Assignment 3

Q1:

a. P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2

b. P(Cavity):

P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2

P(~cavity) = 0.016 + 0.064 + 0.144 + 0.576 = 0.8

Cavity	**P**
cavity	0.2
~cavity	0.8

c. P(Toothache|cavity):

P(toothache|cavity) = (0.108 + 0.012) / (0.2) = 0.6

P(~toothache|cavity) = (0.072 + 0.008) / (0.2) = 0.4

Toothache	**P**
toothache	0.6
~toothache	0.4

d. P(Cavity|toothache V catch):

P(toothache) = (0.108 + 0.012 + 0.016 + 0.064) = 0.2

P(catch) = (0.108 + 0.016 + 0.072 + 0.144) = 0.34

P(toothache, catch) = 0.108 + 0.016 = 0.124

P(toothache V catch) = P(toothache) + P(catch) − P(toothache, catch)

= 0.2 + 0.34 − 0.124

= 0.42

P(cavity|toothache V catch) = (0.108 + 0.012 + 0.072) / (0.42) = 0.46

P(~cavity|toothache V catch) = (0.016 + 0.064 + 0.144) / (0.42) = 0.54

Cavity	**P**
cavity	0.46
~cavity	0.54

Q2:

Legend:

+x = positive test
-x = negative test
+y = infected by virus
-y = not infected by virus

Information given in question:

Test A:

P(+x|+y) = 0.95
P(-x|+y) = (1 - 0.95) = 0.05
P(+x|-y) = 0.10
P(-x|-y) = (1 – 0.10) = 0.90

Test B :

P(+x|+y) = 0.90
P(-x|+y) = 0.10
P(+x|-y) = 0.05
P(-x|-y) = 0.95

P(+y) = 0.01
P(-y) = 0.99

Given a positive test, what is the probability that you have the virus?

Test A:

P(+y|+x) = P(+x|+y)P(+y) / P(+x)

P(+x) = P(+x|+y)P(+y) + P(+x|-y)P(-y)
= (0.95)(0.01) + (0.10)(0.99)
= 0.0095 + 0.099
= 0.109

P(+y|+x) = (0.95)(0.01) / (0.109)
P(+y|+x) = 0.088

Given that someone tests positive with test A, the probability that they are actually infected is 0.088.

Test B:

P(+y|+x) = P(+x|+y)P(+y) / P(+x)

P(+x) = P(+x|+y)P(+y) + P(+x|-y)P(-y)
= (0.90)(0.01) + (0.05)(0.99)
= 0.009 + 0.0495
= 0.0585

P(+y|+x) = (0.90)(0.01) / (0.0585)
P(+y|+x) = 0.154

Given that someone tests positive with test B, the probability that they are actually infected is 0.154.

A probability of 0.154 is greater than 0.088. So Test B returning a positive result is more indicative of someone carrying the virus than Test A.

Q3:

Legend:

+x = positive test
-x = negative test
+y = infected by virus
-y = not infected by virus

Information given:

$P(+x|-y) = 0.05$
$P(-x|-y) = (1 - 0.05) = 0.95$

$P(-x|+y) = 0.02$
$P(+x|+y) = 0.98$

$P(+y) = 0.0001$
$P(-y) = 0.9999$

a) What is the chance that you have the disease?

$P(+y|+x) = P(+x|+y)P(+y) / P(+x)$

$P(+x) = P(+x|-y)P(-y) + P(+x|+y)P(+y)$
$= (0.05)(0.9999) + (0.98)(0.0001)$
$= 0.05$

$P(+y|+x) = (0.98)(0.0001)/(0.05)$
$= 0.002$

Given that you have tested positive, the chance of having the disease is 0.002

b) The result of the second independent test is still positive. What is now your chance of having the disease?

$P(+y|+x,+x) = P(+x,+x|+y)P(+y) / P(+x,+x)$

$P(+x,+x) = P(+x,+x|-y)P(-y) + P(+x,+x|+y)P(+y)$
$= (0.05)^2(0.9999) + (0.98)^2(0.0001)$
$= 0025$

$P(+y|+x,+x) = (0.98)^2(0.0001) / 0.0025$
$= 0.038$

Given two positive tests, there is a 0.038 chance of having the disease.

Q4 Classifiers:

Given the training data in the below table.

Home Owner (H)	Marital Status (M)	Job Experience (1-5) (J)	Defaulted (D)
Yes	Single	3	No
No	Married	4	No
No	Single	5	No
Yes	Married	4	No
No	Divorced	2	Yes
No	Married	4	No
Yes	Divorced	2	No
No	Married	3	Yes
No	Married	3	No
Yes	Single	2	Yes

Predict if Bob will default his loan or not

	Home Owner	Marital Status	Job Experience (1-5)
Bob	No	Married	3

*** Rows 8 and 9 are identical, but with different labels, so for the following classifiers I chose to ignore row 9.**
**** Also note that I added single-letter abbreviations to each column name to be used as variables in the following calculations**

Naïve Bayes Classifier

P(D=No|Bob) = P(Bob|D=No) P(D=No)
= P(H=No|D=No) P(M=Married|D=No) P(J=3|D=No) P(D=No)
= (3/6)(3/6)(1/6)(6/9)
= 0.028

Given Bob's attributes, the probability that he will not default his loan is 0.028.

P(D=Yes|Bob) = P(Bob|D=Yes) P(D=Yes)
= P(H=No|D=Yes) P(M=Married|D=Yes) P(J=3|D=Yes) P(D=Yes)
= (2/3)(1/3)(1/3)(3/9)
= 0.024

Given Bob's attributes, the probability that he will default his loan is 0.024

0.028 > 0.024 Therefore, Naïve Bayes will predict that Bob will not default his loan.

