

Extra credit problems

Math 427

0. Find a mistake or misprint in the book.
1. Describe all the motions of the Manhattan plane.
2. Construct a metric space \mathcal{X} and a distance preserving map $f: \mathcal{X} \rightarrow \mathcal{X}$ which is not a motion of \mathcal{X} .
3. Note that the following quantity

$$\tilde{\angle}ABC = \begin{cases} \pi & \text{if } \angle ABC = \pi \\ -\angle ABC & \text{if } \angle ABC < \pi \end{cases}$$

can serve as the angle measure; that is, the axioms hold if one changes everywhere \angle to $\tilde{\angle}$.

- (a). Show that \angle and $\tilde{\angle}$ are the only possible angle measures on the plane.
 - (b). Show that without Axiom IIc, this is not longer true.
4. Consider triangle $\triangle ABC$ with $D \in (AC)$ such that $(BD) \perp (AC)$, and points N and M such that $AN = DC$, $CM = AD$, $(AN) \perp (AB)$ and $(CM) \perp (BC)$.
Prove that M and N are equidistant from B .
 5. Lines ℓ and m are tangent to two circles of radii r and R on such a way the circles are on one side of ℓ and on different sides of m . Let A and B be tangential points of ℓ and Q be the point of intersection ℓ and m . Show that

$$QA \cdot QB = R \cdot r.$$

6. Given two parallel lines ℓ and m and a point P , use only ruler to construct the line through P parallel to ℓ and m . (You can play with the java applet “Third parallel line” on <http://anton-petrinin.github.io/birkhoff/car/>.)
7. Two points A and B lie on one side of line ℓ . Two points M and N are chosen on ℓ such that $AM + BM$ is minimal and $AN = BN$. Show that points A , B , M and N lie on one circle.
8. Give a ruler-and-compass construction of a circle or a line which perpendicular to each of three given circles. (You may assume any two of three given circles do not intersect.)