

Extra credit problems

Math 427

0. Find a mistake or misprint in the book. (The score depends on the type of mistake.)

1. Describe all the motions of the Manhattan plane.

2. (*solved*) Construct a metric space \mathcal{X} and a distance preserving map $f: \mathcal{X} \rightarrow \mathcal{X}$ that is not a motion of \mathcal{X} .

3. Note that the following quantity

$$\tilde{\angle}ABC = \begin{cases} \pi & \text{if } \angle ABC = \pi \\ -\angle ABC & \text{if } \angle ABC < \pi \end{cases}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere \angle to $\tilde{\angle}$.

(a). Show that \angle and $\tilde{\angle}$ are the only possible angle measures on the plane.

(b). Show that without Axiom IIIc, this is not longer true.

4. (*solved*) Show that a composition of three reflections in the sides of a nondegenerate triangle does not have a fixed point.

5. Consider $\triangle ABC$ with $D \in (AC)$ such that $(BD) \perp (AC)$, and points N and M such that $AN = DC$, $CM = AD$, $(AN) \perp (AB)$ and $(CM) \perp (BC)$.

Prove that M and N are equidistant from B .

6. (*solved*) Lines ℓ and m are tangent to two circles of radii r and R on such a way the circles are on one side of ℓ and on different sides of m . Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m . Show that

$$QA \cdot QB = R \cdot r.$$

7. Given a line segment with marked midpoint, make a ruler-only construction a line through a given point P parallel to the line containing the segment.

8. Let ABC be a nondegenerate triangle and $A' \in (BC)$, $B' \in (CA)$, $C' \in (AB)$ be the points such that

$$2 \cdot \angle AA'B \equiv 2 \cdot \angle BB'C \equiv 2 \cdot \angle CC'A \equiv \frac{2}{3} \cdot \pi.$$

Show that the triangle formed by the lines (BB') , (CC') , and (AA') , is congruent to $\triangle ABC$.

9. Two points A and B lie on one side of a line ℓ . Two points M and N are chosen on ℓ such that $AM + BM$ is minimal and $AN = BN$. Show that points A , B , M and N lie on one circle.
10. Let Γ be a circle with the center O and A and C be two different points on Γ . For any third point P of the circle let X and Y be the midpoints of the segments AP and CP . Finally, let H be the orthocenter of the triangle OXY . Prove that the position of the point H does not depend on the choice of P .
11. Suppose D and E lie on the same side from (AC) , $(AE) \parallel (CD)$, and $AB = BC$. Let K be the intersection of the bisectors of the angles EAB and BCD . Prove that $(BK) \parallel (AE)$.
12. Give a ruler-and-compass construction of an inscribed quadrilateral with given sides.
13. Show that a neutral plane is Euclidean if and only if it has a rectangle.