Decision Tree Classifier:

	Defaulted	
Homeowner	No	Yes
Yes	3	1
No	3	2

	Defaulted	
Marital Status	No	Yes
Single	2	1
Married	3	1
Divorced	1	1

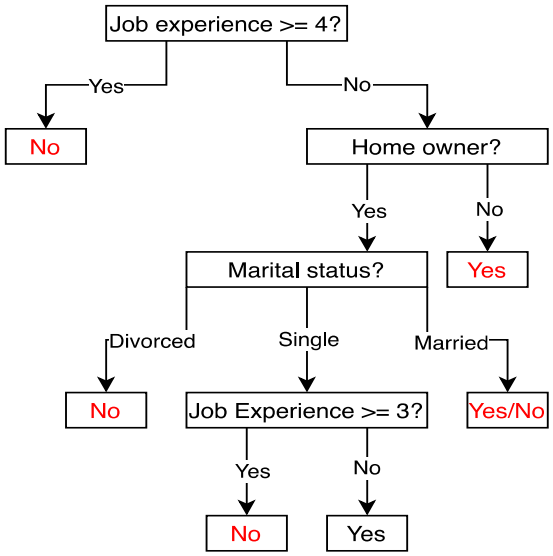
	Defaulted	
Job Experience	No	Yes
2	1	2
3	1	1
4	3	0
5	1	0

From the tables, the only certainty right away is that people with 4 or more years of job experience never defaulted their loans. (Can classify 4 people)

Of the five remaining people, there are two who do not own a home (rows 5 and 8), and both of them defaulted their loans. (Can classify 2 people)

Of the three remaining, #7 is divorced, and they did not default their loan. None are married, so it cannot be predicted whether a married person would default their loan or not.

Of the 2 remaining, #1 with 3 years of job experience defaulted their loan whereas #10 with 2 years did not.



Bob:
Job experience >=4?
No
Home owner?
No
⇨ Yes

The decision tree classifier predicts that Bob will default his loan.

KNN Classifier (k=3)

Distance, D, will be calculated by:

H + M + J

H = 0 if Home owner status matches
H = 1 if Home owner status doesn't match

M = 0 if Marital status matches
M = 1 if Marital status doesn't match

J = |JobExperience_x – JobExperience_y|
(ie. J is the number of years that the job experience differs from Bob)

All distances from Bob:

#1 = 1 + 1 + 0 = 2
#2 = 1
#3 = 3
#4 = 2
#5 = 2
#6 = 1
#7 = 3
#8 = 0
#10 = 3

Closest 3:

1. No, Married, 3, Yes (row 8, Distance = 0)
2. No, Married, 4, No (row 2, Distance = 1)
3. No, Married, 4, No (row 6, Distance = 1)

The first neighbour is obviously the closest to Bob because all their attributes match exactly. Row #2 and #6 differ in only the job experience years by 1 year. There are no other rows that match in at least 2 attributes (Therefore, they have a distance from Bob of at least 2).

Of the three nearest neighbours, 2/3 did not default their loans. Therefore it is predicted that Bob will not default his loan also.

k-Means (k=2)

Distance, D, will be calculated by:
H + M + J

H = 0 if Home owner status matches
H = 1 if Home owner status doesn't match

M = 0 if Marital status matches
M = 1 if Marital status doesn't match

J = |JobExperience_x – JobExperience_y|
(ie. J is the number of years that the job experience differs from the mean)

1st iteration:

Random starting points:
X1 = point 1 (Yes, Single, 3)
X2 = point 5 (No, Divorced, 2)

Distances to cluster centres:
D(point2, X1) = 1 + 1 + 1 = 3
D(point2, X2) = 0 + 1 + 2 = 3
Coin flip to assign… Point2 assigned to cluster 1

D(point3, X1) = 3
D(3, X2) = 4
Point3 assigned to cluster 1

D(4, X1) = 2
D(4, X2) = 4
Point4 assigned to cluster 1

D(6, X1) = 3
D(6, X2) = 3
Point6 assigned to cluster 2

D(7, X1) = 2
D(7, X2) = 1
Point7 assigned to cluster 2

D(8, X1) = 2
D(8, X2) = 2
Point8 assigned to cluster 2

D(10, X1) = 1
D(10, X2) = 2
Point 20 assigned to cluster 1

New clusters:

1. 1,2,3,4,10
2. 5,6,7,8

2nd iteration:

New means (cluster centres):

Means are points in which each attribute is calculated by finding the mode of the points in the cluster. Job experience is the calculated average

Cluster 1:

Home owner: Mode = Yes

Marital status: Mode = Single

Job experience: (3+4+5+4+2)/5 = 3.6

Cluster 2:

Home owner: No

Marital status: Married (random choice because there is a tie with "divorced")

Job experience: 2.8

After calculating distances…

Point1 assigned to cluster 1
Point2 assigned to cluster 2
Point3 assigned to cluster 1
Point4 assigned to cluster 1
Point5 assigned to cluster 2
Point6 assigned to cluster 2
Point7 assigned to cluster 1
Point8 assigned to cluster 2
Point10 assigned to cluster 1

New Clusters:

1. 1,3,4,7,10
2. 2,5,6,8

New means:

Cluster 1:

Home owner = Yes

Marital Status = Single

Job Experience = 3.2

Cluster 2:

Home owner = No

Marital Status = Married

Job experience = 3.3