Extra credit problems

Math 427

0. Find a mistake or misprint in the book.

1. Describe all the motions of the Manhattan plane.

2. Construct a metric space $\mathcal{X}$ and a distance preserving map $f\colon \mathcal{X} \to \mathcal{X}$ which is not a motion of $\mathcal{X}$.

3. Note that the following quantity

$$\tilde{\measuredangle}ABC = \begin{bmatrix} \pi & \text{if} \quad \measuredangle ABC = \pi \\ -\measuredangle ABC & \text{if} \quad \measuredangle ABC < \pi \end{bmatrix}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere $\measuredangle$ to $\tilde{\measuredangle}$.
(a). Show that $\measuredangle$ and $\tilde{\measuredangle}$ are the only possible angle measures on the plane.
(b). Show that without Axiom IIc, this is not longer true.

4. Consider triangle $\triangle ABC$ with $D \in (AC)$ such that $(BD) \perp (AC)$, and points N and M such that $AN = DC$, $CM = AD$, $(AN) \perp (AB)$ and $(CM) \perp (BC)$.
Prove that M and N are equidistant form B.

5. Lines ℓ and m are tangent to two circles of radii r and R on such a way the circles are on one side of ℓ and on different sides of m. Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m. Show that

$$QA \cdot QB = R \cdot r.$$

6. Given two parallel lines ℓ and m and a point P, use only ruler to construct the line through P parallel to ℓ and m. (You can play with the java applet "Third parallel line" on http://anton-petrunin.github.io/birkhoff/car/.)