

Math 429, Extra Credit Problems

These problems are hard and interesting. The solutions should be presented orally before April 16. It might improve your score, but should be used for fun. Only the first solution will be graded.

1. Find three disjoint open sets on the real line that have the same nonempty boundary.
2. Construct a continuous function $f: [0, 1] \rightarrow [0, 1]$ such that f takes every value in $[0, 1]$ an infinite number of times.
3. Describe all the homeomorphisms from the Sierpinski triangle to itself.
4. Prove that $\mathbb{R}^3 \setminus S^1$ is homeomorphic to $\mathbb{R}^3 \setminus (\ell \cup \{p\})$, where ℓ is a straight line and $p \notin \ell$ is a point in \mathbb{R}^3 .
5. Show that any nonempty open star-shaped set in \mathbb{R}^2 is homeomorphic to the open disc.

t.b.c.