

## Extra credit problems

Math 485

0. Find a mistake or misprint in “Extra pearls”. (The score depends on the type of mistake.)

1. Assume  $d_1, \dots, d_p$  is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if  $d_p \geq 0$ , the sum  $d_1 + \dots + d_p$  is even and

$$d_1 \leq d_2 + \dots + d_p.$$

(A sequence of integers  $d_1, \dots, d_p$  is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

2. Assume that a sequence  $d_1, \dots, d_p$  is graphic,  $d_i \geq 1$  for each  $i$  and

$$d_1 + \dots + d_p \geq 2 \cdot (p - 1).$$

Show that there is a *connected* graph  $G$  with the degree sequence  $d_1, \dots, d_p$ .

3. Show that in any connected graph  $G$  there is a vertex  $v$  such that  $G - v$  is connected.

4. Let  $G$  be a connected graph. Show that any two paths of maximum length in  $G$  have a common vertex.

5. Assume two trees  $R$  and  $S$  have the vertices  $r_1, \dots, r_n$  and  $s_1, \dots, s_n$  correspondingly. Assume that  $R - r_i$  is isomorphic to  $S - s_i$  for each  $i$ . Show that  $R$  is isomorphic to  $S$ .