Extra credit problems

Math 485

~~1.~~ *(solved)* Assume that the sequence $d_1, \ldots, d_p$ is graphic, $d_i \geq 1$ for each i and

$$d_1 + \ldots + d_p \geq 2 \cdot (p - 1).$$

Show that there is a connected graph G with the degree sequence $d_1, \ldots, d_p$.

2. Assume $d_1, \ldots, d_p$ is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if $d_p \geq 0$, the sum $d_1 + \ldots + d_p$ is even and

$$d_1 \leq d_2 + \ldots + d_p.$$

(A sequence of integers $d_1, \ldots, d_p$ is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

~~3.~~ *(solved)* Show that in any connected graph G there is a vertex v such that $G - v$ is connected.

4. Let G be a critical graph and $\chi(G) = k + 1$. Show after removing any $k - 1$ edges from G the obtained graph remains connected.

5. Assume both sequences $d_1, \ldots, d_p$ and $d_1 - 1, \ldots, d_p - 1$ are graphic. Show that there is a graph with a 1-factor and with the degree sequence $d_1, \ldots, d_p$.

~~6.~~ *(solved)* Show that any 4-regular graph has a 2-factor.

7. Show that any edge of cubic graph lies in an even number of Hamiltonian cycles.

~~8.~~ *(solved)* Let G be a connected graph. Show that any two paths of maximum length in G have a common vertex.

~~9.~~ *(solved)* Assume that the edges of a complete graph are colored in two colors. Show that there is a Hamiltonian cycle which either monochromatic or consists of two monochromatic paths.

10. Assume two trees R and S have the vertices $r_1, \ldots, r_n$ and $s_1, \ldots, s_n$ correspondingly. Assume that $R - r_i$ is isomorphic to $S - s_i$ for each i. Show that R is isomorphic to S.

11. Assume that G is 3-regular connected graph such that for any two vertices v, w of G there is an isomorphism $G \to G$ which sends v to w. Prove that G remains connected after removing any 2 edges.

12. Let G be a connected graph. Given any two vertices v, w in G, denote by $d(v, w)$ the length of a shortest path containing v to w.

Assume that G has no triangles and

$$d(x, y) + d(v, w) = \max\{\, d(x, v) + d(y, w), d(x, w) + d(y, v) \,\}$$

for any 4 vertices x, y, v, w in G. Show that G is a tree.

13. Suppose $p = r(m, n)$ is the Ramsey number for m and n.

Assume that G results from K_p by deleting a single edge. Show that G has a red/blue edge coloring with no red K_m and no blue K_n.

14. Solve 2.13 from "Extra pearls".