k, res;
    memset(vis, false, sizeof(vis));
    for (res = 0, i = R[0]; i; i = R[i])
    {
        if (!vis[i])
        {
            res++;
            for (j = D[i]; j != i; j = D[j])
            {
                for (k = R[j]; k != j; k = R[k])
                    vis[C[k]] = true;
            }
        }
    }
    return res;
}
void Dance(int now)
{
    if (R[0] == 0)
        ans = min(ans, now);
    else if (now + A() < ans)
    {
        int i, j, temp, c;
        for (temp = INF,i = R[0]; i; i = R[i])
        {
            if (temp > S[i])
            {
                temp = S[i];
                c = i;
            }
        }
        for (i = D[c]; i != c; i = D[i])
        {
            Remove(i);
            for (j = R[i]; j != i; j = R[j])
                Remove(j);
            Dance(now + 1);
            for (j = L[i]; j != i; j = L[j])
                Resume(j);
            Resume(i);
        }
    }
}
void Init(int m)
{
    int i;
    for (i = 0; i <= m; i++)
    {
        R[i] = i + 1;
        L[i + 1] = i;
        U[i] = D[i] = i;
        S[i] = 0;
    }
    R[m] = 0;
    size = m + 1;
}
```

8.4 fibonacci knapsack

```
#include<stdio.h>
#include<stdlib.h>
#include<algorithm>

#define MAXX 71

struct mono
{
    long long weig,cost;
}goods[MAXX];

int n,T,t,i;
long long carry,sumw,sumc;
long long ans,las[MAXX];

bool comp(const struct mono a,const struct mono b)
{
    if(a.weig!=b.weig)
        return a.weig<b.weig;
    return b.cost<a.cost;
}

void dfs(int i,long long cost_n,long long carry_n,int last)
{
    if(ans<cost_n)
        ans=cost_n;
    if(i==n || goods[i].weig>carry_n || cost_n+las[i]<=ans)
        return;
    if(last || (goods[i].weig!=goods[i-1].weig && goods[i].cost>goods[i-1].cost))
        dfs(i+1,cost_n+goods[i].cost,carry_n-goods[i].weig,1);
    dfs(i+1,cost_n,carry_n,0);
}

int main()
{
    scanf("%d",&T);
    for(t=1;t<=T;++t)
    {
        scanf("%d␣%lld",&n,&carry);
        sumw=0;
        sumc=0;
        ans=0;
        for(i=0;i<n;++i)
        {
            scanf("%lld␣%lld",&goods[i].weig,&goods[i].cost);
            sumw+=goods[i].weig;
            sumc+=goods[i].cost;
        }
        if(sumw<=carry)
        {
            printf("Case␣%d:␣%lld\n",t,sumc);
            continue;
        }
        std::sort(goods,goods+n,comp);
        for(i=0;i<n;++i)
        {
            las[i]=sumc;
            sumc-=goods[i].cost;
        }
        dfs(0,0,carry,1);
        printf("Case␣%d:␣%lld\n",t,ans);
    }
    return 0;
}
```

9 Others

9.1 .vimrc

```
set number
set history=1000000
set autoindent
set smartindent
set tabstop=4
set shiftwidth=4
set expandtab
set showmatch

set nocp
filetype plugin indent on

filetype on
syntax on
```

9.2 bigint

```
// header files
#include <cstdio>
#include <string>
#include <algorithm>
#include <iostream>

