Extra credit problems

Math 485

0. Find a mistake or misprint in "Extra pearls". (The score depends on the types of mistakes).

1. Assume $d_1, \ldots, d_p$ is a sequence of integers in a nonincreasing order. Show that it is multigraphic if and only if $d_p \geq 0$, the sum $d_1 + \ldots + d_p$ is even and

$$d_1 \leq d_2 + \ldots + d_p.$$

(A sequence of integers $d_1, \ldots, d_p$ is called *multigraphic* if it appears as a sequence of degrees of a multigraph.)

2. Assume that the sequence $d_1, \ldots, d_p$ is graphic, $d_i \geq 1$ for each i and

$$d_1 + \ldots + d_p \geq 2 \cdot (p-1).$$

Show that there is a connected graph G with the degree sequence $d_1, \ldots, d_p$.

3. Show that in any connected graph G there is a vertex v such that $G - v$ is connected.

4. Let G be a connected graph. Show that any two paths of maximum length in G have a common vertex.

5. Assume two trees R and S have the vertices $r_1, \ldots, r_n$ and $s_1, \ldots, s_n$ correspondingly. Assume that $R - r_i$ is isomorphic to $S - s_i$ for each i. Show that R is isomorphic to S.

6. Let G be a critical graph and $\chi(G) = k + 1$. Show after removing any $k - 1$ edges from G the obtained graph remains connected.