Extra credit problems

Math 485

0. Find a mistake or misprint in "Extra pearls". (The score depends on the type of mistake.)

1. Assume $d_1, \dots, d_p$ is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if $d_p \geq 0$, the sum $d_1 + \dots + d_p$ is even and

$$d_1 \leq d_2 + \dots + d_p.$$

(A sequence of integers $d_1, \dots, d_p$ is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

2. Assume that a sequence $d_1, \dots, d_p$ is graphic, $d_i \geq 1$ for each i and

$$d_1 + \dots + d_p \geq 2 \cdot (p - 1).$$

Show that there is a *connected* graph G with the degree sequence $d_1, \dots, d_p$.

3. Show that in any connected graph G there is a vertex v such that $G - v$ is connected.

4. Let G be a connected graph. Show that any two paths of maximum length in G have a common vertex.

5. Assume two trees R and S have the vertices $r_1, \dots, r_n$ and $s_1, \dots, s_n$ correspondingly. Assume that $R - r_i$ is isomorphic to $S - s_i$ for each i. Show that R is isomorphic to S.

6. Let G be a critical graph and $\chi(G) = k + 1$. Show that after removing any $k - 1$ edges from G the obtained graph remains connected.

7. Assume both sequences $d_1, \dots, d_p$ and $d_1 - 1, \dots, d_p - 1$ are graphic. Show that there is a graph with a 1-factor and degree sequence $d_1, \dots, d_p$.

8. Show that any edge of cubic graph lies in an even number of Hamiltonian cycles.

9. Assume that the edges of a complete graph are colored in two colors. Show that there is a Hamiltonian cycle which either monochromatic or consists of two monochromatic paths.

10. Assume that G is a connected cubic graph such that for any two vertices v, w of G there is an isomorphism $G \to G$ which sends v to w. Prove that G remains connected after removing any 2 edges.

11. Let G be a connected graph. Given any two vertices v, w in G, denote by $d(v, w)$ the length of a shortest path containing v to w.

Assume that G has no triangles and

$$d(x, y) + d(v, w) = \max\{ d(x, v) + d(y, w), d(x, w) + d(y, v) \}$$

for any 4 vertices x, y, v, w in G. Show that G is a tree.

12. Understand the proof of Vizing's theorem (see the references for Theorem 2.2.2 in "Pearls" or find a proof elsewhere).

13. Suppose $p = r(m, n)$ is the Ramsey number for m and n.

Assume that G results from K_p by deleting a single edge. Show that G has a red/blue edge coloring with no red K_m and no blue K_n.