

## Math 312, Extra Credit Problems

These problems are hard and interesting. The solutions should be presented orally before April 14. It might improve your score, but should be used for fun. Only the first solution will be graded.

- 1a. Show that the field of all numbers of the form  $a + b\sqrt{2}$  for  $a, b \in \mathbb{Q}$  admits a nonstandard ordering (that is, not the one coming from the standard ordering of real numbers).
- 1b. Show that the field of rational numbers admits a unique ordering.
- 1c. Show that no finite field admits an ordering.
2. Let  $S$  be a collection of disjoint plane sets such that each set in  $S$  is a union of two intersecting circles. Show that  $S$  is countable.