Extra credit problems

Math 427

0. Find a mistake or misprint in the book. (The score depends on the type of mistake.)

1. Describe all the motions of the Manhattan plane.

2. Construct a metric space $\mathcal{X}$ and a distance preserving map $f\colon \mathcal{X} \to \mathcal{X}$ that is not a motion of $\mathcal{X}$.

3. Note that the following quantity

$$\tilde{\measuredangle}ABC = \begin{bmatrix} \pi & \text{if} \quad \measuredangle ABC = \pi \\ -\measuredangle ABC & \text{if} \quad \measuredangle ABC < \pi \end{bmatrix}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere $\measuredangle$ to $\tilde{\measuredangle}$.
(a). Show that $\measuredangle$ and $\tilde{\measuredangle}$ are the only possible angle measures on the plane.
(b). Show that without Axiom IIIc, this is not longer true.

4. Show that a composition of three reflections in the sides of a nondegenerate triangle does not have a fixed point.

5. Lines ℓ and m are tangent to two circles of radiuses r and R on such a way the circles are on one side of ℓ and on different sides of m. Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m. Show that

$$QA \cdot QB = R \cdot r.$$