struct Bigint
{
    // representations and structures
    std::string a; // to store the digits
    int sign; // sign = -1 for negative numbers, sign = 1 otherwise
    // constructors
    Bigint() {} // default constructor
    Bigint( std::string b ) { (*this) = b; } // constructor for std::string
    // some helpful methods
    int size() // returns number of digits
    {
        return a.size();
    }
    Bigint inverseSign() // changes the sign
    {
        sign *= -1;
        return (*this);
    }
    Bigint normalize( int newSign ) // removes leading 0, fixes sign
    {
        for( int i = a.size() - 1; i > 0 && a[i] == '0'; i-- )
            a.erase(a.begin() + i);
        sign = ( a.size() == 1 && a[0] == '0' ) ? 1 : newSign;
        return (*this);
    }
    // assignment operator
    void operator = ( std::string b ) // assigns a std::string to Bigint
    {
        a = b[0] == '-' ? b.substr(1) : b;
        reverse( a.begin(), a.end() );
        this->normalize( b[0] == '-' ? -1 : 1 );
    }
    // conditional operators
    bool operator < ( const Bigint &b ) const // less than operator
    {
        if( sign != b.sign )
            return sign < b.sign;
        if( a.size() != b.a.size() )
            return sign == 1 ? a.size() < b.a.size() : a.size() > b.a.size();
        for( int i = a.size() - 1; i >= 0; i-- )
            if( a[i] != b.a[i] )
                return sign == 1 ? a[i] < b.a[i] : a[i] > b.a[i];
        return false;
    }
    bool operator == ( const Bigint &b ) const // operator for equality
    {
        return a == b.a && sign == b.sign;
    }

    // mathematical operators
    Bigint operator + ( Bigint b ) // addition operator overloading
    {
        if( sign != b.sign )
            return (*this) - b.inverseSign();
        Bigint c;
        for(int i = 0, carry = 0; i<a.size() || i<b.size() || carry; i++ )
        {
            carry+=(i<a.size() ? a[i]-48 : 0)+(i<b.a.size() ? b.a[i]-48 : 0);
            c.a += (carry % 10 + 48);
            carry /= 10;
        }
        return c.normalize(sign);
    }

    Bigint operator - ( Bigint b ) // subtraction operator overloading
    {
        if( sign != b.sign )
            return (*this) + b.inverseSign();
        int s = sign; sign = b.sign = 1;
        if( (*this) < b )
            return ((b - (*this)).inverseSign()).normalize(-s);
        Bigint c;
        for( int i = 0, borrow = 0; i < a.size(); i++ )
        {
            borrow = a[i] - borrow - (i < b.size() ? b.a[i] : 48);
            c.a += borrow >= 0 ? borrow + 48 : borrow + 58;
            borrow = borrow >= 0 ? 0 : 1;
        }
        return c.normalize(s);
    }
    Bigint operator * ( Bigint b ) // multiplication operator overloading
    {
        Bigint c("0");
        for( int i = 0, k = a[i] - 48; i < a.size(); i++, k = a[i] - 48 )
        {
            while(k--)
                c = c + b; // ith digit is k, so, we add k times
            b.a.insert(b.a.begin(), '0'); // multiplied by 10
        }
        return c.normalize(sign * b.sign);
    }
    Bigint operator / ( Bigint b ) // division operator overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0"), d;
        for( int j = 0; j < a.size(); j++ )
            d.a += "0";
        int dSign = sign * b.sign;
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
            {
                c = c - b;
                d.a[i]++;
            }
        }
        return d.normalize(dSign);
    }
    Bigint operator % ( Bigint b ) // modulo operator overloading
    {
        if( b.size() == 1 && b.a[0] == '0' )
            b.a[0] /= ( b.a[0] - 48 );
        Bigint c("0");
        b.sign = 1;
        for( int i = a.size() - 1; i >= 0; i-- )
        {
            c.a.insert( c.a.begin(), '0');
            c = c + a.substr( i, 1 );
            while( !( c < b ) )
                c = c - b;
        }
        return c.normalize(sign);
    }

    // output method
    void print()
    {
        if( sign == -1 )
            putchar('-');
        for( int i = a.size() - 1; i >= 0; i-- )
            putchar(a[i]);
    }
};



int main()
{
    Bigint a, b, c; // declared some Bigint variables
    /////////////////////////
    // taking Bigint input //
    /////////////////////////

    std::string input; // std::string to take input
    std::cin >> input; // take the Big integer as std::string
    a = input; // assign the std::string to Bigint a

    std::cin >> input; // take the Big integer as std::string
    b = input; // assign the std::string to Bigint b

