

Extra credit problems

Math 427

0. Find a mistake or misprint in the book. (The score depends on the type of mistake.)

1. Describe all the motions of the Manhattan plane.

2. Construct a metric space \mathcal{X} and a distance preserving map $f: \mathcal{X} \rightarrow \mathcal{X}$ that is not a motion of \mathcal{X} .

3. Note that the following quantity

$$\tilde{\angle}ABC = \begin{cases} \pi & \text{if } \angle ABC = \pi, \\ -\angle ABC & \text{if } \angle ABC < \pi. \end{cases}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere \angle to $\tilde{\angle}$.

(a) Show that \angle and $\tilde{\angle}$ are the only possible angle measures on the plane.

(b) Show that without Axiom IIIc, this is not longer true.

4. Show that a composition of three reflections in the sides of a nondegenerate triangle does not have a fixed point.

5. Lines ℓ and m are tangent to two circles of radii r and R on such a way the circles are on one side of ℓ and on different sides of m . Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m . Show that

$$QA \cdot QB = R \cdot r.$$

6. Given a line segment with marked midpoint, make a ruler-only construction a line through a given point P parallel to the line containing the segment.

7. Let ABC be a nondegenerate triangle and $A' \in (BC)$, $B' \in (CA)$, $C' \in (AB)$ be the points such that

$$2 \cdot \angle AA'B \equiv 2 \cdot \angle BB'C \equiv 2 \cdot \angle CC'A \equiv \frac{2}{3} \cdot \pi.$$

Show that the triangle formed by the lines (BB') , (CC') , and (AA') , is congruent to $\triangle ABC$.

8. Give a ruler-and-compass construction of an inscribed quadrilateral with given sides.

9. A circle Γ with marked center O are given. Let ℓ be a line that passes thru O and P be a point on Γ . Make a ruler-only construction of a line thru P that is perpendicular to ℓ .