Extra credit problems

Math 427

1. Describe all the motions of the Manhattan plane.

2. Construct a metric space $\mathcal{X}$ and a distance preserving map $f\colon \mathcal{X} \to \mathcal{X}$ which is not a motion of $\mathcal{X}$.

3. Note that the following quantity

$$\tilde{\measuredangle} ABC = \begin{cases} \pi & \text{if} \quad \measuredangle ABC = \pi \\ -\measuredangle ABC & \text{if} \quad \measuredangle ABC < \pi \end{cases}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere $\measuredangle$ to $\tilde{\measuredangle}$.
(a). Show that $\measuredangle$ and $\tilde{\measuredangle}$ are the only possible angle measures on the plane.
(b). Show that without Axiom IIc, this is not longer true.