Extra credit problems

Math 427

0. Find a mistake or misprint in the book.

~~1.~~ Describe all the motions of the Manhattan plane.

~~2.~~ Construct a metric space $\mathcal{X}$ and a distance preserving map $f\colon \mathcal{X} \to \mathcal{X}$ which is not a motion of $\mathcal{X}$.

~~3.~~ Note that the following quantity

$$\tilde{\measuredangle} ABC = \left[\begin{array}{lll} \pi & \text{if} & \measuredangle ABC = \pi \\ -\measuredangle ABC & \text{if} & \measuredangle ABC < \pi \end{array}\right.$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere $\measuredangle$ to $\tilde{\measuredangle}$.
(a). Show that $\measuredangle$ and $\tilde{\measuredangle}$ are the only possible angle measures on the plane.
(b). Show that without Axiom IIc, this is not longer true.

~~4.~~ Consider triangle $\triangle ABC$ with $D \in (AC)$ such that $(BD) \perp (AC)$, and points N and M such that $AN = DC$, $CM = AD$, $(AN) \perp (AB)$ and $(CM) \perp (BC)$. Prove that M and N are equidistant form B.

~~5.~~ Lines ℓ and m are tangent to two circles of radii r and R on such a way the circles are on one side of ℓ and on different sides of m. Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m. Show that

$$QA{\cdot}QB = R{\cdot}r.$$

~~6.~~ Given two parallel lines ℓ and m and a point P, use only ruler to construct the line through P parallel to ℓ and m. (You can play with the java applet "Third parallel line" on http://anton-petrunin.github.io/birkhoff/car/.)

~~7.~~ Two points A and B lie on one side of line ℓ. Two points M and N are chosen on ℓ such that $AM + BM$ is minimal and $AN = BN$. Show that points A, B, M and N lie on one circle.

~~8.~~ Give a ruler-and-compass construction of a circle or a line which perpendicular to each of three given circles. (You may assume any two of three given circles do not intersect.)