    //////////////////////////////////
    // Using mathematical operators //
    //////////////////////////////////

    c = a + b; // adding a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a - b; // subtracting b from a
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a * b; // multiplying a and b
    c.print(); // printing the Bigint
    puts(""); // newline

    c = a / b; // dividing a by b
    c.print(); // printing the Bigint
    puts(""); // newline
    c = a % b; // a modulo b
    c.print(); // printing the Bigint
    puts(""); // newline

    ////////////////////////////////
    // Using conditional operators //
    ////////////////////////////////

    if( a == b )
        puts("equal"); // checking equality
    else
        puts("not␣equal");

    if( a < b )
        puts("a␣is␣smaller␣than␣b"); // checking less than operator

    return 0;
}
```

9.3 Binary Search

```
//[0,n)
inline int go(int A[],int n,int x) // return the least i that make A[i]==x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
            l=mid+1;
        else
        {
            r=mid-1;
            if(A[mid]==x)
                re=mid;
        }
    }
    return re;
}

inline int go(int A[],int n,int x) // return the largest i that make A[i]==x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
        {
            l=mid+1;
            if(A[mid]==x)
                re=mid;
        }
        else
            r=mid-1;
    }
    return re;
}

inline int go(int A[],int n,int x) // retrun the largest i that make A[i]<x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
        {
            l=mid+1;
            re=mid;
        }
        else
            r=mid-1;
    }
    return re;
}

inline int go(int A[],int n,int x)// return the largest i that make A[i]<=x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
        {
            l=mid+1;
            re=mid;
        }
        else
            r=mid-1;
    }
    return re;
}

inline int go(int A[],int n,int x)// return the least i that make A[i]>x;
{
    static int l,r,mid,re;
    l=0;
    r=n-1;
    re=-1;
    while(l<=r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
            l=mid+1;
        else
        {
            r=mid-1;
            re=mid;
        }
    }
    return re;
}

inline int go(int A[],int n,int x)// upper_bound();
{
    static int l,r,mid;
    l=0;
    r=n-1;
    while(l<r)
    {
        mid=l+r>>1;
        if(A[mid]<=x)
            l=mid+1;
        else
            r=mid;
    }
    return r;
}

inline int go(int A[],int n,int x)// lower_bound();
{
    static int l,r,mid,;
    l=0;
    r=n-1;
    while(l<r)
    {
        mid=l+r>>1;
        if(A[mid]<x)
            l=mid+1;
        else
            r=mid;
    }
    return r;
}
```

9.4 java

```
//Scanner

Scanner in=new Scanner(new FileReader("asdf"));
PrintWriter pw=new PrintWriter(new Filewriter("out"));
boolean       in.hasNext();
String        in.next();
BigDecimal    in.nextBigDecimal();
BigInteger    in.nextBigInteger();
BigInteger    in.nextBigInteger(int radix);
double        in.nextDouble();
int           in.nextInt();
int           in.nextInt(int radix);
String        in.nextLine();
long          in.nextLong();
long          in.nextLong(int radix);
short         in.nextShort();
short         in.nextShort(int radix);
int           in.radix(); //Returns this scanner's default radix.
```

```
Scanner       in.useRadix(int radix);// Sets this scanner's default radix to the specified radix.
void          in.close();//Closes this scanner.

//String

char          str.charAt(int index);
int           str.compareTo(String anotherString); // <0 if less. ==0 if equal. >0 if greater.
int           str.compareToIgnoreCase(String str);
String        str.concat(String str);
boolean       str.contains(CharSequence s);
boolean       str.endsWith(String suffix);
boolean       str.startsWith(String preffix);
boolean       str.startsWith(String preffix,int toffset);
int           str.hashCode();
int           str.indexOf(int ch);
int           str.indexOf(int ch,int fromIndex);
int           str.indexOf(String str);
int           str.indexOf(String str,int fromIndex);
int           str.lastIndexOf(int ch);
int           str.lastIndexOf(int ch,int fromIndex);
//(ry
int           str.length();
String        str.substring(int beginIndex);
String        str.substring(int beginIndex,int endIndex);
String        str.toLowerCase();
String        str.toUpperCase();
String        str.trim();// Returns a copy of the string, with leading and trailing whitespace omitted.

