Extra credit problems

Math 427

0. Find a mistake or misprint in the book. (The score depends on the type of mistake.)

1. Describe all the motions of the Manhattan plane.

2. Construct a metric space $\mathcal{X}$ and a distance preserving map $f\colon \mathcal{X} \to \mathcal{X}$ that is not a motion of $\mathcal{X}$.

3. Note that the following quantity

$$\tilde{\measuredangle}ABC = \begin{bmatrix} \pi & \text{if} \quad \measuredangle ABC = \pi \\ -\measuredangle ABC & \text{if} \quad \measuredangle ABC < \pi \end{bmatrix}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere $\measuredangle$ to $\tilde{\measuredangle}$.
(a). Show that $\measuredangle$ and $\tilde{\measuredangle}$ are the only possible angle measures on the plane.
(b). Show that without Axiom IIIc, this is not longer true.

4. Let M be the midpoint of the side $[AB]$ of $\triangle ABC$ and M' be the midpoint of the side $[A'B']$ of $\triangle A'B'C'$. Assume $C'A' = CA$, $C'B' = CB$, and $C'M' = = CM$. Prove that $\triangle A'B'C' \cong \triangle ABC$.

5. Show that a composition of three reflections in the sides of a nondegenerate triangle does not have a fixed point.