Extra credit problems

Math 485

1. Assume that the sequence $d_1, \ldots, d_p$ is graphic, $d_i \geq 1$ for each i and

$$d_1 + \ldots + d_p \geq 2 \cdot (p - 1).$$

Show that there is a connected graph G with the degree sequence $d_1, \ldots, d_p$.

2. Assume $d_1, \ldots, d_p$ is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if $d_p \geq 0$ and

$$d_1 \leq d_2 + \ldots + d_p.$$

(A sequence of integers $d_1, \ldots, d_p$ is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

3. Show that in any connected graph G there is a vertex v such that $G - v$ is connected.

4. Let G be a critical graph and $\chi(G) = k + 1$. Show after removing any $k - 1$ edges from G the obtained graph remains connected.