

Extra credit problems

Math 485

0. Find a mistake or misprint in “Extra pearls”. (The score depends on the type of mistake.)

1. Assume d_1, \dots, d_p is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if $d_p \geq 0$, the sum $d_1 + \dots + d_p$ is even and

$$d_1 \leq d_2 + \dots + d_p.$$

(A sequence of integers d_1, \dots, d_p is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

2. Assume that a sequence d_1, \dots, d_p is graphic, $d_i \geq 1$ for each i and

$$d_1 + \dots + d_p \geq 2 \cdot (p - 1).$$

Show that there is a *connected* graph G with the degree sequence d_1, \dots, d_p .

3. Show that in any connected graph G there is a vertex v such that $G - v$ is connected.

4. Let G be a connected graph. Show that any two paths of maximum length in G have a common vertex.

5. Assume two trees R and S have the vertices r_1, \dots, r_n and s_1, \dots, s_n correspondingly. Assume that $R - r_i$ is isomorphic to $S - s_i$ for each i . Show that R is isomorphic to S .

6. Let G be a critical graph and $\chi(G) = k + 1$. Show that after removing any $k - 1$ edges from G the obtained graph remains connected.

7. Assume both sequences d_1, \dots, d_p and $d_1 - 1, \dots, d_p - 1$ are graphic. Show that there is a graph with a 1-factor and degree sequence d_1, \dots, d_p .