//StringBuilder
StringBuilder str.insert(int offset,...);
StringBuilder str.reverse();
void          str.setCharAt(int index,int ch);

//BigInteger
compareTo(); equals(); doubleValue(); longValue(); hashCode(); toString(); toString(int radix); max(); min(); mod(); modPow(BigInteger exp,BigInteger m); nextProbablePrime(); pow();
andNot(); and(); xor(); not(); or(); getLowestSetBit(); bitCount(); bitLength(); setBig(int n); shiftLeft(int n); shiftRight(int n);
add(); divide(); divideAndRemainder(); remainder(); multiply(); subtract(); gcd(); abs(); signum(); negate();

//BigDecimal
movePointLeft(); movePointRight(); precision(); stripTrailingZeros(); toBigInteger(); toPlainString();

import java.util.*;

//sort
class pii implements Comparable
{
    public int a,b;
    public int compareTo(Object i)
    {
        pii c=(pii)i;
        return a==c.a?c.b-b:c.a-a;
    }
}

class Main
{
    public static void main(String[] args)
    {
        pii[] the=new pii[2];
        the[0]=new pii();
        the[1]=new pii();
        the[0].a=1;
        the[0].b=1;
        the[1].a=1;
        the[1].b=2;
        Arrays.sort(the);
        for(int i=0;i<2;++i)
            System.out.printf("%d␣%d\n",the[i].a,the[i].b);
    }
}

//fraction
class frac
{
    public BigInteger a,b;
    public frac(long aa,long bb)
    {
        a=BigInteger.valueOf(aa);
        b=BigInteger.valueOf(bb);
        BigInteger c=a.gcd(b);
        a=a.divide(c);
        b=b.divide(c);
    }
    public frac(BigInteger aa,BigInteger bb)
    {
        BigInteger c=aa.gcd(bb);
        a=aa.divide(c);
        b=bb.divide(c);
    }
    public frac mul(frac i)
    {
        return new frac(a.multiply(i.a),b.multiply(i.b));
    }
    public frac mul(long i)
    {
        return new frac(a.multiply(BigInteger.valueOf(i)),b);
    }
    public frac div(long i)
    {
        return new frac(a,b.multiply(BigInteger.valueOf(i)));
    }
    public frac add(frac i)
    {
        return new frac((a.multiply(i.b)).add(i.a.multiply(b)),b.multiply(i.b));
    }
    public void print()
    {
        System.out.println(a+"/"+b); //printf 会 PE 啊尼玛死……
    }
}
```

9.5 others

```
god damn it windows:
#pragma comment(linker, "/STACK:16777216")
#pragma comment(linker,"/STACK:102400000,102400000")


chmod +x [filename]

while true; do
./gen > input
./sol < input > output.sol
./bf < input > output.bf

diff output.sol output.bf
if [ $? -ne 0 ]; then break; fi
done



```

1. nothing to be afraid of, 'cause you love it. isn't it?
2. calm_down();calm_down();calm_down();
3. 读完题目读完题目读完题目
   (a) 认真读题、认真读题、认真读题、认真读题、
   (b) 不盲目跟版
   (c) 换题/换想法
4. 对数/离线/hash/观察问题本身/点 ↔ 区间互转
   (a) 对数调整精度 or 将乘法转换成加法
   (b) 点化区间，区间化点
5. 数组大小……
6. 写解释器/编译器的时候别忘了负数
   (a) 还有 istringstream in <sstream>
   (b) 指令/函数名也可能是变量名
7. vector 比 array 慢很多
8. modPow 比手写快速幂慢很多
9. 对于 bool 数组，memset 快 8 倍