Preliminary list of questions for the midterm

1. Inverse function theorem.

2. Sard's lemma.

3. Degree modulo 2 and integer degree; their homotopy invariance.

4. Construction of partition of unity.

5. Brouwer fixed-point theorem.

6. Thom's traversality theorem, intersection number.

7. Whitney embedding theorem (for closed manifolds).

8. Vector fields as sections of tangent bundle: integral curves, flows, straightening lemma.

9. Vector fields as a differential operator: Lie bracket, Jacobi identity.

10. Straightening lemma for commuting vector fields.

11. Lie derivative of tensor fields: defintions and proof of identities

$$\begin{aligned}
\mathcal{L}_X(\alpha \otimes \beta) &= (\mathcal{L}_X \alpha) \otimes \beta + \alpha \otimes (\mathcal{L}_X \beta), \\
\mathcal{L}_X \text{Contraction} &= \text{Contraction}\, \mathcal{L}_X, \\
\mathcal{L}_{X+Y} &= \mathcal{L}_X + \mathcal{L}_Y, \\
\mathcal{L}_X \mathcal{L}_Y - \mathcal{L}_Y \mathcal{L}_X &= \mathcal{L}_{[X,Y]}.
\end{aligned}$$

12. Grassmann algebra and its existence (tensor interpretation).

13. Differential forms: definition, Lie, external, and internal derivative, their product rules, pullback and its relation to wedge product and external differential.

14. Cartan's magic formula.

15. Stokes' theorem, closed and exact forms.

16. De Rham cohomology algebra: definitions, an example of calculations via symmetry.

17. Homotopy invariance of De Rham cohomology, Poincaré's lemma.

18. Mayer–Vietoris sequence: formulation + an application.

19. Top cohomology. Cohomological definition of degree.

20. Moser's theorem via Moser's trick.

21. Morse theory: degenerate and nondegenerate critical points, existence of Morse function (for closed manifolds), product structure at noncritical levels.

22. Morse lemma.

23. Handle decomposition: rearrangement of handles, handle body decomposition of 3-manifolds (Heegaard splitting).