-. Give an example of a noncomplete vector field.

-. Show that the identity

$$L_X[Y,Z] = [L_X Y, Z] + [Y, L_X Z]$$

holds for any vector fields X, Y, and Z.

-. Let X and Y be vector fields and p be a critical point of a smooth function f; that is, $d_p f = 0$.

Show that $(XYf)(p)$ depends only on the vectors $X(p), Y(p) \in \mathrm{T}_p$. Furthermore, the map $X(p), Y(p) \to (XYf)(p)$ is a symmetric bilinear form on T_p.

-. Show that

$$dL_X\omega = L_X d\omega$$

for any vector field X and any form ω.

-. Let ω be a symplectic form; that is, ω is a 2-form such that $d\omega = 0$ and for any 1-form α there is a vector field X such that $\alpha = \iota_X\omega$.

Suppose that $\iota_X\omega = dh$ for a smooth function h and a vector field X. Show that $L_X\omega = 0$. Conclude that ω is invariant with respect to the flow μ of X; that is, $\mu^t_*\omega = \omega$ for any t.