

Math 312, Extra Credit Problems

These problems are hard and interesting. The solutions should be presents orally before April 14. It might improve your score, but should be used for fun. Only the first solution will be graded.

1a. Show that the field of all numbers of the form $a + b\sqrt{2}$ for $a, b \in \mathbb{Q}$ admits a nonstandard ordereing (that is, not the one coming from the standard ordering of real numbers).

1b. Show that the field of rational numbers admits unique a ordering.

1c. Show that any finite field admits no ordering.

2. Let x, y be positive real numbers such that $\frac{x}{y}$ is irrational. Show that the set

$$\{mx + ny \mid m, n \in \mathbb{N}\}$$

is dense in \mathbb{R} .

3. Let us define θ -set as the union of a circle with one of its chords. Show that any collection of disjoint θ sets in the plane is countable.